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## Article

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Cogent Economics & Finance

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*Suggested Citation:* Keregero, Chirongo Moses (2021) : A theoretical framework for simulating systemic risk and its application to analysis of the banking system, Cogent Economics & Finance, ISSN 2332-2039, Taylor & Francis, Abingdon, Vol. 9, Iss. 1, pp. 1-19, <https://doi.org/10.1080/23322039.2021.1986930>

This Version is available at:

<https://hdl.handle.net/10419/270166>

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To cite this article: Chirongo Moses Keregero | (2021) A theoretical framework for simulating systemic risk and its application to analysis of the banking system, Cogent Economics & Finance, 9:1, 1986930, DOI: [10.1080/23322039.2021.1986930](https://doi.org/10.1080/23322039.2021.1986930)

To link to this article: <https://doi.org/10.1080/23322039.2021.1986930>



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Published online: 05 Nov 2021.



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Received: 26 June 2020  
Accepted: 25 September 2021

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Reviewing editor:  
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## FINANCIAL ECONOMICS | RESEARCH ARTICLE

# A theoretical framework for simulating systemic risk and its application to analysis of the banking system

Chirongo Moses Keregero<sup>1\*</sup>

**Abstract:** Risk of basic defaults and contagious defaults are two main sources of bank systemic risk. In this paper, a theoretical framework is proposed to classify the time evolution of the basic defaults and contagious defaults using sequences of daily financial data. The new theoretical framework combines an existing asset value estimation algorithm and obligation clearing algorithm to calculate the time evolution of systemic risk. The asset value estimation algorithm is used to estimate the asset values of the banks each day and the obligation clearing algorithm is used to calculate systemic risk given the tuples of data each day. This framework is applied to assess the systemic risk of the Nigerian banking system between 2008 and 2014 when the economy was hit by the financial meltdown. The main findings depict that the risk of the basic defaults was high during this period while contagious default seldom appeared. It is also found that the Nigerian banking system was more stable in 2010 and 2012 than in other years, while it was seriously unstable in 2008, 2011, and 2014. The findings would assist in monitoring systemic risk in the Nigerian banking system.

**Subjects:** Economics; Finance; Business, Management and Accounting;

**Keywords:** Systemic risk; Network analysis; Interbank market; Bank defaults · Financial Stability



Chirongo Moses Keregero

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### PUBLIC INTEREST STATEMENT

The global credit crisis that started in 2007 has highlighted the need of addressing incomplete reforms, one of them being the contribution of systemic risk in destabilizing the financial markets. The crisis has shown how systemic risk can propagate in the financial system causing failure of financial institutions and ultimately distress of the global economy. Based on that, this simulation-based study proposes a theoretical framework to assess systemic risk in the Nigerian banking system from 2008 to 2014 using a sequence of daily financial data. Results depict that the Nigerian banking system was more stable in the years 2010 and 2012 than in other years. The approach proposed in study will provide insights for regulators in the management of systemic risk.

## 1. Introduction

The collapse of Lehman Brothers showed that the defaults of large banks can set in motion a chain of insolvencies in the financial markets causing distress and even failure of some key institutions in the financial system leading to financial crisis (Sedunov, 2016). The crisis emphasizes the need to identify the underlying factors that destabilize the financial institutions which could result in systemic risk.

Systemic risk has been defined in different ways by different scholars and policymakers. Schwarz (2008) has reconciled the different definitions of systemic risk and ultimately gives the general definition of it. The risk that an economic shock such as market or institutional failure (i) triggers (through a panic or otherwise) either (x) the failure of a chain of markets or institutions or (y) a chain of significant losses to financial institutions, (ii) results in substantial financial-market price volatility. It is characterized as a way in which the distress of one institution endangers the rest of the financial system. The reason for this is that financial intermediaries are said to be either too big to fail, too interconnected to fail or too important to fail (Bramer et al., 2014). The above four scenarios indicate the many dimensions of systemic risk in the banking market. To be more clear and incorporate the four indicators mentioned above, systemic risk is interpreted as the threat which threaten the stability of the banking sector, generated by both single institutions (too big, too interconnected, too important) as well as group of banks (too many).

Therefore, we can construct two main components to illustrate and model systemic risk based on the definition. One component is basic default that an initial random shock affects even to the point of failing one financial institution and the other is contagious default that a contagion transmits such negative effects to other institutions on the system. Various scholars have addressed the basic defaults, namely, systemic risk of single institutions for instance, Acharya et al. (2010), Adrian and Brunnermeier (2016); Tarashev et al (2010), while others deal with systemic risk of the contagious defaults such as Huang et al. (2011), Illing and Liu (2006).

Nigeria economy has passed different stages of economic crisis, apart from the global financial crisis in 2008; it has also been affected by the effects of crude oil crash in 2014 (Osadume & Mbachu, 2017). All of these could have a significant impact on the Nigerian banking system. Therefore, this paper uses the data of 10 major banks in Nigeria in terms of assets value and test the extent of systemic risk in the banking system. Thus, supported by the empirical evidence in Nigerian banking system, this study evaluates systemic risk in the Nigerian banking system based on the proposed theoretical framework.

The rest of the paper is organized as follows. Section 2 provides literatures on the measures of systemic risk. Section 3 provides the theoretical framework underlying the evolution of basic defaults and contagion defaults. Section 4 provides the data used. Section 5 presents the results and lastly section 6 provides the concluding remarks.

## 2. Literature review

Two approaches pertaining to the identification and measurement of systemic risk have been proposed. The first approach is the traditional approach that analyzes systemic risk based on stock market. This includes measures such as Value at risk (VaR) and Expected Shortfall which measure systemic risk for liquid positions operating under normal market conditions over a short period of time. Adrian and Brunnermeier (2016) proposed the concept of conditional value at risk to measure the contribution to systemic risk ( $\Delta$  CoVaR) from each bank. Acharya et al. (2010) proposed systemic expected shortfall in order to gauge listed financial institutions contributions to systemic risk. Huang et al. (2009; 2010,, 2011) proposed to measure the risk of a financial system by the insurance premium that would be required to cover losses in a distressed condition.



In terms of assessing systemic risk, the traditional approach measures systemic losses conditional on each financial institution being in distress. These measures take into account the size, the probability of default, and the correlation of each financial institution. However, the correlation fails to capture the interconnectedness sufficiently because it does not consider the various interactions such as contagious defaults or the relationship between interconnectedness and systemic importance in a financial system. Besides measures like VaR and MES has been confronted for not capturing the systemic risk of the financial system as a whole as it looks on systemic risk of the banking system in isolation.

Based on that, this paper focuses on the second approach to analyze systemic risk that is network model. The network models examine the interbank network as a contagion channel that is when some banks are not able to honor their promises in the interbank market, they might push other banks into insolvency which might again lead to defaults of other banks. In normal times, the interbank market ensures efficient liquidity redistribution from banks with surplus liquidity to banks with a shortage of liquidity and thus serves as an absorber of idiosyncratic liquidity shocks. In turbulent situation, however, interbank markets can become a channel for liquidity contagion due to liquidity hoarding by banks and/or credit risk contagion due to credit losses on interbank exposures.

The literature on contagion starts with the work of Allen and Gale (2000) who give a model of risk propagation through interbank exposure network. They show that if there is no liquidity shock, all banks can survive; however in liquidity shock case, number of defaulting banks change depending on network completeness. Freixas et al. (2000) consider that for contagion to happen in a system with money-centre banks where the institutions on the periphery are connected to banks at the centre but not to each other depends on the precise values of the models parameters. Upper (2011) describes that the possibility for contagion depends on the precise structure of the interbank market, for the same shocks some structures would result in contagion while others would not, a complete structure of claims, in which every bank has symmetric exposures to all other banks is much more stable than an incomplete structure, where banks are linked only to one neighbor. Disconnected structures are more vulnerable to contagion than complete structures, but they prevent contagion from spreading to all banks. Finally, show that the possibility for contagion in a system with money-centre banks, where the institutions on the periphery are linked to banks at the centre but not to each other, crucially depends on the precise values of the model's parameters.

A number of the studies in this approach are based on Eisenberg and Noe (2001) who define a clearing payment process for interbank networks considering all possible contagion effects in a static network structure. Elsinger et al. (2006) include bankruptcy costs in their simulation of the Austrian banking system and show that the system is able to absorb shocks well for small bankruptcy costs while large dead weight losses can wipe out the banking system. Rogers and Veraart (2013) model clearing in the interbank networks with bankruptcy costs and provide an analysis of the situations in which banks have incentives to bail out distressed bank. Memmel et al. (2011) use the bilateral interbank liabilities of 15 German banks and run simulations on idiosyncratic defaults for each of the institution allowing only one bank to fail at time, failure is explained at Tier 1 capital falling below 6% of risk weighted assets. Bramer et al. (2014) introduce a network model to assess system risk in the banking sector and run simulations based on theoretical data. Their paper build on the network simulation of Memmel et al. (2011) and incorporate an exogenous shock on the market that comprehends the primary default of one or more institutions and examine the degree of volatility of the system. Ebrahimi Kahou and Lehar (2017) give a detailed overview of systemic risk measures.

Despite the fact that the researches on contagion risk in the interbank banking system have made great progress, there are some constraints in existing research. Most of the studies are

based on static network structures (fixed bank lending matrixes) and static bank systems (fixed bank balance sheets). However, an interbank network system portrays high complex dynamics. Thus, this study examines systemic risk based on the dynamic financial networks. For instance, Eisenberg-Noe framework describes simultaneous defaults for one period and not on the dynamic multiperiod which is applies to this study. Therefore, this study extends a framework to multiperiod setting which borrows from the framework of Kanno (2015) and theoretically analyze the Nigerian interbank market. Kanno (2015) considered the multiperiod scenario but didn't take into account the dynamic changing of the interbank market. Through the dynamic changing of the interbank market, both the total asset value and equity value of the bank change dynamically and this can be estimated from real world data instead of theoretical assumption.

Few studies have examined the dynamic model of network structure such as, Georg (2013), Lux (2015), Xu et al. (2016), and Bluhm et al. (2014). But these literatures did not consider the effect of systemic random shocks in the banking system, they almost considered credit and liquidity shocks. Thus, for shock scenarios, this study adds a systemic shock artificially in the system to observe the time evolution of the banking system and hence measure the systemic risk. Stated in other words, the systemic risk is measured by checking whether a bank system can withstand certain strength of systemic shock. Some banks may be bankrupt at a certain time point due to this artificially added systemic shock, therefore both the structure and the state of the bank change dynamically. The systemic risk is measured by recording the number of banks which undergo bankrupt during the time. If the strength of the systemic shock is fixed, then a bank system with more banks which undergo bankrupt during the whole-time course of its evolution is believed to suffer more systemic risk.

The study defines the methodology and conduct analysis of the theoretical framework, and present the results of the systemic risk of Nigeria banking system. Specifically, an optimization procedure is conducted to estimate the bilateral exposures matrix using the aggregate balance sheet data on loans and deposits from African market web site. The study uses the estimated bilateral exposures matrix to theoretically analyze the network structure of the interbank market in Nigeria. From the daily market capitalization data, the market values of assets at each day and the drifts and volatilities of their returns for each bank are estimated using the stochastic model and the maximum likelihood estimation method (Duan et al., 2005). This study also endeavors to examine bank defaults which are classified as basic defaults and contagious defaults which generally trigger domino effect. With regard to the contagious default the study uses the Eisenberg and Noe framework that is based on a mathematical proof showing that there always exists a unique vector for simultaneously clearing the obligations of all participants in the system. The vector is developed under mild regularity using a fixed point theorem. By using the in and out degree and network centrality measures the research explain how banks play an important role in the interbank market.

### 3. Theoretical frameworks

This research considers the interconnectedness of the Nigerian bank network, which allows the analysis of transmission of systemic risk through bilateral exposures, possibly causing contagious defaults that are triggered by banks basic default. Figure 1 and Figure 2 illustrate the theoretical frameworks of this study. Figure 1 show an example bank network structure of ten banks connected with each other through bilateral exposures which are described by matrix  $X$  in Figure 1(a). Figure 1(b) shows the bank balance sheet of bank  $i$ . In the balance sheet,  $v_i$  represents the assets of bank  $i$ ,  $a_i$  stands for the interbank assets of bank  $i$ ,  $D_i$  is the liability of bank  $i$ , and  $l_i$  is the interbank liability of bank  $i$ . If the bank balance sheet of bank  $i$  shows that the assets are smaller than the liability, namely  $v_i + a_i < D_i + l_i$ , then bank  $i$  defaults by the definition of basic default. Through bilateral exposures, the basic default of bank  $i$  can cause the contagious defaults to the banks that connect to bank  $i$ . Figure 2 shows the flowchart of the process of the theoretical frameworks. In section 3.1, the methodology for estimating the matrix of bilateral exposures is described and then the methods of estimating the time



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### 3.1. Estimation of bilateral exposure matrix

In this section the study describes the methodology for estimating the bilateral exposure matrix that is described in Figure 1(a). Usually the interbank market of participants comprises central banks, commercial banks, investment banks, cooperative banks, security companies, private banking, and asset management companies, real estate and mortgage banks, and specialized government institutions. To fully fill the scope of this study, it is required to determine the interbank exposures. Unfortunately, this ideal data quality is rare. The interbank exposures cannot be fully observed but has to be partially estimated from balance sheet data. Initial total interbank assets data  $a_i$  and liabilities data  $l_j$  in the balance sheet described in Figure 1(b) usually correspond with two relevant accounting items, loans and advances to banks ( $a_i$ ) and deposits from banks ( $l_j$ ). The lending relationships in the interbank market will be represented by  $(N \times N)$  nominal interbank matrix  $X$  as shown in Figure 1(a), where  $x_{ij}$  represents the lending loans of bank  $i$  to bank  $j$ ,  $\sum_j x_{ij} = a_i$  represents the total interbank assets of bank  $i$ , and  $\sum_i x_{ij} = l_j$  means the total interbank liability of bank  $j$ . It has to hold that

$$\sum_i a_i = \sum_j l_j = x \quad (1)$$

where  $x$  is size of the interbank market. It is assumed that the diagonal elements of  $X$  have to be zero. Therefore, the prior matrix of  $x^0$  is set as follows:

$$x_{ij}^0 = \begin{cases} 0 & \text{for } i = j \\ a_i * l_j & \text{for } i \neq j \end{cases} \quad (2)$$

However,  $x^0$  values violate the summing constraints expressed in Equation (1). The standard way in the literature to handle this problem is to determine an admissible matrix  $X$  that minimizes the Kullback-Leibler divergence with respect to some specified nonnegative prior matrix  $x^0$ . The Kullback-Leibler divergence for nonnegative but otherwise arbitrary  $X$  is given by

$$\min \sum_{i=1}^n \sum_{j=1}^n \frac{x_{ij}}{x} \log \left( \frac{x_{ij}}{x_{ij}^0} \right) \quad (3)$$

Where  $x^0$  is the sum of all entries in  $x^0$ . It is easy to verify that Equation (3) is also equivalent to Equation (4) as follows:

$$\min \sum_{i=1}^n \sum_{j=1}^n x_{ij} \log \left( \frac{x_{ij}}{x_{ij}^0} \right) \quad (4)$$

Therefore, the estimation of  $X$  conditional on the prior matrix  $x^0$  is given by the solution of

$$\min \sum_{i=1}^n \sum_{j=1}^n x_{ij} \log \left( \frac{x_{ij}}{x_{ij}^0} \right)$$

$$\text{Subject to } \sum_{i=1}^n x_{ij} = a_i, \quad \sum_{j=1}^n x_{ij} = l_j,$$

$$x_{ij} \geq 0 \quad (5)$$

where  $0 \log(0) = 0$ ;  $0 \log(0/0) = 0$ . An algorithm of realizing the theoretical estimation

of  $X$  is summarized as follows.

### 3.1.1. The algorithm for estimating the bilateral exposures matrix

Step 1: Start the iteration by setting  $x_{ij}^0 = a_i * l_j$  if  $i_1 \neq j$  otherwise  $x_{ij}^0 = 0$ .

Step 2: Take the rows constraint and set:

$$x_{ij}^1 = \frac{x_{ij}^0 a_i}{\sum_{i=1}^N \sum_{j=1}^N (a_i * l_j)} \text{ for all } i \in 1, \dots, N. \quad (6)$$

Step 3: Take the columns constraint and set:

$$x_{ij}^2 = \frac{x_{ij}^1 l_j}{\sum_{i=1}^N \sum_{j=1}^N (a_i * l_j)} \text{ for all } j \in 1, \dots, N. \quad (7)$$

The  $K$  iteration runs across the rows and columns constraints show that:

$$x_{ij}^k = \frac{x_{ij}^{k-1} a_i}{\sum_{i=1}^N \sum_{j=1}^N (a_i * l_j)} \text{ for all } i \in 1, \dots, N \quad (8)$$

$$x_{ij}^{k+1} = \frac{x_{ij}^k l_j}{\sum_{i=1}^N \sum_{j=1}^N (a_i * l_j)} \text{ for all } j \in 1, \dots, N \quad (9)$$

The iteration is stopped as soon as some distance measure, e.g., the Euclidean distance, between  $x_{ij}^{k+1}$  and  $x_{ij}^{k-1}$  is smaller than a prespecified  $\varepsilon > 0$ .

### 3.2. Estimation of the time evolution of $v_i$

Asset value is not daily observable. However, asset value can be collected in the bank balance sheet at the end of each year, while the equity market price of banks can be observed by stock price on each day. The time  $t$  is measured in units of day. This paper also gives a method to estimate asset values of each day (time evolution of asset value) according to the equity market data of banks. Assume that the asset value  $v_i$  of bank  $i$  follow a geometric Brownian motion with drift  $\mu_i$  and volatility  $\sigma_i$ :

$$dV_i = \mu_i V_i dt + \sigma_i V_i dz \quad (10)$$

Then equity  $E_i(t)$  can be seen as a call option on the assets of bank  $i$  with a strike price equal to the future notional value of bank  $i$ 's debt  $D_i(t)$ , which is assumed to have a maturity of  $t$ . Then the value of bank equity is given by the Black-Scholes model as follows:

$$E_i(t) = v_i(t)N(d_t) - D_i(t)N(d_t - \sigma_i \sqrt{T}) \quad (11)$$

where  $T = 365$  days,  $t$  represents the evolution of days and

$$d_t = \frac{\ln(V_i(t)/D_i(t)) + (\sigma_i^2/2)T}{\sigma_i\sqrt{T}} \quad (12)$$

In the stock market one can observe a time series of equity prices  $E_i(t)$  and read the face value of bank debt  $D_i(0)$  from the balance sheet. All bank debt is assumed to be insured and will therefore grow at the risk-free rate  $r$ . Then  $D_i(t) = D_i(0)e^{rt}$ . Given the initial data of  $v_i(0)$ , and time series data of  $E_i(0), E_i(1), \dots, E_i(T), D_i(0), D_i(1), \dots, D_i(T)$ , and the arbitrary initial value of  $\mu_i(0), \sigma_i(0)$ , it is easy to get the estimation of  $\hat{V}_i(1), \hat{V}_i(2), \dots, \hat{V}_i(T)$  according to the Equation (11). Then the following maximization likelihood function is used to estimate the parameters  $\mu_i$  and  $\sigma_i$ , which is proposed by Duan et al. (2005)

$$L(U_i, \sigma_i; \hat{V}_i(1), \hat{V}_i(2), \dots, \hat{V}_i(T)) = \frac{T}{2} \ln(2\pi\sigma_i^2 h) - \frac{T}{2} \sum_{k=1}^T \frac{(R_i(k) - (u_i - \frac{\sigma_i^2}{2})h)^2}{\sigma_i^2 h} - \sum_{k=1}^T \ln \hat{V}_i \quad (13)$$

where  $R_i(k) = \ln(\hat{V}_i(t)/\hat{V}_i(t-1))$ ,  $h = 1/365$

After getting the estimation value of  $\mu_i$  and  $\sigma_i$  the evolution of  $v_i(t)$  can be estimated as follows:

$$V_i(t) = v_i(0)e^{u_i - \left(\frac{\sigma_i^2}{2}\right)th + \sigma_i \sqrt{th} z_i(t)} \quad (14)$$

where  $z_i(t)$  obeys normal distribution  $(N(0, 1))$ .

To test the stability of the bank network system, a systemic shock is added to the banking system. If the banks withstand a strong shock, then it can be said that the system is stable. If the banking system collapses when a weak shock applies, then it can be concluded that the bank system is unstable. Thus, this research applies a medium shock to count the numbers of banks which undergoes bankruptcy. If the probabilities of the banks which undergo bankruptcy are large for a medium shock, then the bank system is unstable. The shock is added to the system by replacing  $z_i(t)$  in Equation (14) with  $(1 - \xi) z_i(t) + \xi * \omega(t)$  where  $\xi$  represents the strength of the systemic shock and  $\omega(t)$  is the systemic shock which is the same for all banks,  $\omega(t)$  follows the normal distribution  $(N(0;1))$  where  $\xi = 0.1$ . Thus the evolution of  $V_i(t)$  can be estimated as follows:

$$V_i(t) = v_i(0)e^{u_i - \left(\frac{\sigma_i^2}{2}\right)th + \sigma_i \sqrt{th} [(1-\xi) z_i(t) + \xi * \omega(t)]} \quad (15)$$

### 3.3. The basic default of banks

The basic default of bank  $i$  occur due to its insolvent situation, i.e.

$$v_i(t) + a_i(t) - D_i(t) - l_i(t) < 0 \quad (16)$$

where  $v_i(t)$  is calculated by Equation (14), and  $D_i(t)$  can be described as follows:

$$D_i(t) = D_i(0)e^{rt} \quad (17)$$

where  $r$  is the risk-free rate for each day,  $a_i(t)$  and  $l_i(t)$  will be described in Section 3.4.

### 3.4. The contagion default of banks

The contagion default of banks occurs due to the interbank market. Here, the paper presents a framework to calculate the contagion default of banks based on the clearing payment mechanism proposed by Eisenberg and Noe (2001). The paper extends the clearing payment mechanism of Eisenberg and Noe (2001) to suit the calculation of the time evolution of the contagion default (time evolution of systemic risk). In the present paper, the clearing payment mechanism simultaneously solves the interbank payment amounts of all the banks on daily basis. The banking system is represented as  $(X(t); e(t))$ , where  $X(t)$  is a  $(N \times N)$  bilateral exposures matrix, and  $e(t) = v_i(t) - D_i(t)$  as shown in Figure 1. This study defines a new matrix  $\Pi(t)$  that is derived from  $X(t)$  by normalizing the entries by total interbank liabilities:

$$\Pi_{ij}(t) = \begin{cases} x_{ji}(t)/l_i(t) & \text{if } l_i(t) > 0, \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

where  $l_i(t) = \sum_j x_{ji}(t)$ . The present paper describes the banking system as a tuple  $(\Pi(t); e(t); x(t))$  for which a clearing payment vector  $p^*(t)$  is defined. The clearing payment vector has to respect limited liability of banks and proportional sharing in case of default. **Thus, the clearing payment vector denotes the total payments made by the banks under the clearing mechanism.** It is defined by

$$p_i^*(t) = \begin{cases} l_i(t) & \text{if } \sum_{j=1}^N \Pi_{ji}(t) p_j^*(t) + e_i(t) \geq l_i(t), \\ \sum_{j=1}^N \Pi_{ji}(t) p_j^*(t) + e_i(t) & \text{if } \sum_{j=1}^N \Pi_{ji}(t) p_j^*(t) + e_i(t) > \sum_{j=1}^N \Pi_{ji}(t) p_j^*(t) + e_i(t) \geq 0 \\ 0 & \text{if } \sum_{j=1}^N \Pi_{ji}(t) p_j^*(t) + e_i(t) < 0. \end{cases} \quad (19)$$

The study adopts the default algorithm developed by Eisenberg and Noe (2001) to find a clearing payment vector. They proved that under mild regularity conditions, a unique clearing payment vector always exists for  $(\Pi(t); e(t); x(t))$ . These results apply to the multiperiod setting.

#### 3.4.1. Contagious default

Although Bank  $i$  does not default according to Equation (16), it may default when other banks are not able to keep their promises. In other words, contagious default of bank  $i$  occurs if

$$\sum_{j=1}^N \Pi_{ji}(t) p_j^*(t) + e_i(t) - l_i(t) < 0 \quad (20)$$

The number of contagious defaults of banks can be calculated by Equation (20). At each time step  $t$ , the detailed algorithm of calculating the contagious defaults of banks can be referred to Eisenberg and Noe (2001).

#### 3.4.2. The evolution of the bilateral exposures $X(t+1)$ , $a_i(t+1)$ , and $l_i(t+1)$

After calculating a clearing payment vector according to the algorithm in Eisenberg and Noe (2001) at time step  $t$ , the new matrix of  $X$  at the time step  $t+1$  is calculated. The following is the algorithm of calculating the evolution of the matrix  $X$ .



Step 1:  $i = 1$ . At time step  $t$ , calculate the net claim value of bank  $i$  as  $\Delta v_i(t) = \sum_{j=1}^N \Pi_{ji}(t) p_j^*(t) + e_i(t) - l_i(t)$ . If  $\Delta v_i(t) > 0$  (the clearing payment vector of bank  $i$  will be  $p_i^*(t) = l_i(t)$  in this case), let  $\chi_i = 1$  and go to Step 3. Otherwise, go to Step 2.

Step 2: Contagion default of bank  $i$  occurs when  $\Delta v_i(t) < 0$ , so bank  $i$  can pay only a part of the liabilities to other banks. The ratio is defined as follows:

$$\chi_i = \frac{\sum_{j=1}^N \Pi_{ji}(t) p_j^*(t) + e_i(t)}{l_i} \quad (21)$$

Step 3: Update the assets values and liability values of bank  $j$  from time step  $t + 1$  to  $T$  as follows:

$$V_j(t + 1 : T) = V_j(t) - \chi_i * x_{ij}$$

$$D_j(t + 1 : T) = D_j(t) - \chi_i * x_{ij} \quad (22)$$

Step 4: If  $\Delta v_i(t) < 0$ , then we clear out bank  $i$  from the network bank system,

$$x_{j,i}(t) = 0$$

$$x_{i,j}(t) = 0 \quad (23)$$

Step 5: Let  $i = i + 1$ . Go to step 2.

Step 6: Recalculate the bilateral exposures matrix  $x_{t+1}$  according to the algorithm in Section 3.1.1. Therefore, the evolution of  $a_i(t + 1)$ , and  $l_j(t + 1)$  is described as:

$$a_i(t + 1) = \sum_{j=1}^N x_{ij}(t + 1) \quad (24)$$

$$l_j(t + 1) = \sum_{i=1}^N x_{ij}(t + 1).$$

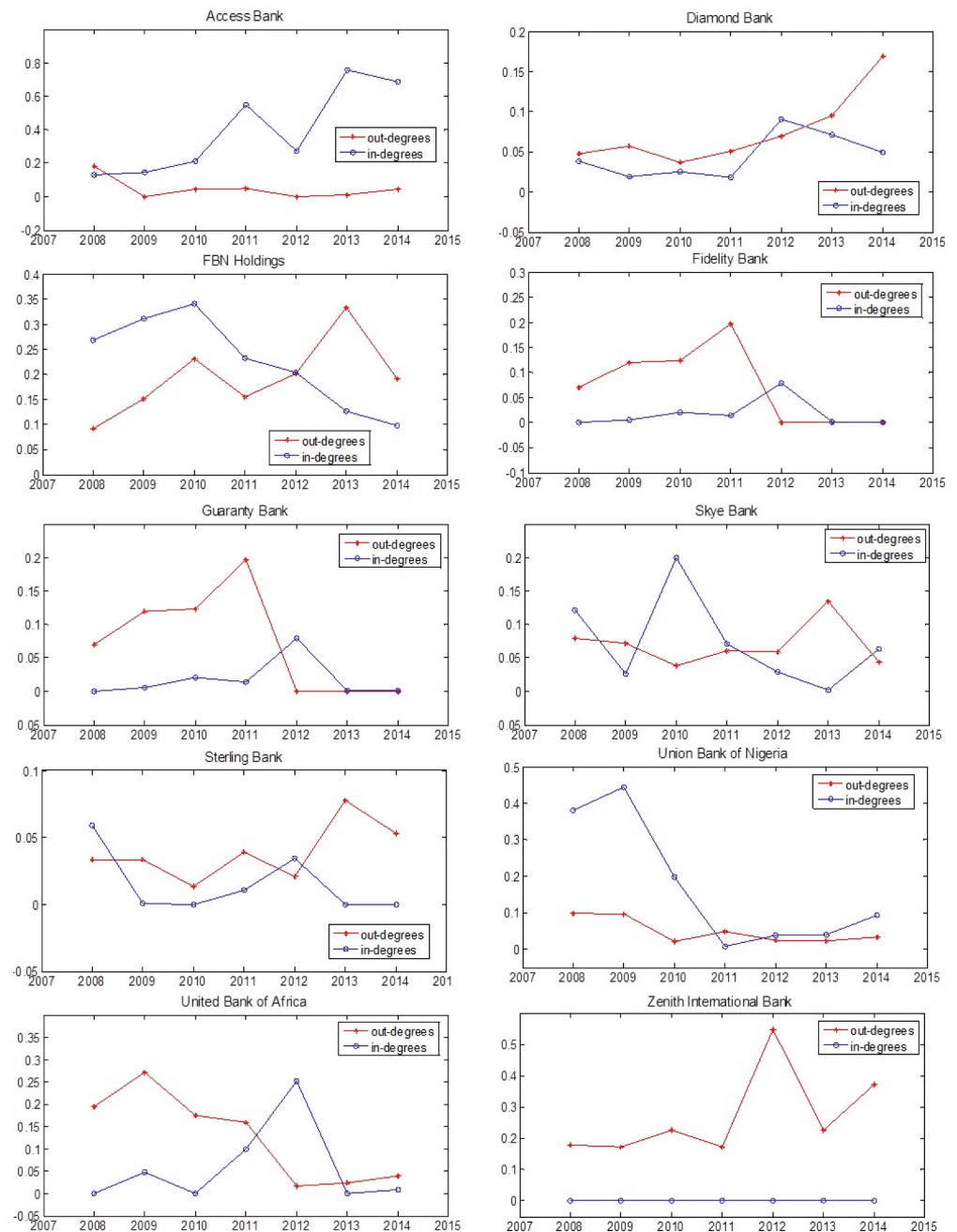
### 3.4.3. The measure of the stability of the bank network system

The probability of the basic default of bank  $i$ ,  $Pb_i$  is calculated as:

$$Pb_i = \frac{Nb_i}{N_s} \quad (25)$$

where  $Nb_i$  is the number of runs of the simulation during which basic default of bank  $i$  occurs and  $N_s$  is the total number of runs of the simulation. Similarly, the calculation of the probability of the contagious default of bank  $i$ ,  $Pc_i$ , is calculated as:

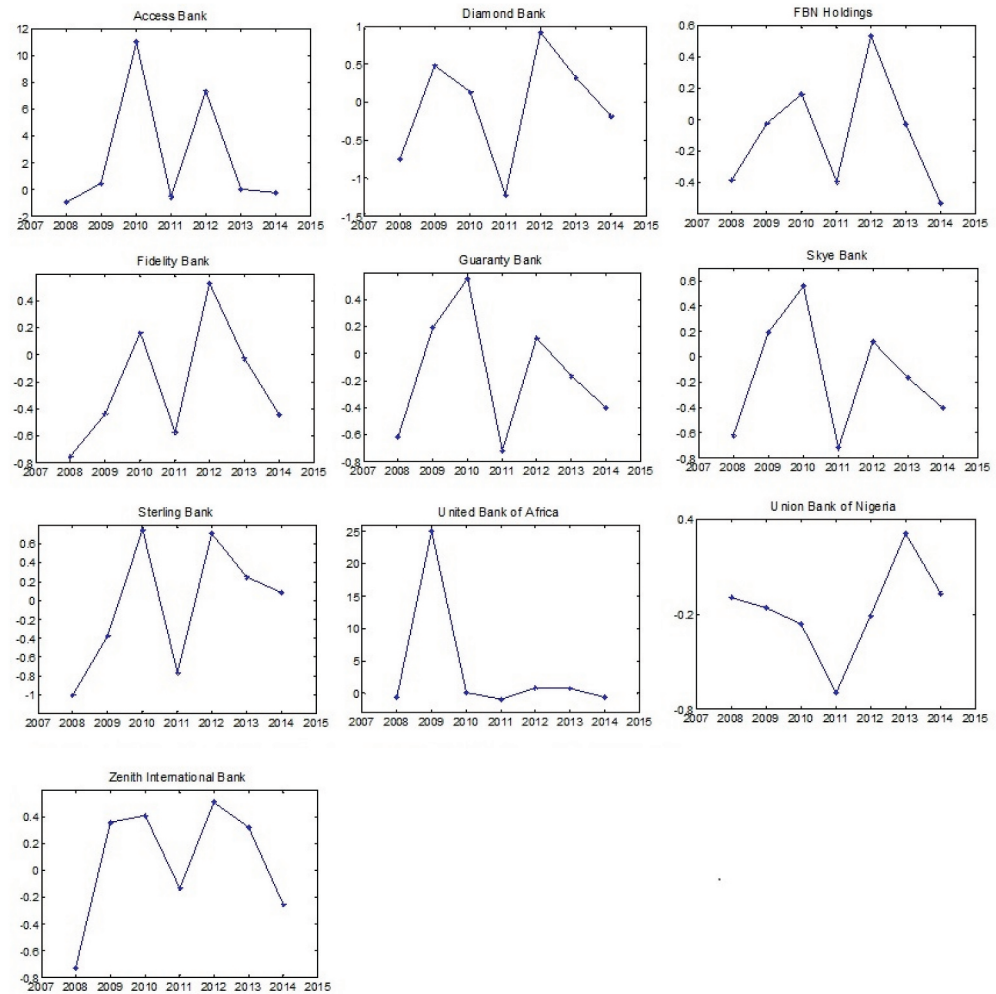
**Figure 3. In-degree and out-degree for each bank (the vertical shows the percentage of the amount of money borrowed or lent by a bank while horizontal axis represents time in years).**



$$PC_i = \frac{Nc_i}{N_s} \quad (26)$$

where  $Nc_i$  is the number of runs of the simulation during which contagious default of bank occurs. The stability of the bank system  $S_i$  is defined as:

**Figure4. Variations of the drift rate (Vertical axis represents drift rate while horizontal axis represents time.**



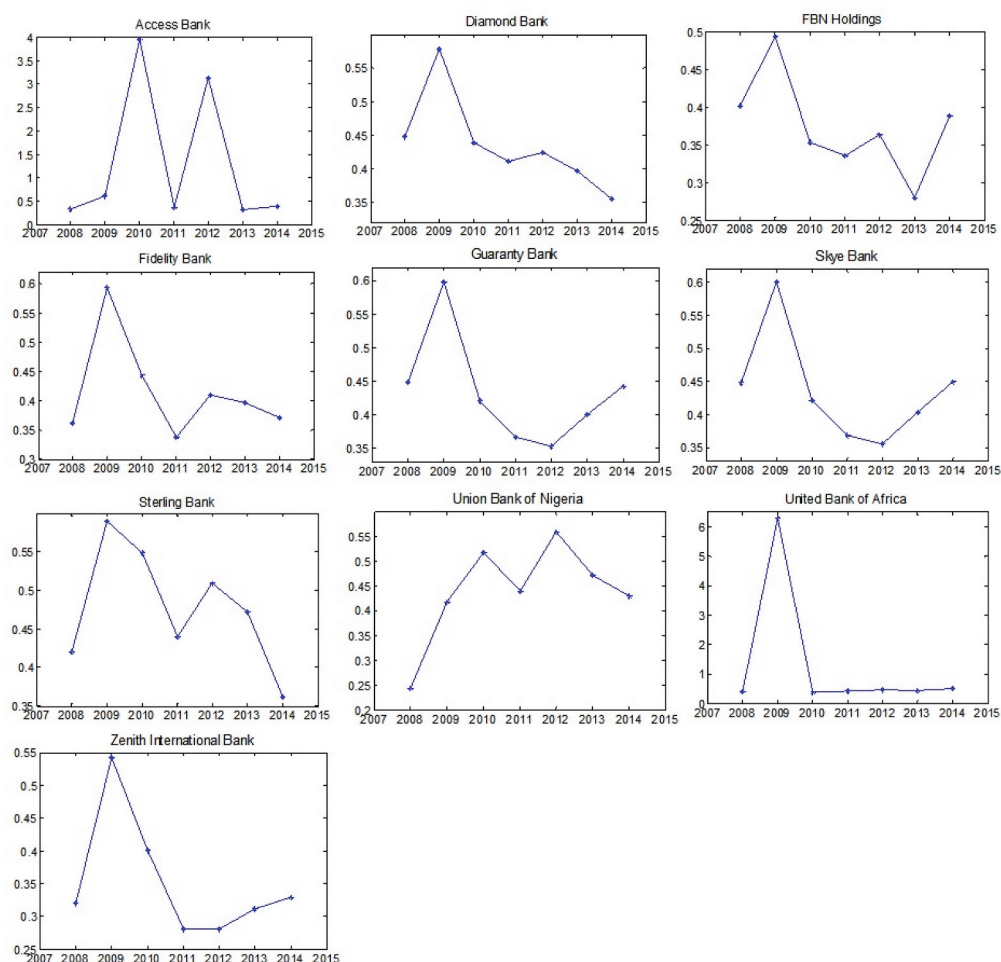
$$S_t = 1 - \frac{\sum_i P b_i + \sum_i P c_i}{N} \quad (27)$$

Where  $N$  is the number of banks in the banking system.

#### 4. Data

In this study the financial data of Nigerian banks are collected from African markets database. The study involves 10 major commercial banks out of 15 publicly traded commercial banks in Nigeria from 2008–2014, namely Access bank, Diamond bank, FBN Holdings, Fidelity bank, Guaranty bank, Skye bank, Sterling bank, Union bank of Africa, United bank of Africa, and Zenith international bank. The sample of 10 banks is selected based on banks with large assets value (NSE). The study concentrates on bank accounts and not on group accounts and thus collect data on bank total assets, total liabilities, interbank lending and interbank borrowing. As the need is to estimate the market value of the assets from equity data, only publicly traded banks are considered. The interest rates of country during the period of 2008–2014 are collected from the Central bank of Nigeria and run simulations based on the collected data.

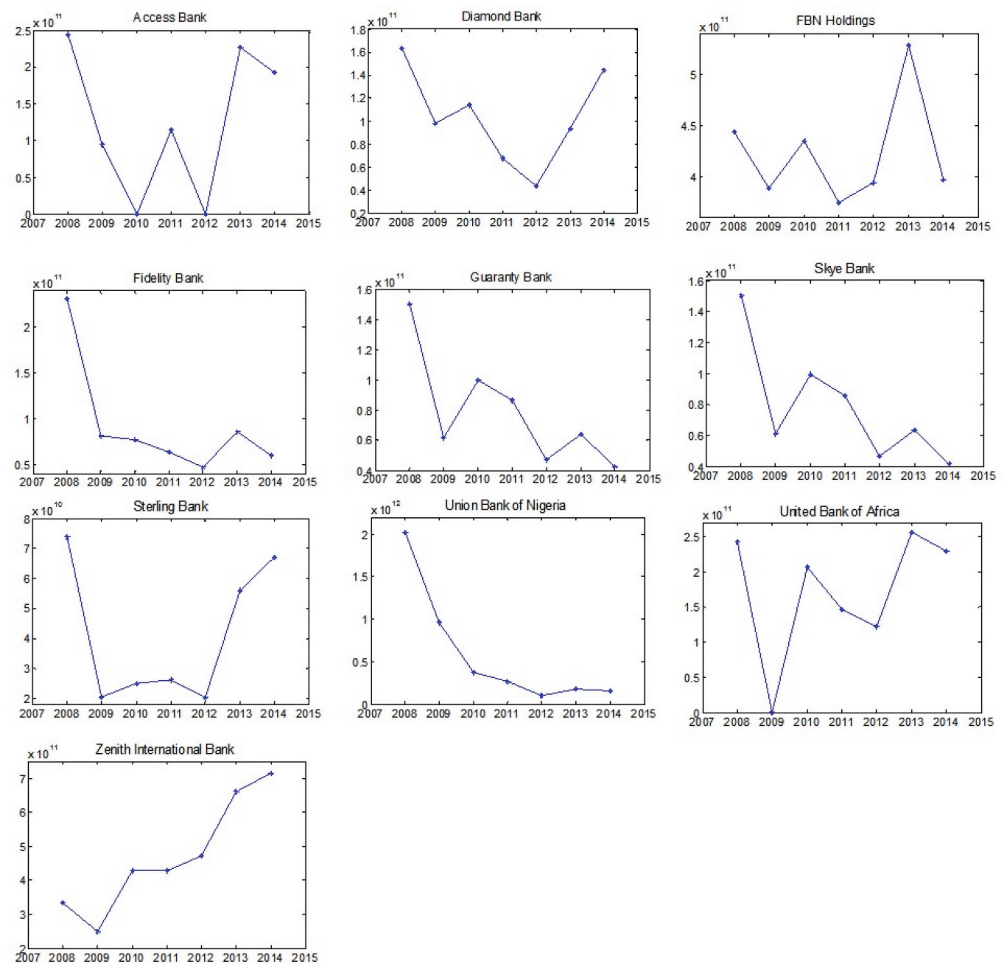
**Figure 5. Variations in the volatility of the assets value.**



**Table 1. The stability of the Nigerian network system; the total number of runs of the simulation  $N_s = 10,000$**

	2008	2009	2010	2011	2012	2013	2014
Probability of BD	1	0.5083	0.1006	1	0.1003	0.4064	0.9572
Probability of CD	0	0	0.1001	0	0	0	0.026
Stability of the system	0	0.4917	0.7993	0	0.8997	0.5936	0.0168

**Figure 6. Variations in the asset values for selected banks.**



## 5. Results

### 5.1. Network structure and estimation of bilateral exposures matrix

The bilateral exposures matrix  $X$  is estimated as stated in Equation (1) and use the matrix to examine the network structure of the Nigerian banking system. For this case, the Nigerian inter-bank network is analyzed using the network centrality measures. This paper uses degree measures to obtain information on the roles played by the market participants. Usually the degree of a node is considered as a proxy variable for interconnectedness and explains the number of edges connected to a node.

The in-degree shows the amount of money going toward (borrowed by) a bank (corresponding to  $a_i$  in Equation (1). For simplicity, this study uses the ratio of the money borrowed by the bank to the total money borrowed by all banks to measure in-degree of bank  $i$ , i.e. the in-degree of bank  $i$  is  $a_i / \sum_i a_i$ . Similarly, the out-degree of bank  $i$  is  $l_i / \sum_i l_i$ . The total degree of a bank is the summation of its in-degree and out-degree. These measures, hence give a sense of investment and funding diversifications. Figure 3 describes the ratio of borrowing and lending for each bank as expressed

**Table 2. The evolution of bank defaults in years 2009–2014**

2009	Feb-05	Feb-14	Mar-04	May-06	
<b>Defaults</b>	s8	s7	s4	s3	
<b>2010</b>	Jan-01				
<b>Defaults</b>	s8				
<b>2011</b>	Jan-20	Jan-22	Jan-24	Jan-30	Feb-08
<b>Defaults</b>	s9	s7	s2	s6	s5
	Feb-25	Feb-28	Mar-08	Mar-13	May-30
	S8	S4	S1	S3	S10
<b>2012</b>	May-04				
<b>Defaults</b>	s8				
<b>2013</b>	Mar-20	Jun-19	Aug-02	Aug-23	
<b>Defaults</b>	s6	s5	s4	s3	
<b>2014</b>	Feb-16	Feb-18	Feb-20	Mar-05	Mar-18
<b>Defaults</b>	s9	s6	s3	s4	s1
	Mar-21	Apr-03	Apr-14	Sep-15	
	S5	S10	S2	S8	

This paper uses s1, s2, s3, s4, s5, s6, s7, s8, s9, and s10 to represent Access bank, Diamond bank, FBNH, Fidelity bank, Guaranty bank, Skye bank, Sterling bank, Union bank of Nigeria, United bank of Africa, and Zenith International bank respectively.

by the in-degree and out-degree relationship. Results depict that there are banks that borrow more than lending while others lend more than borrowing. Banks like Access bank, FBH, Union bank of Nigeria, and Skye bank borrow more than they lend to other banks by average of 43.1 %, 22%, 18.4% and 7.7% respectively from 2008 to 2014. Banks like Zenith International Bank, United Bank of Africa, Fidelity Bank, Guaranty bank, Diamond bank, and Sterling bank lend more than they borrow by the average of 28.4%, 17.2%, 8.6%, 8.4%, 8.3%, and 4.3%, respectively, from 2008 to 2004. Thus, banks which borrow (lend) more than they lend (borrow) in the Nigerian banking system in terms of percentage are examined as follows. The list of banks that borrow in order of preference in the average of seven years includes Access 43.1%, FBH 22%, Union bank of Nigeria 18.4 %, Skye bank 7.7 %, United Bank of Africa 6.1%, Diamond Bank 5.71%, Sterling bank 2.1%, Guaranty bank 1.9%, Fidelity bank 1.7%, and Zenith International bank 0% while banks that funds in order of preference includes Zenith International Bank 28.4%, FBN 20%, United bank of Africa 17.2%, Fidelity 8.6%, Guaranty bank 8.4%, Diamond bank 8.3%, Skye bank 7.3%, Access bank 6.6%, Union Bank of Nigeria 5.3%, and Sterling bank 4.3 %.

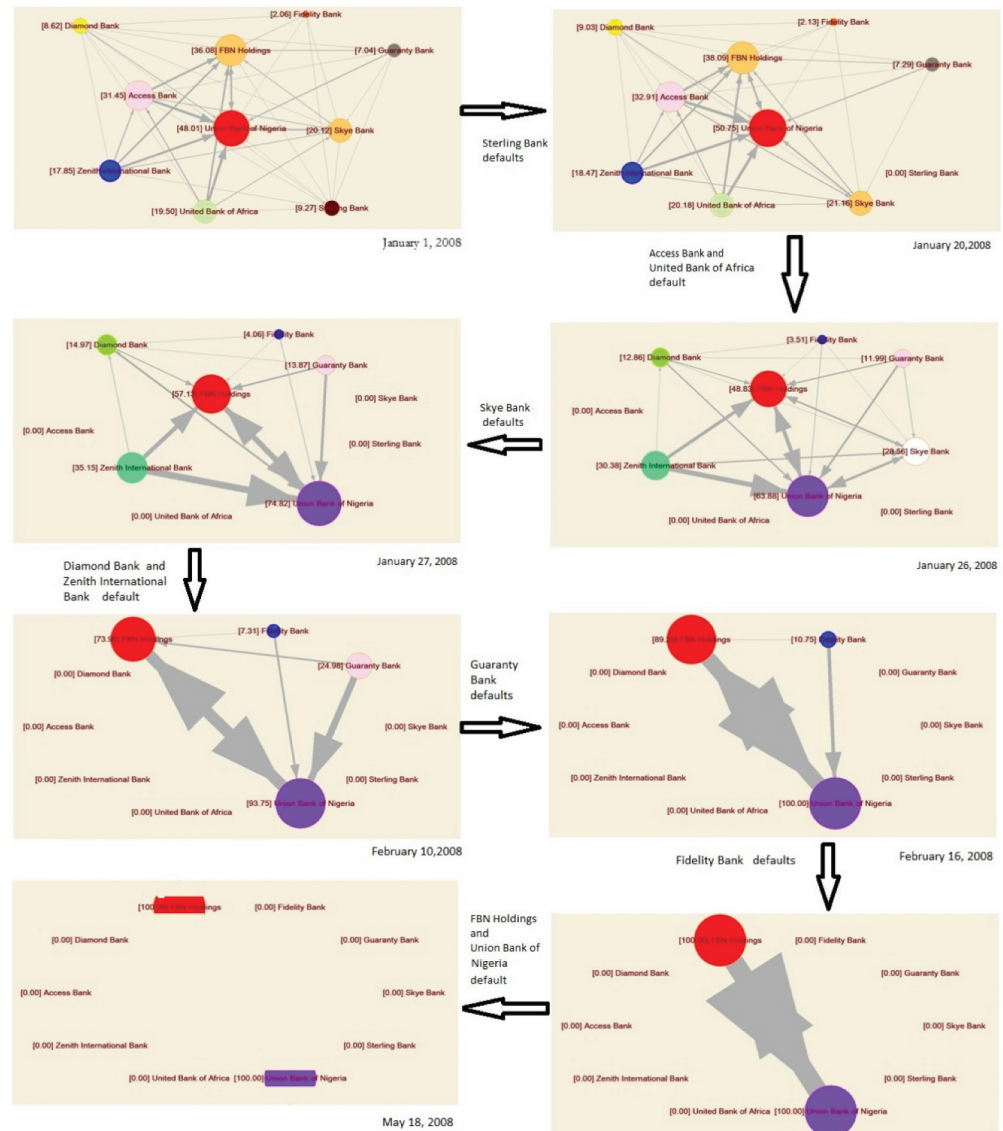
## 5.2. Estimation of market variables

This study estimates the market values of assets and the drifts and volatilities of the assets returns for each bank using the maximum likelihood method as explained earlier.

Drift rate indicates the development of the bank. If the drift rate is greater than 0, the trend of the development of the bank is upward and vice versa. The volatility of the assets value of the bank affects the stability of the bank, if the volatility is small then the development of the bank is more stable compared to when volatility is high. Figure 4 shows the variations of the drift rate. It depicts the variations of the drift rate for the ten banks involved in the study. The general findings show that the banks with the larger drift rate are Access bank, Diamond Bank, United Bank of Africa, Union Bank of Nigeria, Sterling Bank, and Zenith International bank. This implies that these banks are developing better than others. The banks with lower drift rates are FBN Holdings, Fidelity Bank,



**Figure 7. The evolution of the Nigerian network system in 2008.**



Guaranty bank, and Skye bank, indicating that the development trends of these banks are weaker than the earlier ones. Figure 5 portrays volatility of the bank assets and hence reveals the stability of the bank. The banks with the larger volatility comprise United Bank for Africa and Access Bank; this suggests that these banks may be more unstable compared to others. Banks with lower volatility include Diamond bank, FBN Holdings, Fidelity, Guaranty, Skye bank, Sterling bank, Union bank of Nigeria, and Zenith International bank; these banks are more stable.

Assets value implies the price an asset would fetch in the marketplace. Market value is also commonly used to refer to the market capitalization of a publicly traded company. Market value can fluctuate a great deal over periods of time, and is substantially influenced by the business cycle. Market values plunge during the bear markets that accompany recessions, and rise during the bull markets that are a feature of economic expansion. Figure 6 portrays the market values of



the selected banks which shows some banks increasing their market value over time like Zenith International Bank, Sterling bank, Diamond bank, Access bank, and United bank of Africa, while FBN Holdings, Fidelity bank, Guaranty bank, and Skye bank suffer a tremendous decrease of their market value.

### 5.3. The stability of the Nigeria network system

In order to examine the stability of the Nigerian network system data were simulated as shown in Table 1. The findings show that there were seldom contagious defaults in Nigeria, basic defaults seemed to appear. The results also show that the Nigeria bank network system is more stable in years 2010 and 2012 than in other years, while in years 2008, 2011, and 2014, the Nigeria bank network system is seriously unstable.

#### 5.3.1. The evolution of the Nigerian network system

In this subsection, the evolution of the Nigerian network system is analyzed. From Table 1, it is shown that basic defaults of all the banks occurred in 2008, which means a very high systemic risk (It is worth noting that theoretical defaults of these banks do not mean that these banks really went bankrupt, because in reality, banks can go on operating even when its liability is larger than its assets). However, the need is to know the evolution of the Nigerian network system in 2008, so the evolution of the Nigeria network system for one run of the simulation is analyzed, which is shown in Figure 7. It is found that the first bank to default is Sterling bank, although its connectivity is very small. The connectivity is the main reason for contagion. Therefore, small connectivity does not prevent the default of Sterling bank. Then, what caused Sterling bank to default firstly? It is found that from Figure 4 the drift rate of Sterling bank is the largest negative and from Figure 5 the volatility was increasing in 2008, which caused the evolution of assets of Sterling bank to gradually decrease as shown in Figure 6. Therefore, the reason of basic defaults for the Nigerian network system mainly is caused by the drift rate, the volatility, and the evolution of assets.

In 2008, the drift values of most of banks in Nigeria are negative; the volatility of most banks is increasing, and the evolution of assets of most banks also gradually decreasing, which answers the reason of the occurrence of the basic defaults of all the banks in 2008. Figure 7 shows the default sequence of banks in the Nigerian network system while Table 2 shows the evolution of default banks in other years. For individual bank, Union Bank of Nigeria is most unstable bank during the time. However, in 2008 and 2014, Union Bank of Nigeria is more stable than other years. In addition, it is found that its assets achieve the largest value among all the banks in 2008, but gradually decrease from 2008 to 2014 as shown in Figure 6. Until 2013, its assets remained stable. Therefore, in 2014, Union Bank of Nigeria improves its stability largely. Between years 2009 and 2014, Zenith International bank was the most stable bank among all banks; its assets value is gradually increasing year by year between 2009 and 2014 (see Figure 6), which is due to its positive drift rate.

The size of each node represents the total percentage of in-degree and out-degree of each bank and the number of size is marked beside each node.

### 6. Conclusions

This paper proposed a new theoretical framework to reveal the time evolution of the systemic risk by calculating the number of defaults of banks using sequences of daily financial data. The framework combines the asset value estimation algorithm and obligation clearing algorithm to calculate time evolution of the systemic risk. The asset value estimation algorithm was used to estimate the asset values of the banks at each day which are used to compute time evolution of systemic risk. The obligation clearing algorithm was used to calculate the systemic risk given the tuples of data each day.

The theoretical framework proposed in this paper was used to analyze the Nigerian bank system. This paper examined the network structure of the Nigerian interbank market using centrality measures. Besides, it assessed the interconnectedness of each bank in the interbank market using the in-degree and out-degree measures, which helped to identify the most important banks in the Nigeria banking system. The paper also examined the contagious defaults in the Nigeria banking system and theoretically analyzed the mechanisms of contagious defaults conditional on the basic defaults. In contrast to the Western banks, the findings reveal the occurrence of basic defaults while contagious defaults are seldom observable; the reason for this is that the Africa's stock market capitalization is still very low compared to world capitalization, furthermore African banking assets represent only a small portion of global banking assets.

It was also found that the Nigerian bank network system is more stable in years 2010 and 2012 than in other years, while in years 2008, 2011, and 2014, the Nigerian bank network system is seriously unstable. For individual bank, the Union Bank of Nigeria is most unstable, however, in 2014; the Union Bank of Nigeria improves its stability largely. Between years 2009 and 2014, the Zenith International bank is the most stable bank among all banks.

The method of estimating bilateral exposure matrix in this paper assumes that the topology of the interbank network is complete network and does not reproduce incomplete interbank market. Depending on the actual network structure this may negatively or positively bias the results. In future, one can adopt both the maximum entropy estimation method, Upper (2011) and minimum density approach, Anand et al. (2015) to estimate the bilateral exposure matrix, and then analyze the systemic risk in financial networks.

#### Acknowledgments

The author is very thankful for the comments of the Editor and the anonymous reviewers which significantly improved this work.

#### Funding

This research did not receive any specific grant from funding agencies.

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#### Disclosure statement

No potential conflict of interest was reported by the author(s).

#### Citation information

Cite this article as: A theoretical framework for simulating systemic risk and its application to analysis of the banking system, Chirongo Moses Keregero, *Cogent Economics & Finance* (2021), 9: 1986930.

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