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**PERSONALIZED PRICING  
WITH HETEROGENEOUS  
MISMATCH COSTS**

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August 2022

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# Personalized pricing with heterogeneous mismatch costs\*

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## Abstract

Personalized pricing has become a reality through digitization. We examine firms' incentives to adopt one of the three pricing schemes: uniform, personalized, or group pricing in a Hotelling duopoly model. There are two types of consumer groups that are heterogeneous in their mismatch costs. We show that both firms employ personalized pricing in equilibrium regardless of the heterogeneity of consumer groups. If the consumer groups' heterogeneity is significant, the profits are higher when both firms use personalized pricing than when they employ uniform pricing; otherwise, the latter profits are higher than the former. Profits are highest when firms employ group pricing among the three cases. The ranking of consumer welfare among the three cases is opposite to that of profits.

**Keywords:** Personalized pricing, Group pricing, Heterogeneous consumer types, Hotelling model.

**JEL Classification Numbers:** L13, D43

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# 1 Introduction

Personalized pricing has become a reality through digitization (OECD, 2018).<sup>1</sup> In particular, mobile technology allows companies to offer discriminatory prices because customers have individual mobile devices to access online services (Esteves and Resende, 2016). A famous example of such personalized pricing is Uber’s “route-based pricing,” which depends on customers’ willingness to pay (Bloomberg, 2017).<sup>2</sup> Although personalized pricing can help firms capture each consumer’s willingness to pay, the strategic interaction with rivals can also exert opposite effects. This possibility raises questions about whether personalized pricing can improve a firm’s profits and benefit the consumers.

Regarding the influence of personalized pricing on firms’ profits, there has been broad discussion since the seminal paper by Thisse and Vives (1988). They concluded that personalized pricing induces a prisoner’s dilemma where all firms become worse off than the situation when all the firms commit to uniform pricing (Shaffer and Zhang, 1995; Bester and Petrakis, 1996; Zhang, 2011). However, several studies show that personalized pricing does not necessarily result in a prisoner’s dilemma in the case of firm asymmetry (e.g., a quality difference (Shaffer and Zhang, 2002), quality choice (Ghose and Huang, 2009), and initial cost difference with R&D (Matsumura and Matsushima, 2015)). Therefore, discussions on the effect of personalized pricing on firms’ profits are inconclusive.

This study examines consumer heterogeneity and develops a linear market model comprising two consumer groups—one incurring high transportation costs (high-end consumers) and the other incurring low transportation costs (low-end consumers). The model also comprises two symmetric firms competing in this market and choosing one of the pricing

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<sup>1</sup> OECD (2018) “Personalised pricing in the digital era,” 28 November 2018  
[https://one.oecd.org/document/DAF/COMP\(2018\)13/en/pdf](https://one.oecd.org/document/DAF/COMP(2018)13/en/pdf) (Last access: 13 March 2022).

<sup>2</sup> Bloomberg (2017) “Uber starts charging what it thinks you’re willing to pay,” 19 May 2017  
<https://www.bloombergquint.com/markets/uber-s-future-may-rely-on-predicting-how-much-you-re-willing-to-pay> (Last access: 13 March 2022).

schemes: uniform, personalized, or group pricing.

The results are as follows. Both firms always employ personalized pricing in equilibrium, which enhances both firms' profits compared with the situation where both employ uniform pricing if the consumer groups are sufficiently heterogeneous. Furthermore, firms' profits when they both employ group pricing are always the highest compared with the other two cases mentioned before. We show that the consumer surplus in group pricing is always the lowest when comparing the aforementioned three cases. The consumer surplus in uniform pricing is the highest if the consumer groups are sufficiently heterogeneous.

In this regard, the intuition stems from how the low-end consumers influence the uniform prices of firms. When the low-end consumers incur a sufficiently low transportation cost (relative to the high-end consumers), they are price-sensitive. When firms adopt a uniform pricing policy, they offer sufficiently low prices to capture the price-elastic consumers; this diminishes their profits from high-end consumers. However, when both firms adopt personalized pricing, they customize each consumer's price and delink the markets for high- and low-end consumers. This makes the competition mode similar to an asymmetric Bertrand competition for each consumer. Unlike a uniform pricing policy, the adoption of personalized pricing intensifies the competition for low-end consumers (a negative effect). Nevertheless, it enables each firm to set higher prices for high-end consumers located closer to that firm (a positive effect). In summary, personalized pricing triggers market segmentation such that low-end consumers pay less. In contrast, some high-end consumers pay more, which benefits the firms when the heterogeneity between the two consumer groups is sufficiently significant.

Group pricing is the simplest way to delink the two markets for the two consumer types and allows firms to escape fierce Bertrand competition under personalized pricing at each point. However, group pricing is not attainable in equilibrium. The comparison between group and personalized pricing is equivalent to that between uniform and personalized pricing.

ing in Thisse and Vives (1988) when we focus on each consumer group. We can directly apply the conclusion in Thisse and Vives (1988) and find that each firm has an incentive to employ personalized pricing, given that the rival employs group pricing. Therefore, group pricing is not sustainable as an equilibrium outcome, although firms are better off if they employ group pricing.

In a recent study, Esteves (2021) considers a static setting where personalized pricing can become a winning strategy for competing firms.<sup>3</sup> The intuition relies on the heterogeneity in the quantity demanded by consumers. As in Shin and Sudhir (2010), some consumers demand more products than others. We capture consumers' heterogeneity by their horizontal valuation of products (i.e., the transportation cost) instead of the quantity purchased. We also consider the endogenous choices of pricing schemes. Given these differences, our study complements Esteves (2021).

Rhode and Zhou (2022) investigate the effects of personalized pricing on profits and welfare, using a generalized oligopoly model based on Perlof and Salop (1985).<sup>4</sup> The authors generalize the number of firms and the degree of market coverage and show that in the short run (e.g., when firms' number is exogenously given), personalized pricing would benefit firms and harm consumers compared with uniform pricing, only when the market coverage is low. By incorporating consumers' heterogeneity in their mismatch costs, we show that the same argument could hold even under a *full* market coverage. Therefore, we provide a complement to Rhode and Zhou (2022).

The remainder of this paper is organized as follows. Section 2 formulates the model. Then, section 3 analyzes the model and presents the main results. Finally, section 4 concludes the paper. The detailed mathematical procedures are available in the not-for-publication

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<sup>3</sup> Chen et al. (2020) examine a model wherein consumers' identity management allows them to choose the uniform prices set only for new customers by firms. The model shows that personalized pricing can allow firms to fully extract the surpluses of targeted consumers.

<sup>4</sup> The Perlof and Salop (1985) model is also used in the context of mixed bundling (see Zhou, 2021).

Appendix.

## 2 Model

We consider a product characteristic space with interval  $[0, 1]$ . There are two consumer types called consumer  $k$  ( $= L, H$ ). We refer to the markets with consumers  $H$  and  $L$  as markets  $H$  and  $L$ . We normalize the number of consumers to 1 and assume that the shares of consumers  $H$  and  $L$  are  $\lambda$  and  $1 - \lambda$ , respectively. The distribution of consumers in each market is uniform on the interval  $[0, 1]$ .

There are two firms—firms 0 and 1 located at 0 and 1, respectively. We assume that firm  $i$  can choose one of the three pricing schemes—uniform, personalized, or group pricing. In uniform pricing, the firm offers the same price to all consumers regardless of their location or type. In personalized pricing, the firm can customize the price for each consumer located at  $x \in [0, 1]$  in market  $k$ . Finally, in group pricing, the firm offers discriminatory prices to consumers  $H$  and  $L$ .

The utility function for a consumer located at  $x \in [0, 1]$  in market  $k$  is

$$U(x, k) = \begin{cases} v - p_0(x, k) - t_k x & \text{buying from firm 0,} \\ v - p_1(x, k) - t_k(1 - x) & \text{buying from firm 1,} \end{cases}$$

where  $v$  is the willingness to pay for the ideal product,  $p_i(x, k)$  is the customized price offered to consumers located at  $x$  in market  $k$ , and  $t_k$  is the per-distance transportation (mismatch) cost in market  $k$ , as in Armstrong (2006, pp.116-117). Note that, for any  $x \in [0, 1]$  and  $k = H, L$ ,  $p_i(x, k)$  is constant when firm  $i$  uses uniform pricing. We denote the uniform price offered by firm  $i$  by  $p_i$ . When firm  $i$  uses group pricing, it offers a uniform price  $p_i(k)$  to market  $k$ . Assume  $t_H > t_L > 0$  and define  $\tau \equiv t_L/t_H$ . We perform comparative statics on  $\tau$  keeping  $t_H$  fixed throughout the study.

Each firm produces a product without cost. Then, firm  $i$ 's profit is as follows:

$$\begin{aligned}\pi_0[p_0(x, k), p_1(x, k)] &\equiv \lambda \int_0^{x_H} p_0(x, H)dx + (1 - \lambda) \int_0^{x_L} p_0(x, L)dx, \\ \pi_1[p_0(x, k), p_1(x, k)] &\equiv \lambda \int_{x_H}^1 p_1(x, H)dx + (1 - \lambda) \int_{x_L}^1 p_1(x, L)dx,\end{aligned}$$

where  $x_k$  is the location of indifferent consumers in market  $k$ .

We follow the timing structure of Thisse and Vives (1988). In the first stage, each firm selects one of the three pricing schemes. In the second stage, a firm employing a uniform or group pricing offers its observable uniform price. Subsequently, a firm employing personalized pricing offers personalized prices,  $p_i(x, k)$ , determined by the consumer types and locations. If the two firms adopt the same pricing scheme, they simultaneously determine their prices. The timing of pricing offers are in line with those considered in the literature (e.g., Thisse and Vives, 1988; Shaffer and Zhang, 2002; Clavorà Braulin and Valletti, 2016; Choe et al., 2018; Chen et al., 2020; Esteves, 2021). Using backward induction, we solve this game.

### 3 Analysis

We need to solve six types of subgames: (i) Both firms choose uniform pricing; (ii) Both firms choose personalized pricing; (iii) Both firms choose group pricing; (iv) One firm chooses uniform pricing, and the other one chooses personalized pricing; (v) One firm chooses group pricing, and the other one chooses uniform pricing, and (vi) One firm chooses group pricing, and the other one chooses personalized pricing. As the calculations are simple, we provide only the results in the subgames.



### 3.1 Both firms choose uniform pricing

We consider the case where both firms employ uniform pricing. They simultaneously offer their uniform prices. We use the superscript “ $UU$ ” to denote this case.

We classify the outcome in this subgame into two cases: (1) a pure strategy equilibrium exists; (2) no pure strategy equilibrium exists. The former case appears if  $\tau$  satisfies the following inequality, otherwise the latter appears:

$$\tau \geq \frac{-(1-\lambda)(2+\lambda) + 2\sqrt{1-\lambda}}{\lambda(3+\lambda)} \equiv \tau_{uu}. \quad (1)$$

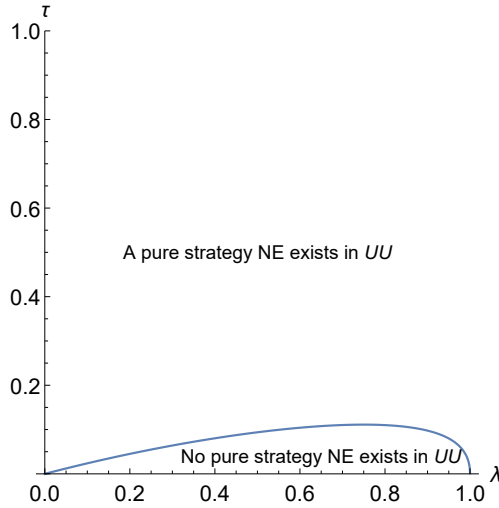


Figure 1: The area in which a pure strategy NE exists in  $UU$

In the first case ( $\tau \geq \tau_{uu}$ ), we obtain each firm’s price and profit as follows:

$$p \equiv \frac{t_H \tau}{1 - \lambda(1 - \tau)}, \quad \pi \equiv \frac{t_H \tau}{2 - 2\lambda(1 - \tau)}. \quad (2)$$

In the second case (when  $\tau < \tau_{uu}$ ), we instead consider a mixed strategy equilibrium where, (i) firm 0 chooses a low price  $p_{0l}$  with probability  $\alpha$  and a high price  $p_{0h}$  with proba-

bility  $1 - \alpha$ , and (ii) firm 1 chooses a low price  $p_{1l}$  with probability  $\beta$  and a high price  $p_{1h}$  with probability  $1 - \beta$ . We use the superscript  $m$  to indicate an outcome in a mixed strategy equilibrium if necessary. We focus on a symmetric mixed strategy NE such that  $x_L(p_{0h}, p_{1l}) < 0$ ,  $x_L(p_{0l}, p_{1h}) > 1$ ,  $x_L(p_{0l}, p_{1l}) = x_L(p_{0h}, p_{1h}) = 1/2$ ,  $x_H(p_{0l}, p_{1h}) \in (0, 1)$ ,  $x_H(p_{0h}, p_{1l}) \in (0, 1)$ ,  $x_H(p_{0l}, p_{1l}) = x_H(p_{0h}, p_{1h}) = 1/2$ , and  $\alpha = \beta \in (0, 1)$ . In this equilibrium, the resulting outcome in market  $L$  is the corner solution in which the low-price firm obtains all type  $L$  consumers' demand if the rival offers the high price.

Therefore, we can numerically obtain a symmetric mixed strategy equilibrium:

$$\begin{aligned}
p_{il}^{UUm} &= \frac{(2\lambda\tau + \alpha^{UU}(1-\lambda)(1+\lambda\tau) + (\alpha^{UU})^2(1-\lambda)^2)t_H\tau}{2\lambda\tau(1-\lambda(1-\tau)) + \alpha^{UU}(1-\alpha^{UU})(1-\lambda)(1-\lambda-2\lambda\tau)}, \\
p_{ih}^{UUm} &= \frac{(\lambda(1+\tau-\lambda(1-\tau)) + \alpha^{UU}(1-\lambda)(1-\lambda(2-\tau)) - (\alpha^{UU})^2(1-\lambda)^2)t_H\tau}{2\lambda\tau(1-\lambda(1-\tau)) + \alpha^{UU}(1-\alpha^{UU})(1-\lambda)(1-\lambda-2\lambda\tau)}, \\
\alpha^{UU} = \beta^{UU} &\in (0, 1) \text{ that satisfy the following two equations:} \\
\pi_0^{UUm} &\equiv p_{0l}^{UUm}[\lambda\{\beta^{UU}x_H(p_{0l}^{UUm}, p_{1l}^{UUm}) + (1-\beta^{UU})x_H(p_{0l}^{UUm}, p_{1h}^{UUm})\} \\
&\quad + (1-\lambda)\{\beta^{UU}x_L(p_{0l}^{UUm}, p_{1l}^{UUm}) + (1-\beta^{UU})\}] \\
&= p_{0h}^{UUm}[\lambda\{\beta^{UU}x_H(p_{0h}^{UUm}, p_{1l}^{UUm}) + (1-\beta^{UU})x_H(p_{0h}^{UUm}, p_{1h}^{UUm})\} \\
&\quad + (1-\lambda)(1-\beta^{UU})x_L(p_{0h}^{UUm}, p_{1h}^{UUm})] \text{ and} \\
\pi_1^{UUm} &\equiv p_{1l}^{UUm}[\lambda\{\alpha^{UU}(1-x_H(p_{0l}^{UUm}, p_{1l}^{UUm})) + (1-\alpha^{UU})(1-x_H(p_{0h}^{UUm}, p_{1l}^{UUm}))\} \\
&\quad + (1-\lambda)\{\alpha^{UU}(1-x_L(p_{0l}^{UUm}, p_{1l}^{UUm})) + (1-\alpha^{UU})\}] \\
&= p_{1h}^{UUm}[\lambda\{\alpha^{UU}(1-x_H(p_{0h}^{UUm}, p_{1l}^{UUm})) + (1-\alpha^{UU})(1-x_H(p_{0h}^{UUm}, p_{1h}^{UUm}))\} \\
&\quad + (1-\lambda)(1-\alpha^{UU})(1-x_L(p_{0h}^{UUm}, p_{1h}^{UUm}))].
\end{aligned} \tag{3}$$

Given the complexity of the last two simultaneous equations, we cannot explicitly derive the values of  $\alpha^{UU}$  and  $\beta^{UU}$  but numerically show those values in Figure 2. The region where  $\alpha^{UU}(\lambda, \tau) \in (0, 1)$  coincides with that of the pure strategy equilibrium is not sustainable.

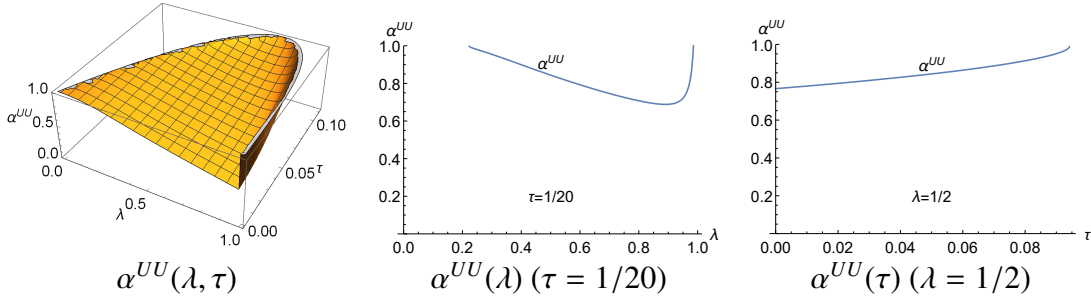


Figure 2: Mixed strategy in which both firms employ uniform pricing

We can summarize the discussion as follows:

$$p_i^{UU} = \begin{cases} \frac{t_H \tau}{1 - \lambda(1 - \tau)} & \text{if } \tau \geq \tau_{uu}, \\ \begin{cases} p_{il}^{UUm} \text{ in (3) with probability } \alpha^{UU} (\beta^{UU}), \\ p_{ih}^{UUm} \text{ in (3) with probability } 1 - \alpha^{UU} (1 - \beta^{UU}). \end{cases} & \text{if } \tau < \tau_{uu}, \end{cases} \quad (4)$$

$$\pi_i^{UU} = \begin{cases} \frac{t_H \tau}{2 - 2\lambda(1 - \tau)} & \text{if } \tau \geq \tau_{uu}, \\ \pi_i^{UUm} \text{ in (3),} & \text{if } \tau < \tau_{uu}. \end{cases} \quad (5)$$

Conducting comparative statics, we find that when  $\lambda$  is large and  $\tau$  is smaller than but sufficiently close to  $\tau_{uu}$ , a decrease in  $\tau$  enlarges the profits of the two firms. If  $\tau < \tau_{uu}$ , each firm sets a high price,  $p_{ih}^{UUm}$ , with a positive probability to escape from fierce competition in market  $L$  and to concentrate on market  $H$  (see  $\alpha^{UUm}(\lambda, \tau)$  in Figure 2). The strategic effect mitigates price competition, although the direct effect of a decrease in  $\tau$  intensifies price competition. If  $\tau$  is smaller than but sufficiently close to  $\tau_{uu}$ , the marginal impact of the strategic effect is larger than the direct effect because the probability of offering  $p_{ih}^{UUm}$  becomes strictly positive, implying that the strategic effect begins to be effective and has a first-order impact on competition. Otherwise, the direct effect of a decrease in  $\tau$  dominates the strategic effect, aggravating competition. We summarize the above finding in the following proposition:

**Proposition 1** *Suppose that both firms employ uniform pricing. When  $\tau$  is smaller than but*

sufficiently close to  $\tau_{uu}$  and  $\lambda$  is larger than a threshold value, a decrease in  $\tau$  increases the expected profits of the firms.

### 3.2 Both firms choose personalized pricing

We consider the case where both the firms use personalized pricing. They simultaneously offer personalized prices for distinct consumers. We use the superscript “PP” to denote this case.

In each market, the firms face an asymmetric Bertrand competition at each location,  $x \in [0, 1]$ . The firms choose the following prices at location  $x$  in market  $k (= H, L)$ .

$$p_0^{PP}(x, k) \equiv \begin{cases} t_k(1-x) - t_k x & \text{for } x \in [0, 1/2], \\ 0 & \text{for } x \in (1/2, 1], \end{cases}$$

$$p_1^{PP}(x, k) \equiv \begin{cases} 0 & \text{for } x \in [0, 1/2], \\ t_k x - t_k(1-x) & \text{for } x \in (1/2, 1]. \end{cases}$$

Then, consumers in  $[0, 1/2]$  and  $(1/2, 1]$  buy from firms 0 and 1, respectively. We obtain the profit of each firm as follows:

$$\pi^{PP} \equiv \frac{t_H(\lambda + \tau(1 - \lambda))}{4}. \quad (6)$$

When  $\lambda = 1$  or  $\tau = 1$ , the outcome is the same as that with the personalized pricing in Thisse and Vives (1988). When  $\tau \neq 1$ ,  $\pi^{PP}$  decreases when  $\lambda$  decreases. However,  $p_i^{PP}(x, k)$  ( $k = H, L$ ) remains the same because the two markets are independent. Similarly, when  $\lambda \neq 1$ ,  $\pi^{PP}$  and  $p_i^{PP}(x, L)$  decrease when  $\tau$  decreases, and  $p_i^{PP}(x, H)$  remains the same.

### 3.3 Both firms choose group pricing

When both firms adopt group pricing, they offer uniform prices to each consumer group. Therefore, they compete in markets  $H$  and  $L$ , respectively. We use the superscript “ $GG$ ” to denote this case.

The prices and the resulting profits are

$$p^{GG}(H) \equiv t_H, \quad p^{GG}(L) \equiv t_L = \tau t_H, \quad \pi^{GG} \equiv \frac{t_H(\lambda + (1 - \lambda)\tau)}{2}. \quad (7)$$

A decrease in  $\lambda$  and a decrease in  $\tau$  lower profits.

### 3.4 One firm chooses uniform pricing, and the other one chooses personalized pricing

We consider the case where one firm (say firm 0) employs uniform pricing and the other (say firm 1) adopts personalized pricing. In the second stage, firms 0 and 1 become the leader and the follower, respectively. We use the superscript “ $UP$ ” to denote this case.

We check the optimal pricing of the firms. Since firm 1 becomes the follower, given  $p_0$ , firm 1 chooses  $p_1^{UP}(x, k) = \max\{p_0 + t_k x - t_k(1 - x), 0\}$  at  $x \in [0, 1]$  in market  $k$ . Next, we consider firm 0’s decision. A consumer who is indifferent between buying from firm 0 or firm 1 must satisfy  $p_0 + t_k x = p_1 + t_k(1 - x)$ , from which we have  $p_1(x, t_k) = p_0 + (2x - 1)t_k$ , which must be non-negative. By solving  $p_0 + (2x_k^{UP} - 1)t_k = 0$ , for  $x_k^{UP}$ , we obtain the location of the indifferent consumer in market  $k$ ,  $x_k^{UP}(p_0) \equiv (t_k - p_0)/(2t_k)$ . Since  $x_k^{UP}(p_0) \in [0, 1]$ , firm 0’s profit function is

$$\pi_0^{UP}(p_0) \equiv \begin{cases} p_0\{\lambda x_H^{UP}(p_0) + (1 - \lambda)x_L^{UP}(p_0)\} & \text{if } 0 \leq p_0 \leq \tau t_H, \\ p_0 \lambda x_H^{UP}(p_0) & \text{if } \tau t_H \leq p_0 \leq t_H, \\ 0 & \text{if } t_H \leq p_0. \end{cases}$$

By solving the profit maximization problem of firm 0, we obtain the following optimal price:

$$p_0^{UP} = \begin{cases} \frac{t_H \tau}{2 - 2\lambda(1 - \tau)} & \text{if } 0 < \lambda \leq \frac{\tau}{1 - \tau} \text{ or } 1/2 \leq \tau, \\ \frac{t_H}{2} & \text{if } \frac{\tau}{1 - \tau} \leq \lambda < 1 \text{ and } \tau < 1/2. \end{cases}$$

If  $\lambda$  is large and  $\tau$  is small, firm 0 will abandon the supply to type  $L$  consumers, and firm 1 will monopolize the market for type  $L$  consumers. Otherwise, firm 0 will serve both consumer types by offering  $p_0^{UP}$ , which is lower than  $t_H/2$  unless  $\tau = 1$ . This pricing of firm 0 implies intense price competition when the ratio of type  $H$  consumers is low ( $\lambda$  is small).

Using this result, we obtain the profit of firm 0:

$$\pi_0^{UP} = \begin{cases} \frac{t_H \tau}{8 - 8\lambda(1 - \tau)} & \text{if } 0 < \lambda \leq \frac{\tau}{1 - \tau} \text{ or } 1/2 \leq \tau, \\ \frac{t_H \lambda}{8} & \text{if } \frac{\tau}{1 - \tau} \leq \lambda < 1 \text{ and } \tau < 1/2. \end{cases} \quad (8)$$

We obtain the profit of firm 1 is as follows:

$$\pi_1^{UP} = \begin{cases} \frac{t_H[9\tau + 4\lambda(1 - \tau)^2 - 4\lambda^2(1 - \tau)^2]}{16 - 16\lambda(1 - \tau)} & \text{if } 0 < \lambda \leq \frac{\tau}{1 - \tau} \text{ or } 1/2 \leq \tau, \\ \frac{t_H(8 + \lambda)}{16} & \text{if } \frac{\tau}{1 - \tau} \leq \lambda < 1 \text{ and } \tau < 1/2. \end{cases} \quad (9)$$

A decrease in  $\lambda$  and a decrease in  $\tau$  lower profits.

### 3.5 One firm chooses group pricing, and the other one chooses uniform pricing

We consider the case where one firm (say firm 0) employs group pricing and offers  $p_0(k)$  to market  $k$  and the other (say firm 1) adopts uniform pricing and offers  $p_1$ . They simultaneously offer their prices. We use the superscript “ $GU$ ” to denote this case. The indifferent consumer in market  $k$  is denoted by  $x_k^{GU}(p_0(k), p_1) = (-p_0(k) + p_1 + t_k)/(2t_k)$ .

We need to classify the outcome in this subgame into the following two cases: (1) a pure strategy equilibrium exists; (2) no pure strategy equilibrium exists. The former case appears if  $\tau$  satisfies the following inequality, otherwise the latter appears:

$$\tau \geq \frac{-8 + 3\lambda + 9\lambda^2 + 4\sqrt{4 - 3\lambda}}{3\lambda(5 + 3\lambda)} \equiv \tau_{gu}. \quad (10)$$

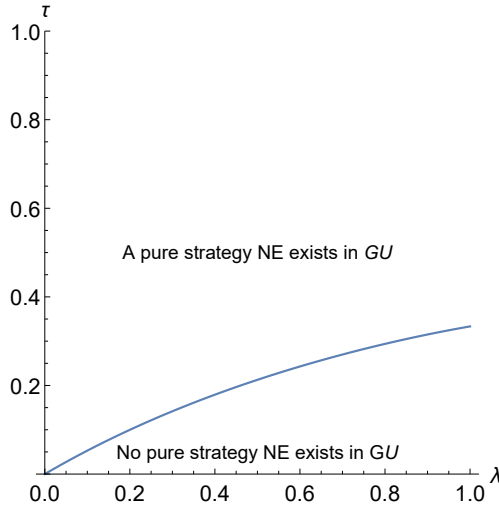


Figure 3: The area in which a pure strategy NE exists in  $GU$

In the first case ( $\tau \geq \tau_{gu}$ ), firms solve  $\max_{p_0(H), p_0(L)} \lambda p_0(H) x_H^{GU}(p_0(H), p_1) + (1 - \lambda) p_0(L) x_L^{GU}(p_0(L), p_1)$  and  $\max_{p_1} p_1 \{ \lambda (1 - x_H^{GU}(p_0(H), p_1)) + (1 - \lambda) (1 - x_L^{GU}(p_0(L), p_1)) \}$ , from which we have

$$\begin{aligned} p_0^{GU*}(H) &\equiv \frac{t_H(1 - \lambda + \tau + \lambda\tau)}{2(1 - \lambda + \lambda\tau)}, \quad p_0^{GU*}(L) \equiv \frac{t_H(2 - \lambda + \lambda\tau)}{2(1 - \lambda + \lambda\tau)}, \quad p_1^{GU*} \equiv \frac{t_H\tau}{1 - \lambda + \lambda\tau} \\ \pi_0^{GU*} &\equiv \frac{t_H\lambda(1 - \lambda + \tau + \lambda\tau)^2}{8(1 - \lambda + \lambda\tau)^2} + \frac{t_H(1 - \lambda)\tau(2 - \lambda + \lambda\tau)^2}{8(1 - \lambda + \lambda\tau)^2}, \quad \pi_1^{GU*} \equiv \frac{t_H\tau}{2(1 - \lambda + \lambda\tau)}. \end{aligned} \quad (11)$$

In the second case ( $\tau < \tau_{gu}$ ), we instead consider a mixed strategy equilibrium where, (i) firm 0 employs a pure strategy and offers  $p_0(H)$  and  $p_0(L)$ , and (ii) firm 1 chooses a low

price  $p_{1l}$  with probability  $\beta$  and a high price  $p_{1h}$  with probability  $1 - \beta$ . We focus on a mixed strategy NE such that  $x_L^{GU}(p_0(L), p_{1l}) \in (0, 1)$ ,  $x_L^{GU}(p_0(L), p_{1h}) > 1$ ,  $x_H^{GU}(p_0(H), p_{1l}) \in (0, 1)$ ,  $x_H^{GU}(p_0(H), p_{1h}) \in (0, 1)$ , and  $\beta \in (0, 1)$ . In this mixed strategy equilibrium, the resulting outcome in market  $L$  is the corner solution where firm 0 obtains all type  $L$  consumers if the rival offers the high price.

We can obtain a mixed strategy NE:

$$\begin{aligned}
p_0^m(H) &= \frac{t_H(9(1 - \lambda) + 4(1 + 2\lambda)\tau - \beta^{GU}(1 - \lambda)(3 - 2\tau))}{3(3 - (3 - 4\tau)\lambda + \beta^{GU}(1 - \lambda))}, \\
p_0^m(L) &= \frac{t_H\tau(12 + \beta^{GU} - (\beta^{GU})^2 - 12\lambda + 5\beta^{GU}\lambda + (\beta^{GU})^2\lambda + 12\lambda\tau - 6\beta^{GU}\lambda\tau)}{3(3 - (3 - 4\tau)\lambda + \beta^{GU}(1 - \lambda))}, \\
p_{1l}^m &= \frac{t_H(6 + 5\beta^{GU} + (\beta^{GU})^2 - 6\lambda + 7\beta^{GU}\lambda - (\beta^{GU})^2\lambda)\tau}{3(3 - (3 - 4\tau)\lambda + \beta^{GU}(1 - \lambda))}, \\
p_{1h}^m &= \frac{t_H(9 - 9\lambda + 2\tau + \beta^{GU}\tau + 10\lambda\tau - \beta^{GU}\lambda\tau)}{3(3 - (3 - 4\tau)\lambda + \beta^{GU}(1 - \lambda))},
\end{aligned} \tag{12}$$

$\beta^{GU} \in (0, 1)$  that satisfies the following equation:

$$\begin{aligned}
\pi_1^{GUm} &\equiv \lambda p_{1h}^m(1 - x_H(p_0^m(H), p_{1h}^m)) \\
&= p_{1l}^m[\lambda(1 - x_H(p_0^m(H), p_{1l}^m)) + (1 - \lambda)(1 - x_L(p_0^m(L), p_{1l}^m))].
\end{aligned}$$

Although we can explicitly derive the value of  $\beta$  which is complex, we numerically show this value in Figure 4. The region where  $\beta^{GU}(\lambda, \tau) \in (0, 1)$  coincides with that of the pure strategy equilibrium is not sustainable.

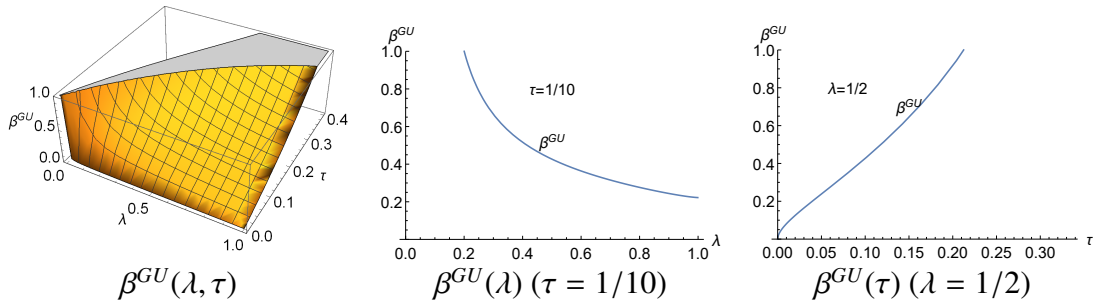


Figure 4: Mixed strategy in which one firm chooses group pricing and the other chooses uniform pricing



We can summarize the discussion as follows:

$$p_0^{GU}(H) = \begin{cases} \frac{t_H(1 + \tau - (1 - \tau)\lambda)}{2(1 - \lambda(1 - \tau))} & \text{if } \tau \geq \tau_{gu}, \\ p_0^m(H) \text{ in (12)} & \text{if } \tau < \tau_{gu}, \end{cases} \quad (13)$$

$$p_0^{GU}(L) = \begin{cases} \frac{t_H(2 - (1 - \tau)\lambda)}{2(1 - \lambda(1 - \tau))} & \text{if } \tau \geq \tau_{gu}, \\ p_0^m(L) \text{ in (12)} & \text{if } \tau < \tau_{gu}, \end{cases} \quad (14)$$

$$p_1^{GU} = \begin{cases} \frac{t_H\tau}{1 - \lambda(1 - \tau)} & \text{if } \tau \geq \tau_{gu}, \\ \begin{cases} p_{il}^m \text{ in (12) with probability } \beta^{GU}, \\ p_{ih}^m \text{ in (12) with probability } 1 - \beta^{GU}, \end{cases} & \text{if } \tau < \tau_{gu}, \end{cases} \quad (15)$$

$$\pi_0^{GU} = \begin{cases} \frac{t_H\lambda(1 - \lambda + \tau + \lambda\tau)^2}{8(1 - \lambda + \lambda\tau)^2} + \frac{(t_H(1 - \lambda)\tau(2 - \lambda + \lambda\tau))^2}{8(1 - \lambda + \lambda\tau)^2} & \text{if } \tau \geq \tau_{gu}, \\ \pi_0^{GUm} \equiv p_0^m(H)\lambda(\beta^m x_H(p_0^m(H), p_{il}^m) + (1 - \beta^m)x_H(p_0^m(H), p_{ih}^m)) \\ \quad + p_0^m(L)(1 - \lambda)(\beta^m x_L(p_0^m(L), p_{il}^m) + (1 - \beta^m)), & \text{if } \tau < \tau_{gu}. \end{cases} \quad (16)$$

$$\pi_1^{GU} = \begin{cases} \frac{t_H\tau}{2 - 2\lambda(1 - \tau)} & \text{if } \tau \geq \tau_{gu}, \\ \pi_1^{GUm} \text{ in (12),} & \text{if } \tau < \tau_{gu}. \end{cases} \quad (17)$$

We find that when  $\tau < \tau_{gu}$ , a decrease in  $\tau$  *increases* firm 1's expected uniform price, mitigating price competition. A decrease in  $\tau$  has direct and strategic effects on profits. First, a decrease in  $\tau$  directly intensifies competition in market  $L$ , which harms firm 1. Second, when  $\tau < \tau_{gu}$ , the lower  $\tau$ , the lower the probability of setting  $p_{1l}$ ,  $\beta^{GU}$  (see Figure 4), because the market for type  $L$  consumers becomes less profitable. The change of  $\beta^{GU}$  induces firm 0 to offer a higher price in market  $H$  because of the strategic complementarity of both firms' prices, which benefits firm 1. The relative significance of the two contrasting effects depends on the pricing scheme of firm 0. When firm 0 adopts group pricing, firm 0 can offer discriminatory prices in the two markets, implying that firm 0 can keep its price in market  $H$  high for any  $\tau$  (see  $p_0^m(H)$  in (12)). Such a high price of firm 0 in market  $H$  allows firm 1 to concentrate on market  $H$  to obtain a positive profit even if  $\tau$  is sufficiently small (see  $\beta^{GU}$  in

Figure 4). Therefore, for firm 1's profit, the strategic effect dominates the direct effect.

However, when  $\tau$  is sufficiently small, the mitigation of price competition becomes weak. Firm 0 is less likely to compete with firm 1 in market  $L$  because of a low  $\beta^{GU}$ , weakening the strategic complement effect on  $p_0^m(L)$ . In fact,  $p_0^m(L)$  increases in  $\tau$  if  $\tau$  is sufficiently small; otherwise,  $p_0^m(L)$  decreases in  $\tau$ . Therefore, when  $\tau$  is smaller than  $\bar{\tau}$  defined in Figure 5, a reduction in  $\tau$  reduces  $p_0^m(L)$  and such a price reduction consequently causes a profit reduction for firm 0, which captures the entire demand in market  $L$  with probability  $1 - \beta^{GU}$ . We summarize the above finding in the following proposition:

**Proposition 2** *Suppose that one firm (firm 0) employs group pricing and the other one (firm 1) employs uniform pricing. When  $\bar{\tau} < \tau < \tau_{gu}$ , a decrease in  $\tau$  increases the expected profit of firm 0. When  $\tau < \tau_{gu}$ , a decrease in  $\tau$  increases the expected profit of firm 1.*

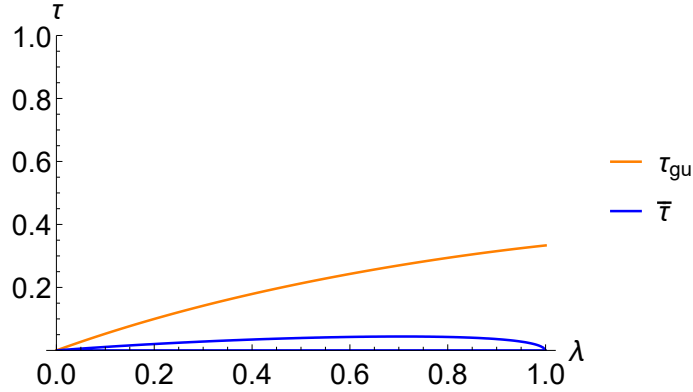


Figure 5: The area in which a decrease in  $\tau$  increases profits

### 3.6 One firm chooses group pricing, and the other chooses personalized pricing

We consider firm 0 employs group pricing and firm 1 employs personalized pricing. In the second stage, firms 0 and 1 become the leader and the follower, respectively. We use

the superscript “ $GP$ ” to denote this case. Given firm 0’s price  $p_0(k)$ , firm 1 offers  $p_1 = p_0(k) + (2x - 1)t_k$ . The indifferent consumer in each market is denoted by  $x_k^{GP}(p_0(k)) \equiv (t_k - p_0(k))/(2t_k)$ . Firm 0 then solves  $\max_{p_0(k)} x_k^{GP}(p_0(k))p_0(k)$ , from which we have

$$p_0^{GP}(k) \equiv \frac{t_k}{2}, \pi_0^{GP} \equiv \frac{t_H((1 - \lambda)\tau + \lambda)}{8}, \pi_1^{GP} \equiv \frac{9t_H((1 - \lambda)\tau + \lambda)}{16}. \quad (18)$$

Each market outcome is the same as in Thisse and Vives (1988) except for the market size. As in the previous arguments, a decrease in  $\lambda$  and a decrease in  $\tau$  lower profits.

## 4 Equilibrium pricing scheme

First, we derive the equilibrium pricing schemes in the first stage and show that both firms employ personalized pricing in equilibrium. Second, we discuss how pricing schemes influence profits and welfare.

### 4.1 Decisions on pricing schemes

The discussions in the previous subsections provide the payoff matrix in the first-stage game.

| Firm 0/Firm 1 | uniform              | personalized         | group                |
|---------------|----------------------|----------------------|----------------------|
| uniform       | $\pi^{UU}$ in (5)    | $\pi_1^{UP}$ in (9)  | $\pi_0^{GU}$ in (16) |
|               | $\pi^{UU}$ in (5)    | $\pi_0^{UP}$ in (8)  | $\pi_1^{GU}$ in (17) |
| personalized  | $\pi_0^{UP}$ in (8)  | $\pi^{PP}$ in (6)    | $\pi_0^{GP}$ in (18) |
|               | $\pi_1^{UP}$ in (9)  | $\pi^{PP}$ in (6)    | $\pi_1^{GP}$ in (18) |
| group         | $\pi_1^{GU}$ in (17) | $\pi_1^{GP}$ in (18) | $\pi^{GG}$ in (7)    |
|               | $\pi_0^{GU}$ in (16) | $\pi_0^{GP}$ in (18) | $\pi^{GG}$ in (7)    |

We solve for the firms’ equilibrium pricing schemes. We find that both firms employ personalized pricing in equilibrium.

**Proposition 3** *Both firms employ personalized pricing as a unique equilibrium outcome.*

The resulting outcome is expected given the outcome in Thisse and Vives (1988). If we pick up only personalized and group pricing, personalized pricing is the dominant strategy. Similarly, if we pick up only personalized and uniform pricing, personalized pricing is still the dominant strategy. Therefore, we find that personalized pricing is attainable in equilibrium.

## 4.2 The effects of pricing schemes on profits and welfare

We numerically compare  $\pi^{UU}$  in (5),  $\pi^{PP}$  in (6), and  $\pi^{GG}$  in (7), due to the mathematical complexity.

**Proposition 4**  $\pi^{UU} < \pi^{PP} < \pi^{GG}$  if and only if the parameter pair  $(\lambda, \tau)$  is in the gray area in Figure 6; otherwise,  $\pi^{PP} < \pi^{UU} < \pi^{GG}$ .

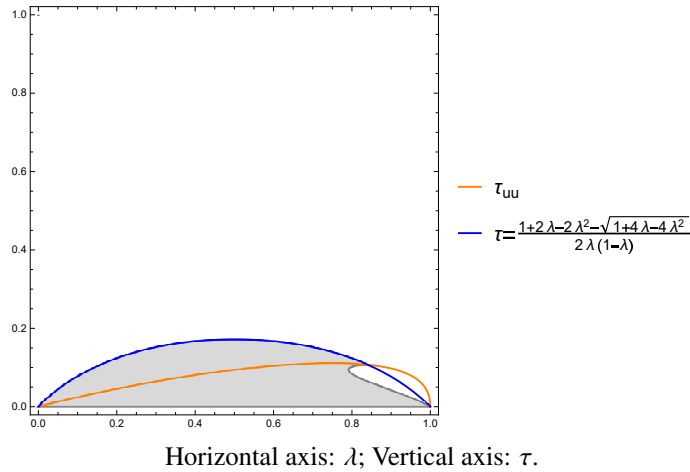


Figure 6: The condition that  $\pi^{UU} < \pi^{PP}$

When the mismatch costs of the two consumer groups are sufficiently heterogeneous, personalized pricing is better than uniform pricing. This contrasts with the standard result in the context of personalized pricing.

We discuss how the demand condition is important to evaluate the profitability of personalized pricing, comparing our result with Esteves (2021), who also shows that personalized pricing can be more profitable than uniform pricing. In her study, the ratio of type  $H$  consumers whose necessary amounts of product are large should be around a critical value (approximately 20-30%), which matches the 80/20 rule (the Pareto Principle) in marketing (Twedt, 1964). Figure 6 shows that the reversal result is more likely to hold if  $\lambda$  (the ratio of type  $H$  consumers) is around 1/2. Our contrasting result implies the demand condition is critical to predict the profitability of personalized pricing.

Finally, we investigate consumer welfare and numerically show that the flip side of Proposition 4 holds.

**Proposition 5** *The consumer surplus in  $UU$  is the highest, and that in  $GG$  is the lowest if and only if the parameter pair  $(\lambda, \tau)$  is in the gray area in Figure 6; otherwise, the consumer surplus in  $PP$  is the highest and that in  $GG$  is the lowest.*

Because of the inelastic demand in the Hotelling model, the welfare ranking between the pricing schemes is opposite to the profit ranking.

Case  $GG$  is always the worst for consumer welfare among the three cases,  $UU$ ,  $PP$ , and  $GG$ . Although  $UU$  can be the best for consumer welfare when the group heterogeneity is high, it is not easy to intervene in personalized pricing because such an intervention could facilitate group pricing, which is acceptable in practice, leading to the worst case.

## 5 Conclusion

We examine firms' incentives to adopt one of the three pricing schemes: uniform, personalized, or group pricing in a Hotelling duopoly model. There are two types of consumer groups that are heterogeneous in their mismatch costs.

We show that regardless of the heterogeneity of consumer groups, both firms employ personalized pricing in equilibrium. Moreover, the profits when both firms employ personalized pricing are higher than those when they use uniform pricing if the heterogeneity of consumer groups is significant. The result is aligned with that of Esteves (2021). However, profits are always the highest among the three cases when the firms employ group pricing.

Additionally, if we focus on the three cases where both firms employ (i) uniform pricing ( $UU$ ), (ii) personalized pricing ( $PP$ ), and (iii) group pricing ( $GG$ ), the consumer surplus in  $UU$  is the highest and that in  $GG$  is the lowest among the three cases if the heterogeneity of consumer groups is significantly high; otherwise, the consumer surplus in  $PP$  is the highest and that in  $GG$  is the lowest among the three cases. Therefore, group pricing is always the most harmful from the consumer welfare perspective. Furthermore, this result implies that it is not easy to intervene in personalized pricing because this could facilitate group pricing, leading to the worst case.

We can incorporate consumers' privacy concerns into our model by using the demand structure in Montes et al. (2019), where there are old and new consumers in a Hotelling duopoly model. In their model, a monopolistic data broker has the locational information of old consumers and can sell it to at least one of the firms. They consider each old consumer's incentive to delete the locational information from the database. As there are two dimensions of consumer characteristics in our model, location  $x$  and type  $t_k$ , each consumer has four options to manage the information characteristics. The richness of the consumer characteristics may lead to interesting results. Considering the extension can be future research.

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## A Appendix: Not for publication

### A.1 Both firms choose uniform pricing ( $UU$ )

We first show condition under which no firm unilaterally deviates from  $(p^{UU}, p^{UU})$ . By symmetry, assume that firm 0 deviates. Since firm 0 must obtain positive share, we exclude the case with  $x_H = x_L = 0$ . i.e.,  $p_0 \geq p^{UU} + t_H$ . In addition, for any  $p_0 \leq p^{UU} - t_H$ , we have  $x_H = x_L = 1$ , which means that demand for firm 0 is constant. Then, for  $p_0 \in [0, p^{UU} - t_H]$ , firm 0's profit is maximized at  $p_0 = p^{UU} - t_H$ . Hence, it suffices to consider two types of deviation: (i)  $p^{UU} - t_H \leq p_0 \leq p^{UU} - \tau t_H$  and (ii)  $p^{UU} + \tau t_H \leq p_0 < p^{UU} + t_H$ .

First, we consider case (i) in which  $x_L = 1$ . Given  $p_1 = p^{UU}$ ,  $\pi_0(p_0, p^{UU}) = \lambda p_0 x_H + (1 - \lambda)p_0$ . The first-order condition yields the deviation price (with superscript  $UD1$ ).

$$p_0^{UD1} = \frac{t_H[\lambda^2(1 - \tau) - 3\lambda(1 - \tau) + 2]}{2\lambda[1 - \lambda(1 - \tau)]}.$$

The necessary condition for this deviation is  $p_0^{UD1} < p^{UU} - t_L$ , which is always violated because

$$(p^{UU} - t_L) - p_0^{UD1} = -\frac{t_H[2\lambda^2\tau^2 + 3(1 - \lambda)\lambda\tau + (2 - \lambda)(1 - \lambda)]}{2\lambda[1 - \lambda(1 - \tau)]} < 0.$$

Hence, firm 0 does not deviate in case (i).

Next, we consider case (ii) in which  $x_L = 0$ . With  $p_1 = p^{UU}$ ,  $\pi_0(p_0, p^{UU}) = \lambda p_0 x_H$ . The first-order condition yields the deviation price (with superscript  $UD2$ ).

$$p_0^{UD2} = \frac{t_H[1 + \tau - \lambda(1 - \tau)]}{2 - 2\lambda(1 - \tau)},$$

The deviation profit is

$$\pi_0^{UD2} = \frac{t_H\lambda[1 + \tau - \lambda(1 - \tau)]^2}{8[1 - \lambda(1 - \tau)]^2}.$$

Comparing  $\pi^{UU}$  with  $\pi_0^{UD2}$  yields

$$\pi^{UU} - \pi_0^{UD2} = \frac{t_H(1 - \lambda)[\lambda^2(1 - \tau)^2 - \lambda(1 + 2\tau - 3\tau^2) + 4\tau]}{8[1 - \lambda(1 - \tau)]^2},$$

which is strictly positive if

$$\frac{2\sqrt{1 - \lambda} - (2 - \lambda - \lambda^2)}{\lambda(3 + \lambda)} \equiv \tau^{UU} < \tau < 1.$$

Notice that under this condition,  $p^{UU} + t_L - p_0^{UD2} \leq 0$  always holds. Hence,  $(p^{UU}, p^{UU})$  is a pure strategy NE if  $\tau > \tau^{UU}$ .

For  $0 < \tau \leq \tau^{UU}$ , we derive the mixed NE as described in Equation (3). We solve four simultaneous equations derived from the following maximization system:

$$\max_{p_{0l}} \pi_{0l}^{UU} \equiv p_{0l}\lambda(\beta x_H(p_{0l}, p_{1l}) + (1 - \beta)x_H(p_{0l}, p_{1h})) + p_{0l}(1 - \lambda)(\beta x_L(p_{0l}, p_{1l}) + (1 - \beta)),$$

$$\max_{p_{0h}} \pi_{0h}^{UU} \equiv p_{0h}\lambda(\beta x_H(p_{0h}, p_{1l}) + (1 - \beta)x_H(p_{0h}, p_{1h})) + p_{0h}(1 - \lambda)(1 - \beta)x_L(p_{0h}, p_{1h}),$$

$$\begin{aligned} \max_{p_{1l}} \pi_{1l}^{UU} &\equiv p_{1l}\lambda(\alpha(1 - x_H(p_{0l}, p_{1l})) + (1 - \alpha)(1 - x_H(p_{0h}, p_{1l}))) \\ &\quad + p_{1l}(1 - \lambda)(\alpha(1 - x_L(p_{0l}, p_{1l})) + (1 - \alpha)), \end{aligned}$$

$$\begin{aligned} \max_{p_{1h}} \pi_{1h}^{UU} &\equiv p_{1h}\lambda(\alpha(1 - x_H(p_{0h}, p_{1l})) + (1 - \alpha)(1 - x_H(p_{0h}, p_{1h}))) \\ &\quad + p_{1h}(1 - \lambda)(1 - \alpha)(1 - x_L(p_{0h}, p_{1h})). \end{aligned}$$

Next, we let  $\pi_{0l}^{UU} = \pi_{0h}^{UU}$  and  $\alpha = \beta$ , and solve for  $\alpha$  and  $\beta$ . The numerical outcome is in Figure 2.

## A.2 One firm chooses uniform pricing and the other one chooses personalized pricing (*UP*)

We first consider the case with  $0 \leq p_0 \leq \tau t_H$ . The first-order condition yields

$$p_0^{UPI} \equiv \frac{t_H \tau}{2 - 2\lambda(1 - \tau)}, \quad \pi_0^{UPI} \equiv \frac{t_H \tau}{8 - 8\lambda(1 - \tau)}, \quad \pi_1^{UPI} \equiv \frac{t_H [9\tau + 4\lambda(1 - \tau)^2 - 4\lambda^2(1 - \tau)^2]}{16 - 16\lambda(1 - \tau)},$$

where the superscript *UPI* denotes the case of  $p_0^{UPI} \in [0, \tau t_H]$ , which can be reduced to  $0 < \lambda < 1/(2 - 2\tau) \equiv \lambda^{UPI}$  or  $\tau \geq 1/2$ .

Next, we consider the case with  $\tau t_H \leq p_0 < t_H$ . The first-order condition yields:

$$p_0^{UPh} \equiv \frac{t_H}{2}, \quad \pi_0^{UPh} \equiv \frac{t_H \lambda}{8}, \quad \pi_1^{UPh} \equiv \frac{t_H(8 + \lambda)}{16},$$

where the superscript *UPh* denotes the case of  $p_0 \in [\tau t_H, t_H]$ . Here, we derive the condition such that  $\tau t_H \leq p_0^{UPh} \leq t_H$ , from which we have  $\tau \leq 1/2$ .

From the above, we have two equilibrium candidates,  $p_0^{UPI}$  and  $p_0^{UPh}$ , if  $0 < \lambda < \lambda^{UPI}$  and  $0 < \tau < 1/2$ , whereas there is a unique candidate,  $p_0^{UPh}$ , otherwise. Comparing  $\pi_0^{UPI}$  with  $\pi_0^{UPh}$  yields

$$\pi_0^{UPI} - \pi_0^{UPh} = \frac{t_H(1 - \lambda)[\tau - \lambda(1 - \tau)]}{8 - 8\lambda(1 - \tau)} > 0 \iff \lambda < \frac{\tau}{1 - \tau} \equiv \lambda^{UP}.$$

Since  $\lambda^{UPI} > \lambda^{UP}$  for  $0 < \tau < 1/2$ , the optimal  $p_0$  is

$$p_0^{UP} = \begin{cases} \frac{t_H \tau}{2 - 2\lambda(1 - \tau)} & \text{if } 0 < \lambda \leq \frac{\tau}{1 - \tau} \text{ or } 1/2 \leq \tau, \\ \frac{t_H}{2} & \text{if } \frac{\tau}{1 - \tau} < \lambda < 1 \text{ and } 0 < \tau < 1/2. \end{cases}$$

Using this result, we obtain the profit of firm 0.

$$\pi_0^{UP} = \begin{cases} \frac{t_H \tau}{8 - 8\lambda(1 - \tau)} & \text{if } 0 < \lambda \leq \frac{\tau}{1 - \tau} \text{ or } 1/2 \leq \tau, \\ \frac{t_H \lambda}{8} & \text{if } \frac{\tau}{1 - \tau} < \lambda < 1 \text{ and } 0 < \tau < 1/2. \end{cases}$$

### A.3 One firm chooses group pricing and the other one chooses uniform pricing ( $GU$ )

Due to symmetry, let firm 0 choose group pricing and offer  $p_0(H)$  to market  $H$  and  $p_0(L)$  to market  $L$ . Let firm 1 choose uniform pricing and offer  $p_1$ . The indifferent consumer in each market is given by  $x_k^{GU}(p_0(k), p_1) = (t_k - p_0(k) + p_1)/(2t_k)$ . Then, firm 0 and firm 1 solve

$$\begin{aligned} & \max_{p_0(H), p_0(L)} p_0(H)\lambda x_H^{GU}(p_0(H), p_1) + p_0(L)(1 - \lambda)x_L^{GU}(p_0(L), p_1), \\ & \max_{p_1} p_1\{\lambda[1 - x_H^{GU}(p_0(H), p_1)] + (1 - \lambda)[1 - x_L^{GU}(p_0(L), p_1)]\}, \end{aligned}$$

which yields the equilibrium prices  $p_0^{GU*}(H)$ ,  $p_0^{GU*}(L)$ ,  $p_1^{GU*}$  as in Eq. (11). Since firm 0's profit function with respect to  $p_0(H)$  and  $p_0(L)$  is globally concave, it has no incentive to deviate. However, given  $p_0^{GU*}(H)$  and  $p_0^{GU*}(L)$ , firm 1's profit function is not globally concave, and may therefore unilaterally deviate from  $p_1^{GU*}$ . It can be confirmed that  $\frac{t_H(1-\lambda)(1-\tau)}{2(1-\lambda+\lambda\tau)} < 0$ , given  $p_0^{GU*}(H)$ ,  $p_0^{GU*}(L)$  and any  $p_1 \in (0, 1)$ ,  $x_H^{GU} > 0$ . Moreover, firm 1 will never offer a price such that both  $x_H = 1$  and  $x_L = 1$ , because its profit would become zero. Therefore, given  $p_0^{GU*}(H)$ ,  $p_0^{GU*}(L)$  firm 1's profit function is

$$\begin{aligned} & \pi_1^{GU}(p_0^{GU*}(L), p_0^{GU*}(H), p_1) \\ \equiv & \begin{cases} p_1\lambda[1 - x_H^{GU}(p_1)] & \text{if } \frac{t_H\tau(4-3\lambda+3\lambda\tau)}{2(1-\lambda+\lambda\tau)} < p_1 \leq \frac{t_H(3-3\lambda+\tau+3\lambda\tau)}{2(1-\lambda+\lambda\tau)}, \\ p_1\{\lambda[1 - x_H^{GU}(p_1)] + (1 - \lambda)[1 - x_L^{GU}(p_1)]\} & \text{if } \frac{t_H\lambda(1-\tau)\tau}{2(1-\lambda+\lambda\tau)} \leq p_1 \leq \frac{t_H\tau(4-3\lambda+3\lambda\tau)}{2(1-\lambda+\lambda\tau)}, \\ p_1\{\lambda[1 - x_H^{GU}(p_1)] + (1 - \lambda)\} & \text{if } 0 < p_1 < \frac{t_H\lambda(1-\tau)\tau}{2(1-\lambda+\lambda\tau)}. \end{cases} \end{aligned}$$

The second case induces the equilibrium price  $p_1^{GU}$ , so we discuss whether firm 1 deviates to remaining two cases: (i)  $\frac{t_H\tau(4-3\lambda+3\lambda\tau)}{2(1-\lambda+\lambda\tau)} < p_1 \leq \frac{t_H(3-3\lambda+\tau+3\lambda\tau)}{2(1-\lambda+\lambda\tau)}$ , and (ii)  $0 < p_1 < \frac{t_H\lambda(1-\tau)\tau}{2(1-\lambda+\lambda\tau)}$ .

First, we consider case (i) in which  $x_L = 1$ . Solving the first-order condition for  $p_1$  yields the deviation price and profit (with superscript  $GU1$ ):

$$p_1^{GU1} = \frac{t_H(3-3\lambda+\tau+3\lambda\tau)}{4(1-\lambda+\lambda\tau)}, \quad \pi_1^{GU1} = \frac{t_H\lambda(3-3\lambda+\tau+3\lambda\tau)^2}{32(1-\lambda+\lambda\tau)^2}$$

Since  $p_1^{GU1}$  always satisfies case (i), we compare  $\pi_1^{GU1}$  with  $\pi_1^{GU}$ , from which we have

$$\pi_1^{GU*} > \pi_1^{GU1} \iff \frac{4\sqrt{4-3\lambda} - (8-3\lambda-9\lambda^2)}{3\lambda(5+3\lambda)} \equiv \tau^{GU} < \tau < 1,$$

Next, we consider case (ii) in which  $x_L = 0$ . Solving the first-order condition for  $p_1$  yields the deviation price (with superscript  $GU2$ )

$$p_1^{GU2} = \frac{t_H[(4-5\lambda+\lambda^2+5\lambda\tau-\lambda^2\tau)]}{4\lambda(1-\lambda+\lambda\tau)}.$$

It can be confirmed that  $p_1^{GU2}$  always violates case (ii). To summarize,  $(p_0^{GU*}(L), p_0^{GU*}(H), p_1^{GU*})$  is a pure strategy NE if  $\tau > \tau^{GU}$ .

For  $0 < \tau \leq \tau^{GU}$ , we derive the mixed NE as described in Equation (12). We solve four simultaneous equations derived from the following maximization system:

$$\begin{aligned} & \max_{p_0(H), p_0(L)} p_0(H)\lambda(\beta x_H(p_0(H), p_{1l}) + (1-\beta)x_H(p_0(H), p_{1h})) \\ & + p_0(L)(1-\lambda)(\beta x_L(p_0(L), p_{1l}) + (1-\beta)); \\ & \max_{p_{1l}} \pi_{1l}^{GU} \equiv p_{1l}(\lambda(1-x_H(p_0(H), p_{1l})) + (1-\lambda)(1-x_L(p_0(L), p_{1l}))); \\ & \max_{p_{1h}} \pi_{1h}^{GU} \equiv p_{1h}(\lambda(1-x_H(p_0(H), p_{1h}))). \end{aligned}$$

Next, we let  $\pi_{1l}^{GU} = \pi_{1h}^{GU}$  and solve for  $\beta$ . The numerical outcome is Figure 4.

## A.4 Equilibrium pricing schemes

We first show that  $GG$ ,  $GP$  ( $PG$ ),  $UP$  ( $PU$ ),  $GU$  ( $GU$ ) and  $UU$  cannot be an NE.

$GG$  is not an NE: Given firm 0 choosing group pricing, firm 1's profit change from choosing group pricing to choosing personalized pricing is

$$\pi_1^{GP} - \pi^{GG} = \frac{t_H(\tau + (1 - \tau)\lambda)}{16} > 0.$$

$GP$  ( $PG$ ) is not an NE: Due to symmetry, we only consider  $GP$ . Given firm 1 choosing personalized pricing, firm 0's profit change from choosing group pricing to choosing personalized pricing is

$$\pi_0^{PP} - \pi^{GP} = \frac{t_H(\tau + (1 - \tau)\lambda)}{8} > 0.$$

$UP$  ( $PU$ ) is not an NE: Due to symmetry, we only consider  $UP$ . From the results in  $UP$  (see Proof A.2), we have (i)  $UPl$  when  $0 < \lambda \leq \tau/(1 - \tau)$  or  $\tau \geq 1/2$ , and (ii)  $UPh$  when  $\tau/(1 - \tau) < \lambda < 1$  and  $0 < \tau < 1/2$ . In case (i), given firm 1 choosing personalized pricing, firm 0's profit change from choosing uniform pricing to choosing personalized pricing is

$$\pi^{PP} - \pi_0^{UPl} = \frac{t_H[2\lambda(1 - \lambda)\tau^2 + (1 - 2\lambda)^2\tau + 2\lambda(1 - \lambda)]}{8(1 - \lambda + \lambda\tau)},$$

which is always positive for any  $\tau \in (0, 1)$  and  $\lambda \in (0, 1)$ . In case (ii), given firm 1 choosing personalized pricing, firm 0's profit change from choosing uniform pricing to choosing personalized pricing is

$$\pi_0^{PP} - \pi_0^{UPh} = \frac{t_H(\lambda + 2(1 - \lambda)\tau)}{8} > 0.$$

$UU$  is not an NE: when  $\tau \geq \tau_{uu}$ , we have pure strategy NE. We want to confirm

that whether firm 1 would unilaterally deviate from choosing uniform pricing to choosing personalized pricing. From the results in *UP* (see Proof A.2), we have (i) *UPI* when  $0 < \lambda \leq \tau/(1 - \tau)$  or  $\tau \geq 1/2$ , and (ii) *UPh* when  $\tau/(1 - \tau) < \lambda < 1$  and  $0 < \tau < 1/2$ . In case (i), given firm 0 choosing uniform pricing, firm 1's profit change from choosing uniform pricing to choosing personalized pricing is

$$\pi_1^{UPI} - \pi^{UU} = \frac{t_H[4\lambda(1 - \lambda)\tau^2 + (1 - 8\lambda + 8\lambda^2)\tau + 4\lambda(1 - \lambda)]}{16(1 - \lambda + \lambda\tau)},$$

which is always positive for any  $\tau \in (0, 1)$  and  $\lambda \in (0, 1)$ . In case (ii), given firm 0 choosing uniform pricing, firm 1's profit change from choosing uniform pricing to choosing personalized pricing is

$$\pi_1^{UPh} - \pi^{UU} = \frac{t_H[(8 - 7\lambda - \lambda^2) - (8 - 8\lambda - \lambda^2)\tau]}{16(1 - \lambda + \lambda\tau)} > 0,$$

which is always positive for any  $\tau \in (0, 1)$  and  $\lambda \in (0, 1)$ .

When  $\tau < \tau_{uu}$ , we have mixed strategy NE. Due to mathematical complexity involved, we can numerically show that each firm always has an incentive to deviate to choosing personalized pricing.

*GU* is not an NE: When  $\tau \geq \tau_{gu}$ , we have pure strategy NE. We want to confirm that whether firm 0 would unilaterally deviate from choosing group pricing to choosing personalized pricing. From the results in *PU* (see Proof A.2), we have (i) *PUI* when  $0 < \lambda \leq \tau/(1 - \tau)$  or  $\tau \geq 1/2$ , and (ii) *PUh* when  $\tau/(1 - \tau) < \lambda < 1$  and  $0 < \tau < 1/2$ . In case (i), given firm 1 choosing uniform pricing, firm 0's profit change from choosing group pricing to choosing personalized pricing is

$$\pi_0^{PUI} - \pi_0^{GU} = \frac{t_H[2\lambda(1 - \lambda)\tau^2 + (1 - 2\lambda)^2\tau + 2\lambda(1 - \lambda)]}{16(1 - \lambda + \lambda\tau)},$$



which is always positive for any  $\tau \in (0, 1)$  and  $\lambda \in (0, 1)$ . In case (ii), given firm 1 choosing uniform pricing, firm 0's profit change from choosing group pricing to choosing personalized pricing is

$$\pi_0^{PUh} - \pi_0^{GU} = \frac{t_H[-2\lambda(1-\lambda)\tau^2 - (8-12\lambda+3\lambda^2)\tau + (8-9\lambda+\lambda^2)]}{16(1-\lambda+\lambda\tau)},$$

which is always positive for any  $\tau \in (0, 1)$  and  $\lambda \in (0, 1)$ .

When  $\tau < \tau_{gu}$ , we have mixed strategy NE. Due to mathematical complexity involved, we can numerically show that each firm always has an incentive to deviate to choosing personalized pricing.

*PP* is an NE: We show that given firm 1 choosing personalized pricing, firm 0 does not deviate to choosing uniform pricing or to choosing group pricing. From the results in *UP* (see Proof A.2), we have (i) *UPI* when  $0 < \lambda \leq \tau/(1-\tau)$  or  $\tau \geq 1/2$ , and (ii) *UPh* when  $\tau/(1-\tau) < \lambda < 1$  and  $0 < \tau < 1/2$ . In case (i), firm 0's profit change from choosing personalized pricing to choosing uniform pricing is

$$\pi_0^{UPI} - \pi^{PP} = \frac{t_H(-2\lambda(1-\lambda)\tau^2 - (1-2\lambda)^2\tau - 2\lambda(1-\lambda))}{8} < 0.$$

In case (ii), firm 0's profit change from choosing personalized pricing to choosing uniform pricing is

$$\pi_0^{UPh} - \pi^{PP} = -\frac{t_H(\lambda + 2(1-\lambda)\tau)}{8} < 0.$$

Moreover, firm 0's profit change from choosing personalized pricing to choosing group pricing is

$$\pi_0^{UPh} - \pi^{PP} = -\frac{t_H(\lambda + (1-\lambda)\tau)}{8} < 0.$$