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## Finance, Trade, Man and Machines: A New-Ricardian Heckscher-Ohlin-Samuelson Model\*

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### ABSTRACT

This paper attempts to build up a Heckscher-Ohlin-Samuelson model of production and trade where capital is introduced outside the production process as a financial capital or credit as per the classical Ricardian wage fund framework. Stock of credit or financial capital as past savings, finances employment and machines or capital goods used in the process of production with Ricardian fixed coefficient technology. Availability of finance does not affect production or pattern of trade only nominal factor prices. International financial flows will not alter pattern of trade, but movement of labour and machines will. Such results change drastically when we consider a model with unemployment and finance dictates real outcomes much more than before. Introducing finance affects trade patterns with unemployment and especially with imperfect credit markets. In a two-period extension with credit demand being allocated for financing R&D expenditure, a rise in interest rate in the subsequent period will motivate perpetual tendencies to invest in machine via R&D so that machine-intensive sector will expand at the expense of the labour-intensive sector. This can account for the secular decline in labour income share as has been observed for some time. Our results are consistent with contemporary empirical evidence and have serious policy implications for role of financial development and quality of institutions for innovation and economic development. Numerical illustration corroborates this.

**JEL Classification:** B12, B13, B17, F11, F63, F65, F16, O12.

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## 1. Introduction

Importance of finance or credit for carrying out production for trade has received much less attention. This is different from common parlance of typical ‘trade-finance’ literature where the focus hinges essentially on financing of trade and commercial transactions so as to ‘mitigate, or reduce, the risks involved in international trade transaction’, involving two parties, exporter and importer.<sup>1</sup> In this paper, based on classical wage-fund theory (Ricardo 1817, Mill 1848) we deal with the issue where credit finances purchase of factor input services like labor costs, capital goods, and other material inputs, and what are consequences of changes in availability of such ‘financial capital’ to alter production structure, trade patterns, and factor returns. Hollander (1984) and Vint (2018) summarise the history and implications of wage-fund doctrine of Mill (1848, 1869). In fact, as wage-fund theory deals with aspects of financing production and trade, this methodological approach is quite pertinent in providing novel and valuable insights on the role of credit-constraints in financing trade and explaining several real-world phenomena like financial crises, and unemployment.

The core reason for engaging in this exercise is that the literature in trade and finance till date has not made use of the wage fund approach in modern trade theory, although wage fund is possibly the earliest framework of introducing financial capital in analysing issues related to trade and growth (Ricardo 1817). Mill (1848, 1869) provided a definitive analysis recognizing the role of finance (capital) to fund labor cost in production. In typical wage-fund doctrine, a fixed amount of fund is available to pay for wages to the workers, i.e., wages are determined by the ratio of capital (wage-fund) available to the employers and the number of employees. However, in this paper, we extend that doctrine to incorporate that the fixed amount of capital or credit is available to fund the costs of production (machinery) and wages (laborers) for their sustenance during the production.

Moreover, the usual way of bringing in the impact of financial problems in trade models has been to consider the role of trade finance. For example, in an empirical paper Chor and Manova (2012) has discussed the adverse impacts of tightened credit conditions and especially the access to trade credit on exports volume. In another paper, Manova (2013) has extended the analysis with similar implications, but in a firm-heterogeneity model with imperfect competition (product differentiation) and highlights the role of financial market imperfections and institutional frictions in shaping trade volume.

All these highlight the significance of introducing financial capital into conventional workhorse of Neo-classical trade models such as, Ricardo, Specific Factor and Heckscher-Ohlin-Samuelson (HOS). Lack of financial capital or barriers to access credit could impair firms’ performance, trade, and could tell upon economic growth and employment. Our papers fill this gap. Incorporating wage-fund theory, entrepreneurial finance and borrowing constraints in the traditional GE model is a novel mechanism. Wage fund theory was developed in the Classical models of Ricardo (1817) and J. S. Mill (1848), but unlike the classical notion, its Neo-Classical treatment considered the features of diminishing returns (DMR)-see Hicks and Hollander (1977), Steedman (1979), Mansechi et al. (1983), Findlay (1984). This paper builds upon precursors, viz., Marjit and Das (2021) and Marjit and Nakanishi (2021) by

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<sup>1</sup> See Global Trade Review for overview <https://www.gtreview.com/what-is-trade-finance/>. The issue of financing of trade, such as, exports and imports are not touched upon in this paper, as our focus is different. However, this could be extended.

incorporating finance in a conventional general equilibrium model of HOS structure using the classical or Ricardian perspective on Wage Fund hypothesis.

The fundamental question we try to address in this paper is –how significant is the role of availability of finance in a standard HOS trade model when financial capital is necessary to purchase the services of labour and machines or capital goods to produce final goods? The answer to this question is critical to assess the mechanism of how finance is likely to affect the key variables of the system. Only then, we can evaluate what financial crisis should do the international trading system in terms of its impact on output, pattern of trade, factor prices, factor mobility etc. In this context, we prove two sets of interesting results in our fully specified general equilibrium HOS model.

Without any distortions such as unemployment or credit market imperfections in the system, shortage of finance does not affect production or pattern of trade in a HOS model. But it does affect nominal factor prices as it affects interest rates and hence can induce factor movements thus affecting the pattern of trade. However, international mobility of finance does not change production or trade but alters income distribution. Mobility of machines and labor will affect the trade pattern.

These results change drastically in a model with unemployment. Finance plays a much more significant role and hence shortage of finance, hallmark of nations under siege of financial crisis, would affect each real variable starting with employment. In a way *without such a distortion*, our framework partly emanates the classical macroeconomic flavour where money does not matter and *with wage distortion*, it has Keynesian outcome.

Relatively modern treatments of classical models with neo-classical flavour such as, Hicks and Hollander(1977), Findlay (1984,1995), Steedman (1979) etc. were usually interested with modelling agriculture with diminishing returns and industry with unlimited supplies of labour at a given level of subsistence real wage. Marjit (2020), Marjit and Das (2021), and Marjit and Nakanishi (2021) have tried to explore the implication of wage fund or stock of credit in a full employment Ricardian trade model, introducing finance or credit in an otherwise well-known text book version of the model. Such an inclusion yields many results of a typical Neo-Classical production theory. It completely replicates Solow (1956) and its antecedents such as optimal growth theory (Ramsey 1928, Cass 1965, Koopmans 1965).

However, here we reconstruct the Jones (1965) framework which is still hailed as a major contribution (See Markusen 2021, Jones 2018), the building block of the trade models, to include ‘Capital (K)’ construed as financial capital used to finance homogeneous labor costs, and machines or capital goods (which embodies the role of physical capital simultaneously working with the labor (L))<sup>2</sup>. In this paper, capital is not used ‘directly’ as a factor of production within the production process, it is represented as finance that makes production possible. Thus, unlike the canonical Neo-classical models capital operates *outside* the production process to replicate the 2X2 model with two-sectors, two inputs. We have a given supply of labour (L) and stock of machines (M) that are needed to be used in the production of two goods X and Y. But the wage bill and the expenses to acquire ‘M’ require finance at the beginning of the period. Total value of payments for wage bills and capital equipment or machines must equal ‘K’, the stock of credit or finance. At the end of the period, outputs (X and Y) are sold, revenues are generated and financiers, owners of ‘K’ are paid back in terms of the principal and interest

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<sup>2</sup> We can conceive this as putty-clay type where flexibility or malleability of capital goods is maintained. ‘M’ is produced means of production, which is used as intermediate input for final good production.

(r). Thus, the system starts with stock of K, L, and M and the general equilibrium generates the wage (w), price of machines ( $p_m$ ) and interest rate 'r', along with outputs of X and Y and relative price  $P_x/P_y$  given a (relative) demand function. The major purpose of this paper is to highlight what role finance, i.e., K plays in such a model, in terms of production, factor prices and trade pattern. This is novel with financial capital. Finance or credit in Neo-Classical trade theory is not very common. This is about how finance alters fundamental trade theorems and related theoretical outcomes. In fact, role of imperfect credit market in a proper trade theoretic framework has been discussed by Jones and Marjit (2001), Matsuyama(2005), Antras and Caballero(2009), Manova (2008), Manova(2013), Manova et al. (2009), Amiti and Weinstein (2011), Egger and Keuschnigg (2015), Egger et al. (2018), Marjit and Misra(2020), etc. Egger et al. (2017) has shown that credit constraints and asymmetric information could deter firms to enter and compete in the global economy while financial development eliminating frictions could would create pro-competitive as well as productivity effects.

Our purpose and framework are entirely different from these papers. *Firstly*, we want to explore what happens when finance is introduced in a standard competitive general equilibrium model. As such it belongs to the class of more recent works on the usefulness of competitive trade models, such as Jones and Marjit(1985, 2003, 2009), Marjit and Kar(2018), Jones(2018), Marjit Mandal and Nakanishi(2020), Das (2013), etc. *Second*, because we do not use standard and neo-classical production model but a framework, which is Ricardian in nature, we bring in classical wage fund hypothesis to introduce financial capital and withdraw capital as a factor of production from within the product process. Another purpose is to compare the theoretical insights derived from this new Ricardian structure with the well-known trade theorems. *Third*, in several extensions of the benchmark model we will explore how this methodological innovation of finance-economics interface could provide some fresh perspectives on some contemporary issues of relevance. For example, how financial crisis or unanticipated financial shocks affect the entire system or, how international financial flows, trade flows and factor flows are interrelated, and how credit market imperfections will affect unemployment are analysed. These have important insights for role of financial development, better institutions, governance, and financial access for economic growth.

Last but not the least, whether men and machines are helped or hurt by trade or finance is also a valuable insight derived in this model. This is very pertinent in the context of emergence of fourth industrial revolution or artificial intelligence (AI) or ICT causing disruptions in sectoral adjustments, and changing the landscape of labour market with secular decline in the labor share of income, which raised clamor in the crisis-laden global economy (see Acemoglu 2003, Caselli and Manning 2019, Korinek and Stiglitz 2017, Acemoglu and Restrepo 2019). In most of the high-income and upper middle-income as well as emerging market, developing economies share of wage to GDP has been monotonically declining over three decades or so at an alarming rates (see Karabarbounis and Neiman 2014a&b, Doan and Wan 2017).<sup>3</sup> As the growing apprehension about the impact of automation (enabled by artificial intelligence) in replacing tasks or routine-jobs and creating jobs polarization between non-routine cognitive skills and non-routine manual workers and those in middle skill looms large, the division of income between labor and capital is a growing concern. How bankers or entrepreneurs financial allocation (via R&D-financing) will induce potential emergence of technical change in a direction favouring machine intelligence rather

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<sup>3</sup> Here we do not distinguish between skill-differentiated labor force and hence consider composite labor. Of course, there will be differential impacts across labor types depending on their absorptive capacity or technology readiness. But for parsimony, we refrain from such analysis. More on this in Section 4.3 to follow.

than humans is discussed. In particular, we show that unlike explicit R&D-based growth—R&D financing in the sector intensive in machines experiencing favourable exogenous shock a la price rise—will lead to unidirectional innovation-driven growth furthering automation, increasing machine premium, a declining labor share (in total), and possible emergence of unemployment. This could be interpreted through the lens of directed technical change theory a la Acemoglu (1998 & 2002).

The results that we derive here are as follows: (i) with full-employment, level of finance does not affect the pattern of trade, only influences factor prices and interest rate; (ii) even without trade in goods, factors of production and financial capital could be traded without any change in the relative returns to labour and machines and relative prices; (iii) direction of financial flow is *reverse* of the direction of factor flows. Goods trade can lead to financial flows in any direction depending on financing intensity of man and machines.

Interestingly, we retain the Stolper–Samuelson and Rybczynski type result or magnification outcomes as formalized in Jones(1965, 2018) as in this framework existence of finance does not disturb these. However, *absolute values* of factor prices are uniquely affected by the availability of finance.

The paper is laid out as follows. In the second section, we develop the model and describe the determination of equilibrium, pattern of trade, role of credit and factor flows, and compare with standard HOS results. The third section considers endowment and price effects, while in the fourth section we introduce four critical extensions of the model in terms of factor flows, fixed wage and unemployment, imperfect credit market and credit rationing, and allocating finance for R&D-investment in machines respectively. Fifth section offers a numerical illustrative simulation with parametric changes. The last section concludes.

## 2. Model and Equilibrium

The economy produces two goods X and Y with labour (L) and machines (M) with *fixed coefficient* production functions. We deliberately abstract from possibilities of substitution to retain the Ricardian flavour, thus, features of DMR and/or, DMP are set aside. Thus, factor-substitution is ruled out. At the beginning of the period the economy inherits K as the stock of credit of finance to be invested in production, a given supply of labour L and a stock of machines.<sup>4</sup> Demand for credit (K) is induced via demand for ‘L’ and ‘M’ and resultant cost. Thus, credit market equilibrium must enforce that ‘K’ is sufficient to finance  $(WL + p_m.M)$  where ‘W’ is the wage rate and  $p_m$  is the price of machines, to be determined via

$$K=WL + p_m.M \quad (1)$$

Production involves time. At the beginning of the period labour is hired and machines are purchased, financed by loans from the bank or financiers. After production and sale are over, the borrowed amount is returned with interest ‘r’.<sup>5</sup> With perfectly competitive markets, price of goods will

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<sup>4</sup> In a dynamic model, ‘K’ can change as in say, via typically perpetual inventory accumulation over time.

<sup>5</sup> However, we do not build intertemporal framework but as in different static equilibrium, production and sales occur at a single point of time. Without sale proceeds to pay interest rate, at a particular time period ‘credit-capital’ can’t be borrowed. Extending the model to 3-Sector with skill-unskilled split would transform the equation (1) as  $K= WL+P_m.M+W_s.S$ , where S is skilled and  $W_s$  is their wage. That does not change the pivotal elements of the paper.

be just sufficient to cover average wage cost, machine cost and interest payments. The notations are as follows:

$a_{ij}$  : fixed unit input requirement of 'i' per unit of  $j^{\text{th}}$  product,  $i \in \{L, M\}$  and  $j \in \{X, Y\}$ .

$a_{Lj}$  : fixed unit labor requirement per unit of  $j^{\text{th}}$  product,  $j \in \{X, Y\}$ .

$a_{Mj}$  : fixed per unit requirement of capital equipment for the  $j^{\text{th}}$  product.  $j \in \{X, Y\}$ .

$\lambda_{ij}$  : endowment shares of  $i^{\text{th}}$  resource in the production of X, Y

$\theta_{ij}$  : Cost-shares of  $i^{\text{th}}$  resource in the production of X, Y

$P_j$  : final good prices for  $j \in \{X, Y\}$ .

$p_M$  : unit price of the machines.

$P$  : relative price of X i.e.  $P = \frac{P_X}{P_Y}$ . 'Y' is the numeraire good (i.e.,  $P_Y = 1$ ).

$\hat{V}$  : proportional changes of any generic variable, V, such that  $\hat{V} = \frac{dV}{V}$

The following system of Competitive price equations [i.e., (2) and (3)], and the full-employment conditions [viz., (4) and (5)] for primary factor 'L and material inputs 'M' determine the supply side of the model as below:

$$[W a_{LX} + p_M a_{MX}] (1+r) = P \quad (2)$$

$$[W a_{LY} + p_M a_{MY}] (1+r) = 1 \quad (3)$$

$$a_{LX}X + a_{LY}Y = L \quad (4)$$

$$a_{MX}X + a_{MY}Y = M \quad (5)$$

where (2)-(3) are competitive price conditions,  $P = AC$  (average cost), and (4)-(5) are full employment constraints. With a given credit size 'K', specific size of the labor force, and fixed stock of machines at a point in time, with full-employment of resources certain level of 'W' and ' $p_M$ ' are paid to the workers and the industrialists owning the machines of certain vintage. Given fixed coefficients  $a_{ij}$ ,  $\bar{L}$ , and  $\bar{M}$ , X and Y are determined by (4) and (5) and depend only on (L, M) and technology, independent of P and K. From (2) and (3), W and  $p_M$  are determined as functions of P and (1+r). Given (exogenous) P, we plug that into (1) to solve for (1+r). Given P, L, M, r and  $a_{ij}$ 's where  $i \in \{L, M\}$ , we determine W,  $p_M$ , X, and Y from Equations (2)–(5). L and M are given as stocks of labor and machine from the last period. Hence,

$$K^d = W(r).L + p_M(r).M \quad (6)$$

where  $K^d$  is demand for capital. 'P' is given and hence suppressed in  $W(r)$  and  $p_M(r)$ .

$$\text{As } W' < 0 \text{ and } p_M' < 0, K_d'(r) = W'(r).L + p_M'(r).M < 0 \quad (7)$$



Financial market equilibrium condition is given by:  $\bar{K} = K_d(r)$  (8)

Implication of (8) is that  $K_d$  is an inverse function of 'r', and 'P' determines 'W' and 'p<sub>M</sub>', and hence 'r'.

Right hand side (RHS) of (8) is demand for credit ( $K_d$ ) to equilibrate with supply ( $\bar{K} = K_s$ ). Eq. (8) determines 'r' in equilibrium. Since (P, L, M, K) are parameters,

$$r = r(P, \bar{L}, \bar{M}, \bar{K}) \quad (9)$$

One can easily show using Caves, Frankel and Jones (2011) and Feenstra (2003) that, with 'Λ' denoting proportional change:

$$\hat{X} - \hat{Y} = \alpha(\hat{M} - \hat{L})$$

where  $\alpha = \frac{1}{\lambda_{MX} - \lambda_{MY}} > 0$ , and due to intensity assumption  $\lambda_{MY} < \lambda_{MX}$ . Or, via integration

$$\left[ \frac{X}{Y} \right]^s = f \left[ \frac{M}{L} \right], f' > 0 \quad (10a)$$

This is relative supply (RS) of X vis-à-vis Y with  $f' > 0$ .

We assume negatively sloped homothetic demand to express Relative Demand (RD) as below:

$$\left[ \frac{X}{Y} \right]^d = D(P), D' < 0 \quad (10b)$$

Using (10a) and (10b) RS-RD conjointly determine market-clearing for X and Y so that equilibrium  $P = P_e$  can be expressed as .

$$P_e = F \left( \frac{M}{L} \right), F' < 0 \quad (10c)$$

This completes the determination of the general equilibrium. Thus, we determine X, Y; W, (1+r), p<sub>M</sub>, P and RD. Equation (1) representing credit-constraint plays crucial role to internalize the demand and supply of 'L' and 'M' as it represents matching demand and supply via two full-employment conditions.

### 3. Comparative Static Changes.<sup>6</sup>

We consider the following parametric changes—changes in P (exogenous) and changes in K—to compare and contrast with the conventional HOS setup.

#### 3.1 Price Effects

Following CFJ (2011)—using (2) and (3) and assuming  $\hat{P} = 0$ —we derive that:

<sup>6</sup> For parsimony, most of the detailed derivations are relegated to the Appendices 1, 2 and 3.

$$\widehat{W}\theta_{LX} + \widehat{p}_M\theta_{MX} = -(\widehat{1+r}) \quad (11a)$$

$$\widehat{W}\theta_{LY} + \widehat{p}_M\theta_{MY} = -(\widehat{1+r}) \quad (11b)$$

$$\text{where } \theta_{LX} + \theta_{MX} = \theta_{LY} + \theta_{MY} = 1$$

Assume X is relatively M-intensive, and Y is relatively L-intensive. Also,  $\theta_{MX} > \theta_{MY}$  and equivalently,  $\theta_{LY} > \theta_{LX}$ . Hence,  $|\theta| = \theta_{LX}\theta_{MY} - \theta_{MX}\theta_{LY} = \theta_{MY} - \theta_{MX} = \theta_{LX} - \theta_{LY} \Rightarrow |\theta| < 0$ . For our results, intensity ranking can be reversed without anticipating any significant changes.

Using (11a) and (11b), by Cramer's rule,  $(\widehat{1+r}) = -\widehat{p}_M = |\theta|$ . It is obvious that given P, from (2) and (3),  $(\widehat{W}, \widehat{p}_M) < 0$  if  $(\widehat{1+r}) > 0$ . As mentioned above (see Appendix 2), following Caves, Frankel and Jones (CFJ 2011) it is straightforward to show that

$$\widehat{W} = \widehat{P} \frac{\theta_{MY}}{|\theta|} \quad (12a)$$

$$\widehat{p}_M = -\widehat{P} \frac{\theta_{LY}}{|\theta|} \quad (12b)$$

which on simplification gives:

$$\widehat{p}_M - \widehat{W} = \frac{-\widehat{P}}{|\theta|} > 0, \text{ as } |\theta| < 0, \widehat{P} > 0$$

Equations (12a) and (12b) occur at a given 'r'. From (12a) and (12b), it can be inferred that:

$$\widehat{p}_M > \widehat{P} > 0 > \widehat{W} \quad (12c)$$

It is well-known magnification effect (Jones 1965, and CFJ 2011).

From (6), demand for credit and its allocation is given by derived demand for credit ( $K_d$ ) to finance costs of 'L' and 'M' such that  $\widehat{K}_d = \lambda_{LK}\widehat{W} + \lambda_{MK}\widehat{p}_M$  (13)

Here,  $\lambda_{LK}$  is share of credit-finance  $K_d$  devoted to wage bill (for labor services) and  $\lambda_{MK}$  is that for purchasing capital goods so that  $\lambda_{LK} + \lambda_{MK} = 1$ . This indirectly affects production via financing equipment purchase.

$$\text{Furthermore, using (12a\&b), } \widehat{K}_d = \frac{\widehat{P}}{|\theta|} [\lambda_{LK}\theta_{MY} - \lambda_{MK}\theta_{LY}] \quad (14)$$

$$\text{Eq. (14) suggests that when } |\theta| < 0, \text{ if } \widehat{P} > 0, \widehat{K}_d < 0 \text{ iff } \lambda_{LK}\theta_{MY} > \lambda_{MK}\theta_{LY} \quad (15)$$

$$\text{Using (13), we write } \widehat{K}_d = -(\widehat{1+r}) = \frac{r\widehat{r}}{(1+r)} = \widehat{K}_d[(\widehat{1+r}), \widehat{W}, \widehat{p}_M], \quad (15a)$$

Demand for credit or finance drops with a rise in 'P' as 'W' drops and share of investment in 'L' is relatively large, i.e., high  $\lambda_{LK}$  and/or, drop in W is large, i.e., high  $\theta_{MY}$ . Thus,  $K_d$  can rise or fall and given  $K_s = \bar{K}$ , 'r' can also rise or fall simultaneously as  $K'_d < 0$  to clear the financial market. Given  $(1+r)$ , with rise in P (=  $P_X$ ),  $\hat{X} > 0$ , demand for 'M' rises (as X is relatively more M-intensive), resulting in rise in 'p<sub>M</sub>' with  $K_s = \bar{K}$ , and eventually  $(1+r)$  will rise as  $K_d$  rises, given  $\lambda_{LK}$ , via Eq. (15). Intuitively speaking, when 'p<sub>M</sub>' rises and 'W' falls higher value of  $\lambda_{MK}$  compared to  $\lambda_{LK}$  will imply 'K<sub>d</sub>' will increase pushing  $(1+r)$  upward to rise. Higher value of  $\lambda_{LK}$  will mean 'K<sub>d</sub>' will shrink and correspondingly,  $(1+r)$  falls.

For better understanding, we assume that  $\lambda_{LK}$  is low, so that  $\hat{P} > 0$  will imply  $\hat{K}_d > 0$  and  $\hat{r} > 0$ .

Hence, as 'P' goes up  $\frac{W}{P_M}$  will fall and 'r' will go up. One can think of the impact in two stages: first,

given 'r', 'W' goes down and p<sub>M</sub> goes up; second, as 'r' goes up both will drop.  $\frac{W}{P_M}$  remains the same at

a given P. Given L and M,  $\bar{K}$  determines the nominal value of these stocks determining 'r' and hence, from given 'P', determining 'W' and 'p<sub>M</sub>'. This leads us to write the following propositions.

**Proposition 1:**  $\hat{r} \geq 0$  iff  $\hat{P} > 0$  and  $\hat{K}_d \geq 0$  iff  $\lambda_{MK}\theta_{LY} - \lambda_{LK}\theta_{MY} \geq 0 \Rightarrow \lambda_{MK}\theta_{LY} \geq \lambda_{LK}\theta_{MY}$

**Proof:** See the discussion above. From equation (14) and (15), effects on 'r' depends on relative-intensity of finance in sectors (i.e., whether M-intensive or L-intensive). See appendix 2 for derivation.

**Proposition 2:** Availability of finance does not affect  $\frac{W}{P_M}$ , but it affects absolute values of 'W' and 'p<sub>M</sub>'

via changes in 'r'.

**Proof:** Higher  $\bar{K}$  will reduce 'r' to equilibrate financial market, increasing (W, p<sub>M</sub>). But at a given P,  $\frac{W}{P_M}$

remains the same (as discussed above). QED.

### 3.2 Endowment effects

**Proposition 3:** Given  $P = \bar{P}$ ,  $\hat{L} = 0, \hat{M} = 0, \hat{K}_s > 0$  does not affect trade patterns.

**Proof:**  $\hat{K}_d = 0, \hat{K}_s > 0$  means supply of credit increases resulting in lower  $(1+r)$  ( $\hat{r} < 0$ ).

As  $\hat{P} = 0$ , from Eq. (11a&b),  $(1+r) < 0$ . 'p<sub>M</sub>' rises along with 'W'. Given  $\hat{L} = 0, \hat{M} = 0$ , via Eq (10c), M/L is unaltered meaning RD-RS remains the same, with no change in relative prices P<sub>e</sub>. Thus,  $\hat{X} = 0, \hat{Y} = 0$ . Demand for L and M rises due to credit availability with fixed supply of resources causing p<sub>M</sub> and W to rise. Real wage is unaffected. This result is analogous to the canonical macroeconomic systems where credit is similar to money supply and it has *neutrality* (i.e., classical dichotomy is valid) in the same sense that it affects absolute factor prices p<sub>M</sub> and W, but not the relative ones and the output itself as X and Y do not change.<sup>7</sup> This is similar to Marjit and Das (2021) in a Ricardian Specific Factor framework. With

<sup>7</sup> See Patinkin (1958), Lucas (1990). Derivations are in Appendix 2 and 3.

perfect financial market, competitive firms can get as much as they want at given 'r' and hence, it does not play an important role *unless imperfection is built in*. Differences in credit availability (i.e., finance) and its allocation across home and the foreign (or, the rest-of-the-world-ROW) will cause changes in factor prices (viz., W and  $p_M$ ) via factor movements discussed in entirely different model in the literature (Mussa 1991, Lucas 1990, etc.).

#### 4. Further extensions:

##### 4.1 Factor Flows.

Denote foreign variables by '\*' Consider two economies-- home and foreign-- with endowments being  $M, M^*; L, L^*$ , and  $K, K^*$ . These endowments could be the same or different. With differences such that  $L \neq L^*$ , and  $M \neq M^*$ , no doubt trade will occur irrespective of  $K \geq K^*$ .

**Proposition 4:** Given  $M=M^*$  and  $L=L^*$ ,  $\widehat{K} > 0$  implies that with  $K > K^*$ , without trade in X and Y and with *no control* on capital outflows and immigration, home will import labour and machines while financial capital will outflow. International mobility of financial capital does not affect pattern of trade.

**Proof:** In the full employment model, with such identical endowments in both the Home and the foreign country, RS of X and Y will be the same ( $RS=RS^*$ ), and  $P=P^*$  being the same, no goods trade will take place. As  $K > K^*$ ,  $\widehat{K} = 0 \Rightarrow \widehat{W} = \widehat{p}_M = -(\widehat{1+r})$  and  $\widehat{K}^* = 0 \Rightarrow \widehat{W}^* = \widehat{p}_M^* = 0$ . As explained earlier, this will mean at a given P,  $(1+r) < (1+r^*)$ ,  $W > W^*$  and  $p_M > p_M^*$ . With higher real wage at Home, immigration opens up. Similarly, imports of machinery will occur. 'K' finances intermediate goods and immigrations. *Without restrictions* on outflows of financial capital (K), capital flight will occur from Home, as it is dearer abroad.<sup>8</sup> Gradually, outflow might make 'K' (relatively) scarcer at Home with upward pressure on 'r' to raise 'r' at home in the long-run (HOS is a long run model), 'W' and 'p<sub>M</sub>' will start falling to arrest imports of machines and workers from abroad. *With restrictions* on financial flows, however, although 'r' will be low initially, but *no restriction* on labor movements or machine imports will cause (due to arbitrage) 'W' and  $p_M$  to fall at home, and '(1+r)' to rise as more  $L+L^*$  and  $M+M^*$  raises demand for K. Here factor trade complementing commodity trade unlike HOS model (QED).

**Proposition 5:** Given  $M=M^*$ ,  $L=L^*$ , and  $\widehat{K} > 0$ , *without capital control*, FPE will hold.

**Proof:** With identical endowments, since P remains the same and so are  $RS(X/Y) = RS^*(X^*/Y^*)$ , from Proposition 3, no trade occurs. As  $\widehat{W} = \widehat{p}_M = -(\widehat{1+r})$ ,  $\frac{\widehat{W}}{\widehat{p}_M} = 0$  i.e., such trade does not disturb  $\frac{W}{p_M}$ .

Thus, if only cross-country financial flows are allowed *absolute factor prices will be equalized*.

Even if with  $K > K^*$ ,  $(1+r) < (1+r^*)$ , absolute factor prices will be different. i.e.  $W > W^*$  and  $p_M > p_M^*$  (a la Proposition 3) and with free financial flows across borders (i.e., *without Capital control*), perfect arbitrage ensures,  $(1+r)=(1+r^*)$  and hence,  $r=r^*$ ,  $W=W^*$ ,  $p_M=p_M^*$ . However, relative factor prices will always be the same, since it does not depend on  $(1+r)$ . Therefore, without trade in goods, factor flows or movement in credit across borders does not generate overall gains from trade. Even if with  $K > K^*$ ,

<sup>8</sup> Lucas (1990) and others on hindrances of capital flow from rich to poor countries despite higher rate of return. In the economic growth literature, several barriers to capital flows including institutional have been mentioned. For a small open economy as price taker, K moves to ROW or foreign and with less than perfect capital mobility FPE does not occur while with free financial flows with perfect mobility, FPE occurs.

$(1+r) < (1+r^*)$ , absolute factor prices will be different. i.e.  $W > W^*$  and  $p_m > p_{m^*}$  (a la Proposition 3) and with free financial flows across borders (i.e., without Capital control), arbitrage ensures, i.e.  $(1+r)=(1+r^*)$  and hence,  $r=r^*$ ,  $W=W^*$ ,  $p_M=p_{M^*}$ . Given  $P=P^*$  in home and foreign country, with free trans-border capital flows,  $\frac{W}{P_M} = \frac{W^*}{P_{M^*}}$  (QED).

## 4.2 Trade, Unemployment and Role of Credit Market

In the context of our benchmark model, we considered full employment without any minimum wage. Unemployment problem is quite common when labor supply exceeds demand. However, in the presence of wage fund or working capital imperfection in the credit market or borrowing constraints could have severe jolt in the labor market and hence, could affect production and trade pattern. Excess demand for funds to be borrowed creates this situation. Thus, depending upon the credit crunch and default risk unemployment problem could be severe (Calvo et al, 2012, Popov et al. 2018). In fact, that issue is quite pertinent for the consideration of financial development and interesting perspectives on the role of financial institutions for inclusive development (Rajan and Zingales 1998, Noack and Costello 2022). For example, Alexandre et al. (2021) has considered the case of financially distressed firms in case of minimum wage increases as it reduces employment growth and profitability, especially after the pandemic eroding the financial condition of firms. Aizenman et al. (2022) explored the role of bank lending in times of pandemic-led shock when government also comes forward with fiscal stimulus for expansionary effects. Egger et al. (2018) is also an important contribution to show empirically that removal of credit constraints via external funding to borrowers and abolishing frictions or information gap could have a joint productivity and competition effects translating into entry of otherwise less productive firms.

### 4.2.1 Unemployment in the benchmark model

First, we consider the case Unemployment in this 2x2-model with no credit constraints. Just to reiterate, we start from a stock of finance or working capital or bank credit generated out of savings in the last period. For this section, we coin the financiers as bankers. There is a fixed minimum wage  $\bar{W}$  for hiring workers. Let  $W = \bar{W}$  and  $L_e$  be the level of employment of labor (L) such that  $(\bar{L} - L_e)$  is unemployment at Home. Following three equations determine  $p_M$ ,  $r$  and  $L_e$ .

$$\frac{P}{(1+r)} = \bar{W}a_{LX} + p_M a_{MX} \quad (16)$$

$$\frac{1}{(1+r)} = \bar{W}a_{LY} + p_M a_{MY} \quad (17)$$

$$K = \bar{W}L_e + p_M \cdot M \quad (18)$$

'M' is still given from last-period production of machines. Once 'L<sub>e</sub>' is known,  $(L_e, M)$  determine X and Y. The interesting question is how the system responds to a hike in wage from  $W$  to  $\bar{W} > W$ , given  $(P, M)$ . From (1) and (2) at a given  $P = \bar{P}$ , assuming a small economy facing  $P$  of the rest of the world (i.e., price-taker), average cost of production of X ( $C_X$ ) and Y ( $C_Y$ ), we can write:

$$\frac{C_X}{C_Y} = \frac{p_M a_{MX} + \bar{W}a_{LX}}{p_M a_{MY} + \bar{W}a_{LY}} = P \quad (19)$$

$$\begin{aligned} \text{Therefore, } \widehat{C}_X - \widehat{C}_Y &= 0 \\ \Rightarrow (\theta_{MX} - \theta_{MY})\widehat{P}_M + (1 - \theta_{MX} - 1 + \theta_{MY})\overline{W} &= 0 \end{aligned}$$

$$\text{Or, } \widehat{P}_M = \widehat{W} > 0 \quad (20)$$

$$\text{Hence, } \widehat{r} < 0 \quad (21)$$

From (18), following CFJ (2011) we can write:

$$\lambda_{LK}(\widehat{W} + \widehat{L}_e) + \lambda_{MK}\widehat{P}_M = 0 \quad (18a)$$

$$\text{Therefore, } \widehat{L}_e < 0 \quad (22)$$

As  $\overline{W}$  and  $P_M$  both rise, given  $\overline{K}$ ,  $L_e$  must fall.

Since  $P$  and  $\overline{W}$  are given,  $C_x$  and  $C_y$  both should change in the same proportion. A rise in  $\overline{W}$ , at a given 'r' increased  $C_x$  and  $C_y$ . But  $C_x$  rises less than  $C_y$  as  $X$  is assumed to be  $M$ -intensive. So the rates will fall and to prevent this  $P_M$  will rise equiproportionately with  $\overline{W}$ . As both  $(P_M, \overline{W})$  rise, 'r' must fall to satisfy (16) and (17). Given  $\overline{K}$ ,  $L_e$  must fall. Thus, higher wage or a minimum wage leads to unemployment. The mechanism is completely different from the diminishing marginal productivity argument (here  $a_{ij}$ s are fixed coefficients).

As  $L_e$  drops,  $\frac{L_e}{M}$  will fall and  $\frac{X}{Y}$  will rise. So, a labour abundant economy will produce less of labor-intensive good. This will lower  $P$  in the large country case, reduce  $P_M$  and raise 'r'. Given  $\overline{K}$  initial fall in  $L_e$  would recover to some extent. *The major result with unemployment is that now higher  $\overline{K}$  will affect the pattern of trade, relative prices, etc.* Given  $\overline{W}$ , hence  $(P_M, r)$  at a given  $P$ , a higher stock of  $K$  must increase  $L_e$  and  $\frac{L_e}{M}$  leading to greater export by the labor-abundant country increasing global production of  $Y$  and  $Y/X$ . Consequent rise in  $P$  ( $P_x$ ) and  $P_M$  will increase demand for 'K' and reduce  $L_e$ . But initial excess supply of 'K' will prevail, and 'r' will drop in ultimate equilibrium.

The main takeaway from this section is that in a world ridden with unemployment, finance plays a pivotal role. Greater credit-finance ( $K$ ) will increase global income and employment. But it will also adversely affect the terms of trade of the labor-abundant economy. Machine producers will be better off. This leads to the following proposition.

**Proposition 6:** Financial boom ( $\widehat{K} > 0$ ) or crisis ( $\widehat{K} < 0$ ) affects patterns of trade, relative price 'P',  $\frac{\overline{W}}{P_M}$ , in a minimum wage driven unemployment equilibrium.

**Proof:** See the discussion above. (QED).

If  $\widehat{K} > 0$  and given  $P$ , in this unemployment model, extra cash-in-advance will increase employment and will determine  $L_e$ ,  $X$  and  $Y$  (i.e., real changes or non-neutrality unlike the full employment scenarios in the previous section). This will increase  $Y$  from the full employment conditions. This country will export

Y and import X. After trade, P will be lower.  $P_M$  will be lower and  $L_e$  will rise furthest. So higher unemployment economy will export the labour-intensive good. But price changes or changes in levels of K will not affect wages (alike Keynesian case). This is an added theoretical feature.

#### 4.2.2 Trade and Unemployment in the presence of Imperfect Credit Market:

Stiglitz and Weiss (1981) seminal paper as well as Williamson (1987) has discussed Credit rationing with imperfect information when borrower's riskiness of default and lenders loan interest as well as the monitoring cost matters. Let us assume that  $K = \bar{K}$  is the total supply of fund in a country, where there are two sources of finance, viz., own entrepreneurial finance as source of internal fund ( $K_e$ ) as well as external funds from banks or other financial sources ( $K_b$ ) so that we write collaterals as:

$$\bar{K} = K_b + K_e \quad (23)$$

This is important for trade-finance and expansion of credit.

$$\text{Also,} \quad K_b = (K_b - B) + B \quad (24)$$

Apparently, this identity tells us that 'B' is the fixed amount such that with credit rationing ( $K_b - B$ ) is not lent out (leakage) due to imperfect credit market, and  $K_b$  is constrained by  $B^{\max}$ .

Suppose there are 'n' entrepreneurs each with identical endowment of internal finance ( $k_e$ ) such that

$$K_e = k_e \times n \quad (23a)$$

Let the lending by the banks be denoted by 'B' with 'R' being the *borrowing rate* (cost of borrowing). For internal finance, *opportunity cost is exogenous 'r'* ('r' = 0 with no other opportunities for investment). This is the deposit rate and  $R > r$ , implying that the entrepreneurs can borrow to augment their financial capital stock by paying  $R > r$  (the deposit or lending rate). For 'n' identical entrepreneurs each with "b" amount of disbursed credit, with total disbursement of 'B' fixed by Credit-rationing, we write:

$$B = K_b = b \times n \quad (23b)$$

'B' is allocated endogenously to X-Y sectors via credit rationing depending on risk of default and corresponding appropriation of funds. Let ' $0 < q < 1$ ' be probability of default and ' $0 < S < 1$ ' be the proportion collateralized from the defaulters by the financiers. 'B' will be more as ' $K_e$ ' rises because the later could be used as collateral in case of default. Bankers—when 'q' is high (risky)—will hedge against risk by charging higher 'R' and hence will have higher 'S'. 'qS' determines the degree of defaulter punishment. It is a parameter (exogenous) in this model. A relation between maximum loanable  $B^{\max}$  and "R" will endogenise 'B'. As 'qS' becomes higher, 'R' is charged low, and 'B' rises. Using no-default constraint, it can be derived that (see Marjit and Das 2021):

$$B = \frac{qS}{(1+R) - qS} K_e \quad (25)$$

with  $\lambda_e = \frac{K_e}{K_e + K_b}$  same across X-Y assuming same economy-wide 'qS'.

Now credit market equilibrium ensures that supply matches the funds required to purchase factor inputs, rewriting (18) as:

$$B + K_e = \bar{W}L_e + p_M M \quad (26)$$

As mentioned before, instead of 'r' now we have two rates –borrowing (R) and deposit (r) –with weighted average of both. Using (26), with dual sources of finance, we now rewrite (16) and (17) as:

$$(\bar{W}a_{LX} + p_M a_{MX}) \left[ \frac{k_b}{k_b + k_e} (1+R) + \frac{k_e}{k_b + k_e} (1+r) \right] = P_X \quad (27)$$

$$(\bar{W}a_{LY} + p_M a_{MY}) \left[ \frac{k_b}{k_b + k_e} (1+R) + \frac{k_e}{k_b + k_e} (1+r) \right] = P_Y \quad (28)$$

where  $\lambda_e = \frac{k_e}{k_e + k_b}$ ,  $\lambda_b = 1 - \lambda_e = \frac{k_b}{k_e + k_b}$

From the benchmark model, full-employment condition (4) is rewritten as:

$$a_{LX}X + a_{LY}Y = L_e \quad (4a)$$

$$a_{MX}X + a_{MY}Y = M \quad (5)$$

From (26), using (25) derive:

$$K_e \left[ \frac{(1+R)}{(1+R) - qS} \right] = \bar{W}L_e + p_M M \quad (29)$$

With these specifications, we have 5 variables:  $P_M$ ,  $R$ ,  $L_e$ ,  $X$  and  $Y$ . Given  $P_X$  and  $P_Y$ , Eqs. (27) and (28) determine  $P_M$  and  $R$ ; then, (29) determines  $L_e$ , and (4a), (5) determine  $X$ ,  $Y$ . Note that with credit-rationing (fixed "B"), increasing  $K_b$  has no role as "B" remains unaltered.

Let us consider two nations with identical endowments of collaterals where in autarkic equilibrium,  $K_b = K_b^*$ ,  $K_e = K_e^*$ , and  $\bar{W} = \bar{W}^*$ . Suppose ceteris paribus,  $(qS) > (qS^*)$  (same 'q' but degree of appropriation due to default differs contingent on rule of law or governance) which implies that probability of penalty of defaulter is higher in the home than in the foreign country thanks to better quality institutions, judiciary, or financial development.

Now from Eqns. (27) and (28), a la Jones (1965), we get:

$$\widehat{p_M} \theta_{MX} + \lambda_b \widehat{(1+R)} = \widehat{P_X} \quad (30)$$

$$\widehat{p_M} \theta_{MY} + \lambda_b \widehat{(1+R)} = \widehat{P_Y} \quad (31)$$

Solving, we get:

$$(\theta_{MX} - \theta_{MY}) \widehat{p_M} = \widehat{P_X} - \widehat{P_Y} = \left[ \frac{P_X}{P_Y} \right] \quad (32)$$

Now, rise in  $L_e$  would affect  $(X, Y)$  production and pattern of trade thanks to financial institution development couched in terms of rise in 'qS'. However, given  $(P_X, P_Y)$  and  $r$ ,  $(R, p_M)$  are not different.



But if Y is  $L_e$  intensive, rise in credit would cause Y, and hence,  $\frac{Y}{X}$  to increase in autarky, resulting in

$\left[\frac{\widehat{P}_X}{\widehat{P}_Y}\right] > 0$ , and hence  $(R, p_M)$  would *now* be different. If  $\left[\frac{\widehat{P}_X}{\widehat{P}_Y}\right] > 0, \widehat{p}_M > 0$  for  $\theta_{MX} > \theta_{MY}$ .

From (30) and (31), we can derive:  $\widehat{(1+R)} = -\frac{\theta_{MX}\widehat{p}_M + \widehat{P}_X}{\lambda_b} = -\frac{\theta_{MY}\widehat{p}_M + \widehat{P}_Y}{\lambda_b}$  (33).

For relative supply changes, following Jones (1965):  $\widehat{X} - \widehat{Y} = -\frac{\widehat{L}_e}{|\lambda|}$  where  $|\lambda| = \lambda_{MX} - \lambda_{LY}$  (34)

Similarly, closing the model from demand relationship, changes in the ratio of X/Y consumption is:

$$\widehat{X}_D - \widehat{Y}_D = -\sigma_D(\widehat{P}_X - \widehat{P}_Y) \quad (35)$$

where  $\sigma_D$  is the elasticity of substitution between X and Y on the demand side. As prices adjust to clear the markets for X and Y in general equilibrium adjustments, we can write:  $-\sigma_D(\widehat{P}_X - \widehat{P}_Y) = -\frac{\widehat{L}_e}{|\lambda|}$  so that:

$$(\widehat{P}_X - \widehat{P}_Y) = \frac{\widehat{L}_e}{|\lambda|\sigma_D} \quad (36)$$

Choosing 'Y' as numeraire good so that  $P_Y = 1$  and relative price  $P = P_X$ , we further rewrite (36) as:

$$\widehat{P} = \frac{\widehat{L}_e}{|\lambda|\sigma_D} \quad (36a)$$

From (32), we can then find where  $(\theta_{MX} - \theta_{MY}) = |\varphi|$ ,  $\widehat{p}_M = \frac{\widehat{P}}{(\theta_{MX} - \theta_{MY})} = \frac{\widehat{L}_e}{|\varphi||\lambda|\sigma_D}$  (37)

Hence, using (33),  $\widehat{(1+R)} = -\frac{\theta_{MY}\widehat{p}_M}{\lambda_b} = -\frac{\theta_{MY}}{\lambda_b} \frac{\widehat{L}_e}{|\varphi||\lambda|\sigma_D} \Rightarrow \widehat{(1+R)} < 0$  (38)

As we have seen before, competitive price equation determines 'R' and given 'qS',  $B^{\max}$  is determined.  $p_M$  is already determined. Intuitively speaking, for a given  $\widehat{P}$  in an economy with higher "qS" (due to better financial development and good quality institutions), credit-rationed is relaxed to supply more credit in keeping with "K<sub>b</sub>", causing "L<sub>e</sub>" to rise. Thus, demand for credit adjusts with rise in employment as supply of credit expands. As "X/Y" falls (Y is L-intensive relatively), with trade the general equilibrium adjustments trigger rise in "P", translating concomitant rise in  $p_M$ . Consequently, as more financing will be necessary for  $p_M M$  further rise in credit demand is envisaged. But fall in  $(1+R)$ —as explained before—will reduce default possibility or incentive. " $B^{\max}$ " will increase further. If  $\gamma_M$  is very high,  $K_b > B^{\max}$  (i.e., demand exceeds supply of credit), causing shrinkage of employment to some extent (i.e.,

$\widehat{L}_e < 0$ ). However, general equilibrium adjustments where  $\widehat{L}_e > 0$  is the trigger of chain of events described so far will stabilize the economy's adjustment to new equilibrium, and secondary effect can't outweigh the primary effect. This leads to:

**Proposition 7:** Given  $\bar{P}$ , as  $(qS) > (qS^*)$ ,  $L_e > L_e^*$

**Proof:** See above discussion.  $L_e > L_e^*$  and right hand side of (26) must rise as increase in  $L_e$  at given  $P_x$ ,  $P_y$  causes changes in  $p_M$ . If trade opens up (due to changes in relative price— $P$ ),  $Y$  will be exported. Even when  $P$  is changing, if  $(qS) > (qS^*)$ ,  $L_e > L_e^*$  (QED).

$$\text{Now, from Eqn. (29), rewriting as: } K_e \left[ \frac{1}{1 - qS / (1 + R)} \right] = \bar{W}L_e + P_M M \quad (29a)$$

Taking total differentials on both sides, we get:

$$-\frac{d[1 - qS / (1 + R)]}{1 - qS / (1 + R)} = \gamma_L \widehat{L}_e + \gamma_M \widehat{P}_M \quad (39)$$

where  $\gamma_L, \gamma_M$  are cost-shares of  $L_e$  and  $p_M$  in finance respectively.

On simplification: we get:

$$[\widehat{qS} - \widehat{(1 + R)}] \cdot \left[ \frac{qS}{(1 + R) - qS} \right] = \gamma_L \widehat{L}_e + \gamma_M \left[ \frac{\widehat{L}_e}{|\varphi||\lambda|\sigma_D} \right] = \widehat{L}_e \left[ \gamma_L + \gamma_M \frac{1}{|\varphi||\lambda|\sigma_D} \right] \quad (40)$$

Plugging in (38) into (40) and using (25), it simplifies to:

$$\left[ \widehat{qS} + \frac{\theta_{MY}}{\lambda_b} \frac{\widehat{L}_e}{|\varphi||\lambda|\sigma_D} \right] \cdot \left[ \frac{B}{K_e} \right] = \widehat{L}_e \left[ \gamma_L + \gamma_M \frac{1}{|\varphi||\lambda|\sigma_D} \right] \quad (41)$$

$$\text{We can write (41) succinctly as: } [\widehat{qS} + \widehat{L}_e \cdot A2] \cdot \left[ \frac{B}{K_e} \right] = \widehat{L}_e \cdot A1 \quad (42)$$

where  $A1 = \left[ \gamma_L + \gamma_M \frac{1}{|\varphi||\lambda|\sigma_D} \right]$  and  $A2 = \frac{\theta_{MY}}{\lambda_b |\varphi||\lambda|\sigma_D}$

$$\text{Further with algebraic manipulation (42) simplifies to: } \widehat{L}_e = \frac{\widehat{qS}(B / K_e)}{A1 - A2 \cdot \frac{B}{K_e}} \quad (43)$$

As  $(qS) > (qS^*)$ ,  $L_e > L_e^*$  and via Proposition 7,  $\hat{p}_M > 0$ ,  $(1+R) < (1+R^*)$ ,  $P > P^*$ , affecting production and trade. With  $(qS) > (qS^*)$ ,  $(B/K_e) > 0$ ,  $\hat{L}_e > 0$  iff  $A1 > A2 \cdot \frac{B}{K_e}$  or  $\frac{A1}{A2} > \frac{B}{K_e}$ . This is the

Stability Condition. This boils down to  $\frac{\gamma_L[|\varphi||\lambda|\sigma_D] + \gamma_M}{\theta_{MY}} > \frac{B/K_e}{\lambda_b} = \frac{qS}{\lambda_b[(1+R) - qS]}$

In other words, despite rise in  $p_M$  might have a ‘choking-off’ impact on  $L_e$ , it cannot overturn as  $(B/K_e)$  or  $\gamma_M$  cannot be very high. Pivotal role is played by share of credit going to machine-sector as well as ratio of external finance to stock of capital ( $\lambda_b = \frac{k_b}{k_e + k_b}$ ). This leads to the following

proposition:

**Proposition 8:** From (42) and (43), it follows that given  $\theta_{MX} > \theta_{MY}$ ,  $|\varphi| > 0$ ,  $\theta_{MY}$  is fairly low enough and  $\gamma_M \rightarrow 0$ , so that  $\gamma_L \rightarrow 1$ , and  $\sigma_D$  is high enough then stability condition will always hold and  $B/K_e$  will not be high enough. In other words, the condition  $A1 > A2 \cdot \frac{B}{K_e}$  ensures that given quite low

values of  $\theta_{MY}$  and  $\gamma_M$  even if  $B/K_e$  is bit high, positive impact on  $\hat{L}_e$  could be insignificantly low, but not negative (i.e., reduced).

**Proof:** See the discussion above. (QED). This is the ‘Stability condition’.

### 4.3 Allocating Finance for R&D-investment

In a modern capitalist economy, invention and innovation is the engine of growth providing dynamics in the process of long-run economic growth (Schumpeter 1934). Neo-Classical growth theory and its exponent Solow (1956&1957) has emphasized the role of capital-accumulation and technical progress (exogenous) for sustained economic growth. Following that tradition, Lucas (1995, 2003), Aghion and Howitt (1992) and Romer (1990), among others, have laid down the foundations of growth process where creation of ideas occurs endogenously with creative destruction a la Schumpeter. One can refer to this strand of literature in Barro and Sala-i-Martin (2003) and Acemoglu (2019a&b). Further, role of innovation network where cumulative technical progress due to scientific progress generates patent growth with further innovation via upstream technology is important for continuous invention, spillovers and growth (Acemoglu, Akcigit and Kerr 2016). Lucas (2003) has mentioned that trade theory lacks the focus of growth-inducing mechanism other than resource allocation. Also, how (biased) technical change can cause inequality with declining labor share of income especially after the onslaught of Artificial intelligence in post-ICT phase has attracted attention (Aghion et al. 2019a&b, Korinek and Stiglitz 2017, Acemoglu and Restrepo 2018a&b, Acemoglu and Restrepo 2019a&b, 2020). This decline has been most prominent since 1987 till now where large displacement effect of rapidly spreading new technology reduced the labor demand by 30% between 1987-2017 (NBER Digest June 2019). Most of these papers have shown the dominance of ‘displacement effect’ or ‘labor-replacing impact’ of new technologies accounting for fall in low-skill labor, assuming skill proxied by human capital is lumped together with capital, and also emergence of new tasks with automation which could have smaller

‘reinstatement effect’. Wage-rental relativities as well as demographics (for example, population aging) determine the extent of automated new tasks and the corresponding effects on labor-capital share in GDP.

Given this backdrop, how invention could affect production and trade depends on the active choice of economic agents reacting to the incentives. However, very few studies have incorporated role of finance and growth in traditional trade models—see Marjit (2020) and Marjit and Nakanishi (2021)—by embedding R&D-funding in a trade model per se. In this section, we extend the benchmark model to highlight the role of finance or entrepreneurs in innovation via R&D investment. This improves productivity of machines as well as labor (Syverson 2011). Durvasula et al. (2019) has discussed role of public vis-à-vis private investment in research. Hasan and Sheldon (2016) has empirically shown the adverse effect of financial or credit constraints on productivity, technology choice and exports of firms-level trade. Egger and Keuschnigg (2015) has emphasized the role of financial development for investment in R&D for comparative advantage for producing ‘innovative goods’. Lower level of financial development (or, higher financial constraint) could affect R&D activities adversely and hence, the TFP growth. Furthermore, rapid spread of advanced technology and investment in automation (AI) for R&D will affect productivity of laborers depending on their endowment of abilities or absorptive capability with chances of technological unemployment and growing within group or between-group inequality. Davis and Dingel (2020) has offered a model where individual comparative advantage based on skill could explain specialization in skill-intensive tasks in larger cities with relatively higher endowment of skilled labor force. Autor et al. (2019) emphasizes the necessity of a framework with general equilibrium features capturing human-machine interactions based on high-quality data on nature of work, occupations, etc. Most recently, Graetz et al. (2022) has explored many facets of changes in cutting-edge technology esp. AI and impacts on labor demand to account for the rise of gig economy, shift in labor demand, and fall in wage share and highlighted the necessity of theoretical underpinning behind such evidences. In particular, the necessity of quantitative general equilibrium modelling to trace the repercussions on wages, employment and labor market ramifications is emphasized.

The benchmark model shows “K” is used to finance wage fund and cost of machine. However, in case of financial allocation there might be a situation where the financiers or bankers consider alternatives whereby they can expropriate more benefits. Innovations occur due to entrepreneurial activities motivated by higher prospective rate of return on R&D. We extend the model to include R&D and resultant technological change keeping in view the incentive of the financiers to invest in R&D rather than in production. From *Proposition 1 and 2*, we saw that when  $\widehat{P} > 0, \widehat{X} > 0$  as X is M-intensive (relatively),  $\widehat{P}_M > 0$  and with fixed supply of “K”, as demand for ‘K’ rises, given  $\lambda_{MK} > \lambda_{LK}, \theta_{MX} > \theta_{MY}, (\widehat{1+r}) > 0, \widehat{W} < 0$ . Thus, rise in ‘P’ rises triggers demand for credit or finance and the bankers will have incentive to invest in R&D in the machine-intensive sector with a prospective higher rate of return.

We introduce a two-period (t =1, 2) simple dynamic model in our basic framework to show how R&D is motivated in such a system. In particular, we demonstrate a case where banks might be interested in investing in R&D only in the machine-intensive sector. Certain fraction of ‘K’ is invested in R&D in period 1. In the second period, as marginal productivity of machines and labor rises, because of technical progress the  $a_{ij}$  coefficients decline monotonically with R&D expenditure ( $R_d$ ) incurred in the 1<sup>st</sup> period. R&D expenditure is important for productivity of firms and makes differences in their

performances (Doraszelski and Jaumandreu 2013). As it's a two-period model, we assume that in the second period ( $t=2$ ), such incentives to undertake R&D further dissipates as economic agents don't survive beyond second period. All the variables of interest will now bear two dimensions such that  $r_t, P_{m_t}, W_t, K_{dt}, \forall t=1,2$ .

Let "R<sub>d</sub>" be the total expenditure in the R&D activities with inputs being labor (L) and machine (M).<sup>9</sup> So, the credit market equilibrium will look like:

$$K = W(L - L_{Rd}) + P_m(M - M_{Rd}) + (WL_{Rd} + P_m M_{Rd}) \quad (44)$$

where  $R_d = WL_{Rd} + P_m M_{Rd}$ . Sheer reallocation of "K" to alternative channel of R&D-investment apart from production does not change the competitive price conditions in the 1<sup>st</sup> period. Also, we keep relative prices frozen in both periods ( $\hat{P} = 0$ ). This *assumption* rules out any feedback of relative price changes in the second period. Note that this is not critical for our results as with stability of equilibrium the feedback effect can never rule out the initial impact and hence it is the *initial impact that will prevail qualitatively*. Only magnitude will matter.

Since competitive price conditions do not change, "W" and "P<sub>m</sub>" remain unaltered compared to the situation *without R&D* expenditure when  $R_d = 0$ . The demand for credit does not change in period 1 and hence with "R<sub>d</sub>" in  $t=1$ ,  $r_1$  does not change too.

Now the fruits of R&D-induced technical progress occur with a gestation lag so that it's realized in 2<sup>nd</sup> period. We compare cases when R<sub>d</sub> could be invested in M-intensive sector (X) or alternatively in the L-intensive sector (Y), or, in M or L in the sector experiencing favourable shock. However, to the banker or entrepreneur the issue is whether  $r_2$  increases with "R<sub>d</sub>", else there will be no incentive to approve the project. Thus, for period 2 the following holds:

$$(1 + r_2)[W_2 a_{L2}^X(R_d) + P_{m2} a_{m2}^X] = P \quad (45)$$

$$(1 + r_2)[W_2 a_{L2}^Y(R_d) + P_{m2} a_{m2}^Y] = 1 \quad (46)$$

Assume uniform rate of technical progress in both machine and labor (i.e., Hicks-Neutral type) in X-sector so that we write:

$$-\frac{\partial a_{L2}^X}{\partial R_d} \cdot \frac{R_d}{a_{L2}^X} = -\frac{\partial a_{m2}^X}{\partial R_d} \cdot \frac{R_d}{a_{m2}^X} = \beta > 0 \quad (47)$$

We know that X is M-intensive,  $P_{m2}$  will rise and hence,  $W_2$  will fall.

In period 2, the credit market equilibrium—as there is no further R&D-investment undertaken ( $R_d=0$ , in  $t=2$ ) -- is written as (back to same level):

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<sup>9</sup> Here we do not distinguish between skilled and unskilled labor while skilled labor goes into R&D sector. However, implicit assumption is that machine (as intermediate input) embodies the boons of technological development of the past based on skilled labor force in general.

$$K = W_2L + P_{m2}M \quad (48)$$

The right-hand side (RHS) is credit demand in period 2, i.e.,  $K_{d2}$

Based on the preceding discussion, the following proposition is immediate that:

**Proposition 9:** If  $K_{d2}$  has a greater allocative share towards M (i.e.,  $\lambda_{m2K} > \lambda_{L2K}$ ), given  $\theta_{m2X} > \theta_{m2Y}$ ,  $\widehat{r}_2 > 0$ .

**Proof:** A rise in  $P_{m2}$  will increase the value of RHS in (48) iff  $\frac{P_{m2}M}{K_{d2}} > \frac{W_2L}{K_{d2}}$ . To clear the market,  $r_2$  will rise. (QED)

In this case, the bank will not invest in R&D for L-intensive Y-sector as that will reduce  $r_2$ . Thus,  $\widehat{R}_d > 0$  in the X-sector (M-intensive) would be consistent with declining share of labor-income, i.e., a drop in  $\frac{W_2}{P_{m2}}$  and  $\frac{W_2}{r_2}$  as well as R&D in the only machine-intensive sector. This will increase production of X and reduce Y in the second period and increasing, in turn, the share of machines even more in the demand for credit, paving the way for further expansion of R&D in the machine-intensive sector.

As R&D causes TFP growth with Hicks-Neutral technical progress, share of long-run growth that is explained by Solow's residual is attributed to this technical progress, and finance boosts further machine-augmenting investment perpetually and hence, would cause secular decline in labor-share.<sup>10</sup> This is kind of 'directed technical change' at the expense of labor-intensive sector where demand for inventive activity affect the production in a trade-theoretic framework unlike that in Acemoglu (1998, 2002). In those papers, increase in relatively skilled workforce induces SBTC with skill-complementary technologies and price effect (incentives for developing technologies for products whose prices rise and uses more expensive goods) and market size effect (for the larger market of those technologies) govern the response of skilled-labor augmenting technical change and movement of factor prices. In an altogether different theoretical setup, Loebbing (2022) has considered the case of automation-induced labor-replacement in a continuum of skill model with skill-capital (machine) complementarity and skill supply induces capital-augmenting investment. All these differ from our main framework where capital (K) as source of finance remains outside the production process.

We propose an alternative theoretical setting to highlight the nexus between relative labor (homogeneous composite) endowments, premium to capital as well as machines, and the economic growth via R&D. As relative supply of machine-intensive goods increase, the market for machines used by labor widens, thereby creating additional incentives to direct or channel R&D activities aimed at those specific automation. As a direct consequence, demand for finance steers towards machine registering higher premium to capital and machine. In fact, due to higher relative demand for machines

<sup>10</sup> If we extend the model beyond two inputs to split labor into skilled-unskilled types then the model becomes specific factor variety alike Marjit and Das (2021). In that case, investment in skill-acquisition or human capital along with R&D-capital or invention-capital could be another option. There too, skilled labor wage could go high along with return to R&D investment so that the unskilled labor could experience a decline in their share of income.

exceeding the increase in its relative supply, our model shows that diversion of wage-fund—propelled by higher returns thanks to favourable external condition for the machine-intensive sector—into financing R&D investment in furthering machine production (a la technical progress) will induce machine-biased technical change (MBTC). This translates into raising premium to machine at the expense of ‘man’. As bankers give more loans for R&D investment, the worker-entrepreneurs will not be happy as their wages decline. Without skill-differentiation in the composite labor and assuming skill is bundled into machine, such embodiment is akin to skill-biased technical change (SBTC) as well where skill-machine complementarity could be accounted for registering further misfortune of low-skilled workers with declining share (Acemoglu 2022)<sup>11</sup>. This is a novel feature, which could accommodate also the traditional theoretical conjectures, for example, trade-openness, and change in real minimum wage, ability-biased technical change, etc., as sources of such bias.

### Section 5: Illustrative Numerical Exercise

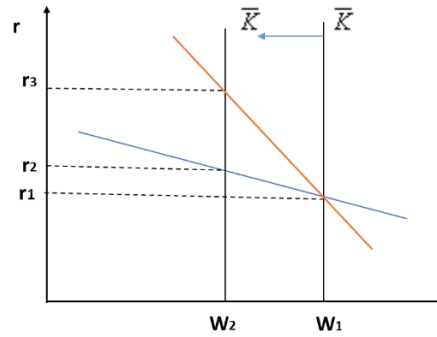
Following the theoretical results in the preceding section, we now proceed to show some numerical counterfactual exercise. We pick some key results that govern our conjectures. Purpose of the simulation is to show the role of financial crisis or boom affecting wage-fund and hence, the demand for credit for production.

For that, we make the following table with core results divided into three blocks based on main building block and extensions. First block is related to Proposition 1 and 3 related to changes in  $K$ . The second one is related to the aspect of credit market constraints and unemployment with emphasis on roles of better financial institution to arrest risk of default (main Proposition 6, 7, 8), and the last one concerns R&D financing for machine-biased technical change (MBTC). As a background, while considering the parametric changes the key assumptions underlying the values will be mentioned.

So far as 1<sup>st</sup> block is concerned as in Section 3, we consider relative prices changes of exogenous  $\widehat{P}_x > 0, \widehat{P}_y = 0$  and changes in demand for wage fund ( $\widehat{K}_d$ ), and  $\widehat{r}$ . Crucial assumptions are:  $\theta_{MX} > \theta_{MY}$ ,  $\lambda_{MK}\theta_{LY} \gtrless \lambda_{LK}\theta_{MY}$ .

For a given  $K_d$ , changes in  $\overline{K}$  will affect ‘ $r$ ’. During financial crisis,  $\overline{K}$  falls and ‘ $r$ ’ rises, while ‘ $W$ ’ and ‘ $p_m$ ’ fall. This causes  $K_d$  to shrink as  $K'_d(r) < 0$  where  $\frac{dK_d}{dr} = \frac{dp_m}{dr}M + \frac{dW}{dr}L$ . ‘ $r$ ’ and ‘ $W$ ’ are inversely related. Now, given  $\overline{K}$ , effect of financial debacle on unemployment in the labor market will depend on the sensitivity or elasticity of  $K_d$  w.r.t. ‘ $r$ ’ and the impact on  $p'_M(r), W'(r)$ . This, in turn, will affect the relative changes in demand for machine and labor depending on fall in ‘ $W$ ’ while ‘ $r$ ’ changes. As  $\overline{K}$  shifts left (during credit shortage or crisis) or shifts right (financial boom), employment impact will depend on how sensitive  $K_d$  to ‘ $r$ ’ is i.e., the steepness. See the diagram 1 below. With the same fall in  $\overline{K}$ , the more elastic (flat) is the  $r$ - $W$  curve would mean ‘ $W$ ’ will be compressed more with respect to small changes in ‘ $r$ ’ rise whereas inelastic (steep) schedule will mean for same decline in ‘ $W$ ’, ‘ $r$ ’ has to increase by more. Unemployment effect will vary accordingly. If  $K_d$  falls by more due to rise in ‘ $r$ ’ thanks to financial crisis, ‘ $L$ ’ could rise or fall depending on extent of fall in ‘ $W$ ’ vis-à-vis ‘ $p_m$ ’ (Hollander 1984).

<sup>11</sup> Daron Acemoglu on US Inflation, Ukraine War and Labor market (May 10, 2022) on Project Syndicate. Accessed at: [https://www.project-syndicate.org/say-more/an-interview-with-daron-acemoglu-2022-05?utm\\_source=Project%20Syndicate%20Newsletter&utm\\_campaign=%E2%80%A6&barrier=accesspaylog](https://www.project-syndicate.org/say-more/an-interview-with-daron-acemoglu-2022-05?utm_source=Project%20Syndicate%20Newsletter&utm_campaign=%E2%80%A6&barrier=accesspaylog)



**Figure 1:** Sensitivity of  $r$ - $w$  and leftward shift of capital supply

For the 2<sup>nd</sup> block, we consider exogenous changes [ $\widehat{P}_x > 0, \widehat{P}_y = 0$ ] with parametric variations—as shown in the Table for each set of simulations—and consider entrepreneurial finance ( $K_e$ ), external credit ( $B$ ) with borrowing cost ( $R$ ) and ‘ $qS$ ’ as indicator of financial development (See section 4.2). Given the benchmark case of parametric configurations, on top of the shares of these two sources of finance with ‘ $R$ ’ as borrowing cost and ‘ $r$ ’ as deposit rate, we consider  $\sigma_D, \lambda$  to determine  $\widehat{L}_e, \widehat{p}_M$ . Also, we see how variations in  $\widehat{qS} > 0$  will affect  $\widehat{L}_e$  positively monotonically so long as cost shares of labor in finance is high,  $\theta_{MY}$  and  $\gamma_M$  low while ratio of credit to entrepreneurial finance [ $B / K_e$ ] is not so high.

The final block for financing R&D vis-à-vis labor, we need to consider three crucial parametric variations, viz., (i) share of finance in machine in the period when R&D investment materializes,  $\lambda_{M2K} > \lambda_{L2K}$ ; (ii) share of machine in X production when X is already M-intensive,  $\theta_{M2X} > \theta_{M2Y}$ ; (iii) share of machine entering into R&D and share of R&D in X as X is intensive in using machine,  $\theta_{MRd} > \theta_{LRd}, \lambda_{RdX} > \lambda_{RdY}$ . Here to opt out feedback effects via relative price changes we hold  $\widehat{P}_x = \widehat{P}_y = 0$ . However, with no factor-bias technical change in X-sector due to R&D, based on Eq. (45) and (46) in Section 4.3 and Proposition 9 following comparative static derivations are imminent for the second period while in the first period only R&D-expenditure occurs.

$$\theta_{L2X} \cdot \widehat{W}_2 + \theta_{M2X} \cdot \widehat{p}_{M2} = -\widehat{(1+r_2)} = \theta_{L2Y} \cdot \widehat{W}_2 + \theta_{M2Y} \cdot \widehat{p}_{M2} \quad (49)$$

Simplification yields, given  $|\theta_2| < 0, \widehat{r}_2 > 0$

$$\widehat{W}_2 = \frac{\widehat{(1+r_2)}}{|\theta_2|} < 0, \widehat{p}_{M2} = -\frac{\widehat{(1+r_2)}}{|\theta_2|} > 0 \Rightarrow \left(\frac{\widehat{W}_2}{p_{M2}}\right) = \frac{2\widehat{(1+r_2)}}{|\theta_2|} < 0, \frac{\widehat{W}_2}{\widehat{r}_2} = \frac{r_2}{|\theta_2|(1+r_2)} < 0 \quad (50)$$

Now given the assumptions we assign some values to the parameters with reasonable range of variations. The results are tabularly presented below in Table 2 to quantify the relationships.

From Table 2, the results exhibit that under some plausible benchmark conditions, the direction of established relationships is consistent and intuitive. In block 1, in keeping with proposition 1 and its analogues we corroborate the effect of 20% change in price of X and under plausible conditions, trace that  $\widehat{p}_M > \widehat{P} > 0 > \widehat{W}$  and  $(\widehat{W} - \widehat{p}_M) < 0, \widehat{K}_d > 0$ . Further, as  $\widehat{P}$  changes, keeping the parameters same,  $\widehat{K}_d$  rises.



**Table 1: Parameters changes and List of Variables.**

<b>1<sup>st</sup> Block: Based on Proposition 1</b>		
<u>Parameters and assumptions</u>	<u>Variables of Interest</u>	<u>Key Equations</u>
$\theta_{MX} > \theta_{MY}, \theta_{LY} > \theta_{LX},$ $\lambda_{LK} = 1 - \lambda_{MK}, \theta_{MY} = 1 - \theta_{LY}, \theta_{MX} = 1 - \theta_{LX}$ $ \theta  = \theta_{MY} - \theta_{MX} = \theta_{LX} - \theta_{LY} < 0,$ $\lambda_{LK}\theta_{MY} > \lambda_{MK}\theta_{LY}, P_x=P (P_y \text{ numeraire}) \text{ is exogenous,}$	$\widehat{K}_d, \widehat{P}_M, \widehat{W}, (\widehat{W} - \widehat{P}_M) \widehat{P} = 20\%,$ 30%	(11a&b), (12a&b), (13), (14), (15)
Sims	Parameter values	Variables
<i>See Table 2</i>		
<b>2<sup>nd</sup> Block: Based on Proposition 6, 7, and 8</b>		
<u>Parameters: Same and assumptions as above,</u>	<u>Variables of Interest</u>	<u>Key Equations</u>
Additional ones: $0 < q < 1, 0 < S < 1, 0 < (qS) < 1,$ $\lambda_b = 1 - \lambda_e,  \lambda  = \lambda_{MX} - \lambda_{LX} > 0,$ $(\theta_{MX} - \theta_{MY}) =  \varphi  > 0, \sigma_D, A1, A2, A1 > A2 \cdot \frac{B}{K_e}$	$\widehat{L}_e = \widehat{P}_M  \lambda   \varphi  \sigma_D, \widehat{P}_M, (\widehat{1+R}),$ $\widehat{L}_e = \frac{\widehat{qS}(B/K_e)}{A1 - A2 \cdot \frac{B}{K_e}}$	(25), (26), (27), (28), (34), (35), (36a), (37), (38), (43)
Sims	Parameter values	Variables
<i>See Table 2</i>		
<b>3<sup>rd</sup> Block: Based on Proposition 9</b>		
<u>Parameters: Same and assumptions as above,</u>	<u>Variables of Interest</u>	<u>Key Equations</u>
<u>Period 1: R&amp;D Production Expenditure</u> $\lambda_{M1K} > \lambda_{L1K}, \theta_{M1X} > \theta_{M1Y}$	$\widehat{W}_2, \widehat{P}_{M2}, \widehat{r}_2 > 0, (\frac{\widehat{W}_2}{P_{M2}}, \frac{\widehat{W}_2}{r_2}, \widehat{P} = 0$	See above (49) and (50).
<u>Period 2: R&amp;D usage. HNTF (<math>\beta &gt; 0</math>) in Sectors,</u> $\lambda_{M2K} > \lambda_{L2K}, \theta_{M2X} > \theta_{M2Y}, \theta_{M2Rd} > \theta_{L2Rd}, \lambda_{RdX} > \lambda_{RdY}$		
Sims	Parameter values	Variables
<i>See Table 2</i>		

For the second block, same configurations of parameters are taken except newly introduced ones ( $qS, \gamma_M, \gamma_L, \lambda_{MX}, \lambda_b, \sigma_D$ ) being assigned values as per the table with reasonable assumptions and approximations. The reason is that here we want to see variations of  $\widehat{L}_e$  with 'qS' (for stability condition) and with price changes of Machine-intensive sector while share of machine being high in X ( $\lambda_{MX} - \lambda_{LX} > 0$ ). As per table 1, we find that in keeping with Proposition 8, when  $A1 > A2 \cdot \frac{B}{K_e} > 0$ , the stability condition is satisfied with insignificantly low positive impact on  $L_e$  whereas with  $\widehat{P} = 20\%$  and rise in 'qS',  $L_e$  rises. Looking at the values of the term  $(A1 - A2 \cdot \frac{B}{K_e})$  contingent on the configurations of the constituent parameters, we easily see that as it turns to positive from negative,  $\widehat{L}_e$  goes up (insignificantly low) with rise in 'qS' while  $\widehat{P}$  increases. Keeping that stability condition requires, as expounded in Proposition 8 and captured in the simulated values in the Table,  $\frac{B}{K_e}$  should not be high and  $\gamma_M \rightarrow 0, \gamma_L \rightarrow 1$  while  $\theta_{MX} - \theta_{MY} = |\varphi| > 0$ .

For the third block,  $\hat{P}=0$ , and in keeping with Proposition 9 we just consider variations in shares of machine in X-sector and shares of finance going into R&D-intensive machine sector and in the machine sector itself. It's just simple variations of  $\theta, \lambda$  values without exogenous price changes. Hence, it is quite similar to block 1 without price variations. We retain those values treating them as 1<sup>st</sup> period when R&D investment is just undertaken without impacting on production. Here from (11a) and (11b),  $(1+r_1) = -\widehat{p}_{M1} = |\theta_1|$  in first period. The configuration of values changes when production involves R&D inputs as per Section 4.3. All these highlight the mechanism numerically and offer policy perspectives such that improving institutions and governance (captured by rise in 'qS' values) is good for employability. Investing in R&D-biased for machine aggravates wage inequality and hence, investing in skills or 'man' versus 'machine' needs to be given prior concern.

Table 2: Parametric changes and Sensitivity of Variables.																
<b>BLOCK 1</b>																
$\hat{P} = 0.2$																
$\theta_{mx}$	$\theta_{Lx}$	$\theta_{my}$	$\theta_{Ly}$	$\theta$	$\lambda_{mx}$	$\lambda_{mk}$	$\lambda_{Lk}$	$\widehat{K}_d$	$\widehat{p}_M$	$\widehat{W}$	$(\widehat{W} - \widehat{p}_M)$					
0.6	0.4	0.2	0.8	-0.4	0.5	0.6	0.4	0.2	0.4	-0.1	-0.5					
0.7	0.3	0.3	0.7	-0.4	0.6	0.7	0.3	0.2	0.35	-0.2	-0.5					
0.8	0.2	0.4	0.6	-0.4	0.7	0.8	0.2	0.2	0.3	-0.2	-0.5					
0.55	0.45	0.45	0.6	-0.1	0.7	0.55	0.45	0.2	1.1	-0.9	-2					
$\hat{P} = 0.3$																
$\widehat{K}_d$ $\widehat{p}_M$ $\widehat{W}$ $(\widehat{W} - \widehat{p}_M)$																
0.3   0.6   -0.2   -0.75																
0.3   0.53   -0.2   -0.75																
0.3   0.45   -0.3   -0.75																
0.3   1.65   -1.4   -3																
<b>BLOCK 2</b>																
$\hat{P} = 0.2$ $\widehat{p}_M$																
$\sigma$	$\gamma_m$	$\gamma_l$	$\lambda_{mx}$	$\lambda_{lx}$	$\varphi$	$\lambda$	A1	A2	qS	B/Ke	A1-A2.B/Ke	Le-hat (stability)	Le-Hat value			
2	0	1	0.6	0.45	0.4	0.1	1.01	4.17	0.2	1	-3.16	-0.063	0.04	0.54		
2	0.01	0.99	0.6	0.4	0.4	0.2	1.05	3.13	0.2	0.5	-0.51	-0.196	0.08	0.54		
2	0.1	0.9	0.7	0.35	0.4	0.3	1.32	2.78	0.2	0.45	0.07	1.35	0.12	0.54		
2	0.11	0.89	0.7	0.3	0.1	0.4	2.27	9.38	0.85	0.2	0.39	0.436	0.16	0.54		
										$\lambda_b$	$\theta_{my}$	$\theta_{mx}$	$\theta_{mx} - \theta_{my}$			
										0.6	0.33	0.7	0.37			
<b>BLOCK 3</b>																
<b>Period 1</b> <u>Benchmark Equilibrium</u>																
$\hat{P} = 0$																
$\theta_{mx}$	$\theta_{Lx}$	$\theta_{my}$	$\theta_{Ly}$	$\theta$	$\lambda_{mx}$	$\lambda_{mk}$	$\lambda_{Rdk}$	$\lambda_{Lk}$	$\lambda_{my}$	$\lambda_{mRd}$	$\lambda_{lx}$	$\lambda_{ly}$	$\lambda_{lRd}$	$\widehat{p}_M$	$\widehat{W}$	$\widehat{K}_d$
0.6	0.4	0.2	0.8	-0.4	0.5	0.5	0.2	0.3	0.2	0.3	0.5	0.3	0.2	0.4	-0.1	0.2
<b>Period 2</b>																
$\theta_{mx}$	$\theta_{Lx}$	$\theta_{my}$	$\theta_{Ly}$	$\theta$	$\lambda_{mx}$	$\lambda_{mk}$	$\lambda_{Rdk}$	$\lambda_{Lk}$	$\lambda_{my}$	$\lambda_{mRd}$	$\lambda_{lx}$	$\lambda_{ly}$	$(1+r_2)$	$\widehat{p}_M$	$\widehat{W}$	$\widehat{K}_d$
0.65	0.35	0.3	0.7	-0.4	0.5	0.5	0.2	0.3	0.2	0.3	0.5	0.3	1.71	0.6	-4.90	0.21
0.7	0.3	0.25	0.8	-0.5	0.5	0.6	0.2	0.2	0.2	0.3	0.5	0.3	1.78	0.8	-3.95	0.23

Source: Authors' own calculations. Fictitious data, for illustration purposes only.

## 6. Concluding Remarks and implications:

In this paper, we have extended the traditional two-sector Neo-Classical trade model—workhorse of Heckscher-Ohlin-Samuelson model couched in the framework of Jones (1965)—by incorporating Ricardian wage fund theory *a la* Marjit and Das (2021). Incorporating finance in general

equilibrium trade model is quite novel as it offers important valuable insights such as role of finance in affecting production and trade patterns. With perfect credit market and full employment, finance does not affect trade pattern; however, it does affect absolute values of wage and price of machines via changes in the market interest rate. In the full employment model, with identical endowments in both the home and foreign country, no trade will occur with same price as in the world market price. With differences in endowments, alike HOS model trade will take place. With differences in availability of capital, and mobility of financial capital factor trade complements commodity trade unlike the HOS model. Trade pattern is not affected. Without capital control, factor prices will be equalized. Higher minimum wage will lead to unemployment without diminishing marginal productivity, and a labor-abundant economy might produce less of labour-intensive goods. With unemployment, however, higher wage fund will affect trade pattern as well as relative prices. Financial crash or boom—as exogenous shocks—affect pattern of trade, relative prices, in a minimum wage-driven unemployment equilibrium. For example, given prices of goods and machines and interest rate, higher working capital will increase labor employment translating into greater export by a labor-abundant economy initially lacking enough credit to finance trade. The main takeaway is that in a world ridden with unemployment, finance plays a pivotal role with impact on global income and employment. However, it will also adversely affect the terms of trade of the labor-abundant economy. Machine producers will be better off. With imperfect credit market and credit-rationing, the story becomes more interesting and of course, realistic because when demand for loanable funds exceed supply to finance production and trade, then risks of default and quality of financial institutions engineering penalty for bringing someone to book will matter to a great extent. The model shows that for two otherwise identical countries with same initial conditions, the country with higher degree of financial development (e.g., judiciary, rule of law, accountability, etc.) will have higher stocks of finance and hence, will lend more at lower rate of interest so that employment growth will occur. The country where entrepreneurs have better access to external finance on top of their own financial resources will be better off in curing unemployment problem and will participate in trade in the world market. From the model, we could also elicit a mechanism where incentive to invest in R&D to induce technical progress in the machine sector could account for the secular decline in the labor-share of income. Additionally, the model develops some empirically testable hypothesis such as, role of financial institutions, and financial development for inclusive growth via job and employment. In fact, Emara and Said (2021) has shown empirically in the context of African economies and emerging markets that improvement in governance, supervisory and regulatory regimes, judicial independence, and contract enforcement coupled with financial access is conducive for economic growth. Empirical validation of our results with numerical simulation of the cost-shares of machines vis-à-vis labor along with changes in prices of goods and factors can be mounted in further extension of this research. However, our conjecture and mechanism differs from the Stiglitz and Weiss (1981) and Williamson (1987) conjectures. Also, in a two period dynamic model we have shown that if rate-of-return on R&D investment is higher than rate of interest plus the cost of undertaking R&D, then the financiers will always invest in the machine-intensive sector. Rate-of-interest in the second period being higher, there will be investment in more machines like directed technical change with machine-bias. Thus, machine-biased R&D-financing might be good for productivity-enhancing growth, but bad for intra- and inter-group or occupational equality. This might have ramifications in the labor-market in general. Ours value-addition lies in developing a theoretical setup, and showing a novel mechanism by blending traditional Neo-Classical 2x2 sector workhorse of trade models with finance, credit rationing, allocation of capital in production vis-à-vis innovation along the lines of classical wage-fund theory.

## References

- Acemoglu, D., 1998. Why do new technologies complement skills? Directed technical change and wage inequality. *Quarterly Journal of Economics* 113 (4), 1055–1089.
- Acemoglu, Daron. 2002. “Directed Technical Change.” *Review of Economic Studies* 69 (4):781–810.
- Acemoglu, D., 2003. Cross-country inequality trends. *Economic Journal* 113 (485), 121–149.
- Acemoglu, D., and P Restrepo (2018a). The Race between Man and Machine: Implications of Technology for Growth, Factor Shares, and Employment. *American Economic Review*. 108 (6), pp. 1488—1542.
- (2018b), “Modeling Automation”, *AEA Papers and Proceedings*, 108, 48–53.
- (2019a). Automation and New Tasks: How Technology Displaces and Reinstates Labor. NBER WP 25684.
- (2019b), “Demographics and Automation” (Working Paper).
- (2020), “Robots and Jobs: Evidence from US Labor Markets”, *Journal of Political Economy*, 128, 2188–2244.
- Acemoglu, D., U. Akcigit, and W. R. Kerr (2016). Innovation Network. *PNAS*. Pp. 1—6.
- Aghion, Philippe, Ufuk Akcigit, Antonin Bergeaud, Richard Blundell, and David Hemous. (2019a). “Innovation and Top Income Inequality.” *Review of Economic Studies* 86 (1):1–45.
- Aghion, Philippe and Peter Howitt. 1992. “A Model of Growth Through Creative Destruction.” *Econometrica* 60 (2):323–351.
- Aghion, Phillippe A., Chad Jones, and Benjamin F. Jones. (2019b). “Artificial Intelligence and Economic Growth.” In *The Economics of Artificial Intelligence: An Agenda*. University of Chicago Press, 237–282.
- Aizenman, Joshua, Yothin Jinjarak, and Mark M. Spiegel (2022). Fiscal Stimulus and Commercial Bank Lending Under COVID-19. NBER Working Paper No. 29882 pp. 1-49.
- Alexandre, F., P. Bacao, J. Cerejeira, H. Costa, M. Portela, (2021). Minimum wage and financially distressed firms: another one bites the dust, *Labour Economics* (2021), doi: <https://doi.org/10.1016/j.labeco.2021.102088>
- Amiti, M. and Weinstein, D. (2011), “Exports and Financial Shocks”, *Quarterly Journal of Economics*, 126, 1841–1877.
- Antras, P. & Caballero R. J. (2009). Trade and Capital Flows: A Financial Frictions Perspective. *Journal of Political Economy*, 117(4) 701–743
- Autor, David., Frank, M., J. Bessen, E. Brynjolfsson, M. Cebrian, David J. Deming, Maryann Feldman, Matthew Groh, Jose´ Lobo, Esteban Moro, Dashun Wang, Hyejin Youn, and Iyad Rahwan (2019). Toward understanding the impact of artificial intelligence on labor. *PNAS*. Edited by Jose A. Scheinkman. Pp. 1-9.
- Beladi, H. and S., Marjit (1992). Foreign Capital and Protectionism. *Canadian Journal of Economics*, vol. 25, issue 1, pp. 233-38
- Caselli, Francesco and Alan Manning (2019). Robot Arithmetic: New Technology and Wages. *American Economic Review: Research Insights*. Vol 1 (1), pp. 1-12.
- Caves, R., J. Frankel and R. W. Jones (2007): *World Trade and Payments* (10th Ed.)
- Calvo GA, Coricelli F, Ottonello P (2012): Labor Market, Financial Crises and Inflation: Jobless and Wageless Recoveries. NBER Working Paper, No. 18480.

Chor, Davin, and K. Manova (2013). Off the cliff and back? Credit conditions and international trade during the global financial crisis. *Journal of International Economics*, Vol. 87, pp. 117-133.

Das, Gouranga G. (2013): "Moving" land across borders: spatial shifts in land demand and immiserizing effects, *Journal of Economic Policy Reform*, 16:1, 46-67

Davis, Donald (2020). The Comparative advantage of Cities. *Journal of International Economics*, Vol. 123.

Doan, H. T. T. and Wan, G. 2017. Globalization and the Labor Share in National Income. Asian Development Bank Institute, Working Paper, No. 639.

Doraszelski, U. and J. Jaumandreu (2013). R&D and Productivity: Estimating Endogenous Productivity. *R. Ec. Stud.* 80. 1338—1383.

Durvasula, Maya, Lisa Larrimore Ouellette, and Heidi L. Williams. (2019). "Private and Public Investments in Biomedical Research." *AEA Papers and Proceedings*.

Egger, P. and C. Keuschnigg (2015). Innovation, Trade, and Finance. *American Economic Journal: Microeconomics*, 7(2), pp. 121-157.

Egger, P., S. Kunert, and T. Seidel (2018). The Competitive Effects of Credit constraints in the Global Economy. *Economica*, 85, pp. 771-792.

Emara, Noha., and Ayah el Said (2021). Financial inclusion and economic growth: The role of governance in selected MENA countries. *International Review of Economics and Finance*. Vol. (75), pp. 34-54.

Findlay, Ronald (1984). Growth and development in trade models. *Handbook of International Economics*, Volume 1, 1984, Pages 185-236

Georg Graetz , Pascual Restrepo , Oskar Nordström Skans (2022). Technology and the Labor Market, *Labour Economics*, doi: <https://doi.org/10.1016/j.labeco.2022.102177>

Hasan, S. and Ian Sheldon (2016). Credit-Constraints, Technology Choice and Exports: A Firm-level Study for Latin American Countries. *Review of Development Economics*, 20 (2), pp. 547-560.

Hicks J & Hollander S. (1977). "Mr. Ricardo & the Moderns." *Quarterly Journal of Economics*, Vol 91, No 03, Pages 351-369.

Hillman, Arye L. & Heinrich W. Ursprung (1988). Domestic politics, foreign interests and international trade policy. *American Economic Review* 78, 729- 745.

Hollander, Samuel (1984). J.S. Mill on "Derived Demand" and the Wage-Fund Theory Recantation. *Eastern Economic Journal*. Vol. 10 (1), pp. 87—98.

Jones, R. W. (1965): "The Structure of Simple General Equilibrium Models," *Journal of Political Economy*, v. 73, pp. 557-72

----- (1985). A theorem on income distribution in a small open economy, *Journal of International Economics*, Volume 18, Issues 1–2, Pages 171-176

----- (2012), General equilibrium theory and competitive trade models. *International Journal of Economic Theory*, 8: 149-164. <https://doi.org/10.1111/j.1742-7363.2012.00183.x>

----- (2018). *International Trade Theory and Competitive Models: Features, Values, and Criticisms*. World Scientific Studies in International Economics: Volume 65

---and Dei, F. (1983). International trade and foreign investment a simple model. *Economic Inquiry*, 21: 449-464.

---and Marjit, S (2003), Economic Development, Trade and Wages. *German Economic Review*, 4: 1-17.

- and Marjit, S (2009). Competitive trade models and real world features. *Econ Theory* 41, 163–74.
- and Marjit,S (1985). A Simple Production Model with Stolper-Samuelson Properties. *International Economic Review*. Vol. 19, pp. 565-567.
- and R. Ruffin (1975). Trade patterns with Capital Mobility. In M. Parkin and A. R. Nobay (eds.). *Current Economic Problems*. Cambridge University Press, pp. 307-32
- and R. Weder (Eds.) (2017). *200 Years of Ricardian Trade Theory: Challenges of Globalization*. Springer Publishers.
- Karabarbounis, L and B Neiman (2014a), “The Global Decline of the Labor Share”, *Quarterly Journal of Economics* 129(1): 61–103.
- Karabarbounis, L and B Neiman (2014b), “Capital Depreciation and Labor Shares Around the World: Measurement and Implications”, Working Paper, University of Chicago.
- Korinek, Anton and Joseph E. Stiglitz (December 2017). Artificial Intelligence and Its Implications for Income Distribution and Unemployment. NBER WP # 24174, pp. 1-45
- Loebbing, Jonas (2022). An Elementary Theory of Directed Technical Change and Wage Inequality. *Review of Economic Studies*, 89, pp. 411-451.
- Lucas, R. (1990). Why Doesn't Capital Flow from Rich to Poor Countries? *American Economic Review*, 80(2), 92-96.
- Maneschi A. (1983). Dynamic Aspects of Ricardo’s International Trade Theory. *Oxford Economic Papers*, Volume 35, Issue 01, Pages 67-80.
- Manova, K. (2008), “Credit Constraints, Equity Market Liberalizations and International Trade”, *Journal of International Economics*, 76, 33–47.
- Manova, K, Wei, S.-J. and Zhang, Z. (2009), “Firm Exports and Multinational Activity under Credit Constraints”. (NBERWorking Paper 16905).
- Marjit, S. (1991) Agro-based industry and rural-urban migration: A case for an urban employment subsidy, *Journal of Development Economics*, Volume 35, Issue 2, Pages 393-398,
- Marjit, S (2020). A New Ricardian Model of Trade, Growth and Inequality. CESifo Working Paper # 8689-2020. November 2020. Germany. ISSN 2364-1428.
- Marjit, S and Gouranga G. Das (2021). The New Ricardian Specific Factor Model. *Journal of Asian Economics*. V.76, 1013628, pp. 1-8.
- Marjit, Sugata & Suryaprakash Mishra, 2020. "Credit Market Imperfection, Lack of Entrepreneurs and Capital Outflow from a Developing Economy," CESifo Working Paper Series 8515, CESifo.
- Marjit, S., B. Mandal, and N. Nakanishi (2020). *Virtual Trade and Comparative Advantage: the Fourth Dimension*. Springer Publisher.
- Marjit, S and N. Nakanishi (2021). The Wage Fund Theory and the Gains from Trade in a Dynamic Ricardian Model. CESifo Working Paper No. 9218. Pp. 1-27.
- and S. Kar (Eds.), 2018. *International Trade, Welfare, and the Theory of General Equilibrium*. Cambridge: Cambridge University Press.
- Matsuyama, K. (2005), “Credit Market Imperfections and Patterns of International Trade and Capital Flows”, *Journal of the European Economic Association*, 3, 714–723.
- Mussa, M, (1982). Imperfect factor mobility and the distribution of income, *Journal of International Economics*, Volume 12, Issues 1–2, 1982, Pages 125-141,

Mill, J. S. (1848). Principles of Political Economy with some of their applications to Social Philosophy. (1st ed.), London: John W. Parker, 7 December 2012. Online version via Project Gutenberg. <http://www.gutenberg.org/files/30107/30107-h/30107-h.html>

Mill, J.S. 1869. Thornton on labour and its claims. *Fortnightly Review* 505–18: 680–700.

Noack, Frederik, and Christopher Costello (2022). Credit Markets, Property Rights, and the Commons. NBER WP # 29889.

Rajan, R. and Zingales, L. (1998), “Financial Dependence and Growth”, *American Economic Review*, 88, 559–586.

Patinkin, D. (1965) *Money, Interest and Prices: An Integration of Monetary and Value Theory*. 2nd Edition, Row, Peterson and Co., Evanston, Harper and Row, New York.

Popov A, Rocholl. J (2018): Do Credit Shocks Affect Labor Demand? Evidence for Employment and Wages during the Financial Crisis. *Journal of Financial Intermediation*, 36:16-27.

Ricardo, David (1817). *Principles of Political Economy and Taxation*. 1st ed. London: John Murray.

Schumpeter, Joseph A. 1934. *The Theory of Economic Development: An Inquiry into Profits, Capital, Credit, Interest, and the Business Cycle*. New York, NY: Oxford University Press.

Solow, Robert M. 1956. “A Contribution to the Theory of Economic Growth.” *Quarterly Journal of Economics* 70 (1):65.

———. 1957. “Technical Change and the Aggregate Production Function.” *Review of Economics and Statistics* 39 (3):312–320.

Steedman, I (1979). *Fundamental Issues in Trade Theory- Palgrave, McMillan*

Stiglitz, J. E., and Andrew Weiss (1981). Credit Rationing in Markets with Imperfect Information. Vol. 71, No. 3 (Jun., 1981), pp. 393-410.

Syversen, C. (2011). “What Determines Productivity?” *Journal of Economic Literature* 49 (2):326–365.

Vint, John (2018). Wages Fund. In: *The New Palgrave Dictionary of Economics*. Palgrave Macmillan, London. [https://doi.org/10.1057/978-1-349-95121-5\\_2129-1](https://doi.org/10.1057/978-1-349-95121-5_2129-1)

Williamson, Stephen D. (1987). Costly Monitoring, Loan Contracts, and Equilibrium Credit Rationing. *The Quarterly Journal of Economics*, Feb., 1987, Vol. 102, No. 1 (Feb., 1987), pp. 135-146

## Appendices

### Appendix 1.

Following Caves, Frankel and Jones (2011) and Feenstra (2003), using (4) and (5)

$$\lambda_{LX} \hat{X} + \lambda_{LY} \hat{Y} = \hat{L}$$

$$\lambda_{MX} \hat{X} + \lambda_{MY} \hat{Y} = \hat{M}$$

$$\text{Using Cramer's Rule: } \hat{X} = \frac{\lambda_{MY} \hat{L} - \lambda_{LY} \hat{M}}{\lambda_{LX} \lambda_{MY} - \lambda_{MX} \lambda_{LY}}, \hat{Y} = \frac{\lambda_{LX} \hat{M} - \lambda_{MX} \hat{L}}{\lambda_{LX} \lambda_{MY} - \lambda_{MX} \lambda_{LY}} \Rightarrow \hat{X} - \hat{Y} = \frac{\hat{L} - \hat{M}}{|D|},$$

where  $|D| = \lambda_{LX} \lambda_{MY} - \lambda_{MX} \lambda_{LY} = \lambda_{LX} - \lambda_{LY} = \lambda_{MY} - \lambda_{MX} < 0$  as X uses more M (by assumption)

$$\text{Thus, } \hat{X} - \hat{Y} = \frac{\hat{L} - \hat{M}}{|D|} = \alpha [\hat{M} - \hat{L}] > 0 \text{ as } \lambda_{MX} > \lambda_{MY}, \text{ and } \alpha = \frac{1}{\lambda_{MX} - \lambda_{MY}} > 0 \text{ (QED).}$$

### Appendix 2.

From (2) and (3), via Jones (1965), when  $\hat{P} \neq 0$

$$\begin{pmatrix} \theta_{LX} & \theta_{MX} \\ \theta_{LY} & \theta_{MY} \end{pmatrix} \begin{pmatrix} \widehat{W} \\ \widehat{p}_M \end{pmatrix} = \begin{pmatrix} \hat{P} - \widehat{(1+r)} \\ -\widehat{(1+r)} \end{pmatrix} \text{ and } |\theta| = \theta_{LX}\theta_{MY} - \theta_{MX}\theta_{LY} = \theta_{MY} - \theta_{MX} = \theta_{LX} - \theta_{LY} < 0 \text{ (by}$$

intensity assumption). Applying Cramer's rule yields:

$$\widehat{W} = \frac{1}{|\theta|} \begin{pmatrix} \hat{P} - \widehat{(1+r)} & \theta_{MX} \\ -\widehat{(1+r)} & \theta_{MY} \end{pmatrix} = \hat{P} \frac{\theta_{MY}}{|\theta|} - \widehat{(1+r)}$$

$$\widehat{p}_M = \frac{1}{|\theta|} \begin{pmatrix} \theta_{LX} & \hat{P} - \widehat{(1+r)} \\ \theta_{LY} & -\widehat{(1+r)} \end{pmatrix} = -\hat{P} \frac{\theta_{LY}}{|\theta|} - \widehat{(1+r)}$$

Using these and (11),

$$\widehat{K}_d = -\frac{\hat{P}}{|\theta|} [-\lambda_{MK}\theta_{LY} + \lambda_{LK}\theta_{MY}] = \lambda_{LK} \left[ \frac{\hat{P}\theta_{MY}}{|\theta|} - \widehat{(1+r)} \right] + \lambda_{MK} \left[ -\widehat{(1+r)} - \frac{\hat{P}\theta_{LY}}{|\theta|} \right]$$

$$\Rightarrow \widehat{K}_d = -\widehat{(1+r)} [\lambda_{LK} + \lambda_{MK}] + \frac{\hat{P}}{|\theta|} [\lambda_{LK}\theta_{MY} - \lambda_{MK}\theta_{LY}] = -\widehat{(1+r)} + \frac{\hat{P}}{|\theta|} [\lambda_{LK}\theta_{MY} - \lambda_{MK}\theta_{LY}] \text{ (QED).}$$

Given P, higher 'K' will imply lower (1+r) or 'r'. With  $\hat{P} > 0$  from (2) and (3), derive:

$$\widehat{W}\theta_{LX} + \widehat{p}_M\theta_{MX} = -\widehat{(1+r)} + \hat{P} = \widehat{W}\theta_{LY} + \widehat{p}_M\theta_{MY} + \hat{P} \Rightarrow \widehat{W}(\theta_{LX} - \theta_{LY}) + \widehat{p}_M(\theta_{MX} - \theta_{MY}) = \hat{P}$$

$$\text{Thus, } \widehat{p}_M - \widehat{W} = \frac{-\hat{P}}{|\theta|} = \frac{\hat{P}}{\beta}, \text{ where } |\theta| = \theta_{LX} - \theta_{LY} = \theta_{MY} - \theta_{MX} < 0, \beta = -|\theta|$$

### Appendix 3.

$$\text{From above, } \widehat{K}_d = 0 \Rightarrow -\widehat{(1+r)} = \frac{\hat{P}}{|\theta|} [\lambda_{LK}\theta_{MY} - \lambda_{MK}\theta_{LY}]$$

$$\text{Thus, } \hat{P} - \widehat{(1+r)} = \hat{P} - \frac{\hat{P}}{|\theta|} [\lambda_{LK}\theta_{MY} - \lambda_{MK}\theta_{LY}] \Rightarrow \hat{P} - \widehat{(1+r)} = \hat{P} \left[ 1 - \frac{\lambda_{LK}\theta_{MY} - \lambda_{MK}\theta_{LY}}{|\theta|} \right]$$

$$\text{And } \hat{P} - \widehat{(1+r)} = \frac{\hat{P}}{|\theta|} [\theta_{MY}(1 - \lambda_{LK}) + \lambda_{MK}\theta_{LY} - \theta_{MX}] = \frac{\hat{P}}{|\theta|} [(\theta_{MY} + \theta_{LY})\lambda_{MK} - \theta_{MX}] = \frac{\hat{P}}{|\theta|} [\lambda_{MK} - \theta_{MX}] \text{ (QED.)}$$