

Ponthiere, Gregory

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Epictetusian Rationality and Evolutionary Stability

Gregory Ponthiere*

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Abstract

An economic interpretation of Epictetus's precept of 'Taking away aversion from all things not in our power' consists of extending the domain of indifference beyond its boundaries under non-ethical preferences, so as to yield indifference between outcomes differing only on things outside one's control. This paper examines the evolutionary dynamics of a population composed of Nash agents and Epictetusian agents matched randomly and interacting in the prisoner's dilemma game. It is shown that, whether or not the types of players are common knowledge, neither the Nash nor the Epictetusian type is an evolutionary stable strategy under perfectly random matching. However, if the matching process exhibits a sufficiently high degree of assortativity, the Epictetusian type is an evolutionary stable strategy, and drives the Nash type to extinction.

Keywords: ethical preferences, evolutionary stability, cooperation, prisoner's dilemma, Epictetus.

JEL classification codes: C73, C62, D60.

*UCLouvain, Hoover Chair in Economic and Social Ethics and Global Labor Organization (GLO). Address: Collège Dupriez, Office D307, 3, Place Montesquieu, 1348 Louvain la Neuve, Belgium. E-mail: gregory.ponthiere@uclouvain.be

1 Introduction

Economists have recently paid increasing attention to ethical preferences. Ethical preferences include Kantian preferences (Laffont 1975, Roemer, 2010, 2019), and, also, a convex combination of Kantian rationality with Nash rationality known as the *homo moralis* (Alger and Weibull, 2013, 2016). Kantian agents differ from standard decision-makers by selecting the best *generalizable* strategy, instead of the best reply to others's strategies taken as given.¹ This feature has key implications on how agents interact in cooperation games. By focussing only on the diagonal of the normal form of the game, Kantian agents choose to cooperate, which allows them to avoid the prisoner's dilemma (Curry and Roemer 2012). Introducing Kantian agents in a population of Nash agents affects also the design of optimal policies. Standard policy instruments can, by reducing the proportion of Kantian agents in the population, worsen the situation with respect to the laissez-faire (Bezin and Ponthiere 2019).

Whereas economists's study of ethical preferences has so far focused mainly on Enlightenment Philosophies, such as Kantianism, there is *a priori* no reason to limit the study of ethical preferences to these philosophies. Actually, philosophers have, across centuries, proposed various precepts aimed to guiding individuals in their everyday life, and the history of ethics spans several millennia (see MacIntyre 1971). Therefore, the economic analysis of ethical behavior does not need to be restricted to Kantian behavior and the underlying idea of generalizability. Other, possibly older, moral philosophies can also allow economists to better understand what ethical preferences are.

From that perspective, Ancient philosophies have remained so far largely ignored by economists. One possible reason explaining this may lie in the fact that these Ancient systems of thought are far more distant from economic thought than 18th century philosophies such as Kantianism.² It should be stressed, however, that in both cases, philosophers did not formulate their ethical precepts in terms of the economic concept of 'preferences', so that in any case a kind of 'translation' or 'formalization' exercise has to be done to incorporate the insights of early ethical thoughts within economic reasoning. Such an exercise of thought is necessary to be able to examine the consequences of ethical behavior on economic outcomes, and, then, on the optimal design of public policies.

This paper proposes to contribute to the study of ethical preferences and of their consequences, by focusing on a particular kind of Ancient ethical thought: Stoicism, and, more precisely, the ethical doctrine defended by the Stoic philosopher Epictetus.³ In the *Manual*, Epictetus argued that, in order to reach mental

¹This behavior is in line with Kant's categorical imperative 'universalize your maxim' (see Kant 1785).

²One important reason explaining this extra distance between Ancient philosophies and economic thought lies in the fact that Ancient philosophies generally define what the 'good life' is in relationship with the position of the person in the Universe or Cosmos (see MacIntyre 1971, chapter 8).

³Epictetus (55 AC - 135 AC) is a Greek philosopher born in Phrygia. He served as a slave for many years under the Roman Empire, and kept, during the remaining of his life, some bodily stigma of bad treatments imposed by his master. While having a miserable life,

freedom and happiness, individuals should distinguish between things that are under their control (their judgements, desires and acts) and things that are not under their control (their body, properties, social reputation, others's acts), and *wish for nothing that is not under one's control*.⁴ Epictetus's precept is best summarized in this quote from the *Manual* (I, II):

Of things some are in our power, and others are not. In our power are opinion, movement towards a thing, desire, aversion, turning from a thing; and in a word, whatever are our acts. Not in our power are the body, property, reputation, offices (magisterial power), and in a word, whatever are not our own acts. [...] Take away then aversion from all things which are not in our power, and transfer it to the things contrary to nature which are in our power.

'Taking away aversion from all things not in our power' can be interpreted in various ways. A plausible economic interpretation consists of interpreting Epictetus's precept as a requirement of *extending the symmetric factor of the preference relation beyond its boundaries under non-ethical preferences*. This interpretation amounts to regard Epictetus's Stoicism as an ethical doctrine recommending the extension of the 'domain of indifference' beyond its boundaries under non-ethical preferences.⁵

While there exist several ways to extend the 'domain of indifference', one way of formalizing Epictetus's precept is the *indifference account*: individuals should be indifferent between outcomes that differ only on circumstances (circumstances being defined as things not under one's control).⁶ This interpretation of Epictetus's precept does justice to the idea according to which things outside control are 'neither good nor bad', and, thus cannot make a person either better off or worse off (*Manual*, XXXII):

For if it is any of the things which are not in our power, it is absolutely necessary that it must be neither good nor bad.

Epictetusian rationality, that is, 'wishing for nothing that is outside control', looks like an intuitive rule of behavior. But what would be the implications of a fraction of the population adopting this rule of behavior in strategic environments? Would Epictetusian players benefit from some evolutionary advantage in interacting with standard Nash players? Or, alternatively, is it the case that Nash players would lead Epictetusian players to extinction?

Epictetus developed some practical wisdom, and gave pieces of advice to a large number of citizens. Epictetus's precepts were transcribed by his follower Arrian. This transcription is at the origin of what is known as the *Manual* or the *Handbook*, which is often regarded as the first textbook in philosophy.

⁴On Epictetus's ethical thought, see Clark (1995), Long (2002) and Graver (2009).

⁵This economic interpretation of Epictetus's precept is developed in Ponthiere (2022).

⁶The indifference account is not the only possible interpretation of Epictetus's precept. Another possible formalization is indifference between outcomes involving the best reply to circumstances. But the indifference account exhibits some clear advantage over this alternative interpretation. In particular, it has the virtue to exclude adaptive and counteradaptive preferences. See Ponthiere (2022).

This paper examines the evolutionary dynamics of a population composed of Nash agents and Epictetian agents matched randomly and interacting in the prisoner’s dilemma game. We first study the outcome of a two-player prisoner’s dilemma game played by Epictetian agents and by Nash agents drawn randomly. We then examine whether the Nash behavior or the Epictetian behavior is an evolutionary stable strategy (ESS) in the sense of Maynard-Smith (1982, 1989). Finally, we relax the assumption of perfectly random matching, and examine the existence of an ESS in that more general framework.

Anticipating our results, it is shown that, whether or not the types of players are common knowledge, neither the Nash type nor the Epictetian type is an ESS under perfectly random matching. No type can drive the other type to extinction. Using the discrete time replicator’s dynamics (Cressman 2003), we show that the equilibrium partition of the population is locally stable, and that the long-run proportion of Epictetian agents is decreasing in the gain of Nash player when they defect while the other player cooperates. However, when examining a more general setting with imperfectly random matching, we show that, if the matching process exhibits a sufficiently high degree of assortativity, the Epictetian type is an evolutionary stable strategy, and drives the Nash type to extinction.

The present paper is related to several branches of the literature. First, it is related to the growing literature on ethical preferences (Laffont 1975, Roemer 2010, Roemer 2019, Alger and Weibull 2013, 2016).⁷ While that literature focused mainly on Kantian preferences or on convex combinations of Kantian preferences with standard preferences, our contribution to that literature is to study another form of ethical preferences, inspired by the Stoic philosopher Epictetus. This other form of ethical preferences relies on the idea of extending the domain of indifference of persons beyond its boundaries under non-ethical preferences. Second, this paper can also be related to the previous study by Ponthiere (2022) on the formalization of Epictetian rationality. That study examined, within a static framework, several plausible formalizations of Epictetus’s precept of ‘wishing for nothing that is not under one’s control’, and analyzed the conditions under which a preference relation on outcomes can satisfy Epictetus’s precept while being complete, reflexive and transitive. The main contribution of the present paper with respect to Ponthiere (2022) is to consider Epictetian rationality in a dynamic setting (instead of a static setting), and, also, to study interactions between Epictetian agents with other agents (instead of focusing only on Epictetian agents). Third, this paper is also linked to the literature on evolutionary stability under diverse kinds of preferences (Bergstrom and Stark 1993, Curry and Roemer 2012, Alger and Weibull 2013, 2016).

This paper is organized as follows. As a preliminary step for our explorations, Section 2 presents key elements of Epictetus’s ethical thought. Section 3 studies the behavior of Epictetian agents playing a prisoner’s dilemma. Section 4 examines the outcomes of prisoner’s dilemmas played by pairs of (either Nash

⁷Other recent works on ethical preferences include Curry and Roemer (2012), Van Long (2015), Bezin and Ponthiere (2019) and De Donder et al (2021).

or Epictetian) players drawn randomly in the population, under two distinct informational settings (with or without common knowledge on the opponent's type). Section 4 also studies the existence of ESS in our framework. Section 5 explores the long-run dynamics of the partition of the population into Nash and Epictetian players under the discrete time replicator's dynamics. Section 6 examines the robustness of our results to assuming imperfectly random matching. Concluding remarks are left to Section 7.

2 Epictetus's ethics

Unlike modern philosophers, Epictetus did not synthesize his ethical thought in a treatise, but the *Manual* - based on the transcription of Epictetus's lectures by his student Arrian - allows us to draw the contours of his ethical thought. Following the Stoic tradition, Epictetus defended a particular conception of the 'good life' known as *apatheia*.⁸ According to Epictetus, a 'good life' is a life that is coherent with what the person is, that is, a life consistent with her status and her position as an infinitely small part of the Universe.⁹

Stoic philosophers - including Epictetus - argue that a 'good life' requires respecting three disciplines of the 'interior discourse', disciplines that allow the person to have a life that is coherent with what she is: (1) the discipline of judgements; (2) the discipline of desires; (3) the discipline of acts.¹⁰

The discipline of judgements The discipline of judgements includes several dimensions. First, this consists of clearly *identifying the boundaries of the self*, that is, of separating things that belong to the self (and are under the control of the person) and things that are external to the self (and are not under her control).¹¹ According to Stoic philosophers, the self is limited to the 'intellect' of the person, and includes her judgements, desires/aversions and impulses to acts or acts. Other things, such as the person's body or emotions, do not belong to the self of the person, that is, to what Hadot (2001) calls, following Marcus Aurelius, the 'inner citadel' of the person.¹² Second, once the self of the person is properly delimited, the person should take some distance with respect to her primary representations and judgements, and learn to be critical with respect to these. According to Stoic philosophers, events do not hurt persons, but only the judgements made by persons on these events can hurt them.¹³ The

⁸See MacIntyre (1971, chapter 8).

⁹Moore (1903) and Sidgwick (1906) interpreted Stoicism as a metaphysical ethical doctrine recommending 'Living with Nature', and criticized that doctrine for its circularity. Stoic philosophers defend this circularity as a form of internal consistency.

¹⁰See Hadot (2001). These three disciplines are present in the works of all Stoic philosophers. However, in Epictetus's works, the last two disciplines are hard to distinguish.

¹¹This task is regarded as a fundamental dogma of Stoicism. Its importance is revealed by the fact that Epictetus's *Manual* begins precisely with the distinction between things under control and things outside control (see *Manual*, I).

¹²See Hadot (2001), chapter 6.

¹³See Epictetus, *Manual*, X.

key aspect of the discipline of judgements consists of always *valuing and judging things from the perspective of the Universe* (the ‘Great Everything’). How to value these things is the object of the discipline of desires.

The discipline of desires The discipline of desires requires to *wish for nothing that is not under one’s control*, that is, ‘indifference to indifferent things’. Epictetus’s precept ‘Take away then aversion from all things which are not in our power’ (*Manual*, II) is Epictetus’s own reformulation of the Stoic discipline of desires. A person should wish for nothing that is not under her control (such as her body, properties, reputation and events), but, rather, should reorient her desires and aversions towards things that are under her control (her judgements and acts). The discipline of desires takes its roots in the requirement of judging things from the perspective of the Universe.¹⁴ The reasoning is the following. A person is an infinitely small part of the Universe, which is composed of myriad of things on which she has no control. Things external to the person were not ‘produced’ by the person, but by the Universe. Hence, the person should be indifferent to these things. Otherwise, by (dis)liking other parts of the Universe, the person would break the coherence of the Universe.¹⁵

The discipline of acts The discipline of acts is strongly related to the disciplines of judgements and desires. This states that *for the things that depend on themselves, persons should act in the way that best serves the interests of the City, i.e., the Common Good*.¹⁶ The discipline of acts requires persons to act in the way that is the most appropriate, that is, that best fits the interests and values of the City. This means that persons should dedicate their time and their energy to activities that contribute, given their own skills and talents, to best serve the interests of the City. The discipline of acts is also based on an ideal of living a life that is coherent with what the person is. Coherency is here achieved by choosing acts that, given the skills and talents received by the person, best serve the whole to which the person belongs, that is, the City. In other words, the discipline of acts guarantees that the person, as a part of the City, does her best to promote the whole to which she belongs. Only this consistency between the whole and each of its parts can contribute to the Common Good.

The good life When a person respects the three disciplines of the ‘interior discourse’, that person is regarded as virtuous. According to Stoic philosophers, virtue is unique and indivisible: one cannot be partly virtuous, but must be

¹⁴See Hadot (2001), chapter 7.

¹⁵This argument relies on the idea of a coherent Universe, to which Stoic philosophers such as Epictetus and Marcus Aurelius adhered. It should be stressed, however, that Stoic philosophers defend also that Stoicism is the only reasonable ethical attitude even in the absence of such a coherent world. Clearly, if the world is a set of purely random things, it is still rational, by means of an appropriate discipline of desires, to preserve one’s ‘inner citadel’ from this chaotic environment. Here again, ‘wishing for nothing that is not under one’s control’ makes a lot of sense. Stoic philosophers recommend that persons do not play at the casino with their happiness. See Hadot (2001, chapter 7).

¹⁶See Hadot (2001), chapter 8.

either virtuous or not.¹⁷ Note that Stoic philosophers associate some virtues to the three disciplines of the ‘interior discourse’: prudence is usually associated with the discipline of judgements, courage and temperance are associated with the discipline of desires, and justice is associated with the discipline of acts.¹⁸ From a Stoic perspective, a virtuous life is regarded as equivalent to a ‘good life’. Stoic philosophers, including Epictetus, reduce goodness to moral goodness and badness to moral badness. This means that *the only way for the life of a person to be good is to be a virtuous life*, that is, the person should satisfy the three Stoic disciplines. There is no other way in which a life can be good.¹⁹

3 Epictetian rationality

How can one translate Epictetus’s ethics into the language of modern economics? A plausible economic interpretation of Epictetus’s thought consists of interpreting this doctrine as recommending persons to extend what we could call the ‘domain of indifference’, that is, as encouraging persons to extend the symmetric factor of the preference relation beyond its boundaries under standard (non-ethical) preferences.²⁰ This extension of the ‘domain of indifference’ is an economic formalization of the Stoic discipline of desires consisting of promoting ‘indifference to indifferent things’.²¹

Following this interpretation, Epictetus’s precept can be formalized as a requirement on the symmetric factor of a preference relation on outcomes defined as pairs (acts, circumstances), where acts are under control, whereas circumstances are not under control. One way of formalizing Epictetus’s precept is the indifference account: *a person should be indifferent between outcomes differing only on circumstances*. The indifference account of Epictetus’s precept does justice to the Stoic discipline of desires: circumstances, being external to the ‘inner citadel’ of the person, cannot make her life better or worse.

Epictetus’s indifference account has important implications in game-theoretical contexts. To see this, let us first turn back to the basic of Stoicism, that is, the discipline of judgements. If one applies the Stoic discipline of judgements, one can see that other persons’s acts lie outside the boundaries of the self of the person. The acts of other players - being ‘produced’ by other persons - are typically outside the control of a player. According to Epictetus, only the own acts of a person are within the self of that person and under her control, but not other persons’s acts.²² Therefore, the Stoic discipline of desires - ‘Take away

¹⁷Note that a virtue is something that is pursued for its own sake, independently from consequences. Stoicism, like Epicureanism, belongs to the ethics of virtue, in contrast with consequentialist ethics (such as utilitarianism). See MacIntyre (1971), chapter 8.

¹⁸See Hadot (2001), chapter 9.

¹⁹Thus an event, even detrimental to a person, cannot make her life bad, because the goodness or badness of a life depends only on whether or not the person is virtuous in her life.

²⁰See Ponthiere (2022) on this interpretation of Stoicism.

²¹That economic interpretation of Stoicism is actually supported by interpretations of contemporary philosophers considering that Stoicism is a doctrine recommending ‘indifference to indifferent things’. See Hadot (2001), chapter 4.

²²See Epictetus, *Manual*, II.

then aversion from all things which are not in our power' - recommends that the person should be indifferent with respect to the acts of other persons. This is precisely what is recommended by the indifference account of Epictetus's precept: *an Epictetusian player should be indifferent between outcomes of the game that involve the same act for himself, but distinct acts for other players.*

At first glance, one may be suspicious of this result: if all players are indifferent between all outcomes of the game where they play the same strategy, this kills out all kinds of 'strategic interactions' among players, which is the main topic of game theory. But the fact that Epictetusian rationality - in particular the discipline of desires requiring indifference to things not under control - rules out the existence of strategic interactions among persons constitutes a result on its own, which is worth being underlined. This result highlights that what game theorists call 'strategic interactions' are not ahistorical phenomena, which would exist under all kinds of representations and rationalities. Actually, persons who adopt the three Stoic disciplines of the interior discourse presented in Section 2, will not give rise to 'strategic interactions' as usually defined by game theorists. Indeed, Epictetusian agents reduce goodness to moral goodness, and badness to moral badness. From that perspective, the only way for a life to be good is to have a virtuous life, independently from what other persons do. This explains that, under Epictetusian rationality, strategic interactions cannot occur, unlike what happens with standard decision-makers.

In order to examine the consequences of adopting Epictetusian rationality in games, let us study a simple two-player simultaneous prisoner's dilemma. A two-player prisoner's dilemma (PD) takes the following form, where numbers in brackets indicate pay-offs for players A and B, respectively:²³

		Player B	
		cooperate	defect
Player A	cooperate	(1, 1)	(a, b)
	defect	(b, a)	(0, 0)

Table 1. A two-person prisoner's dilemma.

A key feature of the PD game is that the outcome of the game where each player cooperates Pareto-dominates the outcome where each player defects. In Table 1, this feature is captured by the fact that (cooperate, cooperate), with pay-offs (1, 1), Pareto-dominates (defect, defect), with pay-offs (0, 0). Throughout this paper, we assume that it is common knowledge to all players of the game that the outcome where everyone cooperates Pareto-dominates the outcome where everyone defects.

Let us now consider how an Epictetusian agent would play a PD game. The indifference account of Epictetus's precept recommends indifference between outcomes that differ only on circumstances. Treating the other player's acts

²³See, for instance, Curry and Roemer (2012).

as circumstances, this implies that an Epictetian player should be indifferent between all outcomes of the game where he plays the same strategy. Assuming that player A is Epictetian, this implies that the pay-off parameters a and b in Table 1 should satisfy:

$$a = 1 \text{ and } b = 0$$

The reason why Epictetian rationality requires $a = 1$ goes as follows. From an Epictetian perspective, things outside one's control are 'neither good nor bad'. *The only thing that matters for a person is how the person herself behaves.* In this game, the only thing that matters for player A is whether player A cooperates or defects. Therefore player A must be indifferent between all outcomes of the game where she cooperates, that is, between, on the one hand, the outcome where all players cooperate, and, on the other hand, the outcome where she cooperates and player B defects. Given that the pay-off for player A under (cooperate, cooperate) is equal to 1, it follows from Epictetus's precept that the player A's pay-off under (cooperate, defect) must also be equal to 1.²⁴

The same rationale explains why the pay-off b obtained by player A under the outcome where player A defects and player B cooperates is, from an Epictetian perspective, equal to the pay-off obtained by player A under the outcome (defect, defect), that is, 0. Otherwise, if $b \neq 0$, player A would not be indifferent between outcomes differing only on things not under one's control, and, hence, would violate the Stoic discipline of desires.

Substituting for $a = 1$ and $b = 0$ in the pay-off matrix, while supposing that both players A and B are Epictetian, the pay-off matrix becomes:

		Player B	
		cooperate	defect
Player A	cooperate	(1, 1)	(1, 0)
	defect	(0, 1)	(0, 0)

Table 2. The game of Table 1 played by 2 Epictetian players.

An immediate implication of Epictetian rationality is that for each player, *cooperating is the dominant strategy.* Indeed, once it is acknowledged that the outcome where every player cooperates is better, for each player, than the outcome where every player defects, the indifference account of Epictetus's precept implies that cooperating is the dominant strategy, and that defect is a dominated strategy for each Epictetian player. This result follows from the Stoic discipline of desires, which reduces goodness to moral goodness, and badness to moral badness (see Section 2). *From a Stoic perspective, the only way to achieve a good life is through one's own acts, which have to be virtuous.* This explains why cooperate is the dominant strategy for Epictetian agents, that is, the

²⁴Otherwise, if $a \neq 1$, player A would not be indifferent between outcomes of the game differing only on circumstances.

best strategy for them no matter what other players do. As a consequence of this, the outcome of the game is (cooperate, cooperate).

Proposition 1 *When two Epictetian players play the game of Table 1, (cooperate, cooperate) is the dominant strategy equilibrium, and the pay-offs are (1, 1).*

Proof. See above. ■

Another implication worth noticing is that the game played by Epictetian players can no longer be regarded as a prisoner’s dilemma: indeed, a prisoner’s dilemma is, by definition, a game where a Nash equilibrium is Pareto-dominated by an outcome of the game that is not a Nash equilibrium. This is not the case in the game described in Table 2, where the unique Nash equilibrium, which is (cooperate, cooperate), cannot be Pareto-dominated by any other outcome of the game. Indeed, the pay-off matrix of Table 2, which was implied by the indifference account of Epictetus’s precept, rules out the possibility where an outcome of the game that is not a Nash equilibrium can Pareto-dominate a Nash equilibrium. Hence, in some sense, we can say that Epictetian rationality, when generalized, makes the prisoner’s dilemma vanish.

Note that the reason why Epictetian rationality makes the prisoner’s dilemma vanish differs from the one prevailing under other ethical preferences, such as Kantian preferences (Curry and Roemer 2012). Under Kantian preferences, agents must choose the best generalizable act, that is, they must focus only on the diagonal of the game. This exclusive focus on generalizable acts makes them choose cooperation. Here, under Epictetian rationality, the mechanism that makes agents cooperate consists of the *extension of the boundaries of the domain of indifference*: agents must regard all outcomes where they cooperate as equally good (whatever the other player does), and, also, they must regard all outcomes where they defect as equally bad (whatever the other player does).²⁵ These indifference relations imply, under our assumption on the pay-off matrix, that cooperation is the dominant strategy. Thus the extension of the domain of indifference provides a path out of the prisoner’s dilemma, which differs from the one prevailing under Kantian preferences.

It should be stressed, however, that we focused here on the outcome of the game when all players are Epictetian. The next section will focus on games where players are not necessarily Epictetian, and explore the implications of the outcome of the game for the dynamics of heterogeneity in the population.

4 Natural selection

Let us consider a large population of agents who are pairwise-matched at random each period of the game. We assume, as a benchmark case, that the matching process is perfectly random. Suppose that a fraction π of agents behave in the

²⁵This follows from reducing goodness to moral goodness and badness to moral badness (see Section 2).

Nash manner (i.e., choosing best reply to the other player’s strategy), while a fraction $1 - \pi$ of agents are Epictetusan individuals (i.e., choosing the best strategy while being indifferent to the opponent’s strategy).

As usual in evolutionary game theory, we consider that agents with a higher expected payoff will grow more quickly in the population, the payoffs being proportional to evolutionary fitness. From an evolutionary perspective, a key issue is whether or not there exists an evolutionary stable strategy (ESS) (see Maynard-Smith, 1982, 1989). An ESS is a strategy that can survive an infinitely small invasion of mutants adopting another strategy.

In our framework, the Nash behavior is an ESS if and only if the expected pay-off of Nash players is larger than the one of Epictetusan players when π is close to 1. In that case, the population is almost entirely Nash, and a mutant Epictetusan agent would do worse than Nash agents, and would be driven to extinction. Alternatively, the Epictetusan behavior is an ESS if and only if the expected pay-off of Epictetusan players is larger than the one of Nash players when π is close to 0. In that case, the population is almost entirely Epictetusan, and a mutant Nash agent would do worse than Epictetusan agents, and would be driven to extinction.

In order to examine whether the Nash or the Epictetusan behavior is an ESS, we will consider two regimes regarding the informational structure of the game. In the first regime, types of agents are common knowledge to all players, whereas, in the second one, agents’s types are private information.

4.1 Natural selection (1): types are common knowledge

In order to compute expected pay-offs for Nash and Epictetusan players when pairs of players of a prisoner’s dilemma are randomly selected in the population, one must first compute the pay-offs in the three possible cases: (i) the game is played by a pair of Epictetusan players; (ii) the game is played by a pair of Nash players; (iii) the game is played by a pair composed of an Epictetusan player and a Nash player. Case (i) was studied in Section 3.

Let us consider case (ii). When the game is played by two Nash players, the pay-off matrix is:

		Player B	
Player A	cooperate	defect	
cooperate	$(1, 1)$	(α, β)	
defect	(β, α)	$(0, 0)$	

Table 3. The game of Table 1 played by two Nash players.

As usual in the literature, we assume that $\beta > 1$, that is, that playing defect when the other player cooperates brings an extra pay-off with respect to the outcome where all players cooperate.²⁶ Thus Nash players have an incentive

²⁶See, for instance, Curry and Roemer (2012).

to deviate and defect. Moreover, we also assume that $\alpha < 0$, that is, that a player who cooperates when the other player defects is worse off than under the outcome where all players defect.

Since $\beta > 1$ and $\alpha < 0$, defect is the dominant strategy for each Nash player. Hence (defect, defect) is a dominant strategy equilibrium. This is the outcome of the game played by a pair of Nash players. Note that this outcome is Pareto-dominated by the outcome where each player cooperates. But the outcome (cooperate, cooperate) is not a Nash equilibrium, since this game admits a (necessarily unique) dominant strategy equilibrium (defect, defect).

Proposition 2 *When two Nash players play the PD game, if $\beta > 1$ and $\alpha < 0$, (defect, defect) is the dominant strategy equilibrium, and the pay-offs are (0, 0).*

Proof. See above. ■

Consider now case (iii): an Epictetusian player A plays against a Nash player B, and each player knows the type of the opponent. The pay-off matrix is:

		Player B	
		cooperate	defect
Player A	cooperate	(1, 1)	(1, β)
	defect	(0, α)	(0, 0)

Table 4. The game of Table 1 played by an Epictetusian player and a Nash player.

The Epictetusian player A knows that player B is a Nash player. But player A plays nonetheless cooperate, since this is the dominant strategy for him. Indeed, from an Epictetusian perspective, only agents's own acts can make their life good, no matter what other agents do (this is just a circumstance, to which one should be indifferent). This motivates player A choosing to cooperate, despite the knowledge that player B will defect. The Nash player B knows that player A is an Epictetusian player. Since $\beta > 1$, player B plays defect, because this is the dominant strategy for him. Thus the outcome (cooperate, defect) is the dominant strategy equilibrium. Pay-offs are (1, β).

Proposition 3 *When an Epictetusian player A and a Nash player B play the PD game, their type being common knowledge, if $\beta > 1$ and $\alpha < 0$, the dominant strategy equilibrium is (cooperate, defect), and the pay-offs are (1, β).*

Proof. See above. ■

Having computed the outcome of the game and the associated pay-off in cases (i), (ii) and (iii), we can now compute the expected pay-offs for the Epictetusian agents and the Nash agents of playing two-person games with a randomly assigned opponent.

An Epictetusian agent has a probability $1 - \pi$ of being matched with another Epictetusian agent, and a probability π of being matched with a Nash agent.

In any case, an Epictetusian agent will cooperate, and enjoy the same pay-off, equal to 1 (since for the Epictetusian agent pay-offs are invariant to the other player's strategy). Hence the expected pay-off for an Epictetusian agent is:

$$E(\text{payoffs})_{\text{Epictetusian}} = (1 - \pi)1 + \pi 1 = 1 \quad (1)$$

For a Nash agent, the pay-off is β when the agent is matched with an Epictetusian player (which occurs with probability $1 - \pi$), and the pay-off is 0 when the agent is matched with another Nash player (which occurs with a probability π). Thus the expected pay-off for a Nash agent is:

$$E(\text{payoffs})_{\text{Nash}} = (1 - \pi)\beta + \pi 0 = (1 - \pi)\beta \quad (2)$$

Interestingly, the higher the proportion of Epictetusian players in the population $1 - \pi$, the higher the expected pay-off of Nash players is. Indeed, playing their dominant strategy (defect) yields a higher pay-off for Nash players when they play against an Epictetusian player in comparison to when they play against another Nash player. Proposition 4 summarizes our results.

Proposition 4 *Suppose that the types of players are common knowledge.*

- *If $1 > (1 - \pi)\beta$, the expected pay-off is larger for the Epictetusian player than for the Nash player.*
- *If $1 = (1 - \pi)\beta$, the expected pay-offs are equal for the Epictetusian player and for the Nash player.*
- *If $1 < (1 - \pi)\beta$, the expected pay-off is larger for the Nash player than for the Epictetusian player.*
- *Neither the Nash type nor the Epictetusian type is an ESS. No type can drive the other type to extinction.*

Proof. The first part of the proposition follows from the calculus of expected pay-offs for the two types of players.

Regarding whether or not the Nash behavior is an ESS, consider the case where π is close to 1. In that case, the expected pay-off of the Nash player is:

$$\lim_{\pi \rightarrow 1} (1 - \pi)\beta = 0 < 1$$

which is lower than the expected pay-off of Epictetusians, which is 1. Hence a population composed (almost) entirely of Nash players cannot resist an infinitely small invasion of Epictetusian players. Thus Nash behavior is not an ESS.

Consider now the case where π is close to 0. In that case, the expected pay-off of the Nash player is:

$$\lim_{\pi \rightarrow 0} (1 - \pi)\beta = \beta > 1$$

which is higher than the expected pay-off of the Epictetusian player, which is 1. Hence a population composed (almost) entirely of Epictetusian players cannot

resist an infinitely small invasion of Nash players. Thus Epictetusian behavior is not an ESS. ■

Proposition 4 states that neither Nash behavior nor Epictetusian behavior is an ESS. No type can drive the other type to extinction. The intuition behind that result is the following. When the population is composed almost entirely of Nash players, expected pay-offs of Nash players are low, because they are trapped in the coordination failure and play defect against each others. The expected pay-offs of Nash players are then lower than the expected pay-offs of the few Epictetusian players invading the population. Thus Nash players cannot resist that invasion, and thus cannot push the Epictetusian to extinction. Alternatively, when the population is composed almost entirely of Epictetusian players, expected pay-offs for Epictetusian players is 1, which is smaller than the expected pay-offs of Nash players, because, under this composition of the population, the small population of invading Nash players benefits a lot from playing defect (since they play mainly against Epictetusian players). Enjoying a higher expected pay-off, Nash players tend then to invade the population. Thus Epictetusian players cannot resist an invasion of Nash players. This explains why neither the Nash nor the Epictetusian types are ESS in our framework.

4.2 Natural selection (2): types are private information

Let us now consider the alternative informational regime, where players do not know the type of their opponent.

Note first that, for an Epictetusian player, knowing or not knowing the type of the opponent playing the game does not affect the best way to behave. Indeed, from an Epictetusian perspective, the pay-offs are invariant to the other player's strategy. Hence, from that perspective, whether the other player is also an Epictetusian player, or a Nash player does not make any difference. In any case, Epictetusian players will play cooperate, as they do under the common knowledge assumption, and they will obtain a pay-off equal to 1, no matter the type of the other player and the strategy chosen by the other player.

As for Nash players, a Nash player knows that the other player cooperates if he is an Epictetusian player, which happens with a probability $1 - \pi$. Moreover, a Nash player knows that the other player defects if he is a Nash player, which happens with a probability π . These pieces of information can be used to compute the expected pay-offs of cooperating and defecting for a Nash player in the absence of knowledge about the opponent's type.

Hence, for a Nash player, in the absence of information about the other player's type, the expected payoff of playing defect is:

$$\pi 0 + (1 - \pi)\beta = (1 - \pi)\beta \quad (3)$$

whereas the expected pay-off of playing cooperate is:

$$\pi\alpha + (1 - \pi)1 = \pi\alpha + 1 - \pi \quad (4)$$

Note that, since $\beta > 1$ and $\alpha < 0$, we have:

$$(1 - \pi)\beta > 1 - \pi > \pi\alpha + 1 - \pi \quad (5)$$

so that the expected payoff of defect is higher than the expected pay-off of cooperation, for all levels of π .

As a consequence of this, a Nash player ignoring the type of his opponent will choose to play defect, because this strategy yields a higher expected pay-off for him. The expected pay-off of the Nash player is thus $(1 - \pi)\beta$. Proposition 5 summarizes our results.

Proposition 5 *Suppose that the type of each player is private knowledge.*

- *If $1 > (1 - \pi)\beta$, the expected pay-off is larger for the Epictetusian player than for the Nash player.*
- *If $1 = (1 - \pi)\beta$, the expected pay-offs are equal for the Epictetusian player and for the Nash player.*
- *If $1 < (1 - \pi)\beta$, the expected pay-off is larger for the Nash player than for the Epictetusian player.*
- *Neither the Nash type nor the Epictetusian type is an ESS: no type can drive the other type to extinction.*

Proof. The first part of Proposition 5 follows from the above calculations.

The second part of Proposition 5 can be proved by showing that the expected pay-off of Nash players is smaller than the expected pay-off of Epictetusian players when π tends to 1, whereas the expected pay-off of Epictetusian players is smaller than the expected pay-off of Nash players when π tends to 0. This is similar to the proof of Proposition 4. ■

Proposition 5 suggests that the previous results, obtained under the assumption that the type of each player is common knowledge, are robust to relaxing that informational assumption, and to supposing that players do not know the type of their opponent. Neither the Nash behavior nor the Epictetusian behavior is an ESS in this game: no type - neither the Nash, nor the Epictetusian - can drive the other type to extinction, and this result holds whether players' types are common knowledge or not.

5 The dynamics of heterogeneity

Up to now, we showed that, under perfectly random matching, neither the Nash type nor the Epictetusian type can drive the other type to extinction. In the light of these results, one may still wonder what the long-run dynamics of heterogeneity looks like. In order to examine that issue, this section studies the dynamics of the partition of the population, by assuming an explicit dynamics of the variable π , the fraction of Nash players in the population.

For that purpose, let us assume that the population partition follows the discrete time replicator dynamics (see Cressman 2003):

$$\pi_{t+1} - \pi_t = \left(\frac{(1 - \pi_t)\beta - [\pi_t(1 - \pi_t)\beta + (1 - \pi_t)1]}{\pi_t(1 - \pi_t)\beta + (1 - \pi_t)1} \right) \quad (6)$$

The left-hand-side (LHS) of the equation is the variation in the proportion of Nash in the population between two successive time-periods. The RHS is the relative gap in expected pay-offs between Nash players and the entire population (relative to the expected pay-off of the population as a whole).

That equation simplifies to:

$$\pi_{t+1} = \pi_t + \left(\frac{\beta(1 - \pi_t) - 1}{\pi_t\beta + 1} \right) \equiv \phi(\pi_t) \quad (7)$$

This expression shows that the proportion of Nash players in the population at time $t + 1$ exceeds the proportion of Nash players in the population at time t when the expected pay-off of Nash players exceeds the expected pay-off of Epictetusian players, that is, when $\beta(1 - \pi_t) > 1$. Alternatively, when $\beta(1 - \pi_t) < 1$, that is, when the expected pay-off of Nash players is smaller than the expected pay-off of Epictetusian players, the proportion of Nash players in the population at period $t + 1$ is smaller than the proportion of Nash players at time t .

Proposition 6 summarizes our results.

Proposition 6 *Assume that the dynamics of π_t follows the discrete time replicator's dynamics. The equilibrium proportion of the Nash type in the population is $\pi^* = 1 - \frac{1}{\beta}$. The equilibrium proportion of Epictetusian type in the population is $1 - \pi^* = \frac{1}{\beta}$. This equilibrium partition is locally stable.*

Proof. The equilibrium partition of the population can be found by setting $\pi_{t+1} = \pi_t$ in the dynamic equation. We then have:

$$\left(\frac{\beta(1 - \pi_t) - 1}{\pi_t\beta + 1} \right) = 0 \iff \pi_t = 1 - \frac{1}{\beta} = \pi^* \quad (8)$$

Thus, at the equilibrium, there is a fraction $1 - \frac{1}{\beta}$ of Nash players, and a fraction $\frac{1}{\beta}$ of Epictetusian players.

We have:

$$\frac{\partial \pi_{t+1}}{\partial \pi_t} = 1 + \left(\frac{-\beta [\pi_t\beta] - \beta\beta(1 - \pi_t)}{(\pi_t\beta + 1)^2} \right) = 1 + \left(\frac{-\beta^2}{(\pi_t\beta + 1)^2} \right) \quad (9)$$

The local stability of the equilibrium requires:

$$\begin{aligned} \left| \frac{\partial \pi_{t+1}}{\partial \pi_t} \right|_{\pi^* = 1 - \frac{1}{\beta}} < 1 &\iff \left| 1 + \left(\frac{-\beta^2}{\left(\left(1 - \frac{1}{\beta} \right) \beta + 1 \right)^2} \right) \right| < 1 \\ &\iff \left| 1 + \left(\frac{-\beta^2}{(\beta - 1 + 1)^2} \right) \right| < 1 \\ &\iff |1 + (-1)| < 1 \\ &\iff |0| < 1 \end{aligned}$$

Thus the equilibrium $\pi^* = 1 - \frac{1}{\beta}$ is stable. ■

Proposition 6 states that, in the long-run, the population is composed of a fraction $1 - \frac{1}{\beta}$ of Nash agents and of a fraction $\frac{1}{\beta}$ of Epictetusian agents, and that this equilibrium partition is locally stable.

As an illustration, Figure 1 shows the form of the function $\phi(\pi_t)$ for distinct values of the parameter β , together with the 45° line. We can see that, as β increases, the equilibrium proportion of Nash agents in the population tends to grow. We can also see that the slope of $\phi(\pi_t)$ at the intersection with the 45° line is always less than 1, so that the equilibrium partition is locally stable.

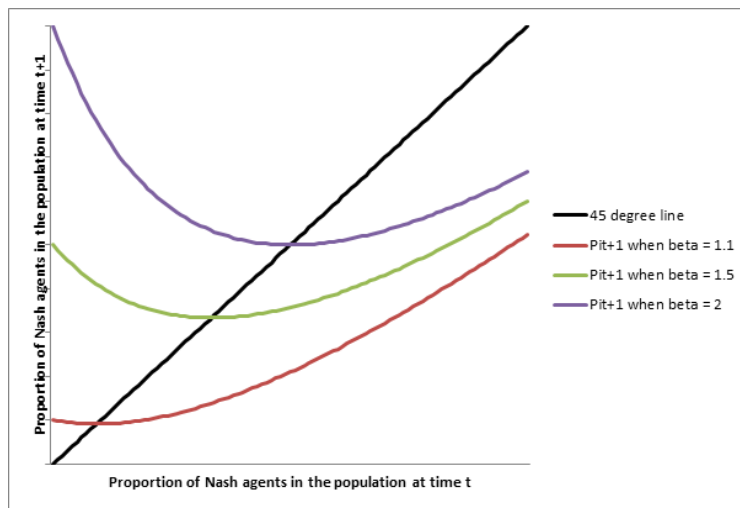


Figure 1. Transition function $\phi(\pi_t)$ for various levels of β .

In sum, this section shows that, under the discrete time replicator’s dynamics, the long-run partition of the population into Nash and Epictetusian agents is locally stable, and includes a proportion of Epictetusian agents that is decreasing with the pay-off gain of defecting for Nash agents when their opponents do not defect.

6 Imperfectly random matching

While the above sections assumed that Nash and Epictetusian agents were matched perfectly randomly to play the prisoner’s dilemma, one may wonder whether our results are robust to allowing for some degree of assortativity in the matching. This section examines that issue, and revisits our results under a more general matching function allowing for some degree of assortativity.

For that purpose, let us first define by $\Pr(N|N, \pi)$ the probability of being matched with a Nash type N conditionally on being of Nash type N when the proportion of Nash agents is π . Similarly, let us define by $\Pr(N|E, \pi)$ the

probability of being matched with a Nash type N conditionally on being of Epictetusian type E when the proportion of Nash agents is π .

Following Alger and Weibull (2013), we can define by $\phi(\pi)$ the difference between the conditional probabilities for an agent to be matched with an agent with type Nash, given that the agent himself either also has the Nash type N , or, alternatively, the Epictetusian type E :

$$\phi(\pi) = \Pr(N|N, \pi) - \Pr(N|E, \pi) \quad (10)$$

The difference $\phi(\pi) : (0, 1) \rightarrow [-1, 1]$ defines the assortment function.

Using the following necessary balancing condition, which states that the number of Nash agents matched with Epictetusian agents must be strictly equal with the number of Epictetusian agents matched with Nash agents:

$$\pi [1 - \Pr(N|N, \pi)] = (1 - \pi) \Pr(N|E, \pi) \quad (11)$$

one can rewrite the conditional probability $\Pr(N|N, \pi)$ as a function of π and $\phi(\pi)$:

$$\begin{aligned} \Pr(N|N, \pi) &= \phi(\pi) + \Pr(N|E, \pi) \\ &= \phi(\pi) + \frac{\pi}{1 - \pi} [1 - \Pr(N|N, \pi)] \\ &= (1 - \pi) \phi(\pi) + \pi \end{aligned} \quad (12)$$

Note that the conditional probability of being matched with an agent of type N collapses to π when the assortment function $\phi(\pi)$ equals 0, which is the case when the matching is perfectly random.

Moreover, the conditional probability $\Pr(N|E, \pi)$ can be rewritten as:

$$\begin{aligned} \Pr(N|E, \pi) &= \Pr(N|N, \pi) - \phi(\pi) \\ &= (1 - \pi) \phi(\pi) + \pi - \phi(\pi) \\ &= \pi(1 - \phi(\pi)) \end{aligned} \quad (13)$$

Note also that $\Pr(N|E, \pi)$ equals π when the assortment function $\phi(\pi)$ equals 0, which is the case when the matching is perfectly random.

The other conditional probabilities $\Pr(E|N, \pi)$ and $\Pr(E|E, \pi)$ can be computed as:

$$\Pr(E|N, \pi) = 1 - \Pr(N|N, \pi) = (1 - \pi)(1 - \phi(\pi)) \quad (14)$$

$$\Pr(E|E, \pi) = 1 - \Pr(N|E, \pi) = 1 - \pi(1 - \phi(\pi)) \quad (15)$$

Following Alger and Weibull (2013), we assume that $\phi(\pi)$ is continuous and that $\phi(\pi)$ converges to some number as π tends to 1:

$$\lim_{\pi \rightarrow 1} \phi(\pi) = \sigma \quad (16)$$

for some real number σ . σ is the index of assortativity of the matching process. It follows from the formula of $\Pr(N|E, \pi)$ that $\sigma \in [0, 1]$. The extreme case

where $\sigma = 0$ corresponds to the case of perfectly random matching. The opposite extreme case, where $\sigma = 1$, corresponds to the case of perfectly correlated matching (each type is matched only with agents of the same type).²⁷

Having defined the assortment function $\phi(\pi)$, let us now reexamine the existence of ESS under imperfectly random matching. For that purpose, let us first consider the case where the types of agents are common knowledge.

Introducing imperfectly random matching has no effect on the expected pay-offs of Epictetian agents in comparison with the perfectly random matching case. The reason is that, for Epictetian agents, their pay-offs do not depend on the other agent's strategy, and, hence, do not depend on the type of agent with whom they are matched. More formally, we have:

$$\begin{aligned} E(\text{payoffs})_{\text{Epictetian}} &= \Pr(E|E, \pi) 1 + \Pr(N|E, \pi) 1 \\ &= [1 - \pi(1 - \phi(\pi))] 1 + [\pi(1 - \phi(\pi))] 1 = 1 \end{aligned} \quad (17)$$

Thus the precise form of the assortment function does not affect the expected pay-offs of Epictetian agents.

For a Nash agent, the pay-off is β when the agent is matched with an Epictetian player (which occurs with probability $\Pr(E|N, \pi)$), and the pay-off is 0 when the agent is matched with another Nash player (which occurs with a probability $\Pr(N|N, \pi)$). Thus the expected pay-off for a Nash agent is:

$$\begin{aligned} E(\text{payoffs})_{\text{Nash}} &= \Pr(E|N, \pi)\beta + \Pr(N|N, \pi) 0 \\ &= (1 - \pi)(1 - \phi(\pi))\beta \end{aligned} \quad (18)$$

The expected pay-off of the Nash type depends on the form of the assortment function. Actually, it is only in the special case where $\phi(\pi) = 0$ (perfectly random matching) that the expected pay-off equals $(1 - \pi)\beta$, as in the baseline model. When $\phi(\pi) > 0$ (some positive degree of assortativity), the expected pay-off for the Nash type is reduced in comparison with the perfectly random matching case. On the contrary, when $\phi(\pi) < 0$ (some negative degree of assortativity), the expected pay-off for the Nash type is increased in comparison with the perfectly random matching case.

Given that the expected pay-off of the Nash type depends on the assortment function, whereas the expected pay-off of the Epictetian type does not, we can expect that the form of the assortment function affects also the issue of whether any of these two types can be an ESS. This issue can be examined by comparing the expected pay-offs of the two types. Proposition 7 summarizes our results.

Proposition 7 *Suppose that the types of players are common knowledge and that the matching process is imperfectly random with the assortment function $\phi(\pi)$.*

²⁷To see why this latter case corresponds to perfectly correlated matching, note that, when $\sigma = 1$, we have:

$$\lim_{\pi \rightarrow 1} \Pr(N|E, \pi) = 0$$

that is, even when the population is almost entirely composed of Nash agents, an Epictetian agent is matched with a Nash agent with a probability equal to 0.

- If $1 > (1 - \pi)(1 - \phi(\pi))\beta$, the expected pay-off is larger for the Epictetusian player than for the Nash player.
- If $1 = (1 - \pi)(1 - \phi(\pi))\beta$, the expected pay-offs are equal for the Epictetusian player and for the Nash player.
- If $1 < (1 - \pi)(1 - \phi(\pi))\beta$, the expected pay-off is larger for the Nash player than for the Epictetusian player.
- The Nash type is not an ESS. When $\lim_{\pi \rightarrow 0} \phi(\pi) \leq \frac{\beta-1}{\beta}$, the Epictetusian type is not an ESS. When $\lim_{\pi \rightarrow 0} \phi(\pi) > \frac{\beta-1}{\beta}$, the Epictetusian type is an ESS.

Proof. The first part of the proposition follows from the calculus of expected pay-offs for the two types of players.

Regarding the second part, consider first the case where π is close to 1. In that case, the expected pay-off of the Nash player is:

$$\begin{aligned}
& \lim_{\pi \rightarrow 1} [(1 - \pi)(1 - \phi(\pi))\beta] \\
&= \beta \lim_{\pi \rightarrow 1} [(1 - \pi)(1 - \phi(\pi))] \\
&= \beta(1 - \sigma)0 = 0 < 1
\end{aligned}$$

which, given that $\sigma \in [0, 1]$, is lower than the expected pay-off of Epictetusians, which is 1. Hence a population composed (almost) entirely of Nash players cannot resist an infinitely small invasion of Epictetusian players. Thus Nash behavior is not an ESS.

Consider now the case where π is close to 0. In that case, the expected pay-off of the Nash player is:

$$\begin{aligned}
& \lim_{\pi \rightarrow 0} [(1 - \pi)(1 - \phi(\pi))\beta] \\
&= \beta \lim_{\pi \rightarrow 0} [(1 - \phi(\pi))] \\
&= \beta \left[1 - \lim_{\pi \rightarrow 0} \phi(\pi) \right]
\end{aligned}$$

When $\beta [1 - \lim_{\pi \rightarrow 0} \phi(\pi)] \geq 1$, the expected pay-off of Nash is higher than or equal to the expected pay-off of the Epictetusian player, which is 1. Hence the Epictetusian type is not an ESS. However, when $\beta [1 - \lim_{\pi \rightarrow 0} \phi(\pi)] < 1$, the expected pay-off of Nash is lower than the expected pay-off of the Epictetusian player, which is 1. Hence the Epictetusian type is an ESS. This condition can be rewritten as: $\lim_{\pi \rightarrow 0} \phi(\pi) > \frac{\beta-1}{\beta}$. ■

Whatever the form of the assortment function is, the Nash type is never an ESS in our framework. When the population is almost entirely composed of Nash agents, it is impossible for them to drive the Epictetusian type to extinction. The intuition goes as follows. When the degree of assortativity σ is low, the expected pay-off of the Nash is lower than the one of Epictetusian

agents, because of the same reason as in the baseline model ($\sigma = 0$).²⁸ However, when the degree of assortativity σ is increased, the situation becomes even worse for the Nash, since in that case the few Epictetusian agents are increasingly matched among themselves, which guarantees them an even higher expected pay-off than Nash agents (in comparison to $\sigma = 0$). Thus increasing the degree of assortativity of the matching does not allow the Nash type to drive the Epictetusian type to extinction.

However, provided the assortment function exhibits a sufficiently high positive assortment when π is close to zero, the Epictetusian type is an ESS. In other words, when the degree of assortativity in the matching is sufficiently high when the Nash type agents are not numerous, the Epictetusian type can drive the Nash type to extinction. The intuition is that, when the degree of assortativity is sufficiently high when π is close to 0, most of Epictetusian agents are matched with Epictetusian agents, which guarantees them higher expected pay-offs than the few Nash being matched, for most of them, with Nash agents. Hence, a higher degree of assortativity of the matching prevents the few invading Nash agents from realizing higher pay-offs than the Epictetusian types, by preventing most of interactions between Nash and Epictetusian agents (which are the only way for Nash agents to have higher pay-offs than Epictetusian agents). Thus, whereas the Epictetusian type could not drive the Nash type to extinction under perfectly random matching, it is possible, when the degree of assortativity of the matching is sufficiently high, that the Epictetusian type can drive the Nash to extinction.

Finally, let us conclude this section by considering, under a general assortment function with imperfect random matching, the case where the type of the opponent is private information.

As under perfectly random matching, the fact that an Epictetusian player knows the type of his opponent or not does not affect the best way to behave for him, because, from an Epictetusian perspective, the pay-offs are invariant to the other player's strategy (see above). In any case, Epictetusian players will play cooperate, as they do under the common knowledge assumption, and they will obtain a pay-off equal to 1, no matter the type of the other player and the strategy chosen by the other player.

As for Nash players, a Nash player knows that the opponent cooperates if the opponent is an Epictetusian player, which happens with a probability $\Pr(E|N, \pi)$. Moreover, a Nash player knows that the opponent defects if he is a Nash player, which happens with a probability $\Pr(N|N, \pi)$. Hence, for a Nash player, in the absence of information about the other player's type, the expected payoff of playing defect is:

$$\begin{aligned}
 & \Pr(N|N, \pi)0 + \Pr(E|N, \pi)\beta \\
 = & \Pr(E|N, \pi)\beta \\
 = & \beta(1 - \pi)(1 - \phi(\pi))
 \end{aligned} \tag{19}$$

²⁸The reason is that most Nash agents are matched with Nash agents (because π is close to 1), and are thus trapped in the prisoner's dilemma. Hence the Nash agents have here a lower expected pay-off than the few Epictetusian agents.

whereas the expected pay-off of playing cooperate is:

$$\begin{aligned}
& \Pr(N|N, \pi)\alpha + \Pr(E|N, \pi)1 \\
= & \Pr(N|N, \pi)\alpha + \Pr(E|N, \pi) \\
= & \alpha[(1 - \pi)\phi(\pi) + \pi] + (1 - \pi)(1 - \phi(\pi))
\end{aligned} \tag{20}$$

Note that, since $\beta > 1$ and $\alpha < 0$, we have:

$$\beta(1 - \pi)(1 - \phi(\pi)) > (1 - \pi)(1 - \phi(\pi)) > \alpha[(1 - \pi)\phi(\pi) + \pi] + (1 - \pi)(1 - \phi(\pi)) \tag{21}$$

so that the expected payoff of defect is higher than the expected pay-off of cooperation, for all levels of π .

Hence, a Nash player ignoring the type of his opponent will choose to play defect, because this strategy yields a higher expected pay-off for him. The expected pay-off of the Nash player is thus $\beta(1 - \pi)(1 - \phi(\pi))$. Proposition 8 summarizes our results.

Proposition 8 *Suppose that the type of each player is private knowledge and that the matching process is imperfectly random with the assortment function $\phi(\pi)$.*

- *If $1 > (1 - \pi)(1 - \phi(\pi))\beta$, the expected pay-off is larger for the Epictetian player than for the Nash player.*
- *If $1 = (1 - \pi)(1 - \phi(\pi))\beta$, the expected pay-offs are equal for the Epictetian player and for the Nash player.*
- *If $1 < (1 - \pi)(1 - \phi(\pi))\beta$, the expected pay-off is larger for the Nash player than for the Epictetian player.*
- *The Nash type is not an ESS. When $\lim_{\pi \rightarrow 0} \phi(\pi) \leq \frac{\beta-1}{\beta}$, the Epictetian type is not an ESS. When $\lim_{\pi \rightarrow 0} \phi(\pi) > \frac{\beta-1}{\beta}$, the Epictetian type is an ESS.*

Proof. The first part of Proposition 8 follows from the above calculations.

The second part of Proposition 8 can be proved by showing that the expected pay-off of Nash players is smaller than the expected pay-off of Epictetian players when π tends to 1, whereas the expected pay-off of Epictetian players is smaller than the expected pay-off of Nash players when π tends to 0 when $\lim_{\pi \rightarrow 0} \phi(\pi) \leq \frac{\beta-1}{\beta}$, but larger than the expected pay-off of Nash players when π tends to 0 when $\lim_{\pi \rightarrow 0} \phi(\pi) > \frac{\beta-1}{\beta}$. This is similar to the proof of Proposition 7. ■

In sum, this section suggests that our previous results are only partly robust to the assumptions made concerning the randomness of the matching process. True, whether or not the matching of players is perfectly random, the Nash type can never drive the Epictetian type to extinction. However, whereas the Epictetian type cannot drive the Nash type to extinction under perfectly

random matching, it is possible that the Epictetian type can drive the Nash type to extinction when the matching exhibits a sufficiently high degree of assortativity. The intuition is that, when the matching is sufficiently assortative, the few invading Nash agents will play with other Nash agents, and the large majority of Epictetian agents will play among themselves, which yields higher expected pay-offs for the Epictetian than for the few Nash. Thus, when the matching process is sufficiently assortative, the Epictetian agents can resist an invasion of a few Nash agents, something that is not true under perfectly random matching. This explains that a sufficiently high degree of assortativity of the matching process makes Stoicism an ESS.

7 Conclusions

This paper proposes to reexamine Epictetus’s precept of ‘Taking away aversion from all things not in our power’ in an evolutionary light. For that purpose, we considered a population composed of Nash agents and of Epictetian agents, the latter being indifferent between outcomes that differ only on things not under one’s control (in line with the Stoic discipline of desires). Pairs of members of that population are randomly drawn and play a prisoner’s dilemma.

When playing a prisoner’s dilemma, Epictetian agents are indifferent between outcomes of the game that differ only regarding the opponent’s strategy, because these agents reduce goodness to moral goodness (only the *own* behavior of Epictetian agents can, if virtuous, make their life good). Hence, if it is common knowledge that the outcome where everyone cooperates Pareto-dominates the outcome where everyone defects, the extension of the domain of indifference to all outcomes of the game where the player plays the same strategy implies that cooperating is the dominant strategy for Epictetian agents. Epictetian rationality, by encouraging cooperation, provides a way out of the prisoner’s dilemma. This path out of the prisoner’s dilemma is based on the extension of the domain of indifference, and, hence, differs from the one prevailing under Kantianism (focusing only on the diagonal of the game).

Our analyses show that, under perfectly random matching, neither Nash behavior nor Epictetian behavior is an evolutionary stable strategy: no type leads the other type to extinction. These results are robust to the informational setting, that is, to whether or not the types of agents are common knowledge or private knowledge. Moreover, assuming the discrete time replicator dynamics, we showed that the (locally stable) equilibrium proportion of Nash players is equal to $1 - \frac{1}{\beta}$, while the one of Epictetian agents is equal to $\frac{1}{\beta}$. Thus the long-run prevalence of Stoicism depends on how large the pay-offs of Nash agents are in case of defection.

However, when considering a more general framework with imperfectly random matching, we showed that, provided the matching process is sufficiently assortative, the Epictetian type is an ESS and can drive the Nash type to extinction. The underlying intuition is that, when the matching process is sufficiently assortative, the few Nash agents invading a population composed almost

entirely of Epictetian agents will play among Nash agents (for most of them), whereas most of Epictetian agents will play among Epictetian agents. This guarantees that only very few Nash agents will be able to defect in presence of Epictetian agents. As a consequence, the Epictetian type exhibits a higher expected pay-off than the Nash type even when the population is almost entirely Epictetian, something that is not possible under perfectly random matching. Thus assortativity of the matching, if sufficiently high, preserves most Epictetian agents from interactions with Nash agents, which makes the Epictetian type drive the Nash type to extinction.

These results have some important implications. First, at the level of history, our analyses allow us to explain the survival of attitudes based on Ancient moral philosophies such as Stoicism, despite the passage of long periods of time. The fact that Nash behavior is not an ESS in our framework - neither under perfectly random matching, nor under imperfectly random matching - provides an explanation for the survival of these ethical attitudes defended more than two millennia ago, and which are orthogonal to modern rationality. Second, also at the level of history, the fact that the Epictetian type could not drive the Nash type to extinction reveals that the degree of assortativity in the matching of interactions was not sufficiently high to make the Epictetian type an ESS. Third, at the level of policy, our results have also a clear implication: in order to deal with climate change and other global disorders due to coordination failures, a possible path is to encourage the prevalence of ethical preferences (such as Epictetian rationality) by reducing, through transfer policies, the gains obtained from defection.

All in all, this paper suggests that Epictetian behavior cannot be driven to extinction by Nash behavior, but can nonetheless be reduced to a small prevalence when the Nash's gains from defection are high enough relative to the Epictetian gains from being virtuous. Moreover, when the matching process is sufficiently assortative, the Epictetian type can drive the Nash type to extinction. It should be stressed, however, that our analysis focused only on a particular class of games: one-shot simultaneous prisoner's dilemmas. In order to have a more exhaustive view of the evolutionary dynamics of Epictetian agents, one should definitely consider also other games. This is at our research agenda.

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