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Latent fragility: Conditioning banks' joint probability of default on the financial cycle

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Latent fragility: conditioning banks' joint probability of default on the financial cycle

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Abstract

We propose the CoJPoD, a novel framework explicitly linking the cross-sectional and cyclical dimensions of systemic risk. In this framework, banking sector distress in the form of the joint probability of default of financial intermediaries (reflecting contagion from both direct and indirect interconnectedness) is conditioned on the financial cycle (reflecting the buildup and unwinding of system-wide balance sheet leverage). An empirical application to large systemic banks in the euro area, US and UK illustrates how the unravelling of excess leverage can magnify banking sector distress. Capturing this dependence of banking sector distress on prevailing financial imbalances can enhance risk surveillance and stress testing alike. An empirical signaling exercise confirms that the CoJPoD outperforms the individual capacity of either its unconditional counterpart or the financial cycle in signaling financial crises – particularly around their onset – suggesting scope to increase the precision with which macroprudential policies are calibrated.

JEL codes: C19, C54, E58, G01, G21

Keywords: Systemic Risk, Financial Crises, Portfolio Credit Risk, Multivariate Density Optimization, Financial Cycle

Non-technical summary

Systemic risk has two dimensions: First, the buildup of systemic vulnerabilities; and second, the propagation of risk. But in practice, these dimensions of systemic risk are not independent. The materialisation of systemic risk is hardly divorced from the buildup of macro excesses in financial credit and asset prices. In this way, there is value in linking the propagation of manifest systemic risk (the cross-sectional dimension of systemic risk) to the prevailing level of vulnerabilities in the financial system (the cyclical dimension of systemic risk).

In this paper, we propose a new method to make this link – and signal systemic stress accompanying the unwinding of macro imbalances. We adapt a well-established measure of systemic risk measurement (the joint probability of default of large banks), allowing it to vary according to prospective financial excesses and their unravelling in national financial cycles. This consideration of latent risk factors in the form of the unwinding of excesses in leverage helps to more accurately gauge the potential distress from interconnectedness and contagion across banks, and associated country banking crises.

The method is applied to analyze the probability multiple that banks will default over the next year depending on forecasted financial cycles, drawing on nearly 30 years of historical data for 7 banks in the euro area, 13 banks in the United States and 4 banks in the United Kingdom. After first providing visual evidence on the prospective benefits of this conditioning, we more formally evaluate the merits of this method in a “signalling exercise” – evaluating the proposed conditional measure of systemic risk in the banking sector against the predictive value of its components (unconditional joint banking sector distress probabilities derived from market prices, and macro-financial imbalances inherent to national financial cycles). We also provide some mathematical proofs on the properties of this conditional indicator, linking it to its unconditional counterpart.

Our empirical results underscore the value of conditioning crisis prediction on latent risk. Put differently, we provide evidence on the benefits of linking the cross-sectional dimension of systemic risk with its cyclical dimension. Whereas usual measures of market price-implied banking sector distress are accurate *thermometers* of systemic risk, insofar as they measure near-term probability of financial stress, they are known to be less accurate in acting as broader *barometers* – that is, predicting how prevailing macrofinancial imbalances condition systemic stress onset. Financial cycles tend to be good at predicting systemic crises one or more years

ahead, but poor at forecasting near-term stress or providing a view of potential propagation. We show that combining the information they provide into a conditional measure of financial market implied banking sector distress is particularly useful upon the onset of crises.

In this sense, explicitly linking the two dimensions of systemic risk may be useful in two ways. It might first provide a more accurate assessment of banking sector health, by accounting for how interconnectedness and contagion varies with system-wide balance-sheet leverage. Second, our approach can also be used to conduct systemic stress tests by conditioning on severe, but plausible, tail outcomes for prospective financial cycle developments.

1 Introduction

In financial crises, relatively small initial financial losses at the banking level can be endogenously magnified to systemic dimensions through interconnectedness and contagion.¹ The prospect of such systemic amplification mechanisms may depend on the degree of leverage in the financial system, which varies with the financial cycle. This implied dependence suggests a clear role for methods which adequately provide a measurement of the buildup of vulnerabilities, upon which changing interconnectedness structures of banks are conditional. However, the empirical literature suffers from an effective separation between papers analyzing cyclical risk buildup and those analyzing structural amplification and contagion (see Benoit et al. (2017)). Such a separation might also inhibit a holistic view on macroprudential policy – whereby methods suitable for informing the setting of structural buffers that buttress the resilience of systemic institutions would accordingly be considered separately from methods used to inform the setting of time varying buffers to counter country financial cycles.²

Conceptually, strong links exist between national financial cycles and associated banking systems. In practice, this is borne out by the link between bouts of systemic stress in banking systems (as measured by banks' joint probability of default, or JPoD, measuring the cross-sectional dimension of systemic risk), and corrections in the buildup of vulnerabilities (as measured by the financial cycle, capturing the cyclical dimension of systemic risk) – see Figure 1, which confirms this relationship for the United States. In particular, systemic risk appears to be most virulent when its cyclical and cross-sectional dimensions coalesce, as indicated by a downturn in the financial cycle and a jump in the JPoD around the period of the Global Financial Crisis, for instance. Research has confirmed this, showing that economic welfare suffers most in deep recessions (see Claessens et al. (2012)) when financial fragility combines with downturns in financial cycles. That is, in contrast to standard recessions, financial recessions occur less frequently, are deeper, and last longer – particularly when combined with a banking crisis. Empirical and theoretical research has confirmed that financial recessions follow credit booms (see Jordà et al.

¹Segoviano (2006) notes the role of intricate structures; i.e., macrofinancial linkages and interconnectedness structures across financial entities and markets (due to contractual exposures across financial entities, exposures to common risk factors and market price channels) that can pave the road for financial contagion and ignite endogenous loops with the possibility of magnifying moderate exogenous shocks into substantial negative financial outcomes with large welfare effects.

²Indeed, improved relating methods examining cyclical and cross-sectional perspectives on systemic risk would permit a needed holistic perspective on systemic risk, analogous to the seminal speech of Crockett (2000) advocating the marriage of the micro- and macro-prudential dimensions of financial stability.

(2013), Schularick and Taylor (2012) and Boissay et al. (2016)), underscoring how credit growth, in particular, is an essential part of the buildup of imbalances presaging financial recessions and their subsequent unwinding. At the same time, financial cycles exhibit considerable persistence, with evidence of marked duration, particularly when compared with business cycle counterparts. This implies lengthy periods of vulnerability building up, which align with the amplification – visible in the JPoD – stemming from this latent buildup in long corrective phases. In this way, unwinding imbalances can suddenly create exposures to latent risk factors that would increase interconnectedness across financial entities and amplify interconnectedness and contagion in the banking system in corrective phases of the financial cycle. A joint evaluation of these factors could, accordingly, allow for better crisis signals and an evaluation of tradeoffs associated with prevention and mitigation of systemic risk.

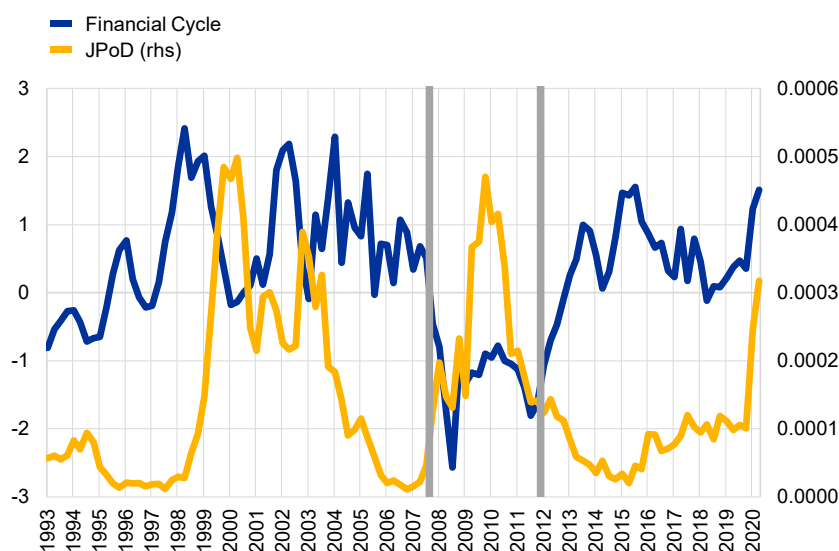


Figure 1: Financial cycle and banks' joint probability of default (JPoD) in the United States
Notes: The financial cycle is estimated using the method of Schüler et al. (2020). The JPoD follows the method of Segoviano (2006) and Segoviano and Goodhart (2009). Horizontal grey lines correspond to the first and last quarter of a systemic banking crisis as given by Laeven and Valencia (2018). For more details, please see Sections 2 and 3.1.

We offer a method to empirically capture the interactions between the cyclical and cross-sectional dimensions of systemic risk. We do so by deriving the JPoD from banks' joint density of equity returns conditional on the state of the financial cycle and term the resulting measure “conditional joint probability of default”, or in brief, CoJPoD. The empirical measure of the cyclical dimension of systemic risk is the financial cycle estimated using the method of Schüler

et al. (2020). This method summarizes aggregate financial conditions by extracting the momentum common to credit and prices in equity, bond and property markets (in this way capturing fundamental balance-sheet leverage at the financial system level).³ The empirical measure of the cross-sectional dimension of systemic risk is the joint default probability among several banks (thereby measuring the propagation of shocks), estimated using the methodology of Segoviano (2006) and Segoviano and Goodhart (2009). This measure aligns well with systemic banking crises. Importantly, our proposed method allows for conditioning on future tail outcomes for the financial cycle at any point in time, even when financial conditions are currently benign, which makes our method a useful tool for scenario analysis. We also propose the ΔCoJPoD , a measure that highlights the systemic risk amplification potential from a deteriorating financial cycle. Specifically, ΔCoJPoD is defined as the difference of the CoJPoD conditional on a realization of the financial cycle in its lower tail compared to the CoJPoD conditional on a realization of the financial cycle at its median.

Overall, we find that our integrated empirical treatment of the cross-sectional and cyclical dimension of systemic risk sharpens the identification of crisis regimes and, in particular, their onsets. Neither the JPoD nor the financial cycle *individually* signal the onset of crises with great precision, with the financial cycle performing worse than the JPoD . For the selection of jurisdictions (euro area, United Kingdom, United States), we find that the ΔCoJPoD significantly outperforms the JPoD in signalling the onsets of crises, improving on the JPoD 's capacity by up to 24 percentage points. But also regarding the identification of crises, more generally, we find that the ΔCoJPoD significantly improves on the capacities of the JPoD . The intuition behind our result is that financial cycle downturns amplify banks' joint probability of default. Similarly, financial cycle upturns dampen banks' joint probability of default. Therefore, linking these measures increases the precision with which one can distinguish systemic events from non-systemic events.

In this sense, we provide initial evidence that linking the cross-sectional dimension of systemic risk and the cyclical dimension of systemic risk can improve the precision with which macro-prudential policies are set. Conditioning the joint density of banks' prospective equity returns on the financial cycle provides a needed policy-relevant perspective on the potential virulence of cross-sectional measures of systemic risk. This is because default probabilities (via dependence

³Several studies show that this proxy measures the buildup of cyclical vulnerabilities and the subsequent systemic risk materialization well (Schüler et al. (2020), Hartwig et al. (2021)).

and potential contagion) are reduced if the financial cycle indicator does not predict risks due to system-wide balance-sheet leverage in the future. On this basis, our approach may provide a valuable perspective for better understanding when time-varying capital requirements, such as countercyclical capital buffers, should be released, as it signals the onsets of financial crises with greater precision, mapped to the amplification of systemic risk. Beyond this risk materialization phase, the *ex ante* phase of prudential policy is also relevant, as our measure highlights the complementary nature of preventative macroprudential measures in financial downturns – that is, measures which reduce the interdependence of banks in stress periods, and measures which address phases in which vulnerabilities buildup in the financial cycle.

The method proposed in this paper complements other methods which measure systemic risk in two ways. First, it complements measures of systemic risk interdependence in the financial sector. In particular, SRISK (Brownlees and Engle (2017)) measures the expected capital shortfall of a financial entity conditional on a prolonged market decline, while CoVaR (Adrian and Brunnermeier (2016)) measures the change in the value-at-risk of the financial system conditional on the distress of an individual financial institution. On the one hand, our indicator differs in the unit of the risk measure – focusing on joint probability of default, rather than correlated financial stress. In this respect, the interdependency focus of the risk measure is directly on interaction of systemic institutions. On the other hand, our measure differs in its conditioning variable – which is system wide leverage, in contrast to individual banks equity losses or banking system-wide equity losses. In this respect, its scope with regard to the conditioning variable includes all asset classes, notably including not only financial assets but also real estate – in tandem with associated credit growth. Second, our method provides a complement to growth-at-risk measures (see Adrian et al. (2019) as well as Adrian et al. (2018)). While these measures are useful for gauging the economic consequences of systemic risk materialization, they do not explicitly account for an intermediate stage of banking sector resilience that tempers the disruptive systemic potential for imbalances to generate endogenous financial stress. Our approach focuses more directly on this intermediate stage.

Furthermore, our CoJPoD relates to other extensions of the JPoD methodology. For one, Radev (2016) introduces a measure for understanding the systemic risk contribution of individual sovereigns to the euro area's sovereign joint probability of default. Radev (2016) therefore extends the JPoD by considering the change in the JPoD due to the default of one entity. For another, Jin and Nadal de Simone (2014) extend the JPoD as well. Among others, the authors

include information from a factor of macro-financial time series when estimating the marginal bank default probabilities. Our approach also embeds macro-financial information in the JPoD, but in a different way, i.e., by conditioning the joint density of banks' equity returns on the financial cycle – explicitly linking the different dimensions of systemic risk. Furthermore, in doing so, our approach can be used as a tool for scenario analysis that may help to guide the process of setting macroprudential policies.

Finally, our measure also relates to extensive literature dealing with the construction and assessment of early warning indicators of financial crises (see, for instance, Demirgüç-Kunt and Detragiache (1998), Kaminsky and Reinhart (1999), Borio and Lowe (2002), Alessi and Detken (2011, 2018), Schularick and Taylor (2012), Anundsen et al. (2016), Behn et al. (2017), Hartwig et al. (2021)). This literature is mainly concerned with the buildup phase of vulnerabilities, so as to inform the activation of countercyclical macroprudential policies. Our approach provides initial evidence that linking the cross-sectional with the cyclical dimension of systemic risk may provide a promising way to discriminate between systemic and non-systemic events. For instance, this approach may be helpful for the early recognition of systemic risk materialization. This is important because early recognition allows for quick and robust interventions that are likely to reduce the costs of crisis (Borio et al. (2010)).

We organize the paper as follows. Section 2 outlines the basis for conditional empirical probabilities of joint bank default. Section 3 contains details about our data and model implementation, while Section 4 presents the empirical results. Finally, Section 5 offers some concluding remarks, as well as avenues for further research. The annex provides additional details on methodological issues and our data set.

2 Modelling the conditional joint probability of default

2.1 Overview

As is standard in structural credit risk modelling (Merton (1974)), the default of an entity is triggered once its asset value falls below some default threshold, determined by its liabilities. The probability of default (PoD) of an entity is the probability of observing such an event for a given time horizon, for example, one year. However, since we are interested in measuring systemic risk, we need to capture the *joint* probability of default (JPoD). Since joint default probabilities cannot be directly observed, they must be inferred from individual asset values and

individual PoDs, for instance.

In this paper, we further extend the *Consistent Information Multivariate Density Optimization* (CIMDO) approach of Segoviano and Goodhart (2009) in order to obtain the joint density of bank equity returns and the financial cycle.⁴ CIMDO is a non-parametric procedure, based on the Kullback (1959) cross-entropy approach. In general, entropy approaches reverse the process of modeling data. Instead of assuming (possibly false) parametric probabilities to characterize the information contained in the data, these approaches use information from the data to infer unknown probability densities. Here, a multivariate density that characterizes equity returns and their interconnectedness structure is inferred from observed (but partial) information on individual financial institutions, i.e., their individual equity returns and individual PoDs.

Specifically, we augment the CIMDO approach by including the financial cycle into the joint density. This allows for the cross-sectional dimension of systemic risk (JPoD) and the cyclical dimension of systemic risk (financial cycle) to be coupled. Using this approach, we can obtain the joint density of bank equity returns conditional on the financial cycle and derive a joint probability of default from this conditional density. We call this measure the “conditional joint probability of default” or, in short, CoJPoD.

2.2 Method

To illustrate our measure, assume there are two banks with equity returns X_t and Y_t and the financial cycle Z_t , where $t = 1, \dots, T$ denotes the time period.⁵ Specifically, X_t , Y_t and Z_t are random variables, and x_t , y_t and z_t denote the realizations of the random variables in t . x , y , z are any possible values of those random variables.

As indicated, the CoJPoD extends the JPoD. Therefore, it is instructive to first define the JPoD. For two banks, it is defined as the integral over the default regions of an estimated joint probability density $p_{t+4|t}^*(x, y)$, measured in t for $t + 4$, meaning for one year ahead when using quarterly data:

$$\text{JPoD}_{t+4|t} = \int_{-\infty}^{x_d} \int_{-\infty}^{y_d} p_{t+4|t}^*(x, y) dy dx, \quad (1)$$

where x_d and y_d denote exogenously fixed default thresholds as we discuss below. *Joint* default

⁴Segoviano and Goodhart (2009) show that density forecasts based on the CIMDO approach perform better than forecasts based on parametric densities calibrated using the same information set when evaluated using an extension of the probability integral transform (PIT) criterion advocated by Diebold et al. (1998)

⁵Note that Segoviano and Goodhart (2009) model the inverted equity returns $-X_t$ and $-Y_t$ as they define the default region on the upper tail of the return distribution.

occurs when both equity returns fall below their respective default thresholds.

We define the CoJPoD on this basis. The CoJPoD is the joint probability of default (again, measured in t for $t + 4$), conditional on a value of the financial cycle:

$$\text{CoJPoD}_{t+4|t}(z) = \int_{-\infty}^{x_d} \int_{-\infty}^{y_d} p_{t+4|t}^*(x, y|z) dy dx, \quad (2)$$

where $p_{t+4|t}^*(x, y|z)$ is the one-year ahead ($t + 4$) conditional joint probability density, estimated using information until t . $p_{t+4|t}^*(x, y|z)$ describes the joint behavior of annual bank equity returns (in $t + 4$) conditional on a value of the financial cycle. Importantly, the conditioning value should conceptually refer to a future value of the financial cycle (in $t + 4$). In this paper, we condition the CoJPoD on different forecasts of the financial cycle, made in t for $t + 4$, denoted as $\hat{z}_{t+4|t}$.

Estimating the conditional joint probability density: We obtain the conditional joint probability density $p_{t+4|t}(x, y|z)$ following the principle of Bayes' law:

$$p_{t+4|t}(x, y|z) = \frac{p_{t+4|t}(x, y, z)}{p_{t+4|t}(z)}. \quad (3)$$

Below, we describe how we obtain each element of the ratio, i.e., $p_{t+4|t}(x, y, z)$ and $p_{t+4|t}(z)$.

$p_{t+4|t}(x, y, z)$: We estimate $p_{t+4|t}(x, y, z)$ in two steps. First, we define a prior probability density $q_t(x, y, z)$, which serves as the starting point to describe the joint distribution of banks' equity returns and the financial cycle. The prior probability density, similar to $p_{t+4|t}(x, y, z)$, can have a time-varying shape as indicated by the t -subscript. Specifically, in our specification, the correlation between the time series is allowed to change over time, reflecting the actual changes present in the data, including correlations between the financial cycle and banks' equity returns.⁶

Second, we minimize the distance between the prior density and $p_{t+4|t}(x, y, z)$ using the Kullback (1959) cross-entropy criterion, $C[p_{t+4|t}(x, y, z), q_t(x, y, z)]$, subject to three constraints. Specifically, let

$$C[p_{t+4|t}(x, y, z), q_t(x, y, z)] = \iiint_{-\infty}^{+\infty} p_{t+4|t}(x, y, z) \ln \left[\frac{p_{t+4|t}(x, y, z)}{q_t(x, y, z)} \right] dz dy dx. \quad (4)$$

⁶See section 3 for a discussion about our data set and model implementation.

Then, we obtain $p_{t+4|t}^*(x, y, z)$ by

$$\arg \min_{p_{t+4|t}(\dots)} C[p_{t+4|t}(x, y, z), q_t(x, y, z)] \quad (5)$$

subject to the three constraints

$$\begin{aligned} \int_{-\infty}^{x_d} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p_{t+4|t}(x, y, z) dz dy dx &= \text{PoD}_{t+4|t}^X, \\ \int_{-\infty}^{+\infty} \int_{-\infty}^{y_d} \int_{-\infty}^{+\infty} p_{t+4|t}(x, y, z) dz dy dx &= \text{PoD}_{t+4|t}^Y, \text{ and} \\ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p_{t+4|t}(x, y, z) dz dy dx &= 1, \end{aligned}$$

which enter the minimization problem via the Lagrange multipliers $\lambda_{t+4|t}^1, \lambda_{t+4|t}^2, \mu_{t+4|t}$.

$\text{PoD}_{t+4|t}^X$ and $\text{PoD}_{t+4|t}^Y$ are the one-year ahead (marginal) probabilities of default for bank X and bank Y , respectively. They are obtained exogenously as we discuss below. The PoDs add the forward-looking dimension to the CoJPoD, such that it measures the one-year ahead conditional joint probability of default. The idea to incorporate the PoDs relates to the fact that the information in equity returns, as captured by the parametric density $q_t(x, y, z)$, is limited and typically inconsistent with the PoDs (Segoviano and Goodhart (2009)). Similarly, the information provided by the PoDs is typically inconsistent with the equity returns. Therefore, an important step in the CIMDO methodology is to consistently link this information. Put differently, the information provided by both the PoDs and the equity returns are of major importance for the usefulness of the estimated joint probability distribution. As a solution to the above minimization problem, one obtains

$$p_{t+4|t}^*(x, y, z) = \exp \left(- \left[1 + \mu_{t+4|t}^* + \lambda_{t+4|t}^{1*} I_{[x < x_d]} + \lambda_{t+4|t}^{2*} I_{[y < y_d]} \right] \right) q_t(x, y, z), \quad (6)$$

where $I_{[x < x_d]}$ and $I_{[y < y_d]}$ are indicator functions that take a value of one, if the respective bank return is below the default threshold, and zero otherwise. $\lambda_{t+4|t}^{1*}$, $\lambda_{t+4|t}^{2*}$ and $\mu_{t+4|t}^*$ represent the Lagrange multipliers that solve the three constraints. They are obtained by substituting $p_{t+4|t}^*(x, y, z)$ into the three constraints and solving the system of equations numerically.

The intuition for the estimate $p_{t+4|t}^*(x, y, z)$ is that the adjustment term $\exp(-[1 + \mu_{t+4|t}^* + \lambda_{t+4|t}^{1*} I_{[x < x_d]} + \lambda_{t+4|t}^{2*} I_{[y < y_d]}])$ compresses and stretches the estimated prior distribution in certain

regions in order to achieve consistency with the three constraints.

$p_{t+4|t}(z)$: The estimated marginal probability density of the financial cycle $p_{t+4|t}^*(z)$ can be obtained by integrating the estimated joint probability density over x and y , i.e.,

$$p_{t+4|t}^*(z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p_{t+4|t}^*(x, y, z) dx dy. \quad (7)$$

However, since the adjustment term $\exp(-[1 + \mu_{t+4|t}^* + \lambda_{t+4|t}^{1*} I_{[x < x_d]} + \lambda_{t+4|t}^{2*} I_{[y < y_d]})$ does not vary over z , it holds that

$$p_{t+4|t}^*(z) = q_t(z), \quad (8)$$

i.e., the marginal probability density is the same as the marginal prior density $q_t(z)$. This is intuitive as (i) the marginal PoDs only add information for X_t and Y_t and (ii) both marginals must integrate to one, since they are proper probability density functions.

Below, in Proposition 1, we summarize one characteristic of the CoJPoD that is important for our empirical analyses.

Proposition 1: The CoJPoD decreases as the financial cycle increases, if bank equity returns correlate positively with the financial cycle.

Proof: For $CoJPoD(z) = \int_{-\infty}^{x_d} \int_{-\infty}^{y_d} p^*(x, y|z) dx dy$ assume that $p^*(x, y, z)$ (and therefore, $p^*(x, y|z)$) is a multivariate standard normal distribution.

More specifically, let $(X, Y, Z)' \sim N(\mu, \Sigma)$ with zero means and covariance matrix

$$\Sigma = \begin{pmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{12} & 1 & \rho_{23} \\ \rho_{13} & \rho_{23} & 1 \end{pmatrix}. \quad (9)$$

ρ_{13} and ρ_{23} denote the correlations of the financial cycle (Z) with the two bank equity returns (X and Y).

Next, let Σ be partitioned as

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma'_{12} & \Sigma_{22} \end{pmatrix}, \quad (10)$$

where $\Sigma_{11} = \begin{pmatrix} 1 & \rho_{12} \\ \rho_{21} & 1 \end{pmatrix}$, $\Sigma_{12} = \begin{pmatrix} \rho_{13} \\ \rho_{23} \end{pmatrix}$, and $\Sigma_{22} = 1$. This implies that

$$\begin{pmatrix} X \\ Y \end{pmatrix} \bigg|_{Z=z} \sim N(\bar{\mu}, \bar{\Sigma}), \quad (11)$$

with

$$\bar{\mu} = (0, 0)' + \Sigma_{12}\Sigma_{22}^{-1}(z - 0) = \begin{pmatrix} \rho_{13}z \\ \rho_{23}z \end{pmatrix} \quad (12)$$

and

$$\bar{\Sigma} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}' = \begin{pmatrix} 1 - \rho_{13}^2 & \rho_{12} - \rho_{13}\rho_{23} \\ \rho_{21} - \rho_{13}\rho_{23} & 1 - \rho_{23}^2 \end{pmatrix}. \quad (13)$$

This shows that the mean of the conditional distribution shifts along ρ_{13} and ρ_{23} as z increases. Therefore, if $\rho_{13} > 0$ and $\rho_{23} > 0$, the conditional distribution shifts to the upper right, away from the joint default region. The CoJPoD decreases as it is the integral of the conditional posterior over that region. ■

Note that Proposition 1 extends to our setting, i.e., when employing the CIMDO approach with multivariate t -distribution for the prior (see Section 3.2) under additional assumptions about the behaviour of the tails as the conditional mean shifts. The proof follows similarly.

Systemic risk amplification: Finally, we introduce a measure of how much a financial cycle downturn could amplify systemic risk relative to median developments, which we label ΔCoJPoD and define as the difference between the CoJPoD with a financial cycle forecast for its lower tail (such as its 1% quantile) and the CoJPoD with the financial cycle forecast for its median:

$$\Delta\text{CoJPoD}(1\%) = \text{CoJPoD}_{t+4|t}(\hat{z}_{t+4|t}^{1\%}) - \text{CoJPoD}_{t+4|t}(\hat{z}_{t+4|t}^{50\%}) \quad (14)$$

where the superscripts of $\hat{z}_{t+4|t}$ indicate that it is a forecast for the 1% or 50% conditional quantile, respectively.

3 Data and CoJPoD implementation

3.1 Data

We analyze conditional systemic stress measures for three jurisdictions: The euro area (EA), United Kingdom (UK) and United States (US). The choice of jurisdictions is motivated by the availability of continuous data for a sufficient number of large listed banks. This section describes our data set and empirical details of the model implementation.

Bank stock returns, marginal probabilities of default and default thresholds: We construct the banking sample for each jurisdiction as follows: We select all institutions with Moody’s Industry classifier N06 (Banks and Savings and Loans corporations)⁷ and at least 50bn USD in total assets for the EA and UK and 100bn USD total assets for the US (as of end-2018).

For the marginal PoDs, we use one-year expected default frequencies (EDFs) from Moody’s Credit Edge from 1992 (1990 for the US and UK) until 2020-Q2.⁸ In the construction of our CoJPoD measure we use default thresholds, such as x_d or y_d in our example above. We set these to the inverse t -CDF with five degrees of freedom (i.e., the inverse prior marginal CDF) evaluated at the time series average of the marginal default probability for each bank. Stock prices are obtained from Bloomberg.

We use end of quarter observations for all data. We require EDF data and stock price data to be available continuously over the sample period to obtain a balanced sample. This requirement significantly restricts the resulting banking samples but facilitates meaningful comparisons of systemic risk measures over time.⁹ Our resulting data set contains most of the largest publicly listed banks in each jurisdiction, with between four (UK) and 13 banks (US). Appendix 6 contains a list of the banks included for each jurisdiction.

Financial cycles: In this paper, we employ the financial cycle estimates – summarizing financial stability relevant common fluctuations in credit and asset prices – proposed by Schuler et al. (2015, 2020). Estimates are available since the 1970s (late 1980s for the euro area) at a

⁷This industry classification deliberately excludes security brokers and dealers such as Goldman Sachs or Morgan Stanley from the analysis.

⁸An alternative to using EDFs would be to obtain marginal default probabilities from CDS spreads. This would allow for some unlisted banks with sufficiently liquid CDS to be included in the analysis but would severely restrict the sample length as most bank CDS series start only in 2004.

⁹In particular, this requirement leads to the omission of some of the largest banks in each jurisdiction, such as HSBC or Credit Agricole, for which EDF data starts too late.

quarterly frequency. These financial cycles relate to the idea of leverage cycles (Geanakoplos (2010)) and capture system-wide balance-sheet leverage. Under this paradigm, risk buildup inherent to common expansions of credit and asset prices, as measured by the indicator of Schüler et al. (2020), may lead to financial instability – similar to the theory of leveraged asset price bubbles discussed by Jordà et al. (2015).

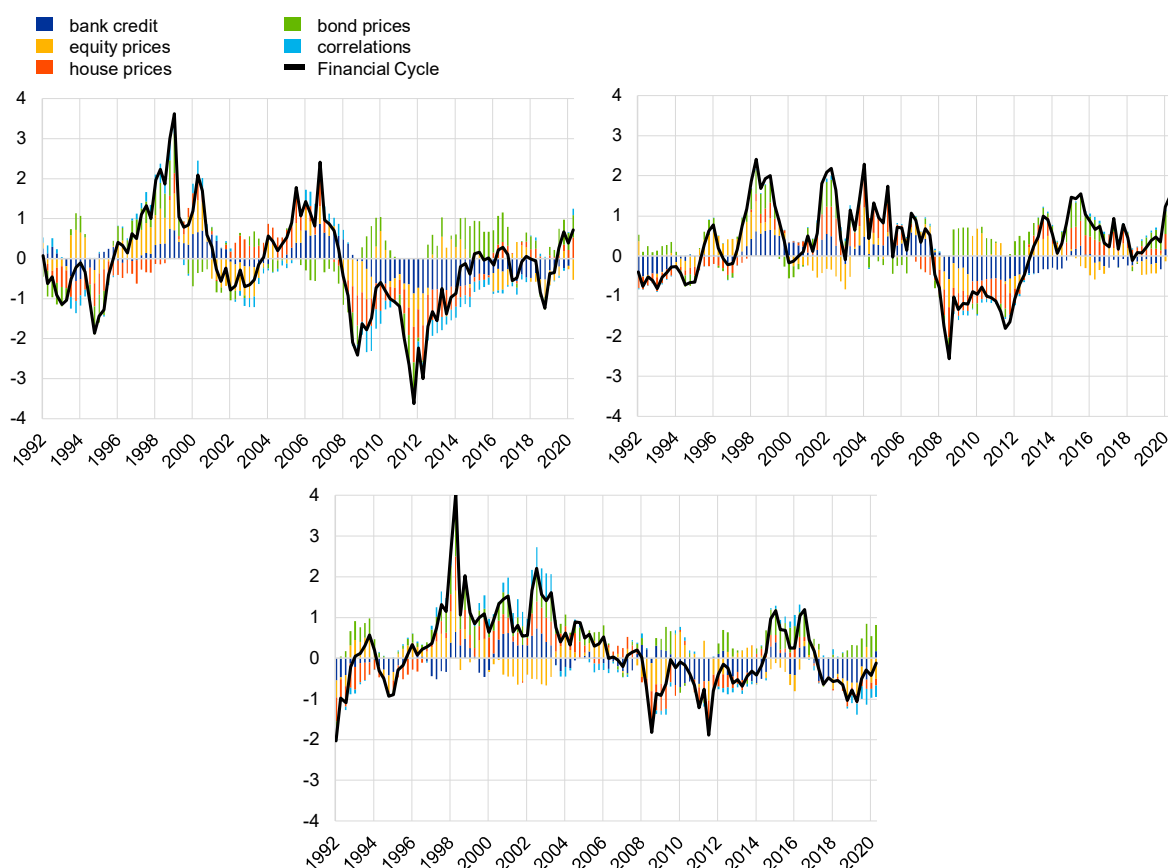


Figure 2: Financial cycle decompositions for the euro area (top-left), United States (top-right) and United Kingdom (bottom).

Notes: Each chart contains (t -transformed) financial cycle estimates based on Schüler et al. (2015) using annual growth rates and a decomposition into its components. The contributions of asset prices and credit correspond to standardized growth rates (divided by four) while the correlation component is obtained as the difference between the correlation-weighted financial cycle estimate and a version with equal weights for the four components. Last observation: 2020-Q2.

Credit is measured by total real bank credit to the non-financial private sector (households and non-financial firms). The set of real asset prices includes house prices, equity prices, and corporate bond prices. Methodologically, the indicator is constructed from standardized growth

rates of the above-mentioned components. Standardization is conducted using each variable's empirical cumulative distribution function in order to align the different equilibrium growth rates and variances of the underlying indicators, before aggregating them into a composite financial cycle. Aggregation is carried out using a time-varying linear combination of the standardized growth rates. The linear combinations take into account pairwise time-varying correlations between components, so as to emphasize the subset of variables that positively co-move strongest.¹⁰ Financial cycles for the three jurisdictions and a decomposition into their components are shown in Figure 2.

The results of Hartwig et al. (2021) suggest that low values of the financial cycle indicator signal high values of systemic risk, i.e., after the turning points of indicators have been observed. This is in line with the JPoD, where high probabilities signal high values of systemic risk. As shown by Figure 1, these phases of the two proxy measures do indeed coincide.¹¹

3.2 CoJPoD implementation

We implement the CoJPoD using a multivariate t -distribution as the prior. The multivariate t -distribution allows for fat tails, especially relevant for modelling the tail dependence between equity returns.

More specifically, we further assume that the prior distribution has zero mean, unit variance and five degrees of freedom (Segoviano and Goodhart (2009)). The time-varying dependency structure is captured via an estimated time-varying correlation matrix. We first estimate the time-varying covariance matrix Σ_t using an exponentially weighted moving average approach involving the time series of yearly bank equity returns and the financial cycle until time period t . To illustrate, consider the example of two banks provided above. For this setup, we obtain dynamic covariances as follows:

$$\hat{\Sigma}_t = (1 - \lambda)(x_t, y_t, z_t)'(x_t, y_t, z_t) + \lambda\hat{\Sigma}_{t-1}. \quad (15)$$

As is common for quarterly data, we choose $\lambda = 0.97$. Furthermore, we initialize $\hat{\Sigma}_t$ using

¹⁰The version of the financial cycle used in this paper is based on year-on-year growth rates of all variables, in line with the yearly horizon underlying PoDs.

¹¹The real-time use of the financial cycle indicator, and therefore our conditional systemic risk measures, is complicated by the substantial reporting lags of the underlying credit and house price series of between two and three months. However, real-time analysis could be facilitated by the use of forecasts for these variables or higher frequency proxies, such as data retrieved from online portals for housing purchases.

the first 22 observations. As a second step, the time-varying covariances are then transformed into correlations. We estimate the covariance and correlation matrices using annual bank stock returns and the t -transformed financial cycle using annual growth rates such that align with the one-year horizon of the PoDs. That is, to fit the prior density, we transform the financial cycle to follow a t -distribution with unit variance and five degrees of freedom before estimating Σ_t .¹² Since the time variation in the prior density is only driven by time-varying correlations, while variances are assumed constant, it implies that the marginal prior distribution of the financial cycle remains static.

We use a one-year ahead forecast of the financial cycle to condition on, such that it aligns with the interpretation of the CoJPoD, i.e., $\text{CoJPoD}_{t+4|t}(\hat{z}_{t+4|t})$. Specifically, we use forecasts for the median, 5% and 1% quantile obtained from an autoregressive process with two lags as conditioning values. The resulting forecasts are depicted in Figure 5 in the appendix.

The estimation of the CoJPoD (for every time period t) requires solving the conditional probability density, which is generally not available in closed form and requires piecewise evaluation of integrals along the default regions. A relatively efficient solution consists in first decomposing the joint prior as $q_t(x, y, z) = q_t(x, y|z)q_t(z)$, then resorting to closed form multivariate CDFs to evaluate $q_t(x, y|z)$ on all piecewise integrals and finally applying the adjustment terms. We found this solution superior to piecewise numerical integration of the joint probability density.

4 Results

4.1 Descriptive analysis

In Figure 3, we plot the CoJPoD conditioning on different forecasts of the financial cycle along with the JPoD (yellow line) and the financial cycle forecast for its median (dark blue line). The three CoJPoD series plotted in the chart relate to (i) the median-forecast of the financial cycle (orange line), (ii) the 5%-quantile forecast of the financial cycle (green line), and (iii) the 1%-quantile forecast of the financial cycle (light blue line). In Figure 4, we plot the ΔCoJPoD for different financial cycle scenarios. The red (yellow) line refers to the scenario with the 1% (5%) quantile forecast of the financial cycle. Here, the blue line refers to the JPoD.

The figures suggest that conditioning on the forecast of the lower tail of the financial cycle

¹²This transformation is achieved by first applying the empirical cumulative distribution function to transform the series to follow a uniform distribution and subsequently the inverse t -CDF.

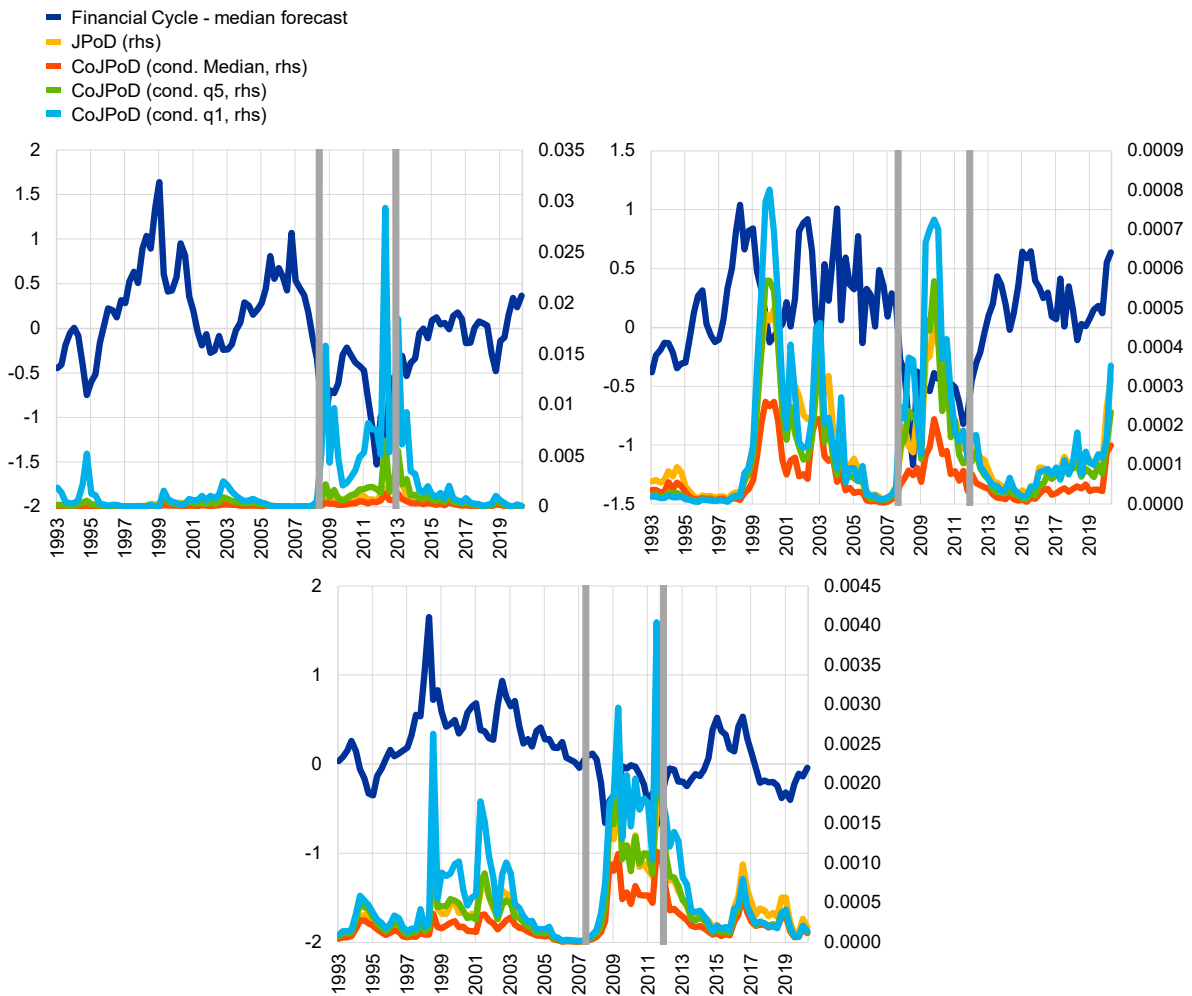


Figure 3: CoJPoD for the euro area (top-left), United States (top-right) and United Kingdom (bottom).

Notes: Each chart contains time series of the one-year ahead median forecast of the financial cycle (dark blue line), the JPoD (yellow line) and three series for the CoJPoD obtained using different one-year ahead forecasts of the financial cycle to condition on: the conditional median (orange line), the conditional 5% quantile (green line) and the conditional 1% quantile (light blue). Horizontal grey lines correspond to the first and last quarter of a systemic banking crisis as given by Laeven and Valencia (2018). Last observation: 2020-Q2.

leads to a strong amplification of the joint probability of default. Put differently, the CoJPoD, conditioned on the lower tail, signals strong amplification potential for the joint probability of default from further deterioration of the financial cycle in the future.

In this way, our approach helps to distinguish systemic events from non-systemic ones or regular periods, i.e., when the financial cycle forecast indicates benign developments. For instance, this amplification is particularly marked for the euro area during its sovereign debt crisis, as well

as the Global Financial Crisis for the euro area, United Kingdom and United States. In contrast, during the burst of the dot-com bubble for the euro area, the benign financial cycle implies that the CoJPoD(1%) remained relatively subdued, even though the JPoD rose relatively strongly, as shown by the dark blue line in Figure 4. Similar observations can also be made for the United States, for instance post 2003, when the financial cycle rises, or for the United Kingdom closely before 2017, again comparing the CoJPoDs from Figure 3 with the JPoDs from Figure 4.

The Δ CoJPoDs best summarize this amplification potential. Figure 4 shows that there are many periods where the JPoD jumps which do not translate into an amplification, given benign financial cycle forecasts.

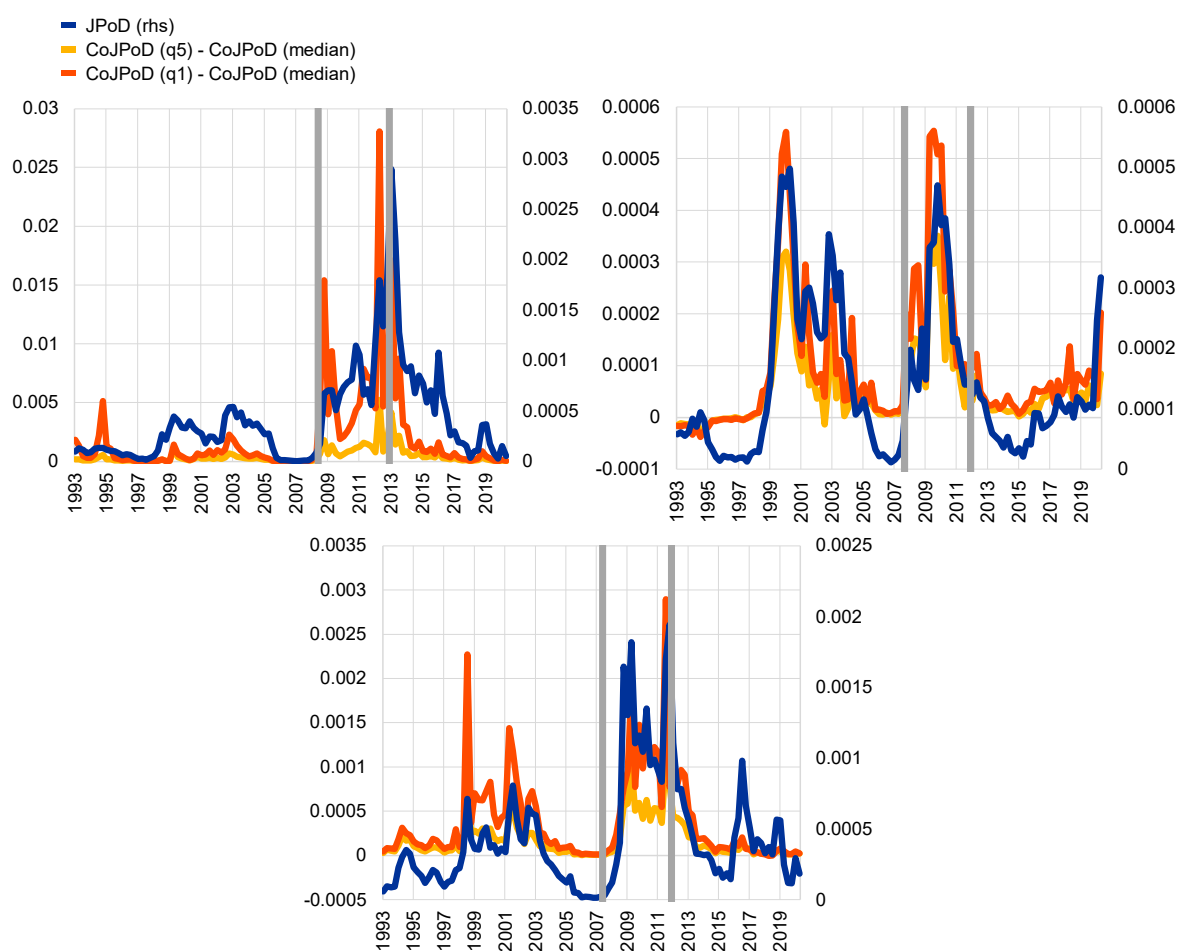


Figure 4: Δ CoJPoD for the euro area (top-left), United States (top-right) and United Kingdom (bottom).

Notes: Each chart contains the time series of the JPoD (dark blue line), Δ CoJPoD(5%) (yellow line) and Δ CoJPoD(1%) (orange line). For a definition of the Δ CoJPoD, please see Section 2. Horizontal grey lines correspond to the first and last quarter of a systemic banking crisis as given by Laeven and Valencia (2018). Last observation: 2020-Q2.

Overall, the visual inspection suggests that the CoJPoD and the Δ CoJPoD can be useful for improving the JPoD's ability to detect systemic crises. It filters out increases in the JPoD that do not have the potential to become systemic, i.e., if the financial cycle forecasts continue to remain high. On the contrary, a deteriorating financial cycle in the future will typically amplify the JPoD. Next, we analyze the properties of our proposed method more formally.

4.2 CoJPoD as an indicator of systemic banking crises

To analyze the CoJPoD's properties as an indicator of systemic banking crises more formally, we conduct a signaling exercise.

Setup: The signaling exercise uses the pooled sample of indicators across the three jurisdictions. The systemic banking crises for which indicators should provide early warning and coincident signals rely on the definition of Laeven and Valencia (2018), covering crises up to (including) 2019-Q4. As signalling events, we consider the entire crisis period (53 out of 332 observations), the onset (3 out of 282 observations), 1 to 4 quarters ahead of the onset (12 out of 279 observations) and 5 to 12 quarters ahead of the onset (24 out of 267 observations). For each of these events, we code a dummy variable that takes a value of 1 exactly on those dates and zero otherwise. For the events ahead of a crisis, we exclude the periods of financial crises (and the periods from the signalling event up to the crisis) so as to avoid a post-crisis bias (see, for example, Anundsen et al. (2016)). To analyze the onset, we exclude the other observations that fall within that crisis.

As is standard (see, for instance, Schularick and Taylor (2012)), we use the area under the curve (AUC) to evaluate the performance of indicators. Broadly speaking, the AUC summarizes an indicator's capacity to discriminate between events and non-events. It is defined to be between zero and one, whereby 0.5 indicates an indicator that performs similarly to a coin toss. A value of one indicates that an indicator can perfectly discriminate between an event and a non-event.

Finally, we test whether the AUC of an indicator at a specific horizon is significantly different to the AUC of the JPoD at that horizon. We do so because we are specifically interested in how the signalling performance of the CoJPoD compares to that of the JPoD. To test this, we use the test proposed by DeLong et al. (1988). It accounts for possible correlation between two curves.

Results: Table 1 reports the AUCs for the JPoD, the financial cycle, the different CoJPoDs, and Δ CoJPoDs across the different events. *, **, and *** indicate that the AUC of the respective indicator is significantly different to the AUC of the JPoD at the 10%, 5%, and 1% significance level.

The results for the JPoD and the financial cycle emphasize their well-known properties. On the one hand, the JPoD performs well with regard to indicating crisis regimes. It can discriminate between crisis and non-crisis periods in 80% of cases. It does not perform particularly well toward the onset of crises, i.e., periods directly preceding them. On the other hand, the financial cycle indicates vulnerability periods well, but not crisis regimes. For periods 5 to 12 quarters ahead, it correctly indicates the buildup of vulnerabilities in 72% of cases, significantly outperforming the JPoD.

However, the main result of this exercise is that our indicator fills an apparent gap in the signalling capacity of the JPoD and the financial cycle. In parallel, it improves on the capacity of the JPoD at all horizons. Specifically, our results suggest that the CoJPoD – but more so the Δ CoJPoD – indicates the onset of a systemic banking crises with higher precision than the JPoD. For instance, the Δ CoJPoD(1%) indicates the onset of systemic banking crises correctly in 62% of cases, significantly outperforming the JPoD at the 5% level.

The Δ CoJPoD(5%) significantly improves on the JPoD at all horizons. Even during crisis regimes – for which the JPoD receives its highest AUC – it significantly outperforms the JPoD.

Lastly, it is noteworthy that the lower the conditioning quantile, the better the performance of the CoJPoD and the Δ CoJPoD for the onset and the vulnerability periods.

Overall, this exercise confirms the usefulness of conditioning the cross-sectional dimension of systemic risk on its cyclical counterpart. It sharpens the identification of crisis regimes and improves our capacity for signalling crises upon their onset – as also highlighted when examining Figure 1 in the previous section. Conditioning the JPoD on the financial cycle allows for better discrimination between systemic and non-systemic events, thereby improving the indicators' signalling capacity. From a policy perspective, such information is particularly useful as it helps to inform the robust and timely release of time-varying prudential requirements.

Table 1: Results of signalling exercise: AUC

Event		Indicator and AUC		
		JPoD	Financial cycle	
During crisis		0.80	0.09***	
Onset		0.38	0.21	
1–4 quarters ahead		0.07	0.45***	
5–12 quarters ahead		0.16	0.72***	
		CoJPoD(50%)	CoJPoD(5%)	CoJPoD(1%)
During crisis		0.79	0.82	0.81
Onset		0.40	0.54	0.58
1–4 quarters ahead		0.09**	0.13***	0.19***
5–12 quarters ahead		0.15	0.15	0.17
		Δ CoJPoD(5%)	Δ CoJPoD(1%)	
During crisis		0.83**	0.81	
Onset		0.59*	0.62**	
1–4 quarters ahead		0.21***	0.25***	
5–12 quarters ahead		0.21**	0.23***	

Notes: This table reports the area under the curve (AUC) for the different indicators. *, **, and *** indicate that the AUC of the respective indicator significantly differs from the AUC of the JPoD at the 10%, 5%, and 1% significance level, using the test proposed by DeLong et al. (1988) that accounts for correlation between two curves. Within the brackets of the CoJPoD, we report which quantile forecast of the financial cycle we use as a conditioning value. For instance, CoJPoD(50%) indicates that we use the median financial cycle forecast. The Δ CoJPoD is the difference between the CoJPoD using the median financial cycle forecast and the CoJPoD using the quantile forecast as specified in brackets.

5 Conclusions

We propose an empirical method to effectively link the materialization of the cross-sectional dimension of systemic risk with latent risk inherent to its prior buildup along a cyclical dimension. We implement this by deriving the JPoD suggested by Segoviano and Goodhart (2009) – a proxy for cross-sectional systemic risk – conditional on the financial cycle proposed by Schuler et al. (2020) – a proxy for the cyclical dimension of systemic risk.

We find that the CoJPoD indicates financial crisis onsets and crisis regimes with greater precision than the JPoD or the financial cycle do individually. In particular, the Δ CoJPoD, a measure for the amplification potential of a deteriorating financial cycle, significantly improves on the JPoD, receiving an AUC of 24 percentage points higher (38% vs. 62%). At the same time, the (Δ) CoJPoD inherits – and significantly improves – the JPoD’s ability to accurately indicate crisis regimes. The intuition behind our result is that financial cycle downturns amplify banks’ joint probability of default. Similarly, financial cycle upturns dampen banks’ joint probability of default. In this way, our approach helps to distinguish systemic events from non-systemic ones, thereby increasing its capacity to signal the likelihood of financial crises.

Our methodology may be used to improve the precision with which macroprudential policies are set – in particular, when countercyclical time-varying prudential measures warrant release.

Conditioning commonly used indicators of banks' joint probability of default on the financial cycle would allow policy makers to better leverage signals obtained from cross-sectional measures of systemic risk, as financial conditions deteriorate. This is because default probabilities are reduced if the financial cycle indicator does not indicate elevated risks in the future. On this basis, our approach provides a useful insight into when countercyclical capital buffers should be released, as linking the two dimensions indicates the onsets of financial crises with greater precision.

The indicator constructed in this paper embeds several avenues for further work. One such avenue regards applications, such as counterfactuals relating banks' default probabilities to financial cycle developments – including stress testing based on extreme, but plausible, financial cycle scenarios. A second area is better optimizing the mix between structural and cyclical macroprudential policies – noting that a timely release of cyclical macroprudential requirements might curb the interplay of cyclical and cross-sectional elements of systemic risk, thereby reducing the severity of financial downturns. All in all, more work on linking the cross-sectional and cyclical dimensions of systemic risk, possibly involving different indicators, would enable a more holistic, encompassing and ultimately effective underpinning of macroprudential policy.

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6 Annex

Further properties of the CoJPoD

This annex contains derivations underpinning the CoJPoD.

Proposition 2: The unconditional JPoD is nested in the joint probability density and is the same as the JPoD derived from the bivariate system.

Proof:

$$\begin{aligned}
 \text{JPoD}_{t+4|t} &= \int_{-\infty}^{x_d} \int_{-\infty}^{y_d} \int_{-\infty}^{+\infty} \exp(-[1 + \mu_t^* + \lambda_{1,t}^* + \lambda_{2,t}^*]) q_t(x, y, z) dz dy dx \\
 &= \exp(-[1 + \mu_{t+4|t}^* + \lambda_{t+4|t}^* + \lambda_{t+4|t}^*]) \int_{-\infty}^{x_d} \int_{-\infty}^{y_d} \int_{-\infty}^{+\infty} q_t(x, y, z) dz dy dx \\
 &= \exp(-[1 + \mu_{t+4|t}^* + \lambda_{t+4|t}^* + \lambda_{t+4|t}^*]) \int_{-\infty}^{x_d} \int_{-\infty}^{y_d} q_t(x, y) dy dx \\
 &= \int_{-\infty}^{x_d} \int_{-\infty}^{y_d} \exp(-[1 + \mu_{t+4|t}^* + \lambda_{t+4|t}^* + \lambda_{t+4|t}^*]) q_t(x, y) dy dx.
 \end{aligned}$$

This is the same definition of the JPoD as in Segoviano and Goodhart (2009) for a bivariate banking system. The property holds because $q(x, y, z)$ has $q(x, y)$ as its bivariate marginal and the adjustment term does not depend on z . ■

Proposition 3: The unconditional JPoD is identical to the expectation of the CoJPoD across all possible values of the financial cycle.

Proof:

$$\begin{aligned}
 \text{JPoD}_{t+4|t} &= \int_{-\infty}^{x_d} \int_{-\infty}^{y_d} \int_{-\infty}^{+\infty} p_{t+4|t}^*(x, y, z) dz dy dx \\
 &= \int_{-\infty}^{x_d} \int_{-\infty}^{y_d} \int_{-\infty}^{+\infty} p_{t+4|t}^*(x, y|z) p_{t+4|t}^*(z) dz dy dx \\
 &= \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{x_d} \int_{-\infty}^{y_d} p_{t+4|t}^*(x, y|z) dy dx \right] p_{t+4|t}^*(z) dz \\
 &= \int_{-\infty}^{+\infty} \text{CoJPoD}_{t+4|t}(z) p_{t+4|t}^*(z) dz \blacksquare
 \end{aligned}$$

Bank sample

EURO AREA:

DEUTSCHE BANK AG	DE0005140008
COMMERZBANK AG	DE000CBK1001
BANCO BILBAO VIZCAYA ARGENTARIA SA	ES0113211835
BANCO SANTANDER SA	ES0113900J37
SOCIETE GENERALE SA	FR0000130809
INTESA SANPAOLO SPA	IT0000072618
UNICREDIT SPA	IT0005239360

UNITED KINGDOM:

STANDARD CHARTERED PLC	GB0004082847
LLOYDS BANKING GROUP PLC	GB0008706128
BARCLAYS PLC	GB0031348658
NATWEST GROUP PLC	GB00B7T77214

UNITED STATES:

THE BANK OF NEW YORK MELLON CORP.	US0640581007
JPMORGAN CHASE & CO.	US46625H1005
FIFTH THIRD BANCORP	US3167731005
REGIONS FINANCIAL CORP.	US7591EP1005
M&T BANK CORP.	US55261F1049
U.S. BANCORP	US9029733048
HUNTINGTON BANCSHARES, INC.	US4461501045
BANK OF AMERICA CORP.	US0605051046
NORTHERN TRUST CORP.	US6658591044
WELLS FARGO & CO.	US9497461015
THE PNC FINANCIAL SERVICES GROUP, INC.	US6934751057
KEYCORP	US4932671088
STATE STREET CORP.	US8574771031

Financial cycle forecasts

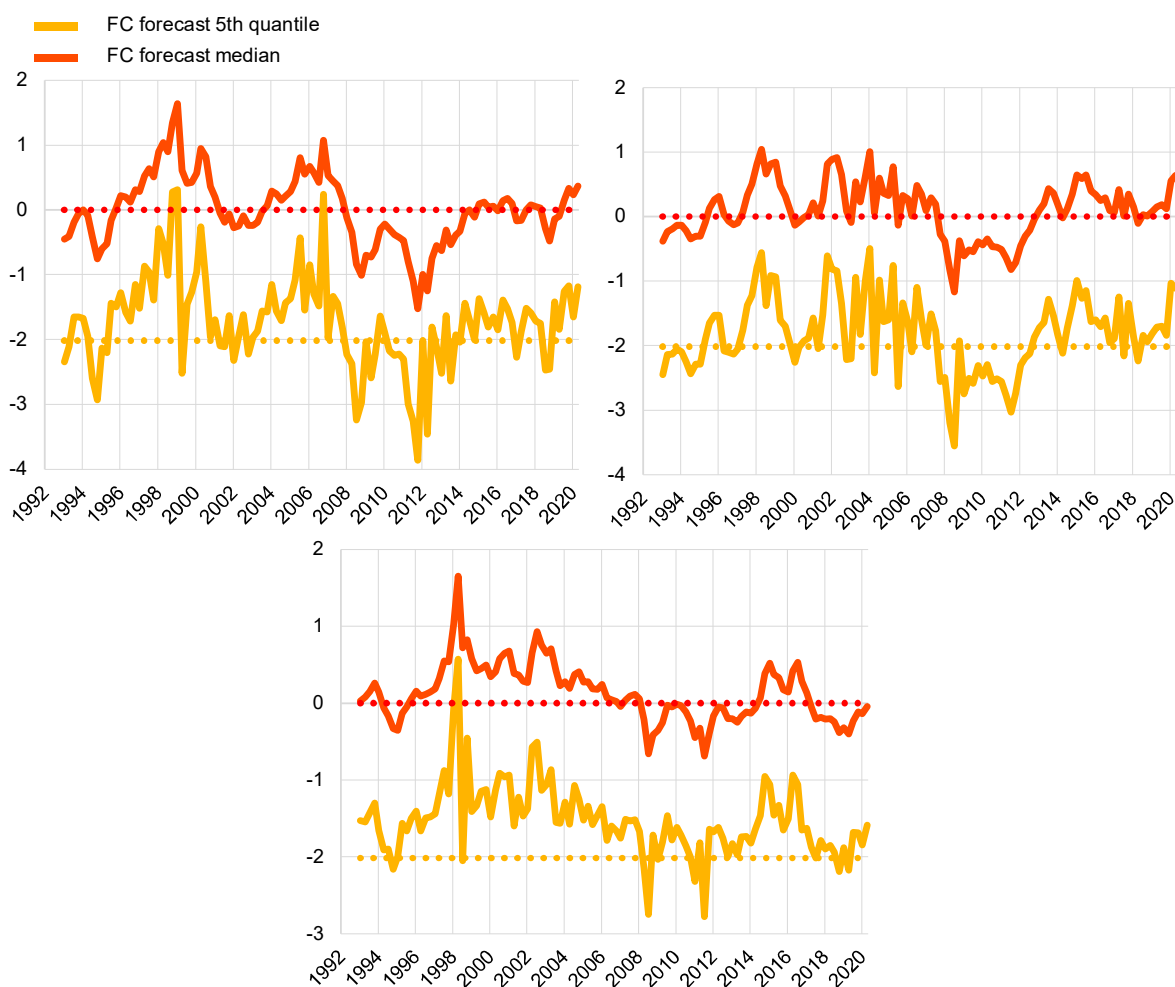


Figure 5: Financial cycle forecasts for the euro area (top-left), United States (top-right) and United Kingdom (bottom).

Notes: Each chart contains one year ahead (t -transformed) financial cycle forecasts for the 5th quantile and median based on quantile autoregressions with two lags. These forecast serve as conditioning values for the estimation of the CoJPoDs. Last observation: 2020-Q2.

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