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Article

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FINANCIAL ECONOMICS | RESEARCH ARTICLE

Nonparametric performance hypothesis testing with the information ratio

Jacque Bon-Isaac Aboy^{1*} and Joselito Magadia²

Abstract: This study proposes a nonparametric bootstrap-based test to compare performances between two portfolios in terms of their information ratio. This serves as an extension to the literature that tests performance between two portfolio investment strategies that uses Sharpe ratio. Monte Carlo experiments show that the test has appropriate sizes and is powerful to most of the scenarios. However, the test does not perform well in highly correlated portfolio returns, but is better when the mean of portfolio return is modeled using an autocorrelated process.

Subjects: Statistics & Computing; Statistics for Business, Finance & Economics; Investment & Securities

Keywords: Information ratio; bootstrap test

1. Introduction

A fund manager or an investor should assess how the portfolio performs in a time period through portfolio performance assessment. There are two major categories of portfolio assessment; the conventional and risk-adjusted methods. Conventional methods simply compare the return of the managed portfolio to the return of the benchmark portfolio. Risk-adjusted methods, on the other hand, calibrate returns in such a way that differences in risk levels between the managed portfolio and benchmark portfolio are taken into account. Some of the traditional risk-adjusted performance measures were influenced by the Capital Asset Pricing Model of Sharpe (1964).

ABOUT THE AUTHOR



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PUBLIC INTEREST STATEMENT

Nowadays, it is highly recommended to invest instead of keeping assets idle. It could be done through portfolio investments that may be done individually or through a fund manager managing a mutual fund. However, an individual investor or a fund manager should always assess the performance of the portfolio being managed as compared to a benchmark portfolio and one way is by calculating portfolio's information ratio. Information ratio measures and compares the active return of an investment compared to a benchmark relative to the volatility of the active return. This study is one of the many answers to the question of how a portfolio compares to another portfolio, by testing whether a portfolio has a statistically significant higher information ratio than the other. The difference is that this study does not assume anything about the nature of the portfolio returns such as normality and constant variance.

For this study, we will use a risk-adjusted performance measure, in particular, the information ratio. It is one of the popular measures being used by investors to assess performance of their portfolio as it is more advanced, because it measures the fund's performance relative to its benchmark and adjusts it for the volatility in the dispersion between the two, compared to Sharpe ratio. It is defined to be the active return of the fund divided by its tracking error, where active return is the difference between the fund's return and that of its benchmark index, and tracking error is the standard deviation of the active return. Rather than take the ratio of the expected difference between the return of managed portfolio and market to the risk of the portfolio, we can also take the ratio of the expected difference to the standard deviation of the difference. According to Kidd (2011), the information ratio tells an investor how much excess return is generated from the amount of excess risk taken relative to the benchmark. The ratio is given by

$$\zeta_p = \frac{E(R_p - R_m)}{\sigma_{p-m}}, \quad (1)$$

where $\sigma_{p-m} := [\text{Var}(R_p - R_m)]^{\frac{1}{2}}$.

This measure is often referred to be a generalized version of Sharpe ratio (generalized Sharpe ratio). There are advantages in using information ratio as compared to Sharpe ratio because information ratio measures extra return an investor or fund manager can obtain from security analysis compared to the firm-specific risk and is best used for measuring active managers against a passive benchmark portfolio. Moreover, according to a report from Dhanorkar (2016), the Sharpe ratio simply tells an investor how much he or she was compensated for taking risks, while the information ratio tells the investor the rewards the fund manager generated by deviating from the benchmark, and also, for those tracking fund manager performance and alpha generation, information ratio gives better insight, provided the investor used the right benchmark or knows how to compare the right funds.

A study of Goodwin (1998) provided a narrative regarding the information ratio in terms of how it should be correctly interpreted and what constitutes a "good" information ratio. He argued that the simplest form and interpretation of the information ratio is the most useful for investors. Furthermore, the study clarified the relationship between the information ratio and a *t*-statistic. Also, numerous applications of information ratio are also seen across literature. For instance in the study of Gupta et al. (1999), they have found that it is more difficult for domestic asset class managers (small-cap is an exception) to outperform their respective benchmarks than it is for international asset class managers. Hence, it is the information ratio—not the alpha and/or tracking error—that is the strongest predictor of persistence of manager performance.

2. The problem

Suppose we have two portfolio investment strategies *p* and *q* whose excess returns over a benchmark at time *t* are given by r_{tp} and r_{tq} , respectively. Risk-free rate as a benchmark is typically used, but for this study, we strictly use a stock index. Then we observe a total of *T* return pairs $(r_{1p}, r_{1q})', \dots, (r_{Tp}, r_{Tq})'$. Here we assume that these observed pairs follow a stationary time series, in other words, the bivariate distribution, denoted by *f*, is constant over time. In particular, the distribution has mean vector μ and covariance matrix Σ , that is,

$$\begin{bmatrix} r_{tp} & r_{tq} \end{bmatrix}' \sim f(\mu, \Sigma)$$

where

$$\mu = \begin{bmatrix} \mu_p \\ \mu_q \end{bmatrix} \quad (2.1)$$

and

$$\Sigma = \begin{bmatrix} \sigma_p^2 & \rho\sigma_p\sigma_q \\ \rho\sigma_p\sigma_q & \sigma_q^2 \end{bmatrix}. \quad (2.2)$$

From the observed samples of f , sample means and sample variances will be denoted by $\hat{\mu}_p, \hat{\mu}_q$ and $\hat{\sigma}_p^2, \hat{\sigma}_q^2$, respectively. Note that μ and σ^2 are the average and variance of excess returns. Then the difference between two information ratios is given by

$$\Delta = \zeta_p - \zeta_q = \frac{\mu_p}{\sigma_p} - \frac{\mu_q}{\sigma_q} \quad (2.3)$$

and its estimator is

$$\hat{\Delta} = \hat{\zeta}_p - \hat{\zeta}_q = \frac{\hat{\mu}_p}{\hat{\sigma}_p} - \frac{\hat{\mu}_q}{\hat{\sigma}_q}. \quad (2.4)$$

We want to test $H_0 : \Delta = 0$, that is, we want to test the null hypothesis that the difference of two information ratios between two portfolio investment strategies is zero, that the performance between the two is not different at all.

According to Goodwin (1998),

$$\sqrt{T} \cdot \hat{\Delta} \stackrel{d}{\sim} t_{T-1}$$

where $T - 1$ is the degrees of freedom. However, the test statistic $\sqrt{T} \cdot \hat{\Delta}$ may not be robust to returns that are not necessarily normally distributed or if the observations are correlated over time. This study therefore will cover other cases where portfolio returns are not the typical normally distributed portfolio returns.

The goal now is to derive a test of comparison between two portfolio investment strategies in terms of their respective information ratios. This paradigm of research is no longer new. For instance, the works of Jobson & Korkie (1981), Lo (2002), and Memmel (2003) are influential in the pursuit of portfolio performance testing based from performance measures, in particular, the Sharpe ratio. They provided a formal statistical comparison between the Sharpe ratios of two portfolios. Then, a robust performance hypothesis testing with the Sharpe ratio was formulated by Ledoit & Wolf (2008). The test took into consideration returns having tails heavier than the normal distribution or are of time-series nature, which has been overlooked by the previous studies. In particular, the study constructed a studentized time-series bootstrap confidence interval for the difference of the Sharpe ratios and to declare the two ratios different if zero is not contained in the obtained interval. As a result, the study has demonstrated the improved finite sample performance compared to existing methods. This study has influenced Auer & Schuhmacher (2013) in the case of hedge funds.

On the other hand, a recent study of Kazak & Pohlmeier (2019) performed one-sided and two-sided tests on out-of-sample performance of portfolios constructed naïvely and through global minimum variance in terms of their Sharpe ratio and certainty equivalent. The methods used were delta and bootstrap methods for which a multivariate normal distribution is assumed to be the distribution of portfolio returns. The study provided an alternative way of using the information of performance test within an algorithmic pre-testing strategy. More related works can also be found in DeMiguel & Nogales (2009) and Gasbarro et al. (2007).

3. Methodology

The tests provided by Jobson & Korkie (1981) and Memmel (2003) have been the standard in the literature in testing for the equality of Sharpe ratios between two portfolio investment strategies. Hence, it is just fitting to use their notations. Note that we want to test the null hypothesis $H_0: \Delta = \zeta_p - \zeta_q = 0$, where $\zeta_p = \mu_p/\sigma_p$ and $\zeta_q = \mu_q/\sigma_q$ are the respective information ratios of two portfolio investment strategies with average excess returns μ_p and μ_q over a market proxy and standard deviations of excess returns σ_p and σ_q (also called the tracking errors). Now given a confidence level α , we reject the null hypothesis if zero lies outside the confidence interval

$$\hat{\Delta} \pm t_{1-\alpha/2} \hat{S}(\hat{\Delta}), \quad (3.1)$$

where $\hat{\Delta} = \hat{\zeta}_p - \hat{\zeta}_q$, $\hat{S}(\hat{\Delta}) = \sqrt{\frac{1}{T} [2 - 2\hat{\rho}_{pq} + \frac{1}{2}(\hat{\zeta}_p^2 + \hat{\zeta}_q^2 - 2\hat{\zeta}_p^2\hat{\zeta}_q^2\hat{\rho}_{pq}^2)]}$, $\hat{\zeta}_p$, $\hat{\zeta}_q$ and $\hat{\rho}_{pq}$ are estimators of the information ratio difference, the standard error of the difference estimator, the information ratios, and the correlation between the investment strategies p and q . Also, T denotes the number of observations of excess returns for each portfolio and $t_{1-\alpha/2}$ represents the $(1 - \alpha/2)$ -quantile of the t distribution with $T - 1$ degrees of freedom. In the event that the null hypothesis is rejected, it will imply that one portfolio outperforms the other.

The problem however with the formulation for the standard error of $\hat{\Delta}$ is that it heavily relies on i.i.d. excess returns from a bivariate normal distribution. It means that the formulation for the standard error is void if the distribution of the excess returns is non-normal or if it follows an autocorrelated process. The following are the remedies that this study will employ to construct a confidence interval whose returns are not necessarily normal.

3.1. Bootstrap inference

Nonparametric bootstrap has been proven to be a very good tool in various statistical analysis and is commonly used by various researches in statistical sciences. It may involve inferences about a parameter, but we use a nonparametric procedure in approximating the parametric distribution using the empirical cumulative distribution function. Nonparametric bootstrap is also used in solving hypothesis testing problems with which this study will be based upon. In doing so, we follow the guidelines that are established in the study of Hall & Wilson (1991).

Suppose $\hat{\zeta}$ is a function of the sample r_1, r_2, \dots, r_n and is an estimator of the unknown quantity ζ , and denote by $\hat{\zeta}^*$ the value of $\hat{\zeta}$ for a resample $r_1^*, r_2^*, \dots, r_n^*$ drawn from the sample with replacement. The first guideline states that we must resample from $\hat{\zeta}^* - \hat{\zeta}$ and not from $\hat{\zeta}^* - \zeta_0$. This first guideline of bootstrap hypothesis testing has the effect of increasing power. The second guideline, on the other hand, reduces error in level of significance. Suppose $\hat{\zeta}$ is an estimate of the scale of $\hat{\zeta}$ and $\hat{\zeta}^*$ denote the value of $\hat{\zeta}$ computed for the resample. The second guideline states that we have to base the test on the bootstrap distribution of $(\hat{\zeta}^* - \hat{\zeta})/\hat{\zeta}^*$ since it estimates the distribution of $(\hat{\zeta} - \zeta_0)/\hat{\zeta}$ under the null hypothesis.

For this study, we approximate the distribution function of the difference through an empirical cumulative distribution function taken from B bootstrap resamples, that is,

$$F_{\hat{\Delta}, B} \approx F_{\hat{\Delta}^*, B} \quad (3.2)$$

where $\hat{\Delta}$ is the observed difference or the estimator for Δ and $\hat{\Delta}^*$ is the observed difference from the bootstrap data, and F denotes the cumulative distribution function. A bootstrap $1 - \alpha$ confidence interval for Δ is then given by

$$\left[F_{\Delta, B}^{-1}\left(\frac{\alpha}{2}\right), F_{\Delta, B}^{-1}\left(1 - \frac{\alpha}{2}\right) \right]. \quad (3.3)$$

For us to generate portfolio resamples, we simply apply that of Efron (1992). Here, we resample residuals from the observed pairs of $(r_{tp}, r_{tq})'$, for all $t = 1, \dots, T$, with replacement. The following is the procedure we use for this resampling technique:

Step 1: Let R_n be an empirical cumulative distribution function of the portfolio returns r_1, r_2, \dots, r_n . Assign a probability of $1/n$ to each of the points in R .

Step 2: From the empirical distribution R_n , draw a random sample of size n with replacement and denote the resample data by $R_n^* = r_1^*, r_2^*, \dots, r_n^*$.

Step 3: Estimate the statistics of interest by using bootstrap sample

$$\zeta_n^* = \zeta_n(r_1^*, r_2^*, \dots, r_n^*).$$

Step 4: Repeat steps 2 and 3 B times to get bootstrap distribution and probability of obtaining a

$$\text{test statistic } \hat{\zeta}_n(\zeta, R_n) = P^*(R_n^* \zeta) = \frac{1}{B} \sum_{b=1}^B \mathbf{I}_{(R_{n,b}^* \zeta)}.$$

3.2. Simulation studies

The purpose of this Monte Carlo experiment is to evaluate finite sample performance of the method we propose. Table 1 shows the simulation boundaries of this study.

There will be 5,000 simulations per scenario each with $M = 10000$ bootstrap resamples. The sample size is $T = 120$ for all scenarios. This is based from empirical applications for which 10 years worth of monthly data or 4 months worth of daily data is usually used.

Moreover, a within-pair correlation of $\rho = 0$ means that the two portfolio investment returns are independent. The value $\rho = 0.75$, however, is based from daily returns of two stock returns of Philippine Stock Exchange, Inc. Composite Index. For the effect sizes, $\Delta = 0$ means that the true null hypothesis of equal information ratio is true. Otherwise, it is false, which is represented by effect sizes of 0.05 and 0.10, respectively. Lastly, we also would want to invoke skewness in our bivariate distributions for there are times where returns are skewed, for instance, during crisis.

3.3. Hypothesis testing procedure for equality of information ratio

The following procedure is proposed:

Step 1: Generate $T \times 2$ samples from bivariate $f(\mu, \Sigma)$. This will represent portfolio returns from two portfolio investment strategies r_p and r_q .

Step 2: Estimate $\Delta = \zeta_p - \zeta_q$ by $\hat{\Delta} = \hat{\zeta}_p - \hat{\zeta}_q$.

Step 3: Take T bootstrap resamples each from r_p and r_q and denote them by r_p^* and r_q^* respectively.

Step 4: Compute $\hat{\Delta}^* = \zeta_p^* - \zeta_q^*$.

Step 5: Iterate steps 3 and 4 M times.

Step 6: Sort $\hat{\Delta}_1^*, \hat{\Delta}_2^*, \dots, \hat{\Delta}_M^*$ in either ascending or descending order.

Table 1. Boundaries of the simulation study

a. Distribution of portfolio returns	Bivariate normal
	Bivariate t_4
b. Mean	Constant
	VAR(1)
c. Variance	Constant
	ECCC-GARCH(1,1)
d. Within-pair correlation	$\rho = 0$
	$\rho = 0.20$
	$\rho = 0.75$
e. Effect size	$\Delta = 0$
	$\Delta = 0.05$
	$\Delta = 0.10$
f. Skewness	$k = -0.05$
	$k = 0$
	$k = 0.05$

Step 7: From the M estimates, get the $\frac{\alpha}{2}100\%$ and the $(1 - \frac{\alpha}{2})100\%$ percentiles from Step 6. These two values comprise the lower and upper limits of the $(1 - \frac{\alpha}{2})100\%$ bootstrap confidence interval, respectively.

Step 8: Reject the null hypothesis of equal information ratio if zero lies outside the percentile bootstrap from step 7.

3.4. The data-generating processes

We assume that portfolio returns follow

$$\mathbf{r} = \boldsymbol{\mu} + \boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\varepsilon} \quad (3.4)$$

where $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are the parameters of a bivariate distribution f whose residuals is $\boldsymbol{\varepsilon}$. With this, we bootstrap from the residual, that is, we get our bootstrap resamples from

$$\hat{\boldsymbol{\varepsilon}} = \left(\hat{\boldsymbol{\Sigma}}^{\frac{1}{2}}\right)^{-1} \cdot (\hat{\mathbf{r}} - \hat{\boldsymbol{\mu}}). \quad (3.5)$$

In our data-generating processes, we have to control the parameters $\boldsymbol{\Sigma}$ and $\boldsymbol{\mu}$ in such a way that it will satisfy the effect sizes Δ stated in Table 3.1. Note that $\Delta = 0$ means that the null hypothesis is true.

First case is when mean and variance are both constant. This is easy since we only need to set-up constant values that will satisfy the Δ s that we desire. Second case is when we have a constant mean but with an ECCC-GARCH(1,1) variance. In this setting, assumption for variance is that of bivariate ECCC-GARCH(1,1). In particular, we use the model of He & Teräsvirta (2004). In this model, we assume that conditional correlation is constant over time and the conditional covariances are caused by changes in each of the corresponding two conditional covariances. Let

$\varepsilon_t = (\varepsilon_p, \varepsilon_q)'$. Then $\varepsilon_t | \mathcal{F}_{t-1} \sim N(\mathbf{0}, \Sigma_t)$, where \mathcal{F}_{t-1} is the σ -field generated by all the available information up to time $t - 1$ and

$$\Sigma_t = \begin{bmatrix} \varsigma_{p,t} & \rho\sqrt{\varsigma_{p,t}\varsigma_{q,t}} \\ \rho\sqrt{\varsigma_{p,t}\varsigma_{q,t}} & \varsigma_{q,t} \end{bmatrix}. \quad (3.6)$$

The conditional covariance matrix Σ_t is positive definite for all t , almost surely, and

$$\varsigma_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta\varsigma_{t-1}. \quad (3.7)$$

We also consider the possibility of a changing mean over time in our data-generating process. To accommodate this, we model the mean as an autocorrelated process, in particular, a VAR(1) process. That is,

$$r_{p,t} = \mu_p^* + \varphi_{11}\mu_{p,t-1} + \varphi_{12}\mu_{q,t-1} + \varepsilon_{p,t} \quad (3.8)$$

$$r_{q,t} = \mu_q^* + \varphi_{21}\mu_{p,t-1} + \varphi_{22}\mu_{q,t-1} + \varepsilon_{q,t} \quad (3.9)$$

where $\varepsilon_t \sim N(\mathbf{0}, \Sigma)$. First, we estimate the parameters μ^* and φ from real data for which we use end-of-month stock returns of Mining Index and Industrials Index from the Philippine Stock Exchange, Inc. Upon estimation, we get the following estimates: $\mu^* = \begin{bmatrix} 0.0102 \\ 0.0076 \end{bmatrix}$ and $\varphi = \begin{bmatrix} 0.0813 & 0.0555 \\ 0.2082 & 0.1420 \end{bmatrix}$. Note that when we use VAR(1) to model the mean, we use the distributions f specified in the simulation boundaries as innovations.

Lastly, we also consider the case where we still generate resamples from VAR(1) with innovation f ; however, the variance of the innovation is that of ECCG-GARCH(1,1) which is referred to as VAR-MGARCH by Carnero & Eratalay (2014).

Tables 2–5 summarize value specifications that will be used in the Monte Carlo procedures.

In the values herewith, we use the autocorrelation matrix φ from what we have estimated in the real data to supply variates from VAR(1).

Table 2. Value specifications for constant mean and constant variance

Value specifications		μ_p	σ_p^2	μ_q	σ_q^2
$\Delta = 0$	$k = -0.05$	0.010	0.025	0.022	$\frac{121}{1000}$
	$k = 0$	0.015	$\frac{1}{64}$	0.032	$\frac{16}{225}$
	$k = 0.05$	0.4	0.6	0.5	$\frac{15}{16}$
$\Delta = 0.05$	$k = -0.05$	0.010	0.025	0.022	2.758704
	$k = 0$	0.015	$\frac{1}{64}$	0.032	0.2089796
	$k = 0.05$	0.4	0.6	0.5	1.149283
$\Delta = 0.10$	$k = -0.05$	0.010	0.025	0.022	0.3582825
	$k = 0$	0.015	$\frac{1}{64}$	0.032	$\frac{64}{25}$
	$k = 0.05$	0.4	0.6	0.5	1.44186

Table 3. Value specifications for constant mean and ECCC-GARCH(1,1) variance

Value specifications		μ_p	ω_p	α_p	θ_p
$\Delta = 0$	$k = -0.05$	0.010	$\frac{29}{4000}$	0.45	0.26
	$k = 0$	0.015	$\frac{29}{6400}$	0.45	0.26
	$k = 0.05$	0.020	$\frac{87}{1000}$	0.45	0.26
$\Delta = 0.05$	$k = -0.05$	0.010	$\frac{29}{4000}$	0.45	0.26
	$k = 0$	0.015	$\frac{29}{6400}$	0.45	0.26
	$k = 0.05$	0.020	$\frac{87}{1000}$	0.45	0.26
$\Delta = 0.10$	$k = -0.05$	0.010	$\frac{29}{4000}$	0.45	0.26
	$k = 0$	0.015	$\frac{29}{6400}$	0.45	0.26
	$k = 0.05$	0.020	$\frac{87}{1000}$	0.45	0.26
		μ_q	ω_q	α_q	θ_q
$\Delta = 0$	$k = -0.05$	0.032	$\frac{1}{25}$	0.36	0.14
	$k = 0$	0.032	$\frac{1}{6}$	0.36	0.14
	$k = 0.05$	0.032	$\frac{6}{25}$	0.36	0.14
$\Delta = 0$	$k = -0.05$	0.032	0.04571429	0.36	0.14
	$k = 0$	0.032	0.01758242	0.36	0.14
	$k = 0.05$	0.032	$\frac{24}{25}$	0.36	0.14
$\Delta = 0$	$k = -0.05$	0.032	$\frac{8}{15}$	0.36	0.14
	$k = 0$	0.032	0.01860465	0.36	0.14
	$k = 0.05$	0.032	$-\frac{12}{25}$	0.36	0.14

Table 4. Value specifications for VAR(1) mean and constant variance

Value specifications		μ_p^*	σ_p^2	μ_q^*	σ_q^2
$\Delta = 0$	$k = -0.05$	0.010219	0.025	0.00759	0.01379135
	$k = 0$	0.010219	$\frac{1}{64}$	0.00759	0.00861959
	$k = 0.05$	0.010219	0.30	0.00759	0.1654962
$\Delta = 0.05$	$k = -0.05$	0.010219	0.025	0.00759	0.2691271
	$k = 0$	0.010219	$\frac{1}{64}$	0.00759	0.05714015
	$k = 0.05$	0.010219	0.30	0.00759	0.0586421
$\Delta = 0.10$	$k = -0.05$	0.010219	0.025	0.00759	0.04604992
	$k = 0$	0.010219	$\frac{1}{64}$	0.00759	0.1730028
	$k = 0.05$	0.010219	0.30	0.00759	0.00870655

4. Results and discussion

For each of the simulation boundaries, results are presented by skewness of the distribution and within-pair correlation of portfolio returns. Note that for all simulations confidence level is $\alpha = 0.05$.

Table 5. Value specifications for VAR(1) mean and ECCC-GARCH(1,1) variance

Value specifications		μ_q^*	ω_p	α_p	θ_p
$\Delta = 0$	$k = -0.05$	0.010219	$\frac{29}{4000}$	0.45	0.26
	$k = 0$	0.010219	$\frac{29}{6400}$	0.45	0.26
	$k = 0.05$	0.010219	$\frac{87}{1000}$	0.45	0.26
$\Delta = 0.05$	$k = -0.05$	0.010219	$\frac{29}{4000}$	0.45	0.26
	$k = 0$	0.010219	$\frac{29}{6400}$	0.45	0.26
	$k = 0.05$	0.010219	$\frac{87}{1000}$	0.45	0.26
$\Delta = 0.10$	$k = -0.05$	0.010219	$\frac{29}{4000}$	0.45	0.26
	$k = 0$	0.010219	$\frac{29}{6400}$	0.45	0.26
	$k = 0.05$	0.010219	$\frac{87}{1000}$	0.45	0.26
		μ_q^*	ω_q	α_q	θ_q
$\Delta = 0$	$k = -0.05$	0.00759	0.009284177	0.36	0.14
	$k = 0$	0.00759	0.00580261	0.36	0.14
	$k = 0.05$	0.00759	0.1114101	0.36	0.14
$\Delta = 0$	$k = -0.05$	0.00759	0.0105781	0.36	0.14
	$k = 0$	0.00759	0.006282946	0.36	0.14
	$k = 0.05$	0.00759	-0.2381301	0.36	0.14
$\Delta = 0$	$k = -0.05$	0.00759	0.0122911	0.36	0.14
	$k = 0$	0.00759	0.006849983	0.36	0.14
	$k = 0.05$	0.00759	-0.05755523	0.36	0.14

4.1. Size of the test

The size of the proposed test is computed for cases with varying skewness and increasing within-pair correlation of portfolio returns. Size is often defined as the probability of committing a Type I error. To compute for the size, we generate the cases under $H_0 : \Delta = 0$, or when the difference between the two information ratios is zero and proportions of Monte Carlo draws where the null hypothesis was rejected is considered in computing the size.

Table 6 shows that the size of the test is affected by the within-pair correlation of portfolio returns. It means that when the behavior of two portfolio returns is almost similar, the tendency of the test is to not reject the null hypothesis. This is not a good indication since at $\alpha = 0.05$ it is expected that the test should reject around 5% of the generated Monte Carlo draws under true null hypothesis. Regardless, the test is correctly sized and performs well under independent and weakly dependent portfolio returns.

Similar results can be seen when distribution used is symmetric and positively skewed ($k = 0, 0.05$). It means to say that the size of the test is really not affected by the skewness of the distribution. Also notice that size is better when we have an autocorrelated mean even at large within-pair correlation (Tables 7 and 8).

It can also be noticed that the size is unaffected by the form of the variance, whether the variance is constant or heteroskedastic.

Table 6. Size of the test at $k = -0.05$

Mean	Constant		VAR(1)	
Variance	Constant	ECCC-GARCH(1,1)	Constant	ECCC-GARCH(1,1)
<i>Bivariate normal:</i>				
$\rho = 0$	0.07837	0.07882	0.06334	0.06656
$\rho = 0.20$	0.07191	0.06987	0.05974	0.05925
$\rho = 0.75$	0.02620	0.02971	0.03221	0.03823
<i>Bivariate t_4:</i>				
$\rho = 0$	0.05013	0.05515	0.05612	0.05916
$\rho = 0.20$	0.04382	0.04524	0.05210	0.05325
$\rho = 0.75$	0.01666	0.01982	0.02007	0.02503

Table 7. Size of the test at $k = 0$

Mean	Constant		VAR(1)	
Variance	Constant	ECCC-GARCH(1,1)	Constant	ECCC-GARCH(1,1)
<i>Bivariate normal:</i>				
$\rho = 0$	0.05575	0.05004	0.06951	0.06339
$\rho = 0.20$	0.04930	0.04995	0.06552	0.05812
$\rho = 0.75$	0.02286	0.01924	0.03113	0.02882
<i>Bivariate t_4:</i>				
$\rho = 0$	0.04024	0.05195	0.05716	0.05505
$\rho = 0.20$	0.03914	0.04834	0.05596	0.05116
$\rho = 0.75$	0.01832	0.02016	0.01112	0.02102

Table 8. Size of the test at $k = 0.05$

Mean	Constant		VAR(1)	
Variance	Constant	ECCC-GARCH(1,1)	Constant	ECCC-GARCH(1,1)
<i>Bivariate normal:</i>				
$\rho = 0$	0.06825	0.06112	0.06806	0.05916
$\rho = 0.20$	0.06019	0.06020	0.06704	0.05623
$\rho = 0.75$	0.02988	0.01790	0.01722	0.02553
<i>Bivariate t_4:</i>				
$\rho = 0$	0.05692	0.05331	0.05105	0.05626
$\rho = 0.20$	0.04980	0.04950	0.05016	0.04805
$\rho = 0.75$	0.02320	0.01678	0.01401	0.01722

4.2. Power of the test

The proposed test is also assessed for its power under similar restrictions of within-pair correlations and skewness. To compute for power, we generate Monte Carlo draws under false null hypothesis, specifically, when $\Delta = 0.05$ and $\Delta = 0.10$. By definition, power of the test is the probability of rejecting a false null hypothesis.

Table 9 shows that power is high and is affected by within-pair correlation. That is, when the null hypothesis is false, the tendency of the test is to reject it. Rejection probability is increasing as Δ is increasing (Table 10).

From Table 9, we see that the test has a higher chance of detecting a difference of information ratio if mean is modeled as an autocorrelated process. Also, note that at $\Delta = 0.10$, chances are, the test will always reject the null hypothesis. Similar results are found under symmetric and positively skewed distributions (Tables 11–14).

Table 9. Power of the test at $k = -0.05$ and $\Delta = 0.05$

Mean	Constant		VAR(1)	
Variance	Constant	ECCC-GARCH(1,1)	Constant	ECCC-GARCH(1,1)
<i>Bivariate normal:</i>				
$\rho = 0$	0.85039	0.85084	0.94256	0.94578
$\rho = 0.20$	0.77150	0.78241	0.92662	0.92613
$\rho = 0.75$	0.34911	0.35262	0.49128	0.39624
<i>Bivariate t_4:</i>				
$\rho = 0$	0.82215	0.82717	0.93534	0.93838
$\rho = 0.20$	0.75636	0.75778	0.91898	0.92013
$\rho = 0.75$	0.33957	0.34273	0.47914	0.38304

Table 10. Power of the test at $k = -0.05$ and $\Delta = 0.10$

Mean	Constant		VAR(1)	
Variance	Constant	ECCC-GARCH(1,1)	Constant	ECCC-GARCH(1,1)
<i>Bivariate normal:</i>				
$\rho = 0$	1.00000	1.00000	1.00000	1.00000
$\rho = 0.20$	1.00000	1.00000	1.00000	1.00000
$\rho = 0.75$	0.98742	0.98662	0.99459	0.99596
<i>Bivariate t_4:</i>				
$\rho = 0$	1.00000	1.00000	1.00000	1.00000
$\rho = 0.20$	1.00000	1.00000	1.00000	1.00000
$\rho = 0.75$	0.98926	0.99248	0.99716	0.99926

Table 11. Power of the test at $k = 0$ and $\Delta = 0.05$

Mean	Constant		VAR(1)	
Variance	Constant	ECCC-GARCH(1,1)	Constant	ECCC-GARCH(1,1)
<i>Bivariate normal:</i>				
$\rho = 0$	0.75706	0.74135	0.87082	0.86470
$\rho = 0.20$	0.72839	0.72904	0.84461	0.83849
$\rho = 0.75$	0.32420	0.32058	0.33247	0.32635
<i>Bivariate t_4:</i>				
$\rho = 0$	0.74155	0.75046	0.85847	0.85585
$\rho = 0.20$	0.71823	0.72743	0.83505	0.83243
$\rho = 0.75$	0.32420	0.32150	0.31246	0.30984

Table 12. Power of the test at $k = 0$ and $\Delta = 0.10$

Mean	Constant		VAR(1)	
Variance	Constant	ECCC-GARCH(1,1)	Constant	ECCC-GARCH(1,1)
<i>Bivariate normal:</i>				
$\rho = 0$	1.00000	1.00000	1.00000	1.00000
$\rho = 0.20$	1.00000	1.00000	1.00000	1.00000
$\rho = 0.75$	0.94273	0.93520	0.95771	0.95160
<i>Bivariate t_4:</i>				
$\rho = 0$	1.00000	1.00000	1.00000	1.00000
$\rho = 0.20$	1.00000	1.00000	1.00000	1.00000
$\rho = 0.75$	0.92929	0.93895	0.94676	0.94414

Table 13. Power of the test at $k = 0.05$ and $\Delta = 0.05$

Mean	Constant		VAR(1)	
Variance	Constant	ECCC-GARCH(1,1)	Constant	ECCC-GARCH(1,1)
<i>Bivariate normal:</i>				
$\rho = 0$	0.83212	0.81499	0.92193	0.93303
$\rho = 0.20$	0.80701	0.74905	0.90350	0.91462
$\rho = 0.75$	0.34984	0.31371	0.39633	0.32631
<i>Bivariate t_4:</i>				
$\rho = 0$	0.82079	0.80718	0.90492	0.93013
$\rho = 0.20$	0.79662	0.79648	0.88662	0.90644
$\rho = 0.75$	0.34316	0.31259	0.39633	0.31800

Table 14. Power of the test at $k = 0.05$ and $\Delta = 0.10$

Mean	Constant		VAR(1)	
Variance	Constant	ECCC-GARCH(1,1)	Constant	ECCC-GARCH(1,1)
<i>Bivariate normal:</i>				
$\rho = 0$	1.00000	1.00000	1.00000	1.00000
$\rho = 0.20$	1.00000	1.00000	1.00000	1.00000
$\rho = 0.75$	0.91957	0.93202	0.97272	0.96383
<i>Bivariate t_4:</i>				
$\rho = 0$	1.00000	1.00000	1.00000	1.00000
$\rho = 0.20$	1.00000	1.00000	1.00000	1.00000
$\rho = 0.75$	0.90871	0.92183	0.95577	0.94829

We still see that at a higher Δ , power is high. We attain the highest power at uncorrelated and weakly dependent portfolio returns. Having high power means that the test will be able to reject the null of equal information ratio.

In the case of positively skewed returns, power is still high and is highest when mean of portfolio returns is modeled as an autocorrelated process. In a similar case from previous values of skewness, test will most likely detect a difference at $\Delta = 0.10$.

4.3. Empirical applications

In this section, we apply the test empirically to end-of-month returns of Services and Mining Indices of the Philippine Stock Exchange Inc. from January 2008 to December 2018. Since better performance is seen where mean is an autocorrelated process, then we use it. We first check the descriptive statistics of the data (Table 15).

EOM (End of Month) returns are both normally distributed with some degree of skewness. Note that we use the difference of natural logarithms to represent returns. Then we fit VAR(1) to the data (Table 16).

Table 15. Descriptive statistics of EOM returns of services index and mining index

	$\log(S_t)$	$\log(M_t)$
Mean	-0.001358	-0.001526
Median	-0.001347	-0.001934
Maximum	0.023006	0.039203
Minimum	-0.035193	-0.047441
Std. dev	0.009816	0.014343
Skewness	-0.350795	0.030802
Kurtosis	3.618144	3.225160
Jarque-Bera	4.335227	0.270189
Probability	0.114450	0.873633

Table 16. Vector autoregression estimates of EOM returns of services index and mining index

	$\log(S_t)$	$\log(M_t)$
$\log(S_{t-1})$	-0.025599	0.063746
	(0.10403)	(0.15325)
	[-0.24607]	[0.41596]
$\log(M_{t-1})$	0.074386	-0.002019
	(0.07039)	(0.10369)
	[1.05680]	[-0.01947]
C	-0.001195	-0.001395
	(0.00092)	(0.00135)
	[-1.30059]	[-1.03125]

We then now check if VAR residuals is normally distributed and if it is heteroskedastic.

Tables 17 and 18 show that the residuals are normally distributed and homoskedastic. Hence, we can use normal distribution as innovation with mean zero and constant variance, as estimated from the residuals,

$$\begin{bmatrix} 9.701312152736229e-05 & 6.286351051125198e-05 \\ 6.286351051125198e-05 & 0.00021052616955669 \end{bmatrix}.$$

Here are the results of the test (Table 19).

The p -value is computed by getting the proportion of bootstrap estimates that are greater than the observed test statistic over the number of bootstrap estimates, as discussed in Davison & Hinkley (1997). Results show that Services Index has significantly higher information ratio

Table 17. VAR residuals normality test

Component	Skewness	χ^2	df	Prob.
1	-0.327400	2.108089	1	0.1465
2	-0.198837	0.777544	1	0.3779
Joint		2.885633	2	0.2363
Component	Kurtosis	χ^2	df	Prob.
1	3.605211	1.800876	1	0.1796
2	-3.374185	0.688403	1	0.4067
Joint		2.489279	2	0.2880
Component	Jarque-Bera	df	Prob.	
1	3.908965	2	0.1796	
2	1.465947	2	0.4805	
Joint	5.374912	4	0.2509	

Table 18. VAR residuals heteroskedasticity test

Joint test					
χ^2		df		Prob.	
10.30145		12		0.5895	
Individual components					
Dependent	R^2	$F(4, 113)$	Prob.	$\chi^2(4)$	Prob.
res1*res1	0.030816	0.898218	0.4676	3.636234	0.4575
res2*res2	0.010623	0.303327	0.8752	1.253533	0.8692
res2*res1	0.065679	1.985855	0.1014	1.750099	0.1012

Table 19. Result of the test

ζ_s	ζ_M	$\hat{\Delta}$	p-Value
0.0279	0.0092	0.0187	0.0019

compared to Mining Index. One may use this as a basis for investing to assets listed in the Services Index for it shows a better performance than the Mining Index.

5. Conclusions

Testing for difference in performance between two investment strategies is important in performance evaluation. This study extends the works of Jobson & Korkie (1981), Lo (2002), and Memmel (2003) by developing a performance hypothesis test that determines whether two portfolios exhibit equal information ratio. Furthermore, this study also does not assume any distribution for the portfolio returns. One may opt to use the test of Goodwin (1998) but only under the assumption of normally distributed returns. In the test we proposed, we considered returns whose tails are heavier than normal and with varying skewness. We also considered returns with time-series characteristics. The selection of this portfolio performance measure is due to its desirable advantages as compared to Sharpe ratio.

Simulation studies suggest that the proposed test is correctly sized in most scenarios. However, the expected size α loses when portfolio returns are highly correlated. In other words, the test has a high tendency to reject a null hypothesis when the behavior of two portfolio returns is almost the same. Moreover, the test exhibits high power, also when portfolio returns are highly correlated. Under highly correlated returns, the power improves more whenever mean is modeled using an autocorrelated process. The overall performance of the test is consistent under varying skewness of the distributions and varying forms of variances.

In the Philippines, the use of the information ratio is relevant since the country is experiencing some volatility in its nation's experience. It is helpful for us to gain feedback and derive insights relative to the additional risks brought about by this volatility. For fund managers (and even for us individual investors), this is helpful since we are "on the lookout" upfront with the risk we are tolerating when we invest. It is also with this motivation that we pursued with this portfolio performance measure and applied it to some of the indices in the Philippine Stock Exchange. We also look forward for this hypothesis testing strategy to be applied in the stock market from developing economies.

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