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Stock Price Wealth Effects and Monetary Policy under Imperfect Knowledge

Adrian Ifrim ^{1,2}

Abstract

Departures from full-information rational expectation models give rise to stock price wealth effects which introduce inefficient cyclical fluctuations in the economy. Waves of optimism/pessimism affect beliefs and asset prices which influence aggregate demand through expectation-driven wealth effects. Monetary policy can play an important role in eliminating the non-fundamental effects of belief-driven asset price cycle: reacting symmetrically and transparently to stock prices increases welfare significantly compared to flexible inflation targeting strategies. A quantitative model estimated on US data shows that increasing interest rates by 12 basis points for every 100% rise in stock prices accomplish this goal. Moreover, a nonlinear reaction to stock prices only when capital gains exceed 7% delivers similar efficiency gains.

Keywords: monetary policy, wealth effects, survey expectations, stock prices, learning, animal spirits

JEL: D84, E32, E44, E52

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1. Introduction

How do asset prices affect the real economy and what is the proper response of central banks in the face of asset price cycles? This paper argues that stock prices have an impact on aggregate demand and business cycle fluctuations through consumption wealth effects that appear as a result of agents' imperfect knowledge of the economic system. Consistent with recent empirical evidence, agents extrapolate historical returns and exhibit slow and persistent movements in expectations. Booms and busts in asset prices, driven by sentiment swings, affect stock prices and the financial position of market participants. Since agents do not know the true law of motion of stock prices they perceive these fluctuations in asset prices as affecting their net wealth which impacts aggregate demand via consumption-wealth effects. Thus, exogenous movements in sentiment (sunspot shocks) can impact the real economy through belief-driven wealth effects introducing inefficient real economic volatility.

In this environment, monetary policy can increase macroeconomic stability and welfare by managing agents' beliefs through responding explicitly and transparently to asset prices. This policy is not accompanied by increased interest rate volatility but on the contrary: eliminating the non-fundamental effects of asset cycles on the real economy reduces macroeconomic volatility and via the Taylor rule also the one of interest rates. Crucial for this result is the assumption that agents understand and internalize into their expectations the response of central banks to asset prices. If on the contrary, the central bank acts in a discretionary manner and does not communicate the reaction to asset prices transparently, the gains of responding to stock prices are economically insignificant.

During the 1987 stock market crash, the aggressive easing of monetary policy that helped the recovery of the economy and reflate asset prices has become to be known as the Greenspan put. The implied promise that the Fed will step in and help the financial markets if needed, has continued over the years and has been relabelled as the Fed put. Recent evidence suggests that, although not explicitly, the Fed does indeed take into account the stance of the stock market when setting interest rates, and moreover the main channel that they consider important is the consumption wealth effect: increases in stock prices make

consumers feel wealthier and as a result adjust their consumption decisions accordingly. The empirical evidence from Di Maggio et al. (2020) also shows that the marginal propensity to consume (MPC) out of (unrealized) capital gains can be as high as 20%. Given the high volatility of stock prices these effects can have a large impact on aggregate demand. Nevertheless, most of the research concerning the optimal response of monetary policy to asset prices does not take into account the actual dynamics of stock prices or the consumption wealth effect as the main channel through which stock prices affect the real economy. The present paper tries to fill this gap.

At the core of the paper is the realistic assumption that agents have imperfect knowledge about the determination of macroeconomic variables. Agents are internally rational, maximizing their utility given their system of beliefs.³ The asset pricing literature has pointed out that survey measures of expectations are positively correlated with actual prices while actual returns tend to display a negative correlation.⁴ Rational expectations (RE) models have the opposite prediction, namely, agents expect lower returns at the top of the cycle. In a lab experiment, Galí et al. (2020) show that agents' asset price beliefs are not consistent with rational expectations and propose that adaptive expectations fit better the experimental data. Moreover, the high volatility of stock prices relative to fundamentals poses additional difficulty for RE macro-finance models. Building on this evidence, agents' belief system is extrapolative using constant gain learning to update their expectations about variables exogenous to their decision-making. When stock prices depart from their fundamental value due to sentiment/expectation swings, consumption-wealth effects appear naturally in this framework since agents interpret their asset position as real wealth and modify the consumption decision accordingly. The proposed theory, therefore, links directly the volatility puzzle of Shiller (1981) with stock price wealth effects.

The stock price consumption wealth effect is incorporated in a quantitative Limited Asset Market Participation (LAMP) New Keynesian model where agents are heterogeneous with respect to their participation in the stock market and have homogeneous imperfect informa-

³In the sense of Adam and Marcet (2011).

⁴See Greenwood and Shleifer (2014) and Adam et al. (2017).

tion about macroeconomic variables. The economy is hit by three types of shocks: supply (cost-push), monetary policy and a sentiment shock which affects the beliefs of the agents on their expected capital gains. The latter will operate as a demand shock influencing stock prices and aggregate demand via the consumption wealth effect. The model is estimated on US data by targeting a standard set of business cycle and financial moments. Although not explicitly targeted, the model is able to capture remarkably well the dynamics of survey expectations regarding capital gains, inflation and interest rates and the joint dynamics of the real economy and financial markets.

The quantitative model is next used to assess the implications of stock price targeting under two policy regimes: transparency vs non-transparency. By transparency, it is understood that agents take into account the reaction of policy to stock prices when forming expectations about future interest rates while the reverse is true under non-transparency. Under each scenario two stock price targeting strategies will be analyzed: the Fed put and the Fed put-call. The first one is a policy of taking into account stock prices in the Taylor rule only when stock prices decrease while the second considers a symmetrical reaction to capital gains. The results show that by reacting symmetrically and transparently to stock prices, monetary policy can increase welfare by 0.15% on average per period while if agents do not internalize the reaction of monetary policy to stock prices the gains are insignificant. The welfare improvement is not accomplished through pricking the bubble but instead by disconnecting sentiment-driven cycles from the real economy. This result emphasizes the key mechanism through which stock price targeting influences the economy in this environment, namely through managing long-term interest rate expectations and linking them to stock prices. Moreover, the central bank does not have to respond linearly to stock price movements in order to implement the optimal policy within the class of simple Taylor rules considered. Responding symmetrically only after capital gains surpass a certain threshold can achieve the same results. I show that in the case of US economy, reacting symmetrically to stock prices only when quarterly capital gains exceed 7% in absolute value maximizes macroeconomic stability and welfare. This last policy has the practical operational advantage of not responding continuously to stock price movements and also avoiding reactions

that might not be granted in the first place.

The rest of the paper is organized as follows. Section II reviews the literature on the interaction between monetary policy and stock prices. Section III presents a simple endowment economy to gain intuition into the origin of stock price wealth effects. Section IV incorporates the mechanism from the previous section in a quantitative monetary model with homogeneous imperfect information and estimates the model on US data. Section V studies the macroeconomic stability and welfare properties of stock price targeting. Lastly, section VI concludes.

2. Related Literature

This paper contributes to the literature that analyses the interaction between monetary policy and stock markets. The seminal papers of Bernanke and Gertler (2000, 2001) use a model with credit market frictions featuring a financial accelerator effect in which exogenous shocks have an amplified effect on the economy. Using a calibrated version of the model they argue that targeting stock prices has no gain and that a central bank is better off, in terms of macroeconomic stability, by sticking to a flexible-inflation targeting regime. In reaching this conclusion their model does not take into account key financial facts like excess volatility of stock prices or market expectations. Carlstrom and Fuerst (2007) analyze the implications of stock price targeting on equilibrium determinacy and conclude that a central bank targeting explicitly stock prices raises the risk of inducing real indeterminacy in the system. Bullard et al. (2002) reach a similar conclusion. Cecchetti et al. (2000), using the same model as Bernanke and Gertler (2001), conclude that central banks can derive some benefit by reacting to stock prices. The main difference between Cecchetti et al. (2000) and Bernanke and Gertler (2000, 2001) is the assumption about the nature of the shock. In Cecchetti et al. (2000) the central bank knows that the swings in stock prices are non-fundamental and, with this knowledge, reacting to stock prices can increase economic performance. In the papers described above and in most of the literature, the effect of stock prices on the economy arises either from the supply side, as in Bernanke and Gertler (2001), or from the central bank reacting explicitly to stock price deviation in the Taylor rule. Nisticò (2012) develops

a NK model with OLG households that features a direct demand effect of stock prices on output in the IS equation.⁵ The author concludes that targeting stock price growth increases macroeconomic stability. In a follow-up paper, Airaudo et al. (2015), using the same model as Nisticò (2012) analyze the stability and learnability of the model and conclude that, if the stock-wealth effect is sufficiently strong, reacting to stock prices increases the policy space for which the equilibrium is both determinate and learnable. Bask (2012) also argues for stock price targeting in a model with both fundamental and technical traders.

The current paper complements the results from Winkler (2019) which uses a similar expectation formation process and analyses the supply side effects of stock prices. In particular, the paper introduces learning in a monetary model with financial frictions, similar to the one in Bernanke et al. (1999) and finds that including a reaction to stock prices in the Taylor rule improves macroeconomic stability. Adam et al. (2017) build a real model of the economy in which agents learn about stock price behavior and which is quantitatively able to reproduce the joint behavior of stock prices and the business cycle but do not proceed in analyzing the interaction between monetary policy and asset prices. Airaudo (2016) studies asset prices in a monetary model in which agents have long-horizon learning and find the existence of a wealth effect. The issue of stability is then analyzed in the context of the central bank responding to stock prices and finds that reacting to stock prices increases the stability of the economy.

In the context of bond markets, Eusepi and Preston (2018a) show that imperfect knowledge about the structure of the economy generates wealth effects arising from long-term bond holdings. In their framework, the steady state level of long-term bonds influences the

⁵The effect appears due to the fact that in each period a fraction of households who own financial wealth die and are replaced by newcomers with zero stock holdings. Therefore, increases in stock prices in period t (which forecast higher financial wealth next period) generate higher consumption due to the desire of households to intertemporally smooth consumption. Once the next period arrives, some households will not be affected by the higher financial wealth (since they were replaced with newcomers who do not hold any) and therefore the increase in aggregate consumption seems higher than granted by the increase in financial wealth. In this sense stock prices affect consumption although this stock wealth effect is artificially generated by the assumption of households being replaced with 0 financial wealth ones.

magnitude of this effect and the effects of monetary policy. Agents have perfect knowledge of how prices of long-term bonds are determined but there exists a wedge between their forecast of the quantity of future bonds and the future level of taxes which make bonds net wealth giving rise to the wealth effect. In a similar spirit, this paper extends their argument to equity markets.

This research builds on the existing empirical literature that provides evidence on the magnitude of stock price wealth effects and the response of monetary policy to stock prices. Aastveit et al. (2017) use a time-varying VAR to show that the Fed has responded significantly and episodically to stock prices, especially in recessions. Cieslak and Vissing-Jorgensen (2020) analyze Federal Open Market Committee (FOMC) transcripts and conclude that the FED officials pay attention to asset prices and perceive the stock market as influencing the economy mainly through a consumption-wealth effect. They show that stock price decreases between 2 consecutive FOMC meetings are one of the best predictors of subsequent federal rate cuts. Case et al. (2005, 2011) and more recently Chodorow-Reich et al. (2019) bring empirical evidence for the existence of this wealth effect. The magnitude of this effect is not insignificant either. Di Maggio et al. (2020) show that unrealized capital gains lead to MPC ranging from 20% for the bottom 50% of the wealth distribution to 3% for the top 30%.

3. Wealth effects in endowment economies

This section lays out a basic endowment economy which shows that incomplete information about stock prices fundamentally changes the equilibrium of the economy. In this environment stock prices affect the endogenous variables due to a wedge between actual stock prices and their expected discounted sum of dividends.

Consider a flexible price endowment economy populated by a continuum of households, indexed by i , who maximize their utility by choosing how much to consume, C_t^i , save in bonds, B_t^i and invest in a risky asset, S_t^i . The risky asset is a claim to an exogenous stream of dividends, D_t . For simplicity assume that $D_t \sim \mathcal{N}(\mu, \sigma^2)$. Specifically, the problem of a typical household i is

$$\begin{aligned}
& \max_{C_t^i, B_t^i, S_t^i} E_0^{\mathcal{P}_i} \sum_{t=0}^{\infty} \delta^t \frac{(C_t^i)^{1-\sigma}}{1-\sigma} \\
& \text{s.t.} \quad P_t C_t^i + B_t^i + S_t^i Q_t \leq B_{t-1}^i (1 + i_{t-1}) + S_{t-1}^i (Q_t + D_t) \\
& \quad 0 \leq S_t^i \leq S^H, \forall t
\end{aligned} \tag{1}$$

where P_t is the aggregate price index, i_t is the nominal interest rate (set exogeneously by the monetary authority) and Q_t is the ex-dividend price of the risky asset. The expectation is taken over the subjective probability measure \mathcal{P}_i which is household specific and different than the rational expectation hypothesis, denoted by E . Furthermore there is a central bank following a Taylor type rule $i_t = \phi_\pi \pi_t$ which is common knowledge among all the agents in the economy. The FOCs are

$$\frac{1}{1 + i_t} = \delta E_t^{\mathcal{P}_i} \left\{ \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \frac{1}{1 + \pi_{t+1}} \right\}, \tag{2}$$

$$Q_t = \delta E_t^{\mathcal{P}_i} \left\{ \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \frac{(Q_{t+1} + D_{t+1})}{1 + \pi_{t+1}} \right\}. \tag{3}$$

Letting $\mathcal{W}_t^i = B_{t-1}^i (1 + i_{t-1}) + S_{t-1}^i (Q_t + D_t)$ and after imposing a transversality condition, the intertemporal BC becomes

$$\frac{\mathcal{W}_t^i}{P_t} = E_t^{\mathcal{P}_i} \sum_{j=0}^{\infty} \delta^j \left(\frac{C_{t+j}^i}{C_t^i} \right)^{-\sigma} C_{t+j}^i \tag{4}$$

which shows that the discounted sum of future consumption equals real wealth.

The equilibrium that is analyzed here has the property that all agents are identical although they do not know this to be true. This will prove to be essential to the pricing of the risky asset and for the existence of the wealth effect. Given that agents have the same preferences, constraints and beliefs they will make the same decisions. Equilibrium implies

$$\begin{aligned}
& \int_0^1 B_t^i di = 0, \\
& \int_0^1 C_t^i di = C_t = d_t, \\
& \int_0^1 S_t^i di = 1.
\end{aligned} \tag{5}$$

Aggregating equation (4), imposing $E^{\mathcal{P}_i} = E^{\mathcal{P}}$ and applying the equilibrium condition (5) yields

$$q_t + d_t = E_t^{\mathcal{P}} \sum_{j=0}^{\infty} \delta^j \left(\frac{C_{t+j}}{C_t} \right)^{-\sigma} C_{t+j}. \quad (6)$$

where q_t and d_t are the risky asset real price and real dividends.

Before moving to the imperfect information equilibrium the RE counterpart will first be derived.

3.1. Rational Expectations

First it will be assumed that agents have RE: $E^{\mathcal{P}} = E$. Given this, the FOC with respect to stock prices can be substituted forward to arrive at

$$q_t = E_t \sum_{j=1}^{\infty} \delta^j \left(\frac{C_{t+j}}{C_t} \right)^{-\sigma} d_{t+j}. \quad (7)$$

which can be substituted in equation (6) to obtain

$$E_t \sum_{j=0}^{\infty} \delta^j \left(\frac{C_{t+j}}{C_t} \right)^{-\sigma} d_{t+j} = E_t \sum_{j=0}^{\infty} \delta^j \left(\frac{C_{t+j}}{C_t} \right)^{-\sigma} C_{t+j}. \quad (8)$$

Applying a first-order approximation around a non-stochastic steady state yields

$$E_t \sum_{j=0}^{\infty} \delta^j \tilde{d}_{t+j} = E_t \sum_{j=0}^{\infty} \delta^j \tilde{C}_{t+j}. \quad (9)$$

Using the fact that $E_t(\tilde{C}_{t+k}) = \tilde{C}_t + \frac{1}{\sigma} E_t \sum_{j=0}^{k-1} (i_{t+j} - \pi_{t+j+1})$ yields the optimal consumption rule for the household.

Lemma 1. *Optimal consumption decision under RE*

$$\tilde{C}_t = (1 - \delta) E_t \sum_{j=0}^{\infty} \delta^j \tilde{d}_{t+j} - \frac{1}{\sigma} \delta E_t \sum_{j=0}^{\infty} \delta^j (i_{t+j} - \pi_{t+j+1}). \quad (10)$$

Equation (10) highlights the standard transmission mechanism of monetary policy which operates through the inter-temporal substitution of consumption which is influenced by the whole future path of real interest rates. Imposing the goods market equilibrium condition $\tilde{C}_t = \tilde{d}_t$, using the interest rate rule and the process for dividends yields the unique RE equilibrium condition.

Proposition 2. *RE Equilibrium*

$$\pi_t = -\frac{\sigma}{\phi_\pi} \tilde{d}_t \quad (11)$$

Similarly to Eusepi and Preston (2018a) inflation is a linear function of the endowment process. Stock prices or beliefs about stock prices do not influence the real economy. Anticipating the next section, this will not be the case under imperfect knowledge.

3.2. Imperfect Knowledge: Learning

In deriving the optimal decision (10) we have used the fact that the price of the risky asset, q_t , can be written as the discounted sum of dividends, as in equation (7). Indeed, under RE this is true. Under imperfect knowledge, we cannot iterate (3) forward since this would imply that any agent would know that either he is the marginal agent forever or that all the other agents in the economy share his beliefs, preferences and constraints.

Since agents have imperfect knowledge about the economy, even if agents know that the other agents share their preferences and constraints but have different beliefs, agent i would not be able to apply the Law of Iterated Expectations (LIE) to his FOC since

$$\begin{aligned} q_t &= \delta E_t^{\mathcal{P}_i} \left\{ \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} (q_{t+1} + d_{t+1}) \right\} \\ &= \delta E_t^{\mathcal{P}_i} \left\{ \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \left(d_{t+1} + \delta E_{t+1}^{\mathcal{P}^{mg}} \left\{ \left(\frac{C_{t+2}^{mg}}{C_{t+1}^{mg}} \right)^{-\sigma} (d_{t+2} + q_{t+2}) \right\} \right) \right\} \end{aligned} \quad (12)$$

and $E_t^{\mathcal{P}_i} E_{t+1}^{\mathcal{P}^{mg}} \neq E_t^{\mathcal{P}_i}$. Here \mathcal{P}^{mg} is the subjective probability measure of the marginal agent which is not known by agent i at time t . The marginal agent is the agent with the highest valuation of the asset which will determine the price of the asset in that period.⁶ Moreover, in line with actual equilibrium dynamics, it will be assumed that under the subjective probability measure, \mathcal{P}_i the expected lifetime asset valuation of each individual agent does not differ up to a first order approximation from the one of a rational agent. The consistency of this assumption in the context of long-horizon learning is discussed in appendix A from the supplementary material. This assumption does not provide enough

⁶See Adam and Marcet (2011) for further details.

information for each individual agent to recover the pricing map from dividends to prices. As a result, the optimality condition for stock prices is of the one-step ahead form which after log-linearization becomes

$$\tilde{q}_t = (1 - \delta)E_t^{\mathcal{P}}(\tilde{d}_{t+1}) + \delta E_t^{\mathcal{P}}(\tilde{q}_{t+1}) + \sigma(\tilde{C}_t - E_t^{\mathcal{P}}\tilde{C}_{t+1}). \quad (13)$$

In this economic environment, the optimal consumption decision of the agent takes the following form.

Lemma 3. *Optimal consumption decision under Imperfect Knowledge*

$$\begin{aligned} \tilde{C}_t \approx & (1 - \delta)E_t^{\mathcal{P}} \sum_{j=0}^{\infty} \delta^j \tilde{d}_{t+j} - \frac{1}{\sigma} \delta E_t^{\mathcal{P}} \sum_{j=0}^{\infty} \delta^j (i_{t+j} - \pi_{t+j+1}) \\ & + \delta \tilde{q}_t - (1 - \delta) \underbrace{\left[E_t^{\mathcal{P}} \sum_{j=1}^{\infty} \delta^j \tilde{d}_{t+j} - \frac{\delta}{1 - \delta} E_t^{\mathcal{P}} \sum_{j=0}^{\infty} \delta^j (i_{t+j} - \pi_{t+j+1}) \right]}_{\text{Wealth Effect}=0 \text{ in RE}}. \end{aligned} \quad (14)$$

The first line from (14) is the standard transmission mechanism as also found under RE (see equation (10)). The second line represents a new channel through which stock prices and beliefs about stock prices affect the consumption decision of the household. The second channel is the difference between actual stock prices and the discounted sum of future dividends. Under RE these terms would sum exactly to 0 since stock prices are exactly equal to the discounted sum of dividends. Under learning, there is no reason for this to be the case. Since beliefs influence stock prices and vice versa, stock prices may drift away from their perceived fundamental value therefore causing agents to feel wealthier and increase consumption. In the current framework stock price wealth effects appear because agents do not have perfect knowledge about the economy and how stock prices are actually determined. What is perhaps interesting in the current theory is that this lack of knowledge is the principal determinant of stock price wealth effects.

In order to determine the learning equilibrium assume the following:

1. similarly to RE, agents have perfect knowledge about the dividend process, therefore

$$E_t^{\mathcal{P}} d_{t+j} = \mu$$

2. agents know the interest rate rule, therefore $E^{\mathcal{P}}i_{t+j} = \phi_{\pi}E^{\mathcal{P}}\pi_{t+j}$
3. agents think that inflation and stock prices follow an unobserved component model

$$\begin{aligned}x_t &= \beta_t^x + \epsilon_t \\ \beta_t^x &= \beta_{t-1}^x + \psi_t\end{aligned}\tag{15}$$

where $x = (\tilde{q}, \pi)'$.

Denoting by $\hat{\beta}_{t-1} = (\hat{\beta}_{t-1}^{\pi}, \hat{\beta}_{t-1}^q)$ period t subjective expectations, agents use the following optimal recursive algorithm to update their beliefs

$$\hat{\beta}_t = \hat{\beta}_{t-1} + \lambda(x_t - \hat{\beta}_{t-1})\tag{16}$$

where λ is the constant gain coefficient which governs the speed at which agents incorporate new information into current beliefs.⁷ Given these assumptions the expectations of real interest rates from equation (14) can be evaluated as

$$E_t^{\mathcal{P}} \sum_{j=0}^{\infty} \delta^j (i_{t+j} - \pi_{t+j+1}) = \phi_{\pi} \pi_t + \frac{\delta \phi_{\pi} - 1}{1 - \delta} \beta_{t-1}^{\pi}.\tag{17}$$

Substituting the forecasts from above into the consumption equation (14), applying assumption 1 and market clearing in the goods market, $\tilde{C}_t = \tilde{d}_t$, gives the data-generating process or the actual law of motion for inflation.

Proposition 4. *Learning Equilibrium*

$$\pi_t = \frac{\delta \sigma}{\phi_{\pi}} \beta_{t-1}^q - \left(\frac{\sigma}{\phi_{\pi}} - \frac{(1 - \sigma)(\delta \phi_{\pi} - 1)}{(1 - \delta)\phi_{\pi}} \right) \beta_{t-1}^{\pi} - \frac{\sigma}{\phi_{\pi}} \tilde{d}_t.\tag{18}$$

The learning equilibrium is fundamentally different from the RE counterpart. The first term in the above equation is totally absent from the RE equilibrium relation. Beliefs about

⁷As it is usually done in the learning literature, in order to avoid the simultaneity formation of beliefs and equilibrium variables, agents form expectations at period t using information from the previous period.

stock prices influence directly inflation in equilibrium through a stock price wealth effect. Eusepi and Preston (2018a) reach a similar conclusion for the case of long-term bonds, although in that case agents are assumed to know the pricing map and learn about taxes and long term bonds.

Having gained this intuition into the origin of the effect of stock prices on consumption, next section lays out a quantitative general equilibrium monetary model in which stock price wealth effects influence the aggregate economy.

4. Monetary Policy and Stock Prices: Quantitative Evaluation

This section describes a heterogeneous agents New Keynesian model with learning in which agents hold subjective beliefs about the variables which are exogenous to their decision-making (from the point of view of an individual agent). There are two types of consumers and the only source of heterogeneity between them is the fact that a constant fraction, \mathcal{O} , of the agents is assumed not to participate in the stock market. This assumption is in line with the empirical evidence on US stock market participation.⁸ Notice that while some of the agents are excluded from saving in stocks all agents have access to the bond market and can smooth consumption by investing in a riskless asset. In essence, the model is a limited asset market participation New Keynesian model (LAMP-NK) with homogeneous imperfect information. The economy is comprised of households, final goods producers, intermediary goods producers, a mutual fund and a central bank conducting monetary policy.

4.1. Households

The economy is populated by a continuum of infinitely lived consumers indexed by i who choose consumption, C_t^i , labor, N_t^i , bond holdings, B_t^i , stock holdings in a mutual fund, S_t^i , and receive income in form of dividends, D_t and wages, W_t . The mutual fund is introduced to abstract from the portfolio choice of the households and its problem will be described in

⁸This does not change qualitatively the main results of the paper and serves in dampening the magnitude of stock price wealth effects.

a later section. Let $i = S^U, S^C$ denote the label for unconstrained/ constrained agents. The problem of the household is to maximize utility subject to a standard budget constraint

$$\begin{aligned} \max_{C_t^i, N_t^i, B_t^i, S_t^i} E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \delta^t \left[\frac{(C_t^i)^{1-\sigma}}{1-\sigma} - \frac{(N_t^i)^{1+\phi}}{1+\phi} \right] \\ \text{s.t. } P_t C_t^i + B_t^i + S_t^i Q_t \leq B_{t-1}^i (1 + i_{t-1}) + W_t N_t^i + S_{t-1}^i (Q_t + D_t) \\ 0 \leq S_t^i \leq S^H, \forall t \end{aligned} \quad (19)$$

where P_t is the aggregate price index, i_t is the nominal interest rate, Q_t is the ex-dividend price of the mutual fund share, W_t is the nominal wage and D_t is the nominal dividend paid by the mutual fund. Short-selling is not allowed and there is an upper bound for stock holdings, S^H , which can be bigger than 1.⁹

The optimality conditions of the household problem are

$$\frac{(N_t^i)^\phi}{(C_t^i)^{-\sigma}} = w_t, \quad (20)$$

$$\frac{1}{1 + i_t} = \delta E_t^{\mathcal{P}} \left\{ \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \frac{1}{1 + \pi_{t+1}} \right\}, \quad (21)$$

$$Q_t = \delta E_t^{\mathcal{P}} \left\{ \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \frac{(Q_{t+1} + D_{t+1})}{1 + \pi_{t+1}} \right\}, \quad (22)$$

where π_{t+1} is the inflation rate between t and $t + 1$ and $w_t = \frac{W_t}{P_t}$ is the real wage. Equation (20) determines the consumption and labor decision, equation (21) is the Euler equation and equation (22) is the asset pricing equation. Also notice that equation (22) holds with equality as long as $S_t^i \in [0, S^H)$.

The only difference from the standard household problem is the operator $E_t^{\mathcal{P}}$. The expectations of the households are determined using the subjective probability measure \mathcal{P} that assigns probabilities to the variables the household is trying to forecast. I proceed in deriving the consumption decision of the household following the anticipated utility framework of Preston (2005).¹⁰ The intertemporal budget constraint of the household reads

⁹It is assumed that the upper bound on stock holdings is never reached.

¹⁰A large body of the literature uses the *Euler Equation* approach to introduce learning in DSGE models.

$$\frac{\mathcal{W}_t^i}{P_t} \approx E_t^{\mathcal{P}} \sum_{j=0}^{\infty} \delta^j \left(\frac{C_{t+j}^i}{C_t^i} \right)^{-\sigma} \left[C_{t+j}^i - w_{t+j}^{\frac{1+\phi}{\phi}} (C_{t+j}^i)^{\frac{-\sigma}{\phi}} \right] \quad (23)$$

where $\mathcal{W}_t^i = B_{t-1}^i(1 + i_{t-1}) + S_{t-1}^i(Q_t + D_t)$ represents nominal wealth at time t .

Log-linearization of equation (A.3) around a steady state characterized by $\pi = 0$, $S = 1$, $C = Y$ yields

$$\tilde{w}_t^i = (1 - \delta) E_t^{\mathcal{P}} \left\{ \sum_{j=0}^{\infty} \delta^j \left[\sigma \tilde{c}_t^i + \Delta_r \tilde{c}_{t+j}^i - \frac{\Delta_i}{1 - \delta} \tilde{w}_{t+j} \right] \right\}. \quad (24)$$

and $w \tilde{w}_t^i = (1 + i)b_{t-1}^i + q(\tilde{S}_{t-1}^i + \tilde{q}_t) + d(\tilde{S}_{t-1}^i + \tilde{d}_t)$ where $w = \frac{d}{1 - \delta}$. In the above expression, any variable \tilde{x} denotes the percentage deviation of real variables from their steady-state values Y , q , d represent steady-state values of aggregate output, real stock price and real dividends.

Substituting the Euler equation in the linearized budget constraint (24) and rearranging results in the decision rule of the household

$$\tilde{c}_t^i = \Delta_i \tilde{w}_t^i + \Delta_w \sum_{j=0}^{\infty} \delta^j E_t^{\mathcal{P}} (\tilde{w}_{t+j}) - \frac{\delta}{\sigma} \Delta_r \sum_{j=0}^{\infty} \delta^j E_t^{\mathcal{P}} (i_{t+j} - \pi_{t+j+1}).^{11} \quad (25)$$

Equation (25) makes clear that the consumption decision of the household at time t depends not only on the next period output and interest rate (as dictated by the standard Euler equation) but on the whole future path of wages, inflation and interest rates, as well as on the current wealth. Therefore, the agent will need to form expectations/forecasts for all future π and \tilde{w} and i using the subjective probability measure \mathcal{P} . The next proposition presents the optimal consumption decision for the two types of agents.

This approach entails that after solving the model using the RE assumption, expectations are replaced mechanically with some subjective expectations. This approach implies that agents are mixing two probability measures, the RE measure, on the one hand, and the subjective one. Furthermore the stock market wealth effect is not present under this approach since agents implicitly know the mapping from dividends to prices. See Preston (2005) for a detailed discussion of this issue and Eusepi and Preston (2018b) for a comparison between these two approaches.

¹¹Appendix B presents detailed derivation results.

Proposition 5. *The log-linearized aggregate consumption decisions at time t for households participating in the stock market (U) and excluded from trading stocks (C) are given by*

$$\begin{aligned}\tilde{c}_t^U &= \Delta_i \left[\frac{(1+i)}{\omega} b_{t-1}^U + \tilde{S}_{t-1}^U + \delta \tilde{q}_t + (1-\delta) \tilde{d}_t \right] + \Delta_w \sum_{j=0}^{\infty} \delta^j E_t^{\mathcal{P}}(\tilde{w}_{t+j}) - \frac{\delta}{\sigma} \Delta_r \sum_{j=0}^{\infty} \delta^j E_t^{\mathcal{P}}(i_{t+j} - \pi_{t+j+1}), \\ \tilde{c}_t^C &= \Delta_i \left[(1+i) b_{t-1}^C \right] + \Delta_w \sum_{j=0}^{\infty} \delta^j E_t^{\mathcal{P}}(\tilde{w}_{t+j}) - \frac{\delta}{\sigma} \Delta_r \sum_{j=0}^{\infty} \delta^j E_t^{\mathcal{P}}(i_{t+j} - \pi_{t+j+1}).\end{aligned}\quad (26)$$

Notice that the only difference between the optimal consumption decisions of the two types of households is given by the first term from both equations, namely the asset position at time t .

4.2. Firms

Intermediate goods producers

There is a continuum of firms indexed by j which produce differentiated goods using the Cobb-Douglas production function with labor input $N_t(j)$

$$Y_t(j) = N_t(j)^{1-\alpha}. \quad (27)$$

Firms are subject to nominal rigidities when setting prices. Following Calvo (1983), each firm cannot reset its price in a given period with probability θ . The problem of the firm is to maximize profits subject to the demand function

$$\begin{aligned}\max_{P_t^*} & \sum_{k=0}^{\infty} \theta^k E_t^{\mathcal{P}_j} \{ Q_{t,t+k} (P_t^* Y_{t+k/t} - \psi_{t+k}(Y_{t+k/t})) \} \\ \text{s.t.} & Y_{t+k/t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} Y_{t+k}\end{aligned}\quad (28)$$

where $Y_{t+k/k}$ denotes output in period $t+k$ for a firm that last reset price in period t , $\psi_t(\cdot)$ is the cost function and $Q_{t,t+k} = \delta^k \left(\frac{Y_{t+k}}{Y_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}}$ is the stochastic discount factor for nominal profits.¹² Notice that compared to the RE framework, the stochastic discount factor is a

¹²It is assumed for simplicity that households and firms share the same belief system.

function of aggregate output and not of consumption. This is because firms do not know the problem of the households or of the mutual fund and therefore, it makes possible for firms to hold subjective beliefs about aggregate outcomes.

The solution to the profit maximization problem yields the optimal price setting decision of the firm

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{k=0}^{\infty} (\theta\delta)^k E_t^{\mathcal{F}_j} [Y_{t+k}^{1-\sigma} P_{t+k}^\epsilon MC_{t+k/k}]}{\sum_{k=0}^{\infty} (\theta\delta)^k E_t^{\mathcal{F}_j} [Y_{t+k}^{1-\sigma} P_{t+k}^{\epsilon-1}]} \quad (29)$$

where $MC_{t+k/k}$ is the real marginal cost of a firm which last updated prices in period t . After log-linearization the previous relation becomes

$$p_t^* = (1 - \delta\theta) \sum_{k=0}^{\infty} (\theta\delta)^k E_t^{\mathcal{F}_j} \left\{ \frac{\alpha}{1 - \alpha + \epsilon\alpha} \tilde{y}_{t+k} + \frac{1 - \alpha}{1 - \alpha + \epsilon\alpha} (\tilde{w}_{t+k} + \epsilon_{t+k}^u) + p_{t+k} \right\} \quad (30)$$

where ϵ_{t+k}^u is an exogenous process interpreted as a cost-push shock.

Final goods producers

The consumption good in this economy is produced by perfectly competitive firms which use intermediary goods as inputs in their CES production function:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{1-\epsilon}} \quad (31)$$

Profit maximization yields the following demand for intermediary goods:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t \quad (32)$$

where $P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$ is the aggregate price index.

4.3. Mutual Fund

For the sake of simplicity, the model abstracts from the portfolio choice of the households and instead assumes the existence of a mutual fund which holds all the intermediary firms in this economy and issues shares with nominal price Q_t which are sold in a perfectly

competitive market to the household sector. The asset pricing equation of the mutual fund is given by:

$$Q_t = \delta E_t^{\mathcal{F}} \left\{ \left(\frac{Y_{t+1}}{Y_t} \right)^{-\sigma} \frac{(Q_{t+1} + D_{t+1})}{1 + \pi_{t+1}} \right\} \quad (33)$$

which in equilibrium will be the same as the asset pricing equation of the households.

4.4. Central Bank

The monetary authority sets the interest rate by following a Taylor rule

$$i_t = \phi_{\pi} \pi_t + \phi_y \tilde{y}_t + \epsilon_t^i \quad (34)$$

where ϵ_t^i is a stochastic process with zero mean which can be interpreted as a monetary policy shock.

4.5. Equilibrium

Defining aggregate consumption of the two types of households as

$$C_t^C = \int_0^1 C_t^{i,C} di, \quad C_t^U = \int_0^1 C_t^{i,U} di,$$

and aggregate labour as

$$N_t^C = \int_0^1 N_t^{i,C} di, \quad N_t^U = \int_0^1 N_t^{i,U} di,$$

the equilibrium conditions are

$$\begin{aligned} \int_0^1 B_t^i di &= 0, \\ \int_0^1 C_t^i di &= C_t = \mathcal{O} C_t^C + (1 - \mathcal{O}) C_t^U = Y_t, \\ \int_0^1 N_t(j) dj &= \int_0^1 N_t^i di = \mathcal{O} N_t^C + (1 - \mathcal{O}) N_t^U, \\ S_t^U &= 1. \end{aligned} \quad (35)$$

First equation is the bond market clearing condition which assumes that bonds are in 0 net supply. The next two equations are the good market and labor market clearing conditions and finally the last equation requires clearing in the equity market.

On the supply side, since producers of intermediate goods are identical, the fraction of firms that will re-optimize each period $(1 - \theta)$ will choose the same price, p_t^* . This fact combined with the definition of the aggregate price level (see Final goods producers section) results in the following aggregate law of motion for inflation:

$$\pi_t = (1 - \theta)(p_t^* - p_{t-1}) \quad (36)$$

Aggregating the household decision rule (25) and combining it with the market clearing condition (35) results in the demand block of the model, the IS equation

$$\tilde{y}_t = \Delta_i \mathcal{O}(\delta \tilde{q}_t + (1 - \delta) \tilde{d}_t) + \Delta_w \sum_{j=0}^{\infty} \delta^j E_t^{\mathcal{P}}(\tilde{w}_{t+j}) - \frac{\delta}{\sigma} \Delta_r \sum_{j=0}^{\infty} \delta^j E_t^{\mathcal{P}}(i_{t+j} - \pi_{t+j+1}). \quad (37)$$

Equation (37) implies that not only current and future wages and real interest rates affect output today but also current stock prices. Agents do not internalize the fact that their pricing equation is determining stock prices today but instead they hold subjective beliefs about its evolution, therefore creating an equity channel effect: an increase in the equity prices today makes the consumers feel wealthier which affects aggregate consumption and output. As discussed in the previous section this stock price wealth effect appears because of the difference between actual stock prices and their fundamental value, determined by the discounted sum of dividends. If the same economy would be studied under the Euler Equation approach the equity channel will be absent.

In this environment, the stock market and the real output gap are determined simultaneously in equilibrium. Suppose that in period t agents are hit by a wave of optimism which causes stock prices to increase in the same period. This in turn triggers the stock price wealth effect and increases output contemporaneously via the IS equation. The central bank reacts to the rise in output by increasing interest rates which has two effects. First, via the intertemporal substitution channel of monetary policy, it lowers consumption and output today. Second, it affects negatively stock prices which trigger stock price wealth effects decreasing aggregate demand. If monetary policy does not react strongly enough, the increase in interest rates might not be sufficient to counteract the initial increase in stock

prices which will trigger a positive revision in stock price beliefs which reinforces further the raise in stock prices. The system is self-referential in the sense of Marcet and Sargent (1989): prices affect beliefs which influence prices resulting in a positive feedback loop. Economic policy can play an important role in breaking or further accommodating this positive feedback loop.

Combining the law of motion of inflation (36) with the pricing equation of the firms (30) results in the supply block of the model, the Phillips Curve equation

$$\begin{aligned} \pi_t = & \Theta_y \sum_{k=0}^{\infty} (\theta\delta)^k E_t^{\mathcal{P}} \tilde{y}_{t+k} + \Theta_w \sum_{k=0}^{\infty} (\theta\delta)^k E_t^{\mathcal{P}} \tilde{w}_{t+k} \\ & + \sum_{k=0}^{\infty} (\theta\delta)^k E_t^{\mathcal{P}} u_{t+k} + (1-\theta)\delta \sum_{k=0}^{\infty} (\theta\delta)^k E_t^{\mathcal{P}} \pi_{t+k+1}. \end{aligned} \quad (38)$$

Log-linearization of equation (22) yields the law of motion of stock prices:

$$\tilde{q}_t = (1-\delta)E_t^{\mathcal{P}}(\tilde{d}_{t+1}) + \delta E_t^{\mathcal{P}}(\tilde{q}_{t+1}) - (i_t - E_t^{\mathcal{P}}(\pi_{t+1})). \quad (39)$$

Given optimal prices, firms supply the desired output which determines the amount of labour

$$N_t = Y_t^{1/(1-\alpha)} e^{d_t} \quad (40)$$

which is obtained by aggregating the individual production technologies. The last term captures price dispersion and is given by $d_t = \int_0^1 \left(\frac{P_t(i)}{P_t} \right)$. Wages are determined by the optimality condition of the households

$$w_t = \frac{N_t^\phi}{Y_t^{-\sigma}}. \quad (41)$$

Finally, real dividends are given by the profits of the firms

$$D_t = Y_t - w_t N_t. \quad (42)$$

4.6. Agents' model of learning

The subjective belief system of the agents can be characterized by the probability space (Ω, \mathcal{P}) with a typical element $\omega \in \Omega$, $\omega = \{Y_t, P_t, Q_t, D_t, W_t, u_t, \epsilon^i\}$. As in Eusepi and

Preston (2018a) the belief model includes the variables (exogenous from the point of view of the individual agents) which agents need to forecast in order to make optimal consumption decision today. These are output, inflation, stock prices dividends and wages.

Agents believe that output, inflation, wages, dividends and equity prices follow an unobserved component model

$$\begin{aligned} z_t &= \beta_t + \zeta_t \\ \beta_t &= \rho\beta_{t-1} + \vartheta_t \end{aligned} \tag{43}$$

where $z_t = (\tilde{y}_t, \tilde{\pi}_t, \tilde{q}_t, \tilde{d}_t, \tilde{w}_t)'$, $\rho \in (0, 1]$, $\zeta_t \sim N(0, \sigma_\zeta^2 I_5)$ and $\vartheta_t \sim N(0, \sigma_\vartheta^2 I_5)$. Agents have also knowledge of the Taylor rule that the central bank is following and uses it to forecast interest rates. As usually done in the learning literature, agents have full knowledge of the exogenous shocks. The optimal filter for $E^\mathcal{P}(\beta_t/g^{t-1}) = \hat{\beta}_t$ is the Kalman filter and optimal updating implies the following recursion of beliefs

$$\hat{\beta}_t = \rho \hat{\beta}_{t-1} + \lambda(z_{t-1} - \rho \hat{\beta}_{t-1}) + e_3 \epsilon_t^\beta \tag{44}$$

where $\hat{\beta}_t = [\hat{\beta}_t^y, \hat{\beta}_t^\pi, \hat{\beta}_t^q, \hat{\beta}_t^d, \hat{\beta}_t^w]'$, $g^{t-1} = \{g_{t-1}, g_{t-2} \dots g_1\}$ denotes information up to time t , $e_3 = (0, 0, 1, 0, 0)'$, ϵ_t^β , is a shock to stock price beliefs (sentiment shock) and λ is the steady state Kalman gain which controls the speed of learning.¹³ Adam et al. (2017) show that survey data regarding price expectations are captured well by an extrapolative updating equation of the form (44). Nagel and Xu (2019) call this "learning with fading memory" and links it to the theoretical biology literature which models memory decay in organisms.

It follows from (43) that agents' forecasts about output, inflation, equity prices, dividends and wages are given by

$$E_t^\mathcal{P} z_{t+k} = \rho^k \hat{\beta}_t \tag{45}$$

where beliefs, $\hat{\beta}_t$, are updated each period according to (44).

¹³The resulting equation of belief updating is optimal given the assumption of agents observing the transitory component with a lag. In that case ϵ_t^q represents the new information about the transitory component. For further details and derivation see Appendix 6 from Adam et al. (2017).

Belief system (43) together with the optimal filtering rule imply that agents learn about the long-run conditional means of the variables in the economy. As argued in Eusepi et al. (2018) the belief system proposed is less restrictive than might be thought since usually the drift term drives the largest deviations from rational expectations predictions.

4.7. Estimation of the model

The model is calibrated/estimated on US quarterly data. The elasticity of substitution among goods, ϵ , is set to 6 and the Frisch elasticity of labor-supply, ϕ , to 0.75 following the recommendation from Chetty et al. (2011). From the supply side, the share of labor, α , equals 1/4 and the probability of not being able to adjust prices, θ , is set to 2/3 implying an average duration of keeping prices fixed of 3 quarters. The Taylor rule response to output-gap is set to 0.5/4 and the one for inflation to 1.5. The response of the central bank to the stock-price gap is set to 0 for now but its effect on financial stability will be discussed in a later section. There are three exogenous shocks that drive the dynamics of the model: cost push shocks, u_t and equity belief shocks, ϵ_t^β and monetary policy shocks, ϵ^i . These shocks follow AR(1) processes:

$$x_t = \rho_x x_{t-1} + \xi_t^x \tag{46}$$

where $x \in \{u, \epsilon^\beta, \epsilon^i\}$ and $\xi_t^x \sim \mathcal{N}(0, \sigma^x)$. Sentiment shocks are assumed to be *i.i.d.*, $\rho_\beta = 0$.

The risk aversion parameter, σ is set to 1 and the discount factor, δ to 0.9928 in order to match the average PD ratio. The stock ownership is set to 0.47 which represents the average stock ownership over the period 1989-2019 according to the Survey of Consumer Finances. The calibration is summarized in the following table.

| Calibrated | Symbol | Value |
|--------------------------------|-------------------|--------|
| Discount factor | δ | 0.9928 |
| Risk aversion coef. | σ | 1 |
| Frisch labor supply elasticity | $\frac{1}{\phi}$ | 0.75 |
| Elasticity of substitution | ϵ | 6 |
| Prob. of not adjusting price | θ | 2/3 |
| Share of labor | α | 0.25 |
| Taylor-rule coef. of inflation | ϕ_π | 1.5 |
| Taylor-rule coef. of output | ϕ_y | 0.5/4 |
| Equity Share Ownership | $1 - \mathcal{O}$ | 0.47 |

Table 1: Calibrated Parameters

The rest of the parameters: standard deviation of cost push shock, σ^u , standard deviation of belief shock, σ^β , standard deviation of monetary policy shock, σ^{ϵ_i} , persistence of cost-push shock, ρ_u , persistence of monetary policy shock, ρ_{ϵ_i} , kalman gain coefficient, λ and autoregressive coefficient of beliefs, ρ , are jointly estimated using the method of simulated moments (MSM) to match a set of eight business cycle and financial moments.

Defining $\theta = (\sigma^u, \sigma^\beta, \sigma^{\epsilon_i}, \rho_u, \rho_{\epsilon_i}, \lambda, \rho)$ as the vector of parameters to be estimated, the MSM estimator is given by

$$\hat{\theta} = \arg \min_{\theta} [\hat{S} - S(\theta)]' \hat{\Sigma} [\hat{S} - S(\theta)] \quad (47)$$

where \hat{S} is the vector of empirical moments to be matched, $S(\theta)$ is the model moments counterpart and $\hat{\Sigma}$ is a weighting matrix.¹⁴ The vector of empirical moments is given by

$$\hat{S} = [\hat{\sigma}_{hp}(y), \hat{\sigma}_{hp}(\pi), \hat{\rho}_{hp}(y, \pi), \hat{E}(P/D), \hat{\sigma}(P/D), \hat{\rho}(P/D), \hat{\sigma}(r^e), \hat{\sigma}(r^f)]' \quad (48)$$

¹⁴In practice the inverse of the diagonal of the estimated variance-covariance matrix of the data moments, \hat{S} is used. The latter is obtained using a Newey-West estimator and the delta method as in Adam et al. (2016). For further details on the estimation of $\hat{\Sigma}$ please refer to online appendix of that paper.

where $\hat{\sigma}_{hp}()$ and $\hat{\rho}_{hp}()$ denote the standard deviation and correlation coefficient applied on filtered data, r^e denotes stock returns and r^f the risk free interest rate.¹⁵

The model is estimated on quarterly US data for the post-war period 1955Q1-2018Q4. The data for the business cycle statistics are obtained from the FRED database: the inflation rate is measured as the % change in the CPI for all urban consumers [CPIAUCSL], output as real GDP [GDPC1] and the fed funds rate [FEDFUNDS] is used for the short term nominal interest rate. The real interest rate is obtained by subtracting the ex-post inflation rate from the nominal short term interest rate. Data on real stock market prices and dividends are obtained from Robert Shiller webpage. Since data is monthly, quarterly variables have been obtained by selecting end of period values and dividends have been computed as average of the current and previous three quarters.

Table 2 summarizes the estimated parameters while table 3 shows the data moments and the model implied counterparts.

| Estimated | Symbol | Value |
|--------------------------------------|-----------------------|--------|
| Std. cost push shock | σ^u | 0.0013 |
| Std. MP shocks | σ^{ϵ_i} | 0.0008 |
| Autoregressive coef. cost push shock | ρ_u | 0.97 |
| Autoregressive coef. MP shocks | ρ_{β_q} | 0.96 |
| <i>Agents' model</i> | | |
| Kalman gain | λ | 0.0013 |
| Autoregressive coef. beliefs | ρ | 0.99 |
| Std. equity belief shocks | σ^{β_q} | 0.062 |

Table 2: Estimated parameters from SMM

¹⁵The business cycle component is extracted using the Hodrick-Prescott filter with a smoothing parameter of 1600 for quarterly data.

| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|--------------------------------------|--------------------------------|-------------|-------------------|-------------------|-------------|--------------|----------------------------|
| | Symbol | Data Moment | Learning Model | Moment t -ratio | RE Model | RE Re-est | Gabaix (2020) Re-est |
| I. Business Cycle | | | | | | | |
| Std. dev. of output | $\sigma(y)$ | 1.45 | 1.53 | -0.39 | 0.91 | 1.66 | 1.29 |
| Std. dev. of inflation | $\sigma(\pi)$ | 0.54 | 0.47 | 1 | 0.31 | 0.8 | 0.64 |
| Correlation output/inflation | $\rho_{y,\pi}$ | 0.29 | 0.26 | 0.36 | -0.8 | 0.18 | 0.29 |
| II. Financial Moments | | | | | | | |
| Average PD ratio | E (P/ D) | 154 | 159 | -0.38 | 139 | 138 | 166 |
| Std. dev. of PD ratio | $\sigma(P/D)$ | 63 | 63 | -0.34 | 15 | 8.9 | 29 |
| Auto-correlation of PD ratio | $\rho(P/D)$ | 0.99 | 0.96 | 0.57 | 0.95 | 0.76 | 0.91 |
| Std. dev. of equity return (%) | $\sigma(r^e)$ | 6.02 | 6.02 | 0.04 | 0.13 | 4.5 | 1.3 |
| Std. dev. real risk free rate (%) | $\sigma(r^f)$ | 0.72 | 0.78 | 0.59 | 0.05 | 0.5 | 1 |
| III. Non Targeted moments | | | | | | | |
| Average Equity Return (%) | $E(r^e)$ | 1.78 | 0.8 | 1.92 | 0.73 | 0.72 | 0.84 |
| Average real risk free rate (%) | $E(r^f)$ | 0.32 | 0.74 | -3.5 | 0.72 | 0.72 | 0.64 |
| volatility ratio stock prices/output | $\sigma(Q)/\sigma(y)$ | 6.7 | 5 | 2 | 0.12 | 3.45 | 1.1 |
| corr. Stock Prices/ output | $\rho(Q, y)$ | 0.5 | 0.43 | 0.53 | -1 | -0.72 | 1 |
| Consumption Wealth Effect | dy/dQ | [0.03-0.2] | 0.09 | - | 0 | 0 | 0 |
| corr. Survey Expect./ PD ratio | $\rho(PD_t, E_t(r_{t,t+1}^e))$ | 0.74 | 0.5 | - | -0.47 | -0.1 | -0.42 |
| Std. dev. Expected Returns(%) | $\sigma(E_t(r_{t,t+1}^e))$ | 0.65 | 2 | - | 0.12 | 4.52 | 1.23 |

Table 3: Model implied moments

Data moments are computed over the period 1955Q1: 2018Q3. Moments have been computed as averages over 1000 simulations, each of 256 quarters as in the data with a burn-in period of 500 periods. Subjective expectations are measured by the UBS Gallup survey for own portfolio returns for the period 1998Q2-2007Q3. t -ratios are defined as (data moment-model moment)/ S.E of data moment. The RE model moments (column 6) are computed using the parameters estimated from the learning model while columns (7) and (8) present the moments after re-estimating the RE and cognitive discounting models. For the latter, the discounting parameter, m , is set to 0.85 similar to the calibration from Gabaix (2020).

The first two horizontal panels from table 3 present moments targeted in the estimation while panel III displays a set of non-targeted moments to which the model will be confronted for validation. The first five columns present moments from the estimated learning model and data while the last three columns present the quantitative performance of the RE version of the model and of the cognitive discounting model of Gabaix (2020) which has

been receiving increasing attention in the literature.

The model matches well business cycle moments, the volatility of financial variables and the persistence of the PD ratio. Although not targeted in the estimation, the model delivers a stock market which is 5.2 times more volatile than the real economy at the business cycle frequency and which has a 0.45 correlation coefficient with the output-gap. Figure F.20 from online appendix F shows that at a business cycle frequency the US stock market is 6.7 times more volatile than the real economy. The model matches remarkably well the dynamics (especially volatility) of the stock market and its joint behavior with survey expectations. Responsible for this success is the combination of learning and sentiment shocks. Appendix D from the supplementary material re-estimates the model without beliefs shocks and shows that the latter are key in matching financial statistics. The consumption wealth effect in the model economy is 0.09 meaning that for every 1% increase in stock market wealth consumption responds by 0.09%. The magnitude delivered by the estimation is well within the bounds usually found in the empirical literature. As mentioned in the introduction, Di Maggio et al. (2020) finds that the consumption wealth effect from unrealized capital gains ranges between 3-20%.

The model implied beliefs about stock prices are positively correlated with the PD ratio and are several orders of magnitude less volatile than actual realised returns, replicating the survey evidence. Similarly to other findings from the learning literature, the model is not able to match the equity premium since although prices are very volatile this volatility is not priced since it comes from subjective beliefs.

Turning to the two RE version of the model and the cognitive framework, neither of these can jointly replicate the volatility of the financial market and the smoothness of the business cycle. Moreover, these models predict a negative correlation between prices and return expectations, at odds with empirical survey data.

Using the estimated and calibrated parameters, panel a) in figure 1 presents one simulation of 260 periods from the learning model for the actual stock price and the ex-post rational price while panel b) plots one realization of the PD ratio. The figure shows that while the rational price (P^*) does not fluctuate much, actual stock prices experience booms

and busts of magnitudes of up to 100 % in absolute values. The figure can be directly compared to figure 1 in Shiller (1981) which is the evidence of excess volatility compared to fundamentals.

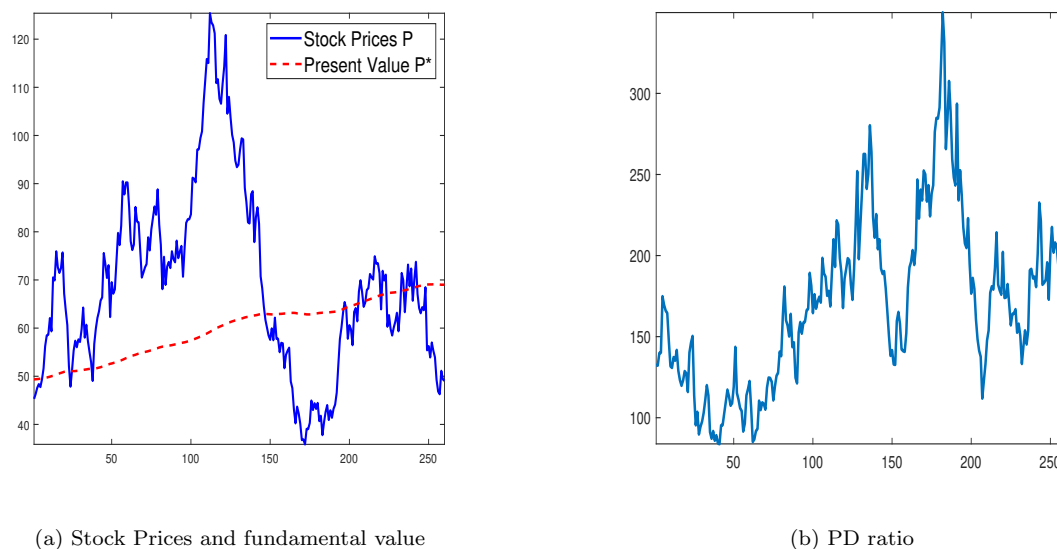


Figure 1: Stock Prices and Discounted Dividends

Panel a) presents one simulation of 260 periods for the time series of stock prices and the corresponding present value of discounted dividends or ex-post rational price in the language of Shiller (1981). Similar to that study, it has been assumed that the end value for the rational price, P^* , is the sample average of the real stock price. Given that, the rest of the time series for the rational price can be backed out by the following recursion $P_t^* = \delta P_{t+1}^* + D_t$ where D_t is the real dividend at time t . Panel b) presents a simulation of the PD ratio.

4.8. Survey Expectations vs Model Beliefs

Coibion and Gorodnichenko (2012, 2015) bring evidence in favor of information rigidity in expectation formation and show that aggregate forecasts of inflation and other macroeconomic variables exhibit under-reaction described by a positive relation between forecast revisions and forecast errors. Let $FR_{t,h}^x = x_{t+h/t} - x_{t+h/t-1}$ and $FE_{t,h}^x = x_{t+h} - x_{t+h/t}$ denote the forecast revision and forecast error for variable x at time t and horizon h . Figure 2 presents the correlation coefficients for forecast errors with prices and revisions in beliefs for stock returns, inflation, real output growth, and nominal interest rates. Under RE both of

these coefficients would be 0.

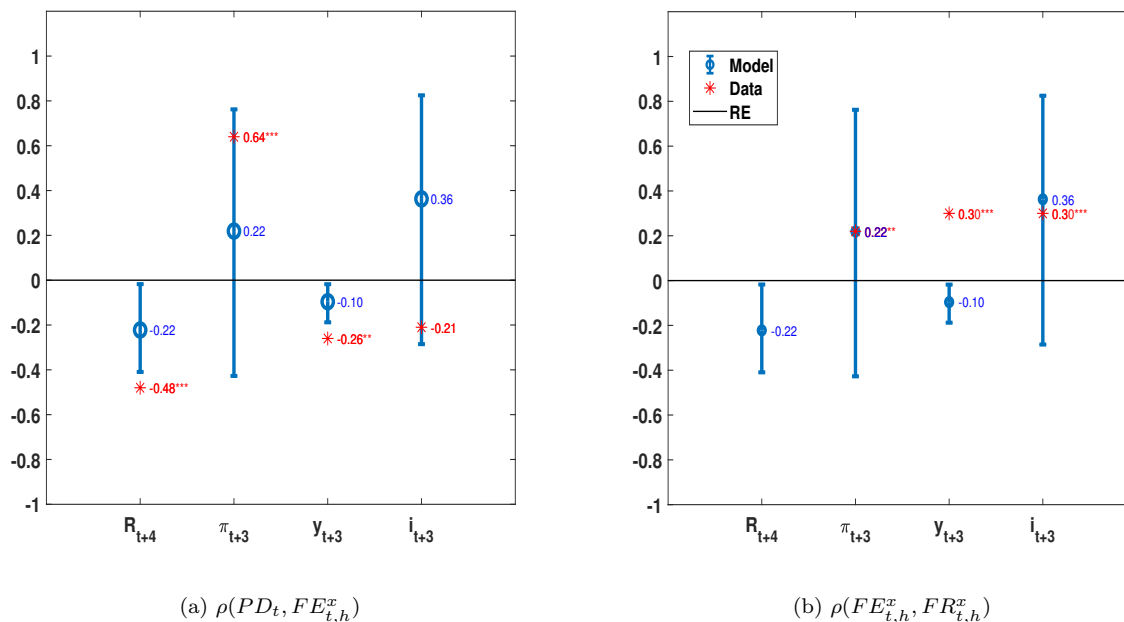


Figure 2: Forecast Error Predictability

The figure shows the correlation coefficient for forecast errors with the PD ratio and the revision in beliefs for 1 year ahead expected capital gains and three quarters ahead inflation, output growth and interest rates. The survey data for the latter three macroeconomic variables comes from SPF and cover the same time period as in Winkler (2019), namely 1981Q1-2012Q4. Expected capital gains are measured by the US Gallup survey (own portfolio) which covers the period 1998Q1-2007Q3. Similar to Coibion and Gorodnichenko (2015) the 3 quarters ahead also includes the nowcast. The series for the revision of beliefs is not available for the US Gallup survey. The model statistics are computed over 1000 simulations each of 256 time periods with a burn-in of 500 quarters. The symbols *, ** and *** denote statistical significance at the 1, 5, 10 % level for the empirical data. Model ranges (blue lines) represent 95% confidence interval and blue dots denote the mean across simulations.

Panel a) shows the correlation coefficient between the PD ratio and the forecast errors for each of the four variables considered. When stock prices are high agents tend to systematically over-predict future capital gains, fact reproduced by the model and supported by the significant negative correlation for stock returns. The model also reproduces the empirical pattern of forecast error predictability for the other three macroeconomic variables.

Following Coibion and Gorodnichenko (2015) panel b) presents the correlation between

the forecast errors and revision in beliefs. The model replicates the positive (under-reaction) correlation between inflation and interest rates. For output growth, the model generates a positive relationship while the data suggests the opposite.¹⁶ It is helpful to compare these results to the ones in Winkler (2019). The model presented there is able to reproduce the patterns of forecast errors for output and other variables but fails in the inflation one. The model outlined here delivers the opposite result: it matches well inflation and other variables and fails regarding the subjective output forecast error dynamics. The mechanism through which stock prices affect the real economy is nevertheless different (supply vs demand) and that could explain at least partially the difference in results.

5. Stock Price Targeting and Macroeconomic Stability

Stock price booms and busts driven by market sentiment affect the real economy through a consumption wealth effect. Since these wealth effects are reflected in output and inflation dynamics, monetary policy could in principle, by responding to just two macroeconomic variables influence or eliminate the non-fundamental effect of stock prices on the real economy. Compared to the RE assumption, in the current economic environment, agents have imperfect information about the structural relations between the real economy and stock prices. To this end, the Taylor rule is augmented with a lagged response to stock prices: $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_{t-1}$.¹⁷ To investigate the effect of monetary policy on the wealth effect, figure 3 plots the magnitude of the wealth effect when the central bank targets only one variable at the time: inflation, output-gap, or stock prices.

¹⁶This result is partly due to the fact that in the model agents form beliefs about the output-gap while in survey data agents predict output growth.

¹⁷Online appendix E presents the results with a contemporaneous response to stock prices and shows that qualitative conclusions remain unchanged.

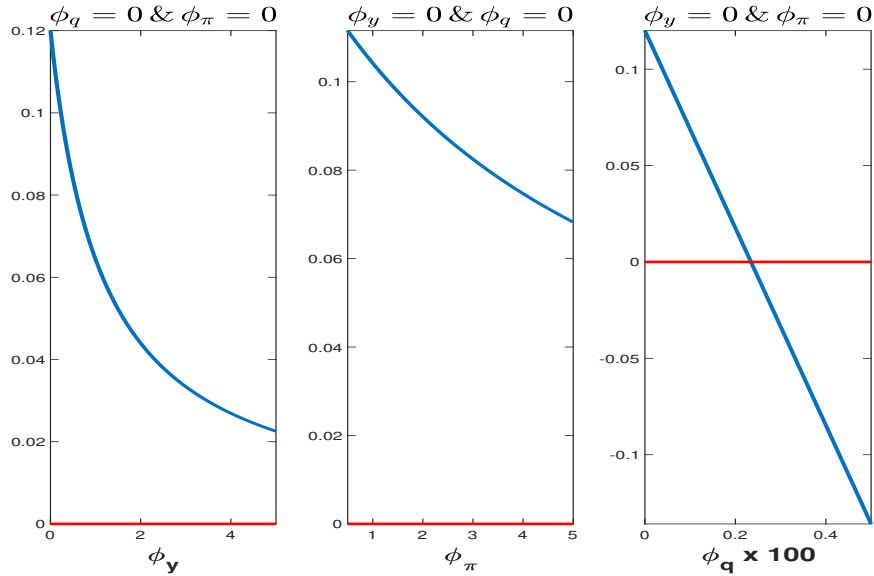


Figure 3: Stock Price Wealth Effects and Monetary Policy

Each panel presents the magnitude of the wealth effects as a function of the central bank response to output, inflation and stock prices while keeping the other coefficients fixed at 0. The Taylor rule is of the following type: $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_{t-1}$

To the extent that the central bank desires to eliminate the effects of stock market fluctuations on output, the only possibility under this simple Taylor rule would be to include an explicit response to stock prices in the monetary policy reaction function. Figure 3 shows that responding stronger to inflation or output has a smaller effect on the stock price wealth effect than by responding directly to asset prices. In fact, no matter how strongly the central bank responds to inflation or output it would not manage to totally neutralize the effects of stock prices on output. The reason for this dynamics lies on the fact that agents do not internalize the relation between stock prices and output and as a consequence, the extra-volatility of stock prices with respect to the real economy would not be internalized if the central bank responds just to output and inflation.

Responding to stock prices might on the other hand introduce additional volatility in the economy which might destabilize the system. Figure 4 shows that this is not necessarily the case for small enough reactions to stock prices and even more when there is in place a

reaction to output.¹⁸

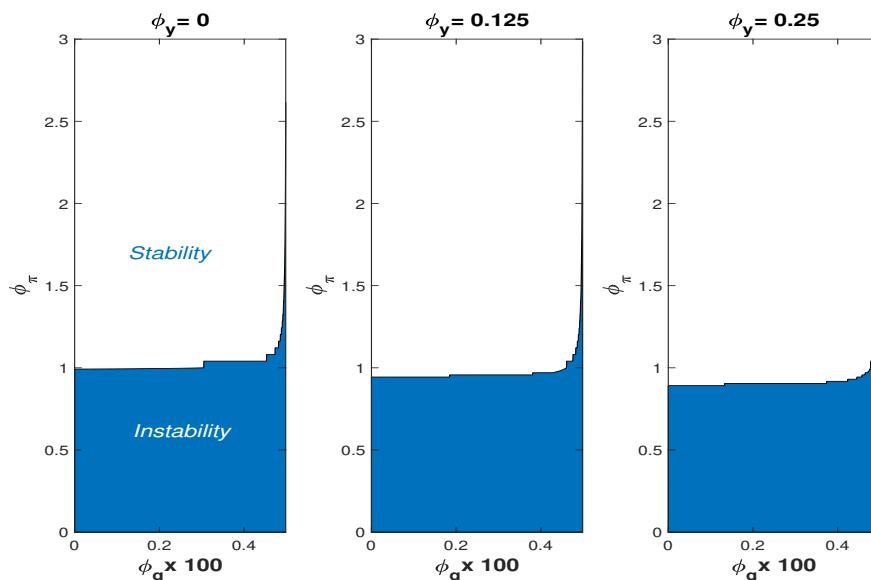


Figure 4: E-Stability and Monetary Policy

The figure presents the stability (white) and instability (blue) regions for different combinations of Taylor rule coefficients. Each panel plots the e-stability regions for different combinations of inflation (Y axis) and stock price (X axis) reaction coefficients while keeping the output reaction fixed. The Taylor rule is of the following type: $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_{t-1}$. The stability of the system is given by the eigenvalues of the matrix $A^{-1}B$. Following Evans and Honkapohja (2012), the dynamical system is e-stable if the largest eigenvalue of the previous matrix has the real part smaller than 1.

To better understand the dynamics of the stock market and its effects on the economy, figure 5 reports the IRFs from a 1% shock in stock prices in the RE model and a 1% shock in the beliefs of agents about the stock prices in the imperfect information model. These two shocks would have a similar effect in a RE model but in the learning model, where agents hold subjective beliefs about the stock market and where these beliefs have a high degree of persistence, the effects of these shocks have very different implications for the dynamics of the stock-market and the real economy.

¹⁸The maximum considered stock price reaction coefficient is 0.05 implying an increase of 1% in the interest rate as a result of a 20% increase in stock prices. This is in line with the findings of Nisticò (2012); Airaudo et al. (2015) and Airaudo (2016).

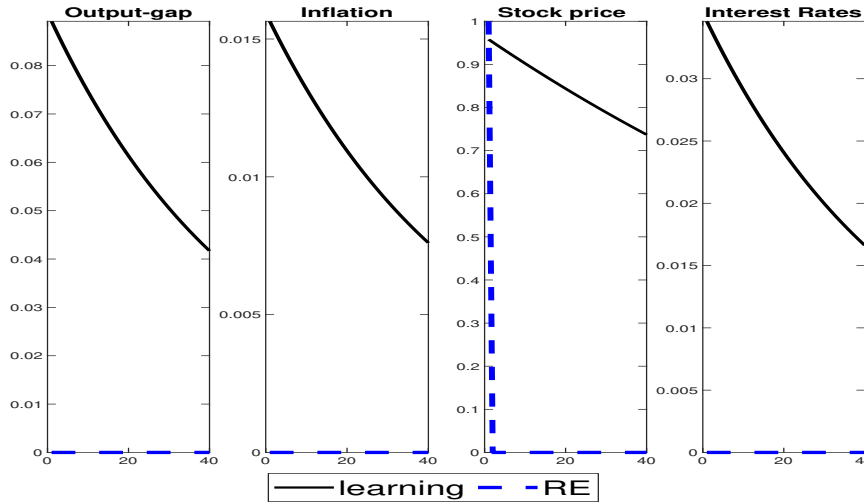


Figure 5: Stock Price vs Belief Shocks

The figure presents the IRF of selected endogenous variables with respect to stock price shocks in RE and shocks to beliefs under the learning framework. Both shocks have an impact magnitude of 1% and persistence 0.

The equity shock increases stock prices contemporaneously and then returns to 0 without affecting any other variable. Belief or sentiment shocks, on the other hand, although being *i.i.d.*, have a persistent effect on stock prices. This is because of the persistence of beliefs which translates into further increases in prices, therefore justifying the initial beliefs. This rise in stock prices is then transmitted through wealth effects on the rest of the economy. Output, inflation and interest rates closely follow the dynamics of the stock market. Although the central bank does not target stock-prices directly, the interest rates rise as a response of increases in inflation and output-gap. Therefore, waves of optimism/ pessimism can affect the real economy without any fundamental change in the economy.

Given that in this environment sentiment shocks can impact the real economy through wealth effects and since monetary policy is rather effective in influencing the magnitude of this effect (see figure 3) it is natural to ask how would the response of the economy, in the face of sentiment shocks, change if monetary policy would include a dedicated response to asset prices. Figure 6 answers this question by plotting the IRFs to sentiment shocks for

different stock price reaction coefficients.

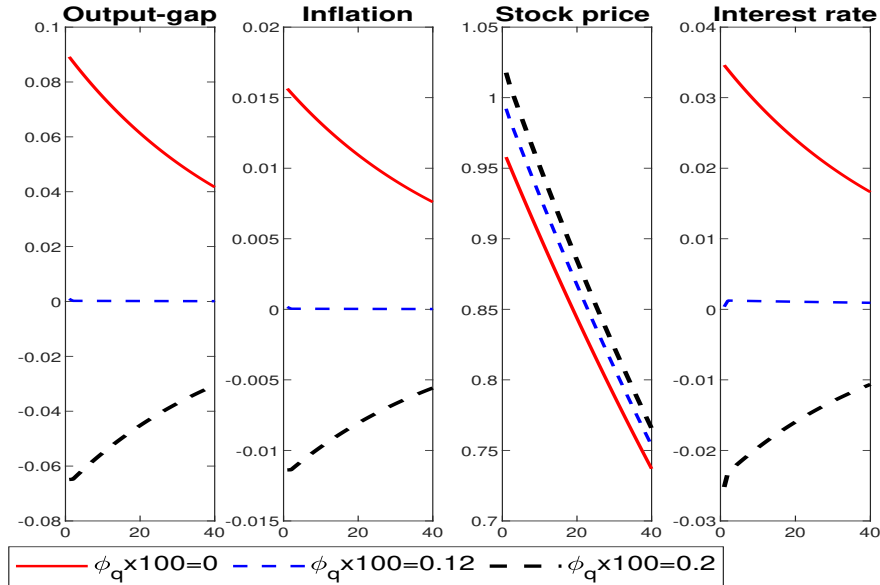


Figure 6: IRFs to Sentiment Shocks

The figure presents the IRF to a 1 % i.i.d shock in equity price beliefs for different reaction coefficients to stock prices. The Taylor rule is of the following form: $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_{t-1}$

The red line denotes the response in the case in which the central bank does not target explicitly stock prices. As the response of the monetary authority to stock prices increases, the effect of sentiment shocks on the economy decreases and is even reversed for large enough coefficients. The blue dotted line shows that the central bank can approximately neutralize the effects of sentiment shocks on the economy by picking a reaction coefficient to stock prices around 0.0012. This response implies that the central bank commits to raising interest rates by 12 b.p. for every 100% increase in stock prices from their steady state or trend value. Reacting too strongly to stock prices (black line) has the effect of reversing the effects of sentiment shocks and causing a recession. While it is true that reacting strongly to stock prices is undesirable since it can cause an economic recession which in turn has to be accommodated by reversing the increase in interest rates, it is not the case that the

response to stock prices should be absent. The key is to fine-tune the response of monetary policy to stock prices such that the real economy is isolated from the effects of belief-driven asset price cycles.

A central bank reacting to stock prices and *communicating clearly this policy* can disrupt the effects of sentiment shocks on the economy by influencing agents' expectations on stock prices and interest rates: agents take into account a possible rate increase in the case of positive stock price beliefs, which given the persistence of beliefs they internalize as a persistent interest rate increase, therefore adjusting the intertemporal consumption decision which counteracts the positive effect on the real economy of the initial optimism wave. The inclusion of stock price targeting in the Taylor rule does not, necessarily, create additional interest volatility in the economy since the sole fact that the central bank *threatens* to not tolerate large stock price swings is enough to influence agents' expectations and consequently the booms and busts would not materialize in the first place.

5.1. Monetary Policy and Welfare

What is the optimal response of monetary policy in the face of belief-driven asset price cycles and does an explicit response to stock prices improve welfare? The current section tries to answer these questions using the quantitative model developed in this paper

To address these questions one would need to specify a welfare criterion under which different monetary policy rules can be examined. The literature on monetary theory uses a second-order approximation of the lifetime utility of the representative agent as a criterion of welfare.¹⁹ The resulting criterion, average welfare loss per period, is an increasing function of the volatility of output and inflation. In the current framework stock prices play an important role as a source of output and inflation variation and standard welfare functions do not apply to this economy.

To this end, it is assumed that the central bank maximizes welfare under the equilibrium probability measure and not under the subjective one held by the agents. Therefore the central bank (social planner) assumes a paternalistic objective for the agents. The learning

¹⁹See Rotemberg and Woodford (1999) and Galí (2015).

literature has also adopted this approach although, compared to RE, here we are dealing with two types of beliefs: subjective vs model or objective beliefs.²⁰ Adopting a criterion of this form implies that we are assuming that the model or objective beliefs are what matter for the overall welfare of the agents. The following lemma presents the welfare function describing the economy presented in section IV.

Lemma 6. *Up to a second order approximation and ignoring terms independent of policy the expected utility in the TANK model with homogeneous imperfect information is proportional to $\sum_{t=0}^{\infty} -\mathcal{L}$ where*

$$\mathcal{L} = \frac{\epsilon}{\psi} \text{var}(\pi_t) + \Upsilon_1 \text{var}(\tilde{y}_t) + \Upsilon_2 \text{var}(\tilde{q}_t) + \Upsilon_3 E(\tilde{y}_t \tilde{q}_t) \quad (49)$$

is the average expected welfare loss per period measured as a fraction of steady-state consumption.

Proof. See Appendix 3 □

Lemma 6 shows that in the current framework the welfare of the agents depends not only on the variances of inflation and output but also on the variability of stock prices and the correlation of stock prices with output.

Stock price targeting rules take the form of Taylor rules augmented with a dedicated response to stock prices. The first rule that is considered is a standard linear and symmetric response to lagged stock prices (rule I in table 4). The empirical evidence shows that the Fed intervenes mostly when stock price decrease while booms are left to their own. Since stock price wealth effects appear both in booms and busts the following two additional non-linear monetary policy rules will be considered. Consistent with the empirical evidence, the second rule implies that the central bank reacts only when lagged stock prices drop under a certain threshold, Q^- , which is the standard Fed put documented in the literature (rule II in table 4). In the third rule, in addition to reacting to stock price decreases, the central bank also

²⁰See Eusepi and Preston (2018b).

reacts when the stock market increases above a specified threshold, Q^+ , which I label the fed put-call rule (rule III in table 4). These rules are summarized in the following table.

| | |
|-------------------|---|
| I. Linear | $i_t = 1.5 \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_{t-1}$ |
| II. Fed put | $i_t = 1.5 \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_{t-1} \mathbb{1}_{\tilde{q}_{t-1} < Q^-}$ |
| III. Fed put-call | $i_t = 1.5 \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_{t-1} (\mathbb{1}_{\tilde{q}_{t-1} < Q^-} + \mathbb{1}_{\tilde{q}_{t-1} > Q^+})$ |

Table 4: Monetary Policy rules under stock price targeting

$\mathbb{1}_{\tilde{q}_t < Q^-}$ is an indicator functions taking a value of one if the condition $\tilde{q}_t < Q^-$ is satisfied and 0 otherwise

The impulse response analysis from the previous section has been performed under the assumption that agents fully understand that monetary policy is responding to stock prices. Therefore, when forming expectations of interest rates agents would have internalized that given the current level of the stock market and their beliefs, the central bank would adjust accordingly the level of interest rates accordingly to the Taylor rule followed. In reality, the Fed does not react explicitly to stock prices although there is increasing evidence that it does intervene when needed. As a result, it is reasonable to assume that agents might not internalize this reaction of the monetary authority to stock prices. To investigate the implications of this lack of information it is assumed (as before) that agents fully understand that the central bank responds to inflation and output but do not take into account the reaction to stock prices. In reality the central bank is responding to stock price deviations even though agents do not internalize this fact. For clarity, the following table summarizes the information set of the central bank and agents under transparency vs non-transparency.

| | Transparency | Non-Transparency |
|--------------|---|---|
| Central Bank | $i_t = 1.5 \pi_t + \phi_y \tilde{y}_t + F(\tilde{q}_{t-1})$ | $i_t = 1.5 \pi_t + \phi_y \tilde{y}_t + F(\tilde{q}_{t-1})$ |
| Agents | $i_t = 1.5 \pi_t + \phi_y \tilde{y}_t + F(\tilde{q}_{t-1})$ | $i_t = 1.5 \pi_t + \phi_y \tilde{y}_t$ |

Table 5: Information set under Transparency vs Non-Transparency

The first line of the table shows the actions of the central bank while the second line shows how agents understand the central bank is responding to macroeconomic variables which is used to forecast future interest rates. $F(\tilde{q}_{t-1})$ denotes the central banks' response to stock prices which can take one of the options presented in table 4.

5.2. Non-Transparency

Figure 7 plots the welfare costs implied by monetary rules considered in table 4 for different thresholds regarding the response to stock prices.²¹ The figures makes clear that even under non-transparency reacting both in bad and good times (Fed put-call) is more efficient that reacting just in bad times (Fed put) for all combinations of threshold variables.

²¹Notice that the the Fed put-call policy with a threshold of 0 (red line in the fist panel in figure 7 is equivalent to the linear Taylor rule.

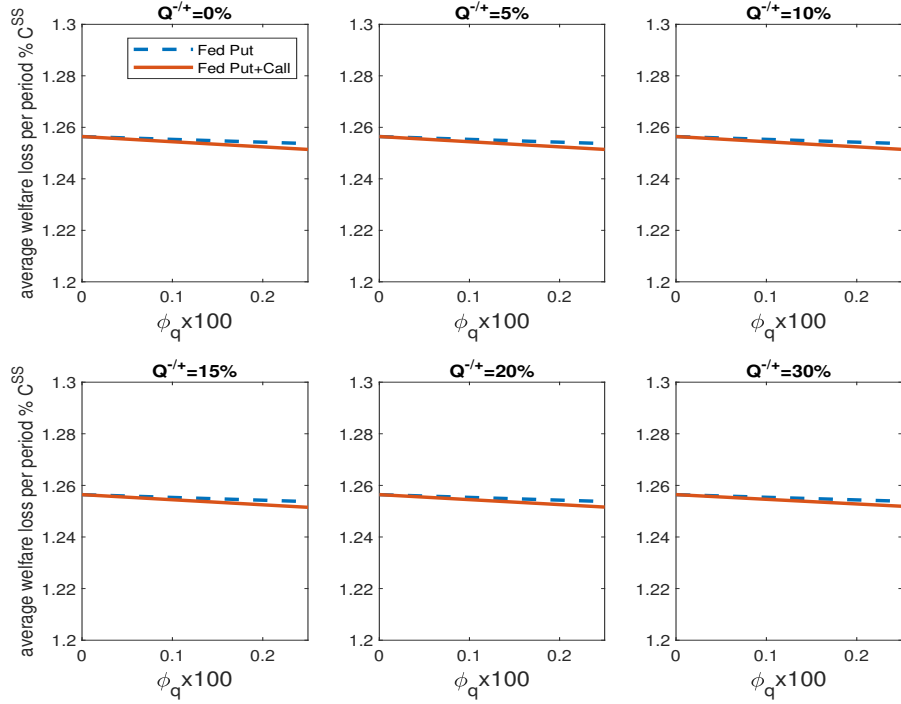


Figure 7: Welfare Implications of Fed put and call under Non-Transparency

It has been assumed that $Q^- = -Q^+$ for the Fed put-call rule. Non-transparency implies that although the central bank is reacting to stock prices using either of the two nonlinear rules considered, agents do not internalize this fact and form beliefs regarding interest rates using the systematic component of the Taylor rule concerning only output and inflation: $i_t = 1.5 \pi_t + 0.125 \tilde{y}_t$. The Fed put is specified as $i_t = 1.5 \pi_t + 0.125 \tilde{y}_t + \phi_q \tilde{q}_{t-1} \mathbb{1}_{\tilde{q}_{t-1} < Q^-}$ while the Fed put-call is $i_t = 1.5 \pi_t + 0.125 \tilde{y}_t + \phi_q \tilde{q}_{t-1} (\mathbb{1}_{\tilde{q}_{t-1} < Q^-} + \mathbb{1}_{\tilde{q}_{t-1} > Q^+})$.

Figure 7 also reveals that a threshold between 5 and 10% attains the highest efficiency gain for both policies considered.²² The welfare costs implied from responding symmetrically to stock prices is minimal for small enough stock price reaction coefficients (of the order 0.005 or smaller). The main conclusion arising from this exercise is that although there are welfare gains from responding symmetrically to stock prices under non-transparency, they are likely to be quantitatively small.

²²See also Figure E.19 from appendix D

5.3. *Transparency*

Moving to the case in which the central bank communicates transparently its policies in such a way that agents internalize this information into their belief system, figure 8 presents the implied welfare loss for different policy parameters in case of linear response to stock prices (rule I from table 4). Each line in the figure corresponds to a different output coefficient in the Taylor rule for different values of the stock price reaction coefficients. Notice first that regardless of the response to output, including a reaction to stock prices is always optimal but the benefit decreases the higher the reaction to output. Nevertheless, reacting too strongly to output variations decreases welfare since it worsens the output-inflation trade-off in the case of cost-push shocks.²³

²³A cost-push shock has the effect of increasing inflation contemporaneously. If the central bank responds strongly enough to inflation deviations, interest rate rise and output gap decreases which counteracts the initial increase in inflation. If at the same time the monetary authority reacts to output gap deviations then the interest rate will not increase as much and the initial impact of the cost push shock will dominate. The overall impact would be a less negative output gap and higher inflation. The optimal policy in standard New-Keynesian models (see for example Galí (2015) chapter 5) is to accommodate the cost-push shock by allowing a negative output-gap.

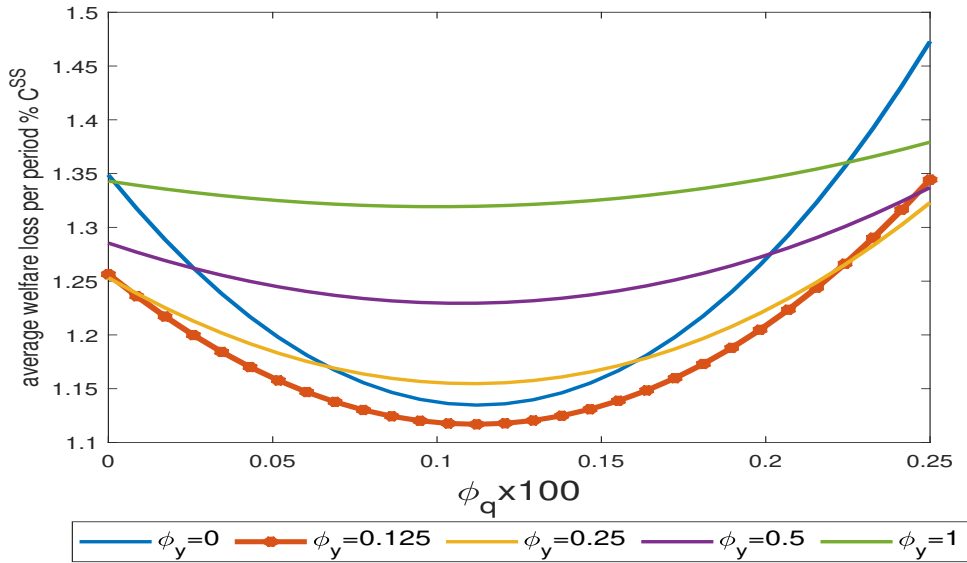


Figure 8: Welfare Maps: Linear response to asset prices

The figure shows the average welfare loss per period as defined in equation (49) for different combinations of Taylor rule coefficients for output and stock prices while keeping the inflation reaction coefficient fixed at 1.5. Welfare losses have been computed as averages over 200 independent short-run simulations, each one including 256 time periods using the estimated parameters from section 4.7.

The shape of the welfare loss as a function of the stock price targeting parameter has a U shape: reacting too strongly to stock prices can in fact decrease welfare by introducing additional volatility in the economy. For the baseline parametrization of the Taylor rule ($\phi_y = 0.125$, red line in figure 8) including a dedicated coefficient to stock prices of 0.12% in the Taylor rule increases welfare by 0.15% on average per period. This reaction implies increasing interest rates by 12 basis points for every 100% deviation of stock prices from their long-run trend. Figure E.15 from appendix E, repeats this exercise for the case in which the economy is solely hit by sentiment shocks and shows that the 0.15% welfare gain comes from counteracting the inefficiencies arising from the waves of optimism/pessimism about capital gains.

Figure 9 decomposes the sources of the welfare gains by plotting the standard deviations of output, inflation and stock prices together with the co-movement between stock prices and output. Including a dedicated reaction to stock prices reduces both the volatility of

inflation and output up to a certain point displaying the same U shape dynamics as the welfare losses. Stock price volatility increases monotonically but the magnitude is relatively small. The penultimate panel shows that responding to stock prices breaks the link between stock prices and output by reducing their co-movement generated by the stock price wealth effect.

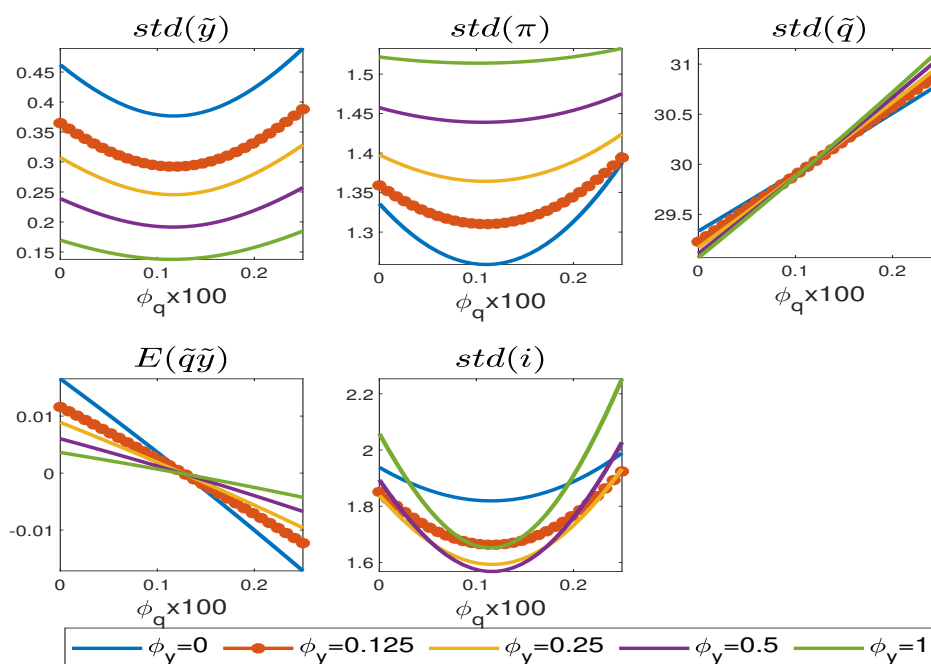


Figure 9: Influence of Monetary policy on Macroeconomic Volatility under Rule 1

Implied volatility of output, inflation, stock prices, co-movement of output with stock prices and interest rates for different combinations of policy parameters. The Taylor rule is specified as $i_t = 1.5 \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_{t-1}$.

Figure 10 shows the welfare losses arising in the case of transparency for the three monetary policy rules included in table 4. Notice that the welfare costs implied by the linear Taylor rule (blue line) are almost identical to the ones in which the central bank starts responding only after stock prices increases/decrease by 7% (yellow line). In reality, a linear policy as described by rule I would imply a continuous adjustment of interest rates in line with asset price movements which could introduce undesirable volatility in the economy through other channels not taken into account in the present framework. The previous result shows that the central bank can reap the welfare benefits by responding symmetrically only

to large asset price movements as long as this policy is communicated transparently to the agents. A central bank following the Fed Put policy rule (red dotted line) increases welfare up to a certain point and requires a stronger response to asset prices compared to a symmetrical policy ($\phi_q = 0.0014$ compared to 0.0011). Nevertheless, such a policy cannot eliminate all of the inefficiencies arising from belief-driven asset price cycles since it does not take into account the effect that stock price booms have on aggregate demand.

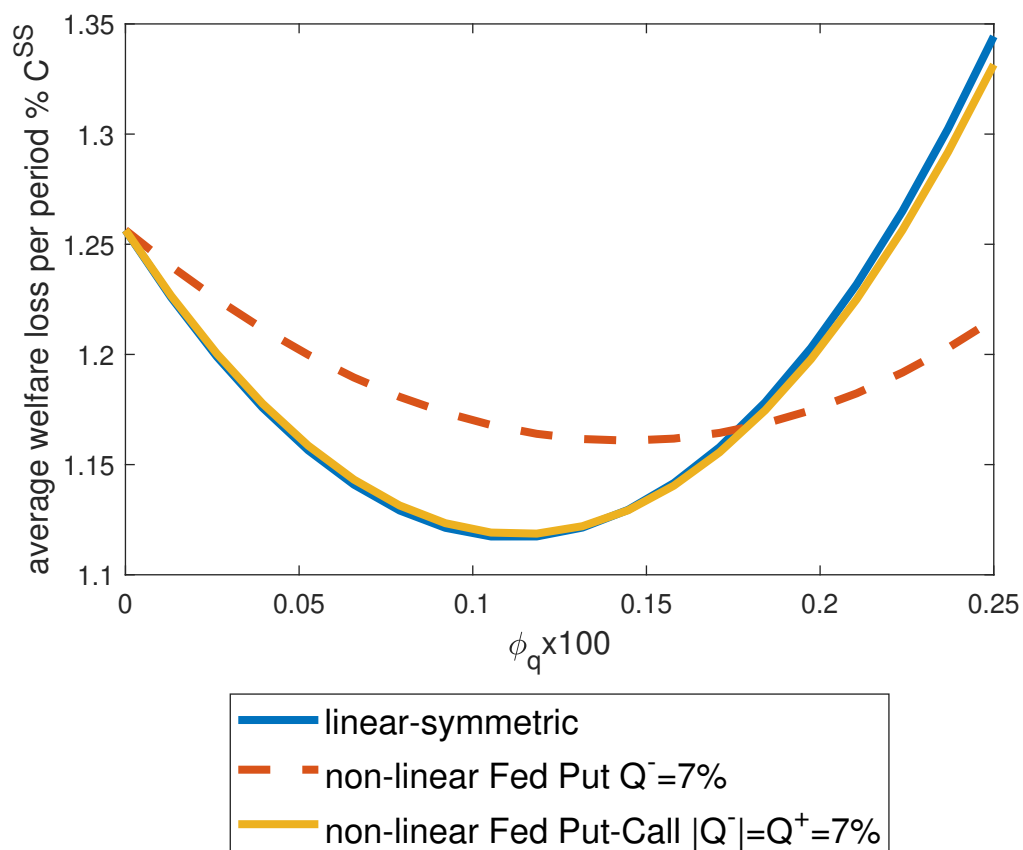


Figure 10: Welfare costs under Transparency

The figure presents the CEV (in %) for the monetary policy rules from table 4 under the case of Transparency for different reaction coefficients for stock prices (ϕ_q).

Table 6 summarises the welfare gains arising from responding to stock prices under the

two information scenarios. It is clear from the table below that announcing transparently the reaction to stock prices is several orders of magnitude more efficient than reacting under non-transparency (0.14% vs 0.002). The benefits of responding to stock prices when agents do not internalize this reaction (column A) are at most limited. This confirms that the management of agents' expectations about capital gains, interest rates and the link between these two is crucial for successfully counteracting the inefficiencies arising from booms and busts in asset prices.

| CEV(%) gain | A. Non-Transparency | B. Transparency |
|---|---------------------|-----------------|
| Monetary Rules | | |
| I. Linear $i_t = 1.5 \pi_t + \frac{0.5}{4} \tilde{y}_t + \phi_q \tilde{q}_{t-1}$ | 0.002 | 0.14 |
| II. Fed put $i_t = 1.5 \pi_t + \frac{0.5}{4} \tilde{y}_t + \phi_q \tilde{q}_{t-1} \mathbb{1}_{\tilde{q}_{t-1} < Q^-}$ | 0.001 | 0.09 |
| III. Fed put-call $i_t = 1.5 \pi_t + \frac{0.5}{4} \tilde{y}_t + \phi_q \tilde{q}_{t-1} (\mathbb{1}_{\tilde{q}_{t-1} < Q^-} + \mathbb{1}_{\tilde{q}_{t-1} > Q^+})$ | 0.002 | 0.14 |

Table 6: Welfare gains from stock price targeting

The values in the table represent CEV differences from the case in which the central bank does not include any response to stock prices ($\phi_q = 0$). The welfare gains have been computed under the optimized Taylor rules under transparency: under rule I and III $\phi_q = 0.0011$ while for rule II $\phi_q = 0.00145$; $Q^+ = |Q^-| = 7\%$.

5.4. (Un)conventional Monetary Policy

The previous analysis considered standard monetary policy which follows a simple Taylor rule when setting interest rates. During the last decade, central banks have adopted several non-conventional policies partly due to the constraint of the zero lower bound on interest rates. Since in this current framework expectations play a key role in determining equilibrium

values of macroeconomic variables one natural question that might be asked is whether monetary policy can eliminate the non-efficiency arising from booms and busts in asset prices through other types of policies. The one that is considered here is Odyssean forward guidance (O-FG)²⁴ in which the central bank announces a change in policy in the future. Consider the following scenario under two alternative policies: in one the central bank is conducting monetary policy via a Taylor rule with an optimal response on stock prices as in the previous section while in the second one the central bank does not respond to stock prices initially but announces a change of policy starting next period in which it includes stock prices in its monetary policy strategy. The following figure plots the impulse-response functions from a sentiment shock of 1% under the two policies.

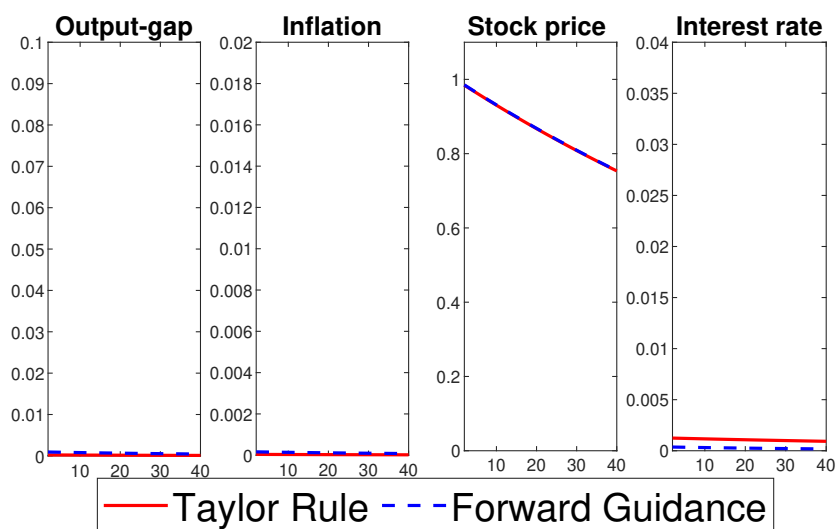


Figure 11: IRFs from a 1% *i.i.d.* sentiment shock under Taylor rule and Odyssean forward guidance

The effect on the economy under the two alternative policies is almost indistinguishable except for the evolution of interest rates which increase more under the Taylor rule regime due to the initial reaction to stock prices. Both policies manage efficiently to eliminate the effect that sentiment waves have on the real economy. The main takeaway from this exercise is that monetary policy operates through expectations about the future reaction to asset prices and not by reacting to asset prices today.

²⁴See Campbell et al. (2012) for a discussion on the different types of forward guidance.

6. Conclusions

The interaction between monetary policy and stock prices has been a long-standing subject both among academic economists and market professionals. Recent evidence suggests that the Fed is responding to stock prices and the main channel that it considers as important as a link between financial markets and the real economy is the consumption wealth effect. The empirical literature has also found that this effect can have magnitudes ranging from 3 to 20%. Given this evidence, this paper first develops a theory of stock price wealth effects arising from agents' imperfect information on the dynamics of equity markets. Departures of stock prices from the expected discounted sum of dividends give rise to a consumption wealth effect through which stock prices influence aggregate demand. The result links directly the volatility puzzle with stock price wealth effects.

The mechanism is embedded in a LAMP-NK model with homogeneous imperfect information where agents differ only regarding their participation in the equity market. The model is estimated on US data using two standard shocks, cost-push and monetary policy and a sentiment shock which affects agents' beliefs about future capital gains. Quantitatively, the model does remarkably well in matching the financial market and the dynamics of survey expectations while producing a smooth business cycle.

Using the quantitative estimated model, this paper first asks whether responding to stock prices can improve macroeconomic stability and welfare. By targeting stock prices the monetary authority does not introduce additional volatility in the economy and furthermore is especially efficient in counteracting the effects that sentiment swings have on aggregate demand via the consumption wealth effect. The optimal monetary policy implies a symmetric and transparent interest rate response of 12bp for every 100% increase in stock prices from the long-run trend which improves welfare by 0.15% on average per period. If on the contrary, the central bank reacts to stock prices in a non-transparent manner, the gains are limited. Moreover, the central bank does not have to respond linearly to stock price movements in order to implement the optimal policy within the class of simple Taylor rules considered. Responding only after stock market returns surpass a certain threshold

can achieve the same results. Reacting symmetrically to stock prices only when quarterly returns exceed 7% in absolute value maximizes macroeconomic stability and welfare.

Central banks can increase macroeconomic stability and welfare by responding explicitly to stock prices and can counteract the effects of asset price movements on the real economy by shutting down the wealth effect channel of stock prices. The analysis is mostly limited to standard monetary policy strategies (e.g. Taylor rules) and further research is needed in order to understand the effects and interactions of non-conventional policies like forward guidance and quantitative easing which have become the norm in the last years. Furthermore, booms and busts in other asset classes (e.g. housing) produce similar effects on the real economy. The inclusion of these in the current framework and the interplay between cycles in multiple asset classes is left for further research.

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Appendix A. Consistency of one-step ahead forecasts

Equation (4) from section III holds with equality from the perspective of the agent only under the Rational Expectation assumption. Under imperfect information agent will not have knowledge of the fact that he will be the marginal agent forever and therefore cannot substitute with equality the FOC 2 in the budget constraint to obtain equation 4. Letting, λ_t denote the lagrange multiplier associated with FOC with respect to stock prices and assuming the agent knows that he will be the marginal agent in the bond market (equation (1) holding with equality) the intertemporal budget constraint becomes

$$\frac{\mathcal{W}_t^i}{P_t} = E_t^{\mathcal{P}_i} \sum_{j=0}^{\infty} \delta^j \left(\frac{C_{t+j}^i}{C_t^i} \right)^{-\sigma} C_{t+j}^i + A_t^i. \quad (\text{A.1})$$

where

$$A_t^i = \sum_{j=1}^{\infty} \delta^j E_t^{\mathcal{P}} E_{t+1}^{\mathcal{P}} \dots E_{t+j-1}^{\mathcal{P}} \left(\frac{C_{t+j}^i}{C_t^i} \right)^{-\sigma} \frac{\lambda_{t+j}^i}{\prod_{s=0}^j (1 + \pi_{t+s})} \quad (\text{A.2})$$

is the term collecting all the Lagrange multipliers λ which take into account that the agent does not know that he will be marginal in all future periods. In steady state $\lambda = 0$ and $A = 0$. Specifically, for $j = 1$

$$\lambda_{t+1}^i = \delta \left[E_t^{\mathcal{P}_{mg}} \left(\left(\frac{C_{t+2}^{mg}}{C_{t+1}^{mg}} \right)^{-\sigma} (P_{t+2} + D_{t+2}) \right) - E_t^{\mathcal{P}_i} \left(\left(\frac{C_{t+2}^i}{C_{t+1}^i} \right)^{-\sigma} (P_{t+2} + D_{t+2}) \right) \right] S_{t+1}^i$$

is the perceived error of agent i with respect to the marginal agent valuation. If A_t^i is sufficiently small up to a first order approximation then we can describe accurately the optimal consumption decision of the agent by equation 14 as if the agent knows he is the marginal agent. I call this the *Average Marginal Agent* assumption.

Average Marginal Agent (AMA) Assumption: *up to a first order approximation*
 $A_t^i \approx 0$

Notice that the AMA assumption is in line with the equilibrium actual law of motion since all agents who have access to the equity market are identical. Moreover, knowledge of this assumption is not enough, from the perspective of the individual agent, to recover the exact mapping from dividends to prices. As a result, in this environment the agent cannot apply the Law of Iterated Expectations when forming beliefs and therefore the linearized FOC with respect to stock prices is of one-step ahead form

$$\tilde{q}_t = (1 - \delta)E_t^{\mathcal{P}}(\tilde{d}_{t+1}) + \delta E_t^{\mathcal{P}}(\tilde{q}_{t+1}) + \sigma(\tilde{C}_t^i - E_t^{\mathcal{P}}\tilde{C}_{t+1}^i). \quad (\text{A.3})$$

This result is a mix between the *long-horizon learning* approach of Preston (2005) and the *Euler Equation* approach. Under the Average Marginal Agent assumption the optimal consumption decision under long-horizon learning given by equation 14 is consistent under Internal Rationality with the one step ahead pricing equation A.3.

Appendix B. Model Derivation Details

Demand Side

Replacing Q_t in the budget constraint with equation (22) and rearranging, I obtain

$$\mathcal{W}_t^i = (P_t C_t^i - W_t N_t^i) + B_t^i + \delta S_t^i E_t^{\mathcal{P}} \left\{ \frac{A_{t+1}}{A_t} \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \frac{(Q_t + D_t)}{1 + \pi_{t+1}} \right\} \quad (\text{B.1})$$

where $\mathcal{W}_t^i = B_{t-1}^i(1 + i_{t-1}) + S_{t-1}^i(Q_t + D_t)$ represents wealth at time t . Adding and subtracting $\delta E_t^{\mathcal{P}} \left\{ \frac{A_{t+1}}{A_t} \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \frac{(B_t^i(1+i_t))}{1+\pi_{t+1}} \right\}$ from the RHS, equation (B.1) becomes

$$\begin{aligned} \mathcal{W}_t^i &= (P_t C_t^i - W_t N_t^i) + \delta E_t^{\mathcal{P}} \left\{ \frac{A_{t+1}}{A_t} \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \frac{\mathcal{W}_{t+1}^i}{1 + \pi_{t+1}} \right\} \\ &\quad + B_t^i \left(1 - \delta E_t^{\mathcal{P}} \left\{ \frac{A_{t+1}}{A_t} \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \frac{1 + i_t}{1 + \pi_{t+1}} \right\} \right) \\ &= (P_t C_t^i - W_t N_t^i) + \delta E_t^{\mathcal{P}} \left\{ \frac{A_{t+1}}{A_t} \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \frac{\mathcal{W}_{t+1}^i}{1 + \pi_{t+1}} \right\} \end{aligned} \quad (\text{B.2})$$

where the second equality follows from the Euler equation of the household. Substituting forward for \mathcal{W}_{t+1} I obtain

$$\mathcal{W}_t^i = E_t^{\mathcal{P}} \sum_{j=0}^{\infty} \delta^j \frac{A_{t+j}}{A_t} \left(\frac{C_{t+j}^i}{C_t^i} \right)^{-\sigma} \frac{(P_{t+j} C_{t+j}^i - W_{t+j} N_{t+j}^i)}{\prod_{s=0}^j (1 + \pi_{t+s})} \quad (\text{B.3})$$

where I have imposed the following transversality condition

$$\lim_{j \rightarrow \infty} E_t^{\mathcal{P}} \frac{A_{t+j}}{A_t} \left(\frac{C_{t+j}^i}{C_t^i} \right)^{-\sigma} \frac{\mathcal{W}_{t+j}^i}{\prod_{s=0}^j (1 + \pi_{t+s})} = 0. \quad (\text{B.4})$$

Dividing equation (B.3) by P_t leads to the following expression for the real wealth

$$\frac{\mathcal{W}_t^i}{P_t} = E_t^{\mathcal{P}} \sum_{j=0}^{\infty} \delta^j \frac{A_{t+j}}{A_t} \left(\frac{C_{t+j}^i}{C_t^i} \right)^{-\sigma} \left[C_{t+j}^i - w_{t+j}^{\frac{1+\phi}{\phi}} (C_{t+j}^i)^{\frac{-\sigma}{\phi}} \right]. \quad (\text{B.5})$$

The steady state (SS) of the model corresponds to the RE SS and is given by

$$\begin{aligned} Y &= \left((1 - \alpha) \frac{\epsilon - 1}{\epsilon} \right)^{\frac{1-\alpha}{\sigma(1-\alpha)+\alpha+\phi}} \\ w &= Y^{\sigma + \frac{\phi}{1-\alpha}} \\ d &= Y - Y^{\sigma + \frac{1+\phi}{1-\alpha}} \\ q &= \frac{\delta}{1 - \delta} d. \end{aligned} \quad (\text{B.6})$$

At the SS equation B.5 becomes

$$\begin{aligned} q + d &= \sum_{j=0}^{\infty} \delta^j (Y - w^{\frac{1+\phi}{\phi}} Y^{\frac{-\sigma}{\phi}}) \\ \frac{q}{\delta} &= \frac{Y^{\sigma + \frac{1+\phi}{1-\alpha}}}{1 - \delta}. \end{aligned} \quad (\text{B.7})$$

Applying a first order Taylor approximation to the IBC around a non-stochastic steady state yields

$$\begin{aligned}
\tilde{w}_t^i &= \frac{\delta}{q} E_t^{\mathcal{P}} \left\{ \sum_{j=0}^{\infty} \delta^j \left[- (Y - Y^{\sigma + \frac{1+\phi}{1-\alpha}}) r_{t+j}^N - \sigma (Y - Y^{\sigma + \frac{1+\phi}{1-\alpha}}) (\tilde{c}_{t+j}^i - \tilde{c}_t^i) \right. \right. \\
&\quad \left. \left. + (Y + \frac{\sigma}{\phi} Y^{\sigma + \frac{1+\phi}{1-\alpha}}) \tilde{c}_{t+j}^i - \frac{1+\phi}{\phi} Y^{\sigma + \frac{1+\phi}{1-\alpha}} \tilde{w}_{t+j} \right] \right\} \\
&= (1-\delta) E_t^{\mathcal{P}} \left\{ \sum_{j=0}^{\infty} \delta^j \left[- r_{t+j}^N + \sigma \tilde{c}_t^i + \left(\frac{Y + \frac{\sigma}{\phi} Y^{\sigma + \frac{1+\phi}{1-\alpha}}}{Y - Y^{\sigma + \frac{1+\phi}{1-\alpha}}} - \sigma \right) \tilde{c}_{t+j}^i - \frac{1+\phi}{\phi} \frac{Y^{\sigma + \frac{1+\phi}{1-\alpha}}}{Y - Y^{\sigma + \frac{1+\phi}{1-\alpha}}} \tilde{w}_{t+j} \right] \right\}.
\end{aligned} \tag{B.8}$$

where $r_{t+j}^N = a_t - a_{t+j}$. Moving from the first line to the third made use of (B.6).

Log-linearization of the Euler equation (21) yields

$$\tilde{c}_t^i = E_t^{\mathcal{P}} \tilde{c}_{t+1}^i - \frac{1}{\sigma} (i_t - E_t^{\mathcal{P}} \pi_{t+1} - r_{t+1}^N) \tag{B.9}$$

which can be rewritten as

$$E_t^{\mathcal{P}} (\tilde{c}_{t+k}^i) = \tilde{c}_t^i + \frac{1}{\sigma} E_t^{\mathcal{P}} \left[\sum_{j=0}^{k-1} i_{t+j} - \pi_{t+j+1} - r_{t+j}^N \right]. \tag{B.10}$$

Substituting equation B.10 in B.8, rearranging and using the fact that

$$\sum_{j=0}^{\infty} \delta^j \sum_{k=0}^{j-1} R_t = \frac{\delta}{1-\delta} \sum_{j=0}^{\infty} \delta^j R_t$$

for any variable R_t , yields

$$\tilde{c}_t^i = \Delta_i \tilde{w}_t^i + \Delta_w \sum_{j=0}^{\infty} \delta^j E_t^{\mathcal{P}} (\tilde{w}_{t+j}) - \frac{\delta}{\sigma} \Delta_r \sum_{j=0}^{\infty} \delta^j E_t^{\mathcal{P}} (i_{t+j} - \pi_{t+j+1} - \Gamma^r r_{t+j}^N). \tag{B.11}$$

where

$$\begin{aligned}
\Delta_i &= \frac{Y - Y^{\sigma + \frac{1+\phi}{1-\alpha}}}{Y + \frac{\sigma}{\phi} Y^{\sigma + \frac{1+\phi}{1-\alpha}}} \\
\Delta_w &= (1-\delta) \frac{1+\phi}{\phi} \frac{Y^{\sigma + \frac{1+\phi}{1-\alpha}}}{Y + \frac{\sigma}{\phi} Y^{\sigma + \frac{1+\phi}{1-\alpha}}}, \\
\Delta_r &= \frac{(1-\sigma)Y + \sigma \frac{1+\phi}{\phi} Y^{\sigma + \frac{1+\phi}{1-\alpha}}}{Y + \frac{\sigma}{\phi} Y^{\sigma + \frac{1+\phi}{1-\alpha}}}, \\
\Gamma^r &= 1 + \frac{1-\delta}{\delta} \sigma \frac{Y - Y^{\sigma + \frac{1+\phi}{1-\alpha}}}{(1-\sigma)Y + \sigma \frac{1+\phi}{\phi} Y^{\sigma + \frac{1+\phi}{1-\alpha}}}.
\end{aligned} \tag{B.12}$$

Evaluating expectations in equation B.11 using the PLM of agents and applying the equilibrium conditions results in the demand side of the model

$$\begin{aligned}
\tilde{y}_t - \Delta_i \delta \tilde{q}_t - \Delta_i (1 - \delta) \tilde{d}_t - \Delta_w \tilde{w}_t + \frac{\delta}{\sigma} \Delta_r i_t &= \frac{\delta}{1 - \rho \delta} \Delta_w \hat{\beta}_{t-1}^w - \frac{\delta^2}{\sigma(1 - \delta \rho)} \Delta_r \phi_y \hat{\beta}_{t-1}^y \\
&- \frac{\delta^2}{\sigma(1 - \delta \rho)} \Delta_r \phi_q \hat{\beta}_{t-1}^q - \frac{\delta^2}{\sigma(1 - \delta \rho)} \Delta_r (\phi_\pi - \frac{1}{\delta}) \hat{\beta}_{t-1}^\pi \\
&- \frac{\delta}{\sigma} \frac{\delta \rho_i}{1 - \delta \rho_i} \delta_r \epsilon_t^i + \frac{\delta}{\sigma} \Delta_r \Gamma_r (1 - \rho_a) \frac{\delta \rho_a}{1 - \delta \rho_a} a_t.
\end{aligned} \tag{B.13}$$

Supply Side

The solution to the profit maximization problem yields the optimal price setting decision of the firm

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{k=0}^{\infty} (\theta \delta)^k E_t^{\mathcal{P}} \left[\frac{A_{t+k}}{A_t} Y_{t+k}^{1-\sigma} P_{t+k}^\epsilon MC_{t+k/k} \right]}{\sum_{k=0}^{\infty} (\theta \delta)^k E_t^{\mathcal{P}} \left[\frac{A_{t+k}}{A_t} Y_{t+k}^{1-\sigma} P_{t+k}^{\epsilon-1} \right]} \tag{B.14}$$

where

$$MC_{t+k/k} = \frac{1}{1 - \alpha} \frac{W_{t+k}}{P_{t+k}} \left(\frac{P_t^*}{P_{t+k}} \right)^{\frac{-\epsilon \alpha}{1 - \alpha}} Y_{t+k}^{\frac{\alpha}{1 - \alpha}} e^{\epsilon_{t+k}^u}. \tag{B.15}$$

In the above equation ϵ_{t+k}^u is a shock to the the marginal costs of the firm and will be interpreted as a cost-push shock.

Log-linearization around a 0 inflation steady state and noting that at SS $\frac{\epsilon-1}{\epsilon} = \frac{1}{1-\alpha} Y^{\sigma + \frac{\phi+\alpha}{1-\alpha}}$ yields the pricing decision rule of the firms

$$p_t^* = (1 - \theta \delta) \sum_{k=0}^{\infty} (\theta \delta)^k E_t^{\mathcal{P}} \left\{ \frac{\alpha}{1 - \alpha + \epsilon \alpha} \tilde{y}_{t+k} + \frac{1 - \alpha}{1 - \alpha + \epsilon \alpha} (\tilde{w}_{t+k} + \epsilon_{t+k}^u) + p_{t+k} \right\}. \tag{B.16}$$

Subtracting p_{t-1} from both sides and taking into account that in equilibrium $\pi_t = (1 - \theta)(p_t^* - p_{t-1})$ results in the equation for inflation

$$\begin{aligned}
\pi_t &= \frac{(1 - \theta \delta)(1 - \theta)}{\theta} \frac{\alpha}{1 - \alpha + \epsilon \alpha} \sum_{k=0}^{\infty} (\theta \delta)^k E_t^{\mathcal{P}} \tilde{y}_{t+k} + \frac{(1 - \theta \delta)(1 - \theta)}{\theta} \frac{1 - \alpha}{1 - \alpha + \epsilon \alpha} \sum_{k=0}^{\infty} (\theta \delta)^k E_t^{\mathcal{P}} \tilde{w}_{t+k} \\
&+ \sum_{k=0}^{\infty} (\theta \delta)^k E_t^{\mathcal{P}} u_{t+k} + (1 - \theta) \delta \sum_{k=0}^{\infty} (\theta \delta)^k E_t^{\mathcal{P}} \pi_{t+k+1}
\end{aligned} \tag{B.17}$$

where $u_{t+k} = \frac{(1-\theta\delta)(1-\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\epsilon} \epsilon_{t+k}^u$ is an exogenous AR(1) process with persistence ρ_u and zero mean.

Evaluating the expectations using agents PLM results in the demand block of the model, the Phillips curve

$$\pi_t - \Theta_y \tilde{y}_t - \Theta_w \tilde{w}_t = \Theta_{\beta^y} \hat{\beta}_{t-1}^y + \Theta_{\beta^w} \hat{\beta}_{t-1}^w + \Theta_{\beta^\pi} \hat{\beta}_{t-1}^\pi + \Theta_u u_t \quad (\text{B.18})$$

where

$$\begin{aligned} \Theta_y &= \frac{(1-\theta\delta)(1-\theta)}{\theta} \frac{\alpha}{1-\alpha+\epsilon\alpha} \\ \Theta_w &= \frac{(1-\theta\delta)(\theta)}{1-\theta} \frac{1-\alpha}{1-\alpha+\epsilon\alpha} \\ \Theta_{\beta^y} &= \frac{(1-\theta\delta)(1-\theta)}{\theta} \frac{\alpha}{1-\alpha+\epsilon\alpha} \frac{\theta\delta}{1-\theta\delta\rho} \\ \Theta_{\beta^w} &= \frac{(1-\theta\delta)(1-\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\epsilon\alpha} \frac{\theta\delta}{1-\theta\delta\rho} \\ \Theta_{\beta^\pi} &= \frac{(1-\theta)\delta}{1-\theta\delta\rho} \\ \Theta_u &= \frac{(1-\theta\delta)(1-\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\epsilon\alpha} \frac{1}{1-\theta\delta\rho_u} \end{aligned} \quad (\text{B.19})$$

Asset Prices

Log-linearization of the FOC wrt to stock holding yields the asset pricing equation

$$\tilde{q}_t = (1-\delta)\hat{\beta}_{t-1}^d + \delta\hat{\beta}_{t-1}^q - (i_t - \hat{\beta}_{t-1}^\pi) \quad (\text{B.20})$$

where ϵ_t^q is a stochastic process with persistence ρ_q and can be interpreted as a equity market fad.

Equilibrium

Labor is demand determined and is obtained by log-linearization of the production function

$$\tilde{n}_t = \frac{\tilde{y}_t}{1 - \alpha}. \quad (\text{B.21})$$

Wages come from the FOC wrt to labor from the households problem which after log-linearization becomes

$$\tilde{w}_t = \phi \tilde{n}_t + \sigma \tilde{y}_t \quad (\text{B.22})$$

Dividends are given are given by the profits of the firms

$$D_t = Y_t - W_t N_t \quad (\text{B.23})$$

which after log-linearization becomes:

$$\tilde{d}_t = \frac{Y}{d} \tilde{y}_t - \frac{WN}{d} (\tilde{n}_t + \tilde{w}_t). \quad (\text{B.24})$$

using the expressions for labor and wages, the above equation can be rewritten only as a function of \tilde{y}_t

$$\tilde{d}_t = \psi_d \tilde{y}_t. \quad (\text{B.25})$$

where $\phi_d = \frac{Y}{d} - \frac{WN}{d} (\sigma + \frac{1+\phi}{1-\alpha})$

Belief System

Let $z_t = (\tilde{y}_t, \tilde{\pi}_t, \tilde{q}_t, \tilde{d}_t, \tilde{w}_t)'$. Agents think that z_t follows an unobserved component model

$$\begin{aligned} z_t &= \beta_t + \zeta_t \\ \beta_t &= \rho \beta_{t-1} + \vartheta_t \end{aligned} \quad (\text{B.26})$$

where β_t is the permanent component. Agents have perfect knowledge about interest rates and about the shock process. Agents form expectations at time t using information up to $t - 1$. Denoting these time t expectations by β_{t-1} , agents update their beliefs following the recursion

$$\hat{\beta}_t = \rho \hat{\beta}_{t-1} + \lambda(z_t - \hat{\beta}_{t-1}). \quad (\text{B.27})$$

Given that agents forecast $E_t^{\mathcal{P}} z_{t+k} = \rho^{k-1} \hat{\beta}_t$ we can evaluate the subjective expectations necessary to compute the Actual Law of Motion (ALM) as

$$\begin{aligned} \sum_{j=1}^{\infty} \delta^j E_t^{\mathcal{P}}(\tilde{w}_{t+j}) &= \sum_{j=1}^{\infty} \delta^j \rho^{j-1} \hat{\beta}_{t-1}^w = \frac{\delta}{1 - \rho\delta} \hat{\beta}_{t-1}^w, \\ \sum_{j=0}^{\infty} \delta^j E_t^{\mathcal{P}}(i_{t+j}) &= \sum_{j=0}^{\infty} \delta^j E_t^{\mathcal{P}}(\phi_{pi}\pi_{t+j} + \phi_y \tilde{y}_{t+j} + \phi_q \tilde{q}_{t+j} + \epsilon_{t+j}^i) \\ &= i_t + \frac{\delta}{1 - \delta\rho} (\phi_{pi} \hat{\beta}_{t-1}^\pi + \phi_y \hat{\beta}_{t-1}^y + \phi_q \hat{\beta}_{t-1}^q) + \frac{\delta\rho_i}{1 - \delta\rho_i} \epsilon_t^i \\ \sum_{j=0}^{\infty} \delta^j E_t^{\mathcal{P}}(\pi_{t+j+1}) &= \frac{1}{1 - \delta\rho} \hat{\beta}_{t-1}^\pi \\ \sum_{j=0}^{\infty} \delta^j E_t^{\mathcal{P}}(r_{t+j}^N) &= \sum_{j=0}^{\infty} \delta^j E_t^{\mathcal{P}}(a_t - a_{t+j}) \\ &= \sum_{j=0}^{\infty} \delta^j E_t^{\mathcal{P}}((1 - \rho_a)\rho_a^{j-1} a_t) = \frac{(1 - \rho_a)\delta}{1 - \delta\rho_a} a_t \\ \sum_{j=0}^{\infty} (\delta\theta)^j E_t^{\mathcal{P}}(\tilde{y}_{t+j}) &= \tilde{y}_t + \frac{\theta\delta}{1 - \theta\delta\rho} \hat{\beta}_{t-1}^y \\ \sum_{j=0}^{\infty} (\delta\theta)^j E_t^{\mathcal{P}}(\tilde{w}_{t+j}) &= \tilde{w}_t + \frac{\theta\delta}{1 - \theta\delta\rho} \hat{\beta}_{t-1}^w \\ \sum_{j=0}^{\infty} (\delta\theta)^j E_t^{\mathcal{P}}(\tilde{u}_{t+j}) &= \frac{\theta\delta}{1 - \theta\delta\rho} \tilde{u}_t \\ \sum_{j=0}^{\infty} \delta\theta^j E_t^{\mathcal{P}}(\pi_{t+j+1}) &= \frac{1}{1 - \delta\rho\theta} \hat{\beta}_{t-1}^\pi \end{aligned} \quad (\text{B.28})$$

System in State-Space form

Equilibrium equations (37), (38), (39), (34), (41) and (42) together with the the learning scheme represented by equations (44) and (45) fully characterize the dynamics of this economy. Substituting agents' subjective forecasts (45) into equilibrium conditions, results

in the following system of equations

$$\begin{aligned} A Z_t &= B (\hat{\beta}_t^Z + e_3 \epsilon_t^\beta) + C \epsilon_t \\ \hat{\beta}_t^Z &= \rho \hat{\beta}_{t-1}^Z + \lambda (Z_{t-1} - \hat{\beta}_{t-1}^Z) \end{aligned} \tag{B.29}$$

where

$$Z_t = (\tilde{y}_t, \pi_t, \tilde{q}_t, i_t, d_t, \tilde{w}_t)',$$

$$\hat{\beta}_{t-1}^Z = (\hat{\beta}_{t-1}^y, \hat{\beta}_{t-1}^\pi, \hat{\beta}_{t-1}^q, \hat{\beta}_{t-1}^i, \hat{\beta}_{t-1}^d, \hat{\beta}_{t-1}^w)',$$

$$\epsilon_t = (u_t, \epsilon_t^i)',$$

$$e_3 = (0, 0, 1, 0, 0, 0)',$$

$$A = \begin{pmatrix} 1 & 0 & -\Delta_i \delta & \frac{\Delta_r \delta}{\sigma} & -\Delta_i (1 - \delta) & -\Delta_w \\ -\Theta_y & 1 & 0 & 0 & 0 & -\Theta_w \\ 0 & 0 & 1 & 1 & 0 & 0 \\ -\phi_y & -\phi_\pi & -\phi_q & 1 & 0 & 0 \\ \psi_d & 0 & 0 & 0 & 1 & 0 \\ -(\sigma + \frac{\phi}{1-\alpha}) & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$B = \begin{pmatrix} -\frac{\delta^2 \Delta_r \phi_y}{\sigma(1-\delta\rho)} & -\frac{\delta^2 \Delta_r}{\sigma(1-\delta\rho)} \left(\phi_\pi - \frac{1}{\delta} \right) & -\frac{\delta^2 \Delta_r \phi_q}{\sigma(1-\delta\rho)} & 0 & 0 & \frac{\Delta_w \delta}{1-\rho\delta} \\ \Theta_{\beta y} & \Theta_{\beta \pi} & 0 & 0 & 0 & \Theta_{\beta w} \\ 0 & 1 & \delta & 0 & 1-\delta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$C = \begin{pmatrix} 0 & -\frac{\delta^2 \Delta_r}{\sigma(1-\delta\rho_i)} \\ \Theta_u & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

Appendix B.1. Lagged response to stock prices

The interest rule is

$$i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_{t-1} + \epsilon_t^i. \quad (\text{B.30})$$

Given this response of monetary policy the forecast of interest rates is given by

$$\begin{aligned}
\sum_{j=0}^{\infty} \delta^j E_t^{\mathcal{P}}(i_{t+j}) &= \sum_{j=0}^{\infty} \delta^j E_t^{\mathcal{P}}(\phi_{\pi}\pi_{t+j} + \phi_y\tilde{y}_{t+j} + \phi_q\tilde{q}_{t+j-1}) + \epsilon_{t+j}^i \\
&= i_t + \frac{\delta}{1-\delta\rho}(\phi_{\pi}\hat{\beta}_{t-1}^{\pi} + \phi_y\hat{\beta}_{t-1}^y) + \delta\phi_q\tilde{q}_t + \frac{\delta^2\phi_q}{1-\delta\rho}\hat{\beta}_{t-1}^q + \frac{\delta\rho_i}{1-\delta\rho_i}\epsilon_t^i
\end{aligned} \tag{B.31}$$

The IS equation becomes

$$\begin{aligned}
\tilde{y}_t - (\Delta_i\delta - \frac{\delta^2\phi_q}{\sigma}\Delta_r)\tilde{q}_t - \Delta_i(1-\delta)\tilde{d}_t - \Delta_w\tilde{w}_t + \frac{\delta}{\sigma}\Delta_r i_t &= \frac{\delta}{1-\rho\delta}\Delta_w\hat{\beta}_{t-1}^w - \frac{\delta^2}{\sigma(1-\delta\rho)}\Delta_r\phi_y\hat{\beta}_{t-1}^y \\
&\quad - \frac{\delta^3}{\sigma(1-\delta\rho)}\Delta_r\phi_q\hat{\beta}_{t-1}^q - \frac{\delta^2}{\sigma(1-\delta\rho)}\Delta_r(\phi_{\pi} - \frac{1}{\delta})\hat{\beta}_{t-1}^{\pi} - \frac{\delta}{\sigma}\frac{\delta\rho_i}{1-\delta\rho_i}\Delta_r\epsilon_t^i.
\end{aligned} \tag{B.32}$$

The system of equations determining \tilde{y}_t , π_t , \tilde{q}_t , i_t , d_t and \tilde{w}_t can be written in a compact form

$$A Z_t = B \hat{\beta}_{t-1}^Z + D Z_{t-1} + C \epsilon_t \tag{B.33}$$

where

$$Z_t = (\tilde{y}_t, \pi_t, \tilde{q}_t, i_t, d_t, \tilde{w}_t)',$$

$$\hat{\beta}_{t-1}^Z = (\hat{\beta}_{t-1}^y, \hat{\beta}_{t-1}^{\pi}, \hat{\beta}_{t-1}^q, \hat{\beta}_{t-1}^i, \hat{\beta}_{t-1}^d, \hat{\beta}_{t-1}^w)',$$

$$\epsilon_t = (\tilde{u}_t, \epsilon_t^i)',$$

$$A = \begin{pmatrix} 1 & 0 & -(\Delta_i \delta - \frac{\delta^2 \phi_q}{\sigma} \Delta_r) \delta & \frac{\Delta_r \delta}{\sigma} & -\Delta_i(1 - \delta) & -\Delta_w \\ -\Theta_y & 1 & 0 & 0 & 0 & -\Theta_w \\ 0 & 0 & 1 & 1 & 0 & 0 \\ -\phi_y & -\phi_\pi & 0 & 1 & 0 & 0 \\ \psi_d & 0 & 0 & 0 & 1 & 0 \\ -(\sigma + \frac{\phi}{1-\alpha}) & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$B = \begin{pmatrix} -\frac{\delta^2 \Delta_r \phi_y}{\sigma(1-\delta\rho)} & -\frac{\delta^2 \Delta_r}{\sigma(1-\delta\rho)} (\phi_\pi - \frac{1}{\delta}) & -\frac{\delta^3 \Delta_r \phi_q}{\sigma(1-\delta\rho)} & 0 & 0 & \frac{\Delta_w \delta}{1-\rho\delta} \\ \Theta_{\beta y} & \Theta_{\beta \pi} & 0 & 0 & 0 & \Theta_{\beta w} \\ 0 & 1 & \delta & 0 & 1 - \delta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$C = \begin{pmatrix} 0 & -\frac{\delta^2 \Delta_r}{\sigma(1-\delta\rho_i)} \\ \Theta_u & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix},$$

$$D = \begin{pmatrix} 0_{2 \times 6} \\ 0 & 0 & \phi_q & 0 & 0 & 0 \\ 0_{3 \times 6} \end{pmatrix},$$

where $0_{a \times b}$ denotes a matrix of zeros of dimension $a \times b$.

Appendix B.2. Non-linear response to stock prices

In the case of the non-linear response to stock prices under transparency, the subjective expectations of agents for the response of monetary policy to stock prices can be computed as follows. Notice that under agents' belief system, stock prices are distributed according to $\tilde{q}_{t+k} \stackrel{\mathcal{P}}{\sim} \mathcal{N}(\mu_{t+k}^q, (\sigma_{t+k}^q)^2)$ with

$$\begin{aligned} \mu_{t+k}^q &= \rho^k \beta_{t-1}^q \\ (\sigma_{t+k}^q)^2 &= (\sigma_0^q)^2 \rho^{2k} + (\sigma_\zeta^q)^2 + (\sigma_\vartheta^q)^2 \frac{1 - \rho^{2k}}{1 - \rho^2} \end{aligned} \tag{B.34}$$

where σ_0^q is the prior subjective belief about the volatility of stock prices, σ_ϑ^q and σ_ζ^q the volatilities of the permanent and transitory components of stock prices under agents' belief

system. The expectation of the response of monetary policy using the Fed put policy under the subjective probability measure can be computed as

$$\begin{aligned}
E_t^{\mathcal{P}}(Q_{t+k}^{put}) &= E_t^{\mathcal{P}}(\tilde{q}_{t+k} \mathbb{1}_{\tilde{q}_{t+k} < Q^-}) \\
&= \int_{-\infty}^{Q^-} \tilde{q}_{t+k} f(\tilde{q}_{t+k}) dq = \int_{-\infty}^{Q^-} \mu_{t+k}^q + (\tilde{q}_{t+k} - \mu_{t+k}^q) dq \\
&= \mu_{t+k}^q \Phi\left(\frac{Q^- - \mu_{t+k}^q}{\sigma_{t+k}^q}\right) - \sigma_{t+k}^q \phi\left(\frac{Q^- - \mu_{t+k}^q}{\sigma_{t+k}^q}\right)
\end{aligned} \tag{B.35}$$

where $\phi()$ and $\Phi()$ are the pdf and cdf of the standard normal distribution. Similarly for the Fed call policy

$$E_t^{\mathcal{P}}(Q_{t+k}^{call}) = \mu_{t+k}^q \left(1 - \Phi\left(\frac{Q^+ - \mu_{t+k}^q}{\sigma_{t+k}^q}\right)\right) + \sigma_{t+k}^q \phi\left(\frac{Q^+ - \mu_{t+k}^q}{\sigma_{t+k}^q}\right). \tag{B.36}$$

Appendix C. Welfare Approximation

Assuming the steady state is efficient under RE equalizes the consumption and labor decision of the two agents. This is ensured by a tax subsidy on sales by the fiscal authority which is rebated back to firms as a lump sum transfer conditional on a balanced budget. This ensures that profits are zero at the steady state but not otherwise since markups will vary over time. At steady steady

$$\begin{aligned}
C^C &= C^U = C \\
N^C &= N^U = N \\
Y &= N^{1-\alpha} \\
w &= N^\phi Y^\sigma \\
\frac{V'(N)}{U'(C)} &= w = (1 - \alpha) \frac{Y}{N}
\end{aligned} \tag{C.1}$$

Following Bilbiie (2008) assume the social planner is maximizing a weighted average of the utility of the agents $U_t(\cdot) = \theta U^C(C_t^C, N_t^C) + (1 - \theta) U^U(C_t^U, N_t^U)$. Up to a second order approximation the utility of type j can be written as

$$\begin{aligned}\hat{U}_t^j(\cdot) &= U^j(C_t^j, N_t^j) - U(C, N) \\ &\approx U_C C \left(\hat{c}_t^j + \frac{1-\sigma}{2} (\hat{c}_t^j)^2 \right) - V_N N \left(\hat{n}_t^j + \frac{1+\phi}{2} (\hat{n}_t^j)^2 \right) + t.i.p + H.O.T\end{aligned}\quad (\text{C.2})$$

where the hat variables denote log deviation from the flexible price RE equilibrium which given the absence of fluctuations in the natural output (e.g. TFP) coincides with the steady state of the model. Explicitly, $\hat{c}_t = \log(C_t) - \log(C)$, *t.i.p* denotes terms independent of policy and *H.O.T* higher order terms (greater than 2). In equilibrium $\hat{c}_t = \hat{y}_t$ and $\hat{n}_t = \frac{1}{1-\alpha} \hat{y}_t + d_t$ where $d_t = \log \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} di$ ²⁵. Given this and aggregating across agents

$$\begin{aligned}\hat{U}_t(\cdot) &\approx U_C C \left[\hat{c}_t + \frac{1-\sigma}{2} (\mathcal{O}(\hat{c}_t^C)^2 + (1-\mathcal{O})(\hat{c}_t^U)^2) \right] \\ &\quad - V_N N \left[\hat{n}_t + \frac{1+\phi}{2} (\mathcal{O}(\hat{n}_t^C)^2 + (1-\mathcal{O})(\hat{n}_t^U)^2) \right] + H.O.T\end{aligned}\quad (\text{C.3})$$

Using the last equation from C.1 we can write $\frac{V_N N}{U_C C} = (1-\alpha)$. The linear terms from the utility approximation boil down to

$$U_C C (\hat{c}_t) - V_N N (\hat{n}_t) = -U_C C [(1-\alpha)d_t] + H.O.T \quad (\text{C.4})$$

Regarding the quadratic terms we can also rewrite them in terms of output-gaps and stock prices. First notice that from Proposition 1 we have

$$\hat{c}_t^U - \hat{c}_t^C = \Delta_i \left[\delta \hat{q}_t + (1-\delta) \hat{d}_t \right] = \Delta_i [\delta \hat{q}_t + (1-\delta) \psi_d \tilde{y}_t].$$

Using the previous relation together with goods market clearing and FOC with respect to labor for the two types of agents we obtain the following

²⁵see Galí (2015) pag 87

$$\begin{aligned}
\tilde{y}_t &= \mathcal{O}\tilde{c}_t^C + (1 - \mathcal{O})\tilde{c}_t^U \\
\tilde{y}_t &= \mathcal{O}\tilde{c}_t^C + (1 - \mathcal{O}) \left\{ \Delta_i \left[\delta\tilde{q}_t + (1 - \delta)\tilde{d}_t \right] + \tilde{c}_t^C \right\} \\
\tilde{c}_t^C &= \tilde{y}_t - (1 - \mathcal{O})\Delta_i \left[\delta\tilde{q}_t + (1 - \delta)\psi_d \tilde{y}_t \right] \\
\tilde{c}_t^C &= [1 - (1 - \mathcal{O})\Delta_i(1 - \delta)\psi_d]\tilde{y}_t - (1 - \mathcal{O})\Delta_i\delta\tilde{q}_t \\
&= \Upsilon_{cy}^C \tilde{y}_t - \Upsilon_{cq}^C \tilde{q}_t \\
\tilde{n}_t^C &= \frac{\tilde{w}_t - \sigma \tilde{c}_t^C}{\phi} = \frac{\left(\frac{\phi}{1-\alpha} + \sigma\right)\tilde{y}_t + \phi d_t - \sigma \tilde{c}_t^C}{\phi} \\
&= \left(\frac{1}{1-\alpha} + \frac{\sigma}{\phi} \right) \tilde{y}_t - \frac{\sigma}{\phi} (\Upsilon_{cy}^C \tilde{y}_t - \Upsilon_{cq}^C \tilde{q}_t) + d_t \\
&= \left(\frac{1}{1-\alpha} + \frac{\sigma}{\phi}(1 - \Upsilon_{cy}^C) \right) \tilde{y}_t + \frac{\sigma}{\phi} \Upsilon_{cq}^C \tilde{q}_t + d_t \\
&= \Upsilon_{ny}^C \tilde{y}_t + \Upsilon_{nq}^C \tilde{q}_t + d_t \tag{C.5} \\
\tilde{c}_t^U &= \Delta_i \left[\delta\tilde{q}_t + (1 - \delta)\psi_d \tilde{y}_t \right] + \Upsilon_y^C \tilde{y}_t - \Upsilon_q^C \tilde{q}_t \\
&= [1 + \mathcal{O}\Delta_i(1 - \delta)\psi_d] \tilde{y}_t + \mathcal{O}\Delta_i\delta\tilde{q}_t \\
&= [\Delta_i(1 - \delta)\psi_d + \Upsilon_{cy}^C] \tilde{y}_t + (\Delta_i\delta - \Upsilon_{cq}^C) \tilde{q}_t \\
&= \Upsilon_{cy}^U \tilde{y}_t + \Upsilon_{cq}^U \tilde{q}_t \\
\tilde{n}_t^U &= \frac{\tilde{w}_t - \sigma \tilde{c}_t^U}{\phi} = \frac{\left(\frac{\phi}{1-\alpha} + \sigma\right)\tilde{y}_t + \phi d_t - \sigma \tilde{c}_t^U}{\phi} \\
&= \left(\frac{1}{1-\alpha} + \frac{\sigma}{\phi} \right) \tilde{y}_t - \frac{\sigma}{\phi} (\Upsilon_{cy}^U \tilde{y}_t + \Upsilon_{cq}^U \tilde{q}_t) + d_t \\
&= \left[\frac{1}{1-\alpha} + \frac{\sigma}{\phi}(1 - \Upsilon_{cy}^U) \right] \tilde{y}_t - \frac{\sigma}{\phi} \Upsilon_{cq}^U \tilde{q}_t + d_t \\
&= \Upsilon_{ny}^U \tilde{y}_t - \Upsilon_{nq}^U \tilde{q}_t + d_t.
\end{aligned}$$

Using these last results we can derive the quadratic terms for consumption and labor in

terms of output gaps and stock prices

$$(\tilde{c}_t^C)^2 = (\Upsilon_{cy}^C)^2 \tilde{y}_t^2 + (\Upsilon_{cq}^C)^2 \tilde{q}_t^2 - 2 \Upsilon_{cy}^C \Upsilon_{cq}^C \tilde{y}_t \tilde{q}_t + H.O.T \quad (C.6)$$

$$(\tilde{c}_t^U)^2 = (\Upsilon_{cy}^U)^2 \tilde{y}_t^2 + (\Upsilon_{cq}^U)^2 \tilde{q}_t^2 + 2 \Upsilon_{cy}^U \Upsilon_{cq}^U \tilde{y}_t \tilde{q}_t + H.O.T \quad (C.7)$$

$$(\tilde{n}_t^C)^2 = (\Upsilon_{ny}^C)^2 \tilde{y}_t^2 + (\Upsilon_{nq}^C)^2 \tilde{q}_t^2 + 2 \Upsilon_{ny}^C \Upsilon_{nq}^C \tilde{y}_t \tilde{q}_t + H.O.T \quad (C.8)$$

$$(\tilde{n}_t^U)^2 = (\Upsilon_{ny}^U)^2 \tilde{y}_t^2 + (\Upsilon_{nq}^U)^2 \tilde{q}_t^2 - 2 \Upsilon_{ny}^U \Upsilon_{nq}^U \tilde{y}_t \tilde{q}_t + H.O.T. \quad (C.9)$$

The aggregate *per-period* approximation of the welfare function is then, up to a second order approximation

$$\hat{U}_t(\cdot) \approx -U_C C [(1 - \alpha)d_t + \Upsilon_1 \tilde{y}_t^2 + \Upsilon_2 \tilde{q}_t^2 + \Upsilon_3 \tilde{q}_t \tilde{y}_t] \quad (C.10)$$

where

$$\begin{aligned} \Upsilon_1 &= \left[\frac{1 + \phi}{2} (1 - \alpha) (\mathcal{O}(\Upsilon_{ny}^C)^2 + (1 - \mathcal{O})(\Upsilon_{ny}^U)^2) - \frac{1 - \sigma}{2} (\mathcal{O}(\Upsilon_{cy}^C)^2 + (1 - \mathcal{O})(\Upsilon_{cy}^U)^2) \right] \\ \Upsilon_2 &= \left[\frac{1 + \phi}{2} (1 - \alpha) (\mathcal{O}(\Upsilon_{nq}^C)^2 + (1 - \mathcal{O})(\Upsilon_{nq}^U)^2) - \frac{1 - \sigma}{2} (\mathcal{O}(\Upsilon_{cq}^C)^2 + (1 - \mathcal{O})(\Upsilon_{cq}^U)^2) \right] \\ \Upsilon_3 &= [(1 + \phi)(1 - \alpha) (\mathcal{O}\Upsilon_{nq}^C \Upsilon_{ny}^C - (1 - \mathcal{O})\Upsilon_{nq}^U \Upsilon_{ny}^U) + (1 - \sigma) (\mathcal{O}\Upsilon_{cq}^C \Upsilon_{cy}^C - (1 - \mathcal{O})\Upsilon_{cq}^U \Upsilon_{cy}^U)]. \end{aligned} \quad (C.11)$$

The price dispersion term, $(1 - \alpha) d_t$, can be rewritten using the arguments from Galí (2015) as $(1 - \alpha) d_t \approx \frac{\epsilon}{\psi} \pi_t^2$ where $\psi = \frac{(1 - \theta)(1 - \delta\theta)}{\theta} \frac{1 - \alpha}{1 - \alpha + \alpha\epsilon}$.

The average welfare loss per period in terms of steady steady consumption is

$$\mathcal{L} = \frac{\epsilon}{\psi} var(\pi_t) + \Upsilon_1 var(\tilde{y}_t) + \Upsilon_2 var(\tilde{q}_t) + \Upsilon_3 E(\tilde{y}_t \tilde{q}_t) \quad (C.12)$$

Appendix D. Do Sentiment Shocks matter?

| | | | Learning | |
|--------------------------------------|--------------------------------|-------------|--------------|-----------------|
| | Symbol | Data Moment | <u>Model</u> | |
| Business Cycle | | | Moment | <i>t</i> -ratio |
| Std. dev. of output | $\sigma(y)$ | 1.45 | 0.62 | 5.5 |
| Std. dev. of inflation | $\sigma(\pi)$ | 0.54 | 0.29 | 3.4 |
| Correlation output/inflation | $\rho_{y,\pi}$ | 0.29 | 8.6 | -3.2 |
| Financial Moments | | | | |
| Average PD ratio | E (P/ D) | 154 | 134 | 1.33 |
| Std. dev. of PD ratio | $\sigma(P/D)$ | 63 | 11 | 4.8 |
| Auto-correlation of PD ratio | $\rho(P/D)$ | 0.99 | 0.84 | 3.2 |
| Std. dev. of equity return (%) | $\sigma(r^e)$ | 6.02 | 0.79 | 12 |
| Std. dev. real risk free rate (%) | $\sigma(r^f)$ | 0.72 | 0.78 | 0.7 |
| Non Targeted Moments | | | | |
| volatility ratio stock prices/output | $\sigma(Q)/\sigma(y)$ | 6.7 | 1.05 | 7.2 |
| corr. Stock Prices/ output | $\rho(Q, y)$ | 0.5 | 0.99 | 3.74 |
| Average Equity Return (%) | $E(r^e)$ | 1.78 | 0.76 | 2.23 |
| Average real risk free rate (%) | $E(r^f)$ | 0.32 | 0.75 | 3.4 |
| Consumption Wealth Effect | dy/dQ | [0.03-0.2] | 0.09 | |
| Std. dev. Expected Returns(%) | $\sigma(E_t(r_{t,t+4}^e))$ | 2.56 | 1.72 | |
| corr. Survey Expect./ PD ratio | $\rho(PD_t, E_t(r_{t,t+4}^e))$ | 0.74 | -0.99 | |

Table D.7: Model implied moments excluding Sentiment Shocks.

Moments have been computed as averages over 10.000 simulations, each of 260 time periods. Subjective expectations are measured by the UBS Gallup survey for own portfolio returns. Survey data covers the period 1998Q1-2007Q3.

Appendix E. Contemporaneous Response to stock prices

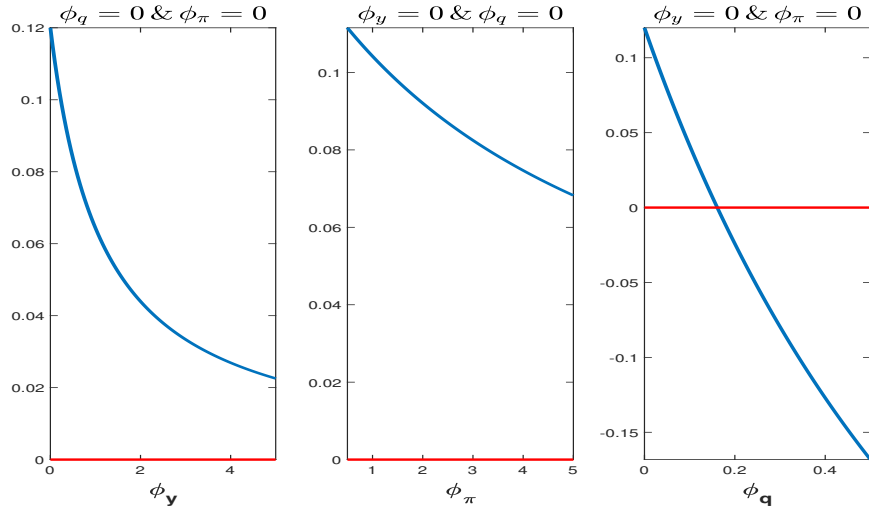


Figure E.12: Stock Price Wealth Effects and Monetary Policy

Each panel presents the magnitude of the wealth effects as a function of the central bank response to output, inflation and stock prices while keeping the other coefficients fixed at 0. The Taylor rule is of the following type: $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_t$

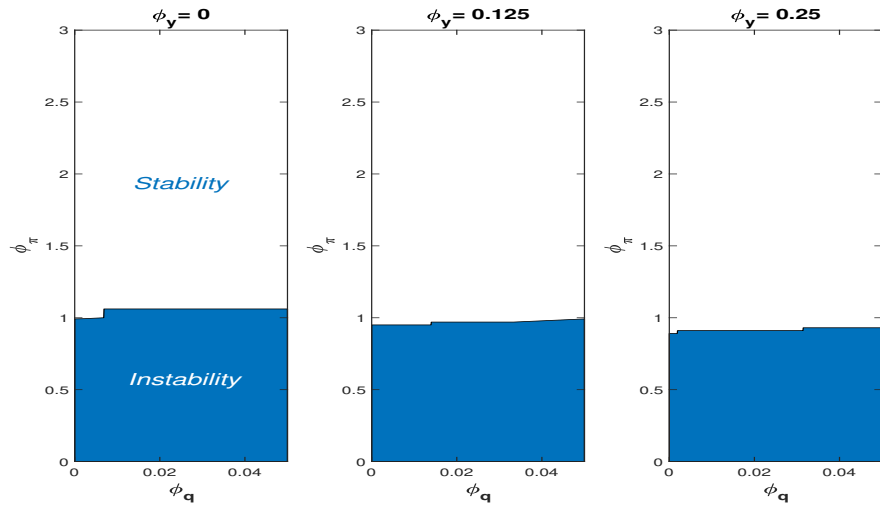


Figure E.13: E-Stability and Monetary Policy

The figure presents the stability (white) and instability (blue) regions for different combinations of Taylor rule coefficients. Each panel plots the e-stability regions for different combinations of inflation (Y axis) and stock price (X axis) reaction coefficients while keeping the output reaction fixed. The Taylor rule is of the following type: $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_t$. The stability of the system is given by the eigenvalues of the matrix $A^{-1}B$. Following Evans and Honkapohja (2012), the dynamical system is e-stable if the largest eigenvalue of the previous matrix has the real part smaller than 1.

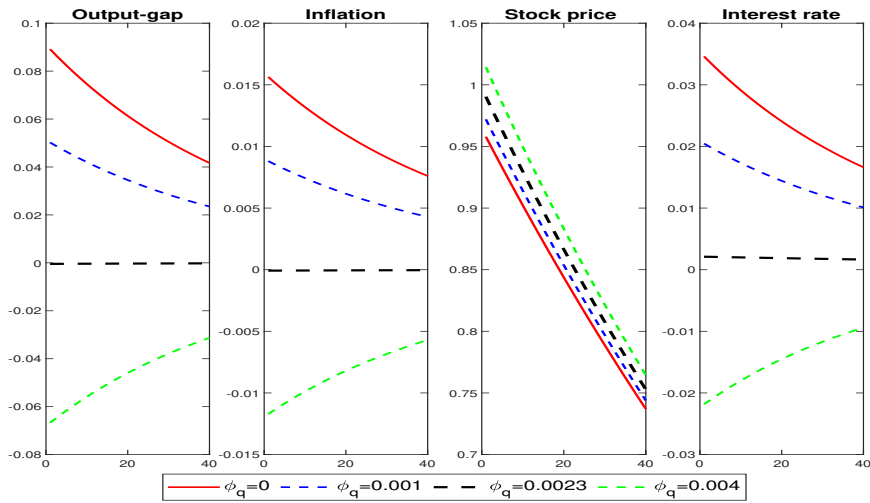


Figure E.14: IRFs to Sentiment Shocks

The figure presents the IRF to a 1 % i.i.d shock in equity price beliefs for different reaction coefficients to stock prices. The Taylor rule is of the following form: $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_t$

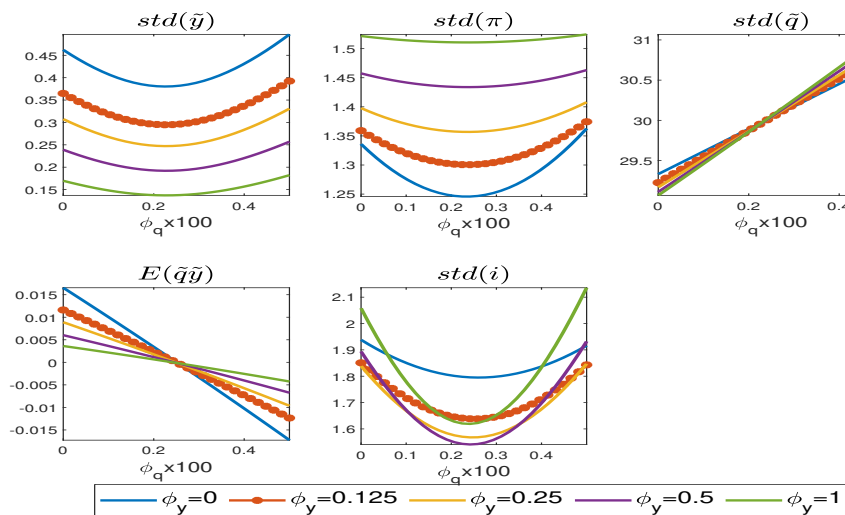


Figure E.16: Influence of Monetary policy on Macroeconomic Volatility

Implied volatility of output, inflation, stock prices, co-movement of output with stock prices and interest rates for different combinations of policy parameters. The Taylor rule is specified as $i_t = 1.5 \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_t$.

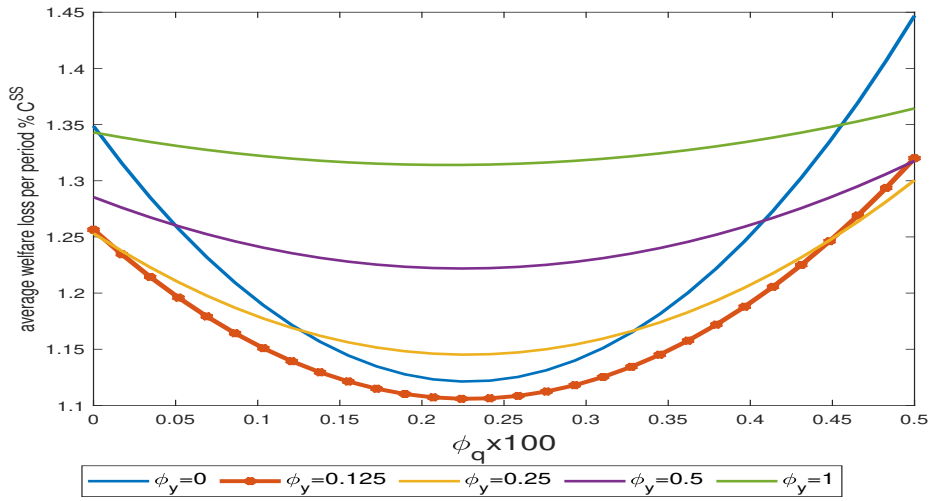


Figure E.15: Welfare Maps

The figure shows the average welfare loss per period as defined in equation (49) for different combinations of Taylor rule coefficients for output and stock prices while keeping the inflation reaction coefficient fixed at 1.5. Welfare losses have been computed as averages over 200 independent simulations, each one including 260 time periods using the estimated parameters from section IV.H

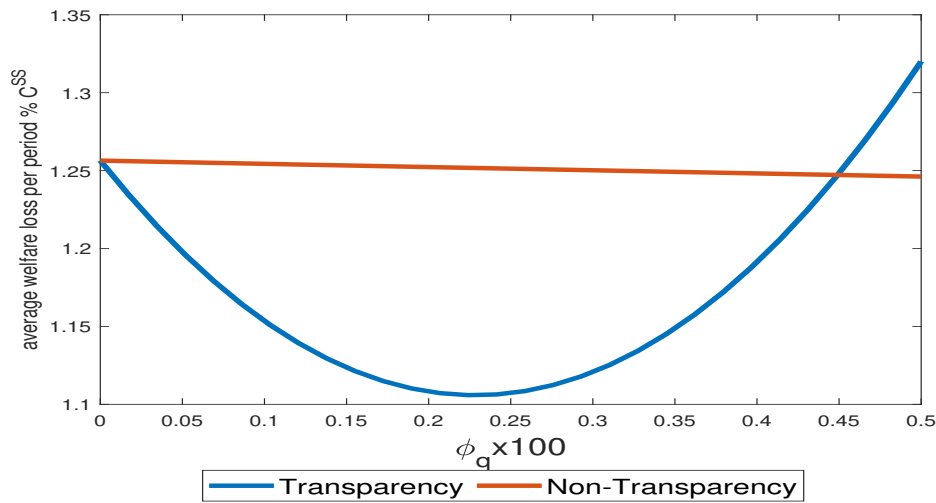


Figure E.17: (Non) Transparency of stock price targeting

Transparency implies that agents internalize the reaction to stock prices while in the non-transparency scenario agents only take into account the response to output and inflation in the Taylor Rule. The latter is specified as $i_t = 1.5 \pi_t + 0.125 \tilde{y}_t + \phi_q \tilde{q}_t$.

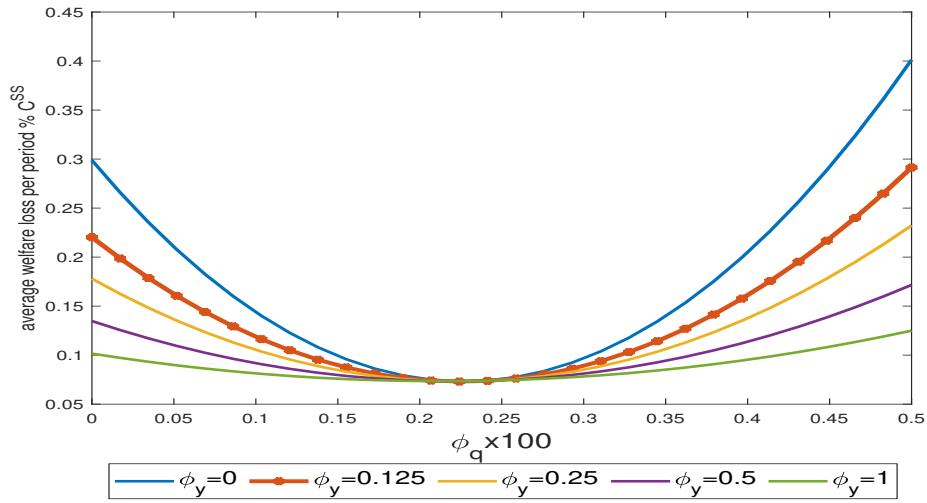


Figure E.18: Welfare maps when the economy is hit only by Sentiment Shocks

The figure shows the average welfare loss per period for different policy parameters for output and stock prices in the case the only source of variation in the economy is given by Sentiment Shocks. The volatility of sentiment shocks is the one estimated in section IV.

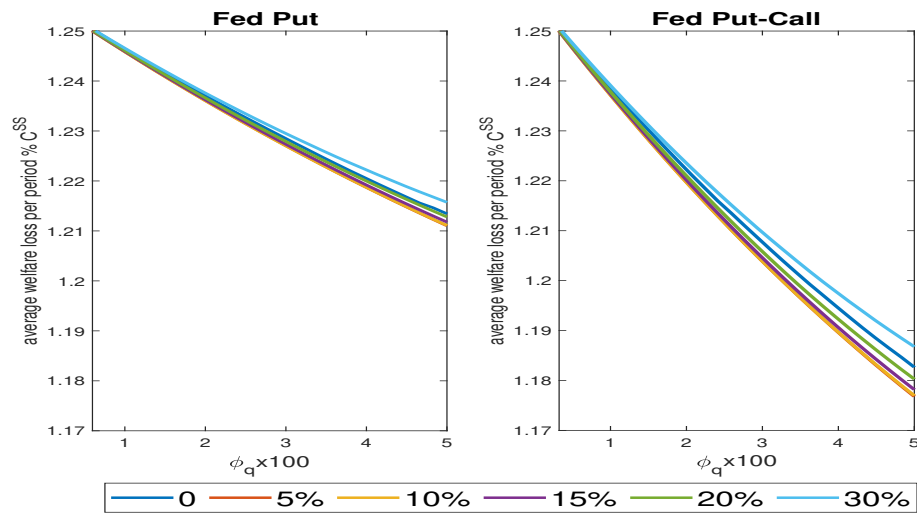


Figure E.19: Welfare Implications of Fed Put and Call under Non-Transparency

$$Q^- = -Q^+$$

Appendix F. Additional Figures

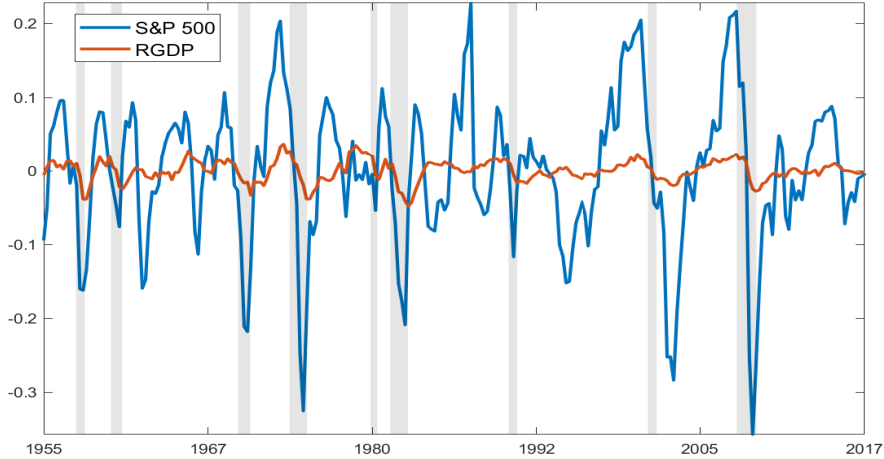


Figure F.20: Real and Financial Volatility at business cycle frequency
HP-filtered quarterly data; shaded bands denote NBER recessions