

Aaberge, Rolf; Langørgen, Audun; Lindgren, Petter Y.

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## A European equivalence scale for public in-kind transfers

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# A european equivalence scale for public in-kind transfers

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DISCUSSION PAPERS

977

Rolf Aaberge, Audun Langørgen and Petter Y. Lindgren

*Rolf Aaberge, Audun Langørgen and  
Petter Y. Lindgren*

## **A european equivalence scale for public in-kind transfers**

**Abstract:**

This paper introduces a theory-based equivalence scale for public in-kind transfers, which justifies comparison of distributions of extended income (cash income plus the value of public services) between European countries. We demonstrate the usefulness of the proposed equivalence scale in an empirical analysis of the effects of public health care, long-term care, education and childcare expenditure on estimates of income inequality and poverty for 24 European countries. The empirical results show significant effects of public in-kind transfers on the level of income inequality and poverty for all countries. Over the period 2006–2018, inequality and poverty estimates display rather different trends across European countries.

**Keywords:** Income distribution, poverty, equivalence scales, needs adjustment, public services, in-kind transfers

**JEL classification:** D30, H40, I30

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**Address:** Rolf Aaberge, Statistics Norway. E-mail: [rolf.aaberge@ssb.no](mailto:rolf.aaberge@ssb.no)

Audun Langørgen, Statistics Norway. E-mail: [audun.langorgen@ssb.no](mailto:audun.langorgen@ssb.no)

Petter Y. Lindgren, Statistics Norway. E-mail: [petter.lindgren@ssb.no](mailto:petter.lindgren@ssb.no)

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## **Sammendrag**

I dette arbeidet utvikler vi metoder for å sammenlikne fordelinger av utvidet inntekt mellom europeiske land. Utvidet inntekt er definert ved disponibel kontantinntekt pluss verdien av offentlige tjenester. Vi studerer effekten av offentlige helsetjenester, pleie og omsorg, utdanning og barnehager på estimater for inntektsulikheter og fattigdom i 24 europeiske land. Verdsettingen av offentlige tjenester og identifikasjon av målgrupper er basert på gruppefordelte regnskaper for hvert av de 24 landene.

Den såkalte EU-skalaen er mye brukt for å bestemme antall forbruksenheter for hushold, der antall forbruksenheter avhenger av husholdets størrelse og sammensetning. For å kunne sammenligne inntektsnivåer på tvers av ulike husholdstyper, deler vi husholdsinntekt etter skatt på antall forbruksenheter i husholdet. Vi introduserer en teoribasert felles ekvivalensskala for europeiske land som vi kaller den behovsjusterte EU-skalaen. Til forskjell fra den ordinære EU-skalaen tar den behovsjusterte skalaen hensyn til behovene som individer og hushold har for offentlige tjenester.

Resultatene fra den empiriske analysen viser signifikante effekter av offentlig tjenesteyting på estimater for inntektsulikheter og fattigdom i de europeiske landene. Utviklingen i ulikhet og fattigdom i perioden 2006–2018 har til dels vært svært forskjellig i de ulike landene.

## 1. Introduction

The increase in economic inequality in European countries during the past four decades has received much attention; not least due to Piketty's (2014) and Scheidel's (2017) dismal predictions of a future class-divided "Downton Abbey" society. This prediction has been justified by historic data records as well as the broadly documented recent rise in wealth and market income inequality. The exclusive focus on the evolution of the distribution of individual pre-tax market income has been dictated by historic data limitations. However, distributions of individual market incomes, even when taxes are subtracted, do not provide a complete picture of present distributions of economic well-being in European countries, since publicly funded welfare services constitute a substantial part of the welfare states' transfers to individuals and households (OECD, 2017). While market income could be considered as an appropriate measure of economic well-being before and shortly after the Second World War, this is clearly not the case for the past four decades. Moreover, there is large variation in the spending on in-kind transfers across European countries. Northern European countries as well as France, Germany, Belgium and the Netherlands spend a relatively large share of GDP on public welfare services, whereas southern and eastern European countries spend much less on public services. However, even the countries with the lowest share spend almost eight per cent of GDP on in-kind transfers. Thus, the omission of public in-kind transfers from a measure of economic well-being may call into question the validity of income comparisons over time for a given country, and across countries with different levels and composition of taxes, cash benefits and publicly provided goods and services. In taking notice of this fact, several researchers have acknowledged the importance of incorporating public in-kind transfers in studies of inequality (Atkinson et al., 2002; Stiglitz et al., 2009; Congressional Budget Office, 2011; OECD, 2011). This shift in focus calls for broader measures of household resources that reflect a comprehensive view of how government redistribution affects household living standards.<sup>1</sup>

The importance of accounting for economies of scale in consumption in analyses of distributions of disposable cash income is universally acknowledged. Equivalence scales designed to account for the consumption needs associated with cash income might however be inappropriate when analysing measures of income that include the value of public services, since these scales ignore the presence of free or subsidised public services such as education and health care. For instance, elderly people have

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<sup>1</sup> For recent studies of the impact of in-kind benefits on the income distribution, see e.g. Figari and Paulus (2015), Aaberge et al. (2010; 2017; 2018; 2021), Piketty et al. (2017) and Aaberge et al. (2019).

higher needs for health care since they in general suffer from poorer health status, whereas children are considered to have high needs for education services. Since such services mainly are received by children and the elderly, it is not plausible to assume that the needs for public services are proportional to the needs for cash income. Consequently, extended income studies relying on equivalence scales designed for cash income might overstate the affluence among the elderly and families with children, since they fail to account for the relatively high needs of such families for public in-kind transfers. Thus, it is required to introduce a separate equivalence scale for public in-kind transfers and to combine it with the EU-scale for cash income to obtain an equivalence scale for extended income.

Pollak and Wales (1979) show that any attempt to recover equivalence scales from household behaviour encounters the problem that household choices are conditional on needs, which makes it inherently difficult to disentangle needs from preferences. As a result, estimation of equivalence scales from demand data requires identifying assumptions that are untestable (Blundell and Lewbel, 1991). Browning et al. (2013) and Chiappori (2016) suggest an empirical approach that makes situational comparisons of the cost of putting the individual on the same indifference curve in different family contexts. However, given that household demand and market prices for in-kind transfers are not observed, such demand-based approaches are ruled out from estimation of equivalence scales that are designed to capture the specific needs associated with public services.

An alternative approach is to base the judgment of recipient needs on the imputation of public values attributed to the government. Despite the difficulties involved in making interpersonal welfare comparisons, government authorities are forced to make such comparisons in decision processes by which in-kind transfers are allocated across individuals and households. Yet, there is little prior work that employs the targeting of in-kind transfers as a source of information on the equivalence scales that are implicit in the transfer system. A notable exception is Olken (2005), who proposes a method for identifying “community equivalence scales” from a subsidised rice program to poor households that is allocated by local authorities in Indonesia. Similarly, Aaberge et al. (2010, 2019) estimate the equivalence scales implicitly used in local government allocations of in-kind transfers in Norway.

A major purpose of the present paper is to introduce a European equivalence scale for extended income by including most of the government spending on in-kind transfers. To this end, we introduce a theory-based equivalence scale for in-kind transfers that can be used as a basis for comparing distributions of extended income across countries. The proposed equivalence scale is derived from a social welfare function which is shown to satisfy a generalised multiple goods’ version of the Pigou-

Dalton principle of transfers. An advantage of our approach is that measures of equivalence scales, welfare, inequality and poverty constitute a coherent framework that ensures internal consistency between the different methodological elements and moreover have a transparent normative justification.

The obtained European equivalence scale for extended income is applied for an empirical analysis that accounts for the value of basic public services in 24 European countries: childcare, education, health care and long-term care. These services amount to a sizeable share of public spending and are services targeted to individuals who belong to well-defined subgroups of the population. Allocation of public expenditure to different services is reported by Eurostat in the Education Database and the System of Health Accounts, while we also utilise spending profiles by age and gender reported by the European Commission. Households' cash incomes have been made available by the European Union Statistics on Income and Living Conditions (EU-SILC).

The paper is organised as follows. Section 2 provides a discussion of the theoretical foundation for needs-adjusted (NA) equivalence scales and presents a common equivalence scale for European countries. Section 3 discusses data, empirical implementation and methods used for analysing income inequality and poverty. Section 4 displays the results of the empirical analysis. Section 5 concludes.

## **2. Needs for public in-kind transfers and equivalence scales**

Equivalence scales are used to justify comparisons of incomes of households who differ in needs. The equivalence scale for a given household shows the scale rate of income that a specific household needs to obtain the same material well-being as the reference household. When disposable cash income is used as a measure of economic welfare, the common practice is to employ the EU scale as a means for achieving interpersonal comparability in analyses of inequality and poverty.<sup>2</sup> While theoretically justified equivalence scales can be constructed from household cost functions, most empirical analyses typically use more pragmatic scales to adjust incomes for differences in household size and composition (see e.g. Coulter et al., 1992). However, since the commonly used equivalence scales are designed to account for differences in household needs for disposable cash income, they are not necessarily appropriate when analysing a measure of economic welfare that includes the value of

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<sup>2</sup> The EU scale is also called the modified OECD scale in the literature. This scale assigns weight 1 to the first adult of the household, 0.5 to each additional member aged 14 and above and 0.3 to children aged under 14. Economies of scale in consumption is the rationale for assigning a higher weight to the household head. Cars and housing are examples of jointly consumed goods, which are assumed to contribute to economies of scale.



public in-kind transfers. For example, the conventional scales do not acknowledge that the needs for health services and education are relatively high among the elderly and households with children. Consequently, the economic welfare of households with relatively high needs for public in-kind transfers might be overrated by studies that apply the conventional equivalence scales as conversion factors for measures of income that include the value of public in-kind transfers.

A major objective of this paper is to introduce an equivalence scale that relaxes the assumption that the relative needs of different household types are unaffected when the definition of income is extended to account for the value of in-kind transfers. To account for heterogeneity in needs for cash income we rely on the conventional EU scale. As is well known, the EU scale assigns relatively low weight to children, simply because children have smaller needs for private consumption than adults. However, when needs are also considered to incorporate public education services, it follows that the equivalence scale factor for children will become larger. Similarly, the equivalence scale should also account for differences in needs between adults of different ages when the income definition includes public health care and long-term care.

## 2.1. Social welfare and needs-adjusted transfers

This subsection describes the social evaluation framework that is used to derive a common European equivalence scale for public in-kind transfers. To this end, it is required to introduce the following notation: Consider a country  $k$  with  $H_k$  households and let  $x_{0hk}$  denote the cash income of household  $h$  that is disposable for consumption of market goods. Let  $(x_{1hk}, x_{2hk}, \dots, x_{Shk})$  denote the values of  $S$  public services received by household  $h$ ,  $\mathbf{x}_{hk} = (x_{0hk}, x_{1hk}, \dots, x_{Shk})$ ,  $h = 1, 2, \dots, H_k$ ;  $k = 1, 2, \dots, K$ , where  $\mathbf{x}_{hk}$  is a vector of cash income (composite consumption) and public in-kind transfers for  $H_k$  households in country  $k$ . To account for heterogeneity in needs of goods and services in comparisons of  $\mathbf{x}_{hk}$ ,  $h = 1, 2, \dots, H_k$ ;  $k = 1, 2, \dots, K$ , we introduce the vector  $\boldsymbol{\gamma}_{hk} = (\gamma_{0hk}, \gamma_{1hk}, \dots, \gamma_{Shk})$  of good-specific needs parameters, which may differ across households as well as across countries. The needs parameters are assumed to form an integral part of the following much used CES-family of measures of well-being<sup>3</sup>

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<sup>3</sup> Blackorby and Donaldson (1982) provide an axiomatic justification of the well-being measure defined by (2.1). As Decancq and Lugo (2012, 2013) and Seth (2013), we assume that the well-being measure is homothetic, which means that the social evaluation is not affected by scale transformations of income.

$$(2.1) \quad W_{hk} = \left[ \sum_{i=0}^S \frac{\gamma_{ihk}}{\gamma_{+hk}} \left( \frac{x_{ihk}}{\gamma_{ihk}} \right)^{1-\tau} \right]^{\frac{1}{1-\tau}}, \quad h = 1, 2, \dots, H_k, \quad k = 1, 2, \dots, K,$$

where  $W_{hk}$  is the well-being obtained by household  $h$  living in country  $k$ , and  $\gamma_{+hk} = \sum_{i=0}^S \gamma_{ihk}$ ,

$\gamma_{++k} = \sum_{i=0}^S \sum_{h=1}^{H_k} \gamma_{ihk}$ . The well-being index is defined as a weighted mean of order  $1 - \tau$  of the

normalised goods ( $x_{ihk} / \gamma_{ihk}$ ). The shape of the iso-well-being curves is determined by the commodity-specific weights ( $\gamma_{ihk} / \gamma_{+hk}$ ) and by the degree of complementarity between goods ( $\tau$ ). The case with  $\tau = 0$  corresponds to perfect substitutes, whereas  $\tau \rightarrow \infty$  corresponds to perfect complements.

When the  $\gamma_{ihk}$ -parameters depend on the size and the demographic composition of households, the well-being measure defined by (2.1) belongs to the family that satisfies “demographic scaling” (Pollak and Wales, 1981), which was first proposed by Barten (1964).<sup>4</sup> Since the need parameter  $\gamma_{ihk}$  is included in the welfare function as a scaling parameter, it can be interpreted as the number of “equivalent adults” associated with good  $i$ . The rescaling of goods by needs parameters means that the normalised goods are all measured in units of need. Moreover, the commodity-specific weights are chosen to account for different relative needs for different goods of different households.<sup>5</sup>

Next, social welfare is aggregated across households within a country by employing the following additive separable function:

$$(2.2) \quad W_k = \frac{1}{\gamma_{++k}} \sum_{h=1}^{H_k} \gamma_{+hk} V(W_{hk}),$$

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<sup>4</sup> This means that the impact of family size and composition on well-being is treated as analogous to a price distortion. Chiappori (2016) provides a theoretical justification of commodity-specific scaling of the Barten type.

<sup>5</sup> Hence, needs parameters provide sufficient information to identify both the commodity-specific weights and the normalising transformation. Note, however, that by allowing the needs parameters to differ across households, the specification (2.1) is more flexible than conventional well-being indices which ignore heterogeneity in needs.

where  $V(W_{hk})$  is the contribution to total welfare from the well-being of household  $h$  in country  $k$ . Note that the assumption of a common welfare function  $V$  for all households justifies comparisons of the welfare levels  $V(W_{hk})$  across households in country  $k$ . The welfare weights of the social welfare function are assumed to be proportional to the total needs of each household ( $\gamma_{+hk}$ ).<sup>6</sup> This means that households with equal needs are treated symmetrically and moreover that a higher welfare weight is assigned to a household with higher needs than to a household with lower needs.

The common welfare function  $V$  is assumed to increase in household well-being, which implies that social welfare increases if a household gets a partial increase in the consumption of any market good or any in-kind transfer. For households with identical needs parameters, satisfaction of Pigou-Dalton's principle of transfers requires that  $V$  is concave. Moreover, as demonstrated by Ebert (1995, 1997, 1999) and Ebert and Moyes (2003) the principle of transfers for households with identical needs parameters can be generalised to the case with heterogeneous households by invoking a principle of between type transfers (BTT). When households differ in needs-related characteristics, a transfer of income from a richer to a poorer household will not necessarily imply a reduction in well-being inequality (see Glewwe, 1991). This paradox, which arises because the interaction between needs and income might offset the effect of differences in incomes on differences in well-being, was used by Ebert (1995) as a justification for introducing a transfer principle defined in terms of needs-adjusted incomes. In the case with one composite good, Ebert (1997) proved that the welfare function defined by (2.1) and (2.2) for increasing concave  $V$  satisfies the BTT principle, which means that the BTT principle provides a normative justification for employing needs dependent welfare weights in analyses of the distribution of income for individuals living in heterogeneous households.

As a reference point for evaluating distributions of extended income, the ethical observer is assumed to rely on the optimal allocation of household consumption according to (2.1).<sup>7</sup> Conditions of allocative efficiency allow us to treat in-kind transfers as non-distorting and fungible with cash income. A concern with this assumption is that there are quantity constraints associated with public provision of in-kind transfers, which might lead to efficiency losses in intra-household allocations.

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<sup>6</sup> If government decision rules are affected by other factors besides pure welfare maximisation, the welfare weights and equivalence scales that are implicit in government targeting reflect the combined impact of needs assessment and those other factors on the allocations (Olken, 2005).

<sup>7</sup> It is not necessary to assume that households are consuming optimal vectors of goods, only that the ethical observer is using optimal vectors as a benchmark for comparing the welfare of different households. Alternatively, when  $\tau = 0$ , in-kind transfers and cash incomes are treated as perfect substitutes by the ethical observer.

However, the effect of this source of inefficiency may diminish if households have access to private substitutes for public in-kind transfers. Indeed, the evidence presented in Cunha (2014), Fraker et al. (1995), Hoynes and Schanzenbach (2009), Moffitt (1989) and Slesnick (1996) suggests relatively small consumption distortions from in-kind transfers.

As indicated above we will obtain a measure of well-being by maximising  $W_{hk}$  defined by (2.1) with respect to consumption of market goods and public in-kind transfers, and subject to the budget constraint

$$(2.3) \quad C_{hk} \equiv x_{+hk} = \sum_{i=0}^S x_{ihk} ,$$

which yields the first order conditions

$$(2.4) \quad \frac{x_{ihk}}{\gamma_{ihk}} = \frac{x_{+hk}}{\gamma_{+hk}} = \frac{C_{hk}}{\gamma_{+hk}}, \quad i = 0, 1, \dots, S.$$

When (2.4) is satisfied for all households, then  $W_{hk} = C_{hk}/\gamma_{+hk}$  for all  $h$ , which means that  $C_{hk}/\gamma_{+hk}$  can be considered as an inter-household comparable measure of material living standard. A scale transformation of  $C_{hk}/\gamma_{+hk}$  will be called equivalent income (or needs-adjusted income) below. If any two households obtain equal needs-adjusted incomes it follows that they also obtain equal well-being.

To extend the single good BTT principle to the case of multiple goods it is convenient to introduce a definition of *needs-adjusted progressive transfers*.

**Definition I.** Let  $E(C, \gamma)$  denote the equivalent income of a household with extended income  $C$  and needs parameter  $\gamma$ . Then the distribution  $(\tilde{C}_{1k}, \tilde{C}_{2k}, \dots, \tilde{C}_{H_kk})$  is said to be obtained from the distribution  $(C_{1k}, C_{2k}, \dots, C_{H_kk})$  by a *needs-adjusted progressive transfer*  $\delta > 0$  if for households  $g$  and  $j$  in country  $k$  the following conditions are satisfied

$$\tilde{C}_{gk} = C_{gk} + \delta \text{ and } \tilde{C}_{jk} = C_{jk} - \delta,$$

$\tilde{C}_{hk} = C_{hk}$ , for all  $h \neq g, j$ , and

$$E(C_{gk}, \gamma_{+gk}) < E(\tilde{C}_{gk}, \gamma_{+gk}) \leq E(\tilde{C}_{jk}, \gamma_{+jk}) < E(C_{jk}, \gamma_{+jk}).$$

Definition I, which can be considered as an extension of the definition of progressive transfers to the case with heterogeneous households, forms a useful basis for a multiple good extension of the BTT principle.

Definition II. (*The principle of between type transfers of multiple goods (BTT)*). Let

$(\mathbf{x}_{I_k}(C_{I_k}, \gamma_{+I_k}), \dots, \mathbf{x}_{H_k k}(C_{H_k k}, \gamma_{+H_k k}))$  be the vector of goods that maximises the welfare of household  $h$  in country  $k$  for given extended income  $C_{hk}$ ,  $h = 1, 2, \dots, H_k$ , when welfare is evaluated according to the social welfare function  $W_k(\mathbf{x}_{I_k}, \mathbf{x}_{2k}, \dots, \mathbf{x}_{H_k k})$ . The welfare function  $W_k$  is said to satisfy the principle of between type transfers if

$$W_k(\mathbf{x}_{I_k}(\tilde{C}_{I_k}, \gamma_{+I_k}), \dots, \mathbf{x}_{H_k k}(\tilde{C}_{H_k k}, \gamma_{+H_k k})) > W_k(\mathbf{x}_{I_k}(C_{I_k}, \gamma_{+I_k}), \dots, \mathbf{x}_{H_k k}(C_{H_k k}, \gamma_{+H_k k}))$$

when  $(\tilde{C}_{1k}, \tilde{C}_{2k}, \dots, \tilde{C}_{H_k k})$  is obtained from  $(C_{1k}, C_{2k}, \dots, C_{H_k k})$  by means of a sequence of needs-adjusted progressive transfers.

The following proposition shows that the principle of BTT imposes the condition of concave  $V$  on the social welfare functions defined by (2.1) and (2.2).

Proposition I. The social welfare functions defined by (2.1) and (2.2) satisfy the principle of BTT for strictly increasing concave  $V$ .

The proof is given in Appendix I.

## 2.2. Needs-adjusted equivalence scales

Inserting (2.4) in (2.1) yields

$$(2.5) \quad W_{hk}^*(C_{hk}) = \frac{C_{hk}}{\gamma_{+hk}}, \quad h = 1, 2, \dots, H_k, k = 1, 2, \dots, K,$$

where  $W_{hk}^*$  is the maximum welfare that can be obtained for household  $h$  for given extended income  $C_{hk}$ . Furthermore, the solution of the dual problem of minimising the cost to obtain the welfare level  $W_{hk}$  for household  $h$  is given by the cost function

$$(2.6) \quad C_{hk}^*(W_{hk}) = W_{hk} \gamma_{+hk}, \quad h = 1, 2, \dots, H_k, k = 1, 2, \dots, K.$$

By employing the cost functions (2.6) we get the following family of relative equivalence scales:

$$(2.7) \quad NA_{hk} = \frac{C_{hk}^*(W_{rk})}{C_{rk}^*(W_{rk})} = \frac{\gamma_{+hk}}{\gamma_{+rk}}, \quad h = 1, 2, \dots, H,$$

where  $NA_{hk}$  is the scale factor for household  $h$  and  $C_{rk}^*(\cdot)$  is the cost function of the reference household  $r$  in country  $k$ . Equivalent income is defined by  $E_{hk} = C_{hk} / NA_{hk}$ , which represents a money measure of material living standard, and is interpreted as the minimum cost required for the reference household to attain the same welfare level as household  $h$  enjoys from extended income  $C_{hk}$ .

The equivalence scale  $NA_{hk}$  is called a needs-adjusted scale since it can be considered as an ordinary scale for cash income adjusted for the needs of services provided by local and central governments, which is demonstrated by the following decomposition of (2.7),

$$(2.8) \quad NA_{hk} = \theta_{rk} CI_{hk} + (1 - \theta_{rk}) NC_{hk},$$

where  $CI_{hk} = \gamma_{0hk} / \gamma_{0rk}$  is the equivalence scale for cash income,  $NC_{hk} = (\gamma_{+hk} - \gamma_{0hk}) / (\gamma_{+rk} - \gamma_{0rk})$  is the non-cash scale for the value of public service provision, and  $\theta_{rk} = \gamma_{0rk} / \gamma_{+rk}$  is the weight assigned to cash income in the composite  $NA$  scale for extended income. The weight  $\theta_{rk}$  is equal to the share attributed to cash income in the needs for extended income of the reference household  $r$ .

An equivalence scale is said to be exact if it does not depend on the income level. The commonly used scales are exact. Under the assumption that needs parameters depend solely on household size and composition, the equivalence scale defined by (2.7) satisfies *relative equivalence scale exactness*

(Lewbel 1989; Blackorby and Donaldson 1993).<sup>8</sup> This is due to income-ratio comparability of the welfare function (2.5), which requires that equality of well-being is preserved under common rescaling of the household's income (Blackorby and Donaldson 1993).

Ebert and Moyes (2003) provide a normative justification for using relative equivalence scales when the inequality concept is relative. They employ an axiomatic approach to justify the use of income-independent relative equivalence scales. By invoking the Between Type Transfer (BTT) principle and the conditions of scale invariance, income monotonicity, type monotonicity and path independence, the equivalent income function is shown to satisfy relative equivalence scale exactness. Importantly, Ebert (2010) also uses an axiomatic approach to demonstrate that relative poverty measures require use of relative equivalence scales.

### **2.3. A common European needs-adjusted equivalence scale**

The standard approach in empirical analyses based on income after tax (cash income) is to use one common scale for all countries in comparative cross-national research on income distribution and poverty. However, to the extent that our equivalence scale estimates for public services differ across countries, the associated composite equivalence scale for extended income will also differ across countries. By contrast, if we rely on the standard assumption that the relative needs of different household types are the same in all countries, it is required to use a common equivalence scale.

As is standard for equivalence scales of cash income, we impose the conditions of unit consistency and reference independence to derive a common scale for extended income. *Unit consistency* means that the equivalence scale is invariant with respect to changes in measurement unit or currency. This condition implies that equivalence scale factors as well as measures of inequality and poverty are independent of the choice of measurement unit for a given country. *Reference independence* means that measures of (relative) inequality and poverty are independent of choice of reference household in the definition of the equivalence scale.

It follows from expression (2.7) that the NA scale for a given country satisfies unit consistency and reference independence. A proportional change in all needs parameters cancels out in (2.7), whilst a change of reference household will merely lead to a scale transformation of the country-specific NA scale. Accordingly, measures of (relative) inequality and poverty are independent of choices of

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<sup>8</sup> This property is termed *independence of base utility* by Blundell and Lewbel (1991).

measurement unit and reference household for a given country. However, these properties do not necessarily carry over to any common equivalence scale derived from the country-specific scales.

The weighted average of the country-specific equivalence scale rates emerges as a relevant candidate of a common scale. The construction of this scale requires assessment of needs-adjusted scales for each of the European countries in question. Next, the country-specific needs-adjusted scales are assigned to all households in the total population formed by all countries,<sup>9</sup> which requires evaluation of the needs of household  $h$  as measured by the needs parameters associated with each of the countries. To this end it will be convenient to introduce an alternative notation to the one used in Sections 2.1 and 2.2. Whilst  $NA_{hk}$  in Sections 2.1 and 2.2 denotes the scale factor for a household ( $h$ ) living in country  $k$ ,  $NA_h^k$  denotes the scale factor for a household ( $h$ ) living in any of the countries in question when its needs are judged according to the needs parameters of country  $k$ . Thus, in the former case  $h=1, 2, \dots, H_k$ , whereas in the latter case  $h=1, 2, \dots, H$ , and  $H = \sum_{k=1}^K H_k$  is the total number of households living in the  $K$  countries. Moreover  $\gamma_{ih}^k$  is the needs associated with income component  $i$  for household  $h$  when needs are judged according to the needs parameters of country  $k$ ,

$\gamma_{+h}^k = \sum_{i=0}^S \gamma_{ih}^k$  is the total need of household  $h$  according to the needs parameters of country  $k$ , where  $h=1, 2, \dots, H$ . Thus, to assess the scale factors of the common equivalence scale it is required to calculate the equivalence scale factors according to each of the  $K$  different national service standards for all households in the  $K$  countries. The common NA scale for European countries is thus defined by a weighted average of the  $K$  country-specific NA scales for every household living in these countries,

$$(2.9) \quad NA_h = \sum_{k=1}^K q_k NA_h^k = \sum_{k=1}^K q_k \frac{\gamma_{+h}^k}{\gamma_{+r}^k}, \quad h = 1, 2, \dots, H,$$

where  $q_k$  is the weight assigned to the equivalence scale for country  $k$ . In general, the country-specific weights may depend on the needs parameters of all household types and on the choice of reference household  $r$ ,

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<sup>9</sup> Household types are defined by household size and different compositions of members from different target groups. Since some of the household types do not exist in all countries, it is convenient to simulate scale rates for any household according to the needs parameters of different countries, irrespective of where the household lives. This method implies that households of equal type are given equal scale rates.



$$(2.10) \quad q_k = q_k(\gamma^1, \gamma^2, \dots, \gamma^K; r),$$

where  $\gamma^k = (\gamma_{+1}^k, \gamma_{+2}^k, \dots, \gamma_{+H}^k)$  is the vector of total needs for different households derived from the spending profile in country  $k$ . Each country-specific weight is assumed to be the same for all household types. Next, we impose the following conditions on the country-specific weights:

**Condition I:** (*Unit consistency*). The country-specific weights are invariant with respect to scale transformations of the needs parameters in any given country, i.e. for  $\lambda_k > 0, k = 1, 2, \dots, K$ ,

$$q_k(\lambda_1 \gamma^1, \lambda_2 \gamma^2, \dots, \lambda_K \gamma^K; r) = q_k(\gamma^1, \gamma^2, \dots, \gamma^K; r), \quad k = 1, 2, \dots, K.$$

Condition I requires that the country-specific weights are not affected by a change of currency or measurement unit for any country.

**Condition II:** (*Reference independence*). Change of reference household implies that the common equivalence scale will change by a constant scale parameter, i.e. for  $m \neq r$  there exists a constant parameter  $\xi_{rm}$  such that

$$\sum_{k=1}^K q_k(\gamma^1, \gamma^2, \dots, \gamma^K; m) \frac{\gamma_{+h}^k}{\gamma_{+m}^k} = \xi_{rm} \sum_{k=1}^K q_k(\gamma^1, \gamma^2, \dots, \gamma^K; r) \frac{\gamma_{+h}^k}{\gamma_{+r}^k}.$$

Condition II assures that measurement of relative inequality and poverty in the distribution of equivalent extended income will not be affected by a change in the choice of reference group for the common NA scale.

**Proposition II.** Let  $\gamma_{++}^k = \sum_{h=1}^H \gamma_{+h}^k$ , and let  $w_k, k = 1, 2, \dots, K$  be country-specific weights that are constant and independent of the household-specific needs parameters and the reference household. Then the following weight functions associated with the NA scale

$$(2.11) \quad q_k(\gamma^1, \gamma^2, \dots, \gamma^K; r) = \frac{w_k \frac{\gamma_{+r}^k}{\gamma_{++}^k}}{\sum_{k=1}^K w_k \frac{\gamma_{+r}^k}{\gamma_{++}^k}}, \quad k = 1, 2, \dots, K,$$

satisfy conditions I and II.

The proof is given in Appendix I.

Propositions I and II show that the weighted average of the national NA scales with weights defined by (2.11) satisfies the Between Type Transfer principle as well as the conditions of reference independence and unit consistency and thus emerges as an appropriate common equivalence scale for comparison of inequality and poverty.

Choosing  $w_l > 0$  and  $w_k = 0$  for all  $k \neq l$  means that country  $l$  is treated as a reference country, i.e. the NA scale derived for country  $l$  is applied as a common scale for a group of countries.

Alternatively, we may assign equal weights to all countries or use weights that are proportional to population size. The latter method forms the basis of the empirical analysis presented in Section 3.

## 2.4. Assessment of the equivalence scale

To quantify the needs parameters for European countries we use mean public spending targeted to different population subgroups defined by age and gender.<sup>10</sup> Mean spending per person received by different target groups, such as children and the elderly is used as indicators of the population groups' needs for childcare, education, health care and long-term care. The mean in-kind transfers received by different target groups are assumed to reflect the relative needs of the target groups. Since the needs parameters for public services are connected to individuals, household specific needs parameters are obtained by aggregating the needs parameters of the individuals in each household.

For cash income we use the median of the distribution of equivalent income in a given country as a basis for determining the needs parameter for the reference group. We use the EU scale to account for

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<sup>10</sup> Aaberge et al. (2010, 2019) use detailed accounting data from Norwegian municipalities as a basis for estimating the NA scale for local public services. With such detailed data, they exploit minimum quantity parameters as measures of the local governments' assessment of the need of different services for different population subgroups. Detailed municipal accounting data are however not available for all European countries.

differences in needs of cash income for households who differ in size and composition.<sup>11</sup> Thus, the needs parameter of cash income for individuals of the reference household in country  $k$  is defined by

$$(2.12) \quad \gamma_{0r}^k = \text{median}(\mathbf{x}_{0k}^{EU}),$$

where  $\mathbf{x}_{0k}^{EU}$  is the vector of equivalent cash incomes in country  $k$ . Note that the vector  $\mathbf{x}_{0k}^{EU}$  includes one component for every individual in country  $k$ . Thus,  $\text{median}(\mathbf{x}_{0k}^{EU})$  is the median equivalent cash income in country  $k$ .<sup>12</sup> For households that are not of the reference type we use the chosen EU scale to assess the need for cash income in the following way:

$$(2.13) \quad \gamma_{0h}^k = \gamma_{0r}^k EU_h,$$

where  $EU_h$  is the scale factor for cash income pertaining to household  $h$ .

### 3. Empirical implementation

This section discusses methodological issues related to measurement of publicly financed in-kind transfers, where Sections 3.2 and 3.3 give an account of data and methods for valuation and allocation of public services. Section 3.4 explains the assessment of the common equivalence scale and defines three alternative measures of income. Inequality measures and poverty thresholds are defined in Section 3.5. Appendix B contains more details on the data and methods used in this study.

#### 3.1. Population

The study relies on the EU-SILC cross-sectional data for five different reporting years, which cover the income years 2006, 2009, 2012, 2015 and 2018 and 27 EU member states as well as Iceland, Norway, and Switzerland. Six EU-SILC countries were omitted from the study due to lack of satisfactory data on public services. Table B.1 in Online Appendix B reports the population composition in European countries by household type.

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<sup>11</sup> The EU scale is designed to be particularly relevant for European countries, but the non-cash (NC) scale can be combined with any alternative exact relative equivalence scale for cash income.

<sup>12</sup> In this study the reference household type is defined by a single childless male aged 35 – 44 years.

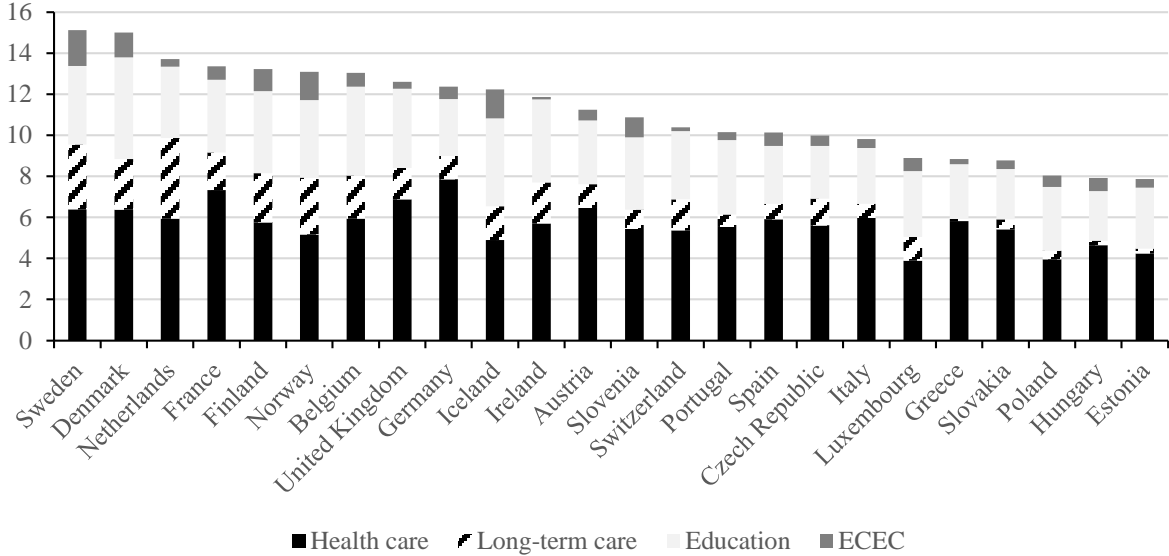
The population in our study is classified by target groups defined by age and gender. Adults aged 18 years and above are classified by seven different age groups, where the elderly groups consist of ages 65–74 and 75 years and above. There are four age groups for school-age children, which we use to account for the allocation of government expenditures to different levels of education (primary, lower secondary and upper secondary level). Moreover, the participation rate in ECEC (Early Childhood Education and Care) varies by age. Children in pre-education age are divided into three target groups: 0 years, 1–2 years and 3 years to primary education age. Since the age intervals for attending different education levels vary between countries, the age group classification accounts for country-specific differences in the structure of the education system. Table B.2 in Appendix B shows the 14 age groups used in this study.

### **3.2. The value of public services**

In studies of extended income, the value of public services is normally assumed to be equal to the cost of providing them (see e.g. Smeeding et al. 1993). Yet, the cost approach does neither account for differences in quality and efficiency in the production of public services nor for possible welfare losses due to quantity constraints in the consumption of public services. Nevertheless, in line with previous studies, this study employs the cost approach as a benchmark for accounting for the distributional impact of public in-kind transfers. However, in Section 4 we perform a sensitivity analysis that relies on available estimates of public sector Technical Efficiency (TE) in developed countries.

Data on public expenditures by functions of government are made available by Eurostat, see Appendix B for details. The data are net public expenditure, which means that households' out-of-pocket payments and other financial sources than government sources are subtracted.

**Figure 1: Public expenditure on welfare services as a proportion of GDP, 2012. Percent**



*Notes.* The figure displays public expenditure on ECEC, primary and secondary education, long-term care and health care, in percent of GDP, 2012. Public expenditure excludes out-of-pocket payments. Countries are sorted by descending public expenditure as a proportion of GDP. Source: Eurostat.

Figure 1 shows public expenditure on four welfare services by country in 2012. Expenditures are normalised by the country-specific gross domestic product (GDP). Netherlands, France, Belgium, UK, Germany and the Nordic countries allocate a relatively large proportion of GDP to public service provision. Countries in Southern and Eastern Europe employ a smaller fraction of GDP on public welfare services. Nordic countries display high spending on ECEC and long-term care. The Netherlands spend a high fraction on long-term care, whilst Germany, France and UK spend a high fraction of GDP on public health services. The cross-country variation in GDP-shares and composition of public spending displayed by Figure 1 demonstrate the importance of accounting for in-kind transfers when comparing distributions of economic well-being across European countries.

**3.3. Allocation of public services**

Government authorities are assumed to target public services to specific demographic groups based on an evaluation of relative needs for public services. Education services are provided to children because of needs for developing skills, whereas most of health care and long-term care spending is targeted to the elderly because they are more exposed to illness and disablement. Since the government decides both the selection of recipients and the type and intensity of treatment, our study accounts for the targeting policies of different governments.

### ***Education and childcare services – the actual consumption approach***

Two methods are used to calculate the value of public services received by individuals. Either the value is based on (i) actual consumption or (ii) expected consumption of the service. The former method is applied for the value of education and ECEC services. By dividing total expenditure by the total number of pupils per education level (primary, lower secondary, and upper secondary), we get estimates per pupil for three education levels. For ECEC services, we also exploit information about utilisation of such services per child in the EU-SILC data. We calculate the cost per hour of ECEC services and allocate an annual value depending on average utilisation per week by age and country.

### ***Expected spending on health and long-term care services – the insurance approach***

Health and long-term care services will be treated as insurance arrangements, i.e. the values of these services are assessed on an ex-ante basis, which means that members of the target groups benefit from the expected expenditure rather than the actual expenditure on recipients. This approach has previously been applied by e.g. Smeeding et al. (1993) and Garfinkel et al. (2006). The insurance value approach is used when the type of service is considered as insurance against poor outcomes, such as poor health. Moreover, the alternative to public service provision would be to buy private insurance in the market. The expected expenditure on health and long-term care is assumed to depend on demographic characteristics such as age and gender. Since there are different age profiles of utilisation, the allocation procedure is carried out separately for health services and long-term care.

## **3.4. Measurement of equivalence scale and material well-being**

Measurement of cash income is consistent with EU-SILC measurement of disposable income and is defined to be equal to gross income subtracted social insurance contributions and taxes on income and wealth. Gross income includes employee income, self-employment income and property income, which is constituted by interests, dividends and rents. Moreover, both public cash transfers and *net* inter-household transfers are treated as components of gross income.<sup>13</sup>

Extended income is defined by the sum of disposable income and in-kind transfers (childcare, education, health care and long-term care). To account for different needs for disposable income, we divide disposable income by the EU equivalence scale. In studies that include in-kind transfers, the common practice is to apply the same scale for extended income as is applied for cash income. As indicated in Section 2, needs for public services might differ from needs for cash income. The needs-

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<sup>13</sup> Net inter-household transfers constitute the difference between such transfers received and paid. Our measure of gross income differs slightly from the definition used in EU-SILC by including gross inter-household transfers.

adjusted (NA) equivalence scale used in this study accounts for differences in needs for public services and needs for cash income. Dividing extended income by the NA scale provides an income definition that allows income comparisons between people who differ with respect to needs of public services like childcare, education, health care and long-term care. Table 1 displays four alternative definitions of equivalent income that are considered in this study.

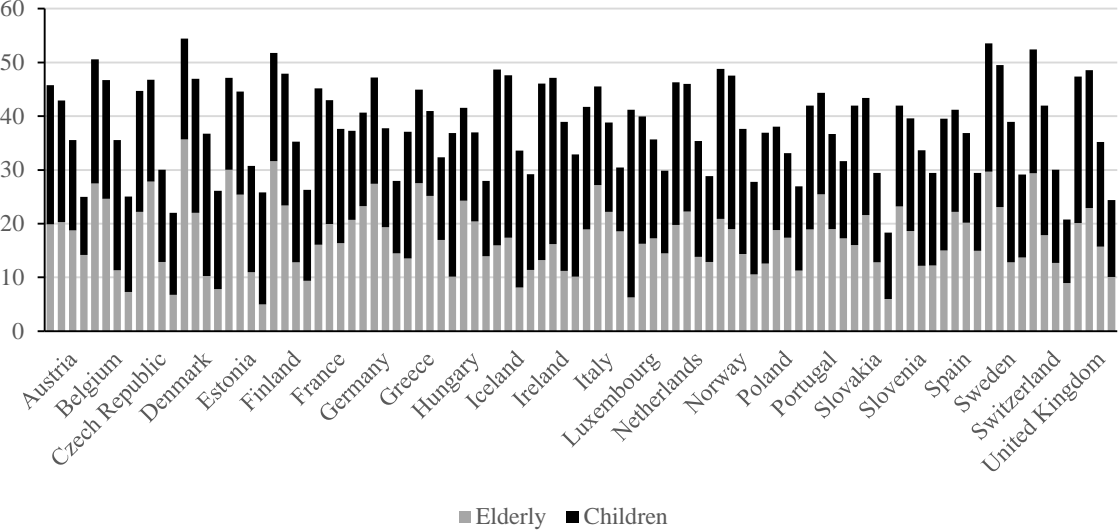
**Table 1: Definitions of equivalent income**

Income components	Equivalence scale	Equivalent income definition
Gross income – tax	EU scale	Disposable income (EU)
Disposable income + public in-kind transfers	EU scale	Extended income (EU)
Disposable income + public in-kind transfers	NA scale	Extended income (NA)
Disposable income + public in-kind transfers	SNA scale	Extended income (SNA)

In Appendix C, we develop a simplified version of the NA scale, denoted the SNA scale. Since the SNA scale is found to be highly correlated with the NA scale, it will for practical purposes be convenient to replace the NA scale by SNA. Note that the assessment of the SNA scale can be based on data for household size and composition by age groups, which makes it straightforward to apply for analyses of the distribution of extended income in any country.

We rely on the conventional practice for income distribution analyses by assuming that the household’s cash income is distributed equally among household members. This assumption applies also to extended income when the in-kind transfers are included in the income definition, although this does not mean that each public in-kind transfer is consumed in equal amounts by all household members.

**Figure 2: Proportion of elderly and children by quartiles in the distribution of disposable income, 2012. Percent**



*Notes.* The figure displays the proportion of children (aged under 18) and elderly (aged above 64) by quartiles in the distribution of disposable income in 2012. From left to right by country: quartile group 1, 2, 3, 4. Countries are ranked by alphabetical order. Source: Eurostat.

Table A.1 in Appendix A presents country-specific relative distributions of extended income by income components in 2012. Although cash income is the dominating income component for all countries, its share of extended income differs significantly across countries.

Most of the in-kind transfers are targeted to elderly people (long-term care and health care) and families with children (ECEC and education). Therefore, the effects of in-kind transfers on income inequality will depend on the association between household disposable income and the age of household members. To illustrate this relationship, Figure 2 displays proportions of children and elderly by quartiles of the distribution of household income for each country. In most European countries, we find that the proportion of children and elderly decreases with household income. For some countries, including Germany, the proportion of elderly is however larger in the second than the first quartile. The tendency of decreasing proportions of elderly and children with household income suggests that inequality in the distribution of extended income is lower than inequality in the distribution of disposable income.

**3.5. Measuring inequality and poverty**

This section discusses and presents the methods for measuring relative inequality and poverty for country-specific distributions of economic well-being. The study of country-specific distributions can be traced back to Adam Smith who argued in *The Wealth of Nations* that poverty is the inability to



afford “whatever the custom of the country renders it indecent for creditable people to be without”. We refer to Brandolini (2007) and Brandolini and Rosolia (2019) for a discussion of the conceptual and empirical challenges in analysing income distribution at the supranational level as in the case of EU.

### ***Inequality***

Empirical analyses of income inequality are normally based on the Lorenz curve. To summarise the information content of the Lorenz curve and to achieve rankings of intersecting Lorenz curves the standard approach is to employ the Gini coefficient, which is equal to twice the area between the Lorenz curve and its equality reference. Appendix H provides results based on two complementary rank-dependent measures of inequality; one that is particularly sensitive to changes that occur in the lower part of the income distribution and the other that pays more attention to changes that take place in the upper tail of the income distribution.

### ***Poverty thresholds***

In most studies of poverty in developed countries, poverty is usually understood as a relative phenomenon. This perspective suggests that people compare their material situation with that of other citizens and adjust their expectations and demands for material well-being relative to the living standards of people in the same society. This study follows such reasoning and applies a relative poverty threshold to measure whether people are at-risk-of poverty or not. According to the EU method, 60 percent of the median equivalent income defines the poverty line, see Atkinson et al. (2002). Each country has its own poverty line for each concept of income.

## **4. Empirical results**

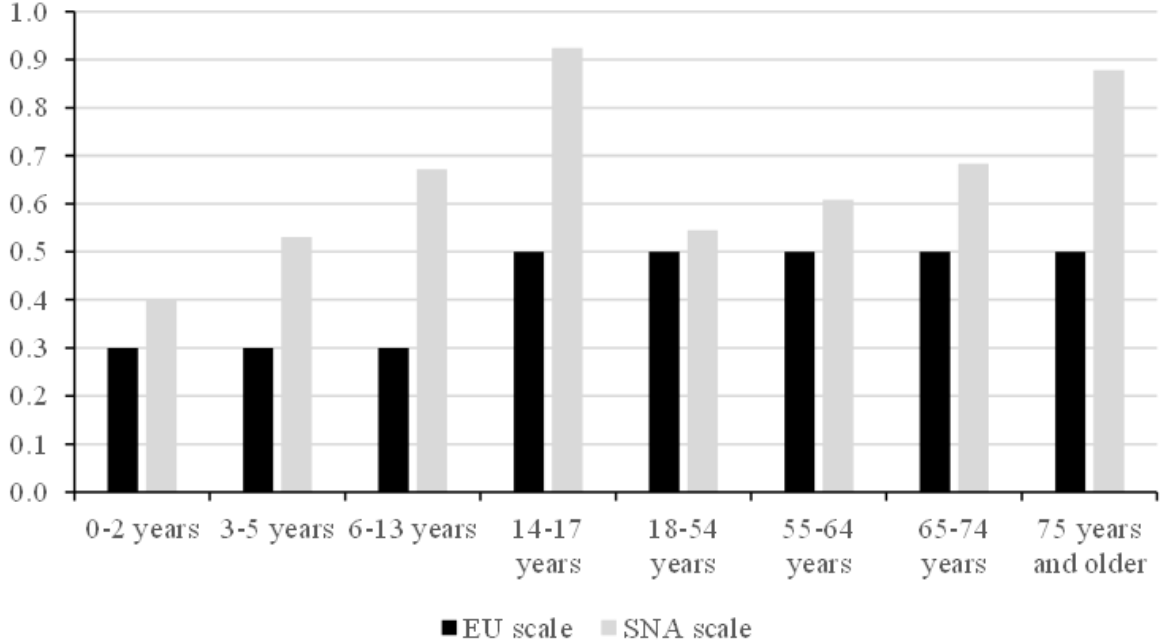
This section examines the impact on inequality and poverty estimates of accounting for publicly financed in-kind transfers, while adjusting for differences in needs for such services across individuals and households.

### ***NA and SNA scale***

Table C.1 in Appendix C displays the EU, NC and NA scales by household types. While the EU scale accounts for economies of scale in composite consumption of market goods and services and gives different weights to children and adults in the household, the NC scale accounts for the needs of public welfare services for the household members. The NA scale combines the NC scale for public in-kind transfers with the EU scale for disposable income, while the SNA scale represents a convenient simplified version of the NA scale.

Appendix C reports the scale factors of the SNA scale, which solely depends on the number of household members in different age groups. Thus, the SNA scale can be computed for any micro-dataset that includes information on household size and composition. The SNA scale is shown to be highly correlated with the NA scale and will therefore work as an appropriate approximation of the NA scale. Moreover, the scale factors of the SNA scale prove to be stable over the period 2006–2012. Figure 3 shows estimates for the SNA scale based on data for 2012. The SNA-scale accommodates needs for childcare, primary and secondary education, health care and long-term care in addition to private consumption paid for out of cash income.

**Figure 3: EU and SNA scales for eight age groups**



*Notes.* The figure displays additive weights that are assigned to individuals in different age groups. The scale factor is equal 1 for a single adult in the reference age group, which is 14 years and above for the EU scale and 18–54 years for the SNA-scale. The bars show the increase in the EU scale and the SNA scale by age group for each extra person that is added to the household. The EU scale assigns weight 1 to the household head, 0.5 to each member aged 14 and above and 0.3 to each member aged below 14. The Simplified Needs-Adjusted (SNA) scale assigns additive weights that differ by eight age groups. In our computations, the school-age groups are adjusted to account for different rules of progression by country. Source: Eurostat, authors’ calculations.

A common practice is to use the EU scale to convert household incomes for broader as well as narrower measures of income. By contrast, the present study applies a different equivalence scale when we consider inequality and poverty for extended income. To obtain information on the effect of replacing the EU scale with the SNA scale, we will also estimate inequality and poverty for extended

income based on the EU scale.<sup>14</sup> For a given household, the change from EU scale to SNA scale represents a re-scaling of extended income that is expressed by:

$$(4.1) \quad \frac{\text{Extended income}}{\text{SNA scale}} = \frac{\text{Extended income}}{\text{EU scale}} \times \frac{\text{EU scale}}{\text{SNA scale}},$$

where the factor of re-scaling equals the ratio of EU scale to SNA scale. Since the re-scaling factor differs across households, this transformation introduces re-ranking and relative income changes that depend on the interaction between the factor of re-scaling and households' relative positions in the distribution of extended income (EU). Intuitively, households with lower (higher) ratio of EU scale to SNA scale will tend to obtain a lower (higher) income rank when the EU scale is replaced by the SNA scale in the equivalent income definition.

**Table 2: Ratio of EU scale to SNA scale by household type**

<b>Household type</b>	<b>Age</b>	<b>Singles</b>	<b>Couples</b>
Childless	18–54	1.00	0.97
	55–64	0.94	0.89
	65–74	0.87	0.82
	75 +	0.75	0.68
1 child, adult(s) age 18–54	0–2	0.93	0.92
	3–school age	0.85	0.87
	School age (under 14)	0.78	0.81
	School age (over 13)	0.78	0.81
2 children, adult(s) age 18–54	0–2	0.89	0.89
	3–school age	0.77	0.80
	School age (under 14)	0.68	0.73
	School age (over 13)	0.70	0.74

*Notes.* The table reports the ratio of EU to SNA scale for a selection of household types. The reference household type for the SNA scale consists of single adults aged 18–54, whereas the EU scale includes all single adults in the reference household type. The age group 18–54 years includes only persons above secondary education age. For households with children, household types are constructed for the case where adults are in the reference age group (18–54). For households with 2 children, both are assumed to belong to the same age group. Source: Eurostat, authors' calculations.

Table 2 displays the ratio of the EU scale to the SNA scale for a selection of household types. For childless adults, we find that the ratio is decreasing with age, and that the relative reduction is larger for couples than for singles. For households with children, the ratio is decreasing with the number of children, and is reduced more for children in school ages compared to pre-school ages. Among the household types in Table 2, the smallest re-scaling factor is found for elderly couples aged 75 and

<sup>14</sup> Note that the SNA scale and the NA scale produce almost identical results.

above, and moreover for single parents with 2 children in school age. Therefore, such households may obtain relatively large reductions in income rank when the EU scale is replaced by the SNA scale. Conversely, non-elderly households without children may obtain higher income ranks because their needs are relatively low according to the SNA scale.

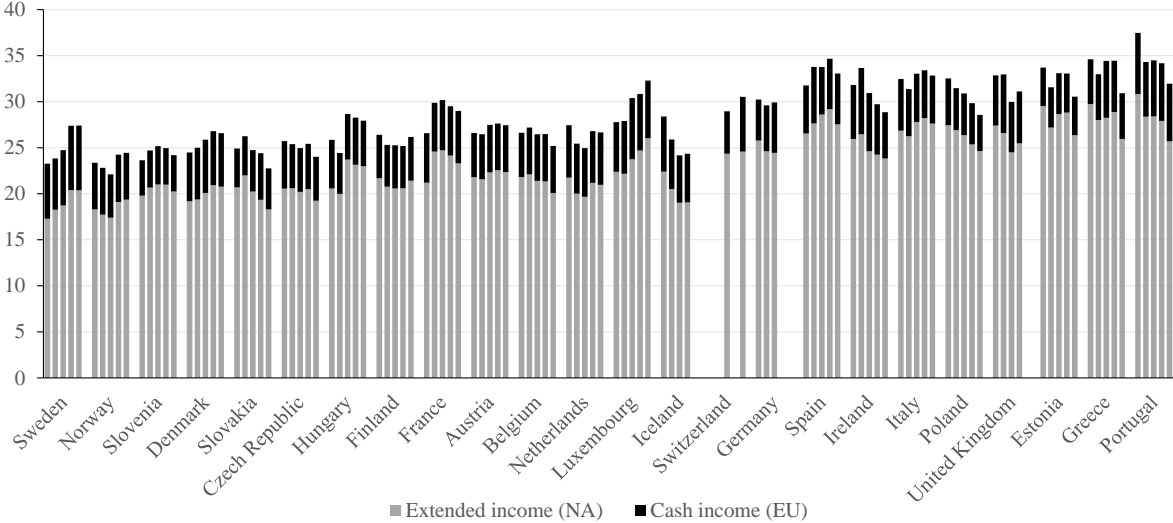
#### *Trends in inequality and poverty*

Figure 4 displays country-specific trends in the Gini-coefficient over the period 2006–2018 for both disposable income (EU) and extended income (NA). Grey bars show the inequality trends for extended income (NA), whereas the sum of grey and black bars show the inequality trends for disposable income (EU). Hence, we find that inclusion of spending on welfare services in the income definition reduces inequality estimates by 10–30 percent. The percentage reduction in inequality estimates tends to be larger among countries with smaller income inequality according to disposable income (EU).

We find that the levels as well as trends in income inequality varies significantly across European countries. Sweden, Denmark and Luxembourg have experienced rising inequality over the period 2006–2018. During the same period, inequality has trended downwards in Iceland, Ireland, Poland and Portugal. In other countries, the trends are unstable or rather flat. The inequality trends are mostly parallel when comparing inequality in disposable income (EU) versus extended income (NA).

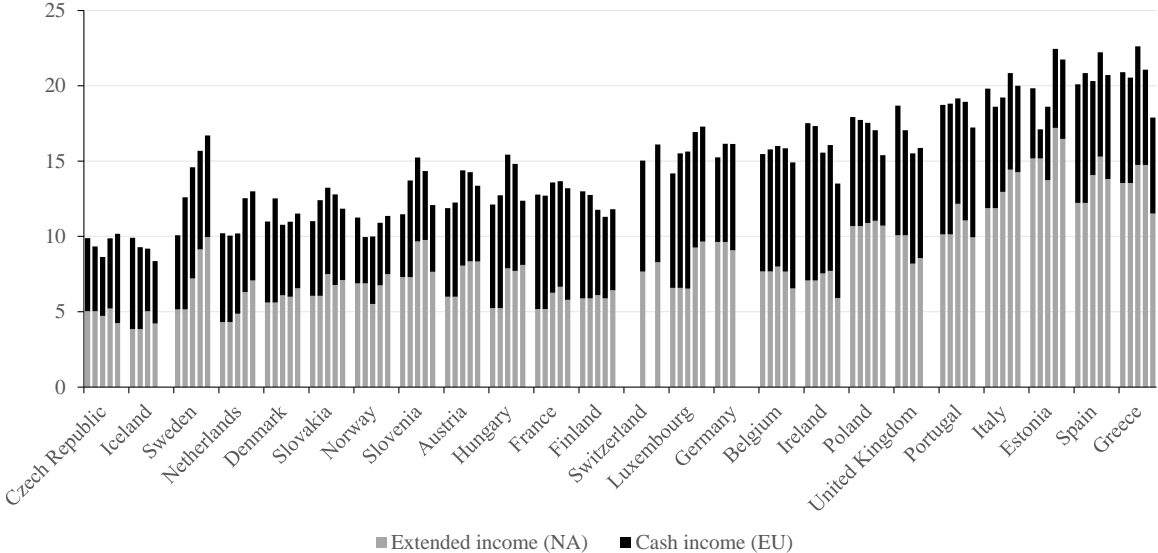
Figure 5 displays country-specific trends in poverty rates over the period 2006–2018 for disposable income (EU) and extended income (NA). Grey bars show the poverty trends for extended income (NA), whereas the sum of grey and black bars show the poverty trends for disposable income (EU). We find that inclusion of spending on welfare services in the income definition reduces poverty rate estimates by 30–60 percent. The percentage reduction in poverty estimates tends to be larger among countries with smaller poverty rates according to disposable income (EU), and moreover in Belgium and Ireland. We find that the levels as well as trends in poverty rates vary significantly across European countries. Sweden, Netherlands, Hungary and Luxembourg have experienced rising poverty rates over the period 2006–2018. In other countries, the poverty trend is ambiguous over time or when comparing poverty in different income definitions. For instance, Finland displays decreasing poverty in disposable income (EU) and increasing poverty in extended income (NA).

**Figure 4: Gini coefficients in European countries for disposable income (EU) and extended income (NA), 2006–2018. Percent**



*Notes.* The figure displays estimates of the Gini coefficient (in percent) by country, year and income definition. From left to right by country: 2006, 2009, 2012, 2015, 2018. Inequality estimates for extended income (NA) are represented by grey bars. The sum of grey and black bars represent inequality estimates for disposable income (EU). Germany is excluded in 2015 and 2018 due to limitations on data quality. UK and Iceland are missing in 2018 because of delayed reporting of EU-SILC data. Switzerland is excluded in 2006, 2009, and 2015 due to missing data on public expenditures. Source: Eurostat, authors’ calculations.

**Figure 5: Poverty rates in European countries for disposable income (EU) and extended income (NA), 2006–2018. Percent**



*Notes.* The figure displays estimates of poverty rates (in percent) by country, year and income definition. From left to right by country: 2006, 2009, 2012, 2015, 2018. Poverty estimates for extended income (NA) are represented by grey bars. The sum of grey and black bars represent poverty estimates for disposable income (EU). Germany is excluded in 2015 and 2018 due to limitations on data quality. UK and Iceland are missing in 2018 because of delayed reporting of EU-SILC data. Switzerland is excluded in 2006, 2009 and 2015 due to missing data on public expenditures. Source: Eurostat, authors’ calculations.

### *Inequality and poverty within groups in three different stages of the life cycle*

Empirical studies typically find that incomes follow a hump-shaped profile over the life cycle. Whilst earnings are relatively low when entering the labour market, yearly incomes tend to increase up to middle ages relatively to cross-cohort averages. Later, income growth starts to stagnate and fall behind that of younger cohorts. This relative loss continues after retirement, when elderly people rely on public and private pensions as their main sources of cash income. Besides changes in average incomes by age, intra-generational income inequality may also change over the life cycle. On the one hand, income inequality could grow with age, because of cumulative differences in the effects of luck and ability on income. On the other hand, redistribution of resources through the tax and transfer system of the welfare state might mitigate the association between age and the accumulation of income disparities.

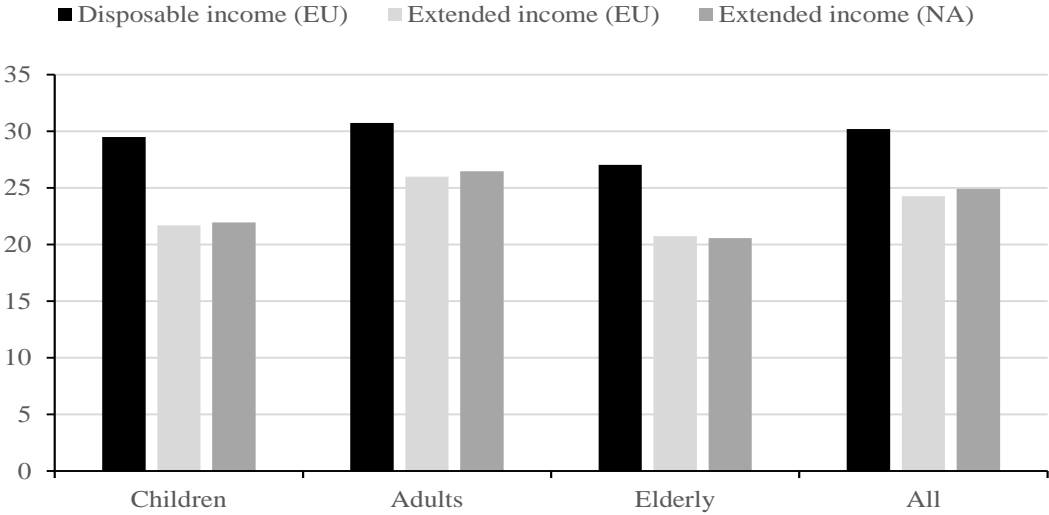
Since the cross-sectional EU-SILC datasets do not follow individuals over time, this paper focuses on distributions of yearly snapshots of income. As a result, we are unable to distinguish between (i) inequality between individuals that persist over the life cycle, and (ii) inequality that follows mechanically when comparing individuals who are in different life-cycle stages, even when their income profiles by age are equal.

To alleviate the concern that our results may confound individual income differences with income changes over the life cycle, we will consider income distributions within three main stages of the life cycle; (1) childhood (age 0–17), (2) adulthood (age 18–64) and (3) retirement (elderly age 65 and above). Hence, while members of a given household are assumed to share incomes and obtain an equal amount of equivalent income, they are assigned to different sub-populations depending on their age. This breakdown allows us to study income distributions within the major target groups receiving public in-kind transfers, such as children and the elderly. We contrast the distributions within those directly targeted groups with working-age adults who benefit more indirectly to the extent that their households include children or elderly people.

Figure 6 shows the average Gini-coefficient in European countries within three age groups and for the total population. Moreover, inequality estimates are compared for different definitions of equivalent income. Extended income (NA) displays lower inequality estimates within each age group than disposable income (EU). Since elderly and children are major recipients of public in-kind transfers, these groups obtain large reductions in inequality estimates when including in-kind transfers in the income definition. By contrast, the reduction in within-group inequality is relatively small for non-elderly adults. The latter group displays higher within-group inequality than the three other household

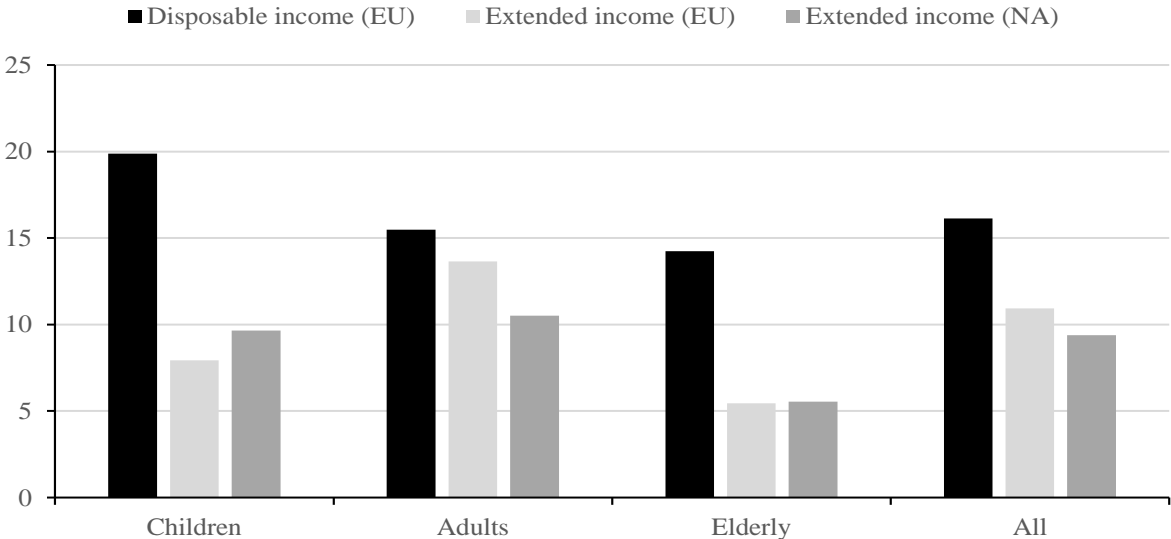
types. A reason for this is that the working-age group is relatively heterogeneous by including non-elderly singles and couples, and moreover childless adults as well as parents.

**Figure 6: Gini coefficient in population groups by life-cycle stages and income definition. Averages across European countries, 2012. Percent**



*Notes.* The figure displays estimates of the Gini coefficient (in percent) within groups of children (age 0–17), adults (age 18–64) and elderly (age 65 and above), and for the total population. Inequality estimates are reported for disposable income (EU), extended income (EU) and extended income (NA). Gini coefficients are first estimated for different European countries. The figure displays weighted average estimates across European countries, where country-specific estimates are weighted by total population size. Source: Eurostat, authors’ calculations.

**Figure 7: European poverty rates by population groups and income definition, 2012. Percent**



*Notes.* The figure displays estimates of the poverty rate (in percent) within groups of children (age 0–17), adults (age 18–64) and elderly (age 65 and above), and for the total population. Inequality estimates are reported for disposable income (EU), extended income (EU) and extended income (NA). Poverty rates are first estimated for different European countries. The figure displays weighted average estimates across European countries, where country-specific estimates are weighted by total population size. Source: Eurostat, authors’ calculations.

Notice that within-group inequality does not account for inequality between the different household types. As a result, within-group inequality does not add up to inequality for the total population. However, we will report group-specific poverty rates which refer to the poverty lines that are derived from the median equivalent income in the total population. Thus, the total poverty rate is a population-weighted average of poverty rates across population subgroups.

Figure 7 displays the average poverty rate in European countries within three age groups and for the total population. Extended income (NA) displays lower poverty estimates within each household type than disposable income (EU). Specifically, there is a large reduction in poverty rates in the groups of elderly and children when including in-kind transfers in the income definition. In summary, the tax and transfer systems in European countries appear to play an important role in reducing inequality and poverty, not least in groups of children and elderly who are main recipients of public in-kind transfers.

Tables A.2 and A.3 in Appendix A show inequality and poverty estimates by country and age groups in different life-cycle stages for disposable income (EU) and extended income (NA). Most European countries display relatively large reductions in estimates of inequality and poverty when moving from disposable income (EU) to extended income (NA). Yet, there is considerable variation in the percentage reduction in poverty and inequality estimates for different countries. For instance, the reduction in child poverty rates varies between 37 percent (Italy) and 77 percent (Luxembourg).

#### *Sensitivity of inequality and poverty estimates to NA versus EU equivalence scale*

We now investigate the sensitivity of inequality and poverty estimates to our method that makes use of the NA scale (or SNA scale) instead of the EU scale to convert household extended incomes into equivalent incomes. Figure 6 shows that when using NA scale instead of EU scale, inequality estimates for extended income are somewhat larger in the total population and within groups of children and adults, on average for European countries. Poverty estimates in Figure 7 display an increase in poverty among children and a decrease in poverty among non-elderly adults when making use of our equivalence scale. Thus, the change in total poverty when changing equivalence scale is more muted in comparison to changes in child poverty and poverty among working-age adults. The reduced poverty rate among non-elderly adults when comparing extended income (NA) to extended income (EU) reflects that individuals in this group have relatively low needs for public in-kind transfers.



In Appendix D, we provide more comprehensive results regarding the equivalence scale sensitivity of inequality and poverty estimates as well as the extent of re-ranking when changing the equivalence scale for extended income. We find that children and elderly are typically under-represented below the median (and over-represented above the median) when using the EU scale instead of the NA scale to equivalise extended income. By making use of the EU scale instead of the NA scale, poverty estimates are significantly affected in different ways for different household types. Thus, the aggregate change in poverty tells only part of the story when evaluating the impact of our proposed NA scale in comparison to the EU scale.

#### *Sensitivity of inequality and poverty estimates to public sector inefficiency*

As discussed in Section 2, it would be rather demanding to account for intra-household allocative efficiency in the measurement of well-being, since this would require information about individuals' willingness to pay for publicly provided services. Technical efficiency is another aspect of efficiency that is just as demanding to account for, since it is often difficult to find valid output measures for public services. Nonetheless, inefficiency in public service production may reduce the value of public services received by households.

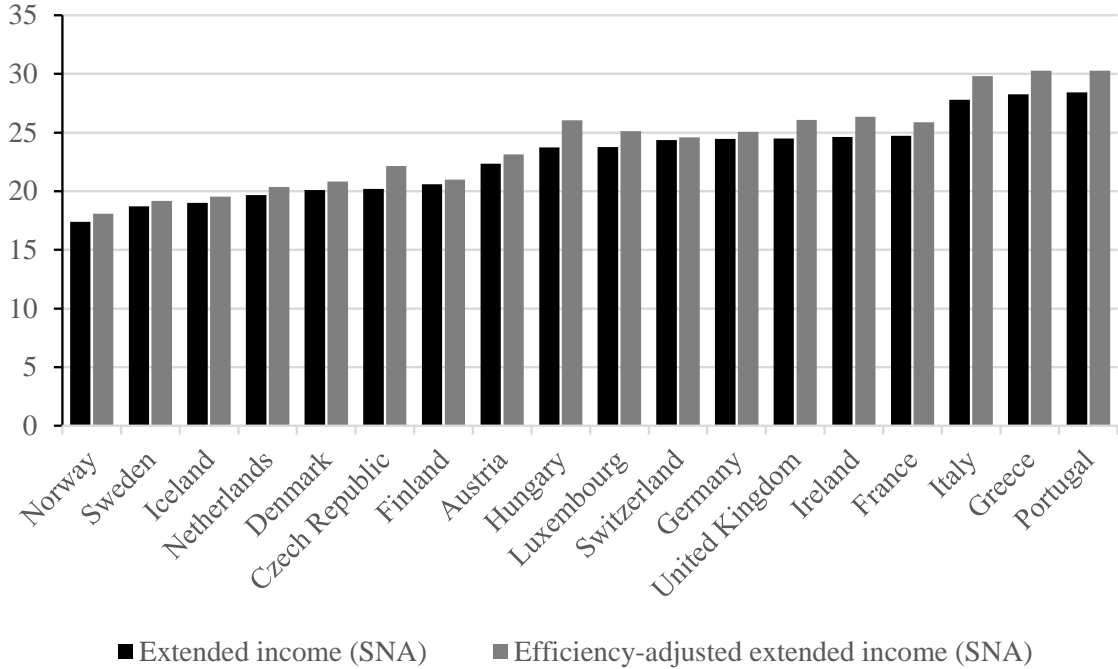
To examine the sensitivity of inequality and poverty estimates to differences in efficiency, we utilise estimates of technical efficiency of public spending reported in Angelopoulos et al. (2008). They estimate public sector efficiency in a sample of 52 countries, of which 19 countries are overlapping with our study of European countries.<sup>15</sup> Although such estimates of efficiency suffer from significant uncertainty, they will provide a helpful basis for illustrating how efficiency adjustment of public spending might change the picture of inequality and poverty across European countries.

As a robustness check, we multiply public expenditure by national efficiency scores for the public sector. Thus, the value of in-kind transfers is deflated in accordance with the estimated inefficiency in service production. The results of this exercise are reported in Figure 8 and in Appendix E. According to the estimates, public sector efficiency is in the higher range in Austria, Germany, Netherlands, and in the Nordic countries, whereas efficiency is lower in Eastern and Southern European countries and in Ireland and the UK.

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<sup>15</sup> The efficiency estimates are derived from a stochastic frontier model, where the measures of public sector output are developed by Afonso et al. (2005).

**Figure 8: Country-specific Gini coefficient estimates for extended income (baseline) versus efficiency-adjusted extended income, percent, 2012**



*Notes.* The figure displays the Gini coefficient (in percent) in the distribution of extended income when the value of in-kind transfers is equal to the production cost (baseline), and when the production cost is adjusted by public sector efficiency scores in Angelopoulos et al. (2008), see their Table A.2. Belgium, Estonia, Latvia, Lithuania, Poland, Slovakia and Slovenia are not included because of missing efficiency scores. For both income measures, household extended incomes are adjusted by the NA equivalence scale. Countries are ranked from lower to higher inequality in the baseline measure. Source: Eurostat, authors' calculations.

The adjustment of public expenditure by efficiency scores leads to increases in inequality and poverty estimates first and foremost in countries with a low level of efficiency. If anything, we uncover a positive association between inefficiency and inequality which implies that the difference between high and low performing countries could be understated by not accounting for cross-country variation in public sector efficiency. However, the increase in the cross-country dispersion of inequality estimates when relying on efficiency adjustment of public spending is rather modest. Appendix E provides decompositions of poverty estimates by household type. The results show that poverty estimates are more sensitive to efficiency adjustment for families with children and elderly households who live in countries with low public sector efficiency.

*Income decomposition*

Extended income can be expressed in terms of income components such as cash income and the value of public services. Furthermore, cash income can be divided into three main income components; market income, public cash transfers and taxes. Public cash transfers include pensions, unemployment

benefits, child allowances, etc. However, since the EU-SILC data do not allow a division of disposable income into market income and public cash transfers, we are left with two components: gross income and taxes. Total value of public services is included as a third component when we consider the distribution of extended income.

To decompose income inequality by income components, we make use of the decomposition method developed by Rao (1969), which is explained in Appendix F. The Rao method shows that the inequality share of an income component is equal to the product of the income share and the concentration coefficient. The income share is the component's share of total income, while the concentration coefficient is defined by the conditional Gini coefficient of the component given the rank order in total income.

Our decomposition of the effect of the income components – gross income, taxes and public in-kind transfers – shows that the tax system has an equalising effect in all countries. The equalising effect of taxes are highest in Denmark, Iceland, and the Netherlands, and lowest in Estonia, Lithuania, and Slovakia. The equalising effect of public services shows to be much smaller than the equalising effect of taxes.

#### *Further checks of robustness*

In Appendix G, the Gini-coefficient is estimated for 24 European countries using the equivalent individual rather than the individual as unit of analysis. The results show that inequality estimates based on the two different methods do not change materially for given income definitions. In Appendix H, we show that this conclusion is also valid for two alternative rank-dependent measures of inequality that are closely associated with the Gini-coefficient.

## **5. Summary and discussion**

This paper introduces a method for analysing income inequality and incidence of poverty when the definition of income is extended to include the value of public services. The method establishes an exact relative and income-independent equivalence scale that ensures comparability between households with different needs. The number of household members and their age define needs for cash income while household composition by age and gender determines the households' needs for public in-kind transfers. Importantly, the proposed needs-adjusted (NA) equivalence scale accounts for the fact that the distribution of needs for public in-kind transfers differs from that of cash incomes. Moreover, the equivalence scale for public services satisfies two basic axioms: unit consistency and

reference independence. When we focus attention on relative inequality and poverty it is required to use an equivalence scale that satisfies unit consistency and reference independence, which means that the equivalence scale does neither depend on the measurement unit of income nor on the choice of reference household.

The results from the empirical analysis show that the level of economic inequality varies substantially across the 24 European countries. On the one hand, Estonia, Greece, Italy, Portugal, Spain and the UK exhibit relatively high Gini coefficients, both when applying cash income and extended income as measures of economic welfare. On the other hand, Austria, Belgium, the Czech Republic, the Netherlands, Slovakia, Slovenia, and the Nordic countries have relatively low Gini coefficients. A similar pattern is found for the poverty rates. Notice, however, that there are changes in country rankings over the period 2006–2018 due to differential trends in poverty and inequality estimates. Inequality and poverty has been rising in Sweden and Luxembourg and declining in Iceland, Ireland, Poland and Portugal.

Consistent to standard practice, our baseline method assumes that the value of public services is equal to the production cost. As an alternative, we make use of public sector efficiency estimates to adjust the value of in-kind transfers for technical efficiency. We find that the dispersion across countries in inequality and poverty estimates according to extended income measure is larger when accounting for differences in public sector efficiency. The increased dispersion is due to a positive association between estimates of inequality, poverty and inefficiency of the public sector across European nations.

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## Appendix A: data and supplementary results

### Additional tables

**Table A.1: Mean extended income shares by income components and country, percent, 2012**

<b>Country</b>	<b>Cash income</b>	<b>ECEC</b>	<b>Education</b>	<b>Health care</b>	<b>Long-term care</b>
Austria	76.3	1.6	7.3	12.9	1.9
Belgium	76.8	1.7	7.2	11.1	3.1
Switzerland	79.2	0.5	7.2	10.5	2.6
Czech Republic	76.8	1.4	6.8	12.3	2.6
Germany	75.6	1.5	6.0	15.2	1.8
Denmark	73.4	2.7	8.5	11.6	3.8
Estonia	79.3	1.3	7.4	11.5	0.6
Spain	77.8	1.7	6.3	12.7	1.4
Finland	77.3	2.2	6.6	10.3	3.7
France	76.7	1.4	6.8	12.2	2.8
Greece	75.1	0.8	7.8	16.0	0.3
Hungary	77.7	2.3	6.9	12.5	0.5
Ireland	73.5	0.3	9.9	12.6	3.6
Iceland	75.4	3.5	9.1	9.5	2.6
Italy	76.9	1.3	6.9	13.5	1.5
Luxembourg	73.9	2.5	11.0	10.6	2.0
Netherlands	73.9	1.0	7.4	11.7	6.1
Norway	74.3	3.2	8.9	9.8	3.8
Poland	79.3	1.5	7.7	10.4	1.1
Portugal	76.5	1.1	8.4	12.7	1.3
Sweden	72.2	4.4	7.2	11.8	4.5
Slovenia	79.1	2.2	7.4	9.8	1.6
Slovakia	76.3	1.2	8.0	13.4	1.1
UK	76.1	0.9	7.6	13.1	2.4

Source: EU-SILC, Eurostat.

**Table A.2: Gini coefficients in the distribution of disposable income (EU) and extended income (NA) for children, adults and elderly, 2012. Percent**

Country	Children		Adults		Elderly	
	Disposable income (EU)	Extended income (NA)	Disposable income (EU)	Extended income (NA)	Disposable income (EU)	Extended income (NA)
Norway	20.7	15.1	22.6	19.3	20.0	12.5
Sweden	23.1	16.4	24.3	20.3	25.3	15.9
Iceland	23.1	16.6	24.3	20.3	23.4	17.0
Netherlands	23.6	16.8	25.1	21.1	24.3	16.3
Denmark	23.3	16.1	26.2	22.0	22.7	15.6
Czech Republic	25.1	18.8	25.4	21.6	18.4	13.2
Slovakia	25.8	18.8	24.8	20.9	17.5	13.8
Finland	23.0	17.5	25.0	21.7	24.8	17.5
Slovenia	24.0	17.9	25.2	21.8	25.1	19.8
Belgium	25.2	18.6	26.4	22.8	23.2	16.0
Austria	25.1	17.7	27.7	23.7	26.6	19.6
Hungary	30.2	21.8	29.7	25.3	21.5	17.9
Luxembourg	30.2	20.5	30.4	25.2	27.6	20.7
Switzerland	26.9	20.4	28.0	24.8	29.8	21.8
Germany	27.8	20.3	31.1	26.6	26.7	19.6
United Kingdom	29.0	21.7	30.2	26.1	26.3	18.2
Ireland	29.1	21.1	31.4	26.4	30.6	21.6
France	28.5	21.3	30.4	26.0	30.2	23.4
Poland	31.7	24.6	31.7	27.7	24.5	20.3
Italy	33.0	24.6	33.8	29.3	29.6	24.4
Greece	37.3	27.2	35.8	30.3	26.8	21.7
Portugal	33.5	25.0	34.8	30.0	33.3	25.3
Spain	34.9	26.7	34.4	30.0	29.0	23.6
Estonia	33.1	25.4	33.1	29.2	24.1	20.4

Source: EU-SILC, authors' calculations. Countries are sorted by the Gini coefficients for the population, measured by extended income (NA).

**Table A.3: Poverty rates in the distribution of disposable income (EU) and extended income (NA) for children, adults and elderly, 2012. Percent**

Country	Children		Adults		Elderly	
	Disposable income (EU)	Extended income (NA)	Disposable income (EU)	Extended income (NA)	Disposable income (EU)	Extended income (NA)
Czech Republic	11.6	5.5	8.3	5.6	6.7	0.9
Netherlands	13.9	4.5	9.9	6.2	6.6	0.7
Iceland	12.2	5.5	8.7	5.6	4.4	1.5
Norway	11.3	4.2	9.2	7.2	10.9	1.5
Denmark	9.7	3.9	10.4	7.9	13.3	3.3
Finland	10.5	3.4	10.5	7.8	17.1	4.0
France	18.0	7.0	13.0	7.2	10.0	2.5
Luxembourg	23.5	5.5	14.4	8.0	6.7	2.0
Sweden	16.5	6.3	12.5	9.2	18.1	2.9
Slovakia	21.2	11.9	12.0	7.4	6.8	1.6
Ireland	18.8	6.5	15.0	8.7	10.6	4.5
Switzerland	16.8	6.5	10.1	6.1	30.9	14.6
Hungary	24.5	7.6	15.6	9.4	5.2	2.5
Belgium	17.5	7.0	13.9	9.5	21.6	4.1
Austria	18.7	6.9	12.6	8.7	15.8	7.1
United Kingdom	18.6	8.3	13.7	9.3	18.1	4.1
Germany	14.8	4.8	16.8	11.4	15.6	6.2
Slovenia	15.1	5.3	13.5	9.7	21.8	15.0
Poland	23.7	11.2	16.7	11.3	12.7	8.6
Portugal	25.2	14.9	18.9	13.8	14.3	4.5
Italy	25.1	15.8	18.7	13.6	15.8	8.9
Estonia	17.8	8.0	16.6	12.5	26.6	24.5
Spain	28.4	19.4	19.9	14.6	13.3	6.7
Greece	28.9	17.3	23.3	16.4	15.2	7.6

Source: EU-SILC, authors' calculations. Countries are sorted by the poverty rates for the population, measured by extended income (NA).

## **Appendix B: Data, methods and descriptive statistics**

This appendix provides details about the datasets and methods that we employ in this study. Moreover, we explain methodological changes relative to our prior work in Aaberge et al. (2013), which contains further details on data and methods. Section A.1 describes the population of study. Section A.2 gives an account of data and methods for valuation of public services. Section A.3 describes how the value of public services is allocated to individuals. Section A.4 provides the income definitions in this study and summary statistic on the composition of in-kind transfers and extended incomes by country.

### **B.1. Population of study**

This study uses the EU-SILC cross-sectional data for 2007, 2010, 2013, 2016, and 2019, which cover the income years 2006, 2009, 2012, 2015, and 2018, respectively. The cross-sectional data contains information about income from the previous year, while household and demographic refer to the point in time when the interviews were conducted: during 2007, 2010, 2013, 2016, and 2019. The interviews are mostly performed in the first quarter of the reporting year, where the dates may vary between countries. We rely on information from the time of interview for measuring incomes during the previous year. The data provides demographic information such as individuals' age. Since the income year is our year of analysis, we exclude observations of children born in the interview year. We assume that there has been no change in household composition from the income year to the time of the interview in the following year.

Students aged 18 to 24 are not included in the population that forms the basis of the poverty and inequality estimates. We exclude also these students' personal incomes in households with adult members who are not students. Taxes in EU-SILC are only reported consistently on household level. Therefore, we account for students' taxes by assuming that their average tax rate equals the average tax rate of the household in total. We define students who are excluded in accordance with other studies using the EU-SILC data. Thus, students aged under 25 in post-secondary education are excluded from the sample.<sup>16</sup> The exclusion of students means also that we exclude the whole household if the students are the only adults in the household. As the cross-sectional survey data do not follow individuals over time, we exclude both persons with student status in the interview year (and assume they were students also in the income year) as well as persons without current student status but who finished a post-secondary education during the income year.

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<sup>16</sup> In previous research, we excluded all students older than secondary education age.

**Table B.1: Population of study by household type and country, percent of individuals, 2012**

	Household type										
	18–64		18–64		65–74		75+		18+		Other
	1	2	1	2	1	2	1	2	3+	3+	1-2
Children	No	No	Yes	Yes	No	No	No	No	No	Yes	No/ Yes
Austria	11	15	4	29	3	4	3	2	14	9	6
Belgium	10	16	5	35	2	4	3	3	10	5	5
Czech Republic	7	16	3	31	3	4	3	2	16	9	7
Denmark	14	16	5	37	3	5	4	2	3	4	5
Estonia	10	15	4	29	3	3	4	1	12	11	7
Finland	12	19	4	34	3	5	4	3	5	5	6
France	10	17	5	38	2	4	4	3	6	4	5
Germany	14	19	4	29	4	4	3	4	8	3	7
Greece	5	11	1	31	2	3	3	3	24	9	8
Hungary	8	17	3	28	3	3	3	1	15	12	7
Iceland	8	14	7	38	2	3	3	2	9	11	4
Ireland	5	14	8	45	2	3	2	1	10	6	5
Italy	7	11	3	29	2	3	4	3	21	9	7
Latvia	7	15	5	22	3	2	4	1	18	16	8
Lithuania	9	13	5	27	4	3	4	2	14	12	7
Luxembourg	10	14	4	35	2	3	2	2	12	11	5
Netherlands	11	18	4	37	2	5	3	3	7	3	5
Norway	13	16	7	38	3	4	3	2	4	4	5
Poland	4	12	1	24	2	2	2	1	21	25	4
Portugal	4	13	3	30	2	4	3	3	20	12	6
Slovakia	4	9	1	27	3	2	2	1	27	19	4
Slovenia	7	13	2	33	3	3	3	2	19	9	6
Spain	6	15	2	33	1	3	3	2	19	9	7
Sweden	11	16	6	36	3	5	4	3	4	4	6
Switzerland	9	20	3	33	2	5	3	2	11	7	6
UK	7	18	7	33	2	4	3	3	11	6	6

Source: EU-SILC. Notes. Children are defined as aged below 18 years. EU-SILC cross-sectional weighting is used to produce estimates for the population. Students are not included in the population.

The summary statistics in this paper are calculated based on cross-sectional sampling weights which are available in the EU-SILC data set. The purpose of weighting is to reduce biases in the estimation and to draw inference from the EU-SILC sample to the whole population. For obtaining population estimates, respondents are given weights which are inversely proportional to the probability of being selected. Moreover, the sample weights are adjusted to counterbalance non-response.

Table B.1 displays the population composition in each country, where households are classified by the age group that adults belong to, the number of adults in the household, and by having or having not children. This classification includes 11 household types, which offers more detailed results than our main breakdown of households in four different types. A substantial proportion of the individuals belongs to households that are constituted by 2 adults with one child or more. Single households are more common in Scandinavia and Germany than in the remaining European countries. Households with more than 2 adults are more common in Czech Republic, Greece, Hungary, Italy, Poland, Portugal, Slovakia, Slovenia and Spain.

Table B.2 shows the 14 age groups used in this study. When the age groups are combined with gender (males and females), the classification includes 28 target groups.

**Table B.2: Age groups in the study**

Category	Age group
1	0 years
2	1–2 years
3	3 years–education age
4	Primary education age
5	Lower secondary education age (13 years and below)
6	Lower secondary education age (14 years and above)
7	Upper secondary education age
8	18–24 years, but not in upper secondary education age
9	25–34 years
10	35–44 years
11	45–54 years
12	55–64 years
13	65–74 years
14	75 years and above

## **B.2. The value of public services**

Eurostat and OECD collaborate on the collection of public expenditure data. Together, they provide the System of Health Accounts and Education data. The two institutions differ slightly in how they report the annual expenditure. Eurostat provides the data on a January to December basis, which is relevant for our purpose. Hence, we use the public expenditure data as presented by Eurostat.<sup>17</sup>

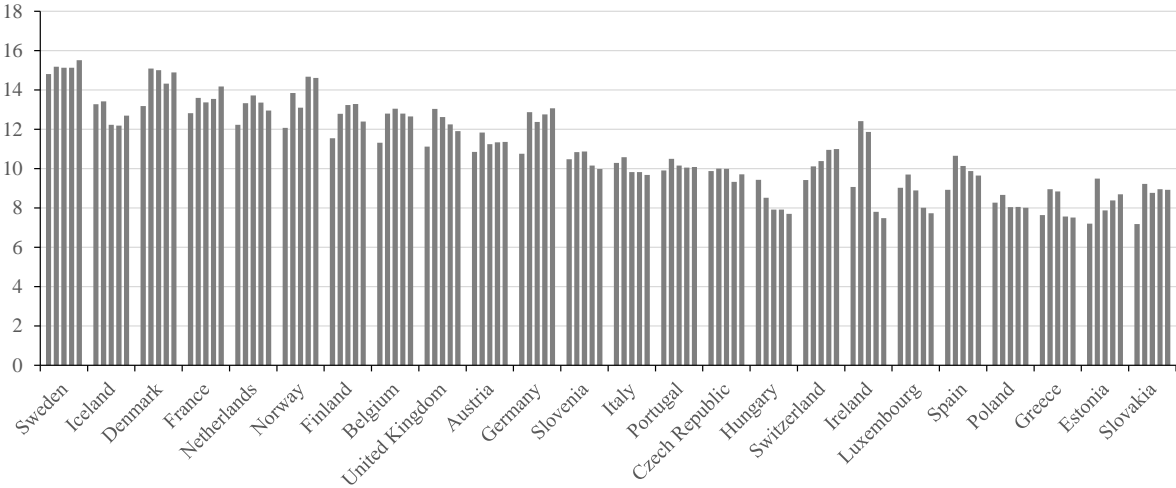
The Eurostat System of Health Accounts provides expenditure data on health and long-term care. In the System of Health Accounts, long-term care spending comprises both health and social support

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<sup>17</sup> There are only minor differences between the datasets provided by OECD and Eurostat. We are grateful to OECD and Eurostat for their help in explaining measurement methods and data definitions.

services to people suffering from chronic conditions and disabilities who need care on an ongoing basis. Since the reporting practices on the allocation of long-term care spending between the health and social components may differ between countries, we have chosen to include total spending on both components to facilitate comparability across countries.

**Figure B.1: Public expenditure on four welfare services as a proportion of GDP. Percent, 2006, 2009, 2012, 2015, 2018**



*Notes.* The figure displays public expenditure on Early Childhood Education and Care (ECEC), primary and secondary education, long-term care and health care, in percent of GDP. From left to right by country: 2006, 2009, 2012, 2015, 2018. Public expenditure excludes out-of-pocket payments. Countries are sorted by descending public expenditure as a proportion of GDP in 2006. Source: Eurostat.

Education expenditure is available from Eurostat under ‘Education and training’.<sup>18</sup> The data on education statistics are collected by UNESCO-OECD-Eurostat (UOE), and compiled from national administrative sources, which are reported by Ministries of Education or National Statistical Agencies. The data apply the International Standard Classification of Education (ISCED) 2011, and are separated into primary, lower secondary and upper secondary education. When countries lack data points for a specific education level, we have estimated the total expenditure to be the average of the year before and the year after. Furthermore, Eurostat provides public expenditure data on pre-primary education in the same database as for education expenditure. The Eurostat expenditure requires the service to include minimum 2 hours of pedagogical content per week. However, comparing the Eurostat numbers to the ECEC expenditure data available from the OECD Social Expenditure Database (SOCX) shows modest differences between the two sources.<sup>19</sup> The 2018 numbers were not available

<sup>18</sup> Specifically, we use the ‘educ\_uoe\_fine’ data.

<sup>19</sup> In Aaberge et al (2013), we used the OECD Family Database to calculate the value of public ECEC services. Our main results are almost unaffected by this change of dataset on ECEC expenditures.

when this study was conducted. We used 2017 data inflated with the growth in the gross domestic product as a proxy for education expenditure in 2018.

Public services in this article include welfare services that are publicly funded, which means that the service producer might be a public or private entity.

Figure B.1 displays country-specific trends in public in-kind transfers as a proportion of GDP over the period 2006-2018. It appears that European countries follow rather different trends with respect to expenditure on welfare services as a proportion of GDP.

### **B.3. Allocation of public services**

The value of education and ECEC services per user is allocated to actual users. Enrollment numbers in each education level (primary, lower secondary and upper secondary) is accessible from Eurostat. Total expenditure divided by the enrollment number provides an estimate of the value received per pupil. For each education level and country, we allocate the spending per pupil to children by age when they attend mandatory schooling. In the EU-SILC data, actual participation in education institutions is only known for people aged 16 years or above. For younger children, however, education participation is largely compulsory and we therefore assume 100 percent participation rates for these children.

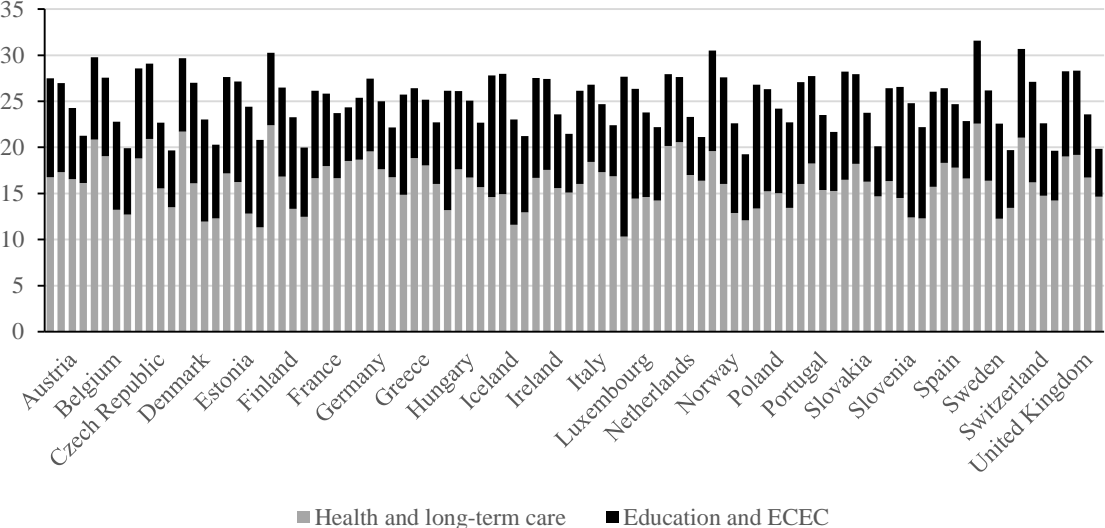
We have assigned children to the three education levels based on information from the Eurydice network, a strategic mechanism established by the European Commission and Member States to support education cooperation in Europe. From their webpage, we have derived the age structure of national education systems. We have assumed for simplicity that children in a given age cohort belong only to one education level throughout the income year. This holds also for children in the last class of upper secondary education. For countries which split children born in the same calendar year into two subsequent years of school entry, we have assigned children into school entry based on year and quarter of birth combined with national cutoff dates for school entry. We do not have access to birth dates, only year and quarter. A few countries have reformed their education system with respect to progression by age, and this study accounts accordingly for such changes.

Our method assumes that the value of childcare and pre-primary education is allocated to users only. The calculation from total public expenditure to per hour value is based on actual participation. Since there are no reliable data on children's total use of ECEC services in European countries, we have



estimated total use in a country based on a combination of sample weights and individual participation rates. For most countries, the sample weight is specifically designed for making the children under school age in the survey representative for the population. For a few countries in selected years, this sample weight is not available. We have then applied the overall sample weight in EU-SILC. The EU-SILC data include variables that provide information about the average hours of participation per week in childcare and pre-primary schooling. We estimate the public expenditure per hour per week given to children in each country, and allocate this value multiplied by the number of hours attended in ECEC services to the actual recipients registered in the EU-SILC.

**Figure B.2: Proportion of public expenditures by quartiles of the disposable income distribution. Percent, 2012**



*Notes.* The figure displays the percentages of public spending on health, long-term, education and ECEC services that is allocated to the population in different quartile groups in 2012. Individuals are grouped in quartiles by country according to the distribution of disposable household income divided by the EU scale. From left to right by country: quartile group 1, 2, 3, 4. Countries are sorted in alphabetical order. Source: Eurostat.

A limitation in the data is that information on the participation in public versus private education and childcare institutions is not accessible. Therefore, we allocate public in-kind benefits to all children receiving ECEC services, and it is assumed that every pupil at a certain education level receives the same amount of government funding for a given country. Note that this method might bias our results for some countries to the extent that schools and ECEC institutions are privately funded.

Regarding health and long-term care services, on the other hand, the value of the public services is understood in terms of being insured, i.e. the value is assessed on an ex-ante basis. The European Commission (2009, 2012, 2015, 2018) have established user profiles by age and gender for both health

and long-term care services. The reported numbers are from three years prior to publication. For instance, this means that European Commission (2018) provides data for 2015. We have been able to apply updated health care user profiles for most countries for 2012 and 2015, but for six countries we use 2009 profiles. For the 2018 income year, we apply the same data as for 2015. For long-term care, we have only been able to access user profiles from 2006. For countries without long-term care profiles, we apply an average across all countries. By combining the user profiles with population data, the relative provision to each citizen is calculated. Multiplication with the total expenditure gives the value of individual health and long-term care insurance.

Figure B.2 displays the proportion of spending on public welfare services that is allocated to different quartiles of the disposable income distribution. Equal allocation requires that each quartile group receives 25 percent of expenditure on in-kind transfers. However, we find that this proportion is decreasing for higher quartile groups in most European countries. This means that a relatively large proportion of public in-kind transfers is allocated to the population in the middle and lower parts of the disposable income distribution.

## Appendix C: Estimation and simplified representation of the NA scale

**Table C.1: Equivalence scales, 2012**

Type	Age	EU	NA	NC
Single males	18–24	1.00	0.98	0.77
	25–34	1.00	0.98	0.78
	35–44	1.00	1.00	1.00
	45–54	1.00	1.03	1.36
	55–64	1.00	1.07	2.07
	65–74	1.00	1.15	3.21
	75 +	1.00	1.32	5.54
Single females	18–24	1.00	0.98	0.77
	25–34	1.00	1.00	1.06
	35–44	1.00	1.01	1.18
	45–54	1.00	1.03	1.45
	55–64	1.00	1.06	1.91
	65–74	1.00	1.14	2.96
	75 +	1.00	1.35	6.02
Couples	18–24	1.50	1.50	1.53
	25–34	1.50	1.52	1.84
	35–44	1.50	1.55	2.18
	45–54	1.50	1.59	2.81
	55–64	1.50	1.67	3.98
	65–74	1.50	1.83	6.16
	75 +	1.50	2.20	11.56
Couples with one child	0	1.80	1.91	3.43
	1–2	1.80	1.98	4.38
	3–education age	1.80	2.08	5.87
	Primary education	1.80	2.20	7.53
	Lower secondary education	1.80	2.25	8.23
	Upper secondary education	2.00	2.46	8.63
	0	2.10	2.28	4.67
Couples with two children	1–2	2.10	2.41	6.57
	3–education age	2.10	2.62	9.57
	Primary education	2.10	2.85	12.89
	Lower secondary education	2.10	2.95	14.28
	Upper secondary education	2.50	3.38	15.08
	0	1.30	1.38	2.43
	1–2	1.30	1.45	3.38
Single mothers with one child	3–education age	1.30	1.55	4.87
	Primary education	1.30	1.67	6.53
	Lower secondary education	1.30	1.71	7.23
	Upper secondary education	1.50	1.93	7.63
	0	1.60	1.74	3.67
	1–2	1.60	1.88	5.57
	3–education age	1.60	2.09	8.57
Single mothers with two children	Primary education	1.60	2.32	11.89
	Lower secondary education	1.60	2.42	13.28
	Upper secondary education	2.00	2.84	14.08

*Notes.* Household types with children in lower secondary education level include only children below 14 years of age. The age group 18–24 years includes only persons above secondary education age. The NA scale is a weighted average of the EU scale and the NC scale, where the EU scale is the equivalence scale for cash income, and the NC scale is the equivalence scale for non-cash income. The NC scale accounts for heterogeneity in needs for childcare, pre-primary, primary and secondary education, health care and long-term care. Results are based on a sample of 24 European countries. Source: Eurostat, authors' calculations.

The EU equivalence scale and our estimates for the NA and NC scales based on data from 2012 are reported by household type in Table B.1. The results differ from those presented in Aaberge et al. (2013), since those previous results were based on data from 2009 for a smaller sample of European countries, and because the datasets and imputation methods have been revised in this study (see Appendix A). Notice that our methods allow us to estimate the NA scale for all household types and every European household in our sample. For the sake of brevity, we only report results for a selection of household types with 2 or less children in Table C.1.

To estimate the NA scale as outlined in Section 2, it is insufficient to have data on household size and composition. It is also required to estimate the  $\gamma$ -parameters that account for the relative needs for disposable income and public services as a function of household characteristics. As explained in section 2.3 these estimates are based on median disposable income and on spending levels as well as spending profiles by age and gender for different public services. Since the computational complexity may reduce the practicability and therefore prevent utilisation of the NA scale, Aaberge et al. (2013, 2017) develop a simplified representation of the NA scale, termed the SNA scale. The SNA scale requires only data for household size and composition by age groups and is easily computed for any dataset with household information that includes age of the household members. The SNA scale is computed in the same way as the EU scale, except that the SNA scale includes several age groups and assigns weights to the age groups that differ from the EU scale.

The SNA scale is derived from a linear regression (OLS) of the NA scale on the number of household members in different age groups:

$$(C.1) \quad NA_h = \alpha_0 + \sum_{j=1}^8 \alpha_j n_{hj} + \varepsilon_h,$$

where  $NA_h$  is the estimated NA scale for household  $h$  (included in the EU-SILC sample),  $n_{hj}$  is the number of members of household  $h$  in age group  $j$ , and  $\varepsilon_h$  is the error term in the regression. The SNA scale is defined as the predicted NA scale from the regression model (C.1), i.e.

$SNA_h = \hat{\alpha}_0 + \sum_{j=1}^8 \hat{\alpha}_j n_{hj}$ , where  $\hat{\alpha}_j$  are parameter estimates ( $j=0,1,\dots,8$ ). Some of the age groups have been merged in the regression model, which is why the model in (3.1) includes only 8 different

age groups. The SNA scale is also simplified in the sense that it does not distinguish between females and males, since it turns out that the effect of gender on the NA scale is modest.<sup>20</sup>

Economies of scale in household consumption are captured by a positive estimate for the intercept  $\alpha_0$  in the regression equation (B.1), while a zero estimate for the constant term implies that there are no economies of scale. When a similar regression as (C.1) is performed with the EU scale on the left hand side, the parameter  $\alpha_0$  is estimated equal to 0.5, since the first adult is assigned a weight 1, which is 0.5 higher than the weight of other adults in the EU scale.<sup>21</sup> However, since the NA scale is normalised to 1 for the reference household type, we impose the restriction  $\alpha_0 = 1 - \alpha_r$ , where  $r$  is the age group of the (single) reference household type.<sup>22</sup> This restriction secures that the SNA scale is equal to 1 for the reference household type.<sup>23</sup>

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<sup>20</sup> The NA scale is estimated based on 28 target groups (14 age groups times 2 genders). When all 28 target groups are included in the regression model for the NA scale, we find that the model explains 100% of the variation in the NA scale. Thus, the reduction in the number of target groups is the reason why the SNA scale is not an exact representation of the NA scale.

<sup>21</sup> Furthermore, the parameter estimate for adults is 0.5 and the parameter estimate for children is 0.3 in a similar regression with the EU scale on the lefthand side.

<sup>22</sup> While the reference household type for the NA scale includes single males aged 35–44 years, the reference household type for the SNA scale is broader by including single households of both genders above education age to 54 years of age.

<sup>23</sup> When healthcare is included in the definition of extended income, the estimate of  $\alpha_0$  is below 0.5. This owes to the fact that healthcare is the most important service received by the reference household. A positive need for public services for the reference household implies that economies of scale are less important in the NA scale than in the EU scale, since the NC scale does not include economies of scale.

**Table C.2: SNA scale estimation results, including different public services in the scale, 2012**

Variable	ECEC	Education	Health care	Long-term care	Education and health care	All 4 services
Constant	0.50	0.50	0.46	0.50	0.46	0.46
0–3 years	0.38	0.30	0.32	0.30	0.32	0.40
3 years to education age	0.52	0.30	0.33	0.30	0.32	0.53
Education age (below 14 years)	0.30	0.66	0.33	0.30	0.67	0.67
Education age (above 13 years)	0.50	0.92	0.53	0.50	0.93	0.93
Above education age–54 years	0.50	0.50	0.54	0.50	0.54	0.54
55–64 years	0.50	0.50	0.60	0.51	0.60	0.60
65–74 years	0.50	0.50	0.66	0.52	0.66	0.68
75 years and above	0.50	0.50	0.73	0.64	0.73	0.87
R <sup>2</sup> adjusted	1.000	1.000	0.999	0.999	0.999	0.999

*Notes.* Figures are estimated weights assigned to individuals in different age groups. When all 4 services are included, a person aged 75 contributes (additively) with a weight of 0.87. The constant weight is added to the sum of weights over individuals in a given household to derive the household equivalence scale. The reference household scale equals unity, where the reference group is childless single adults below 55 years and above education age. Estimation is based on a sample of 24 European countries.

To study the sensitivity of results and allow for flexible applications, we have estimated the NA scale and the SNA scale for each of four different public services, and moreover for different combinations of the public services that are included in this paper. This procedure also provides information about the contribution of different public services to the SNA scale.

The estimation results are reported in Table C.2. As a measure of model fit R<sup>2</sup>-adjusted shows that the goodness of fit is almost perfect for the six different models for different combinations of public services. The results show that children and elderly are given higher weights in the SNA scale than in the EU-scale, depending on which public services are included in the NA scale. Including childcare and education increases the weights of children, while including long-term care and health care increases the weights of the elderly.

The SNA scale provides a close approximation of the NA scale that can be easily applied by scholars interested in examining the distribution of extended income when services such as childcare, long-term care, health care and/or education are included in the analysis.

Table C.3 shows results from estimating the SNA scale for all 4 services in five different years. We find that the estimation results for the SNA scale are quite stable over time. In 2018, both UK and Germany are missing from the analysis. In particular, the relatively low public funding of childcare in

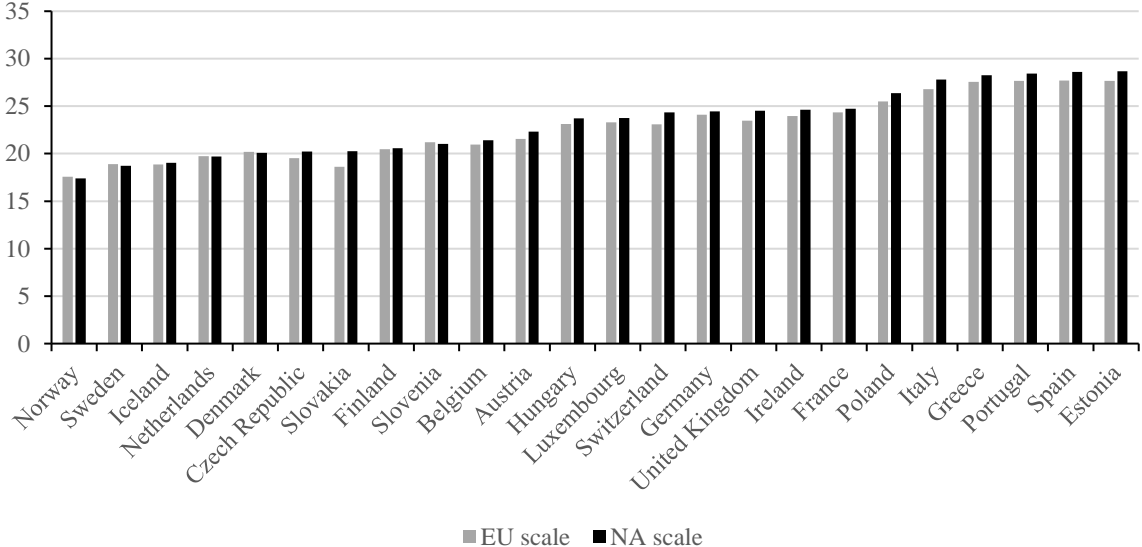
the UK has an impact on the needs estimate for the age groups 0–2 years and 3 years to education age. When reporting inequality and poverty estimates for the years 2015 and 2018, we employ our 2012 NA scale and SNA scale estimates. The reason is that the omission of Germany (2015, 2018), Switzerland (2015), the UK (2018) and Iceland (2018) reduces comparability of the equivalence scale estimates over time.

**Table C.3: SNA scale estimation results for all four services in three different years**

<b>Age group</b>	<b>2006</b>	<b>2009</b>	<b>2012</b>	<b>2015</b>	<b>2018</b>
0–2 years	0.390	0.400	0.400	0.410	0.430
3 years to education age	0.537	0.539	0.531	0.529	0.550
Education age (below 14 years)	0.669	0.678	0.672	0.687	0.678
Education age (above 13 years)	0.928	0.945	0.924	0.909	0.920
Above education age–54 years	0.544	0.546	0.545	0.542	0.542
55–64 years	0.601	0.605	0.608	0.597	0.599
65–74 years	0.685	0.693	0.684	0.671	0.674
75 years and above	0.874	0.890	0.877	0.865	0.854

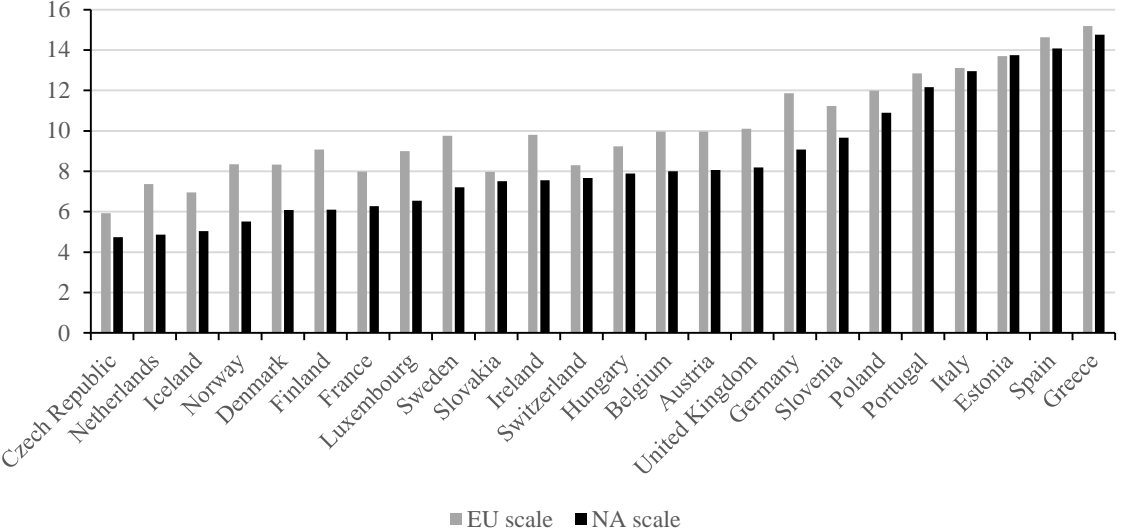
# Appendix D: Sensitivity of inequality and poverty estimates to NA versus EU equivalence scale

**Figure D.1: Gini estimates by country for extended income adjusted by NA scale versus EU scale, percent, 2012**



*Notes.* The figure displays the Gini coefficient (in percent) in the distribution of extended income when the value of in-kind transfers is adjusted by the NA scale or the EU scale, respectively. Countries are ranked from lower to higher economic inequality in extended income adjusted by the NA equivalence scale.

**Figure D.2: Poverty estimates by country for extended income adjusted by NA scale versus EU scale, percent, 2012.**



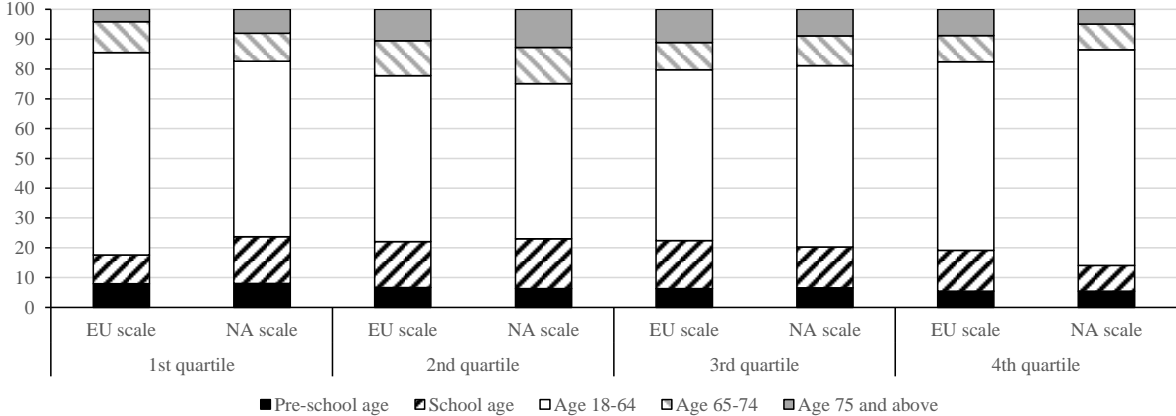
*Notes.* The figure displays the poverty rate (in percent) in the distribution of extended income when the value of in-kind transfers is adjusted by the NA scale or the EU scale, respectively. Countries are ranked from lower to higher poverty in extended income adjusted by the NA equivalence scale.



Figures D.1 and D.2 display inequality and poverty estimates for extended income by country and equivalence scale. The results show that for most countries, poverty estimates are more sensitive than inequality estimates to changing the equivalence scale. When applying the EU scale instead of the NA scale to adjust our extended income measure for differences in household needs, the poverty estimates increase in all countries. The increase is relatively large in Belgium, Ireland and the Nordic countries. Thus, the difference between countries with high and countries with low poverty rates is more accentuated when using the NA scale.

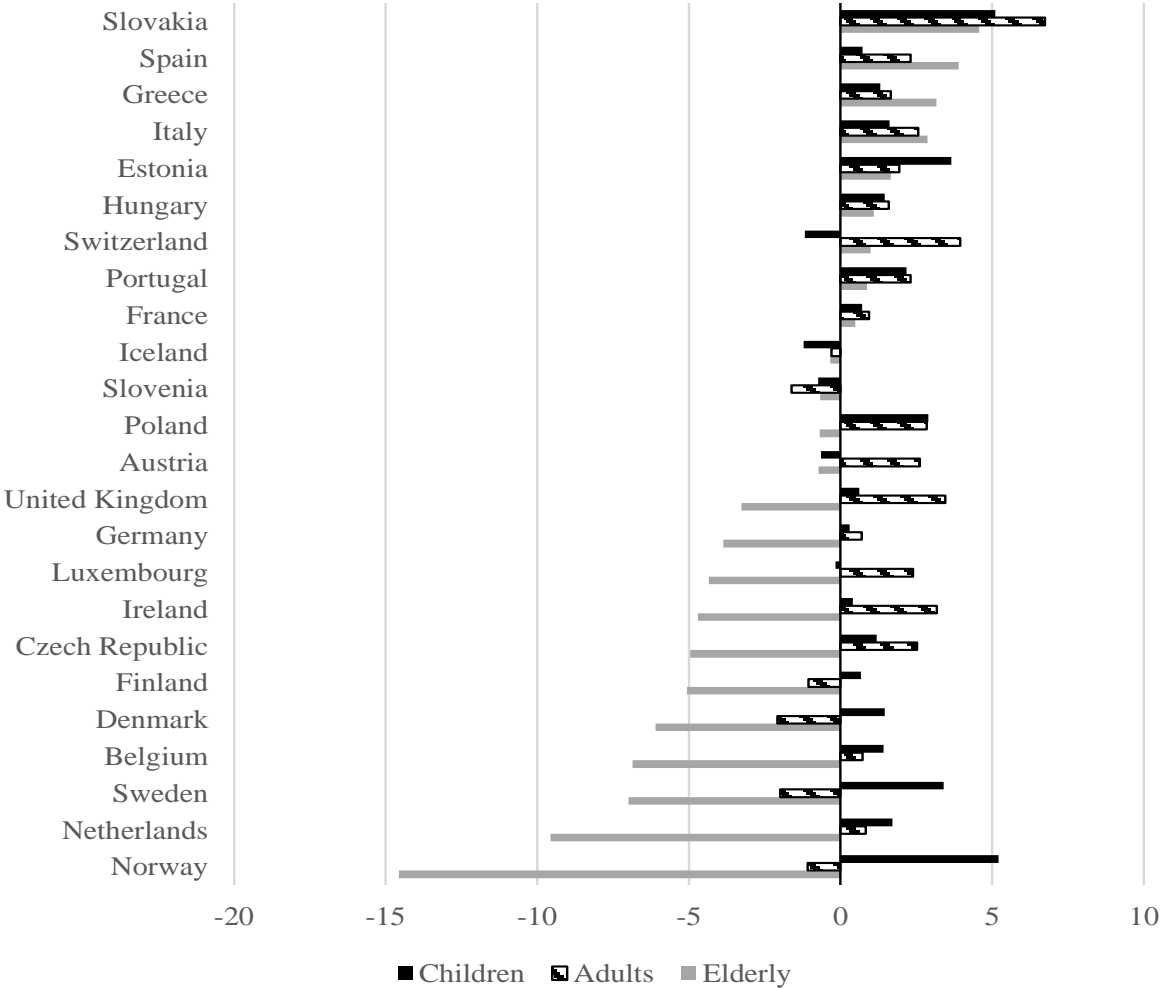
Next, we consider the extent of re-ranking of individuals in different age groups when replacing the NA scale for the EU scale. To this end, we rank individuals first by extended income (EU) and second by extended income (NA). This is used to calculate the proportions of children, adults and elderly within quartile groups of the two versions of the equivalent income distribution. Figure D.3 shows that children and elderly are typically under-represented in the first and second quartile groups and over-represented in the third and fourth quartile groups when using the EU scale instead of the NA scale. The effects on the composition by quartiles appear to be rather symmetric for children and elderly in Figure 8. Considered in conjunction with Figure 7 and Table 2, our findings may suggest that, around poverty thresholds, there is a relatively high frequency of families with children which obtain a sizeable reduction in equivalent incomes when replacing NA scale for the EU scale.

**Figure D.3: Population proportions of age groups by quartiles in the distribution of extended income, converted by EU scale versus NA scale. Averages across European countries, 2012. Percent**



*Notes.* The figure displays the percentages of the population within quartile groups that belong to five different age groups. Individuals are grouped in quartiles by country according to the distribution of extended household income making use of two different equivalence scales. This shows the effect of re-ranking of individuals on age composition within quartiles when replacing NA scale for EU scale. Population proportions within quartiles are first estimated for different European countries. The figure displays weighted average estimates across European countries, where country-specific estimates are weighted by total population size. Source: Eurostat, authors' calculations.

**Figure D.4: Relative change in the Gini coefficient for extended income distribution when replacing NA scale for the EU scale, by country and within age groups, 2012. Percent**



*Notes.* The figure displays percentage change in Gini coefficient within groups of children (age 0–17), adults (age 18–64) and elderly (age 65 and above), when using the NA scale to convert extended income instead of the EU scale. Source: Eurostat, authors’ calculations.

Figure D.4 displays relative changes in inequality estimates by country and age group when replacing NA scale for the EU scale. In most countries, within-group inequality estimates for children and working-age adults increase when introducing the NA scale. In countries with more generous welfare states, there are inequality reductions within the group of elderly people. Conversely, in Slovakia, Spain, Greece and Italy, there are instead increases in inequality among the elderly. Apart from generosity of public pensions and in-kind transfers, these cross-country differences might be associated with differences in household structure. For example, the effects of needs-adjustment among the elderly will depend on whether they live alone, in couples, or in multi-generational households.

Tables D.1 and D.2 report inequality and poverty estimates by country and age group when comparing extended income (EU) to extended income (NA). We find significant differences across countries in the sensitivity of inequality and poverty estimates when changing the equivalence scale.

**Table D.1: Gini coefficients in the distribution of extended income (EU) and extended income (NA) for children, adults and elderly, 2012. Percent**

Country	Children		Adults		Elderly	
	Extended income (EU)	Extended income (NA)	Extended income (EU)	Extended income (NA)	Extended income (EU)	Extended income (NA)
Czech Republic	4.3	5.5	7.7	5.6	1.5	0.9
Netherlands	3.7	4.5	10.5	6.2	1.0	0.7
Iceland	5.0	5.5	9.1	5.6	2.1	1.5
Norway	3.9	4.2	12.0	7.2	2.3	1.5
Denmark	2.9	3.9	11.8	7.9	4.5	3.3
Finland	3.3	3.4	12.0	7.8	6.8	4.0
France	5.1	7.0	10.7	7.2	2.6	2.5
Luxembourg	5.2	5.5	11.9	8.0	3.0	2.0
Sweden	4.8	6.3	13.4	9.2	5.5	2.9
Slovakia	8.9	11.9	8.9	7.4	2.1	1.6
Ireland	6.1	6.5	12.7	8.7	4.8	4.5
Switzerland	4.9	6.5	8.6	6.1	11.1	14.6
Hungary	5.6	7.6	12.0	9.4	2.7	2.5
Belgium	5.4	7.0	12.9	9.5	5.6	4.1
Austria	6.4	6.9	11.8	8.7	7.5	7.1
United Kingdom	7.6	8.3	12.5	9.3	4.6	4.1
Germany	4.4	4.8	15.7	11.4	7.3	6.2
Slovenia	4.8	5.3	13.2	9.7	11.7	15.0
Poland	9.6	11.2	13.6	11.3	8.0	8.6
Portugal	12.2	14.9	15.8	13.8	4.4	4.5
Italy	12.3	15.8	15.6	13.6	6.9	8.9
Estonia	6.5	8.0	14.6	12.5	18.9	24.5
Spain	15.6	19.4	17.0	14.6	5.5	6.7
Greece	13.8	17.3	18.8	16.4	5.8	7.6

Source: EU-SILC, authors' calculations. Countries are sorted by the Gini coefficients for the population, measured by extended income (NA).

**Table D.2: Poverty rates in the distribution of extended income (EU) and extended income (NA) for children, adults and elderly, 2012. Percent**

Country	Children		Adults		Elderly	
	Disposable income (EU)	Extended income (NA)	Disposable income (EU)	Extended income (NA)	Disposable income (EU)	Extended income (NA)
Czech Republic	4.3	5.5	7.7	5.6	1.5	0.9
Netherlands	3.7	4.5	10.5	6.2	1.0	0.7
Iceland	5.0	5.5	9.1	5.6	2.1	1.5
Norway	3.9	4.2	12.0	7.2	2.3	1.5
Denmark	2.9	3.9	11.8	7.9	4.5	3.3
Finland	3.3	3.4	12.0	7.8	6.8	4.0
France	5.1	7.0	10.7	7.2	2.6	2.5
Luxembourg	5.2	5.5	11.9	8.0	3.0	2.0
Sweden	4.8	6.3	13.4	9.2	5.5	2.9
Slovakia	8.9	11.9	8.9	7.4	2.1	1.6
Ireland	6.1	6.5	12.7	8.7	4.8	4.5
Switzerland	4.9	6.5	8.6	6.1	11.1	14.6
Hungary	5.6	7.6	12.0	9.4	2.7	2.5
Belgium	5.4	7.0	12.9	9.5	5.6	4.1
Austria	6.4	6.9	11.8	8.7	7.5	7.1
United Kingdom	7.6	8.3	12.5	9.3	4.6	4.1
Germany	4.4	4.8	15.7	11.4	7.3	6.2
Slovenia	4.8	5.3	13.2	9.7	11.7	15.0
Poland	9.6	11.2	13.6	11.3	8.0	8.6
Portugal	12.2	14.9	15.8	13.8	4.4	4.5
Italy	12.3	15.8	15.6	13.6	6.9	8.9
Estonia	6.5	8.0	14.6	12.5	18.9	24.5
Spain	15.6	19.4	17.0	14.6	5.5	6.7
Greece	13.8	17.3	18.8	16.4	5.8	7.6

Source: EU-SILC, authors' calculations. Countries are sorted by the poverty rates for the population, measured by extended income (NA).

The sensitivity by household types of country-specific poverty estimates to alternative equivalence scales is reported in Tables D.3–D.5. These tables employ a more detailed breakdown of poverty rates on 11 different household types. By making use of the EU scale instead of the NA scale, poverty estimates are significantly affected in different ways for different household types. Thus, the aggregate change in poverty tells only part of the story when evaluating the impact of our proposed NA scale in comparison to the EU scale.

**Table D.3: Poverty rates by 10 household types, extended income (EU), 2012**

	Household type										
	18–64		18–64		65–74		75+		18+		Other
	1	2	1	2	1	2	1	2	3+	3+	1–2
Children	No	No	Yes	Yes	No	No	No	No	No	Yes	No/yes
Austria	27.9	12.5	9.0	7.1	17.6	6.8	5.7	1.8	5.3	5.8	5.5
Belgium	35.6	10.7	10.1	5.0	21.9	5.2	0.6	0.0	6.5	10.6	6.1
Czech Republic	22.9	7.7	9.5	4.0	5.8	0.0	0.0	0.0	6.0	3.7	2.1
Denmark	31.7	8.2	2.4	3.5	11.7	4.0	1.8	0.0	2.3	1.4	4.4
Estonia	32.0	16.0	15.1	5.4	55.0	1.2	24.5	1.0	13.5	6.3	11.6
Finland	36.7	7.8	6.5	3.4	33.8	1.3	0.3	0.0	3.3	2.1	2.8
France	26.5	9.1	9.4	4.6	12.8	1.8	0.4	0.0	7.5	12.5	2.3
Germany	39.2	12.8	11.4	4.3	27.7	3.1	2.2	0.6	5.2	1.3	5.8
Greece	26.6	17.0	19.2	12.8	11.5	1.8	3.1	0.5	19.3	21.5	9.9
Hungary	26.1	12.7	3.8	4.4	6.6	0.0	1.9	0.0	9.8	14.6	3.6
Iceland	35.1	8.1	13.5	3.8	8.1	3.2	0.0	0.0	2.2	1.5	0.7
Ireland	46.4	14.3	10.6	6.1	16.2	3.4	1.2	0.0	8.7	4.6	6.8
Italy	27.9	15.1	22.6	10.7	23.5	3.9	5.5	1.6	12.6	16.7	7.6
Luxembourg	27.5	11.7	10.9	4.9	10.1	1.0	2.0	0.0	8.7	8.0	3.9
Netherlands	32.2	7.5	5.0	3.9	3.7	0.4	0.0	0.0	5.9	0.6	2.4
Norway	35.4	7.8	9.2	2.9	10.4	0.7	0.4	0.0	5.1	0.6	2.1
Poland	30.5	14.0	12.4	7.7	26.0	1.3	4.7	0.0	12.9	12.4	9.4
Portugal	27.3	19.9	9.6	10.5	14.6	3.1	2.5	0.1	13.4	18.8	6.6
Slovakia	27.2	8.2	9.0	7.4	6.8	0.0	1.2	0.0	5.9	10.4	2.4
Slovenia	40.4	15.8	8.9	5.5	36.5	2.9	10.9	1.2	8.6	5.2	12.4
Spain	29.9	17.2	20.1	14.0	8.1	2.8	5.9	1.7	11.5	22.9	9.3
Sweden	40.7	10.2	7.9	3.5	31.3	1.8	0.9	0.0	7.4	3.3	4.8
Switzerland	19.3	8.1	7.3	5.0	24.0	14.5	7.7	0.0	7.3	3.5	11.5
United Kingdom	35.0	11.7	11.5	7.7	18.2	3.8	1.6	0.0	5.9	8.2	4.4

**Table D.4: Poverty rates by 10 household types, extended income (NA), 2012**

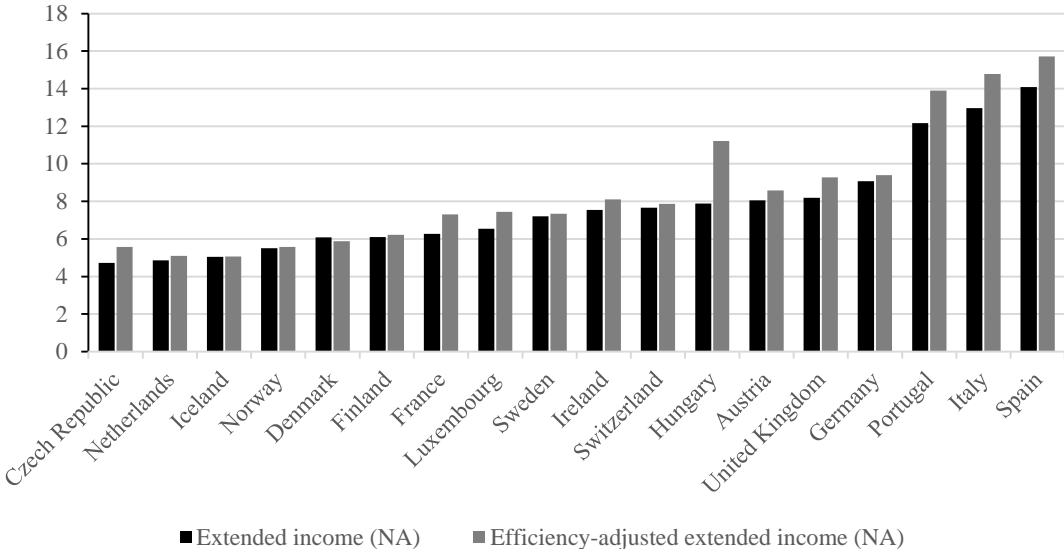
	Household type										
	18–64		18–64		65–74		75+		18+		Other
	1	2	1	2	1	2	1	2	3+	3+	1–2
Children	No	No	Yes	Yes	No	No	No	No	No	Yes	No/yes
Austria	19.0	8.0	10.2	7.0	14.0	6.8	6.1	1.8	4.0	5.6	5.8
Belgium	22.0	7.4	14.2	5.4	11.9	4.9	0.6	0.5	5.0	10.9	6.0
Czech Republic	13.3	4.9	12.9	3.8	2.3	0.0	0.0	0.0	4.3	6.9	1.6
Denmark	20.3	4.8	4.2	3.8	5.3	4.0	1.8	0.0	0.1	1.5	4.4
Estonia	27.3	12.0	16.9	6.1	29.1	1.2	69.7	5.9	11.6	7.4	10.8
Finland	24.3	4.0	6.8	2.9	17.6	1.3	0.8	0.0	1.8	3.0	2.5
France	14.0	4.9	15.0	5.2	8.5	1.8	1.6	0.6	4.6	11.8	2.3
Germany	29.0	7.9	13.9	4.1	20.4	2.4	4.8	1.2	3.4	1.2	4.7
Greece	20.9	12.7	23.6	15.2	7.4	1.8	13.8	3.2	15.0	24.7	9.5
Hungary	17.9	8.7	10.3	5.7	3.7	0.0	4.0	1.4	6.6	14.1	3.0
Iceland	18.9	4.5	16.0	3.2	3.7	3.2	0.0	0.0	1.5	1.9	1.8
Ireland	34.0	6.3	12.5	5.7	13.1	3.4	3.3	0.0	6.0	2.6	5.7
Italy	21.0	10.9	29.9	12.9	18.3	3.9	15.6	4.8	10.0	17.3	8.1
Luxembourg	18.1	6.1	11.8	4.6	5.6	1.0	2.0	0.0	4.3	8.4	2.9
Netherlands	15.7	3.8	6.5	4.3	1.5	0.4	0.0	0.0	4.3	0.6	2.4
Norway	20.8	3.9	8.9	2.7	5.7	0.7	0.4	0.0	2.8	1.5	2.1
Poland	22.3	9.7	12.6	8.6	20.3	1.3	13.4	0.2	9.6	13.3	8.6
Portugal	19.0	15.1	11.1	13.1	8.8	2.7	6.7	0.3	10.2	18.5	7.7
Slovakia	15.6	6.2	17.0	9.1	1.8	0.0	5.4	0.0	4.0	11.7	2.4
Slovenia	26.3	11.2	10.9	5.3	27.0	2.9	32.0	8.8	6.1	5.3	12.2
Spain	24.0	10.3	29.9	16.3	7.6	3.1	8.2	4.1	8.7	24.9	10.7
Sweden	25.6	6.9	11.1	3.9	13.6	1.4	0.9	0.0	4.5	4.5	4.2
Switzerland	12.1	4.4	10.1	5.7	19.2	14.3	20.5	8.1	4.4	5.6	13.6
United Kingdom	23.4	7.9	11.9	7.6	13.6	3.8	3.2	0.1	3.4	8.5	3.5

**Table D.5: Difference in poverty rates between extended income adjusted by EU versus NA scale, by 10 household types, 2012**

	Household type										
	18–64		18–64		65–74		75+		18+		Other
	1	2	1	2	1	2	1	2	3+	3+	1–2
Children	No	No	Yes	Yes	No	No	No	No	No	Yes	No/yes
Austria	8.9	4.5	-1.2	0.1	3.6	0.0	-0.4	0.0	1.3	0.3	-0.3
Belgium	13.6	3.3	-4.1	-0.4	10.0	0.3	0.0	-0.5	1.5	-0.3	0.1
Czech Republic	9.6	2.8	-3.5	0.2	3.5	0.0	0.0	0.0	1.7	-3.1	0.5
Denmark	11.4	3.4	-1.8	-0.3	6.5	0.0	0.0	0.0	2.1	-0.1	0.0
Estonia	4.8	4.0	-1.8	-0.7	26.0	0.0	-45.3	-4.9	1.9	-1.1	0.7
Finland	12.4	3.8	-0.3	0.5	16.2	0.0	-0.5	0.0	1.5	-0.9	0.4
France	12.5	4.2	-5.6	-0.6	4.3	0.0	-1.3	-0.6	2.9	0.7	0.0
Germany	10.2	4.9	-2.4	0.3	7.3	0.7	-2.7	-0.6	1.8	0.0	1.2
Greece	5.7	4.4	-4.4	-2.4	4.1	0.0	-10.7	-2.7	4.2	-3.2	0.4
Hungary	8.1	4.0	-6.6	-1.3	2.9	0.0	-2.1	-1.4	3.2	0.6	0.6
Iceland	16.2	3.6	-2.4	0.6	4.4	0.0	0.0	0.0	0.7	-0.4	-1.0
Ireland	12.4	8.1	-1.9	0.4	3.1	0.0	-2.1	0.0	2.8	1.9	1.1
Italy	6.9	4.1	-7.3	-2.3	5.2	0.0	-10.1	-3.2	2.6	-0.5	-0.5
Luxembourg	9.4	5.6	-0.8	0.3	4.5	0.0	0.0	0.0	4.3	-0.4	1.0
Netherlands	16.5	3.6	-1.4	-0.4	2.3	0.0	0.0	0.0	1.7	0.0	0.0
Norway	14.6	3.8	0.3	0.2	4.6	0.0	0.0	0.0	2.3	-0.8	0.0
Poland	8.2	4.3	-0.2	-0.9	5.7	0.0	-8.8	-0.2	3.2	-0.9	0.8
Portugal	8.3	4.9	-1.5	-2.6	5.9	0.3	-4.1	-0.2	3.2	0.3	-1.1
Slovakia	11.6	2.0	-8.0	-1.7	5.0	0.0	-4.2	0.0	2.0	-1.3	0.0
Slovenia	14.1	4.7	-2.0	0.2	9.5	0.0	-21.2	-7.6	2.5	0.0	0.2
Spain	5.9	6.9	-9.8	-2.3	0.4	-0.2	-2.3	-2.4	2.8	-2.0	-1.4
Sweden	15.1	3.3	-3.2	-0.4	17.7	0.4	0.0	0.0	3.0	-1.3	0.6
Switzerland	7.1	3.7	-2.7	-0.7	4.8	0.2	-12.8	-8.1	2.9	-2.1	-2.1
United Kingdom	11.6	3.8	-0.4	0.1	4.5	0.0	-1.6	-0.1	2.5	-0.3	0.9

# Appendix E: Sensitivity of inequality and poverty estimates to adjustment for public sector efficiency

**Figure E.1: Poverty estimates by country for extended income (baseline) versus efficiency-adjusted extended income, percent**



*Notes.* The figure displays the poverty rate (in percent) in the distribution of extended income when the value of in-kind transfers is equal to the production cost (baseline), and when the production cost is adjusted by public sector efficiency scores in Angelopoulos et al. (2008), see their Table A.2. Belgium, Estonia, Poland, Slovakia and Slovenia are not included because of missing efficiency scores. For both income measures, household extended incomes are adjusted by the NA equivalence scale. Countries are ranked from lower to higher poverty in the baseline measure.



**Table E.1: Inequality and poverty estimates for efficiency-adjusted extended income (NA) and difference from unadjusted extended income (NA), 2012**

Country	Efficiency score	Gini coefficient		Poverty rates (percent)	
		Estimate	Difference	Estimate	Difference
Austria	87	23.1	0.8	8.6	0.5
Czech Republic	65	22.1	1.9	5.6	0.8
Denmark	89	20.8	0.7	5.9	-0.2
Finland	93	21.0	0.4	6.2	0.1
France	80	25.9	1.2	7.3	1.0
Germany	90	25.1	0.6	9.4	0.3
Greece	69	30.3	2.0	17.0	2.3
Hungary	57	26.1	2.3	11.2	3.3
Iceland	91	19.5	0.5	5.1	0.0
Ireland	73	26.3	1.7	8.1	0.6
Italy	66	29.8	2.0	14.8	1.8
Luxembourg	79	25.1	1.4	7.4	0.9
Netherlands	87	20.4	0.7	5.1	0.2
Norway	86	18.1	0.7	5.6	0.1
Portugal	71	30.3	1.9	13.9	1.7
Spain	70	30.3	1.7	15.7	1.6
Sweden	93	19.2	0.4	7.3	0.1
Switzerland	97	24.6	0.3	7.9	0.2
United Kingdom	75	26.1	1.6	9.3	1.1

Source: Efficiency scores are from Angelopoulos et al. (2008), see their Table A.2.

*Notes.* Efficiency adjusted extended income is defined by multiplying public expenditure by the country-specific efficiency score and allocating the resulting value to recipients of public services. Belgium, Estonia, Poland, Slovakia and Slovenia are not included because of missing efficiency scores.

**Table E.2: Poverty rates by 10 household types, efficiency-adjusted extended income (NA)**

	Household type										
	18–64		18–64		65–74		75+		18+		Other
	1	2	1	2	1	2	1	2	3+	3+	1–2
Children	No	No	Yes	Yes	No	No	No	No	No	Yes	No/yes
Austria	18.6	7.9	13.9	7.7	14.3	7.2	7.3	1.8	3.9	7.1	7.3
Czech Republic	11.9	4.8	21.9	4.8	4.1	0.0	1.7	0.0	4.3	9.2	2.8
Denmark	19.0	4.2	4.2	3.9	6.1	4.0	1.8	0.0	0.1	2.2	4.4
Finland	23.6	4.0	6.8	3.5	17.3	1.3	0.8	0.0	1.8	3.0	2.5
France	13.5	4.9	19.5	6.4	9.0	1.9	6.0	2.3	4.6	15.4	2.7
Germany	28.6	7.6	16.2	4.3	21.1	3.1	7.4	2.5	3.6	1.6	5.1
Greece	20.3	12.4	32.3	18.6	9.1	2.8	21.7	6.0	15.6	30.4	11.1
Hungary	17.6	8.6	21.8	12.6	5.4	0.7	6.3	3.2	6.3	21.5	4.2
Iceland	18.5	4.5	16.0	3.2	5.1	3.2	0.0	0.0	1.5	2.2	1.8
Ireland	30.5	5.8	16.7	6.2	13.5	4.4	3.7	0.0	5.6	6.8	6.5
Italy	20.1	10.9	32.3	15.6	19.7	5.6	21.7	8.6	10.4	20.4	11.0
Luxembourg	16.7	6.1	14.0	7.0	5.6	1.0	3.6	0.0	4.0	9.5	2.9
Netherlands	15.0	3.9	6.9	4.8	2.1	0.4	0.6	0.5	4.3	0.6	2.9
Norway	19.5	3.6	11.2	2.7	5.7	0.7	0.4	1.2	2.6	3.5	2.3
Portugal	18.5	14.4	15.9	15.6	11.6	3.4	12.8	2.0	10.1	21.8	11.0
Spain	23.5	10.6	32.7	18.8	8.1	3.1	9.5	14.0	9.1	27.8	12.6
Sweden	25.2	6.8	12.3	4.2	13.6	1.4	0.9	0.0	4.5	4.5	4.2
Switzerland	12.1	4.4	10.1	5.9	19.6	14.6	21.6	10.1	4.4	5.6	13.9
United Kingdom	22.4	7.8	13.5	9.7	14.9	4.7	7.4	0.1	3.4	11.2	4.0

Source: Efficiency scores are from Angelopoulos et al. (2008), see their Table A.2.

Notes. Efficiency adjusted extended income is defined by multiplying public expenditure by the country-specific efficiency score and allocating the resulting value to recipients of public services. Belgium, Estonia, Poland, Slovakia and Slovenia are not included because of missing efficiency scores.

**Table E.3: Difference in poverty rates between unadjusted and efficiency-adjusted extended income (NA), by 10 household types**

	Household type										
	18–64		18–64		65–74		75+		18+		Other
	1	2	1	2	1	2	1	2	3+	3+	1–2
Children	No	No	Yes	Yes	No	No	No	No	No	Yes	No/yes
Austria	0.5	0.1	-3.7	-0.7	-0.2	-0.3	-1.2	0.0	0.1	-1.5	-1.5
Czech Republic	1.4	0.2	-8.9	-1.0	-1.7	0.0	-1.7	0.0	0.0	-2.4	-1.2
Denmark	1.4	0.6	0.0	-0.1	-0.9	0.0	0.0	0.0	0.0	-0.6	0.0
Finland	0.7	0.1	0.0	-0.6	0.4	0.0	0.0	0.0	0.0	0.0	0.0
France	0.5	0.0	-4.5	-1.1	-0.5	-0.1	-4.3	-1.7	-0.1	-3.6	-0.4
Germany	0.3	0.2	-2.4	-0.2	-0.7	-0.7	-2.6	-1.3	-0.2	-0.4	-0.4
Greece	0.7	0.3	-8.6	-3.4	-1.7	-0.9	-7.9	-2.8	-0.6	-5.7	-1.6
Hungary	0.4	0.0	-11.5	-6.9	-1.6	-0.7	-2.2	-1.8	0.3	-7.5	-1.2
Iceland	0.4	0.0	0.0	0.0	-1.4	0.0	0.0	0.0	0.0	-0.3	0.0
Ireland	3.5	0.4	-4.2	-0.5	-0.4	-1.0	-0.4	0.0	0.4	-4.1	-0.8
Italy	0.9	0.0	-2.4	-2.7	-1.4	-1.7	-6.1	-3.7	-0.4	-3.2	-2.8
Luxembourg	1.4	0.0	-2.3	-2.3	0.0	0.0	-1.6	0.0	0.3	-1.1	0.0
Netherlands	0.6	-0.1	-0.5	-0.5	-0.7	0.0	-0.6	-0.5	0.0	0.0	-0.5
Norway	1.3	0.3	-2.3	0.0	0.0	0.0	0.0	-1.2	0.2	-2.0	-0.3
Portugal	0.5	0.7	-4.8	-2.6	-2.8	-0.7	-6.2	-1.6	0.0	-3.3	-3.3
Spain	0.5	-0.3	-2.8	-2.5	-0.4	0.0	-1.3	-9.9	-0.4	-2.9	-1.9
Sweden	0.5	0.1	-1.2	-0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Switzerland	0.0	0.0	0.0	-0.2	-0.4	-0.4	-1.0	-2.0	0.0	0.0	-0.3
United Kingdom	1.0	0.1	-1.6	-2.0	-1.2	-0.9	-4.3	0.0	0.0	-2.7	-0.6

## Appendix F: Decomposition of inequality by income source

Following Rao (1969), the Gini coefficient ( $G$ ) admits the following decomposition

$$(F.1) \quad G = \sum_i v_i(G) = \sum_i \frac{\mu_i}{\mu} \kappa_i,$$

where  $\mu_i$  is the mean of income component  $i$ ,  $\mu$  is the overall mean income, and the ratio is the income share of component  $i$ . The concentration coefficient  $\kappa_i$  can be interpreted as the conditional Gini coefficient of component  $i$  given the rank order in extended income  $x_{+hk}$ . The inequality contribution  $v_i(G)$  is the product of the income share and the concentration coefficient. If the mean of an income component is positive ( $\mu_i > 0$ ), then a negative value of the concentration coefficient represents an equalising contribution from the income component. A positive concentration coefficient implies that the contribution is disequalising. A third case appears when  $\kappa_i = 0$ , which corresponds to the case where an equal amount of component  $i$  is received by every individual. The inequality share ( $\tau_i$ ) of an income component is defined by

$$(F.2) \quad \tau_i = \frac{\frac{\mu_i}{\mu} \kappa_i}{G}.$$

The decomposition method (F.1) is applied for three income components – gross income, direct taxes and publicly funded services. The income share  $\mu_i/\mu$  and the concentration coefficient  $\kappa_i$  represent the basic parameters of the decomposition.

### *Decomposition on gross income, taxes and public in-kind transfers*

The counterfactual approach provides information of the total distributional effect of taxes and public services, where the counterfactual distribution forms the benchmark. The purpose of this appendix is to identify the separate effects of taxes and public services on inequality in the distribution of extended income, where complete equality of extended income forms the benchmark. Thus, the income components in question are gross income, taxes and public in-kind transfers (childcare, education, health care and long-term care). Gross income includes earnings, self-employment income, capital income, inter-household transfers and public cash transfers.

Disposable income is defined by

$$(F.3) \quad x_{0hk} = \tilde{x}_{0hk} - t_{hk},$$

where  $\tilde{x}_{0hk}$  is gross income, and  $t_{hk}$  is the sum of social contributions and taxes on income and wealth paid by household  $h$  in country  $k$ . Extended income is defined by

$$(F.4) \quad x_{+hk} = \tilde{x}_{0hk} - t_{hk} + z_{hk},$$

where  $z_{hk} = \sum_{i=1}^S x_{ihk}$  is the assessed value of public services received by household  $h$  living in country  $k$ . To account for different needs for cash income, we divide cash income by the EU equivalence scale.

Table E.1 displays the decomposition of inequality in extended income measure adjusted by the NA scale on the three components. While the concentration coefficient shows whether an income component is equalising or disequalising (compared to complete equality), the income share shows the share this income component constitutes of extended income.

As expected, gross income exhibits a strong disequalising effect, whereas taxes have a strong equalising effect in all countries. By contrast, the concentration coefficient of public services is very small in all countries. This means that public services show to have a similar effect on the Gini coefficient as a lump-sum transfer.<sup>24</sup> These features are reflected by the inequality share as well. Accordingly, the decomposition method demonstrates that taxes have a much stronger equalising effect than in-kind transfers, which also means that the progressive tax profiles exhibit the largest equalising effects captured by the counterfactual approach.

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<sup>24</sup> For comparison, we perform a counterfactual analysis where the public in-kind transfers are redistributed as equal lump-sum transfers to all individuals within each country. As compared to the actual distribution, the “lump sum” associated distribution yields higher inequality and poverty estimates for extended income (NA). Moreover, the composition of poverty is changed, with more poverty among families with children and elderly people. Inequality and poverty estimates based on the lump-sum distribution are available upon request.

**Table F.1: Decomposition of the Gini coefficient for extended income (NA) by three income components, 2012**

Country	Inequality share			Incomeshare			Concentration coefficient		
	Gross income	Tax	Public services	Gross income	Tax	Public services	Gross income	Tax	Public services
Austria	1.598	-0.566	-0.032	1.082	-0.280	0.198	0.330	0.452	-0.037
Belgium	1.543	-0.515	-0.028	1.055	-0.254	0.199	0.313	0.434	-0.030
Czech Republic	1.362	-0.329	-0.034	0.929	-0.131	0.202	0.296	0.508	-0.034
Denmark	1.642	-0.641	-0.002	1.134	-0.367	0.234	0.291	0.350	-0.001
Estonia	1.289	-0.289	0.000	1.003	-0.164	0.161	0.368	0.505	0.000
Finland	1.532	-0.531	-0.001	1.052	-0.254	0.201	0.300	0.431	-0.001
France	1.347	-0.334	-0.012	1.001	-0.195	0.194	0.332	0.424	-0.015
Germany	1.529	-0.523	-0.006	1.105	-0.308	0.203	0.338	0.415	-0.007
Greece	1.524	-0.511	-0.013	1.132	-0.322	0.190	0.381	0.449	-0.020
Hungary	1.332	-0.320	-0.012	1.010	-0.196	0.187	0.313	0.387	-0.015
Iceland	1.635	-0.619	-0.016	1.095	-0.314	0.219	0.284	0.375	-0.014
Ireland	1.507	-0.500	-0.007	0.982	-0.202	0.220	0.378	0.608	-0.008
Italy	1.515	-0.498	-0.018	1.114	-0.289	0.175	0.378	0.479	-0.028
Luxembourg	1.438	-0.418	-0.020	1.010	-0.229	0.219	0.338	0.433	-0.022
Netherlands	1.746	-0.765	0.020	1.187	-0.421	0.234	0.289	0.358	0.017
Norway	1.497	-0.547	0.050	1.035	-0.274	0.239	0.252	0.348	0.036
Poland	1.344	-0.338	-0.005	1.073	-0.241	0.167	0.330	0.371	-0.009
Portugal	1.445	-0.427	-0.018	1.025	-0.207	0.182	0.401	0.587	-0.029
Slovakia	1.183	-0.127	-0.056	0.882	-0.083	0.201	0.272	0.311	-0.056
Slovenia	1.549	-0.566	0.016	1.061	-0.247	0.185	0.307	0.482	0.019
Spain	1.326	-0.311	-0.015	0.992	-0.158	0.166	0.383	0.564	-0.027
Sweden	1.478	-0.503	0.025	1.013	-0.264	0.251	0.273	0.357	0.019
Switzerland	1.457	-0.412	-0.045	1.158	-0.327	0.168	0.306	0.307	-0.066
United Kingdom	1.570	-0.540	-0.030	1.039	-0.235	0.196	0.370	0.565	-0.037

*Notes.* The unit of analysis is individuals. The tax component includes all taxes on income and wealth and social contributions.

## **Appendix G: Gini estimates with equivalent individuals as unit of analysis**

When estimating the Gini coefficient and other measures of inequality in a heterogeneous population, there are different methods for weighting different household types. The standard approach, favoured for instance by Shorrocks (2004), assigns a weight given by household size (number of household members) to each household. This means that the unit of analysis is given by individuals, and the Lorenz curve is defined over the population of individuals and equivalent incomes assigned to individuals. An alternative method is proposed by Ebert (1997, 1999) and justified by Trannoy (2003). In this alternative method, households are weighted by household needs as measured by the equivalence scale. This means that the unit of analysis is given by “equivalent individuals”. Ebert and Moyes (2003) and Shorrocks (2004) argue that the two weighting methods are supported by different ethical principles. In this paper we follow the standard approach weighting households by their size, which means that individuals are treated as the unit of analysis. As a sensitivity test, we report inequality estimates based on the alternative approach weighting households by their equivalence scale and treating equivalent individuals as the unit of analysis. We refer to Decoster and Ooghe (2002) for a previous empirical analysis comparing the different approaches. However, the choice between individuals and “equivalent individuals” as the unit of analysis has received less attention in the empirical literature than it deserves, given the controversy over conflicting ethical principles in the theoretical literature on income distribution. The empirical results show that inequality estimates for European countries do not differ significantly between the two different weighting methods.

**Table G.1: Gini-coefficients for equivalent individuals in the distributions of income by income definition and country**

<b>Income definition</b>	<b>Cash income (EU)</b>	<b>Extended income (EU)</b>	<b>Extended income (NA)</b>
Austria	27.9	22.3	22.5
Belgium	26.8	21.5	21.4
Czech Republic	25.1	19.7	20.0
Denmark	26.6	21.2	20.4
Estonia	33.6	28.4	29.1
Finland	25.9	21.2	20.8
France	30.4	24.9	24.8
Germany	30.4	24.9	24.5
Greece	34.1	27.6	27.9
Hungary	28.4	23.4	23.6
Iceland	24.6	19.7	19.3
Ireland	31.6	24.8	24.8
Italy	33.2	27.2	27.9
Luxembourg	30.5	24.1	24.0
Netherlands	25.3	20.6	19.8
Norway	22.7	18.6	17.5
Poland	31.0	25.9	26.4
Portugal	34.7	28.0	28.4
Slovakia	24.7	18.8	20.2
Slovenia	25.8	22.0	21.5
Spain	33.8	28.0	28.6
Sweden	25.4	19.8	18.9
Switzerland	29.3	23.6	24.6
United Kingdom	30.3	23.9	24.4

*Notes.* The unit of analysis is equivalent individuals.



**Table G.2: Decomposition of the Gini-coefficient for extended income (NA) by three income components**

Income component	Inequality share			Income share			Concentration coefficient		
	Cash	Tax	In-kind	Cash	Tax	In-kind	Cash	Tax	In-kind
Austria	1.61	-0.58	-0.03	1.09	-0.29	0.20	0.33	0.45	-0.03
Belgium	1.54	-0.52	-0.03	1.05	-0.25	0.20	0.31	0.43	-0.03
Czech Republic	1.37	-0.33	-0.04	0.93	-0.13	0.20	0.30	0.50	-0.04
Denmark	1.64	-0.64	0.00	1.13	-0.37	0.23	0.30	0.36	0.00
Estonia	1.29	-0.30	0.01	1.00	-0.17	0.16	0.37	0.52	0.01
Finland	1.55	-0.54	-0.01	1.07	-0.26	0.19	0.31	0.44	-0.01
France	1.35	-0.34	-0.01	1.01	-0.20	0.19	0.33	0.43	-0.02
Germany	1.52	-0.52	0.00	1.10	-0.30	0.21	0.34	0.42	0.00
Greece	1.52	-0.51	-0.01	1.12	-0.31	0.19	0.38	0.45	-0.02
Hungary	1.33	-0.32	-0.01	1.01	-0.20	0.19	0.31	0.38	-0.01
Iceland	1.63	-0.61	-0.02	1.10	-0.32	0.22	0.28	0.37	-0.01
Ireland	1.48	-0.48	0.00	0.97	-0.19	0.23	0.38	0.62	0.00
Italy	1.52	-0.50	-0.02	1.11	-0.29	0.18	0.38	0.48	-0.03
Latvia	1.29	-0.35	0.06	1.02	-0.20	0.19	0.39	0.53	0.10
Lithuania	1.23	-0.23	0.00	0.96	-0.13	0.17	0.39	0.54	0.00
Luxembourg	1.43	-0.41	-0.01	1.01	-0.24	0.22	0.34	0.42	-0.02
Netherlands	1.73	-0.76	0.04	1.20	-0.43	0.23	0.29	0.36	0.03
Norway	1.48	-0.54	0.06	1.03	-0.27	0.24	0.25	0.35	0.04
Poland	1.35	-0.34	-0.01	1.08	-0.24	0.17	0.33	0.37	-0.01
Portugal	1.45	-0.43	-0.02	1.02	-0.21	0.19	0.40	0.59	-0.03
Slovakia	1.54	-0.56	0.01	1.06	-0.24	0.18	0.31	0.49	0.02
Slovenia	1.47	-0.50	0.03	1.01	-0.26	0.25	0.28	0.36	0.02
Spain	1.33	-0.32	-0.02	1.00	-0.16	0.17	0.38	0.55	-0.03
Sweden	1.19	-0.13	-0.06	0.89	-0.08	0.20	0.27	0.32	-0.06
Switzerland	1.47	-0.42	-0.05	1.17	-0.34	0.17	0.31	0.31	-0.07
United Kingdom	1.57	-0.54	-0.03	1.05	-0.24	0.19	0.37	0.56	-0.04

*Notes.* The unit of analysis is equivalent individuals.

## **Appendix H: Alternative inequality measures**

Empirical analyses of inequality in income distributions are normally based on the Lorenz curve. To summarise the information content of the Lorenz curve and to achieve rankings of intersecting Lorenz curves the standard approach is to employ the Gini coefficient, which is equal to twice the area between the Lorenz curve and its equality reference. However, since a single measure of inequality cannot capture all aspects of the inequality exhibited by a Lorenz curve, we supplement the information provided by the Gini coefficient by applying two closely related measures of inequality  $C_1$  and  $C_2$  discussed by Aaberge (2007). Together with the Gini coefficient these two measures form Gini's Nuclear Family of inequality measures. Whilst it can be shown that the Gini coefficient normally pays particular attention to changes that occur in the middle part of the income distribution, the  $C_1$  is shown to be sensitive to changes that occur in the lower part and  $C_2$  to the upper parts of the income distribution, respectively. This sensitivity test ensures that a broader understanding of the distribution of income is acquired.<sup>25</sup>

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<sup>25</sup> See Aaberge (2007) for further details.

**Table H.1: C<sub>1</sub>-coefficients for individuals in the distributions of income by income definition and country, 2012**

<b>Income definition</b>	<b>Cash income (EU)</b>	<b>Extended income (EU)</b>	<b>Extended income (NA)</b>
Austria	39.3	31.8	32.1
Belgium	26.5	30.9	30.7
Czech Republic	24.9	28.2	28.4
Denmark	25.9	30.4	29.4
Estonia	33.1	38.6	39.2
Finland	25.3	30.0	29.3
France	30.2	33.3	33.2
Germany	29.9	34.4	33.9
Greece	34.4	38.9	39.2
Hungary	28.7	32.9	33.0
Iceland	24.2	28.1	27.5
Ireland	30.9	33.8	33.8
Italy	33.0	38.1	38.8
Luxembourg	30.4	32.9	32.7
Netherlands	24.9	28.9	28.1
Norway	22.1	27.4	26.4
Poland	30.9	35.7	36.2
Portugal	34.5	38.3	38.7
Slovakia	24.7	27.9	29.5
Slovenia	25.2	31.2	30.5
Spain	33.8	39.1	39.8
Sweden	24.7	29.3	28.4
Switzerland	29.0	32.2	33.2
United Kingdom	30.0	33.5	34.0

**Table H.2: C<sub>1</sub>-coefficients for equivalent individuals in the distributions of income by income definition and country, 2012**

<b>Income definition</b>	<b>Cash income (EU)</b>	<b>Extended income (EU)</b>	<b>Extended income (NA)</b>
Austria	39.8	32.9	32.3
Belgium	38.0	31.8	30.7
Czech Republic	34.9	28.5	28.1
Denmark	37.9	31.8	29.8
Estonia	45.6	39.4	39.6
Finland	36.1	31.0	29.5
France	40.6	34.0	33.3
Germany	41.9	35.4	34.1
Greece	47.3	39.0	38.8
Hungary	39.6	33.3	32.8
Iceland	34.9	29.3	27.8
Ireland	43.1	34.9	34.1
Italy	46.3	38.7	38.8
Luxembourg	41.5	34.0	33.1
Netherlands	35.6	30.1	28.3
Norway	33.4	29.0	26.7
Poland	42.6	36.3	36.2
Portugal	46.7	38.7	38.5
Slovakia	36.1	28.2	29.3
Slovenia	36.9	32.2	31.0
Spain	47.1	39.6	39.8
Sweden	37.2	30.7	28.7
Switzerland	40.2	32.8	33.4
United Kingdom	41.8	34.3	33.9

*Notes.* The unit of analysis is equivalent individuals.

**Table H.3: C<sub>3</sub>-coefficients for individuals in the distributions of income by income definition and country, 2012**

<b>Income definition</b>	<b>Cash income (EU)</b>	<b>Extended income (EU)</b>	<b>Extended income (NA)</b>
Austria	22.2	17.2	18.1
Belgium	21.2	16.6	17.2
Czech Republic	20.5	15.8	16.6
Denmark	21.1	16.0	16.3
Estonia	27.2	22.5	23.5
Finland	20.6	16.3	16.7
France	25.3	20.3	20.7
Germany	24.6	19.5	20.1
Greece	28.1	22.3	23.1
Hungary	23.4	18.8	19.5
Iceland	19.6	15.0	15.4
Ireland	25.5	19.5	20.3
Italy	26.9	21.7	22.7
Luxembourg	25.2	19.1	19.6
Netherlands	20.3	15.8	16.0
Norway	17.6	13.6	13.7
Poland	25.3	20.7	21.7
Portugal	28.6	22.7	23.6
Slovakia	19.5	14.6	16.1
Slovenia	20.1	16.7	16.7
Spain	27.4	22.4	23.3
Sweden	19.6	14.6	14.7
Switzerland	23.9	18.9	20.1
United Kingdom	24.6	19.0	20.1

**Table H.4: C<sub>3</sub>-coefficients for equivalent individuals in the distributions of income by income definition and country, 2012.**

<b>Income definition</b>	<b>Cash income (EU)</b>	<b>Extended income (EU)</b>	<b>Extended income (NA)</b>
Austria	22.5	17.8	18.2
Belgium	21.6	17.0	17.2
Czech Republic	20.6	16.0	16.5
Denmark	21.7	16.9	16.6
Estonia	27.6	23.2	24.0
Finland	21.0	16.9	16.9
France	25.5	20.8	20.9
Germany	25.0	20.1	20.2
Greece	27.8	22.4	22.9
Hungary	23.3	19.0	19.4
Iceland	20.0	15.7	15.7
Ireland	26.1	20.1	20.4
Italy	27.1	22.1	22.8
Luxembourg	25.2	19.7	19.9
Netherlands	20.6	16.4	16.0
Norway	18.1	14.4	13.8
Poland	25.5	21.1	21.7
Portugal	28.8	23.1	23.6
Slovakia	19.5	14.8	16.1
Slovenia	20.6	17.4	17.1
Spain	27.4	22.7	23.3
Sweden	20.1	15.3	14.9
Switzerland	24.1	19.3	20.3
United Kingdom	24.8	19.3	20.0

*Notes.* The unit of analysis is equivalent individuals.

## Appendix I: Proofs

*Proof of Proposition I.* Inserting (2.4) into (2.1) and (2.2) yields

$$(I.1) \quad W_k(C_{1k}, C_{2k}, \dots, C_{H_k k}) = \frac{1}{\gamma_{++k}} \sum_{h=1}^{H_k} \gamma_{+hk} V\left(\frac{C_{hk}}{\gamma_{+hk}}\right).$$

A marginal progressive needs-adjusted transfer from household  $j$  to household  $g$  in country  $k$  leads to the following change in welfare:

$$(I.2) \quad \frac{W_k(C_{1k}, C_{2k}, \dots, C_{gk} + \delta, \dots, C_{jk} - \delta, \dots, C_{H_k k}) - W_k(C_{1k}, C_{2k}, \dots, C_{gk}, \dots, C_{jk}, \dots, C_{H_k k})}{\delta} \\ \xrightarrow{\delta \rightarrow 0} \frac{1}{\gamma_{++k}} \left[ V\left(\frac{C_{gk}}{\gamma_{+gk}}\right) - V\left(\frac{C_{jk}}{\gamma_{+jk}}\right) \right].$$

Since the common utility function  $V(\cdot)$  is strictly increasing and concave, and

$(C_{gk}/\gamma_{+gk}) < (C_{jk}/\gamma_{+jk})$ , it follows from (I.2) that  $W_k$  satisfies the BTT principle.

*Proof of Proposition II.* Since  $(\lambda_k \gamma_{+r}^k / \lambda_k \gamma_{++}^k) = (\gamma_{+r}^k / \gamma_{++}^k)$  for all  $k$  we have that  $q_k$  defined by (2.11) satisfies Condition I. Next, inserting (2.11) into (2.9) yields the following common equivalence scale:

$$(I.3) \quad NA_h(r) = \frac{\sum_{k=1}^K w_k \frac{\gamma_{+h}^k}{\gamma_{++}^k}}{\sum_{k=1}^K w_k \frac{\gamma_{+r}^k}{\gamma_{++}^k}}, \quad h = 1, 2, \dots, H,$$

where the notation  $NA_h(r)$  indicates that the NA scale might depend on the chosen reference household  $r$ . Furthermore, let

$$\xi_{rm} = \frac{\sum_{k=1}^K w_k \frac{\gamma_{+r}^k}{\gamma_{++}^k}}{\sum_{k=1}^K w_k \frac{\gamma_{+m}^k}{\gamma_{++}^k}}.$$

By changing reference household from  $r$  to  $m$  we get the following expression for the common equivalence scale:

$$(I.4) \quad NA_h(m) = \frac{\sum_{k=1}^K w_k \frac{\gamma_{+h}^k}{\gamma_{++}^k}}{\sum_{k=1}^K w_k \frac{\gamma_{+m}^k}{\gamma_{++}^k}} = \frac{\sum_{k=1}^K w_k \frac{\gamma_{+r}^k}{\gamma_{++}^k}}{\sum_{k=1}^K w_k \frac{\gamma_{+m}^k}{\gamma_{++}^k}} NA_h(r) = \xi_{rm} NA_h(r), \quad h = 1, 2, \dots, H,$$

which demonstrates that the NA scale with weights defined by (2.11) satisfies Condition II.  $\square$