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On the optimal size of a joint savings association

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ZEF-Discussion Papers on Development Policy No. 317

Oded Stark, Wiktor Budzinski, Marcin Jakubek, and Grzegorz Kosiorowski

On the optimal size of a joint savings association

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Abstract

We develop a formula for the optimal size of a joint savings association between individuals who share the same financial goal and who can save towards that goal at the same rate. Our motivating example and the core of our analysis is a Rotating Savings and Credit Association (ROSCA). We measure the efficiency of a ROSCA by the expected waiting time that it takes a participant to attain his goal when no participant reneges on his commitment to contribute to the common fund, and when each of the participants receives (once) the funds needed to meet his goal. Given this criterion, we define the optimal size of a ROSCA as the number of participants that results in the minimal expected waiting time. We show that an optimal size of a ROSCA exists, that it is limited, and that it is a multiple of the number of time periods that it takes an individual to save on his own. Somewhat surprisingly, we find that when treated as a function of the size of a ROSCA, the expected waiting time is not monotonic when the size builds up from an individual saving on his own to the optimal size. A similar result obtains when we study cases where a ROSCA is enlarged beyond the optimal size. Our findings help explain the limited size as well as other features of ROSCAs observed in developing countries all over the world.

Keywords: Joint savings associations; A Rotating Savings and Credit Association (ROSCA); Minimal expected waiting time; Optimal size of a ROSCA; Limited size of a ROSCA

JEL classification: D01; D02; D16; D23; D71; D86; G23; O12; O17; P13

1. Introduction

Joint savings associations in general, and Rotating Savings and Credit Associations (ROSCAs) in particular, are intriguing pieces of social-financial engineering, a form of microfinancing by which poor people who have limited or no access to the formal financial market can avoid being dependent on banks and other formal lending institutions, and they are also attractive ways to speed up the acquisition of a desired (often an indivisible) good. Succinctly described as “an association formed upon a core of participants who make regular contributions to a fund, which is given, in whole or in part, to each contributor in rotation” (Ardener, 1964), and as “a voluntary grouping of individuals who agree to contribute financially at each of a set of uniformly-spaced dates towards the creation of a fund, which will then be allotted in accordance with some prearranged principle to each member of the group in turn” (Calomiris and Rajaraman, 1998), ROSCAs were and remain particularly popular in developing countries (see the reviews by Geertz, 1962; Bouman, 1995; Anderson et al. 2009; and the references cited in Besley et al., 1993). There is evidence that the numbers and incidence of ROSCAs in the developing world have been, respectively, large and widespread. (See, for example, Bouman, 1977, and Ahn et al., 2017.)

Suppose that it takes 12 pesos to buy a tin roof, and that I can save one peso per week, so that it will take me 12 weeks to build up the funds needed to get the roof; and suppose that it will take you that long too. Each of us receives pay, say a wage payment, or a “paycheck” at the end of each week. (We hasten to add that a “week” here is just a name of a period of time; contributions can be made monthly, or following harvests, which occur less frequently than monthly.) Suppose that instead of saving individually we save jointly, and that by the time we have jointly accumulated 12 pesos, we toss a coin to determine who will receive the amount in the “pot.” The winner of the draw will be able to install the roof after 6 weeks rather than 12 weeks, and the other co-saver - 6 weeks afterwards.¹ Our expected waiting time is reduced from 12 weeks to 9 weeks,

¹ In this paper we consider ROSCAs in which the participant who will receive the “pot” each time the “pot” amount is reached is determined by means of a draw. This way, the expected waiting time of each participant is the same. Other ways of allocating the funds include “bidding ROSCAs” where a participant can contribute more than fellow participants to the common pool in order to bring forward his turn to

namely by a quarter. Suppose that there are, say, four of us saving jointly. Then, the points in time at which the “pot” becomes “full” are three, six, nine, and twelve weeks from the start. Applying the rule of random allocation of the accumulated funds to the participants, the expected waiting time is $7\frac{1}{2}$ weeks, a reduction in the expected waiting time from 12 weeks by $\frac{3}{8}$. This rate of reduction is more than a quarter. To check whether this is a precursor to *a trend* in reductions in the expected waiting time, we let the number of co-savers rise from four to five who together accumulate 5 pesos per week. The five co-savers will obtain the first “full pot” in 3 weeks from the start (with 3 pesos carried over), the second “full pot” in 5 weeks from the start (with 1 peso as a surplus), the third “full pot” in 8 weeks from the start (with 4 pesos as a surplus), the fourth “full pot” in 10 weeks from the start (with 2 pesos as a surplus), and the fifth “full pot” in 12 weeks from the start. The expected waiting time in the case of joint saving by five participants, which is $7\frac{3}{5}$ weeks, is longer than the expected waiting time in the case of joint saving by four participants, which as we have shown is $7\frac{1}{2}$ weeks. So, assuming that all the members in a ROSCA abide by the terms of participation (continued saving at the agreed rate until each of them reaches the goal of being allocated the funds needed for buying the tin roof), what number of members will reduce the expected waiting time to a minimum?

Students of ROSCAs have been aware of the limited number of participants in these schemes. Adams and Canavesi de Sahonero (1989) report that in Bolivia, the average number of participants in ROSCAs is ten. Drawing on a review of 142 ROSCAs in India, Guha and Gupta (2005) infer that the number of participants in a ROSCA ranges from eight to thirteen. A similar finding is reported by Velez-Ibanez (2010) for Mexico, where very few ROSCAs have more than twenty participants. Etang et al. (2011) report

receive the “pot.” (A comparison between random ROSCAs and “bidding ROSCAs” can be found in Besley et al., 1993.)

that in Cameroon ROSCAs had at most 45 participants. Kedir and Ibrahim (2011) report that the median number of ROSCA participants in Ethiopia is 24. Acquah and Dahal (2018) note that the median size of an Indonesian ROSCA is 37.

The existing literature lists three main reasons for the limited size of a ROSCA. Each of these reasons invites a short reflection.

One explanation is purely technical: managing a large ROSCA is demanding (Armendariz and Morduch, 2005; Velez-Ibanez, 2010). This explanation is similar to an argument that ROSCAs are not larger because smaller ROSCAs allow for easier monitoring of the participants. However, the monitoring rationale may not be that much of an issue. In many villages across the developing world, the organization of a ROSCA is simple and transparent; the participants deposit their contributions with the organizer, say a shopkeeper who then, with a chalk on a piece of cardboard, marks an extra “+” next to the name of contributing member. There need not be much to monitor, and compliance or lack thereof is visible to everyone. (In his vivid description of the operation of a ROSCA, Bouman (1979) remarks: “[ROSCAs] admirably fit into the scenery of a low-keyed economy. There is no office, transactions being conducted at the house of the organizer There is a blissful lack of bureaucracy and paperwork. The only visible expense is a soiled exercise book to keep accounts.”)

A second reason is of a “moral hazard” type: as noted by Besley et al. (1993), for a *given* duration of the scheme, the higher the number of participants, the shorter the waiting time for one of the participants to be the first to receive the “pot,” thus the greater the temptation for this participant to default: winning the draw to receive the “pot” earlier implies that the duration of the commitment to contribute to the common pool will be longer. As we show in this paper, it is not optimal to fix the duration of a ROSCA and then match that duration with different numbers of participants; *the number of participants is a direct derivative of the optimal expected waiting time*. In addition, the “moral hazard” reasoning ignores the flip side of the same coin: when the purpose of participation in a ROSCA is to speed up the acquisition or installation of an implement that boosts production, then getting the “pot” early translates into earlier higher

productivity and earnings, which renders honoring the commitment to continue to contribute easier; longer need not then mean harder. Forming a ROSCA to expedite the installation of a tin roof has nice second-round effects beyond the reduction of the expected waiting time. Living in a better insulated house provides protection against bad weather, colds, and other causes of morbidity. This allows for increased productivity and a boost to earnings. And, of course, the installation of a tin roof (investment in housing) is an example. The purpose of savings could also be the installation of a farm-specific implement such as a tube well (investment in agriculture) with a direct gain to farm production and earnings.

Another reflection on this second reason is that a good way to see to it that the participants in a ROSCA will honor the ROSCA's terms of agreement is to team up with people who are close to each other in social space so as to provide the trust needed for collaborative saving. In the developing world, a ROSCA is typically a village or community arrangement, not an inter-village or inter-community arrangement. Geertz (1962) writes that "[ROSCAs] are usually formed by a *local* group of three or four householders through a common agreement, and other households are then invited to join" (emphasis added). And Besley et al. (1994) write that "the typical scenario for a rosca is a group of individuals who work in the same office block or belong to the same community." That said, concern that a participant in a ROSCA might renege may not be all that realistic when we bear in mind, first, that as evidence suggests, participation in a ROSCA is not a once in a lifetime experience;² the life span of a ROSCA can be years, with renewals within and across years.³ As is well known from game theory, the expectation or likelihood of repetition is a powerful disciplining device, a reason to

² Bouman (1979) remarks: "One must bear in mind that RoSCA are not one-time affairs. As soon as one cycle is finished, the next begins, usually with the same players. RoSCA with a "recycled lifetime" of 10 to 15 years are reported for Sri Lanka and Nigeria." In a similar vein, Anderson et al. (2009) who allude to exclusion from future saving schemes as an enforcement device report that out of their sample of 374 participants in ongoing ROSCAs, about half of the participants had taken part in earlier ROSCAs.

³ For example, Gugerty (2007) reports that the average ROSCA in Kenya completed 5.35 cycles (rounds), and Etang et al. (2011) report that the average life span of a ROSCA in a Cameroon village is eight years, with the average cycle lasting six months. Each cycle is a renewal of a ROSCA, with fresh funds being collected and allocated to the participants. In a regime of sequential cycles, renegades will find it particularly hard to gain a foothold.

establish a good reputation. And second, by and large, and as already noted, a ROSCA is set up *within* a village or community where bad behavior can result in swift and painful social sanctions.⁴

There are good reasons to assume that participants in a ROSCA will not tend to default, and evidence supports this conjecture. For example, Handa and Kirton (1999) report that only 2.5% of the ROSCA members that they surveyed witnessed any incidence of delinquent contributions. What incentive does an early winner of a draw have not to renege? What can participants who contributed to the savings pool but did not win the first “pot” do to effectively dissuade the co-saver who received the “pot” from renegeing? If they use means that help cement joint saving, the perceived risk involved in joint financing can be moderated, and this form of financing will be attractive. Conversely, when such means are not available, lone saving will be more appealing. A standard list of responses to the preceding two questions includes social deterrents, reputational concerns, and repeat transactions, with obvious linkages between the three.

Compliance can be strengthened by applying social pressure, for example in the form of sanctions. The option of sanctioning will be effective when a participant is close in social space to co-savers who have yet to receive funding, but will not have teeth when the contracting parties are distant in social space. Furthermore, sanctioning will be more effective when a participant wants to keep open the option of “re-ROCSA-ing.” In the “grand” scheme of things, this implies that participation in a ROSCA in a given point in time is not a final event, not the last act in a sequence of moves; rather, it is a stage in a process, part of a broader, lasting, and dynamic relationship. (Recall footnote 2.) This line of reasoning can be taken a step further. Suppose that people in a village who seek to form a ROSCA and who want to lower the likelihood of “default” differ in the likelihood that they will want to re-enter ROSCAs. Then, if people with a low likelihood team up

⁴ Bouman (1979) brings up four points. First, because ROSCAs are self-selected peer groups, wilful default is rare. Second, because membership of a ROSCA is local, social pressure and the threat of public ostracism appears sufficient to prevent default. Third, defaulters would find it difficult to join another ROSCA. And fourth, in the rare event in which public ostracism does not have much effect, sanctions may take the form of none too subtle warnings such as a threat to set fire to the defaulter’s barn or granary, or to steal or kill his livestock.

with other people of low likelihood, and people with high such likelihood team up with people of high likelihood, the overall incidence of default will be lower than if people with low likelihood team up with people with high likelihood, and people with high likelihood team up with people with low likelihood.⁵

A third reason relates to the frequency with which the participants receive their income (Anderson et al., 2009). To see this, suppose that the size of a ROSCA “pot” is fixed and is equal to the cost of the good to be acquired, which is 12 pesos, and that a ROSCA of four members will have a weekly rotation that matches the points in time at which the members receive incomes (with weekly contributions equal to 4, and as per our example above, four rotations to completion). If instead a ROSCA of five members is contemplated, members will not be able to increase the frequency of the rotation, thus not exhausting the potential to maximize the gain from increased membership.

Whether a ROSCA is formed endogenously (by the participants) or exogenously (by entities such as charities and commercial banks that offer savings accounts that mimic a ROSCA arrangement), we assume that the creators are benevolent in the sense that their aim is to maximize the participants’ gain. In particular, in concert with the basic notion of time preferences, they all seek to minimize the expected waiting time for receiving the “pot.” Clearly, participation in a ROSCA shortens the waiting period to obtain the desired good for all the participants except for the last one who, anyhow, does not wait longer than if saving alone.

People resort to participation in a ROSCA because if they do not, the benefit (welfare gain) from mastering the funds needed to install, implement, acquire and so on is delayed. The *raison d’être* of participation is to gain time, to expedite. Given this, what architecture will *on average* best deliver it? The first order and natural statistic to use is the expected mean. This is the measure that we use. Imagine that we visit a village and meet people who fit the preceding description, and who have already formed a ROSCA.

⁵ An interesting study could be to analyze cases in which ROSCAs failed (experienced defaults) in order to establish the reasons for this, perhaps common reasons. Such a study could also seek to establish whether the reasons for a breakdown are related to deviations from the optimality condition identified in this paper.

If we find that the number of participants is lower than the optimum, then we will recommend adding new members. For scholars who study the formation and functioning of informal financial institutions, we provide a rigorously derived rule for limiting the size of these institutions. However, if ROSCAs were to function merely as a mechanism that facilitates saving, then there would be no reason for their special design; people could deposit, say with the village elder, some funds, and at a future point in time the accumulated sum will be divided between them.⁶

In this paper we draw on the intuition behind the third reason discussed above. We conduct a formal analysis showing how the “discreteness” of the frequency of ROSCA rotations feeds into the optimal size of the membership of a ROSCA.⁷ What we show, which to the best of our knowledge has not been reported before, is that an optimal size of a ROSCA exists, that it is (relatively) limited, and that it is a multiple of the number of time periods that it takes an individual to save on his own. Our result is far from being “relatively obvious:” to us, instinctively, it would seem that if we were to neglect management costs and moral hazard considerations, then larger ROSCAs would generally be more efficient due to scale.⁸ Establishing the existence of an optimal size is then counterintuitive, and proving this existence was not done before.

⁶ In addition to gaining time as the core reason, there can be other benefits from joining a ROSCA as opposed to saving alone. For example, saving regularly on one’s own, even for a self-disciplined person, could be a challenge. Participating in a ROSCA can help to develop a habit of regular saving, in a sense a voluntary undertaking to enter a regime of enforced saving.

⁷ The typical ROSCA operates under three characteristics of discreteness (“lumpiness” and “indivisibility”): the depositing of contributions is done at specific points in time (weekly, monthly, or even seasonally following harvests); the good to be purchased is often bulky (a tube well, a cow), not divisible, and even when it is (a bullock cart the components of which can be bought incrementally), it does not confer value until it is complete; and the number of participants is naturally discrete.

⁸ There is an additional reason why a large ROSCA could be considered more attractive than a small one. Suppose that there is a group of people who seek to form a ROSCA, where one of them is a defaulter, meaning that this person will default once he receives the “pot.” While the others suspect that there is a defaulter, they do not know who this person is. The biggest benefit to the defaulter will be conferred when he is drawn to be the first to receive the “pot.” In a small ROSCA, say of three participants, the probability of this occurring is one third; in a big ROSCA, say of ten participants, the probability of this occurring is one tenth. Thus, joining a small ROSCA will be preferred by a defaulter to joining a big ROSCA. It then follows that, as a defense against the presence of a defaulter in their midst, people will prefer to have a big ROSCA as opposed to having a small one.

The financial architecture that we study in this paper is a random ROSCA. Our study is not an all-encompassing study of all types ROSCAs. We do know that random ROSCAs are prevalent, and frequent. For example, Kedir and Ibrahim (2011) find that random ROSCAs are the common type of a ROSCA in Ethiopia. In the sample of Kenyan ROSCAs studied by Gugerty (2007), a large fraction of ROSCAs (42%) are random, while in the Kenyan sample of Anderson et al. (2009), 29% of the ROSCAs are random. According to Rutherford (2001), most of the ROSCAs in Bangladesh are of the random type.

We assume that the participants in a given ROSCA have a similar need, save in a similar manner, and are allocated the “pot” in the same manner. This is the reference setting that we study. In reality needs can differ, the ability to set aside a given sum on a regular basis can differ, and “pot” allocations can be governed not by a draw but, rather, by bidding where the participants bid competitively for the “pot” which is then given to the highest bidder, or allocations can be guided by considerations such as need, age, and compassion. (When the latter, then a ROSCA also assumes the role of an insurance provider.) To the extent that heterogeneity in need and the ability to save can interfere with the smooth functioning of ROSCAs and detract from their stability, we would expect the formation of several ROSCAs in a given village, each with a considerable degree of homogeneity rather than the formation of mixed ROSCAs. Women who save towards buying a cow will form one ROSCA, men who save for the installation of a tube well will form another ROSCA. All in all, our paper does not strive to be a Handbook of ROSCAs. We focus on a case of considerable homogeneity and uniformity which, as we state repeatedly, is not only an appropriate characterization or approximation of a good part of reality; it is also the basis for the construction of a plethora of ROSCA architectures. Particularly because of this, the attributes of the type of case that we study need to be fleshed out and ascertained rigorously. And the methodology and criteria that we develop can serve as a foundation for the study of other cases.

Our findings wield major consequences for the design of ROSCAs. An idea that could come to mind is to exploit the effect of scale to increase the efficiency of ROSCAs:

if we were to sufficiently reduce the cost of management (monitoring, enforcement, and other transaction costs) and the temptation to default, then a large ROSCA would be deemed more efficient than a smaller one. However, what we show is that even if the cost of management was negligible and the moral hazard was eliminated entirely, increasing the size of ROSCA beyond the optimum will not be beneficial to the participants.

2. The optimal size of a ROSCA

2.1. An example.

We develop further the example presented in the Introduction where we assumed that it takes 12 pesos to buy a tin roof (a lumpy investment), and that saving is at the rate of 1 peso per week. In Figure 1 we plot the expected waiting time for ROSCAs with different numbers of participants as a function of these numbers.

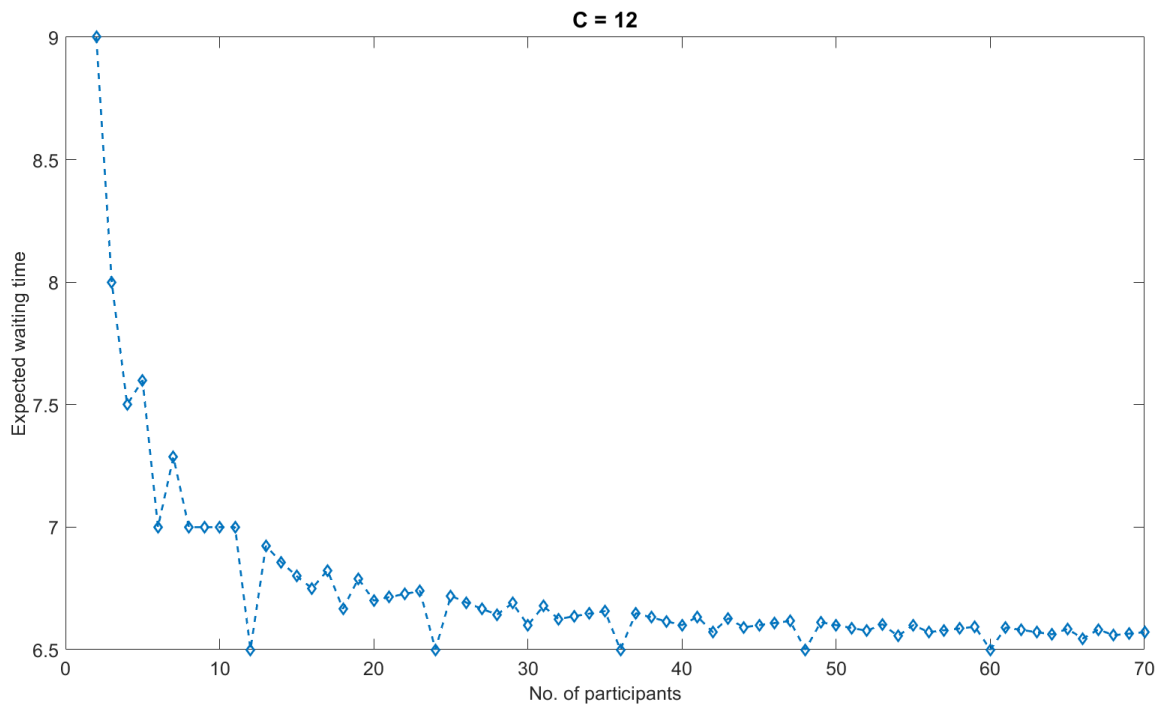


Figure 1. The expected waiting time in weeks for ROSCAs of different sizes when the cost of a tin roof is 12 pesos, and when each participant saves at the rate of one peso per week.

Four features of the Figure strike the eye. (i) When there are twelve participants, the expected waiting time is $6\frac{1}{2}$ weeks. For fewer participants, the expected waiting time is longer than this. (ii) The expected waiting time of $6\frac{1}{2}$ weeks is recurrent for ROSCA sizes that are multiples of 12: the function of the expected waiting time has multiple minima obtained at $k = 1, 2, \dots$ multiples of 12, which in this example occur in weeks 12, 24, 36, 48, 60, and so on. (iii) The expected waiting time does not change monotonically with respect to the size of the ROSCA. (iv) For numbers of participants that differ from multiples of 12, there is an overall downward trend in the expected waiting time which, in turn, converges to $6\frac{1}{2}$ weeks (the same expected waiting time as the one yielded by 12 participants) as the number of participants grows to infinity.

We next state and prove a claim showing that the minimal expected waiting time is a simple linear function of the number of time periods it takes an individual who saves on his own to accumulate the required funds.

2.2 A generalization

Let $C \in \mathbb{R}_+$ be the cost of the tin roof. The individual is assumed to save at the constant rate of 1 unit per week; for ease of reference we call that amount 1 peso. Instead of saving alone, a ROSCA of $N \in \mathbb{R}_+$ individuals is formed where all the participants save in the same way as the individual referred to above, namely at the end of every week each of them puts his savings into the common “pot.” Whenever there are at least C pesos in the common “pot,” as many as possible (randomly chosen) individuals receive funds from the common “pot” to facilitate the purchase of the tin roof. (No individual gets the funds twice.) As a result, every participant receives funds not later than C weeks after starting the ROSCA. By T_N we denote the expected number of weeks that it takes to receive the funds needed for installing the tin roof. What is the *minimal* expected waiting time in weeks, denoted by T_{min} , that it will take for a participant to receive the funds needed for installing the tin roof? For what number of participants in a ROSCA is the

minimal waiting time to be obtained? The essence of the following claim is that the minimal expected waiting time for getting the funds needed to install the tin roof, achieved when the number of participants in a ROSCA is a multiple of the cost C of the tin roof (namely when this number is kC , $k = 1, 2, \dots$), is $\frac{C+1}{2}$ weeks.

Claim 1. Case (i): We assume that N is a multiple of C . Then $T_N = T_{min} = \frac{C+1}{2}$.

Case (ii): We assume that N is not a multiple of C . Then $T_N > T_{min} = \frac{C+1}{2}$.

Proof. In the Appendix.

Claim 1 informs that the minimal expected waiting time, T_{min} , is a simple linear function of C , the number of the time periods that it takes an individual to save on his own. We can say that the minimal expected waiting time is “slightly over half” the number of time periods that it takes an individual to save on his own.⁹

By itself, the observation that the minimal expected waiting time is obtained when the number of participants in a ROSCA is kC , $k = 1, 2, \dots$, is quite telling. Referring to our “example of 12,” suppose that we “visit” a village in which we observe that there are two ROSCAs, each of the same size of 12 participants, and each yielding the same minimal expected waiting time of $6\frac{1}{2}$ weeks. If we were to point out to the villagers that, to expedite matters, instead of two ROSCAs of 12 participants the villagers will do better by forming a single ROSCA of 24 participants, then we will be proved wrong: other things being held the same, in terms of the expected waiting time such a rearrangement will confer no gain.

⁹ In a ROSCA of an optimal size, *all the participants except one* receive the funds needed to install the tin roof earlier than if saving alone. Only the participant who is drawn to receive the funds last ends up experiencing a waiting time that is the same as the waiting time that he would have experienced if saving alone.

Two remarks shed additional insight into the attractiveness of having a ROSCA of size C .

Remark 1. When lone saving is replaced by a ROSCA saving, the gain in terms of the reduction in the expected waiting time that obtains right at the outset, namely when lone saving is replaced by a two-person ROSCA, is larger, or at least not smaller, than the gain that obtains when the two-person ROSCA is replaced by the ROSCA size that confers the minimal expected waiting time. We present the reasoning that yields this observation in two parts: when C is an even number, and when C is an odd number.

(i) We consider first the case in which C is an even number. The expected waiting time when saving alone is C and when, instead, saving is with one other person, the expected waiting time is $\frac{3C}{4}$, so the gain is $\frac{C}{4}$. From Claim 1 we know that the minimal expected waiting time is $\frac{C+1}{2}$. So if we were to shift from a two-person ROSCA to the ROSCA size that delivers the minimal expected waiting time, the gain would be $\frac{3C}{4} - \frac{C+1}{2} = \frac{C-2}{4}$. This number is smaller than $\frac{C}{4}$.

(ii) We next consider the case in which C is an odd number. The expected waiting time when saving alone is C , and when saving with one other person the expected waiting time is $\frac{3C+1}{4}$, so the gain is $\frac{C-1}{4}$. From Claim 1 we know that the minimal expected waiting time is $\frac{C+1}{2}$. So if we were to shift from a two-person ROSCA to the ROSCA size that delivers the minimal expected waiting time, the gain would be $\frac{3C+1}{4} - \frac{C+1}{2} = \frac{C-1}{4}$. This number is the same as the gain obtained when moving from lone saving to a two-person ROSCA.

Remark 2. Suppose that instead of an individual saving on his own, we start with a ROSCA of two, and then expand it incrementally; that is, we increase the size of the ROSCA from two participants to three, then from three participants to four, and so on. In

terms of convergence with the minimal expected waiting time, such changes do not take us *monotonically* closer to the minimal expected waiting time.¹⁰ As shown in Figure 1, an initial decrease in the expected waiting time can be followed by a subsequent increase.

Thus: the biggest increment to the efficiency of a ROSCA occurs right at the outset, namely upon a change from lone saving to joint saving by two individuals, and the efficiency of a ROSCA does not change monotonically, as when a downward trend in the expected waiting time is followed by an increase. Consequently, when a process of enlarging a ROSCA from size C to size kC is underway, there is a risk of being “stuck” in a non-optimal size “in between” the multiples of C . This observation supports the conclusion that out of ROSCA sizes that yield the lowest expected waiting time (kC , $k = 1, 2, \dots$), the smallest size C will be preferred, so it is to this size that we refer as the “optimal size of a ROSCA.”

The expected waiting time is not the only measure for which a ROSCA of size C or, for that matter, of a size that is a multiple of C , outperforms ROSCAs of other sizes. For a ROSCA of size N we denote the waiting time to obtain the “pot” by the random variable τ_N . The following claim reveals that with respect to waiting time, in terms of stochastic dominance ROSCAs of a size that is a multiple of C are better than ROSCAs of other sizes.

Claim 2. For any $N_1 \in \mathbb{N}_+$ that is a multiple of C , and for any $N_2 \in \mathbb{N}_+$ that is not a multiple of C , it holds that $P(\tau_{N_1} \leq k) \geq P(\tau_{N_2} \leq k)$, where $k \in \{1, \dots, C\}$. For a k such that kN_2 is not a multiple of C , the inequality is strict.

Proof. In the Appendix.

We next study in greater detail characteristics of sizes of ROSCAs that differ from the ones that yield the minimal expected waiting time.

2.3. Progression outside the minima of the expected waiting time of ROSCAs of different sizes

¹⁰ It turns out that this feature pertains to values of C that are not prime numbers.

The preceding considerations still leave open the question of what can be said about the pattern or the progression of the expected waiting time for ROSCAs of different sizes *outside* the minima $N = kC$? For example, in terms of the expected waiting time, is there a gain from having $2C - 1$ participants as opposed to $C - 1$ participants? (In the example of 12, is having 23 participants better than having 11 participants?) Is it the case that being anywhere between $(k + 1)C$ and kC is better than being anywhere between $(m + 1)C$ and mC where k and m are natural numbers, and $k > m$? (Again, “better” stands for a shorter expected waiting time.)

Revisiting Figure 1, for $N \neq kC$, there is an overall downward trend of the expected waiting time which, in turn, converge to T_{min} .

This feature is the subject of Claim 3 which identifies and explains an overall decreasing pattern in the expected waiting time, T_N . The meaning of the finding reported in the claim is that when we are out of the minima of the expected waiting time, a larger number of participants generally results in a lower expected waiting time (although obviously it never takes the expected waiting time below the minima). This consideration is important: in terms of obtaining the minimal expected waiting time, even the largest possible ROSCA will not do better than a ROSCA of size C . The reason for the pattern described is that for any $N, k \in \mathbb{N}_+$, T_{N+kC} is a particular convex combination of T_N and of T_{min} such that the value of T_{kC+j} and the difference between T_{kC+j} and T_{min} (for $j \in \{1, 2, \dots, C - 1\}$) decrease when k increases. Ultimately, the sequence $(T_N)_{N=1}^\infty$ converges to T_{min} .

Claim 3. The following properties of the sequence $(T_N)_{N=1}^\infty$ hold.

(a) For any $N, C, k \in \mathbb{N}_+$, the term T_{N+kC} is given by

$$T_{N+kC} = \frac{N}{N+kC} T_N + \frac{kC}{N+kC} T_{min}.$$

(b) A decreasing trend: for any fixed $C \in \mathbb{N}_+$ and for any $j \in \{1, 2, \dots, C-1\}$, the sequence $(T_{j+kC})_{k=1}^{\infty}$ is strictly decreasing.

(c) Convergence: for any fixed $C \in \mathbb{N}_+$ it holds that $\lim_{N \rightarrow \infty} T_N = T_{min}$.

Proof. In the Appendix.

3. Discussion and conclusions

We develop a formula for the optimal size of a ROSCA, using as our measure of efficiency the expected waiting time that it takes to obtain the funds needed to acquire a desirable good. We find that maximal efficiency is reached for a ROSCA of size C , where C is the number of time periods that it takes an individual to save on his own. We also find that the same level of maximal efficiency is achieved by ROSCAs of sizes that are multiples of C . In addition, we find that the relationship between the efficiency of a ROSCA and its size is not monotonic. Although a change from lone saving to joint saving by two individuals always increases efficiency, the same does not apply to other incremental increases: when the size of an existing ROSCA increases by one, the expected waiting time may actually become longer. Nonetheless, an indefinite increase of the size of a ROSCA results in efficiency converging to the maximal efficiency. Because, as we have shown, maximal efficiency is obtained for size C , this indefinite increase confers no gain. In addition, in the close neighborhood of any kC , $k = 1, 2, \dots$, efficiency is lower than at kC . Our results refute the presumption that when management issues and moral hazard considerations are controlled for, a large ROSCA is naturally more efficient than a small ROSCA.

Consider the mirror image of the theme examined above. While the cost of the tin roof is fixed at C , the number of individuals who will want to participate in a ROSCA, N , is also fixed. How many ROSCAs to form, and how to distribute the individuals between them? To gain an easy handle on this question, imagine that $C = 4$, and that there are $N = 6$ individuals who seek to expedite getting a tin roof, to which end they are inclined to resort to “a ROSCA solution.” Because 6 is not a multiple of 4, what

configuration will yield the overall minimal expected waiting time? For example, one ROSCA of six individuals? Two ROSCAs of three individuals each? Three ROSCAs of two individuals each? One ROSCA of four individuals and one ROSCA of two individuals? Intuitively, the last and the first possibilities appear particularly appealing. So, we run a quick check. From Claim 1 we know that the expected waiting time in a ROSCA of 4 individuals is $2\frac{1}{2}$, and from Remark 1 we know that the expected waiting time in a ROSCA of 2 individuals is 3. Thus, the weighted average expected waiting time in a split of the numbers of individuals into four and two is $\frac{2}{3} \cdot \frac{5}{2} + \frac{1}{3} \cdot 3 = \frac{8}{3}$. In a ROSCA of six participants, the expected waiting time is $\frac{1}{6} \cdot 1 + \frac{2}{6} \cdot 2 + \frac{1}{6} \cdot 3 + \frac{2}{6} \cdot 4 = \frac{16}{6} = \frac{8}{3}$ weeks, which is the same as in the case of the four and two split. (The expected waiting time when the split is three and three is 3 weeks, and the expected waiting time when the split is two, two and two is 3 weeks.) The outcome of a formal analysis, available on request, is that the way to go is as follows: when the number of the would-be participants is bigger than a multiple of C , namely when $N = kC + l$, $k = 1, 2, \dots$ and $l \in \mathbb{N}_+$, $l < C$, then in terms of the minimal expected waiting time the following distributions yield the same result: (i) a “grand” ROSCA of N individuals; (ii) k ROSCAs of C individuals each, along with one ROSCA of the residual number l ; (iii) any numbers of ROSCAs where each is of the same multiple of C or of different multiples of C , and one ROSCA is of l individuals.

There is another way of expressing the gain from having a ROSCA of the size that delivers the minimal expected waiting time. This is to calculate a ratio as follows. In the numerator we write the difference between the total waiting time in weeks that it will take all the participants to amass the funds needed when each of them saves alone, minus the total number of weeks of waiting when they save via participation in a ROSCA of an optimal size. And in the denominator we write the sum total of the waiting time in weeks that it will take the participants to amass the funds needed when each of them saves

alone. This ratio, a measure of the time saved, is $\frac{C-1}{2C}$, namely about a half. So we can say that the gain from participation in a ROSCA of an optimal size as opposed to saving alone is to shorten the aggregate saving time by about half.

A limitation of our analysis arises from the simplifying assumption that the rate of saving is constant at 1 peso per week. This assumption implies that the cost of the desired good, C , is always divisible by the saving rate. Suppose that we generalize the saving rate to any number $s \in \mathbb{R}_+$, $s < C$. It is easy to see that then Claim 1 holds just as well for cases in which C is divisible by s . An analysis of the case in which C is not a multiple of s is to be undertaken in future research.

Related to this, the development and analysis of a “richer framework” such as flexible savings where participants can adapt their savings to adjust to the number of participants, given their objective C will require a dedicated treatment that differs from the one provided in this paper, and will be the subject of a separate inquiry. It will be particularly interesting to find out whether the possibility of participants adjusting the level of their savings to the number of participants interferes with the optimality and efficiency established in our paper.¹¹

A typical way in which a ROSCA operates is to make lump sum distributions from the “pot” at each meeting in which the accumulated funds can cover the purchase of an indivisible good. The fact that different people can have different needs and will want to purchase different goods does not interfere much with our core analysis. By and large, ROSCAs are created and managed endogenously. Therefore, it is rather unlikely that people with vastly different needs and interests will band together in a given ROSCA. Moreover, suppose that a person wants to purchase a good that costs more than C , say $2C$. To this end, he can join two ROSCAs, or within a single ROSCA save for two cycles. The former option requires saving at a higher rate, because he will have to

¹¹ A preliminary analysis, available on request, indicates that our findings are robust in that they do not change much when we assume alternatively that the members of a ROSCA choose their rate of saving endogenously.

contribute to two “pots” simultaneously. (For example, Acquah and Dahal, 2018 report that in Indonesia, 24% of ROSCA members participate simultaneously in more than one ROSCA.) In such a case, as this person will want to get the “pot” as early as possible in both ROSCAs, our analysis applies. If this person cannot afford to participate simultaneously in two ROSCAs, then in order to be able to obtain the required amount he will need to wait for the second cycle of his single ROSCA. Indeed, and as we have already noted (recall footnote 2), it is often the case that a ROSCA is entered into multiple times. Then, during the course of the first cycle, it will not matter to that person when he would get the “pot,” whereas he will care about the expected waiting time during the course of the second cycle. Then, the reasoning that we presented earlier on in our paper will still apply.

Appendix: Proofs of Claims 1, 2, and 3

For ease of reference, the claims are replicated here.

Claim 1. Case (i): We assume that N is a multiple of C . Then $T_N = T_{min} = \frac{C+1}{2}$.

Case (ii): We assume that N is not a multiple of C . Then $T_N > T_{min} = \frac{C+1}{2}$.

As preliminaries to proving the claim, we introduce a notation, and we state and prove a Lemma.

Notation: For $x \in \mathbb{R}$, $[x]$ denotes the largest integer that is not bigger than x . In particular, for every $x \in \mathbb{R}$, $[x] \leq x$, and if x is not an integer, then this inequality is strict.

Lemma. Let a_j denote the number of the ROSCA participants who receive funds exactly at the end of the j -th week, $j \in \{1, 2, \dots, C\}$. Then,

$$a_j = \left[\frac{jN}{C} \right] - \left[\frac{(j-1)N}{C} \right].$$

Proof of the Lemma. Over a time period of j weeks, the ROSCA participants jointly put into the common “pot” jN pesos. Each time that they can issue C pesos to a participant, they do so. Thus, the number of times funds can be distributed up to the end of the j -th week is equal to the largest integer that is not bigger than $\frac{jN}{C}$, namely to

$\left[\frac{jN}{C} \right]$. By analogous reasoning, the number of funds distributed up to the end of the

$(j-1)$ -th week is equal to $\left[\frac{(j-1)N}{C} \right]$. Therefore, the number of participants who receive

funds exactly at the end of the j -th week is the difference between the preceding two numbers, namely it is

$$a_j = \left\lfloor \frac{jN}{C} \right\rfloor - \left\lfloor \frac{(j-1)N}{C} \right\rfloor.$$

Q.E.D.

Proof of Claim 1. We calculate the expected waiting time T_N drawing on the definition of the expected value. From the Lemma, it follows that

$$\begin{aligned} T_N &= \frac{1}{N} \sum_{j=1}^C j \cdot \left(\left\lfloor \frac{jN}{C} \right\rfloor - \left\lfloor \frac{(j-1)N}{C} \right\rfloor \right) \\ &= \frac{1}{N} \cdot \left\{ \left\lfloor \frac{N}{C} \right\rfloor + 2 \cdot \left(\left\lfloor \frac{2N}{C} \right\rfloor - \left\lfloor \frac{N}{C} \right\rfloor \right) + \dots \right. \\ &\quad \left. + (C-1) \cdot \left(\left\lfloor \frac{(C-1)N}{C} \right\rfloor - \left\lfloor \frac{(C-2)N}{C} \right\rfloor \right) + C \cdot \left(\left\lfloor \frac{CN}{C} \right\rfloor - \left\lfloor \frac{(C-1)N}{C} \right\rfloor \right) \right\} \\ &= \frac{1}{N} \cdot \left\{ -\left\lfloor \frac{N}{C} \right\rfloor - \left\lfloor \frac{2N}{C} \right\rfloor - \dots - \left\lfloor \frac{(C-1)N}{C} \right\rfloor + CN \right\} = C - \frac{\sum_{j=1}^{C-1} \left\lfloor \frac{jN}{C} \right\rfloor}{N}. \end{aligned}$$

We consider two cases.

Case (i). N is a multiple of C . Then $\frac{N}{C}$ is an integer and, thus, for each $j \in \{1, 2, \dots, C-1\}$

$$\left\lfloor \frac{jN}{C} \right\rfloor = \frac{jN}{C}.$$

Therefore,

$$\sum_{j=1}^{C-1} \left\lfloor \frac{jN}{C} \right\rfloor = \sum_{j=1}^{C-1} \frac{jN}{C} = \frac{N}{2C} \cdot (C-1) \cdot C = \frac{N(C-1)}{2}.$$

It then follows that

$$T_N = C - \frac{\sum_{j=1}^{C-1} \left\lfloor \frac{jN}{C} \right\rfloor}{N} = C - \frac{N(C-1)}{2N} = C - \frac{C-1}{2} = \frac{C+1}{2}.$$

Case (ii). N is not a multiple of C . Then, $\frac{N}{C}$ is not an integer and, thus, $\left[\frac{N}{C} \right] < \frac{N}{C}$.

Moreover, for each $j \in \{2, \dots, C-1\}$, $\left[\frac{jN}{C} \right] \leq \frac{jN}{C}$.

Because

$$\sum_{j=1}^{C-1} \left[\frac{jN}{C} \right] = \left[\frac{N}{C} \right] + \sum_{j=2}^{C-1} \left[\frac{jN}{C} \right] < \frac{N}{C} + \sum_{j=2}^{C-1} \frac{jN}{C} = \sum_{j=1}^{C-1} \frac{jN}{C} = \frac{N}{C} \cdot \frac{(C-1)C}{2} = \frac{N(C-1)}{2},$$

we conclude that

$$T_N = C - \frac{\sum_{j=1}^{C-1} \left[\frac{jN}{C} \right]}{N} > C - \frac{N(C-1)}{2N} = C - \frac{C-1}{2} = \frac{C+1}{2}.$$

Q.E.D.

Claim 2. For any $N_1 \in \square_+$ that is a multiple of C , and for any $N_2 \in \square_+$ that is not a multiple of C , it holds that $P(\tau_{N_1} \leq k) \geq P(\tau_{N_2} \leq k)$, where $k \in \{1, \dots, C\}$. For a k such that kN_2 is not a multiple of C , the inequality is strict.

Proof. In proving this claim, we make use of the lemma that featured in the preliminary stage of the proof of Claim 1. For any $N \in \square_+$, it holds that

$$\begin{aligned} P(\tau_N \leq k) &= \frac{1}{N} \sum_{j=1}^k \left(\left[\frac{jN}{C} \right] - \left[\frac{(j-1)N}{C} \right] \right) \\ &= \frac{1}{N} \left(\left[\frac{N}{C} \right] + \left[\frac{2N}{C} \right] - \left[\frac{N}{C} \right] + \dots + \left[\frac{kN}{C} \right] - \left[\frac{(k-1)N}{C} \right] \right) \\ &= \frac{1}{N} \left[\frac{kN}{C} \right]. \end{aligned}$$

Clearly, if N_1 is a multiple of C , then we get that $P(\tau_{N_1} \leq k) = \frac{k}{C}$. If N_2 is not a multiple of C , then

$$P(\tau_{N_2} \leq k) = \frac{1}{N_2} \left[\frac{kN_2}{C} \right] \leq \frac{1}{N_2} \frac{kN_2}{C} = \frac{k}{C} = P(\tau_{N_1} \leq k).$$

The inequality in the preceding displayed expression follows from an attribute of the $[\cdot]$ function. For k such that kN_2 is not a multiple of C , this inequality is strict (as, for example, when $k=1$).

Q.E.D.

Claim 3. The following properties of the sequence $(T_N)_{N=1}^\infty$ hold.

(a) For any $N, C, k \in \mathbb{N}_+$, the term T_{N+kC} is given by

$$T_{N+kC} = \frac{N}{N+kC} T_N + \frac{kC}{N+kC} T_{min}. \quad (\text{A1})$$

(b) A decreasing trend: for any fixed $C \in \mathbb{N}_+$ and for any $j \in \{1, 2, \dots, C-1\}$, the sequence $(T_{j+kC})_{k=1}^\infty$ is strictly decreasing.

(c) Convergence: for any fixed $C \in \mathbb{N}_+$ it holds that $\lim_{N \rightarrow \infty} T_N = T_{min}$.

Proof. To begin with, we prove property (a).

For any $j \in \{1, 2, \dots, C\}$ and $k \in \mathbb{N}_+$ it holds that

$$\left[\frac{j(N+kC)}{C} \right] = \left[\frac{jN}{C} + k \right] = \left[\frac{jN}{C} \right] + k.$$

Drawing on the definition of the expected value and recalling from the proof of Claim 1

that $T_N = C - \frac{\sum_{j=1}^{C-1} \left[\frac{jN}{C} \right]}{N}$, it follows that

$$\begin{aligned}
T_{N+kC} &= \frac{1}{N+kC} \sum_{j=1}^C j \cdot \left(\left[\frac{j(N+kC)}{C} \right] - \left[\frac{(j-1)(N+kC)}{C} \right] \right) \\
&= \frac{1}{N+kC} \cdot \left\{ \left[\frac{N}{C} \right] + k + 2 \cdot \left(\left[\frac{2N}{C} \right] - \left[\frac{N}{C} \right] + k \right) + \dots + C \cdot \left(\left[\frac{CN}{C} \right] - \left[\frac{(C-1)N}{C} \right] + k \right) \right\} \\
&= \frac{1}{N+kC} \cdot \left\{ - \left[\frac{N}{C} \right] - \left[\frac{2N}{C} \right] - \dots - \left[\frac{(C-1)N}{C} \right] + CN + \frac{kC(C+1)}{2} \right\} \\
&= \frac{N}{N+kC} \cdot \left(C - \frac{\sum_{j=1}^{C-1} \left[\frac{jN}{C} \right]}{N} \right) + \frac{1}{N+kC} \cdot \frac{kC(C+1)}{2} = \frac{N}{N+kC} T_N + \frac{kC}{N+kC} \cdot T_{min},
\end{aligned}$$

so (A1) holds.

We can now prove properties (b) and (c).

Property (b). First, we note that if $x, y, \alpha, \beta \in \square$ such that $x < y$ and $\alpha < \beta$, then $(\alpha - \beta)(x - y) > 0$, which can be rewritten as

$$\alpha x - \alpha y > \beta x - \beta y,$$

and upon adding y to each side,

$$\alpha x + (1 - \alpha)y > \beta x + (1 - \beta)y. \quad (\text{A2})$$

Let $j \in \{1, 2, \dots, C-1\}$, $k \in \square_+$, and $T_i < T_j$. Then, by (A1)

$$T_{j+kC} = \frac{j}{j+kC} T_j + \frac{kC}{j+kC} T_{min}$$

and

$$T_{j+(k+1)C} = \frac{j}{j+(k+1)C} T_j + \frac{(k+1)C}{j+(k+1)C} T_{min}.$$

Applying (A2) for $x = T_{min}$, $y = T_j$, $\alpha = \frac{kC}{j+kC}$, and $\beta = \frac{(k+1)C}{j+(k+1)C}$, we get that

$$T_{j+kC} = \frac{kC}{j+kC} T_{min} + \frac{j}{j+kC} T_j > \frac{(k+1)C}{j+(k+1)C} T_{min} + \frac{j}{j+(k+1)C} T_j = T_{j+(k+1)C}.$$

Thus, the sequence $(T_{j+kC})_{k=1}^{\infty}$ is strictly decreasing.

Property (c). Let $\varepsilon > 0$ be fixed. Then, we define

$$n_0 \equiv C \cdot \left(\left\lceil \frac{(T - T_{min})}{\varepsilon} \right\rceil + 1 \right),$$

where $T = \max\{T_j : j \in \{1, 2, \dots, C\}\}$.

We assume that $N > n_0$. Then, there exist $k \in \mathbb{N}_+$ and $j \in \{1, 2, \dots, C\}$ such that $N = j + kC$. Moreover, from the formula of n_0 ,

$$j + kC > C \cdot \left(\left\lceil \frac{(T - T_{min})}{\varepsilon} \right\rceil + 1 \right)$$

or equivalently,

$$k + \frac{j}{C} > \left\lceil \frac{(T - T_{min})}{\varepsilon} \right\rceil + 1.$$

We know that $k \in \mathbb{N}_+$, that $\left\lceil \frac{(T - T_{min})}{\varepsilon} \right\rceil + 1 \in \mathbb{N}_+$, and that $\frac{j}{C} \leq 1$. The preceding inequality can then be transformed into

$$k + \frac{j}{C} \geq \left\lceil \frac{(T - T_{min})}{\varepsilon} \right\rceil + 1$$

and next into

$$k \geq \left\lceil \frac{(T - T_{min})}{\varepsilon} \right\rceil + 1 > \frac{(T - T_{min})}{\varepsilon}.$$

Therefore,

$$\begin{aligned}
|T_N - T_{min}| &= T_N - T_{min} = T_{j+kC} - T_{min} = \frac{j}{j+kC}T_j + \frac{kC}{j+kC}T_{min} - T_{min} \\
&= \frac{j}{j+kC}(T_j - T_{min}) \leq \frac{j}{j+kC}(T - T_{min}) \leq \frac{C}{C+kC}(T - T_{min}) = \frac{1}{k+1}(T - T_{min}) \\
&\leq \frac{1}{\frac{T - T_{min}}{\varepsilon} + 1}(T - T_{min}) = \frac{\varepsilon}{1 + \frac{\varepsilon}{T - T_{min}}} < \varepsilon.
\end{aligned}$$

In sum: for any $\varepsilon > 0$ there exists $n_0 > 0$ such that $|T_N - T_{min}| < \varepsilon$ for every $N > n_0$.

Therefore, $\lim_{N \rightarrow \infty} T_N = T_{min}$.

Q.E.D.

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