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Inequality of educational opportunities: Evidence from Brazil

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Abstract

This paper aims to estimate the inequalities of educational opportunities of 5th and 9th-grade students in Brazil and its states, verifying the relative contribution of each analyzed variable to this inequality. For this purpose, we used the new methodology developed by Ferreira and Gignoux (2014) considering the data of standardized tests of proficiency in Portuguese Language and Mathematics. Overall, the results show that more than 15% of the inequality of educational opportunities in Brazil is explained by circumstances unrelated to individual effort. The poorest regions of Brazil are also those with the highest inequality of educational opportunities and the circumstances with the greatest power to explain inequalities being parental education and socioeconomic status.

JEL classifications: D63; I24; O12

Keywords: Educational achievement; Inequality of opportunity; Brazil

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1. Introduction

Equity in educational achievement has been the focus of many recent contributions to the economics of education literature. There is a great consensus that schooling has a strong explaining power in future incomes, and therefore inequalities in education may reproduce income inequalities, especially in developing countries. Although a large part of the variations in school outcomes are attributed to individual merit, there is a significant influence of factors associated with lack of opportunities (Martins and Veiga, 2010).

Since the Coleman report (Coleman et al., 1966), several researchers have investigated the effects that socioeconomic characteristics have on students' outcomes. The report suggests socioeconomic status has more impact on student achievement than the quality of schools and teachers. Therefore, school funding would not have sufficient strength to reduce educational inequalities. Although there is still a discussion in the literature about the findings of the report (Hanushek, 1992), the influence that the socioeconomic background has on educational inequalities is undeniable.

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There are some mechanisms discussed in the literature that establish the relationship between socioeconomic background and school outcomes. A first explanation would be that parents with better socioeconomic background are more successful in investing in their children's education (Becker, 1964). Another explanation discussed in the literature is the well-known Bourdieu's theory of transmission of cultural capital. Bourdieu (1977) attributes social reproduction to education which in turn depends on parent level of cultural capital. Finally, there are authors who believe that the school characteristics can also contribute to educational inequalities. Since school quality is correlated with the socioeconomic status of the students, students with higher socioeconomic level would have higher educational achievements. Some studies have attempted to decompose school inequalities to separate these effects. Through a common decomposition technique in the area of health economics, Martins and Veiga (2010) evaluate and decompose the socioeconomic inequality in Mathematics achievements using the 2003 PISA dataset. According to the authors, the socioeconomic level can explain up to 34.6% of educational inequality in the 15 countries analyzed.

Also using the PISA database, Ferreira and Gignoux (2014) propose a new measure of opportunity inequality based on the decomposition of variance to explain the disparities in school achievement among 15-year old students from 57 countries that carried out PISA examinations in 2006. This methodology of inequality estimation has the advantages of adapting to the characteristics of school achievement data besides allowing the decomposition of the estimates by circumstances. The evidence is that up to 35% of all educational achievement disparities are explained by opportunity inequality. The results also suggest that Latin American countries have higher levels of opportunity inequality.

Even though there are studies that use this methodology to measure educational opportunity inequalities in different countries (Ferreira et al., 2011; Gamboa and Waltenberg, 2012; Gamboa and Waltenberg, 2015; Contreras and Puentes, 2017; Madden, 2018) and also studies that discuss the educational inequality in Brazil (Lorel, 2008; Scorzafave and Ferreira, 2011; Procópio et al., 2015), there are no papers that analyze the specific role of each circumstance in the inequalities of Brazilian educational opportunities. This paper aims to follow this new methodology to estimate the inequalities of 5th and 9th-grade students' achievements in Brazil and its states, verifying the relative contribution of each of the circumstances considered. For this purpose, we use an extensive database that includes, in addition to the students' socioeconomic characteristics, their results in standardized proficiency tests of Portuguese Language and Mathematics. Brazil, due to its extension and its heterogeneity, becomes a particularly interesting case study. Thus, this paper proposes to fill this gap in the literature, analyzing not only the temporal evolution of inequalities but also exploring the different regions of a country recognized as unequal as Brazil.

The paper proceeds as follows. In the next section we present the Methodology used. The following section describes the Brazilian data used in the empirical exercise. Our main findings are described in the Results section and the conclusion draws the article to a close.

2. Methodology

This study considers the *circumstances approach* that inequality of educational opportunities is given by issues beyond individual effort. Following Bourguignon et al. (2007) and Ferreira and Gignoux (2014) we can define the school achievements of a given student, y_i , as:

$$y_i = f(C_i, E_i(C_i, v_i), u_i)$$
 (1)

where C_i is the vector of circumstances and E_i , vector of efforts, denote the set of variables under student control that impact school achievement. v_i and u_i are unobserved factors associated with individual performance. According to the sense proposed by Roemer (2008), equality of opportunities implies that the conditional distribution of educational achievements must be independent of circumstances, $F(y_i|C) = F(y_i)$. For this condition to be true, it is required that (1), the circumstances do not affect student performance, therefore $\partial f(C, E, u)/\partial C = 0$, and that (2) the circumstances do not affect individual effort, G(E|C) = G(E).

Although efforts can be determined by circumstances, circumstances are totally exogenous to the individual. By omitting effort variables, even if they were observed, the β of the regression below measures the direct and indirect (through effort) effects of circumstances on school achievement:

$$y_i = C'_i \beta + \epsilon_i \tag{2}$$

Since vector C may be correlated with unobserved variables, the $\hat{\beta}$ of the regression cannot be interpreted as the causal effect of the circumstances on the student's performance. Even if this regression has omitted variable bias, the

coefficient of determination (R^2) of the estimation of this equation by ordinary least squares is the proportion of the variation of y that is explained by the circumstances. Therefore, the measure of inequality of educational opportunity proposed by Ferreira and Gignoux (2013) is simply the R^2 of the estimation of Eq. (2) and can be obtained parametrically by:

$$\hat{\theta}_{IOp} = \frac{Var(C_i, \hat{\beta})}{Var(y_i)}$$
(3)

where $\hat{\beta}$ are the OLS estimators of Eq. (2).

As shown in Ferreira and Gignoux (2014) this estimate of opportunity inequality has important advantages over other methods. First, it is calculated by an OLS regression of the students' achievements on a set of individual circumstances variables. Second, it has a simple interpretation as the lower bound (coefficient of determination is not reduced by adding variables) of the share of inequality of opportunity in total inequality. Thus, since $0 \le R^2 \le 1$, as 0 represents perfect equality and 1 represents total opportunity inequality. The third advantage is that $\hat{\theta}_{IOp}$ can be decomposed in each of the circumstances of vector *C*, which allows us to identify the role of each circumstance in the total inequality of opportunity. Using the Shapley–Shorrocks method,¹ Ferreira et al. (2011) prove that the inequality measure can be decomposed as follows:

$$\hat{\theta}_{IOp} = \sum_{j} \hat{\theta}^{j} = \sum_{j} (vary)^{-1} \left[\beta_{j}^{2} varC_{j} + \frac{1}{2} \sum_{k} \beta_{k} \beta_{j} cov(C_{k}, C_{j}) \right]$$

The school achievements measure, y, used in this study were the 5th and the 9th-grade students' scores on standardized Portuguese Language and Mathematics proficiency tests. The vector of individual circumstances C is composed of dummy variables split into six groups of characteristics.

This division is made up by groups: (1) municipality characteristics, such as if the municipality where the school is situated in is urban and/or capital city, (2) gender of the children who applied for the test, (3) race of the children, (4) ownership of valuables, such as television, radio, DVD player, car, computer and bathrooms (which serve to measure the socioeconomic status of the student),² (5) mother's characteristics, the highest education level completed, and (6) father's characteristics, also, the highest education level completed. Since the variables of the set of circumstances cannot be determined in any way by the individuals, as explained previously, we do not use variables related to the students' schools. This decision is based on the hypothesis that children may have some interference in the decision of where to study.

3. Dataset

The dataset used to perform the measurement of opportunity inequality in education was the *Brazilian Educational Assessment System*, known as SAEB, performed by the *National Institute of Educational Research*, INEP, governed by the Brazilian Ministry of Education. This evaluation is taken every two years, with the purpose of measuring the achievement of 5th grade, 9th grade and 12th-grade students.

The main reason to use this education dataset, when analyzing Brazilian education, is that of its fullness of information, and it's coverage of the Brazilian students in rural and urban regions. This dataset includes, aside from the standardized test results, a socioeconomic questionnaire, which allows researchers to understand how the environment where the students are inserted may have affected their respective outcomes.

The following reason to use the SAEB instead of other tests is that it follows the same rules as the PISA test (standardized scores and item response theory), but it covers more Brazilian students than the PISA. As pointed out by Ferreira and Gignoux (2013), the PISA has a coverage problem in countries such as Brazil. Whilst SAEB covers more than two million students from 5th and 9th grades, making this test more feasible to perform measurement tests such as the one proposed in this paper.

¹ Shorrocks (2013).

² Since it is the children who answer the questionnaires, there are no questions related to the parents' income.



Fig. 1. Inequality of opportunity in every Brazilian state - 2015.

Source: prepared by the authors.

The SAEB is composed of two main spheres, the first one having a more embracing view, and the second one a more specific. This happens because the first sphere of SAEB's comprehensiveness gathers almost every student enrolled in public schools, in 5th and 9th grades, being mentioned as ANEB or, in Portuguese, *Prova Brasil*. The second sphere is related to private schools in 5th and 9th grades and for students enrolled in 12th-grade since schools at this spectrum are not bound to do this test, they enroll as a sample of schools, which receive different treatments and weights when performing any kind of statistical measurements. This second sphere, the sample test, is often referred to as Anresc.

For the purpose of this study, we used the entire extension of SAEB, including ANEB and Anresc, since we gathered information about students enrolled in public and private schools.

4. Results

4.1. Spatial analysis

In this subsection, we analyze the results of Inequality of Opportunity in Education in different Brazilian regions, developing an image of the distribution of inequality throughout Brazilian states, regions, and the country as a whole.

4.2. 5th grade students

We started by presenting the results for 5th grade students, considering different regional analyzes. We also extend the analysis to both subjects tested by SAEB, Mathematics and Portuguese Language.

As can be seen in Fig. 1, the inequality in Brazil seems to affect fewer states when analyzing Mathematics results, rather than Portuguese Language scores. The color scheme shows that states with a red color have higher inequality measured by circumstances apart from the students' effort. This analysis gives a graphical visualization of how Brazilian education can be uneven within its states. This can be corroborated by Table 1, that shows the descriptive statistics in Mathematics and Portuguese Language scores for every state and region of Brazil, including the country itself.

As can be seen in Table 1, the lowest mean in Portuguese Language scores,³ in 2015, for 5th-grade students is held by the state of Maranhão, whilst the highest score for the same test is held by the state of Santa Catarina. When we look at to this means of the Mathematics test, we find that the state of Maranhão holds the lowest score in this test

 $^{^{3}}$ This analysis is made using only the students that qualified for our Inequality of Opportunity analysis, so the results may differ from other sources.

| Table 1 |
|--|
| Descriptive statistics for Portuguese language and Mathematics scores for 5th-grade students – 2015. |

| | Portuguese la | inguage | Mathematics | | Number of observations |
|---------------------|---------------|---------|-------------|-------|------------------------|
| | Mean | SD | Mean | SD | |
| Brazil | 207.57 | 48.66 | 219.30 | 46.99 | 2,071,581 |
| North | 190.26 | 45.21 | 201.22 | 41.91 | 248,732 |
| Rondônia | 203.18 | 44.53 | 217.55 | 44.12 | 23,756 |
| Acre | 205.80 | 45.04 | 207.61 | 45.17 | 11,751 |
| Amazonas | 197.08 | 46.98 | 208.17 | 43.83 | 58,217 |
| Roraima | 193.20 | 45.70 | 193.98 | 38.43 | 6678 |
| Pará | 183.04 | 43.05 | 191.89 | 36.84 | 114,452 |
| Amapá | 181.74 | 42.70 | 205.38 | 42.76 | 11,841 |
| Tocantins | 195.15 | 45.72 | 205.15 | 42.53 | 22,037 |
| Northeast | 192.31 | 47.42 | 203.30 | 43.63 | 557,040 |
| Maranhão | 178.41 | 43.31 | 188.59 | 37.39 | 82,417 |
| Piauí | 190.04 | 47.54 | 202.46 | 44.10 | 33,594 |
| Ceará | 212.64 | 48.34 | 220.87 | 47.67 | 81,387 |
| Rio Grande do Norte | 189.74 | 47.43 | 199.85 | 41.20 | 35,220 |
| Paraíba | 192.76 | 46.64 | 203.73 | 42.23 | 36,618 |
| Pernambuco | 195.36 | 46.69 | 207.08 | 43.69 | 85,830 |
| Alagoas | 184.72 | 46.33 | 198.39 | 42.72 | 40,076 |
| Sergipe | 187.75 | 45.00 | 201.01 | 40.25 | 21,995 |
| Bahia | 189.08 | 46.01 | 200.62 | 41.81 | 139,903 |
| Southeast | 219.50 | 47.46 | 232.10 | 46.84 | 798,925 |
| Minas Gerais | 220.74 | 49.22 | 232.37 | 48.86 | 196,836 |
| Espírito Santo | 213.65 | 47.18 | 224.80 | 45.15 | 42,360 |
| Rio de Janeiro | 211.68 | 45.17 | 221.02 | 42.35 | 131,071 |
| São Paulo | 222.44 | 47.15 | 236.83 | 46.91 | 428,658 |
| South | 218.27 | 45.79 | 231.26 | 44.66 | 294,185 |
| Paraná | 221.10 | 44.14 | 236.12 | 44.27 | 115,253 |
| Santa Catarina | 223.09 | 45.75 | 235.89 | 44.68 | 73,067 |
| Rio Grande do Sul | 212.61 | 46.75 | 223.78 | 43.97 | 105,865 |
| Center-West | 212.10 | 45.88 | 221.31 | 44.22 | 172,699 |
| Mato Grosso do Sul | 210.53 | 43.29 | 220.48 | 43.10 | 34,177 |
| Mato Grosso | 205.52 | 46.71 | 215.93 | 44.45 | 37,886 |
| Goiás | 212.69 | 46.42 | 221.14 | 44.32 | 71,166 |
| Distrito Federal | 219.88 | 44.99 | 228.68 | 43.88 | 29,470 |

as well, but the highest score goes to the state of Paraná. This trend indicates some Brazilian characteristics that are recurrent throughout several education evaluations.

The main analysis proposed by this paper is related to the inequality of opportunity, and its decomposition, within Brazilian states. As we can see in Table 2, the Brazilian inequality index for Mathematics is 0.143, of an index that varies from 0 to 1. This table shows that 22 states have indexes lower than the national index, being the lowest, from the State of Ceará (0.049), and the highest from the state of Piauí (0.198), with almost 20% of the inequality explained by circumstance factors apart from the students' efforts.

The group analysis shows that, in average, parenting conditions, such as higher education have almost 41% of influence on the total of the inequality of opportunity index, followed by the ownership of valuables, that represents, in average, 38% of the impact on the inequality of opportunity index, and that municipality characteristics represents, in average, almost 16% of the index. This result shows that the main circumstances that impact students' outcomes are related to their parent level of education, and the ownership of valuables that they possess at home.

The second analysis performed for the 5th-grade student's group is related to their inequalities of opportunities in Portuguese Language. This result, as seen on Table 3, are higher than the ones related to the subject of Mathematics, where it can be inferred that the circumstances of students' lives, may affect more their scores in Portuguese Language tests, than in Mathematics tests.

| Table 2 | |
|---|-----|
| Inequality of opportunity in Mathematics scores for 5th-grade students - 20 | 15. |

| | IOP | Group 1 | Group 2 | Group 3 | Group 4 | Group 5 | Group 6 | Number of observations |
|---------------------|-------|---------|---------|---------|---------|---------|---------|------------------------|
| Brazil | 0.143 | 0.015 | 0.001 | 0.010 | 0.070 | 0.028 | 0.020 | 1,655,990 |
| North | 0.120 | 0.030 | 0.001 | 0.000 | 0.043 | 0.029 | 0.016 | 168,828 |
| Rondônia | 0.103 | 0.023 | 0.001 | 0.001 | 0.044 | 0.024 | 0.010 | 16,379 |
| Acre | 0.106 | 0.027 | 0.001 | 0.000 | 0.030 | 0.031 | 0.016 | 7854 |
| Amazonas | 0.150 | 0.050 | 0.003 | 0.000 | 0.040 | 0.036 | 0.021 | 38,575 |
| Roraima | 0.187 | 0.073 | 0.002 | 0.000 | 0.036 | 0.048 | 0.027 | 5098 |
| Pará | 0.085 | 0.018 | 0.001 | 0.000 | 0.034 | 0.021 | 0.012 | 77,144 |
| Amapá | 0.081 | 0.015 | 0.002 | 0.000 | 0.028 | 0.023 | 0.014 | 8008 |
| Tocantins | 0.158 | 0.042 | 0.000 | 0.000 | 0.058 | 0.039 | 0.019 | 15,768 |
| Northeast | 0.095 | 0.015 | 0.001 | 0.000 | 0.039 | 0.026 | 0.014 | 451,857 |
| Maranhão | 0.123 | 0.027 | 0.000 | 0.000 | 0.049 | 0.028 | 0.017 | 57,592 |
| Piauí | 0.198 | 0.108 | 0.000 | 0.000 | 0.037 | 0.026 | 0.027 | 26,740 |
| Ceará | 0.049 | 0.009 | 0.001 | 0.001 | 0.015 | 0.017 | 0.007 | 71,154 |
| Rio Grande do Norte | 0.133 | 0.023 | 0.000 | 0.000 | 0.054 | 0.032 | 0.023 | 31,976 |
| Paraíba | 0.136 | 0.017 | 0.004 | 0.002 | 0.046 | 0.043 | 0.024 | 30,990 |
| Pernambuco | 0.092 | 0.008 | 0.000 | 0.001 | 0.038 | 0.031 | 0.013 | 76,178 |
| Alagoas | 0.097 | 0.011 | 0.001 | 0.000 | 0.035 | 0.029 | 0.021 | 31,464 |
| Sergipe | 0.124 | 0.021 | 0.003 | 0.000 | 0.051 | 0.029 | 0.020 | 18,831 |
| Bahia | 0.112 | 0.019 | 0.002 | 0.001 | 0.047 | 0.026 | 0.017 | 106,927 |
| Southeast | 0.093 | 0.003 | 0.001 | 0.008 | 0.040 | 0.025 | 0.015 | 659,727 |
| Minas Gerais | 0.144 | 0.014 | 0.000 | 0.007 | 0.060 | 0.038 | 0.025 | 136,236 |
| Espírito Santo | 0.108 | 0.003 | 0.002 | 0.009 | 0.047 | 0.029 | 0.017 | 31,677 |
| Rio de Janeiro | 0.086 | 0.022 | 0.004 | 0.008 | 0.026 | 0.015 | 0.010 | 129,450 |
| São Paulo | 0.083 | 0.003 | 0.002 | 0.007 | 0.035 | 0.023 | 0.014 | 362,363 |
| South | 0.113 | 0.004 | 0.004 | 0.014 | 0.045 | 0.028 | 0.018 | 239,949 |
| Paraná | 0.099 | 0.010 | 0.004 | 0.007 | 0.040 | 0.025 | 0.013 | 91,788 |
| Santa Catarina | 0.122 | 0.002 | 0.003 | 0.016 | 0.050 | 0.030 | 0.021 | 60,448 |
| Rio Grande do Sul | 0.129 | 0.002 | 0.004 | 0.025 | 0.044 | 0.032 | 0.022 | 87,711 |
| Center-West | 0.103 | 0.004 | 0.003 | 0.003 | 0.047 | 0.030 | 0.016 | 135,629 |
| Mato Grosso do Sul | 0.101 | 0.004 | 0.004 | 0.006 | 0.044 | 0.028 | 0.016 | 25,837 |
| Mato Grosso | 0.105 | 0.001 | 0.001 | 0.004 | 0.053 | 0.032 | 0.014 | 29,611 |
| Goiás | 0.096 | 0.003 | 0.002 | 0.002 | 0.044 | 0.030 | 0.013 | 55,658 |
| Distrito Federal | 0.105 | 0.002 | 0.006 | 0.003 | 0.036 | 0.029 | 0.029 | 24,521 |

The Brazilian inequality of opportunity index for Portuguese Language, in 2015, is 0.153, but the analysis also shows that even though the scores are different, we still have 22 states that remain below the Brazilian index, showing that these states are consistent with remaining below the national threshold of inequality of opportunity in Education. The states that occupy the lowest and highest inequality of opportunity indexes are the same both for Mathematics and Portuguese Language scores, the lowest being the state of Ceará (0.066) and the highest being the state of Piauí (0.237).

Although, the distribution of the circumstance of components that make up the inequality of opportunity index differs from the previous analysis, results show that parental characteristics represent, in average, almost 39% of the index, ownership of valuables, in average, represent 32.5%, and municipality characteristics represent, in average, almost 15% of the index.

These results refer only to students enrolled in 5th-grade, in the year 2015. We also highlight that the analysis gathers students from both, public and private schools, since the choice of a students' school is not defined only by circumstances, as well may be defined by its own effort.

Since both interpretations (Mathematics and Portuguese Language) show that, for 5th-grade students, the two states that hold the highest and the lowest inequality of opportunity indexes, are located in the same region, we performed an exercise, to understand how the Northeast inequality of opportunity index would change if we removed the state of Ceará, which held the lowest index of the whole country.

Table 3 Inequality of opportunity in Portuguese language scores for 5th-grade students – 2015.

| | IOP | Group 1 | Group 2 | Group 3 | Group 4 | Group 5 | Group 6 | Number of observations |
|---------------------|-------|---------|---------|---------|---------|---------|---------|------------------------|
| Brazil | 0.153 | 0.017 | 0.012 | 0.009 | 0.064 | 0.030 | 0.021 | 1,655,990 |
| North | 0.153 | 0.046 | 0.011 | 0.000 | 0.044 | 0.032 | 0.020 | 168,828 |
| Rondônia | 0.120 | 0.020 | 0.019 | 0.001 | 0.042 | 0.024 | 0.014 | 16,379 |
| Acre | 0.139 | 0.038 | 0.014 | 0.000 | 0.035 | 0.034 | 0.017 | 7854 |
| Amazonas | 0.200 | 0.077 | 0.007 | 0.000 | 0.048 | 0.042 | 0.026 | 38,575 |
| Roraima | 0.197 | 0.069 | 0.012 | 0.000 | 0.037 | 0.051 | 0.027 | 5098 |
| Pará | 0.123 | 0.035 | 0.011 | 0.000 | 0.036 | 0.024 | 0.015 | 77,144 |
| Amapá | 0.118 | 0.023 | 0.013 | 0.001 | 0.031 | 0.029 | 0.022 | 8008 |
| Tocantins | 0.158 | 0.036 | 0.017 | 0.000 | 0.050 | 0.036 | 0.019 | 15,768 |
| Northeast | 0.125 | 0.022 | 0.014 | 0.001 | 0.042 | 0.029 | 0.018 | 451,857 |
| Maranhão | 0.140 | 0.030 | 0.017 | 0.000 | 0.041 | 0.033 | 0.018 | 57,592 |
| Piauí | 0.237 | 0.118 | 0.020 | 0.000 | 0.040 | 0.032 | 0.026 | 26,740 |
| Ceará | 0.066 | 0.004 | 0.012 | 0.002 | 0.017 | 0.019 | 0.013 | 71,154 |
| Rio Grande do Norte | 0.163 | 0.031 | 0.015 | 0.001 | 0.057 | 0.035 | 0.024 | 31,976 |
| Paraíba | 0.140 | 0.015 | 0.009 | 0.002 | 0.048 | 0.041 | 0.026 | 30,990 |
| Pernambuco | 0.136 | 0.013 | 0.017 | 0.003 | 0.050 | 0.033 | 0.019 | 76,178 |
| Alagoas | 0.146 | 0.024 | 0.013 | 0.000 | 0.043 | 0.039 | 0.026 | 31,464 |
| Sergipe | 0.145 | 0.022 | 0.011 | 0.001 | 0.055 | 0.035 | 0.022 | 18,831 |
| Bahia | 0.134 | 0.026 | 0.012 | 0.000 | 0.048 | 0.029 | 0.019 | 106,927 |
| Southeast | 0.102 | 0.002 | 0.013 | 0.009 | 0.038 | 0.025 | 0.016 | 659,727 |
| Minas Gerais | 0.147 | 0.008 | 0.016 | 0.007 | 0.058 | 0.033 | 0.025 | 136,236 |
| Espírito Santo | 0.112 | 0.003 | 0.015 | 0.007 | 0.040 | 0.028 | 0.019 | 31,677 |
| Rio de Janeiro | 0.084 | 0.009 | 0.012 | 0.010 | 0.027 | 0.014 | 0.011 | 129,450 |
| São Paulo | 0.097 | 0.001 | 0.012 | 0.009 | 0.034 | 0.026 | 0.015 | 362,363 |
| South | 0.113 | 0.002 | 0.010 | 0.015 | 0.039 | 0.026 | 0.021 | 239,949 |
| Paraná | 0.101 | 0.006 | 0.010 | 0.009 | 0.038 | 0.022 | 0.015 | 91,788 |
| Santa Catarina | 0.120 | 0.003 | 0.010 | 0.015 | 0.040 | 0.030 | 0.023 | 60,448 |
| Rio Grande do Sul | 0.126 | 0.001 | 0.009 | 0.022 | 0.039 | 0.030 | 0.025 | 87,711 |
| Center-West | 0.108 | 0.007 | 0.012 | 0.004 | 0.043 | 0.030 | 0.014 | 135,629 |
| Mato Grosso do Sul | 0.095 | 0.005 | 0.011 | 0.007 | 0.040 | 0.023 | 0.010 | 25,837 |
| Mato Grosso | 0.116 | 0.001 | 0.014 | 0.004 | 0.048 | 0.034 | 0.016 | 29,611 |
| Goiás | 0.109 | 0.008 | 0.013 | 0.003 | 0.042 | 0.030 | 0.013 | 55,658 |
| Distrito Federal | 0.099 | 0.002 | 0.008 | 0.004 | 0.033 | 0.030 | 0.023 | 24,521 |

This analysis shows that, with all the states, the Northeast region holds an index of 0.095 when analyzing Mathematics inequality, and of 0.125 when analyzing Portuguese Language scores. Having removed the state of Ceará, we found that this index rises to 0.117 in Mathematics and to 0.146 in Portuguese Language.

With regard to the inequality of opportunity by regions, the north and northeast are the ones that present the highest indexes, especially when we do not consider the state of Ceará. These are also recognized as the poorest regions in Brazil.

4.3. 9th-grade students

The results below are related to 9th-grade students, the graphical analysis and the further table show insights about the inequality of opportunity in education in higher grade students. These results also comprehend both subjects tested by SAEB, Portuguese Language and Mathematics.

Analyzing Fig. 2, we can perceive that, it maintained the same thresholds for 5th and 9th-grade, there are visually more states that show a problematic development about inequality of opportunity in education. The analysis of this image reveals that, not only, more states of Brazil are less equal, but also shows that in Mathematics this inequality is extended.

Table 4 corroborates this hypothesis, showing that the Brazilian distribution of educational achievements is uneven throughout the country's states. The descriptive statistics are shown for every Brazilian state and each region. As seen in



Fig. 2. Inequality of opportunity in every Brazilian state - 2015.

Source: prepared by the authors.

5th-grade students, the lowest mean belongs to the state of Maranhão both for Mathematics and Portuguese Language scores. Regarding the highest score, the state of Santa Catarina holds this position also for both Mathematics and Portuguese Language scores.

The subsequent analysis is the development of inequality of opportunity in education, and how it can be decomposed throughout groups. The results below are the findings of this inequality for 9th-grade students both in Mathematics and Portuguese Language scores. As can be seen in Table 5, the inequality of opportunity index, for 9th-grade Brazilian students in Mathematics was 0.152 in 2015. This analysis shows that 22 states have indexes below the Brazilian one, being the highest for the state of Piauí (0.219) and the lowest for the state of Rondônia (0.085).

The group analysis of the Brazilian indicator shows that parental variables, as higher education, represent almost 52% of the inequality of opportunity index, followed by ownership of valuables, which represent almost 28% of the index. Compared to the same results developed for 5th-grade students, we can infer that parental education has a higher influence on students' inequality when their children are older.

The same analysis can be done when we look at Portuguese Language Scores. We can note that eight states have indexes higher than the Brazilian one (0.138), even though the state with higher inequality of opportunity for the year of 2015 remains being Piauí (0.206), and the state with lower inequality index remains being Rondônia (0.100), as can be seen in Table 6.

The group analysis presents characteristics similar to those of other analysis, with parental education representing almost 45% of the inequality index, followed by ownership of valuables which represents 24% of the inequality index and the municipality characteristics.

These results were developed considering scores both in Mathematics and Portuguese Language tests, for students enrolled in 9th-grade, in the year 2015. This analysis is more comparable to the results found in Ferreira and Gignoux (2014), since it gathers almost the same age group as tested by Ferreira and Gignoux in 2014.

The differences rely on a few factors that must be considered, first, as was pointed by the authors in their paper, the PISA test has a coverage problem in a few countries such as Brazil, that may bias the final result. Second, the variables used by the authors contain information that cannot be gathered for Brazil, such as Father's Occupation and possession of arts at home, questions not addressed by SAEB. Excluding this variables, we find that the results are pretty similar to what we found, specially showing the important role performed by the parents education when their children gets older.

| Table 4 |
|--|
| Descriptive statistics for Portuguese language and Mathematics scores for 9th grade students – 2015. |

| | Portuguese la | nguage | Mathematics | | Number of observations |
|---------------------|---------------|--------|-------------|-------|------------------------|
| | Mean | SD | Mean | SD | |
| Brazil | 251.53 | 49.95 | 255.76 | 48.07 | 1,842,034 |
| North | 241.41 | 46.05 | 241.97 | 42.28 | 179,985 |
| Rondônia | 252.15 | 44.81 | 255.22 | 42.12 | 15,546 |
| Acre | 246.28 | 44.17 | 245.94 | 41.47 | 8508 |
| Amazonas | 247.63 | 46.18 | 245.93 | 45.16 | 47,454 |
| Roraima | 234.19 | 47.44 | 237.55 | 41.54 | 5557 |
| Pará | 236.76 | 45.14 | 236.99 | 39.56 | 76,854 |
| Amapá | 231.56 | 45.53 | 233.58 | 38.93 | 8810 |
| Tocantins | 242.81 | 47.74 | 247.40 | 45.08 | 17,256 |
| Northeast | 241.84 | 48.48 | 245.20 | 45.35 | 500,871 |
| Maranhão | 230.93 | 46.30 | 232.10 | 40.46 | 73,629 |
| Piauí | 243.28 | 47.97 | 248.42 | 47.41 | 29,867 |
| Ceará | 255.70 | 47.67 | 256.73 | 48.49 | 81,434 |
| Rio Grande do Norte | 244.23 | 48.45 | 247.83 | 44.72 | 23,363 |
| Paraíba | 240.03 | 49.03 | 244.66 | 45.75 | 31,332 |
| Pernambuco | 244.24 | 47.60 | 248.33 | 45.09 | 85,918 |
| Alagoas | 235.14 | 47.17 | 239.84 | 43.12 | 29,426 |
| Sergipe | 242.66 | 47.89 | 247.52 | 43.06 | 17,583 |
| Bahia | 238.45 | 48.87 | 242.75 | 44.07 | 128,319 |
| Southeast | 257.11 | 51.29 | 262.63 | 49.73 | 803,745 |
| Minas Gerais | 258.63 | 50.67 | 264.64 | 49.19 | 223,616 |
| Espírito Santo | 256.53 | 50.67 | 263.50 | 49.44 | 33,064 |
| Rio de Janeiro | 254.08 | 52.94 | 260.45 | 49.56 | 116,702 |
| São Paulo | 257.37 | 51.06 | 262.29 | 50.02 | 430,363 |
| South | 258.33 | 47.22 | 263.39 | 45.94 | 218,521 |
| Paraná | 254.80 | 47.69 | 260.85 | 46.41 | 99,330 |
| Santa Catarina | 266.60 | 46.10 | 272.69 | 46.12 | 59,538 |
| Rio Grande do Sul | 256.65 | 46.61 | 259.38 | 43.96 | 59,653 |
| Center-West | 256.83 | 48.68 | 260.10 | 47.35 | 138,912 |
| Mato Grosso do Sul | 262.96 | 44.93 | 264.56 | 44.91 | 23,075 |
| Mato Grosso | 242.43 | 49.75 | 247.45 | 45.29 | 32,076 |
| Goiás | 260.99 | 47.20 | 262.96 | 47.97 | 58,808 |
| Distrito Federal | 259.46 | 50.35 | 264.90 | 47.85 | 24,953 |

4.4. Temporal analysis

A further analysis would be to understand how the inequality of opportunity index behaved through time, so we ran the same tests for the years of 2011, 2013 and 2015, the last year with data available. These results can be seen in Fig. 3 for the 5th grade, and in Fig. 4 for the 9th grade. The results are displayed with the Brazilian index, and the unique indexes for each Brazilian region.

As we can perceive, the national index is superior in every regional index, showing that inside each region there are outliers that may not be highlighted, but that push the Brazilian value when joined together. Moreover, we can conclude that the inequality of opportunity index increases until 2013 and decreases in 2015 for Brazil and most of its regions. In other words, this recent drop means that inequality explained by the circumstances vector decreased in the last year of available data.

| Table 5 | |
|---|-----|
| Inequality of opportunity in Mathematics Scores for 9th grade students - 20 | 15. |

| | IOP | Group 1 | Group 2 | Group 3 | Group 4 | Group 5 | Group 6 | Number of observations |
|---------------------|-------|---------|---------|---------|---------|---------|---------|------------------------|
| Brazil | 0.152 | 0.006 | 0.008 | 0.017 | 0.042 | 0.040 | 0.039 | 1,582,491 |
| North | 0.111 | 0.015 | 0.013 | 0.002 | 0.037 | 0.023 | 0.021 | 142,450 |
| Rondônia | 0.085 | 0.001 | 0.016 | 0.005 | 0.020 | 0.021 | 0.020 | 12,658 |
| Acre | 0.105 | 0.009 | 0.012 | 0.000 | 0.035 | 0.025 | 0.024 | 7822 |
| Amazonas | 0.132 | 0.026 | 0.014 | 0.002 | 0.040 | 0.030 | 0.022 | 34,096 |
| Roraima | 0.133 | 0.014 | 0.009 | 0.002 | 0.041 | 0.034 | 0.033 | 4913 |
| Pará | 0.103 | 0.017 | 0.014 | 0.001 | 0.033 | 0.019 | 0.018 | 62,894 |
| Amapá | 0.101 | 0.013 | 0.015 | 0.002 | 0.028 | 0.020 | 0.024 | 6874 |
| Tocantins | 0.138 | 0.028 | 0.010 | 0.003 | 0.036 | 0.031 | 0.030 | 13,189 |
| Northeast | 0.131 | 0.010 | 0.016 | 0.002 | 0.040 | 0.033 | 0.030 | 426,940 |
| Maranhão | 0.135 | 0.013 | 0.013 | 0.000 | 0.052 | 0.030 | 0.027 | 58,550 |
| Piauí | 0.219 | 0.064 | 0.012 | 0.001 | 0.057 | 0.043 | 0.043 | 26,291 |
| Ceará | 0.109 | 0.005 | 0.015 | 0.002 | 0.033 | 0.030 | 0.024 | 71,604 |
| Rio Grande do Norte | 0.130 | 0.006 | 0.020 | 0.001 | 0.034 | 0.041 | 0.028 | 24,675 |
| Paraíba | 0.164 | 0.008 | 0.018 | 0.003 | 0.038 | 0.049 | 0.048 | 26,197 |
| Pernambuco | 0.119 | 0.003 | 0.015 | 0.003 | 0.031 | 0.039 | 0.028 | 73,072 |
| Alagoas | 0.128 | 0.006 | 0.018 | 0.001 | 0.036 | 0.036 | 0.031 | 25,929 |
| Sergipe | 0.146 | 0.020 | 0.025 | 0.001 | 0.035 | 0.031 | 0.033 | 16,690 |
| Bahia | 0.139 | 0.010 | 0.016 | 0.003 | 0.041 | 0.034 | 0.035 | 103,927 |
| Southeast | 0.139 | 0.001 | 0.004 | 0.020 | 0.025 | 0.046 | 0.042 | 700,235 |
| Minas Gerais | 0.130 | 0.004 | 0.003 | 0.014 | 0.034 | 0.040 | 0.036 | 172,172 |
| Espírito Santo | 0.142 | 0.003 | 0.005 | 0.026 | 0.027 | 0.042 | 0.039 | 27,539 |
| Rio de Janeiro | 0.157 | 0.019 | 0.008 | 0.023 | 0.026 | 0.042 | 0.038 | 116,859 |
| São Paulo | 0.161 | 0.007 | 0.004 | 0.023 | 0.025 | 0.052 | 0.050 | 383,663 |
| South | 0.137 | 0.002 | 0.012 | 0.018 | 0.026 | 0.041 | 0.039 | 191,377 |
| Paraná | 0.155 | 0.009 | 0.007 | 0.013 | 0.027 | 0.053 | 0.046 | 86,588 |
| Santa Catarina | 0.135 | 0.001 | 0.010 | 0.015 | 0.027 | 0.043 | 0.039 | 49,286 |
| Rio Grande do Sul | 0.118 | 0.000 | 0.027 | 0.021 | 0.020 | 0.022 | 0.027 | 55,501 |
| Center-West | 0.136 | 0.007 | 0.010 | 0.008 | 0.029 | 0.044 | 0.039 | 121,488 |
| Mato Grosso do Sul | 0.134 | 0.007 | 0.016 | 0.008 | 0.033 | 0.039 | 0.031 | 20,584 |
| Mato Grosso | 0.129 | 0.008 | 0.003 | 0.010 | 0.033 | 0.044 | 0.031 | 27,298 |
| Goiás | 0.134 | 0.010 | 0.014 | 0.004 | 0.024 | 0.042 | 0.039 | 50,531 |
| Distrito Federal | 0.147 | 0.001 | 0.009 | 0.012 | 0.020 | 0.056 | 0.049 | 23,073 |



Fig. 3. Inequality of opportunity in 5th grade evolution 2013–2015.

Source: prepared by the authors.

Table 6 Inequality of opportunity in Portuguese language scores for 9th grade students – 2015.

| | IOP | Group 1 | Group 2 | Group 3 | Group 4 | Group 5 | Group 6 | Number of observations |
|---------------------|-------|---------|---------|---------|---------|---------|---------|------------------------|
| Brazil | 0.138 | 0.005 | 0.024 | 0.014 | 0.033 | 0.031 | 0.031 | 1,582,491 |
| North | 0.116 | 0.017 | 0.023 | 0.002 | 0.031 | 0.023 | 0.020 | 142,450 |
| Rondônia | 0.100 | 0.006 | 0.030 | 0.005 | 0.019 | 0.020 | 0.020 | 12,658 |
| Acre | 0.111 | 0.005 | 0.030 | 0.000 | 0.027 | 0.025 | 0.024 | 7822 |
| Amazonas | 0.134 | 0.030 | 0.011 | 0.002 | 0.042 | 0.029 | 0.020 | 34,096 |
| Roraima | 0.141 | 0.034 | 0.026 | 0.003 | 0.027 | 0.030 | 0.022 | 4913 |
| Pará | 0.104 | 0.013 | 0.026 | 0.001 | 0.025 | 0.021 | 0.018 | 62,894 |
| Amapá | 0.132 | 0.015 | 0.024 | 0.002 | 0.037 | 0.030 | 0.024 | 6874 |
| Tocantins | 0.145 | 0.019 | 0.037 | 0.002 | 0.032 | 0.030 | 0.025 | 13,189 |
| Northeast | 0.123 | 0.011 | 0.017 | 0.003 | 0.037 | 0.027 | 0.028 | 426,940 |
| Maranhão | 0.145 | 0.020 | 0.023 | 0.000 | 0.047 | 0.026 | 0.029 | 58,550 |
| Piauí | 0.206 | 0.067 | 0.020 | 0.001 | 0.054 | 0.031 | 0.032 | 26,291 |
| Ceará | 0.103 | 0.003 | 0.017 | 0.003 | 0.032 | 0.024 | 0.024 | 71,604 |
| Rio Grande do Norte | 0.116 | 0.005 | 0.015 | 0.003 | 0.035 | 0.033 | 0.026 | 24,675 |
| Paraíba | 0.145 | 0.011 | 0.015 | 0.003 | 0.035 | 0.041 | 0.040 | 26,197 |
| Pernambuco | 0.131 | 0.007 | 0.018 | 0.004 | 0.033 | 0.036 | 0.034 | 73,072 |
| Alagoas | 0.135 | 0.010 | 0.017 | 0.001 | 0.040 | 0.036 | 0.032 | 25,929 |
| Sergipe | 0.112 | 0.017 | 0.009 | 0.001 | 0.033 | 0.027 | 0.024 | 16,690 |
| Bahia | 0.118 | 0.007 | 0.019 | 0.002 | 0.036 | 0.025 | 0.029 | 103,927 |
| Southeast | 0.130 | 0.001 | 0.029 | 0.018 | 0.020 | 0.033 | 0.030 | 700,235 |
| Minas Gerais | 0.123 | 0.001 | 0.037 | 0.012 | 0.023 | 0.026 | 0.025 | 172,172 |
| Espírito Santo | 0.110 | 0.001 | 0.027 | 0.017 | 0.013 | 0.027 | 0.024 | 27,539 |
| Rio de Janeiro | 0.114 | 0.011 | 0.022 | 0.015 | 0.018 | 0.027 | 0.021 | 116,859 |
| São Paulo | 0.158 | 0.005 | 0.028 | 0.022 | 0.023 | 0.041 | 0.039 | 383,663 |
| South | 0.130 | 0.002 | 0.028 | 0.017 | 0.022 | 0.029 | 0.032 | 191,377 |
| Paraná | 0.149 | 0.009 | 0.033 | 0.012 | 0.022 | 0.038 | 0.035 | 86,588 |
| Santa Catarina | 0.131 | 0.004 | 0.027 | 0.013 | 0.019 | 0.032 | 0.036 | 49,286 |
| Rio Grande do Sul | 0.101 | 0.001 | 0.022 | 0.020 | 0.018 | 0.017 | 0.023 | 55,501 |
| Center-West | 0.122 | 0.004 | 0.028 | 0.006 | 0.022 | 0.033 | 0.029 | 121,488 |
| Mato Grosso do Sul | 0.115 | 0.007 | 0.020 | 0.008 | 0.021 | 0.030 | 0.027 | 20,584 |
| Mato Grosso | 0.143 | 0.003 | 0.044 | 0.007 | 0.028 | 0.036 | 0.025 | 27,298 |
| Goiás | 0.117 | 0.007 | 0.024 | 0.004 | 0.019 | 0.035 | 0.029 | 50,531 |
| Distrito Federal | 0.111 | 0.000 | 0.024 | 0.004 | 0.013 | 0.036 | 0.033 | 23,073 |



Fig. 4. Figure of inequality of opportunity in 9th grade evolution 2013–2015.

Source: prepared by the authors.

5. Conclusion

The main purpose of this paper was to analyze and to estimate the inequality of educational opportunities in Brazil, its regions and its states, using the methodology proposed by Ferreira and Gignoux (2014). To perform the estimations we used an extensive database that contains information about 5th grade and 9th grade student's achievements as well as their social characteristics.

There are four main findings in the paper. First, more than 15% of the inequality of educational opportunities in Brazil are explained by circumstances unrelated to individual effort. Second, the most relevant circumstances to explain inequality are parental education and socioeconomic status. This is remarkable evidence, since this tends to feed the perverse cycle of inequalities reproduction. Third, the results, in general, indicate that the states belonging to the poorest regions of Brazil (North and Northeast) are also the ones with the highest inequality of opportunity. The exception is the state of Ceará, which has shown recent success in education outcomes. Fourth, after a rise in 2013, the inequality of educational opportunities index has dropped in almost all regions by 2015. The value for 5th grade students, however, is not lower than the one at the beginning of the analysis in 2011.

We believe that this study fills a gap in the inequality of opportunity literature in Brazil for two reasons. First, it provides an index of inequality of educational opportunities by state in Brazil. Second, it highlights the circumstances that contribute most to this inequality. Therefore, many studies may arise, based on the findings of this paper, to explain the different Brazilian realities with regard to school achievements.

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