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Capital Gains Taxation under Different Tax Regimes

Abstract

This paper investigates the influence of different systems of current income and capital gains taxation on investor’s decision to either carry out an investment in corporate shares or to invest funds alternatively on the capital market. Three basic tax systems are analyzed, a classical corporate tax system with double taxation of profits on corporate and personal level, a shareholder relief system, that reduces double taxation and finally a tax system with full imputation of corporate tax that avoids double taxation completely. It can be shown that general analytical solutions for the investment problem for different categories of tax regimes, even under certainty, cannot be derived. Applying a growth model, we find under rather restrictive assumptions that the shareholder relief system invokes more severe distortions than the full imputation system. Trying to prove this in a more realistic setting with uncertainty we employ Monte Carlo Simulation for random rates of return and random income tax rates. In many cases, the degree of tax-induced uncertainty is significantly lower under a shareholder relief system than under full imputation. Furthermore, it can be shown that under uncertainty full imputation causes more severe distortions than shareholder relief whenever personal income tax rates are low. In light of international tax competition this is an important result as a reduction of tax rates is taking place or is likely to take place in several countries. Furthermore, the simulation clarifies the trade-off of the opposing effects, i.e. tax and interest rate effects, and the overwhelming impact of capital gains taxation. Apart from tax parameters, we identify the dividend rate and the point in time of selling the shares as important value drivers.

Keywords: capital gains taxation, dividend policy, investment decisions, tax neutrality, timing decisions

JEL classification: H25, H21
1 Introduction

This study investigates the influence of capital gains tax and current income taxes on investor’s behavior under different tax systems. Since tax reforms as well as the discussion of optimal tax systems and tax neutrality are an ongoing process\(^1\) it is important to find out whether under realistic assumptions there are classes of tax systems that induce less distortions than others. We focus on the influence of taxation on investment decisions and leave the fundamental dispute on consumption based systems versus income based systems aside. Since income and profit, respectively, are common tax bases in most real-world tax systems we analyze distortions caused by capital gains taxation for different types of income-based tax systems.

Neutral tax systems that do not affect investment decisions are often considered desirable from a tax policy perspective and may serve as a yardstick for the analysis of real-world tax systems. In contrast to the well-known neutral tax systems\(^2\), real-world tax systems usually are not neutral; rather, they distort investment decisions. Therefore, we investigate the impact of different tax regimes on the performance of the investment and in turn on the decision of either investing into a corporation or alternatively on the capital market, earning the market rate. The most common three basic tax systems are analyzed, a classical corporate tax system with double taxation of profits on corporate and personal level, a shareholder relief system, that reduces double taxation and finally a tax system with full imputation of corporate tax that avoids double taxation completely.\(^3\)

The major German tax reform in 2001 abolished the full imputation system of corporate income tax and introduced a shareholder relief system. Now, profits are due to corporate tax that cannot be credited against shareholder’s personal income tax on dividends and thereby becomes unrefundable. In contrast to a classical corporate tax system, under shareholder relief 50% of the dividends are exempt from shareholder’s income tax. Austria, Belgium, Denmark, UK, Luxembourg, the Netherlands and Sweden have implemented similar systems\(^4\). Furthermore, Finland and France recently discussed reforming its corporate tax system accordingly.

We take the German tax reform and these two systems as examples of different real-world corporate tax systems. Additionally, we analyze a classical tax system with neither full imputation nor shareholder relief, i.e. double taxation of dividend payments\(^5\).


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\(^1\) See Kaplow (1986, p. 60); Auerbach, Hines (1988); Hammond (1990, p. 26).
\(^2\) The cash flow tax and the taxation of true economic profits are prominent examples for such neutral tax systems. See Brown (1948); Samuelson (1964); Johansson (1969).
\(^3\) We refrain analyzing a tax system where profits are taxed solely on corporate level irrespective of dividend policy. Such a system still exists in Greece.
\(^4\) Some of these systems, like the UK’s, do create shareholder relief by partial imputation instead of partially tax exempt dividends as in the German system.
gains tax system that eliminates the incentive to defer the realization of gains which does not require unobservable knowledge. Bradford (1996) extends this work with respect to financial instruments.

Scholz (1988) analyzes how changes in relative tax treatment of dividends and capital gains influence individual behavior and shows that the dividend clientele effect is significantly reasonable. Klein (1999, 2001) and Viard (2000) demonstrate that the disincentive to sell an investment project increases with shareholders’ capital gains tax exposure. Ayers, Lefanowicz and Robinson (2003) test empirically whether capital gains taxes affect premiums paid in taxable corporate acquisitions. Their evidence suggests that shareholder-level taxes have a significant price effect on taxable acquisitions which varies with the tax status of the target’s shareholder. Recently, Keuschnigg and Nielsen (2004) analyze the influence of capital gains tax on start-up finance with double moral hazard empirically. Corresponding with the findings of Poterba (1989a, 1989b), they point out that capital gains tax particularly discourages entrepreneurial efforts. Sinai and Gyourko (2004) investigate the effect of a capital gains tax reduction on the share prices of real estate firms.

Schreiber (2000) discusses the economic and legal aspects of the German 2001 corporate income tax reform with respect to cross-border investment. Sorensen (2002) examines the effects of this major German Tax Reform on the German economy, especially on welfare and distribution. Applying a general equilibrium analysis and considering international effects, his simulation indicates that the new system will raise domestic economic activity and welfare mainly induced by lower tax rates.

In what follows we assume simple tax systems with personal and corporate taxation. Furthermore, reflecting the continuing controversial discussion on capital gains taxation we integrate capital gains taxation. On the basis of a simple asset pricing model under certainty we analyze the influence of current and capital gains taxation on the decision to invest in a corporation. Implementing the various income-based tax regimes above mentioned this decision depends on future after-tax cash flow from the investment. This cash flow consists of post-tax dividend payments and post-tax capital gains, realized when the investor sells his stock.

Furthermore, we explore how the investor’s decision calculus is influenced by effects of taxes on profit distribution and on the decision when to abandon the investment project. In general, optimal dividend and timing strategies for the underlying tax systems cannot be determined analytically, even under the restrictive assumptions of certainty and infinite life of the corporation.

In recent years, public economists were especially interested in tax effects under uncertainty. Since analytical approaches fail Monte Carlo Simulation provides rather faithful descriptions on a numerical basis. In order to extend the scenario to uncertainty, to cases with a finite life of the investment project, and thereby to gain information on profitable investment behavior under more realistic assumptions, we apply a Monte Carlo Simulation. Simulating various expected cash flow streams allows to forecast the extent of distortion caused by the investigated tax regimes. Conclusions about the influence of ta-

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6 E.g., Harbaou, Lasserre (1996); Niemann (1999); Jou (2000); Pennings (2000); Agliardi (2001); Sureth (2002).
xation on dividend and timing strategy can be drawn. These results will be helpful for the analysis and design of tax systems that aim to comply with the neutrality postulate. The remainder of this paper begins with the model design in chapter 2. We present the pre-tax case in 2.1, introduce taxes in section 2.2, and present some analytical solutions under rather restrictive assumptions. Applying a Monte Carlo Simulation in chapter 3 allows to derive results under uncertainty. Chapter 4 closes with a summary.

2 The Model

If we want to analyze tax-induced distortions we need to refer to a yardstick. Under specific assumptions the concept of tax neutrality provides neutral tax systems. They serve as reference systems and furthermore permit to identify the influence of taxation on investors’ decisions. The concept of neutrality of taxation can be interpreted as a partial analytic efficiency measure of tax systems\(^7\) on the investor level.

We apply the well-known growth model introduced by Gordon, Shapiro (1956) and Gordon (1962) that has been enriched by tax aspects\(^8\). Taking account of the ongoing discussion on capital gains taxation and we employ this model as a starting point for investigating the influence of dividend and capital gains taxation on asset pricing and in turn on investor’s behavior. In the following, we abstract from cross-border effects for simplicity.

Introducing such a simplified framework for the analysis of tax influence is justifiable since more elaborated approaches do not permit the derivation of general solutions due to tax specific complications\(^9\).

We assume the (non-institutional) investor A invests an amount \(I_0\) at time \(t = 0\) to either found a corporation or buy shares. A sells his stock at time \(t = z \geq 0\) to the non-institutional investor B at the price \(V_z\). While holding the investment A and B both receive dividend payments. \(\gamma\) denotes a constant fraction of retained earnings from the profit \(P_t\) in each period \(t\) with \(0 \leq \gamma \leq 1\). \(i\) describes the market rate of return. Further, we assume perfect capital markets under certainty, where \(i\) is assumed constant. The debit and credit rates are identical. Neglecting tax effects, this leads to dividend payments to both A and B:

\[
D_t = (1 - \gamma) P_t. \tag{1}
\]

Here, dividends and profits grow at rate \(\gamma i\):

\[
D_{t+1} = (1 - \gamma) (1 + \gamma i) P_t = (1 + \gamma i) D_t. \tag{2}
\]

We start modelling under the rather restrictive assumption of \(I_0\) earning a rate of return before taxes which equals \(i\). This implies that \(i\) describes the internal as well as the external pre-tax rate of return on the investment and

\[
P_t = i I_0. \tag{3}
\]

\(^7\) See e.g. Atkinson, Stiglitz (1980, pp. 350-360).

\(^8\) See e.g. Pye (1972).

\(^9\) E.g., this has been shown by Niemann, Sureth (2004, 2005) applying real option theory.
In chapter 3, we introduce uncertainty into calculus by means of a Monte Carlo Simulation and assume e.g. stochastic internal returns. Thereby, we will relax the assumption of marginal investment and implicitly allow differing rates of return: the expected value of $i^{in}$, the internal rate, may differ from deterministic $i^{ex}$, the external rate of return.

### 2.1 Tax-free Setting

The net present value of A’s investment $V_0$ at time $t = 0$ is a function of dividend payments during the interval $0 < t \leq z$ and the price $V_z$ investor B is willing to pay at time $t = z$. $V_z$ consists of the present value of future cash flows.

Assuming $\gamma < 1$ and $T = \infty$ we find:

$$V_z = \sum_{t=z+1}^{\infty} D_t (1 + i)^{-(t-z)}$$

$$= I_0 (1 + \gamma i)^z.\quad (4)$$

Considering internal growth and interest effects, the present value of dividends at time $t = z$ equals the value of $I_0$. From equation (4) follows:

$$V_0 = \sum_{t=1}^{z} D_t (1 + i)^{-t} + V_z (1 + i)^{-z}$$

$$= I_0.\quad (5)$$

The relation $V_0 = I_0$ holds even for $T < \infty$.

Assuming identical internal and external rates of return the investor is indifferent towards investing into the corporation or on the capital market. Concentrating on marginal investment, i.e. on an investment project earning a pre-tax return equal to $i$, a tax system is neutral whenever the equilibrium condition (5) remains unaffected. In this case, an investor won’t adjust his investment decision due to tax reasons. Consequently, the above described tax-free model can be interpreted as a yardstick for neutral taxation under certainty.

In contrast, relaxing the assumption of marginal investment implies losing the possibility to derive neutral tax systems and to analyze real-world tax distortions in detail. Then, we still will be able to determine the direction of the tax-induced effects. Conclusions on whether the influence of the underlying tax system on investment behavior is desirable from an efficiency point of view in this framework are no longer possible.

### 2.2 Integrating Taxation

Introducing taxation, profits are assumed to be subject to the corporate tax rate $\tau_c$, which may differ from the income tax rate $\tau$ levied on personal income. Dividends are subject to personal income tax $\tau_A$ and $\tau_B$, A and B’s marginal income tax rates. For simplicity we assume$^{10}$

$$\tau_A = \tau_B = \tau.\quad (6)$$

$^{10}$ This implies that we exclude clientele effects from our investigation. In the context of capital gains taxation such effects have been analyzed e.g. by Scholz (1988).
Depending on the tax system, corporate taxes may or not may not be imputed.

Liquidating the corporation at time $t = T$ the investor B receives the originally invested funds $I_0$ and dividends from retained earnings less income tax. $T$ may be finite or infinite. Capital gains may be realized by sale or liquidation and are taxed at a specific tax rate $\tau_c = \beta \cdot \tau_g$, where $\beta$ is a coefficient of the underlying tax system that determines the fraction of the capital gains or dividends, respectively, that is due to income taxation with $\beta \leq 1$. Further, $\tau_g$ with $0 \leq \tau_g < 1$ denotes the capital gains tax rate that may or may not be equal to $\tau$. E.g., $\beta = 0.5$ denotes a tax system with a 50% shareholder relief for capital gains.

Assuming an infinite time horizon excludes cases from the analysis where investor B sells his share of the corporation. He is willing to pay a price $V_z^*$ anticipating all future tax effects\footnote{Concerning the tax capitalization view of dividend taxation see e.g. King (1977); Auerbach (1979); Bradford (1981). Further see Klein (1999) and Viard (2000) who show that equilibrium stock prices reflect the cost of capital gains taxes.}.

$$V_z^* = \sum_{t=z+1}^{\infty} (1 - \alpha \tau_c) (1 - \beta \tau) D_t (1 + i_T)^{-(t-z)},$$

with $0 \leq \alpha$. $\alpha$ denotes the fraction of corporate tax that cannot be imputed for income tax purposes. If $\alpha = 0$, the model describes a full imputation system, for $\alpha = 1$, corporate tax is excluded from imputation and the corporate tax burden becomes refundable. Post-corporate-tax dividends can be described by

$$D^*_t = (1 - \alpha \tau_c) D_t = (1 - \gamma) (1 - \alpha \tau_c) P_t$$

and are due to income taxation. Further, the after-tax discount factor of the investor $i_T$ can be written as

$$i_T = (1 - \beta^F \tau) (1 - \gamma \alpha \tau_c) i,$$

where $0 \leq \beta^F \leq 1$ indicates whether the alternative financial investment on the capital market is subject to privileged taxation, e.g., enjoys a 50% shareholder relief ($\beta^F = 0.5$) or has to be taxed without any privileges ($\beta^F = 1$) such as in the case of an investment into bonds. $\chi$ shows whether an alternative investment is realized in the corporation ($\chi = 1$) or as a private investment on personal level ($\chi = 0$). For reasons of simplicity in the following we focus on A considering an alternative personal level investment in bonds. Thereby, we set $\chi = 0$ and $\beta^F = 1$. The after-tax discount factor reduces to

$$i_T = (1 - \tau) i.$$

At the corporate level, the relevant after-tax discount factor is given by:

$$i_{\tau_c} = (1 - \tau_c) i.$$

Hence, an amount

$$R^*_t = \gamma (1 - \tau_c) P_t$$
may be retained for internal investment and thereby for future corporate growth:
\[
P_{t+1} = P_t + i (1 - \tau_c) \gamma P_t \\
= (1 + \gamma i_{\tau_c}) P_t. \tag{13}
\]
Profit in period \(t+1\) consists of profit from the preceding period enhanced by growth from retained earnings after taxes. Dividends have to be taxed using the individual marginal income tax rate. From equations (8) and (13) follows:
\[
D^c_{t+1} = (1 + \gamma i_{\tau_c}) D^c_t. \tag{14}
\]
In order to focus on economically plausible values of \(\gamma\) we restrict the retention rate for cases with \(T = \infty\) as follows\(^\text{12}\):
\[
\gamma < 1 \quad \land \quad \gamma < \frac{i_{\tau_c}}{i_{\tau_c}}
\]
and finally receive
\[
V_0^\tau = \sum_{i=1}^{\infty} (1 - \beta \tau) D^c_t (1 + i_r)^{-t} + V_z^\tau (1 + i_r)^{-z} - \beta \tau_g (V_z^\tau - I_0) (1 + i_r)^{-z} \tag{15}
\]
\[
= I_0 \cdot \frac{(1 - \beta \tau) (1 - \gamma) (1 - \alpha \tau_c)}{(1 - \tau) - \gamma (1 - \tau_c)} \left(1 - \beta \tau_g \cdot \frac{(1 + \gamma i_{\tau_c})^z - \frac{(1-\tau) - \gamma (1-\tau_c)}{(1-\beta \tau) (1-\gamma) (1-\alpha \tau_c)}}{(1 + i_r)^z}ight)
\]
\[
= I_0 \cdot \phi \cdot \left(1 - \beta \tau_g \cdot \frac{(1 + \gamma i_{\tau_c})^z - \frac{1}{\phi}}{(1 + i_r)^z}\right),
\]
where
\[
\phi = \frac{(1 - \beta \tau) (1 - \gamma) (1 - \alpha \tau_c)}{(1 - \tau) - \gamma (1 - \tau_c)}. \tag{16}
\]
Thereby, present values with
\[
V_0^\tau \gg 1
\]
are possible.

### 2.2.1 Full Imputation System

Setting \(\alpha = 0\) and \(\beta, \beta^F = 1\), we determine the present value of A’s corporate investment under a full imputation system:
\[
V_0^\tau = \sum_{i=1}^{\infty} (1 - \tau) D_t (1 + i_r)^{-t} + V_z^\tau (1 + i_r)^{-z} \tag{17}
\]
\[
= I_0 \cdot \phi \left(1 - \tau_g \cdot \frac{(1 + \gamma i_{\tau_c})^z - \frac{1}{\phi}}{(1 + i_r)^z}\right)
\]
\(^\text{12}\) For \(i_r < i_{\tau_c}\) and additionally \(\gamma \geq \frac{i_{\tau_c}}{i_{\tau_c}}\), we receive – in the case of an infinite time horizon – an infinite present value of future cash flow from the investment for investor B. Since infinite growth does not exist under realistic assumptions, but rather at an finite point in time growth will slow down, cases with \(\gamma \geq \frac{i_{\tau_c}}{i_{\tau_c}}\) have to be considered as economically irrelevant and are excluded from the analysis.
with

\[ \rho = \phi|_{\alpha=0, \beta=1, \beta^F=1} = \frac{(1-\tau)(1-\gamma)}{(1-\tau) - \gamma (1-\tau_c)}. \] (18)

Investor A will compare this present value with the funds to be invested, \( I_0 \).

It is obvious that distortion is due to

- capital gains taxation introduced by \( \tau_{cg} = \beta \cdot \tau_g = \tau_g > 0 \) and
- the factor \( \rho \), which incorporates effects from diverging corporate and income tax rates for \( \gamma > 0 \).

If \( \gamma < \frac{\delta}{\beta \tau_c} \), we find:

\[ \rho > 1 \iff \tau > \tau_c. \] (19)

In order to focus on the effects from current profit taxation we neglect capital gains taxation and assume \( \tau_g = 0 \). Then, from (19) follows that whenever the individual income tax rate \( \tau \) is lower than the corporate tax rate \( \tau_c \), we find \( \frac{V^*_0}{I_0} < 1 \). This indicates that investing into the corporation earns less than investing on the capital market. If \( \tau = \tau_c \), we find \( \frac{V^*_0}{I_0} = I_0 \) and hence neutral taxation. For instance for \( \tau = \tau_c = 0.4 \) the multiplier \( \frac{V^*_0}{I_0} \) becomes 1. A lower corporate tax rate leads ceteris paribus to an unprofitable investment.

As long as capital gains taxation is ignored all these conclusions hold regardless of when A sells his stock, i.e. the decision is unaffected by interest influences.

The rate of retention influences the size of the tax-induced distortions significantly (cf. figure 1). The higher \( \gamma \), the higher the fraction of profit subject to corporate growth and thereby the higher either the amount suffering from a relatively high corporate tax rate or enjoying relative tax privileges at corporate level. Therefore, higher \( \gamma \) indicates a more intensive distortional influence of diverging corporate and income tax rates.

**Figure 1:** \( \frac{V^*_0}{I_0} \) for \( \tau_c = 0.4 \), various \( \gamma \) and \( \tau \) without capital gains taxation

If \( \tau_g > 0 \), a dividend policy with \( \gamma = 0 \) prevents capital gains taxation. Corporate growth does not occur and furthermore, at time \( t = z \) there are no capital gains to be realized by
sale. In contrast, retained earnings, i.e. \( \gamma > 0 \), lead to \( \rho \neq 1 \) which indicates a distortions. The direction of the tax influence depends on various parameters. Unique conclusions cannot be deduced either for \( \tau < \tau_c \) or for \( \tau > \tau_c \). It has to be figured out whether the benefit from preferential capital gains taxation exceeds effects from current taxation in factor \( \rho \). If \( \tau < \tau_c \), follows \( \rho < 1 \), if \( \tau = \tau_c \), follows \( \rho = 1 \) and for \( \tau > \tau_c \) finally \( \rho > 1 \), respectively. Only for \( \tau = \tau_c \) and \( z > 0 \) an unambiguous conclusion about the distortion can be deduced: \( V_0^z < I_0 \).

**Figure 2:** \( \frac{V_z^T}{I_0} \) under capital gains taxation for \( \tau_c = 0.4 \), various \( \tau \) and \( z \)

Figure 2 exemplifies the tax-induced distortions for \( \tau_c = 0.40 \), \( i = 0.1 \), \( \gamma = 0.5 \) and \( \tau_g = \frac{5}{6} \) depending on time \( z \). Only high income tax rates allow unique decisions since then \( \frac{V_z^T}{I_0} > 1 \) for all \( z \). If \( \tau = \tau_c \), the property of tax neutrality holds only for an immediate sale of shares. Selling the shares in \( z = 0 \) avoids capital gains and thereby averts capital gains taxation. For \( z > 0 \) and identical corporate and income tax rates, investing into the (growing) corporation is less attractive than investing on the capital market.

This disadvantage is due to double taxation of retained earnings by capital gains taxation. Buyer B pays an amount for the shares at time \( z \) anticipating the present value of all future after-tax cash flows. From this follows \( (V_z^T - I_0) \neq 0 \). This difference is the tax base for A’s capital gains taxation at time \( z \). Consequently, future profits are taxed on the level of investors B and A. Assuming an infinite time horizon the double taxation with corporate and capital gains taxes will be adjusted at time \( T = \infty \), i.e. never, and hence becomes unrefundable. It can easily be shown that for \( T < \infty \) this distortion is less severe but still exists. Double taxation will be eliminated at time \( T > z \) when B liquidates the corporation and realizes a tax relevant capital loss of \( (V_z^T - I_0)^{14} \). Since the compensation for double taxation at time \( z \) is not effective before time \( T \) temporary double taxation, i.e. a rate of interest effect, remains.15

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13 For an analysis of preferential capital gains taxation see e.g. Pye (1972); Scholz (1988).
14 Investor B will receive the amount originally invested, \( I_0 \), which is less than the price he paid for the investment, \( V_z^T \).
15 For the lock-in effect of capital gains taxation see e.g. Auerbach (1989, 1991); Klein (1999, 2001); Viard (2000).
The function $\frac{V_T^{	au}}{I_0}$ in figure 2 is mainly influenced by growth and discount factors. Concentrating on the term in bracket in equation (17), we realize that this term reflects the impact of capital gains taxation and always reduces $I_0 \cdot \rho$. When $z$ is increased the impact of the term, which incorporates double taxation of capital gains at time $z$, lessens. Furthermore, figure 2 shows that in case of a rather early sale of shares increasing $z$ involves a bigger impact on growth (numerator) than if future cash flows (denominator) are discounted. The tax base for capital gains taxation increases and therefore, we find increasing present values of the capital tax burden, i.e. double taxation, and also decreasing values of $\frac{V_T^{	au}}{I_0}$. Selling sufficiently late, the discount effect dominates the growth effect, invoking higher $\frac{V_T^{	au}}{I_0}$ for rising $z$.

Income tax rates that are lower than the corporate tax rate, e.g., $\tau = 0.30$, reduce $V_z$ and $V_0$ additionally. Finally, privileges from the tax rate $\tau_g$ may be overcompensated.

Obviously, simultaneous dividend and capital gains taxation usually distort investment decisions. Therefore, investing into a corporation will often not be attractive for investor A.

For $T < \infty$ we find:

$$V_T^\tau = \rho \cdot I_0 \cdot \left( \frac{1 - \tau_g \cdot \left(\frac{1+i_{\tau_e}}{1+i_{\tau}}\right)^z}{\tau_g - \frac{1}{1+i_{\tau}}} \cdot \frac{Q_{\tau} - 1}{Q_{\tau} - \tau_g} + \frac{1}{\tau_g - \frac{1}{1+i_{\tau}}} \right) \cdot \left( \frac{\tau_g \cdot \rho^{-1} \cdot Q_{\tau}^{-1} \cdot \rho^{-1} \cdot \left(1 - \gamma \right) (1 - \tau) z + \frac{1}{\tau_g - \frac{1}{1+i_{\tau}}} \right) \cdot \left( (1 + \gamma i_{\tau_e})^T - \frac{(1 - \tau) (1 - \tau)}{1 - \tau} \right)$$

with

$$Q_{\tau} = (1 + i_{\tau})^{T - z}.$$  

(21)

General results concerning the magnitude and direction of the tax-induced distortions for diverging corporate and income tax rates and $T < \infty$ cannot not be determined analytically.

2.2.2 Shareholder Relief System

Assuming $\alpha = 1$ and $\beta = 0.5$, we describe a tax system with a 50% shareholder relief. Furthermore, for reasons of simplicity we concentrate again on cases where alternative investment does not enjoy a corresponding relief and thereby $i_{\tau} = (1 - \tau) i_{16}$. Capital gains are taxable with fraction $\beta = 0.5$ and $\tau_g = \tau$, i.e. $\tau_{cgt} = 0.5 \tau$. We find

$$V_T^\tau = \sum_{t=1}^{\tau} \left( D_t^{\tau_e} - 0.5 \tau D_t^{\tau_e} \right) (1 + i_{\tau})^{-t} + V_z^\tau (1 + i_{\tau})^{-z}$$

$$-0.5 \tau (V_z^\tau - I_0) (1 + i_{\tau})^{-z}$$

$$= I_0 \cdot \theta \cdot \left( 1 - 0.5 \tau \cdot \left( \frac{(1 + \gamma i_{\tau_e})^z}{(1 + i_{\tau})^z} \right) - \frac{1}{\tau} \right) .$$

with

$$D_t^{\tau_e} = (1 - \gamma) (1 - \tau_e) P_t$$

(23)

16 E.g. personal level alternative investment in bonds, see equation (10).
and

\[ \theta = \phi_{|_{\alpha=1, \beta=0.5, \beta^p=1}} = \frac{(1 - 0.5\tau)(1 - \gamma)(1 - \tau_c)}{(1 - \tau) - \gamma (1 - \tau_c)}. \] (24)

Figure 3 shows the difference between shareholder relief and full imputation system for \( \tau_c = 0.4, \ z = 5, \ i = 0.1, \) and \( \gamma = 0.5. \)

**Figure 3:** \( \frac{V^r_r}{I_0} \) under full imputation or shareholder relief for \( \tau_c = 0.4 \) and various \( \tau \)

For \( \tau_c = 0.25 \) we find

**Figure 4:** \( \frac{V^r_r}{I_0} \) under full imputation or shareholder relief for \( \tau_c = 0.25 \) and various \( \tau \)

For constant \( \tau_c = 0.25 \) we find \( \theta > \rho \) if \( \tau = 0.4. \) This relation describes the tax rate induced advantages indicated by cutting the tax base under shareholder relief. For both tax regimes high income tax rates invoke more severe distortions than lower ones.

Another interesting scenario is assuming \( \tau_c = 0.25 \) for an income tax rate \( \tau = 0.4, \) which can be regarded as a probably average marginal income tax rate for shareholders, \( \gamma = 0 \) and \( \tau_g = 0. \) Then, both systems lead to identical non-distorting tax burdens under both system. This setting as well as an alternative financial investment may serve as a yardstick for measuring the tax effects. The indifference of shareholder relief, full imputation and the taxation of income from capital investment can be seen for \( \tau = 0.4 \) in figure 5. If we
increase the rate of retention up to e.g. \( \gamma = 0.5 \) this leads to an indifference of the tax systems for a lower personal tax rate of \( \tau = 0.3076923 \).

\[
\text{Figure 5: } \frac{V^*}{I_0} \text{ for } \gamma = 0, \tau_c = 0.25 \text{ and various } \tau
\]

This figure clarifies again that the shareholder relief system emphasizes the extent of distortions. Obviously, equation (22) is more complex than the one under full imputation. Even for the most simple scenario with \( \tau = \tau_c \) no general conclusions can be derived analytically. Whereas the influence of \( z \) on \( V^*_0 \) corresponds with the interaction observed in figure 2, capital gains taxation now reduces the extent of distortion.

The influence of taxation of capital gains becomes even more obvious in figure 6.

\[
\text{Figure 6: } \frac{V^*}{I_0} \text{ and capital gains taxation under full imputation or shareholder relief for } \gamma = 0.5, z = 5, \tau_c = 0.25 \text{ and various } \tau
\]

The following table gives an overview of scenarios that allow analytical conclusions for \( \alpha = 0, \beta = 1 \), i.e. full imputation, and alternatively for \( \alpha = 1 \) and \( \beta, \beta^F = 0.5 \), i.e. Germany’s shareholder relief system:
Table 1: Analytical results for full imputation and shareholder relief

<table>
<thead>
<tr>
<th></th>
<th>dividend and capital gains taxation</th>
</tr>
</thead>
<tbody>
<tr>
<td>full imputation</td>
<td>$V_0^\gamma = I_0$, if $\gamma = 0$</td>
</tr>
<tr>
<td></td>
<td>$V_0^\gamma \leq I_0$, if $\tau \leq \tau_c$ and $\gamma &gt; 0$</td>
</tr>
<tr>
<td>shareholder relief</td>
<td>$V_0^\gamma = I_0$, if $\gamma = 0$ and $\tau = \frac{2\tau_c}{(1+\tau_c)}$</td>
</tr>
<tr>
<td></td>
<td>all other cases: no general analytical results</td>
</tr>
</tbody>
</table>

This table points out that the shareholder relief system makes tax planning more difficult. Assuming $T < \infty$, $V_0^\gamma$ becomes:

$$V_0^\gamma = \theta \cdot I_0 \left( 1 - 0.5\tau \cdot \frac{(1+\gamma\tau_c)^2}{(1+\tau_c)^2} \cdot \frac{Q - 1}{Q - 0.5\tau} \cdot \frac{1 - 0.5\tau}{1 - \gamma(1 - Q)(1 - 0.5\tau)} \cdot \left( \frac{\tau}{1 - \tau_c} (1 + \gamma\tau_c)^2 \right) \right)$$

(25)

which is even more complex. The investor can only decide on the basis of single-case numerical analyses.

2.2.3 Classical Corporate Tax System

A classical corporate tax system can be easily modelled assuming $\alpha = 1$, $\beta = 1$ and $\beta^F = 1$. This setting increases the distortions observed for shareholder relief.

Again, analytical solutions for the investment problem cannot be derived.

3 Monte Carlo Simulation

General analytical solutions to the investor’s decision problem cannot be found. In order to gain results in the more realistic setting of uncertainty we apply a Monte Carlo Simulation. This method allows to determine the likelihood of the forecast variable and thereby to identify tax-induced effects on the investor’s decision under uncertainty. We integrate uncertainty introducing random parameters into the calculus. Since cash flow from an investment is usually subject to uncertainty we concentrate in section 3.1 on a random internal rate of return, $i^{in}$. We assume its expected value is equal to the deterministic value $i^{ex}$ and therefore relax the assumption of marginal investment, implicitly allowing different rates of return. Since empirical data on corporate rates of return are usually limited to specific industries and general data is not available, a Monte Carlo Simulation seems to be an appropriate instrument for deriving substantiated conclusions on investment behavior.

In a second step we extend the analysis with respect to stochastic income tax rates in section 3.2. Apart from $i^{in}$ and $\tau$ all parameters remain deterministic. Since in reality tax rates frequently change due to ongoing tax reforms, we approximate real-world tax rate jumps by varying the variable $\tau$. 

13
3.1 Random Return

In this simple scenario we analyze the influence of taxation on investment behavior, assuming random annual internal rates of return $\tilde{r}^{in}$ to be normally distributed with drift $\mu_{r^{in}} = \hat{r}^{ex}$ and standard deviation $\sigma_{r^{in}} = 0.05$. Consecutive rates of return are assumed independently and identically distributed. Thereby profits and dividends grow periodically at the stochastic rate $\gamma_{t}^{\tilde{r}^{in}}$. Assuming that the expected internal rate of return equals the external rate of return restricts the analysis to (expected) marginal investment. $\sigma$ determines the range of deviation from a marginal project. A standard deviation of 0.05 covers very profitable investment projects that yield high rates of return as well as projects that are unprofitable, delete invested funds or even require an additional contribution\textsuperscript{18}.

Focussing on the expected internal rate of return as the decision variable and assuming $\mu_{r^{in}} = \hat{r}^{ex}$ we implicitly restrict to risk neutral investors. This assumption and simplification allows to abstract from risk premiums and to concentrate on the expected values as decision criteria\textsuperscript{19}.

Furthermore, we assume:

invested funds : $I_{0} = 1$
external rate of return : $\hat{r}^{ex} = 0.1$
number of trials : $n = 25,000$
time horizon : $T = 30$
time of sale : $z = 10$
expected internal rate of return : $\mu_{r^{in}} = \hat{r}^{ex} = 0.1$
corporate tax rate : $\tau_{c} = 0.4$

Before switching from full imputation to shareholder relief system in Germany, the corporate tax rate was $\tau_{c} = 0.4$. Under the new system the corporate tax rate has been reduced to $\tau_{c} = 0.25$. In the following we focus on these two rates as representative examples.

Since we assume independent random $\tilde{r}^{in}$ for each period $0 < t \leq T$, $T$ gives the number of simulated random variables with identical probability distribution, too\textsuperscript{20}.

$FI, SR, CC$ indicate full imputation, shareholder relief and classical corporate tax system, respectively. $V_{0}^{*}(.)$ denotes the deterministic present value for the underlying tax system and $P(.)$ describes the probability of realizations of $V_{0}^{*} \geq 1$ for the underlying tax system.

We determine mean $m$ and variance $\sigma^{2}$ of our forecast variable $\hat{V}_{0}^{*}$ and the probability $P(.)$. Further, we analyze skewness, $skew = \frac{\mu_{3}}{\sigma}$, where $\mu_{3}$ is the third moment about the mean and $\sigma$ is the standard deviation as a measure of the asymmetry of the probability distribution, and additionally determine kurtosis, $kurt = \frac{\mu_{4}}{\sigma^{4}} - 3$, as a measure of its peakedness.

---

\textsuperscript{17} See equation (2).

\textsuperscript{18} In reality, empirical analyses prove that stochastic return rates of stocks show probability distributions with fat tails and thereby are rather $t$-distributed than normally distributed. In order to expand the analysis to simulations in a more realistic setting $t$-distributed random return should be assumed alternatively. See e.g., Glosten, Jagannathan, Runkle (1993); Theodossiou (1998); Verhoeven, McAleer (2003).

\textsuperscript{19} See Nienmann, Sureth (2004, 2005) who point out the limits of capital budgeting with taxes under risk aversion and irreversibility.

\textsuperscript{20} A time horizon of 30 periods may be interpreted as the endurance of one generation.
Simulations show\textsuperscript{21} that normally distributed returns symmetrically influence the present value of the investment and thus indicate (almost) identical distributed probabilities for $V_0^\tau (SR)$. Variance, skewness and kurtosis have very small values. The retention rate $\gamma$ does not distort this probability distribution. We find corresponding results for varying $z$, $\tau_g$ or the level and relation of the underlying tax rates. 

$m$ depends essentially on the ratio of the corporate, i.e. internal, tax rate to the personal income tax rate. This effect has already been observed in the deterministic case (see figure 1) in chapter 2.

Variance seems to be rather unaffected by random returns. The forecast variable is obviously less volatile than the return itself. Since by assumption a fraction of the profit is paid as dividends to the investor in every period, this fraction of the profit does not increase internal growth of profit after the dividends have been paid and is therefore not influenced by stochastic return. This leads to lower $\sigma^2$ for $V_0^\tau$ than for $i^m$.

Furthermore, low (high) $\gamma$ causes lower (higher) values of skewness and kurtosis. However, the deviation from the normal distribution is very low.

As an example for these findings table 2 shows for $\tau = 0.3 < \tau_c = 0.4$ and $\tau_{egt} = \beta \cdot \tau_g = 0.2$ that high rates of retention increase asymmetry of the probability distribution of $V_0^\tau$ under shareholder relief:

\begin{table}[h]
\centering
\begin{tabular}{c|c|c|c|c|c}
$\gamma$ & $V_0^\tau$ & $m$ & $\sigma^2$ & skew & kurt \\
\hline
0.0 & 0.776 & 0.776 & 0.004 & 0.037 & -0.029 \\
0.1 & 0.767 & 0.768 & 0.004 & 0.037 & -0.029 \\
0.2 & 0.757 & 0.757 & 0.005 & 0.105 & 0.099 \\
0.3 & 0.747 & 0.747 & 0.005 & 0.068 & -0.024 \\
0.4 & 0.736 & 0.735 & 0.009 & 0.155 & 0.031 \\
0.5 & 0.723 & 0.723 & 0.005 & 0.164 & 0.005 \\
0.6 & 0.710 & 0.710 & 0.010 & 0.219 & 0.086 \\
0.7 & 0.695 & 0.695 & 0.011 & 0.261 & 0.162 \\
0.8 & 0.680 & 0.680 & 0.006 & 0.328 & 0.181 \\
0.9 & 0.662 & 0.662 & 0.007 & 0.371 & 0.220 \\
\end{tabular}
\caption{Shareholder Relief System for $\tau = 0.3$, $\tau_{egt} = 0.2$, and different $\gamma$}
\end{table}

### 3.2 Random Return and Random Personal Tax Rates

Referring to frequently changing real-world tax rates we simulate independent positive random tax rates $\tilde{\tau}$ in each period which are normally distributed with $\mu_\tau = 0.4$ or $\mu_\tau = 0.25$ and a standard deviation $\sigma_\tau = 0.05$. These cases cover high income investors with high expected marginal income tax rates or, alternatively investors with rather low expected marginal income tax rates ($\mu_\tau = 0.25$)\textsuperscript{22}. Negative personal tax rates are excluded from

\textsuperscript{21} See e.g. table 2.

\textsuperscript{22} Randomization may reflect tax law uncertainty as well as tax rate uncertainty which might be due to stochastic changes in personal income under a progressive income tax. Simulation of dependant random tax rates rather than independent ones might be favorable for specific settings. For reasons of simplicity such interdependencies will be neglected here.
simulation by assumption. With \( \sigma_T = 0.05 \) the high personal income tax rate may fluctuate mainly between 0.25 and 0.55. These “boundaries” are near to both the lowest and highest German marginal tax rates, including all income tax surcharges. The lower expected income tax rate of \( \mu_T = 0.25 \) produces a realization of \( \hat{\tau} \) mainly between 0.1 and 0.4. These rates correspond to tax rates in several countries that already have reduced their rates as a consequence of international tax competition.

We analyze several scenarios with different values for \( z \) and \( \gamma \) under the following assumptions:

- Invested funds: \( I_0 = 1 \)
- External rate of return: \( \tau^{ex} = 0.1 \)
- Number of trials: \( n = 25,000 \)
- Time horizon: \( T = 100 \)
- Retaining rate: \( \gamma = 0.5 \)
- Expected internal rate of return: \( \mu_{in} = \tau^{ex} = 0.1 \)
- Corporate tax rate: \( \tau_c = 0.25 \)
- Capital gains tax rate: \( \tau_{glt} = 0.2 \)

If one period equals one year, the assumption \( T = 100 \) implies a time horizon that exceeds one generation\(^{23}\). This enables to figure out long-term tax effects, which from our findings, e.g. in section 2.2.1, are important in the context of timing decisions. For reasons of simplicity and transparency we exclude inheritance tax aspects from the analysis, instead focussing on current profit taxation.

It can be shown by simulation that all probability measures follow a similar pattern. Table 3 clarifies this for full imputation and shareholder relief.

*Table 3: Full imputation and shareholder relief system for \( \tau_c = 0.25 \), \( \mu_T = 0.4 \), \( \gamma = 0.5 \) and various \( z \)*

<table>
<thead>
<tr>
<th>( z )</th>
<th>tax</th>
<th>( m )</th>
<th>( \sigma^2 )</th>
<th>skew</th>
<th>kurt</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>( FI )</td>
<td>1.49</td>
<td>1.27</td>
<td>2.87</td>
<td>15.73</td>
</tr>
<tr>
<td></td>
<td>( SR )</td>
<td>1.49</td>
<td>1.24</td>
<td>2.62</td>
<td>12.82</td>
</tr>
<tr>
<td>30</td>
<td>( FI )</td>
<td>1.42</td>
<td>0.89</td>
<td>2.20</td>
<td>8.72</td>
</tr>
<tr>
<td></td>
<td>( SR )</td>
<td>1.43</td>
<td>0.93</td>
<td>2.44</td>
<td>12.65</td>
</tr>
<tr>
<td>50</td>
<td>( FI )</td>
<td>1.44</td>
<td>0.92</td>
<td>2.37</td>
<td>10.79</td>
</tr>
<tr>
<td></td>
<td>( SR )</td>
<td>1.44</td>
<td>0.90</td>
<td>2.36</td>
<td>11.35</td>
</tr>
<tr>
<td>70</td>
<td>( FI )</td>
<td>1.46</td>
<td>0.92</td>
<td>2.46</td>
<td>14.07</td>
</tr>
<tr>
<td></td>
<td>( SR )</td>
<td>1.47</td>
<td>0.91</td>
<td>2.07</td>
<td>7.70</td>
</tr>
<tr>
<td>90</td>
<td>( FI )</td>
<td>1.48</td>
<td>0.94</td>
<td>2.42</td>
<td>12.30</td>
</tr>
<tr>
<td></td>
<td>( SR )</td>
<td>1.49</td>
<td>0.91</td>
<td>2.07</td>
<td>7.66</td>
</tr>
</tbody>
</table>

Analyzing the observed realizations of \( \hat{V}_0^\tau \) we find \( m, \sigma^2, \text{skew} \) and \( \text{kurt} \) to have a minimum in the interval \( 20 < z_{\text{min}} < 40 \). We receive the highest mean for early selling, i.e. \( z = 1^24 \), under all tax systems. Since capital gains increase with time \( z \) this indicates that capital

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\(^{23}\) One generation is assumed to be 30 years.

\(^{24}\) Under full imputation we find \( m = 1.59 \) and for shareholder relief follows \( m = 1.57 \).
gains taxation distorts and decreases the profitability of the investment relative to an alternative equivalent investment without capital gains, e.g. investing into a bond, until time \( z = z_{\text{min}} \). The discounting effect overcompensates the capital gains effect for later points of sale, i.e. \( z > z_{\text{min}} \), and then leads to increasing mean \( m \).

Regarding a risk averse investor instead of a risk neutral one and referring e.g. to \( \sigma^2 \) as an indicator for the degree of risk whenever capital gains taxation invokes cuts in \( V^\tau_0(\cdot) \) it simultaneously reduces risk. This is shown by \( \sigma^2 \) which has a minimum at the same point in time \( z = z_{\text{min}} \). Furthermore, the probability distribution of \( V^\tau_0(\cdot) \) is skewed right and leptokurtic in every scenario (see e.g. figure 7). Again, these deviations from the normal distribution reach a minimum for \( z = z_{\text{min}} \). Thus, whenever an effect from capital gains taxation dominates discounting effects \(^{25}\) this reduces a) the mean of the present value and b) variance and c) skewness and kurtosis simultaneously. Thereby under risk aspects, the investment project becomes relatively more attractive for rather late selling.

\textbf{Figure 7: Probability distribution of } V^\tau_0(SR) \text{ for } \tau_c = 0.25, \mu_r = 0.4, \gamma = 0.5, \text{ and } z = 50

Concentrating on the influence of the underlying tax system, an investor will suffer significant losses under a classical corporate tax system due to double taxation of his dividend payments. As a trade-off, less volatile values for \( V^\tau_0 \) occur than under the other tax regimes. In most cases \( \sigma^2 \) is lower than under full imputation or under a shareholder relief system. This relation holds for \( \mu_r = 0.25 \) and \( \mu_r = 0.4 \). Consequently, shareholder relief always provides less uncertainty than full imputation, as can be seen in table 4.

In contrast to shareholder relief, under full imputation finally, i.e. when profits are either distributed or realized by selling the stock, the whole income from the investment is due to personal income taxation. Under shareholder relief instead, part of distributed income is solely taxed on the corporate level. Consequently, a full imputation system is more sensitive towards random income tax rates than the other systems. This result holds even if we assume stochastic corporate tax rates additionally, especially if we take into account that corporate tax rates usually fluctuate less than personal income tax rates.

\(^{25}\) We find this for rather small values of \( z \), e.g. \( z = 20 \).
Table 4: Full imputation, shareholder relief and classical corporate tax system for $\tau_c = 0.25$, $\mu_\tau = 0.25$, $\gamma = 0.5$ and various $z$

<table>
<thead>
<tr>
<th>$z$</th>
<th>tax</th>
<th>$m$</th>
<th>$\sigma^2$</th>
<th>skew</th>
<th>kurt</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$FI$</td>
<td>1.18</td>
<td>0.72</td>
<td>2.68</td>
<td>13.77</td>
</tr>
<tr>
<td></td>
<td>$SR$</td>
<td>1.16</td>
<td>0.58</td>
<td>2.72</td>
<td>14.36</td>
</tr>
<tr>
<td></td>
<td>$CC$</td>
<td>1.15</td>
<td>0.73</td>
<td>2.84</td>
<td>17.80</td>
</tr>
<tr>
<td>30</td>
<td>$FI$</td>
<td>1.17</td>
<td>0.54</td>
<td>2.23</td>
<td>10.53</td>
</tr>
<tr>
<td></td>
<td>$SR$</td>
<td>1.00</td>
<td>0.41</td>
<td>2.12</td>
<td>8.63</td>
</tr>
<tr>
<td></td>
<td>$CC$</td>
<td>0.86</td>
<td>0.31</td>
<td>2.23</td>
<td>10.53</td>
</tr>
<tr>
<td>50</td>
<td>$FI$</td>
<td>1.15</td>
<td>0.52</td>
<td>2.24</td>
<td>10.35</td>
</tr>
<tr>
<td></td>
<td>$SR$</td>
<td>1.01</td>
<td>0.42</td>
<td>2.28</td>
<td>11.18</td>
</tr>
<tr>
<td></td>
<td>$CC$</td>
<td>0.86</td>
<td>0.30</td>
<td>2.24</td>
<td>10.36</td>
</tr>
<tr>
<td>70</td>
<td>$FI$</td>
<td>1.16</td>
<td>0.57</td>
<td>2.17</td>
<td>8.48</td>
</tr>
<tr>
<td></td>
<td>$SR$</td>
<td>1.02</td>
<td>0.43</td>
<td>2.11</td>
<td>7.99</td>
</tr>
<tr>
<td></td>
<td>$CC$</td>
<td>0.87</td>
<td>0.32</td>
<td>2.17</td>
<td>8.48</td>
</tr>
<tr>
<td>90</td>
<td>$FI$</td>
<td>1.17</td>
<td>0.55</td>
<td>1.98</td>
<td>6.65</td>
</tr>
<tr>
<td></td>
<td>$SR$</td>
<td>1.02</td>
<td>0.42</td>
<td>2.03</td>
<td>7.17</td>
</tr>
<tr>
<td></td>
<td>$CC$</td>
<td>0.88</td>
<td>0.31</td>
<td>1.98</td>
<td>6.64</td>
</tr>
</tbody>
</table>

A general yardstick for tax neutrality under uncertainty has not been derived until now. Therefore, we analyze the influence of random taxation on the extent of tax distortion. We compare simulation results for mean and variance for stochastic return in case of deterministic income tax rates with those in case of stochastic income tax rates. As these outcomes are not identical, the differences between the observed values, i.e. the difference between the means $\Delta m$ and the difference between the variances $\Delta \sigma^2$, indicate a distortion caused by uncertainty in income tax rates. Focussing on full imputation and shareholder relief, we find the following deviations from the scenarios with deterministic taxation for low or high personal tax rates.

Figure 8: Individual tax neutrality for $\tau_c = 0.25$ and $\tau, \mu_\tau = 0.25$ or $\tau, \mu_\tau = 0.4$, $\gamma = 0.5$, $\mu_{cin} = 0.1$ and various $z$
The larger the deviation from zero in figure 8 the larger the distortions evoked by stochastic income tax rates. Again, we see that full imputation tends to be significantly more sensitive towards random taxation in mean and variance for low income tax rates.\(^{26}\)

In order to focus on the relationship between the underlying tax systems we have plotted \(P(.)\) for the three systems with \(\tau_c = 0.25\) for \(\mu_r = 0.25\) in figure 9. The fourth graph shows the probability that can be derived for a classical corporate tax system with \(\tau_c = 0.1562\). This is the corporate tax rate that leads to identical values of deterministic \(V_0^r\) of all three systems in this setting. Then, the three tax systems generate the same tax revenue, ceteris paribus and the same present value and are therefore indifferent in a setting with \(\mu_r = 0.4\) and \(\gamma = 0.5\). Comparing the probabilities for these indifferent tax systems with deterministic \(V_0^r\), which serves as yardstick or reference point, emphasizes again that uncertainty does not change the pattern of time-dependency under the given set of assumptions. Only a classical system with the non-revenue-neutral corporate tax rate \(\tau_c = 0.25\) that can be seen at the bottom of this figure leads to significantly lower realizations.

![Figure 9: Influence of uncertainty on \(V_0^r\)](image_url)

If \(P(.) = 0.5\) this corresponds to deterministic \(V_0^r = 1\). Both values indicate (expected) neutral taxation. Although the probability distribution of the stochastic present value is not symmetric in the underlying setting simulation evokes values for \(P(.)\) that are close to its deterministic counterpart \(V_0^r\).

In order to investigate the influence on profit distributions we perform a simulation using two different retention rates, i.e. \(\gamma = 0.1\) and \(\gamma = 0.9\), and receive the values in tables 5 and 6. The first row for each tax system shows values for \(\gamma = 0.1\), the second row the corresponding results if \(\gamma = 0.9\).

Obviously, high retention rates increase variance, skewness and kurtosis. Risk grows even more if the personal tax rate is relatively high (table 6) compared to the corporate tax rate. These findings are principally due to the same interdependencies as described by figure 1 under certainty. High rates of \(\gamma\) amplify the distorting effects that are mainly caused

---

\(^{26}\) Low tax rates may be a result of international tax competition.
by tax rate differences. For $\tau_c = 0.25 < \tau = 0.4$ we can observe the well-known lock-in growth effect. Consequently, more income is subject to stochastic internal growth. The investment project therefore becomes more risky. Additional Monte Carlo Simulations for various rates of $\gamma$ confirm these findings.

Table 5: Full imputation, shareholder relief and classical corporate tax system for $\mu_\tau = 0.25$, $\tau_c = 0.25$, $\gamma = 0.1$ or 0.9 and various $z$

<table>
<thead>
<tr>
<th>$z$</th>
<th>tax</th>
<th>$m$</th>
<th>$\sigma^2$</th>
<th>skew</th>
<th>kurt</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$FI$</td>
<td>1.22</td>
<td>0.69</td>
<td>2.48</td>
<td>12.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.17</td>
<td>0.89</td>
<td>2.91</td>
<td>16.15</td>
</tr>
<tr>
<td></td>
<td>$SR$</td>
<td>1.08</td>
<td>0.54</td>
<td>2.46</td>
<td>12.86</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.03</td>
<td>0.69</td>
<td>2.91</td>
<td>15.80</td>
</tr>
<tr>
<td></td>
<td>$CC$</td>
<td>0.94</td>
<td>0.41</td>
<td>2.47</td>
<td>12.52</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.90</td>
<td>0.52</td>
<td>2.89</td>
<td>15.93</td>
</tr>
<tr>
<td>30</td>
<td>$FI$</td>
<td>1.18</td>
<td>0.55</td>
<td>2.15</td>
<td>9.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.05</td>
<td>0.62</td>
<td>2.50</td>
<td>11.26</td>
</tr>
<tr>
<td></td>
<td>$SR$</td>
<td>1.03</td>
<td>0.42</td>
<td>2.09</td>
<td>7.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.93</td>
<td>0.48</td>
<td>2.47</td>
<td>10.77</td>
</tr>
<tr>
<td></td>
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<td>0.55</td>
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<td>12.84</td>
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<tr>
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<td>0.31</td>
<td>2.34</td>
<td>9.69</td>
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Table 6: Full imputation, shareholder relief and classical corporate tax system for $\mu_\tau = 0.4$, $\tau_c = 0.25$, $\gamma = 0.1$ or 0.9 and various $z$

<table>
<thead>
<tr>
<th>$z$</th>
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<td>13.60</td>
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</tbody>
</table>

Assuming rather low personal income tax rates and parity between personal and corporate tax rates (table 5), an investor will benefit from a full imputation system. Both shareholder

---

relief and the classical system generate lower values of $V_0^r$ than full imputation. This disadvantage emerges from “double” taxation in case of the classical system and from a relatively low tax privilege\(^{28}\) under shareholder relief, caused by the assumed low income tax rates.

Furthermore, these tables allow conclusions about probably profitable investment decisions for a specific setting under uncertainty. If we look at the decision on the time of selling the investment, long-term investments often should be held for longer than $\frac{T}{3}$. Selling e.g. at time $z = 50$ may lead to a minimal realization of the present value\(^{29}\). If the investor decides to delay the sale e.g. until $z = 90$, risk increases (see table 6). The chance of receiving a higher present value of the investment by delaying the sale grows if high personal and low corporate tax rates coincide and a large fraction of profits is retained in each period. Probabilities for a profitable investment under this setting, i.e. the probability that $V_0^r$ will have a realization of 1 or greater, $P(\cdot)$, have been plotted in figure 9.

Analyzing the range of means, simulation indicates that uncertainty leads to higher expected values of $V_0^r$ than those that can be determined for non-stochastic $V_0^r$. E.g., assuming $\gamma = 0.1$, $\mu_r = 0.25$ and full imputation we find for $0 < z < 100$ deterministic present values between 0.99 and 1.00 whereas, based on stochastic internal and random personal tax rates, realization of $V_0^r$ from 1.17 to 1.28 emerge\(^{30}\). Table 7 exemplifies this:

**Table 7: Range of values and probabilities for $\tau_c = 0.25$, $\mu_r = 0.25$, $0 < z < 100$, and various $\gamma$**

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>tax</th>
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<th>$m$</th>
<th>$P(\cdot)$</th>
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<tr>
<td>0.1</td>
<td>$FI$</td>
<td>0.99 - 1.00</td>
<td>1.17 - 1.28</td>
<td>0.49 - 0.50</td>
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<tr>
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<td>0.87 - 1.00</td>
<td>1.02 - 1.15</td>
<td>0.40 - 0.44</td>
</tr>
<tr>
<td></td>
<td>$CC$</td>
<td>0.75 - 0.80</td>
<td>0.88 - 1.02</td>
<td>0.30 - 0.37</td>
</tr>
<tr>
<td>0.9</td>
<td>$FI$</td>
<td>0.86 - 1.00</td>
<td>1.03 - 1.28</td>
<td>0.39 - 0.49</td>
</tr>
<tr>
<td></td>
<td>$SR$</td>
<td>0.75 - 0.88</td>
<td>0.91 - 1.15</td>
<td>0.31 - 0.43</td>
</tr>
<tr>
<td></td>
<td>$CC$</td>
<td>0.65 - 0.80</td>
<td>0.78 - 1.02</td>
<td>0.23 - 0.37</td>
</tr>
</tbody>
</table>

Neutral taxation is often considered desirable from a tax policy and efficiency point of view. Consequently, distortions, i.e. in case of a risk neutral investor deviations from $V_0^r = 1$, even if they favor the individual investor, should be avoided. Focussing on the mean as decision criterion indicates neutral attitude towards risk. Relaxing this assumption requires a more sophisticated capital market equilibrium model that integrates risk aversion into capital budgeting and endogenously determines the appropriate risk premium\(^{31}\). Referring to $\sigma^2$, skewness and kurtosis only provides information about the degree of risk involved, but not about its functional impact on the investor’s decision calculus.

---

\(^{28}\) I.e. tax base multiplied with the relief factor and the income tax rate.

\(^{29}\) E.g. figures 2 and 9.

\(^{30}\) See table 5 and figure 9.

\(^{31}\) See e.g. the equilibrium models developed by Constantinides (1983) and Basak, Gallmeyer (2003) that both rely on the existence of an exogenously-given risk-free bond which is tax exempt. Further, see Niemann, Sureth (2004) who highlight the need for a general equilibrium approach which takes into account the combined corporate and personal tax wedge resulting from real and financial investment.
Since the analyzed tax systems cause higher present values and increase deviation from the
tax neutral value \( V_0^* = 1^{32} \), they apparently tend to be more inefficient under uncertainty
and the given set of assumptions than under certainty in this partially analytic frame-
work. Furthermore, as the rate of return is stochastic we do not analyze solely marginal
investment. Consequently, beyond statements on the direction of the distortion, conclu-
sions on whether the influence of the underlying tax system on investment behavior is
desirable from an efficiency point of view\(^{33}\) are not possible as an appropriate yardstick
for inframarginal investments has not been deduced until now.

4 Summary and Conclusion

This paper examines the influence of current profit and capital gains taxation on in-
vestments in corporate shares. We focus on three specifications of one parametric tax
model. These specific tax systems are full imputation, shareholder relief and the classical
corporate tax system with double taxation of dividends at corporate and personal level.

Applying a growth model under certainty we find under rather restrictive assumptions
that the shareholder relief system induces more severe distortions than the full imputa-
tion system. Unfortunately, general analytical solutions for the different categories of tax
regimes, even under certainty, cannot be derived.

Trying to prove the distortive power of a tax system with shareholder relief in a more
realistic setting with uncertainty, we employ a Monte Carlo Simulation. The degree of
tax-induced uncertainty in many cases is significantly lower under a shareholder relief
system than under full imputation. In contrast to the analysis under certainty, the results
suggest that under uncertainty full imputation probably causes more severe distortions
than shareholder relief whenever personal income tax rates are low. This is an important
result as in light of international tax competition, a reduction of tax rates is taking place
or is likely to take place in several countries.

Furthermore, the simulation highlights the trade-off of the opposing effects, i.e. tax and
interest rate effects, and the overwhelming impact of capital gains taxation. Apart from
tax parameters, we identify the dividend rate and the point in time of selling the stock
as important value drivers.

All results of this analysis are limited to the underlying assumptions. Obviously, even in
this simple setting taxation has a nonuniform influence on investors behavior. Genera-
lizations are hardly possible. Nevertheless, we found probabilities for specific profitable
investment strategies depending on the tax system.

\(^{32}\) See table 7.

\(^{33}\) Similar to the conclusions under certainty.
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