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Network-based macro fluctuations: what about an open economy?

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ABSTRACT

Do input–output linkages of intermediate products affect the spread of sectoral shocks at the aggregate level in Lithuania, a small and open economy? What role does openness play in the empirical exercise? We answer these questions by: (i) constructing the Lithuanian input–output transactions tables with domestic-only and domestic and imported sector-by-sector direct requirements, and (ii) applying Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi [(2012). The network origins of aggregate fluctuations. *Econometrica*, 80(5), 1977–2016] network-based methodology and Gabaix and Ibragimov's [(2011). Rank-1/2: A simple way to improve the ols estimation of tail exponents. *Journal of Business & Economic Statistics*, 29(1), 24–39] modified log rank-log size regression. Our results indicate that the structure of input–output linkages cause aggregate economic volatility to decay at a rate lower than the established theoretical prediction. Indirect linkages play an equally important role for both domestic-only and aggregated domestic and import transactions.

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1. Introduction

The diversification argument of Lucas (1977), similar in spirit to the portfolio diversification argument put forward by Markowitz (1952), indicates that, following the materialization at the sectorial level of a number of economic disturbances (expected to occur independently of each other), aggregate output reverts to its mean at a known rate, computed to be \sqrt{n} , where n is the number of sectors in the economy. When n becomes large (thus an increasing number of sectors is present in the economy), sectoral economic shocks become less important at the macro level and their impact vanishes quickly.

However, a growing literature, for instance Acemoglu, Carvalho, Ozdaglar, Tahbaz-Salehi (2012); Acemoglu, Ozdaglar, Tahbaz-Salehi (2010); Carvalho, Gabaix (2013); Carvalho (2008); di Giovanni, Levchenko, Mejean (2014); Gabaix (2011); Johnson (2014); and Atalay (2017), has argued that micro and sectorial shocks may have a non-negligible impact at the aggregate level under specific circumstances. For

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instance, according to Gabaix (2011), firm-level shocks can transform into aggregate fluctuations if firm size distribution has a heavy tail and firms contribute unequally to the final aggregate output. Acemoglu et al. (2012) and Carvalho (2008), by taking input–output linkages into consideration, provide novel network-based explanations of the limited validity of the diversification argument. Using U.S. data, the authors show that sectorial shocks do not cancel out and have a non-trivial aggregate impact due to unbalanced network of intermediate inputs.

Furthermore, Acemoglu, Akcigit, and Kerr (2016) and Ozdagli and Weber (2017) decompose the overall effect of various types of shocks into a direct effect and a network effect and find that the later plays a larger role than the former. This paper is closely related to Acemoglu et al. (2012) and Gabaix (2011). So far, the existing literature such as Acemoglu et al. (2016, 2012); Carvalho and Gabaix (2013); Ozdagli and Weber (2017), has provided evidence based primarily on U.S. data.

Lithuania, a small and open economy with a trade/GDP ratio of around 150% and an unbalanced structure of the input–output matrix, offers an interesting case-study. Constantinescu and Proskute (2018) indicate that only a small fraction of the firms present in the Lithuanian economy are engaged in trade, with pronounced heterogeneity present across different industries and size categories. 4% of industry value-added in Electricity was imported in 2014, while in Manufacturing the percentage is as high as 30%. Regarding size categories, a much smaller share of small firms export and import, around 10%, as compared to large firms where the values lie around 50%. These results are in line with the findings for the U.S. by Antràs, Fort and Tintelnot (2017), for Argentina by Gopinath and Neiman (2014) and Tintelnot, Kikkawa, Mogstad, and Dhyne (2017) for Belgium and hint towards the importance of large firms coupled with the presence of a few highly connected industries as conduits of external shocks. In particular, Tintelnot et al. (2017) indicate that for Belgium, 97% of firms acquire imported goods either directly or indirectly through their domestic network of suppliers. This number stands in stark contrast to the share of firms importing goods directly, which the authors compute to be 15%. Most of the exposure to potential foreign shocks comes not directly but through secondary effects driven by the structure of domestic input–output relationships. In the case of exports, this has been quantitatively confirmed as relevant in driving aggregate volatility by di Giovanni et al. (2014).

Substitutability of imported vs. local intermediates and their weight in the production process have been indicated by Halpern, Koren, and Szeidl (2015) as important parameters in driving productivity growth in Hungary while Gopinath and Neiman (2014) highlight the impact imported intermediaries play in determining fluctuations of aggregate TFP following the 2001 Argentinean FX crisis.

Although we lack data needed to compute the firm-level input–output matrix for Lithuania, we conjecture that the similar distributional characteristics observed in the shares of firms engaged in trade provide sufficient conditions to consider the previously uncovered mechanisms potentially at work in Lithuania as well. Bowing to the data constraints, we focus on the most disaggregated level of data available, in our case, industry input–output. To this end, we compare the structure of input–output matrices using domestic only vs. domestic and imported inputs. If a different mix of inputs is used when imports are considered as compared to the domestic-only matrix, this will be captured by the different technical coefficients and subsequently reflect different network structures of intermediates. The sharpness of the results is influenced by the level of sectorial

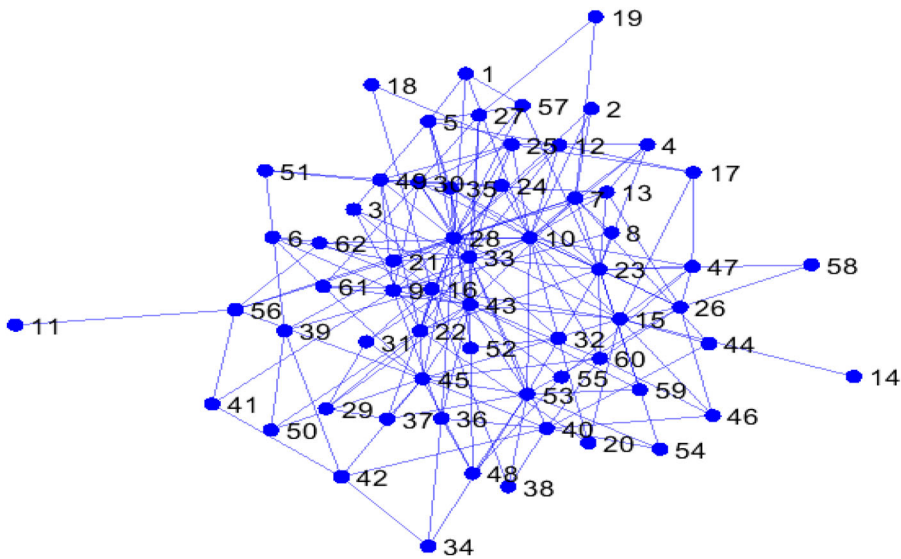


Figure 1. Intersectoral Network in Lithuania in 2010.

Note: Figure presents the network of inter-sectoral linkages in Lithuania in 2010. Each node represents one of the 62 Lithuanian sectors. If a sector purchases intermediate inputs from another sector for more than 1% of the value of its final output, the link between that producing sector and the sector of intermediate inputs is drawn.

disaggregation: inputs from different sub-sectors may not be recognized as different if only highly aggregated sectorial data is present.

Figure 1 represents the network of inter-sectoral linkages in 2010. Each node represents one of the 62 Lithuanian sectors (see Appendix 2 for the full list of sectors). If a sector purchases intermediate inputs from another sector for more than 1% of the value of its final output, a link is drawn. As presented in Figure 1, there are several sectors in Lithuania that are connected with a large number of other sectors via the production of intermediate inputs such as *Electricity, gas, steam and air-conditioning* (sector 23), *Wholesale trade services, except of motor vehicles and motorcycles* (sector 28), or *Warehousing and support services for transportation* (sector 33). At the same time, there are several sectors that are weakly connected with other sectors via the production of intermediate inputs such as *Basic pharmaceutical products and pharmaceutical preparations* (sector 11) or *Basic metals* (sector 14).

It is interesting to highlight the unbalanced nature of the input–output matrix through a visual representation of the most and least connected sectors. We plot in Figure 2 for the year 2010, the directed weighted graph of *Electricity, gas, steam and air-conditioning*, one of the sectors with the highest number of in/out degrees. Sectors with a large number of connections act as potential conduits of economic fluctuations as they transmit sector-specific shocks downstream to firms purchasing its output.

Figure 3 shows the directed weighted graph of *Basic pharmaceutical products and pharmaceutical preparations*, the sector with the lowest number of weighted out-degrees for year 2010.

It is worth mentioning that the U.S. data, i.e. commodity-by-commodity direct requirements tables, used for this type of analysis, is derived from the commodity-by-commodity total requirements tables available from the Bureau of Economic Analysis. However, this

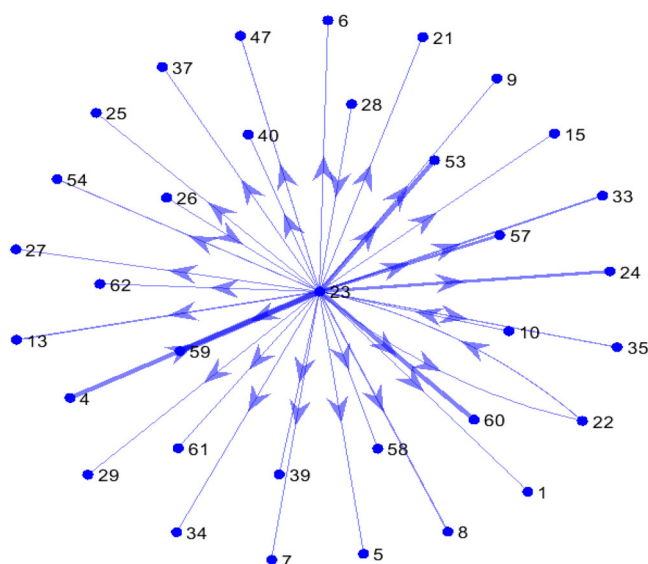


Figure 2. Weighted in/out degrees of electricity, gas, steam and air-conditioning.

Note: Figure presents one of the most connected sectors in the Economy – Electricity, gas, steam and air-conditioning sector (sector 23) with weighted connections to/from other sectors. If other sectors purchase/produce intermediate inputs from/to 23rd sector for more than 1% of the value of purchasing sector final output, the link between them is drawn. The thicker lines present stronger links between sectors.

type of data is not available for Lithuania, therefore we need to construct the domestic as well as the aggregated¹ sector-by-sector direct requirements table using the Lithuanian input-output transactions table.

Our results indicate the presence of first-order and second-order inter-sectoral connections, causing aggregate volatility to decay at a rate lower than \sqrt{n} . Aggregate volatility decays at a rate smaller than $n^{0.41}$ when considering first-order effects while taking into

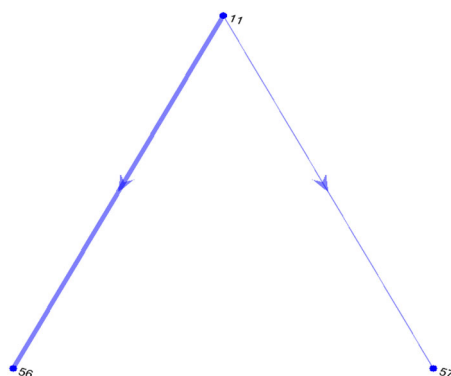


Figure 3. Weighted in/out degrees of basic pharmaceutical products and pharmaceutical preparations sector.

Note: Figure presents one of the least connected sectors in the Economy – Basic pharmaceutical products and pharmaceutical preparations sector (sector 11) with weighted connections to other sectors. If other sectors purchase intermediate inputs from sector 11 for more than 1% of the value of its final output, the link between them is drawn. Sector 11 is not purchasing intermediates from any other sector in the economy for more that 1% of the value of its final output.

account also second-order connections, the aggregate volatility decays at a rate smaller than $n^{0.22}$. This is in line with the argument that indirect linkages play an important role in the propagation of shocks. Due to these connections and the unbalanced structure of the input–output intermediate production networks, sectoral shocks to the one of the dominant sectors would propagate through its downstream sectors and thus lead to fluctuations at the aggregate level.

The paper is structured as follows. Section 2 presents a brief overview of the research methodology and the calculation of domestic direct requirements table using input–output transactions table at basic prices. Data availability allows us to analyse the inter-sectoral linkages between 62 industries in Lithuania. Section 3 presents the main empirical results and robustness checks. Lastly, Section 4 concludes.

2. Methodology

In this section we briefly present the intuition behind the network-based methodology to facilitate the interpretation of the results. A detailed review is available in Appendix 1.

Table 1 shows a stylized input–output matrix of a hypothetical 3 sector economy, along with the definition of total in-degree and out-degree². For example, entry a_{31} in the matrix represent the amount of input sector 3 sold to sector 1. A simple graphical representation translates the matrix entries for Sector 2 in an equivalent network representation in Figure 4.

First-order connections between sectors are computed using the out-degree links of the sectors. First-order connections between sectors capture how shocks propagate from sector i to other sectors that are directly connected with the sector i and use i 's goods as inputs in their production. The larger number of sectors that use i 's goods as inputs, the larger the first-order effect. Meanwhile, the higher-order inter-connectivity captures how shocks propagate from sector i to those sectors that are using inputs of the sectors using i 's goods

Table 1. Input–output linkages.

From/to	1	2	3	Total out-degrees
1	a_{11}	a_{12}	a_{13}	$\sum_{j=1}^n a_{1j}$
2	a_{21}	a_{22}	a_{23}	$\sum_{j=1}^n a_{2j}$
3	a_{31}	a_{32}	a_{33}	$\sum_{j=1}^n a_{3j}$
Total in-degrees	$\sum_{j=1}^n a_{j1}$	$\sum_{j=1}^n a_{j2}$	$\sum_{j=1}^n a_{j3}$	

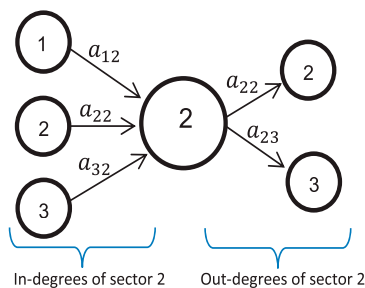


Figure 4. Degrees of sector 2.

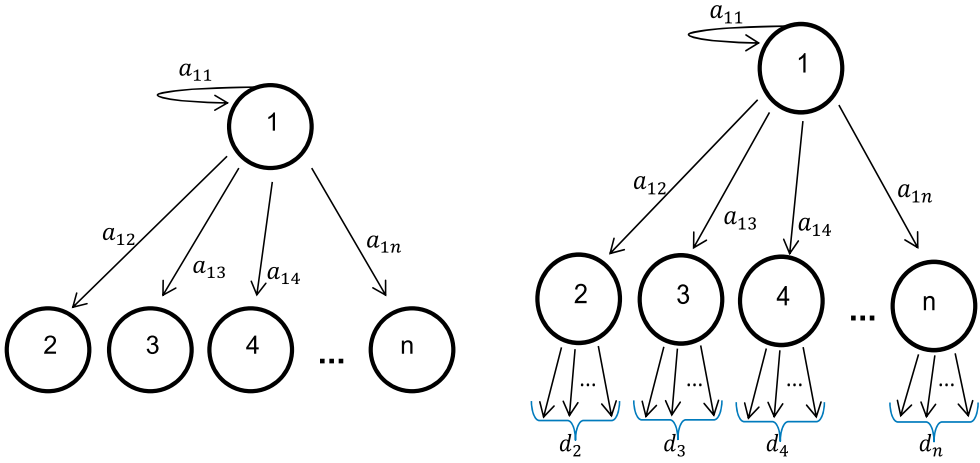


Figure 5. First and second-order degrees.

as inputs in their production. Such higher-order inter-connectivity is referred to as the second-order connections between sectors. Figure 5 presents the first and second-order connections for sector 1 of a hypothetical n -sector economy. In the next section, the formal definitions will be introduced along with their intuitive explanation.

2.1. First-order degree interactions

The influence of the first-order degree connections on aggregate volatility depends on the asymmetry between sectors, which is measured by the coefficient of variation (CV_n) (Acemoglu et al., 2012). The degree (or weighted out-degree) of sector i , denoted as d_i , shows the share of sector i 's output (normalized by the constant $1 - \alpha$) in the input supply of the entire economy presented in Equation (1)³:

$$d_i \equiv \sum_{j=1}^n w_{ji}. \quad (1)$$

For each economy ξ_n with sectoral degrees $\{d_1^n, d_2^n, \dots, d_n^n\}$, the coefficient of variation (CV_n) is defined as:

$$CV_n \equiv \frac{1}{\bar{d}_n} \left[\frac{1}{n-1} \sum_{i=1}^n (d_i^n - \bar{d}_n)^2 \right]^{1/2}, \quad (2)$$

where $\bar{d}_n = (\sum_{i=1}^n d_i^n)/n$ denotes for the average degree of the economy n .

Based on Equation (A6), the volatility of aggregate output becomes:⁴

$$(\text{var } y_n)^{1/2} = \Omega \left(\frac{1}{n} \sqrt{\sum_{i=1}^n (d_i^n)^2} \right) \quad (3)$$

and

$$(\text{var } y_n)^{1/2} = \Omega \left(\frac{1 + CV_n}{\sqrt{n}} \right). \quad (4)$$

Equations (2)–(4) show that an increase in the asymmetry between weighted out-degrees leads to an increase in the coefficient of variation, causing aggregate volatility to decay at the rate slower than \sqrt{n} . A high value for CV_n indicates that a small number of sectors in the economy provides the inputs for most of the remaining sectors.⁵ A shock to one of these dominant sectors would propagate through all the downstream sectors.

At the same time, Equation (3) describes the aggregate volatility in terms of the statistical degree distribution. Fluctuations in aggregate volatility are larger the heavier the tail of the degrees' distribution.

A sequence of economies $\{\xi_n\}_{n \in \mathbb{N}}$ has power law degree sequence if the following assumptions are satisfied:

- (a) There exists a constant $\beta > 1$ showing that the tail of the empirical degree distribution has scaling behaviour. The lower the value of β , the heavier the tail of the empirical degree distribution that leads to the higher differences between the degrees of different sectors in the economy.
- (b) There exists a slowly varying function $L(\cdot)$ that satisfies the following:

$$\begin{aligned} \lim_{t \rightarrow \infty} L(t)t^\delta &= \infty \\ \lim_{t \rightarrow \infty} L(t)t^{-\delta} &= 0 \end{aligned} \tag{5}$$

for all $\delta > 0$.

- (c) A sequence of positive numbers $c_n = \Theta(1)$ that for all $n \in \mathbb{N}$ and all $k < d_{\max}^n = \Theta(n^{1/\beta})$, where d_{\max}^n is the maximum degree in the economy ξ_n .

Based on these assumptions, the empirical counter-cumulative distribution function (CCDF) may be derived in Equation (6):⁶

$$P_n(k) = c_n k^{-\beta} L(k). \tag{6}$$

Taking into account the first-order degree intersectoral network, the aggregate volatility is defined in Equation (7) as a function of the shape parameter $\beta \in (1, 2)$. The shape parameter describes the scaling behaviour of the tail of the empirical degree distribution.

$$(\text{var } y_n)^{1/2} = \Omega(n^{-(\beta-1)/\beta-\delta}), \tag{7}$$

where δ is a constant. Equation (7) suggests that if the heavy tail of the first-order intersectoral network degree sequence is captured in the economy, the aggregate volatility decays at a rate smaller than $n^{(\beta-1)/\beta}$, which in turn should be lower than \sqrt{n} .

2.2. Second-order degree interactions

It is important to mention that sectors with identical first-order degrees might have different impact on the aggregate volatility. This effect depends on the second-order inter-connectivity that indicates how sectors are related indirectly with downstream sectors in the economy. For example, two sectors r and u are selling their output (as intermediate products) to two other sectors in the economy (r sells to l and m (both small sectors) while u sells to m and g (the later having the highest degree in the

economy)). Even if both r and u have the same first-order degrees, u will affect the economy more, since it is selling products to g that is connected with many other sectors in the economy.

Based on Acemoglu et al. (2012), the second-order inter-connectivity coefficient of the economy ξ_n is defined as follows:

$$\tau_2(W_n) = \sum_{i=1}^n \sum_{j \neq i} \sum_{k \neq i,j} w_{ji}^n w_{ki}^n d_j^n d_k^n. \quad (8)$$

The coefficient τ_2 measures how highly-connected sectors are related in the economy through the same suppliers of inputs. This coefficient is higher if the same supplier is being shared between two highly connected sectors (when d_j^n and d_k^n are both comparably high).

The coefficient τ_2 affects aggregate volatility in the following way:

$$(\text{var } y_n)^{1/2} = \Omega \left(\frac{1}{\sqrt{n}} + \frac{CV_n}{\sqrt{n}} + \frac{\sqrt{\tau_2(W_n)}}{n} \right). \quad (9)$$

Comparing Equation (9) with Equation (4) we can see that even if the first-order degrees of economies are the same, the second-order relations between different sectors play an important role in explaining the dynamics of aggregate volatility. At the same time, Equation (9) captures the possibility of cascade effects. A disturbance in one sector may impact not only its downstream sectors but also all other interconnected sectors in the economy.

In a similar fashion to the first-order degree definition, the second-order intersectoral network may be recast as a function of the tail of the degree distribution. Here, the second-order degree of sector i is defined as Equation (10):

$$q_i^n = \sum_{j=1}^n d_j^n w_{ji}^n, \quad (10)$$

where second-order degree of sector i is calculated as the weighted sum of degrees of those sectors that use inputs from sector i .

Taking into account the second-order degrees, aggregate volatility follows Equation (11), if economies follow a power law degree sequence and shape parameter $\zeta \in (1,2)$, where ζ shows the scaling behaviour of the tail of the degree distribution:

$$(\text{var } y_n)^{1/2} = \Omega(n^{-(\zeta-1)/\zeta-\delta}), \quad (11)$$

where $\delta > 0$ is a constant. Equation (11) shows that if the heavy tail of intersectoral network second-order degree sequence is accounted for, the aggregate volatility decays at a rate smaller than $n^{(\zeta-1)/\zeta}$.

2.3. Estimation of shape parameters

As indicated in the previous section, first and the second orders are described by a power law degree sequences that can be generally defined as:

$$P(Z > s) \sim Cs^{-\vartheta}, \quad C, s > 0, \quad (12)$$

where ϑ is a tail index (shape parameter), $\{Z_1, Z_2, \dots, Z_n\}$ stand for observations satisfying the power law and C is a positive constant.

By estimating the Pareto exponent, we obtain the first-order and the second-order shape parameters (β and ζ accordingly). The OLS log rank-log size regression is one of the most popular tools for estimation of Pareto exponent:

$$\log(\text{Rank}) = a - b \log(\text{Size}), \quad (13)$$

where b is the estimate of the tail index. To prevent the small-sample bias, Gabaix and Ibragimov (2011) introduced the modified log ranklog size regression. The regression is presented in Equation (14) and is used for estimation of β and ζ .

$$(\log(\text{Rank} - 1/2) = a - b \log(\text{Size}), \quad (14)$$

where b is the estimate of the tail index (β and ζ accordingly), $\log(\text{Rank})$ stands for empirical log-CCDF, and $\log(\text{Size})$ stands for the log-outdegree sequence. According to Gabaix and Ibragimov (2011), the shift by $1/2$ is optimal and reduces the bias.

Due to the small sample size of Lithuanian data ($n=62$), we use 60% as a cut-off value. This value is larger than the one used in Acemoglu et al. (2012) where $n \approx 480$. In other words, we take the tails of the counter cumulative distributions equal to 60% of the sectors with the largest first-order and second-order degrees (d and q accordingly). We investigate the robustness of the results with different cut-off values, the main insights of the analysis remain.

2.4. Data

The direct requirements table gathers the technical coefficients as follows:

$$\mathbf{A} = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix}, \quad (15)$$

where a'_{ij} s show the flow of products from industrial sectors (i 's), to the same sector and all others (j 's). Technical coefficient a_{ij} can also be explained as the share of inputs from industry i needed to produce 1 Euro output by industry j . These direct requirements tables A are computed as $A = Z\hat{x}^{-1}$, where Z is the statistical input-output transactions table given by:

$$\mathbf{Z} = \begin{bmatrix} z_{11} & \dots & z_{1n} \\ \vdots & \ddots & \vdots \\ z_{n1} & \dots & z_{nn} \end{bmatrix}, \quad (16)$$

where z_{ij} shows the monetary values of transactions of intermediate products from sectors i (rows) to sectors j (columns).

The matrix \hat{x}^{-1} presents the inverse diagonal matrix with elements of the vector of the total outputs along the main diagonal:

$$\hat{\mathbf{x}}^{-1} = \begin{bmatrix} 1/x_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1/x_n \end{bmatrix}, \quad (17)$$

where x_i shows the total output of each sector i . The total output of each sector is equal to the sales to sectors as intermediate products plus sales of the production for the final demand.

We use the data from the Lithuanian transactions tables at basic prices for the year 2010. This data are available from Lithuanian Statistics and the WIOD database. Data from Lithuanian Statistics represents inputs aggregated from both national and international suppliers while the WIOD transactions tables contain only domestic inputs. The data from Lithuanian Statistics is available for the year 2005 and 2010, therefore we perform a robustness check with the 2005 dataset as well. Furthermore, in the analysis we use the WIOD dataset for 2014, the latest available dataset. It is worth mentioning that Lithuanian Statistics 2010 dataset allows analysing inter-sectoral linkages between 62 industries in Lithuania. At the same time, all remaining sets are more aggregated and allow an investigation of only 54 industries.

3. Empirical results

3.1. Weighted in-degrees and out-degrees

Figures 6 and 7 compare the nonparametric estimates of the relative frequencies of weighted first-order and second-order out-degrees in 2010, using both the WIOD input-output matrix (only national inputs) and the Lithuanian Statistics aggregated matrix (with both local and imported inputs). These figures suggest that first-order (d_i) and second-order (q_i) out-degree empirical distributions are skewed with right tails and that openness does not fundamentally alter the nature of the observed

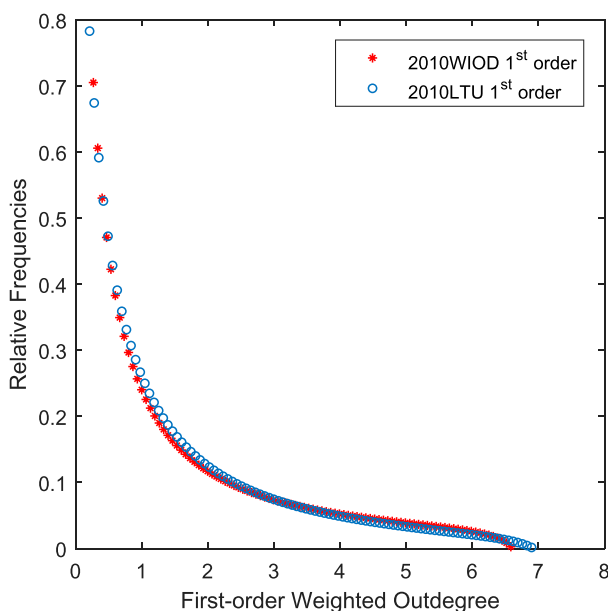


Figure 6. First-order weighted out-degree for Lithuanian industries.

Note: Figures present the nonparametric estimates of empirical densities of the weighted first-order (Figure 6) and second-order (Figure 7) out-degrees in 2010. Both of them are skewed with right tails.

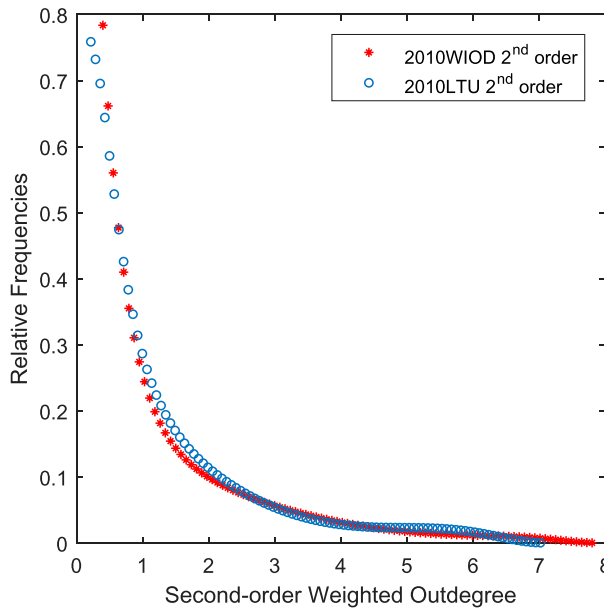


Figure 7. Second-order weighted out-degree for Lithuanian industries.

Note: Figures present the nonparametric estimates of empirical densities of the weighted first-order (Figure 6) and second-order (Figure 7) out-degrees in 2010. Both of them are skewed with right tails.

topological features of the network. This however does not preclude the possibility that different combinations of sectors may have similar in-degree and out-degree distributions.

Given the comparable empirical relative frequencies, shocks are propagated in a similar way regardless whether they emerge from internal sources or are transmitted through the network of import partners. This result is further confirmed by the numerical estimates of the shape parameters using the two different matrices. According to Acemoglu et al. (2012), this type of distributions indicates that: (i) some sectors produce ‘general purpose’ products used as inputs in many other industries, or (ii) some sectors produce inputs to other sectors that produce ‘general purpose’ inputs.

Figures 8 and 9 also indicate the first-order and second-order heavy-tailed distributions by presenting the empirical CCDFs (i.e. 1 minus the empirical cumulative distribution function) of the first-order and second-order degrees on a log–log scales.

The first-order and second-order heavy-tailed distributions for the Lithuanian Statistics aggregated matrix are also presented in Appendix 3 in Figures 1 and 2. Using the Nadaraya (1964) and Watson’s (1964) kernel regression with the least squares cross-validation bandwidth, nonparametric estimates for the empirical counter-cumulative distributions are obtained. The tails of both the first-order and second-order distributions are well approximated by a power law distribution, as shown by the approximate linear relationship.

Figure 10 presents the nonparametric estimate of the relative frequencies of the intermediate input shares for 2010 using the input–output matrix which accounts for both local and imported intermediates as well as the WIOD data accounting only for

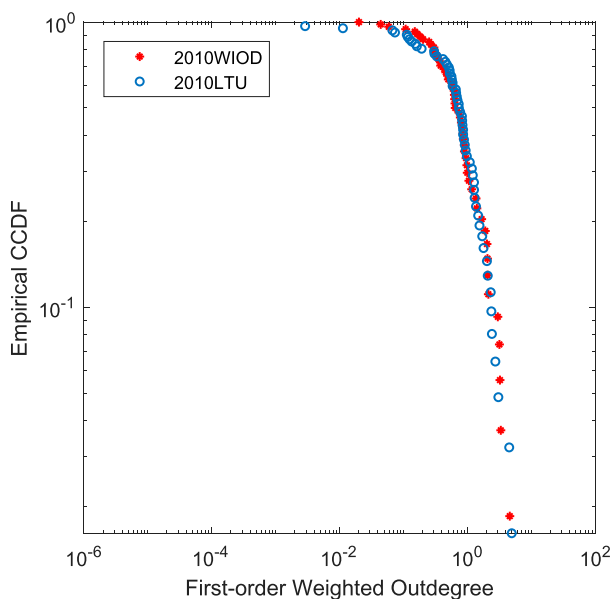


Figure 8. CCDF of the first-order degree.

Note: Figures present the empirical CCDFs of the first-order (Figure 8) and second-order (Figure 9) degrees on a log–log scales.

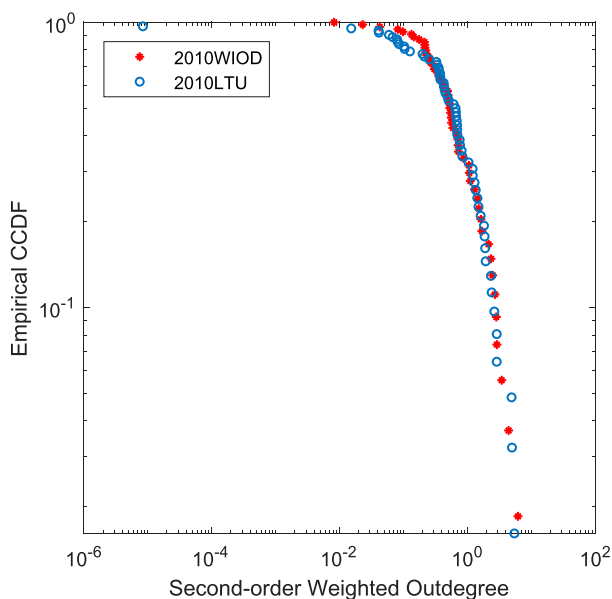


Figure 9. CCDF of the second-order degree.

Note: Figures present the empirical CCDFs of the first-order (Figure 8) and second-order (Figure 9) degrees on a log–log scales.

domestic transactions. Though some industries have more interindustrial connections than others, around 70% of industries are within one standard deviation of the mean input share.⁷

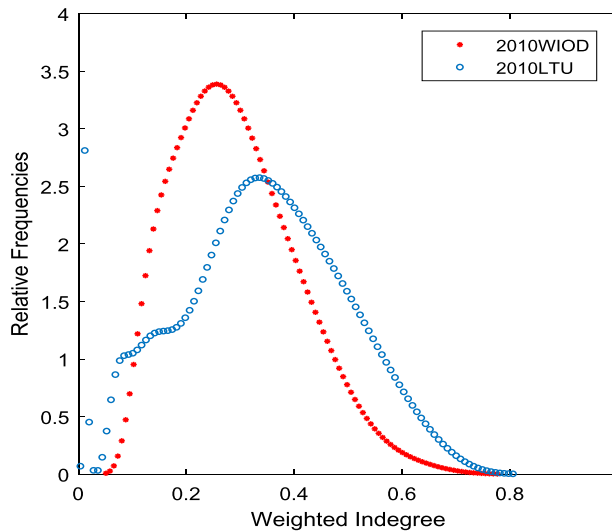


Figure 10. Weighted in-degree for Lithuanian industries.

Note: Figure presents weighted indegrees of industries in Lithuania in 2010. It shows the importance of intermediate products in production of final goods in different sectors.

3.2. Estimation of the shape parameters β and ζ

For the estimation of the shape parameters, we use the modified log rank-log size regression as described in Section 2.3.

In Table 2, we present the OLS estimates of the first-order and second-order degree parameters β and ζ , respectively, with corresponding standard errors (in the brackets), for the 2010 input–output matrix of aggregated data. The total number of sectors is denoted by n while the cut-off value presents the percentage of sectors used in estimating the shape parameters. It is worth mentioning that ζ is smaller than β (1.28 and 1.70 accordingly), which is inline with the argument that second-order connections in the economy play an important role in explaining fluctuations at the aggregate level.

When it comes to the first-order degree, the estimated shape parameter $\hat{\beta} = 1.70$ suggests that the aggregate volatility decays at a rate smaller than $n^{(1.70-1)/1.70} = n^{0.41}$. Regarding the second-order degree, the estimated shape parameter $\hat{\zeta} = 1.28$ indicates that the aggregate volatility decays at a rate smaller than $n^{(1.28-1)/1.28} = n^{0.22}$. The standard error of $\hat{\zeta}$ indicates that the aggregate volatility decay rate of $(n^{0.22})$ is significantly lower than \sqrt{n} .

Table 2. Estimation of β and ζ .

Year	β	ζ	Cut-off value	n
2010	1.70 (0.40)	1.28 (0.30)	0.6	62

Notes: The table presents OLS estimates of the first-order and second-order degrees (β and ζ accordingly) with standard errors in the brackets. The cut-off value presents the percentage of sectors used in the estimation of the shape parameters and n denotes the total number of sectors in the economy.

Table 3. Estimation of β and ζ .

Year	β	ζ	Cut-off value	n
2005	1.71 (0.42)	1.41 (0.35)	0.6	54
2010	1.70 (0.40)	1.28 (0.30)	0.6	62
2010 WIOD	1.54 (0.39)	1.22 (0.30)	0.6	54
2014 WIOD	1.57 (0.39)	1.20 (0.30)	0.6	54

Notes: This table presents OLS estimates of the first-order and second-order degrees (β and ζ accordingly) with standard errors in the brackets. The cut-off value presents the number of sectors used in estimation of the shape parameters and n denotes the total number of sectors in the economy.

Consequently, both the first-order and second-order connections imply that the aggregate volatility decays at the rate lower than \sqrt{n} – as predicted by the standard diversification argument ($n^{0.22} < n^{0.41} < \sqrt{n}$), with the second-order connections playing a more important role. Due to the second-order connections and unbalanced structure of the input–output intermediate production networks, sectoral shocks to dominant sectors would propagate through all the downstream sectors by creating substantial fluctuations at the aggregate level.

We also calculate the shape parameters for the WIOD dataset for both 2010 and 2014 as presented in Table 3. Estimates and standard errors remain similar to the original 2010 value indicating the robustness of the results for the open economy case. Although imports may represent an additional source of shocks, the transmission channels do not change as compared to internal shocks. This is expected given that the degree of substitutability across such broad sectorial definitions is limited by the nature of the production process. Naturally, more diverse inputs may be obtained from external providers yet it is interesting to observe that the parametric estimates of the tail (and their corresponding standard errors) do not change substantially. This may be purely the effect of aggregation (within a particular sector, firms may source from a larger number of external sub-sectors yet, given the available data, this cannot be observed) or it may reflect the homogeneity of the production function with either domestic or imported intermediates.

3.3. Robustness checks

As a further check, we also compare the values to the aggregated internal and imported input matrix for 2005. Some of the observed variation in the estimates may be assigned to the different number of available sectors (54 vs. 62), a fact in line with theoretical predications that indicate that more aggregated data captures lower network effects. Table 4 in Appendix 3 further shows the sensitivity of the parameters to different cut-off values.

Furthermore, in Figure 11 we compare the total intermediate input shares within industries (weighted in-degrees for each of the industry) in Lithuania in 2005 and 2010 for the data provided by Statistics Lithuania. The average share of the intermediate inputs in the production of the final products in Lithuania in 2005 and 2010 is almost the same and equal to 0.354 (35.4%) and 0.337 (33.7%) accordingly. Though some industries have more interindustrial connections than others, around 70% of industries are within one standard deviation of the mean input share in 2005, the same as in 2010.

Figures 12 and 13 present the nonparametric estimates of the relative frequencies of weighted first-order and second-order out-degrees in 2005 and 2010, suggesting that

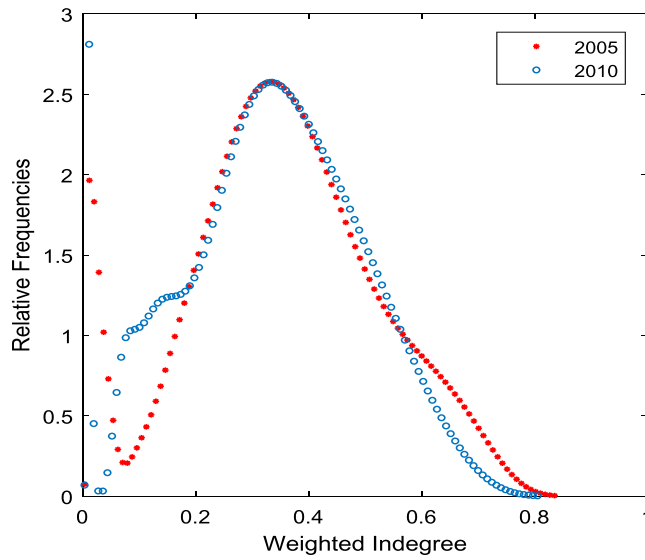


Figure 11. Weighted in-degree for Lithuanian industries in 2005 and 2010.

Note: Figure presents weighted indegrees of industries in Lithuania in 2005 and 2010. It shows the importance of intermediate products in production of final goods in different sectors.

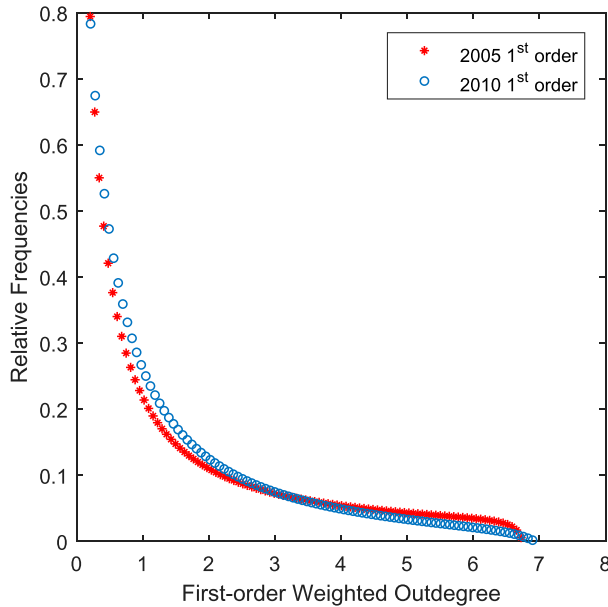


Figure 12. First-order weighted out-degree for Lithuanian industries.

Note: Figures present the nonparametric estimates of empirical densities of the weighted first-order (Figure 12) and second-order (Figure 13) out-degrees in 2005 and 2010. Both of them are skewed with right tails.

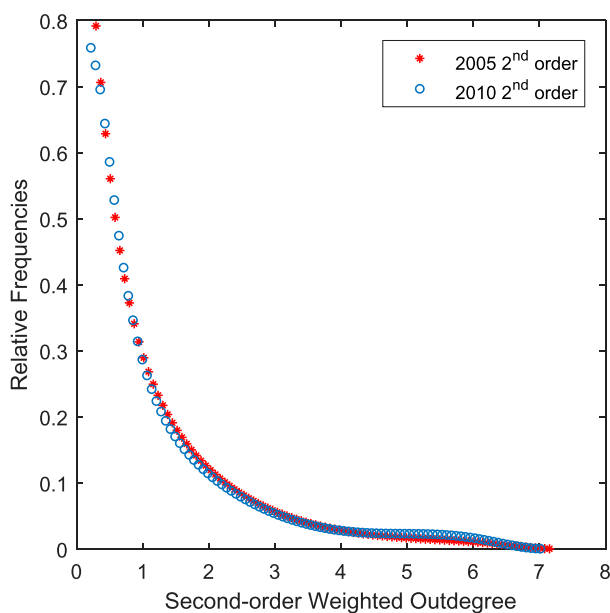


Figure 13. Second-order weighted out-degree for Lithuanian industries.

Note: Figures present the nonparametric estimates of empirical densities of the weighted first-order (Figure 12) and second-order (Figure 13) out-degrees in 2005 and 2010. Both of them are skewed with right tails.

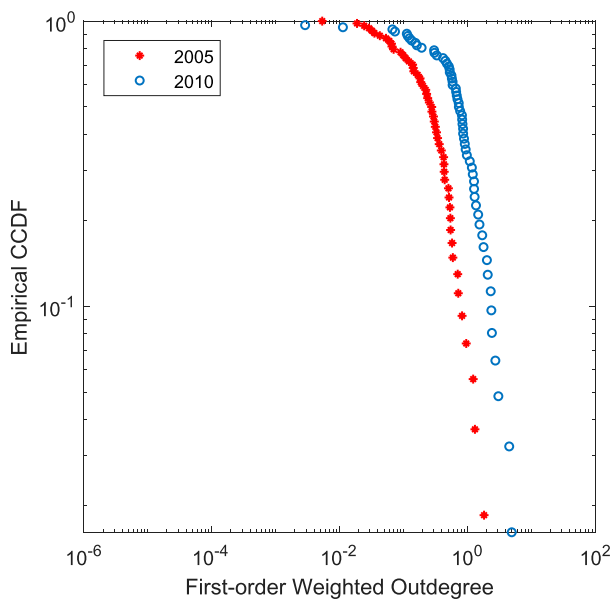


Figure 14. CCDF of the first-order degree.

Note: Figures present the empirical CCDFs of the first-order (Figure 14) and second-order (Figure 15) degrees on a log-log scales in 2005 and 2010. The tails of both distributions are well approximated by a power law distribution.

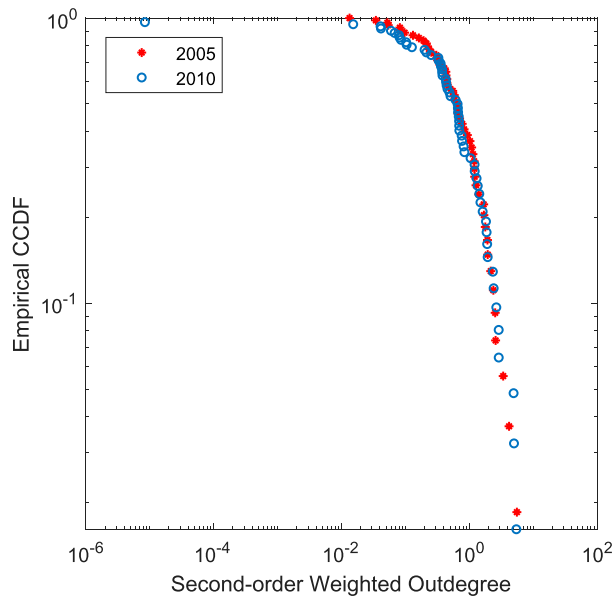


Figure 15. CCDF of the second-order degree.

Note: Figures present the empirical CCDFs of the first-order (Figure 14) and second-order (Figure 15) degrees on a log–log scales in 2005 and 2010. The tails of both distributions are well approximated by a power law distribution.

all first-order (d_i) and second-order (q_i) outdegree empirical distributions are skewed with right tails.

Figures 14 and 15 present the empirical CCDFs of the first-order and second-order degrees on a log–log scales that captures the first-order and second-order heavy-tailed distributions in 2005, the same as in 2010. The tails of the first-order and second-order distributions are well approximated by a power law distribution for both data sets.

4. Conclusion

The current study investigates the importance of inter-sectoral linkages of intermediate products as conduits of sectoral shocks at the aggregate level in Lithuania. We refine the analysis by considering the relevance of imported intermediate products and how these may alter the conclusions of the exercise as compared to the domestic-only case. To do so, we construct a domestic only sector-by-sector direct requirements table using the WIOD data, and a domestic and imported direct requirements table using the Lithuanian input–output transactions table. We then employ Acemoglu et al.’s (2012) network-based methodology and Gabaix Ibragimov’s (2011) modified log rank-log size regression to uncover the structural parameters of the distribution of inter-sectoral linkages and compare the results for the two sets of matrices.

The results show that the first-order and second-order degree distributions are skewed to the right. The network of intermediate products in Lithuania is unbalanced with a small number of sectors playing a dominant role in the economy. The direct and indirect inter-sectoral linkages imply that aggregate volatility decays at a rate lower than \sqrt{n} as implied by the standard diversification argument. The results are confirmed both for the domestic-

only and the aggregated (domestic and imported intermediates) data. This paper provides evidence that the Lithuanian inter-sectoral network of intermediate inputs represents an important propagation channel for idiosyncratic shocks which does not fundamentally change when considering domestic-only or domestic and imported inputs. We further contribute to the literature by providing some preliminary evidence of the suitability of Acemoglu et al.'s (2012) network-based methodology and Gabaix Ibragimov's (2011) modified log rank-log size regression in analysing the input–output structure of open economies.

Notes

1. Aggregated sector-by-sector requirements will refer to the input–output matrix accounting for both domestic as well as imported intermediates.
2. a'_{ij} s show the flow of products from industrial sectors (i 's), to the same sector and all others (j 's). Total in-degrees capture the amount of intermediate goods particular sector needs to purchase from all sectors in the economy while producing its output. Total out-degrees capture how much of its final output sector sells as intermediates to all sectors in the economy.
3. $\sum_{j=1}^n w_{ji}$ is the sum of weighted out-degrees of sector i , capturing how much of its final output sector i sells as intermediates to all sectors j in the economy.
4. $y_n = \Omega(x_n)$ if $\liminf_{n \rightarrow \infty} y_n/x_n > 0$, when $\{y_n\}_{n \in \mathbb{N}}$ and $\{x_n\}_{n \in \mathbb{N}}$ are sequences of real positive numbers.
5. If balanced intersectoral network exists in the economy, all sectors are equally connected between each other, CV is equal to zero. Then Equation (4) implies that aggregate volatility decays at the rate \sqrt{n} – the one predicted by the standard diversification argument – due to sectoral shocks.
6. The empirical CCDF represents the probability of observing a sector with more than k degrees in the economy.
7. In this model the intermediate input share is constant and equal to $1 - \alpha$.
8. In this model, the normalization constant A affects only the mean of aggregate output without affecting aggregate volatility or any other distributional parameters. For further analysis regarding normalization constant, see Acemoglu et al. (2012).
9. Without normalization constant A , the aggregate output would be equal to $y = v'\varepsilon + \mu$.
10. According to Bonacich (1987), the most central sectors in the network have the most connections within the network. A number of connections within the network presents number of sectors that one particular sector is connected with.
11. $y_n = \Theta(x_n)$ if $\limsup_{n \rightarrow \infty} y_n/x_n < \infty$ and $\liminf_{n \rightarrow \infty} y_n/x_n > 0$, when $\{y_n\}_{n \in \mathbb{N}}$ and $\{x_n\}_{n \in \mathbb{N}}$ are sequences of real positive numbers.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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Appendix 1. Review of methodology

The theoretical model of Acemoglu et al. (2012) is based on the real business cycle's multi-sectoral model of Long and Plosser (1983). In this model, the representative household has inelastic one unit of labour and Cobb–Douglas preferences for different n goods as in Equation (A1):

$$u(c_1, c_2, \dots, c_n) = A \prod_{i=1}^n (c_i)^{1/n}, \quad (\text{A1})$$

where c_i presents consumption of good i and A is a normalization constant.⁸

Competitive sectors produce goods in the economy that can be used as intermediate inputs by sectors for their production or consumed by final users. The output of sector i , x_i , is given by:

$$x_i = z_i^\alpha l_i^\alpha \prod_{j=1}^n (x_{ij})^{(1-\alpha)w_{ij}}, \quad (\text{A2})$$

where l_i is labour input in sector i , α is a share of labour, x_{ij} presents the amount of good j used in the production of good i , z_i is idiosyncratic productivity shock to sector i , w_{ij} is the share of goods of sector j needed in the production of i goods.

The input–output table is used in this Cobb–Douglas function as w'_{ij} s, where it shows the needed expenditure on input j per dollar of output of sector i . Assumption $\sum_{j=1}^n w_{ij} = 1$ in this model implies that sectoral production functions have constant returns to scale. Productivity shocks z_i are independent with $\varepsilon_i = \log(z_i)$ having the distribution F_i . An economy is defined as $\xi = (I, W, \{F_i\}_{i \in I})$, where I denotes the set of sectors, W denotes the input–output matrix.

With this specification, normalized aggregate output can be derived as:⁹

$$y \equiv \log(\text{GDP}) = v' \varepsilon, \quad (\text{A3})$$

where $\log(\text{GDP})$ is aggregate output, sectoral shocks $\varepsilon \equiv [\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n]'$ and v is the n -dimensional influence vector.

The influence vector v is related to Bonacich centrality vector corresponding to the inter-sectoral network.¹⁰ Sectors with higher centrality in the network are more important in determining aggregate output as these sectors have more connections, and shocks to these sectors might propagate to other sectors in the economy. On the other hand, sectors with low influence have little or no connections with other sectors. Therefore, shocks to these sectors might weakly influence other sectors in the economy. In detail, the influence vector v is written as:

$$v \equiv \frac{\alpha}{n} [I - (1 - \alpha)W']^{-1} \mathbf{1}. \quad (\text{A4})$$

Equations (A3) and (A4) imply that aggregate output depends on the network of inter-sectoral linkages via the Leontief inverse $[I - (1 - \alpha)W']^{-1} \mathbf{1}$. This term captures how idiosyncratic productivity shocks propagate downstream to other sectors through the input–output matrix.

In order to derive the aggregate volatility, we need the following assumptions regarding the sectoral level shocks:

⁸In this model, the normalization constant A affects only the mean of aggregate output without affecting aggregate volatility or any other distributional parameters. For further analysis regarding normalization constant, see Acemoglu et al. (2012).

⁹Without normalization constant A , the aggregate output would be equal to $y = v' \varepsilon + \mu$.

¹⁰According to Bonacich (1987), the most central sectors in the network have the most connections within the network. A number of connections within the network presents number of sectors that one particular sector is connected with.

- (a) $\mathbb{E}(\varepsilon_{in}) = 0$,
- (b) $\mathbb{E}(\varepsilon_{in}, \varepsilon_{jn}) = 0$,
- (c) $\text{var}(\varepsilon_{in}) = \sigma_{in}^2 \in (\underline{\sigma}^2, \bar{\sigma}^2)$, where $0 < \underline{\sigma} < \bar{\sigma}$.

Assumption (a) is needed for normalization of the shocks (the mean of shocks is equal to zero). Assumption (b) implies that all idiosyncratic productivity shocks are independent of each other. Assumption (c) implies that variance of idiosyncratic productivity shocks is bounded from zero when $n \rightarrow \infty$. While using assumptions (a) and (b) with Equation (A3), we can derive that:

$$(\text{var } y_n)^{1/2} = \sqrt{\sum_{i=1}^n \sigma_{in}^2 v_{in}^2}, \quad (\text{A5})$$

where v_{in} denotes i th element of v_n . With assumptions (b) and (c), we obtain:¹¹

$$(\text{var } y_n)^{1/2} = \Theta(\|v_n\|_2), \quad (\text{A6})$$

where $\|v_n\|_2 = \sqrt{\sum_{i=1}^n v_{in}^2}$.

Appendix 2. List of Lithuanian sectors in 2010

Here is presented the list of 62 sectors in Lithuania in 2010:

- (1) Products of agriculture, hunting and related services
- (2) Products of forestry, logging and related services
- (3) Fish and other fishing products; aquaculture products; support services to fishing
- (4) Mining and quarrying
- (5) Food products, beverages and tobacco products
- (6) Textiles, wearing apparel and leather products
- (7) Wood and of products of wood and cork, except furniture; articles of straw and plaiting materials
- (8) Paper and paper products
- (9) Printing and recording services
- (10) Coke and refined petroleum products; Chemicals and chemical products
- (11) Basic pharmaceutical products and pharmaceutical preparations
- (12) Rubber and plastics products
- (13) Other non-metallic mineral products
- (14) Basic metals
- (15) Fabricated metal products, except machinery and equipment
- (16) Computer, electronic and optical products
- (17) Electrical equipment
- (18) Machinery and equipment n.e.c.
- (19) Motor vehicles, trailers and semi-trailers
- (20) Other transport equipment
- (21) Furniture; other manufactured goods
- (22) Repair and installation services of machinery and equipment
- (23) Electricity, gas, steam and air-conditioning
- (24) Natural water; water treatment and supply services
- (25) Sewerage; waste collection, treatment and disposal activities; materials recovery remediation activities and other waste management services
- (26) Constructions and construction works

¹¹ $y_n = \Theta(x_n)$ if $\limsup_{n \rightarrow \infty} y_n/x_n < \infty$ and $\liminf_{n \rightarrow \infty} y_n/x_n > 0$, when $\{y_n\}_{n \in \mathbb{N}}$ and $\{x_n\}_{n \in \mathbb{N}}$ are sequences of real positive numbers.

- (27) Wholesale and retail trade and repair services of motor vehicles and motorcycles
- (28) Wholesale trade services, except of motor vehicles and motorcycles
- (29) Retail trade services, except of motor vehicles and motorcycles
- (30) Land transport services and transport services via pipelines
- (31) Water transport services
- (32) Air transport services
- (33) Warehousing and support services for transportation
- (34) Postal and courier services
- (35) Accommodation and food services
- (36) Publishing services
- (37) Motion picture, video and television programme production services, sound recording and music publishing; programming and broadcasting services
- (38) Telecommunications services
- (39) Computer programming, consultancy and related services; information services
- (40) Financial services, except insurance and pension funding
- (41) Insurance, reinsurance and pension funding services, except compulsory social security
- (42) Services auxiliary to financial services and insurance services
- (43) Real estate activities excluding imputed rents
- (44) Imputed rents of owner-occupied dwellings
- (45) Legal and accounting services; services of head offices; management consulting services
- (46) Architectural and engineering services; technical testing and analysis services
- (47) Scientific research and development services
- (48) Advertising and market research services
- (49) Other professional, scientific and technical services; veterinary services
- (50) Rental and leasing services
- (51) Employment services
- (52) Travel agency, tour operator and other reservation services and related services
- (53) Security and investigation services; services to buildings and landscape; office administrative, office support and other business support services
- (54) Public administration and defence services; compulsory social security services
- (55) Education services
- (56) Human health services
- (57) Social work services
- (58) Creative, arts and entertainment services; library, archive, museum and other cultural services; gambling and betting services
- (59) Sporting services and amusement and recreation services
- (60) Services furnished by membership organizations
- (61) Repair services of computers and personal and household goods
- (62) Other personal services

Appendix 3. Further robustness checks

Table A1. Estimation of β and ζ .

Year	β	ζ	Cut-off value
2010	1.70 (0.40)	1.28 (0.30)	0.6
2010 WIOD	1.54 (0.39)	1.22 (0.30)	0.6
2010	1.80 (0.46)	1.41 (0.36)	0.5
2010 WIOD	1.62 (0.44)	1.27 (0.34)	0.5
2010	1.90 (0.54)	1.52 (0.43)	0.4
2010 WIOD	1.70 (0.51)	1.42 (0.43)	0.4

OLS estimates of the first-order and second-order degrees (β and ζ accordingly) with standard errors in the brackets for different cut-off values.

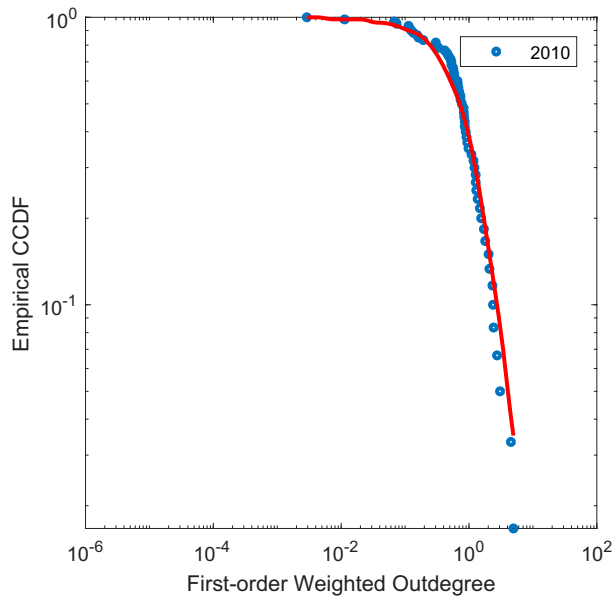


Figure A1. CCDF of the first-order degree.

Note: Figures present the empirical CCDFs of the first-order (Figure 16) and second-order (Figure 17) degrees on a log–log scales together with nonparametric estimates for the empirical counter-cumulative distributions by Nadaraya–Watson kernel regression (solid lines in both Figures 16 and 17). The tails of both distributions are well approximated by a power law distribution, as shown by the approximate linear relationships.

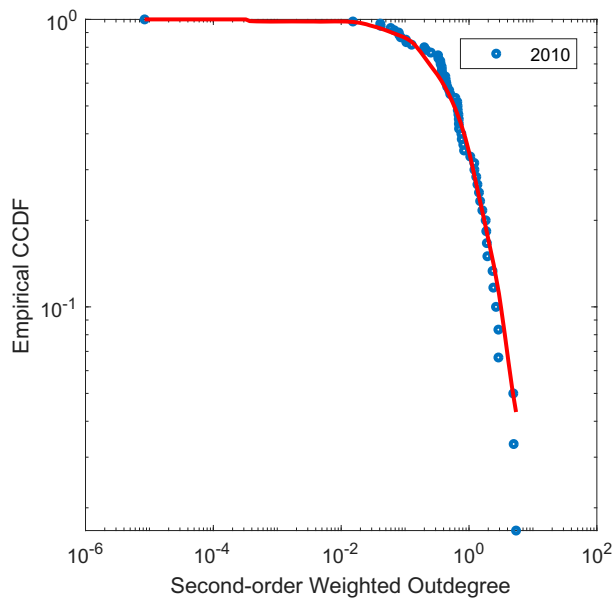


Figure A2. CCDF of the second-order degree.

Note: Figures present the empirical CCDFs of the first-order (figure 16) and second-order (Figure 17) degrees on a log–log scales together with nonparametric estimates for the empirical counter-cumulative distributions by Nadaraya–Watson kernel regression (solid lines in both Figures 16 and 17). The tails of both distributions are well approximated by a power law distribution, as shown by the approximate linear relationships.