

Rys, Przemyslaw; Sobolewski, Maciej

Working Paper

Two-sided platforms: Dynamic pricing and multiple equilibria

JRC Digital Economy Working Paper, No. 2020-14

Provided in Cooperation with:

Joint Research Centre (JRC), European Commission

Suggested Citation: Rys, Przemyslaw; Sobolewski, Maciej (2020) : Two-sided platforms: Dynamic pricing and multiple equilibria, JRC Digital Economy Working Paper, No. 2020-14, European Commission, Joint Research Centre (JRC), Seville

This Version is available at:

<https://hdl.handle.net/10419/266520>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



<https://creativecommons.org/licenses/by/4.0/>



JRC TECHNICAL REPORT

JRC Digital Economy Working Paper 2020-14

Two-sided platforms: dynamic pricing and multiple equilibria

Rys, Przemyslaw
Sobolewski, Maciej

2020

This publication is a Technical report by the Joint Research Centre (JRC), the European Commission's science and knowledge service. It aims to provide evidence-based scientific support to the European policymaking process. The scientific output expressed does not imply a policy position of the European Commission. Neither the European Commission nor any person acting on behalf of the Commission is responsible for the use that might be made of this publication. For information on the methodology and quality underlying the data used in this publication for which the source is neither Eurostat nor other Commission services, users should contact the referenced source. The designations employed and the presentation of material on the maps do not imply the expression of any opinion whatsoever on the part of the European Union concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries.

Contact information

Name: European Commission, Joint Research Centre, Digital Economy Unit (JRC/B6)
Address: Edificio Expo, c/ Inca Garcilaso 3, 41092 Seville (Spain)
Email: maciej.sobolewski@ec.europa.eu

EU Science Hub

<https://ec.europa.eu/jrc>

JRC123533

Seville: European Commission, 2020

© European Union, 2020



The reuse policy of the European Commission is implemented by the Commission Decision 2011/833/EU of 12 December 2011 on the reuse of Commission documents (OJ L 330, 14.12.2011, p. 39). Except otherwise noted, the reuse of this document is authorised under the Creative Commons Attribution 4.0 International (CC BY 4.0) licence (<https://creativecommons.org/licenses/by/4.0/>). This means that reuse is allowed provided appropriate credit is given and any changes are indicated. For any use or reproduction of photos or other material that is not owned by the EU, permission must be sought directly from the copyright holders.

All content © European Union, 2020

How to cite this report: Rys, P., Sobolewski, M., *Two-sided platforms: dynamic pricing and multiple equilibria*, European Commission, Seville, 2020, JRC123533.

Contents

- Foreword..... 1
- Acknowledgements 2
- Abstract 3
- 1 Introduction..... 4
- 2 Related literature..... 5
- 3 The model 7
 - 3.1 Basic setup..... 7
 - 3.1.1 Notation..... 8
 - 3.1.2 Demand, profits and equilibrium conditions..... 8
 - 3.1.3 Existence of equilibrium 9
 - 3.2 Dynamisation 10
 - 3.2.1 Evolution process..... 10
 - 3.2.2 Platform’s decision making process 11
- 4 Main results 12
 - 4.1 Dynamic pricing strategies 12
 - 4.1.1 Two-period pricing..... 14
 - 4.1.2 Two-part tariff 15
 - 4.1.3 Continuous price adjustment 16
 - 4.1.4 Asymmetric two-part tariff 17
 - 4.2 Welfare analysis..... 19
 - 4.2.1 Regulated platform..... 20
- 5 Discussion and conclusions..... 20
- Appendix..... 24
- References..... 26
- List of figures and tables 27

Foreword

This study has been conducted within the work programme of Digital Economy Unit of the Joint Research Centre. It is an exploratory and collaborative research on dynamic models for multi-sided markets. It draws on the prior work of the authors, undertaken at the Faculty of Economics of the University of Warsaw.

For other studies within the JRC Working Papers on Digital Economy visit:

<https://ideas.repec.org/s/ipt/decwpa.html>

Acknowledgements

The authors express their gratitude to Nestor Duch-Brown and Bertin Martens from the Joint Research Centre of the European Commission for providing valuable comments and support for this study.

The scientific output and views expressed in this study do not represent the official position of the European Commission, but are solely those of the authors.

Authors

Maciej Sobolewski (PhD) is an industrial economist working at the Joint Research Centre of the European Commission. At the JRC, his research focuses on the regulatory and competition aspects of online platforms and the economic impact of emerging technologies, such as blockchain and artificial intelligence.

Przemyslaw Rys (MA) is a mathematician graduated from the University of Warsaw. He specializes in financial mathematics. Currently he works at Allegro.pl as a credit risk expert.

Abstract

The static model of two sided markets proposed by Rochet and Tirole analyses optimal pricing of a monopolistic platform at the equilibrium point. Their framework implicitly assumes that for each prices set by the platform, the equilibrium number of users on each side will be unique. However, under general conditions, the uniqueness of market equilibrium is not guaranteed. Optimal static prices do not ensure convergence to the preferred full market outcome, as platform may face failure-to-launch or failure-to-grow problems. Hence, to study problems around multiplicity of equilibria, a different framework is required. We propose a dynamic model of monopolistic platform and demonstrate the effects of different dynamic pricing strategies for equilibrium selection and convergence. The main conclusion from the study is that emerging platform can reach the preferred equilibrium by using tariffs with subsidies for early stage users. We give examples of dynamically adjusting tariffs that minimize subsidies. Finally, the dynamic setting reveals a trade-off between the platform profits and social welfare, related to the speed of user base growth.

1 Introduction

Multiple equilibria are a rule rather than exception in the analysis of multi-sided markets. They arise in a coordination game between different sides of the market, whose payoffs are interdependent via indirect network effects. Multiplicity of equilibria introduces indeterminacy that cannot be dealt with in the static framework with constant prices. Thus, static models pick one equilibrium and focus on the analysis of the optimal level and structure of pricing in that point. The problem of equilibrium selection is by-passed in one way or another. For example, rational expectations of users that give unique participation levels or the capacity of a platform to make credible commitments are assumed. Some papers introduce technical regularity conditions which guarantee uniqueness of the equilibrium or claim to focus only on grown-up platforms. Mature platforms have already brought sufficient number of users on board and can leverage positive indirect network effects. Static models ignore that platforms often face a failure-to-grow problem and thus are incapable to reach the full market coverage or other preferred point.

The importance of equilibrium selection has been mostly acknowledged in the context of chicken and egg problem which refers to the imperative of a start-up platform to build a critical mass of users in order to stay in business. [Evans and Schmalensee \(2010\)](#) show that the severity of this constraint depends on the strength of network effects and heterogeneity of users with respect to benefits from participation on the platform. They use a dynamic approach based on time-continuous diffusion of innovation model. In their framework, in some cases it is sufficient to get only very little users on board in order to enter the path towards fully covered equilibrium. In other cases, a new entrant will have to make a huge investment to get away from a zero participation stable equilibrium. An important insight from their work is that the choice of equilibrium may be a relevant problem also for the established platforms. The critical mass logic applies not only to the choice between zero and full participation but equally to low and high participation. Recently [Cabral \(2019\)](#) proposed a discrete time dynamic model which focuses on the evolution of platform's size. In his setting there exists always a unique equilibrium to which a platform converges. Platform's size in this unique equilibrium may be different, including a combination of low and high participation levels on both sides. The size of the installed base on each side in equilibrium depends on the intensity of indirect network effects and the level of membership benefits. A big advantage of dynamic pricing is that it allows to escape from coordination of expectations towards decisions based on current prices and the actual size of the platform. This approach is closer to real-world conditions where platforms deal with large numbers of myopic users and can adjust prices over time.

Dynamic models are well suited for the analysis of failure-to-launch and failure-to-grow phenomena on multi-sided-markets. These concepts offer an explanation of why there are so few successful attempts and many more failures to establish a platform and also why a large installed user base can be an entry blocking factor. Despite its realism, the dynamic approach is less willingly adopted in literature because of its complexity and difficulty to obtain general results. The present paper aims to contribute to the literature on equilibrium selection in multi-sided markets. We propose a continuous time dynamic model of a monopolistic platform which builds on the framework of [Tirole and Rochet \(2006\)](#) and allows for multiple equilibria and dynamic pricing. In our model a platform achieves a specific equilibrium given the initial market conditions and a given momentary pricing scheme composed of interaction and membership fees. We apply time-dependent price schemes. A change of fees leads to a new equilibrium point to which a platform converges. The focus of the paper is on the economic effects and trade-offs which arise under different forms of dynamic pricing as such. Hence we are not solving for profit-maximizing dynamic prices. Doing so would require strong additional assumptions to make optimal control problem tractable. We deliberately decided to avoid it at this stage.

In Rochet and Tirole's model, the regularity conditions which guarantee uniqueness are assumed implicitly. This is a routine practice in static models. However, as the present paper shows even under very simple forms of users heterogeneity the uniqueness of equilibrium is not guaranteed. We argue that multiple equilibria are not a technical anomaly but rather a prevalent feature of multi-sided markets that has far reaching practical implications. First, platform operator has to identify different equilibrium points and deal with the choice of one of them. Second, this choice is constrained by initial conditions. Not all equilibrium points can be reached by a platform

starting with a particular number of users on each side. Third, setting the optimal prices that correspond to the static solution in principle will not allow a platform to reach the most preferred market equilibrium. We formally show that for each momentary price levels there exists at least one equilibrium in participation rates. We provide examples in which there are few or continuum of equilibria. Our paper focuses on the behavior of a platform *before* reaching any non-empty equilibrium point, namely on the equilibrium selection process, in which dynamic pricing plays a key role. By engaging in a dynamic pricing a platform can leapfrog from one equilibrium to another avoiding failure to launch or failure to grow. We allow for pricing strategies to be composed of interaction and membership fees set, possibly asymmetrically, for each side of the market. We start from a *two-period strategy* in which fees are changed once from a subsidy to a positive level. Next, we demonstrate a single *two-part tariff* which automatically adjusts the size of subsidy and eventually turns it into profits at a later stage. Finally, we show a *continuous adjustment strategy* that increases revenues while maintaining a highest possible platform growth. Different economic effects arise from various pricing strategies. For example, a platform may set a subsidy such that the least wanting user chooses to enter. This is an insulation tariff (Weyl, 2010) which ensures a full speed growth towards a desired equilibrium path. However this boosted growth comes at a cost of low or even negative momentary profits. On the other hand, insulation maximizes momentary demand and leads to a maximal consumer surplus. We compare different strategies in terms of these effects. Conceptually our setting can be considered a dynamic extension of a static framework from Tirole and Rochet (2006). We do not restrict heterogeneity of users with regards to the interaction and membership benefits. We propose a reasonable dynamic system that governs the evolution of platform’s user base. Each stationary point has its basin of attraction which defines a set of initial conditions leading to a particular equilibrium. Since our focus is on equilibrium selection, we use exemplary pricing strategies are not necessarily optimal. As already explained, computation of profit maximizing strategies is beyond the scope of the current study.

The rest of the paper is organized as follows: In section 2 we briefly review literature on monopolistic platform operating a two-sided market. In section 3 we outline our dynamic modeling setup and motivate it with some examples of non-uniqueness of equilibria. In section 4 we demonstrate equilibrium selection process with several pricing schemes and discuss their implications for platform’s profits and social welfare. Section 5 concludes.

2 Related literature

Our paper is closely related with the discussion of the coordination problem on multi-sided markets. Multiple equilibria is a rule rather than exception in multi-sided markets, even for relatively simple forms of user heterogeneity. Due to indirect network externalities, for a given prices there may exist several participation levels on both sides that will form an equilibrium. The inherent problem of discrimination between different equilibria has been solved in static models by introducing user expectations or reputation-driven favorable beliefs (Caillaud & Jullien, 2001, 2003). The criticism of expectation approach boils down to limited cognitive capacities of users who cannot be reasonably expected to coordinate their network choices, especially under heterogeneity of preferences and externalities. There have been some attempts in literature to escape from a coordination failure, such as price commitment and design. Hagiu (2006) introduces a sequential-move coordination game in which sellers come before buyers, as is the case in credit cards or video console markets. In such situations a platform needs to coordinate only the sellers. The buyers will adapt accordingly. On the other hand, a platform can choose to credibly commit *ex ante* to the price for the buyers, signaling to the sellers a large *ex-post* participation rate.

Weyl (2010) discusses an alternative approach in which platform can induce participation behavior of consumers by designing a price mechanism. Weyl uses the same participation constraint as Tirole and Rochet (2006) but focuses on a different pricing problem. A platform chooses an optimal allocation of users rather than maximizes profits. The preferred allocation can be induced by charging insulating tariffs. These tariffs guarantee a certain utility level for users and therefore achieve coordination by making a desired behavior a dominant strategy. In particular, by using an insulation tariff a platform may allocate a positive utility to the equilibrium number of users on side A irrespective of the participation of anyone from another side B . For a given number of \widetilde{N}_A users there is one-to-one correspondence between price and the number of users on B side. Hence,

a platform may trigger the equilibrium demand response on that other side by setting a unique value for P_B .¹ Insulation tariffs may indeed help to reach a preferred equilibrium. The problem with insulation tariffs however is that after the platform achieves an equilibrium on side B , the participation on side A will almost certainly change (increase) because of positive network externalities that now will induce even more users. Hence a price adjustment on the insured side will be needed anyway and this is infeasible in a purely static framework. The costs of insuring side A might be prohibitively large so that a platform may be interested in a different pricing strategy. In another paper [White and Weyl \(2016\)](#) formulate a concept of insulated equilibrium in the context of platform competition to address potential coordination failure. Insulated equilibrium is a refined Nash equilibrium in which each user has a dominant strategy to enter. Thus the concept rules out equilibria that are supported by partial participation rates. The way the insulation tariffs can be implemented depends on the details of user heterogeneity with regards to valuation of network effects, membership and interaction benefits. In any case, as argued above, the concept of insulation is not compatible with a static framework. We show how it could be implemented in practice when a platform can explicitly use dynamic prices. A somehow similar concept to insulation tariffs, put forward in a context of chicken and egg problem is "divide and conquer" pricing ([Evans, 2003](#)). This strategy is widely observed in practice and consists of platform subsidizing the more price sensitive side in order to incentivize entry on the profit side. Depending on the parameters, sometimes it is sufficient to offer arbitrarily low subsidy to move a platform from an empty (0,0) equilibrium and reach a trajectory towards stable full market equilibrium. Hence, the objective of this strategy is to compensate early adopters for the low utility and then let the market grow organically. The effects of price adjustments can only be studied in a dynamic framework, such as the one introduced in this paper.

The issue of multiplicity of equilibria seems to be inherent to the multi-sided markets. It is persistent regardless of the maturity of a platform: new entrant may fail to launch, while an established platform may fail to grow. The problem of equilibrium selection cannot be satisfactorily solved in static models. Dynamic models introduce an explicit possibility for the platform to change prices and manage the evolution of their user base. The problem of expectations is ruled out by assuming myopic users who decide about participation based on the current prices and size of the platform, as for example in [Evans and Schmalensee \(2010\)](#) or [Cabral \(2019\)](#). Such setting can be realistic when the costs of reversing participation is negligible, as is often the case with online platforms. The dynamic model developed in this paper elaborates on the selection of equilibrium by using time dependent strategies. We show some examples of insulation which ensures maximal momentary demand and adjusts fees to a changing participation constraint on each side. Maintaining a maximal growth of the market is socially desirable but usually will not result in the highest profits for the platform. On the other hand, it turns out that some forms of subsidization beyond critical mass may trigger higher profits than if a platform grows organically.

Theoretical foundations of multi-sided markets have been laid down in the seminal paper of [Rochet and Tirole \(2003\)](#). This paper analyzes a monopolistic intermediary in the specific context of card payment market, where both merchants and cardholders have heterogeneous interaction benefits. In this particular application 'quasi' demand of one side depends only on the price set for that side (page 995). Hence, there is no coordination game involved. Rochet and Tirole show that in the unique equilibrium, total price set by the platform is given by the markup formula containing the sum of elasticities. The price structure is defined by the ratio of demand elasticities on both sides. The generic model of monopolistic platform has been laid down in [Tirole and Rochet \(2006\)](#) and [Armstrong \(2006\)](#). Both papers differ with regards to assumptions about user heterogeneity in interaction and membership benefits. For the perfect homogeneous case, Armstrong shows that in equilibrium the price for each side decreases with elasticity of demand and is corrected downwards for the external benefit this side generates to the users on a different side. This result implies that a platform might set a price for one side, say A below marginal cost, possibly offering a subsidy. This occurs for example in dating or nightclub platforms, if group A (women) has high elasticity of demand and generates large external benefit to the second group B (men). Armstrong avoids multiplicity of equilibria in his model by assuming monotone in utility demand function. Tirole and Rochet develop equilibrium analysis for the case of heterogeneous

¹Specifically, Weyl shows that the function $N_B(P_B, \widetilde{N}_A)$ is reversible, leading to a unique set of prices $P^{A,B}(\widetilde{N}_A, \widetilde{N}_B)$, profits and welfare that is consistent with the equilibrium $(\widetilde{N}_A, \widetilde{N}_B)$, see [Weyl \(2010, p. 1648\)](#).

interaction benefits and heterogeneous membership benefits. Participation constraint accounts for cross-side externalities proportional to the number of users. Importantly, for the derivation of the main result they implicitly assume that the resulting system of demand equations has a unique solution characterizing memberships as a function of prices.² This assumption in turn leads to the correspondence between prices and profits and ensures a unique equilibrium.

We argue that the assumption about uniqueness of positive participation equilibrium is quite restrictive and unrealistic. We demonstrate on a few examples that, for a given momentary prices, multiple equilibria exist. Platform's choice of a given equilibrium point is constrained by its current size (initial conditions). The major implication of this observation is that it might be impossible to reach the full market equilibrium point, even if a platform sets optimal static prices. This point of departure motivates our analysis of how a platform may use dynamic pricing to reach the full market equilibrium. We assume that users are heterogeneous in both dimensions (membership and interaction benefits). We also provide some insights into the interplay between social welfare and platform's profits and the speed of growth.

3 The model

Our framework builds on [Tirole and Rochet \(2006\)](#) and can be considered a dynamic extension of their original model. In the static formulation platform sets prices only once and the resulting number of users on each side, given by demand function is fixed. The impact of network externalities on i -side user's utility is proportional to the number of users on side j . The total number of interactions on a platform equals to a product of number of users: $N_i \cdot N_j$. Because the number of users on each side is invariable in the static model, so is the number of interactions. This allows to reduce dimensionality of the price vector by including a fixed fee into the per interaction unit price. In the dynamic variant, user base changes over time and hence all four prices are relevant and useful for constructing pricing strategies.

3.1 Basic setup

A monopolist operates a two-sided platform on which two different groups of users i, j interact. In a specific context of market places, these groups will be called buyers and sellers. Users on each side derive intrinsic membership or participation benefits B_i from subscribing to the platform and also interaction benefits b_i from concluding a single transaction with a person on other side of the market. Interaction benefits can also be interpreted as a valuation of indirect network effect. The market place operator charges i side users with a variable fee p_i for a unit of interaction and a fixed fee P_i for the membership. These fees can be negative, which implies that users are (temporarily) subsidized by the platform. In reality, subsidization can be realized in money, for example by providing rebates or promotional codes, or any equivalent in-kind benefit, such as access to a premium service or additional functionality.

With cross-side network effects, the utility of a particular user on side i from using a platform depends not only on his intrinsic benefits and the level of fees set by the operator but also on the number of participants on the other side: N_j . In line with [Evans and Schmalensee \(2010\)](#) we assume that users are myopic and the costs of reversing participation decision are negligible. Consequently, a user can step in or out from the platform depending on a momentary net utility being non-negative. His decision can change at any time for example in reaction to a change in the momentary price or platform's size. To maximize profits, the platform operator has to balance the level of fees with the incentives for participation on each side.

We normalize the total number of users on the i -side to one and denote it by $N_i \in [0, 1]$. Additionally, we assume that the utility function is quasi-linear in money, which means that additional unit of money increases utility always by the same amount. This assumption introduces common measurement scale for utilities of different users, allowing for a direct comparison of utilities, and facilitates welfare analysis.

²Literary they write: "Under regularity conditions, the system (4) has a unique solution characterizing memberships N_B and N_S as functions of (p_B, p_S) ", see [Tirole and Rochet \(2006, p. 653\)](#).

We introduce continuous time horizon $[0, T]$. Prices are momentary, and hence they can be changed at any point in time. User is charged for a total number of interactions available to him at any given point. Users can enter and leave the platform at any time they wish based on the sign of the net momentary utility from participating on a platform.

3.1.1 Notation

Below we introduce a full notation of variables in the model:

B, S – buyers and sellers, the two sides of market, interacting on the platform. For general considerations one side is denoted by i the and respectively the other side by j .

N_i – a total normalized number of users on the side i . In fact it is a percentage share of i -side users on the platform in the set of all potential users, but will be called "number of users".

C_i – a fixed cost to a platform of getting on board a single user on i -side.

c – a constant marginal cost to a platform for providing an interaction between users.

B_i – i -side user's utility from participation/membership, given by a constant or a draw from a continuous random variable.

b_i – i -side user's utility from a unit of interaction given by a constant or a draw from a continuous random variable.

p_i – a per interaction fee paid by the user on i -side.

P_i – a fee for the membership paid by the user on i -side.

N_i^D – a normalized number of users from i -side, who are willing to participate on a platform for given fees and a number of users on the other side.

t – a point from continuous time horizon $[0, T]$.

U_i – a net utility, i -side user gains in a specified point of time t .

π – momentary platform's profit in a specified point of time t .

Π – total aggregated platform's profit in the entire time horizon.

W – total aggregated social welfare in the entire time horizon.

All variables including number of users, costs, benefits and prices, except Π and W , represent *momentary* values i.e values in the specified point of time. For simplicity unit cost incurred by the platform as well as interaction and membership benefits of a particular user are *stationary*, i.e they do not change over time. A quadruple (p_B, p_S, P_B, P_S) is a price scheme set by a platform in any given moment of time. A dynamic pricing strategy is a mapping of price schemes into time.

3.1.2 Demand, profit and equilibrium

A momentary utility of an i -side user is equal to:

$$U_i = (b_i - p_i)N_j + B_i - P_i \quad (1)$$

Normalizing the utility of an outside option to zero, a user will participate on a platform in a given moment if and only if the net utility is non-negative. Users with zero utility from participation are indifferent, but we will assume that they enter the platform.³ The momentary demand function of i -side represents a number of users willing to participate on the platform on side i for a given number of j -side users and can be presented as follows:

$$N_i^D(N_j, p_i, P_i) = Pr(U_i \geq 0) = Pr((b_i - p_i)N_j + B_i - P_i \geq 0). \quad (2)$$

³Due to the continuity of the considered random variables a set of zero-utility users has measure zero: $Pr(U_i = 0) = 0$.

We note that the above expression is bounded by 0 and 1, since it is a probability. Momentary demand N_i^D depends not only on prices p_i, P_i and the distributions of benefits B_i, b_i , but also on the total number of users who joined the platform on the other side. For a specified price scheme (p_B, p_S, P_B, P_S) the function $N_i^D(N_j, p_i, P_i)$ will be denoted for simplicity by $N_i^D(N_j)$. Momentary profit is given by:

$$\pi = (p_B + p_S - c)N_B N_S + (P_B - C_B)N_B - (P_S - C_S)N_S. \quad (3)$$

A momentary platform's profit depends on the total numbers of users on both sides (N_B, N_S) and momentary unit prices (p_B, p_S, P_B, P_S) chosen by the platform. Note that because the number of users on each side is momentary, the number of transactions is momentary as well.

A pair N_S, N_B forms an equilibrium if and only if it satisfies conditions given by the following system:

$$\begin{cases} N_B = N_B^D(N_S), \\ N_S = N_S^D(N_B). \end{cases} \quad (4)$$

Each solution to system (4) is an equilibrium in the Nash sense. For a given momentary prices nobody new will enter the platform and none of the current users will want to exit. Hence, if the platform already is in equilibrium and keeps momentary prices constant, its size and momentary profits will be stationary in time. The dynamic approach allows us to analyze the process of getting into a particular equilibrium point from a given initial size of the platform, including the starting point with no customers. Moreover, because a platform may freely change momentary prices, it can move from one equilibrium point to another.

3.1.3 Existence of equilibrium

In this subsection we show that at least one equilibrium pair of N_S, N_B exists for any set of momentary prices. Let us start with definition of a relevant strategic game, based on the static model of Rochet-Tirole. For a fixed price scheme, the utility of each user depends only on the total number of users on the other side. Thus, it is reasonable to model a two-sided market as the game between two players $i \in \{B, S\}$, representing the two sides. These representative players have strategy spaces N_i , formed from shares of users who enter the platform. The payoff of each player is the sum of payoffs of all platform participants on the respective side, represented by the integral $\text{Payoff}(N_B, N_S)_i = \int_{N_i} \pi(B_i, b_i, N_j) d(B_i, b_i)$, where N_i denotes the set of N_i users with the highest benefit from using platform among all users of i side.

Definition 1. For the specified price scheme (p_B, p_S, P_B, P_S) let the strategic game G between the buyers and sellers be defined as follows:

$$G_{(p_B, p_S, P_B, P_S)} = \langle \{B, S\}, [0, 1] \times [0, 1], (\text{Payoff}_B(N_B, N_S), \text{Payoff}_S(N_B, N_S)) \rangle$$

The demand functions from expression (2) are the best responses of the players, which implies that Nash equilibrium is achieved whenever the system (4) is satisfied.

Theorem 1. For each (p_B, p_S, P_B, P_S) in a strategic game $G_{(p_B, p_S, P_B, P_S)}$ there exists at least one Nash equilibrium (N_B, N_S) .

Proof. See appendix A. □

System (4) possibly has multiple solutions, which collapses predictive power of static models. The existence of ambiguous solutions means that for the fixed price scheme multiple equilibria exist, possibly with different profit levels. Such situation, with best response functions $N_B^D(N_S), N_S^D(N_B)$ intersecting three times in the cube $[0, 1] \times [0, 1]$, is illustrated by the example below.

Let $P_B = P_S = 0, b_B = b_S = 2, p_B = p_S = 1$ and $B_B \sim \text{Unif}([-1, 0]), B_S \sim \mathcal{N}(-0.5, 0.3)$. The interaction benefits b_B, b_S are constant, therefore the demand functions have relatively simple formulas:

$$N_B^D(N_S) = \Pr((b_B - p_B)N_S + B_B - P_B \geq 0) = \Pr(N_S \geq -B_B) = F_{-B_B}(N_S)$$

$$N_S^D(N_B) = \Pr((b_S - p_S)N_B + B_S - P_S \geq 0) = \Pr(N_B \geq -B_S) = F_{-B_S}(N_B)$$

The demand functions are cumulative distribution functions of random variables $-B_B$ and $-B_S$. As shown in Fig.1 both functions intersect three times at points $(0.082, 0.082)$, $(0.5, 0.5)$, $(0.918, 0.918)$, indicating three Nash equilibria.

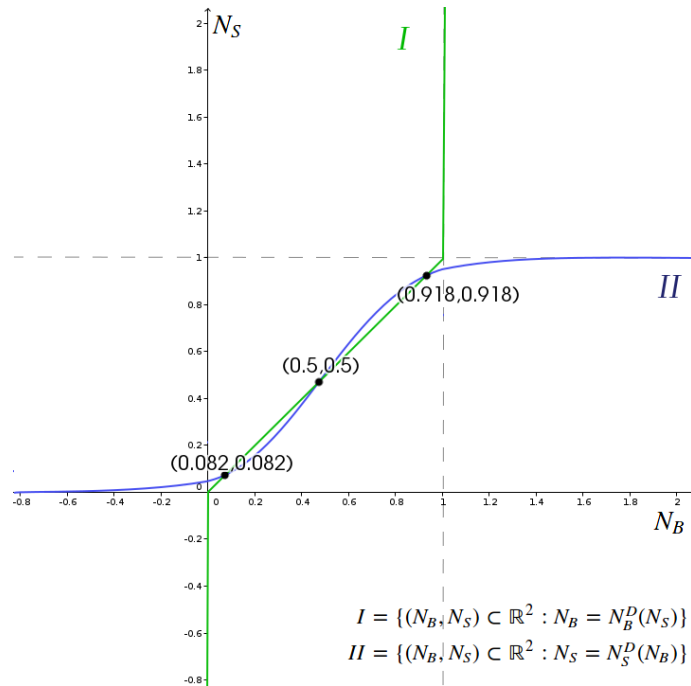


Figure 1: Demand functions $N_B^D(N_S), N_S^D(N_B)$, such that three equilibria exist.

There can be infinite equilibria even in simple conditions. Let us consider the previous example, but with both B_S, B_B from a uniform, independent distributions. Let $P_B = P_S = 0, b_B = b_S = 2, p_B = p_S = 1$ and $B_B \sim Unif([-1, 0]), B_S \sim Unif([-1, 0])$. In this case, both demand functions are given by $N_S^D(N_B) = N_B$ and $N_B^D(N_S) = N_S$. Therefore the system (4) has the following form:

$$\begin{cases} N_B = N_S, \\ N_S = N_B. \end{cases} \quad (5)$$

In consequence, equal market participation rates: $N_B = N_S$ form Nash equilibria. This case is illustrated in the Fig.2. Multiple equilibria can also arise when users are perfectly homogeneous in benefits.

3.2 Dynamisation

As noted in section 2 existing literature predominantly focuses on the optimal pricing at a single equilibrium point. We argue that multi-sided markets can have multiple equilibria, and hence focus on equilibrium selection and convergence process. Equilibrium selection can be modeled with dynamic prices which allow to account for an evolution of customer base and platform profits over time.

3.2.1 Evolution process

Let assume that a price scheme (p_B, p_S, P_B, P_S) is specified. Let $N_B(t), N_S(t)$ denote the normalized number of users on respective side B and S , who participate on the platform in a specified point of time t . We need to specify how platform's size adjusts over time. For that purpose we assume that participation on each side is ruled by the following system of differential equations:

$$\begin{cases} \dot{N}_S(t) = \alpha_S(N_S^D(N_B(t), p_B, p_S, P_B, P_S) - N_S(t)), \\ \dot{N}_B(t) = \alpha_B(N_B^D(N_S(t), p_B, p_S, P_B, P_S) - N_B(t)). \end{cases} \quad (6)$$

All stationary points of the above system are pairs of points (N_B, N_S) , which meet conditions $N_B^D = N_B$ and $N_S^D = N_S$. Hence, they correspond to Nash equilibria. From (6) it follows that

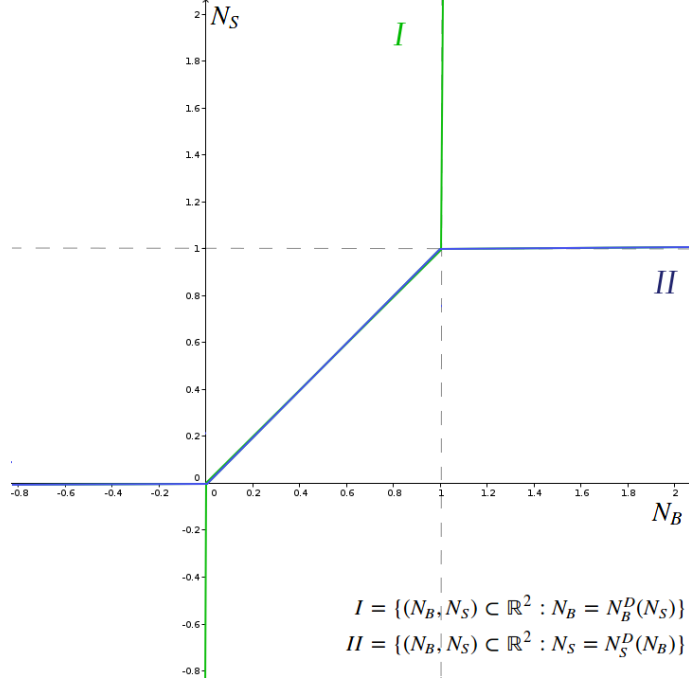


Figure 2: Demand functions $N_B^D(N_S), N_S^D(N_B)$, such that infinite equilibria exist.

N_i increases, if it is below and decreases if it is above demand N_i^D . The type of dynamics is compatible with a basic economic intuitions and rationality of the users. If N_i is below the current demand level then there are i -side users, who can increase (maximize) their utility by joining the platform. Similarly, if $N_i^D < N_i$ then there are users on the platform, who receive a negative net utility. Such users are going to leave the platform. Let us point out that a speed of changes in membership is proportional to the demand gap - a difference between the number of type i users who are willing to use the platform and the number of i -users, using the platform in a specific moment. The proportionality ratio, denoted by α_i , can be interpreted as a *speed* of membership adjustment to changes in the demand.

According to the system (6) and the previous considerations, derivatives \dot{N}_B, \dot{N}_S converge to zero, so N_S, N_B converge to a stationary point - the market equilibrium. The equilibrium which a market converges to depends on the initial conditions set in the beginning of the time horizon, i.e. the values of $N_S(0), N_B(0)$. The convergence process according to initial conditions can be illustrated by the phase portrait, such as the one in Fig.3. It is important to note that each phase portrait is drawn for a specific vector of prices. If a platform changes momentary price levels, the location of stationary points change.

3.2.2 Platform's decision-making process

We introduce a *pricing strategy* function which assigns price schemes to the specified points of time: $PS : t \mapsto (p_B(t), p_S(t), P_B(t), P_S(t))$, where $t \in [0, T]$, $(p_B(t), p_S(t), P_B(t), P_S(t)) \subset \mathbb{R}^4$ and $[0, T] \subset \mathbb{R}$. Let us point out that this definition encompasses fixed fees and subsidies (negative fees). We assume that at the beginning of the time horizon no one is on the platform, which implies that initial conditions of the system (6) are $N_B(0) = N_S(0) = 0$. Momentary profit in a specified point of time t is given in equation (3) and will be denoted by π . For a given pricing strategy the platform earns total profits in the period $[0, T]$, discounted by the continuous interest rate r . Total profits will be written as:

$$\Pi(p_B, p_S, P_B, P_S) = \int_0^T \pi(p_B(t), p_S(t), P_B(t), P_S(t)) e^{-rt} dt.$$

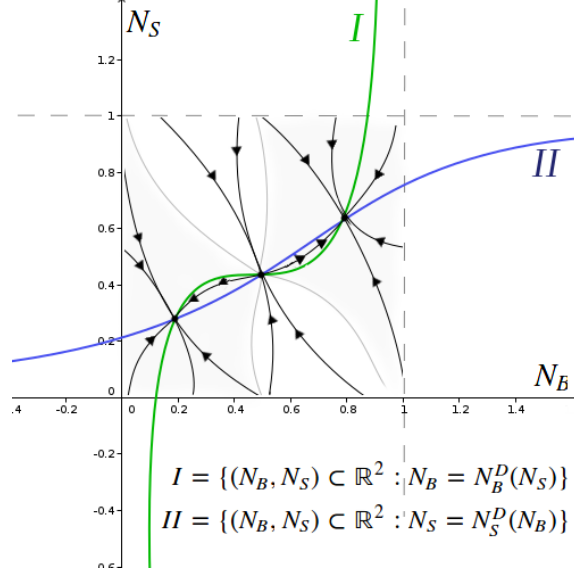


Figure 3: Phase portrait of the (6) system with stationary points and graphs of functions $N_B^D(N_S), N_S^D(N_B)$.

After substitution of π according to equation (3) this is equivalent to:

$$\Pi(p_B, p_S, P_B, P_S) = \int_0^T ((p_B(t) + p_S(t) - c)N_B N_S + (P_B(t) - C_B)N_B + (P_S(t) - C_S)N_S) e^{-rt} dt \quad (7)$$

where N_B, N_S are functions of the price scheme and time in accordance with equation (2) and system (6) for the specified initial conditions. The arguments of function Π - pricing strategies are integrable functions.

A momentary social welfare is the sum of platform's momentary profit and consumer surpluses on both sides:

$$\pi + \int_{\mathcal{N}_B} U_B(\xi) d\xi + \int_{\mathcal{N}_S} U_S(\xi) d\xi \quad (8)$$

where $\mathcal{N}_i := \{\xi \in [0, 1]^2 : U_i(\xi) \geq 0\}$ and $\xi = (\xi_1, \xi_2)$ specifies values of the random variables b_i, B_i i.e. $U_i(\xi) = (b_i(\xi_1) - p_i)N_j + B_i(\xi_2) - P_i$. \mathcal{N}_i stands for the set of i -side users, using platform at present. The measure of set \mathcal{N}_i is equal to a normalized number of users on side i - N_i .

4 Main results

In this subsection, we illustrate how the model operates under specified preference parameters and initial conditions. Consider a startup platform that aims to build a critical mass in order to start developing itself organically via leveraging network externalities. We will present several pricing strategies, which work in this situation by subsidizing users in the initial period of platform's activity. All considered strategies allow a platform to make a transition from a failure to launch (empty) equilibrium to the full market equilibrium point $N_S = N_B = 1$. The strategies differ in construction, costs of subsidization paid by the platform and convergence time to the equilibrium, but all generate equal momentary profit in that point. The objective of the analysis is to compare the performance of strategies with regards to the aggregated profits and social welfare.

4.1 Dynamic pricing strategies

We start from specifying the common market conditions. Consumers are heterogeneous with regards to interaction and membership fees. The heterogeneity is captured by the two uniform

distributions: $B_B, B_S \sim Unif([-1, 0])$, $b_B, b_S \sim Unif([10, 20])$ We assume that fixed membership benefits from joining a platform are negative, because users must devote time to register. Marginal costs of providing the membership and a unit of interaction are zero. A membership adjustment speed is set to one and future profits are not discounted: $\alpha_B = \alpha_S = 1, r = 0$ and $C_B = C_S = c = 0$. A time horizon in which the evolution of the market will be analyzed is set to five: $T = 5$.

For the benchmark we choose a 'static' pricing strategy $(p_B, p_S, P_B, P_S) = (9, 9, 0, 0)$. According to this strategy a platform sets positive and constant interaction fees and no membership fees. For the defined market conditions, the benchmark pricing strategy produces two equilibria as shown in Fig.4.

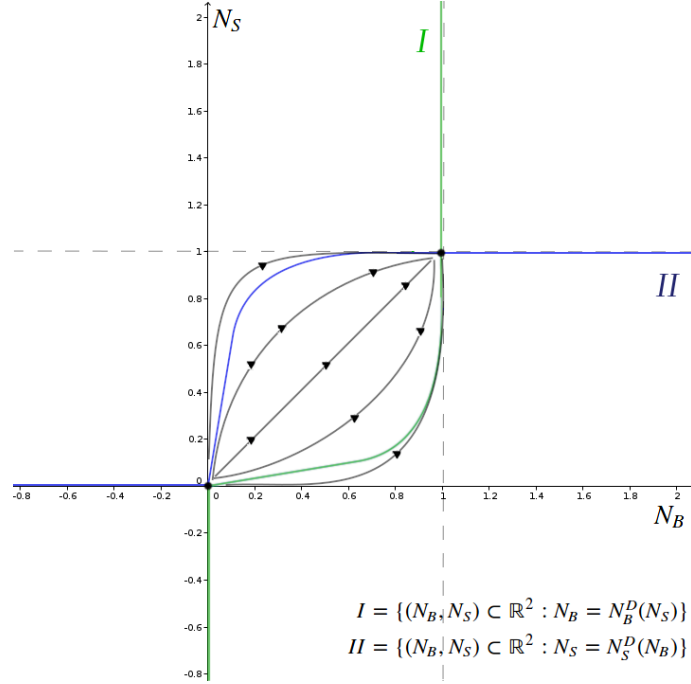


Figure 4: Phase portrait and equilibria for the benchmark unit prices.

We verify these two equilibria. First, because almost certainly $B_B, B_S < 0$, then the pair $(N_B, N_S) = (0, 0)$ is an equilibrium for every single pricing strategy, composed only of variable fees. To see this we check the demand function: $N_i^D(0) = Pr(B_i - P_i \geq 0) = Pr(B_i \geq 0) = 0$. The second equilibrium is the point $(N_B, N_S) = (1, 1)$. To show this we check that for lower bounds of variables b_i, B_i , respectively -1 and 10 , the system (4) is satisfied by $(N_B, N_S) = (1, 1)$:

$$1 \geq N_i^D(1) = Pr((b_B - p_B) + B_B \geq 0) \geq Pr((10 - 9)1 - 1 \geq 0) = Pr(0 \geq 0) = 1.$$

Let us point out that if a platform uses price scheme composed of only variable fees $(p_B, p_S, 0, 0)$ in the entire time horizon $[0, 5]$, then always $N_B = N_S = 0$. Since nobody enters, a momentary platform profit is constantly equal to zero:

$$\begin{cases} \dot{N}_B = 1(N_B^D(0) - 0) = 0, \\ \dot{N}_S = 1(N_S^D(0) - 0) = 0. \end{cases}$$

Consequently, the total platform's profits are also equal to zero. In particular the above remark holds for the price scheme $(p_B, p_S, P_B, P_S) = (9, 9, 0, 0)$. Clearly, in a static framework with only fixed unit prices, a platform would fail to launch under the specified market conditions. In what follows we demonstrate four different types of pricing strategies which can be used to depart from the initial 'empty market' equilibrium point $(N_B, N_S) = (0, 0)$ and move to a 'full market' equilibrium $(N_B, N_S) = (1, 1)$. In all four cases the platform earns the same momentary profit in the full equilibrium as for the benchmark pricing strategy $(9, 9, 0, 0)$. These strategies differ in how fast they attract new users and in the level of total profits earned by the platform.

4.1.1 Two-period pricing

As shown above, the static pricing benchmark with only variable fees is ineffective when a platform initially does not have users. It fails to collect a critical mass of users who start to generate positive network externality. An obvious way of boost early membership is to provide time-limited subsidies. This is the main idea behind a *two-period pricing strategy*. In the first, shorter period this strategy offers a subsidy in form of a negative membership fee. In the second period, it charges everybody with positive interaction fees.

Let us consider a two-period pricing strategy, composed of the pricing scheme $(0, 0, -1, -1)$ in the initial period $[0, 0.15)$ and the pricing scheme $(9, 9, 0, 0)$ in the subsequent period $[0.15, 5]$. In the period $[0, 0.15)$ a momentary demand on each side is constantly equal to 1, independently of the number of users on the other side. This follows from a substitution of random variables B_i and b_i with their lower bound values given by -1 and 0 :

$$1 \geq N_i^D(N_j) = Pr((b_i - p_i)N_j - (P_i) + B_i \geq 0) = Pr((b_i)N_j - (-1) + B_i \geq 0) = Pr(0 \geq 0) = 1.$$

Hence, in the initial period the system (6) has two autonomic linear equations:

$$\begin{cases} \dot{N}_B = 1 - N_B, \\ \dot{N}_S = 1 - N_S. \end{cases}$$

The general solution to this system is a pair of functions $N_B = N_S = C_1 e^{-t} + 1$. After taking into account initial conditions we obtain the specific solution in the initial period:

$$\begin{cases} N_B(t) = -e^{-t} + 1, \\ N_S(t) = -e^{-t} + 1. \end{cases} \quad (9)$$

In the moment $t = 0.15$ the platform changes a price scheme to $(9, 9, 0, 0)$. Momentary demand is no longer equal to one, but remains symmetric. Because distributions of b_i and B_i are uniform and independent, the demand formulas can be calculated as the area of the specified part of rectangle divided by the area of the whole rectangle. Both cases are presented in Fig.5.

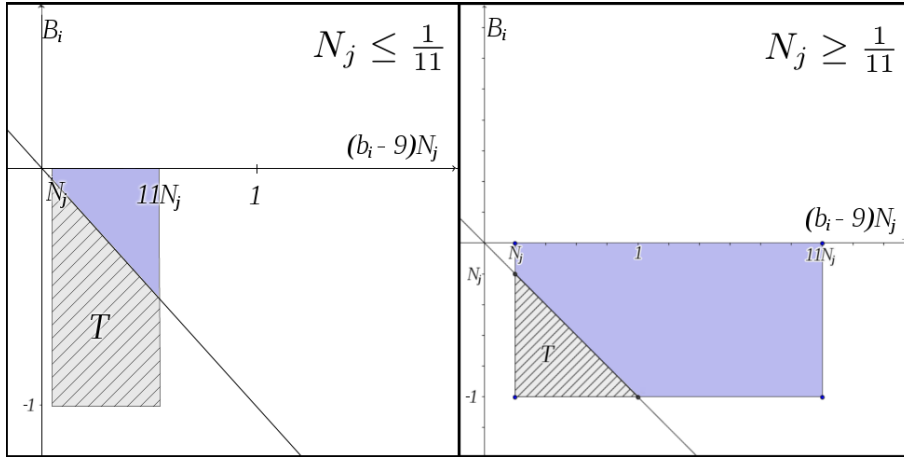


Figure 5: Demand functions as filled parts of rectangles.

Hence, in the second period the momentary demand function $N_i^D(N_j) = Pr((b_i - 9)N_j + B_i \geq 0)$ take the form:

$$N_i^D(N_j) = \begin{cases} 6N_j & \text{for } N_j \leq \frac{1}{11}, \\ 1 - \frac{(1-N_j)^2}{20N_j} & \text{for } N_j \geq \frac{1}{11}. \end{cases} \quad (10)$$

Because in $t = 0.15$ we have $N_j = N_i = 1 - e^{-0.15} \approx 0.1393 > \frac{1}{11}$, only the bottom formula applies. The solution for the total number of users on the platform in the period $[0.15, 5)$ can be approximated by Runge-Kutta 4th order algorithm.⁴ The results of the numerical simulation are presented in Fig.6:

⁴This algorithm approximates the value of an unknown function of time recursively by a weighted average of increments over a small sub-interval of time. See appendix B for more details.

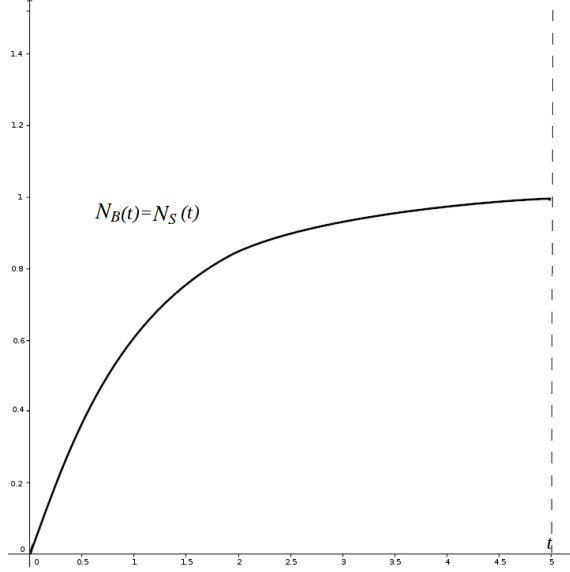


Figure 6: Demand functions $N_B(t), N_S(t)$ for the two-period pricing strategy from subsection 4.1.1, approximated by the Runge-Kutta algorithm on 200 sub-intervals.

The momentary platform's profit can be written as follows:

$$\pi(t) = \begin{cases} -N_B(t) - N_S(t) & \text{for } t \in [0, 0.15] \\ 18N_B(t)N_S(t) & \text{for } t \in [0.15, 5] \end{cases}$$

Total platform profit is an integral of momentary profits on the interval $[0, 5]$. The value of this integral is approximately equal to $\Pi = 61.9073$. Let us emphasize that the total loss incurred in the initial period $[0, 0.15]$ is approximately equal to 0.002. The cost of attracting the first 13.9% users is very low in comparison to the stream of profits earned at a later stage. This example illustrates that the initial investment in a critical mass initiates a positive loop of network externalities. Late users enter on a platform despite the positive interaction fees. In the above example, in contrast to the static benchmark, a small subsidy was effective in unlocking a 'full coverage' equilibrium.

4.1.2 Two-part tariff

The two period strategy required a change of the price scheme. This may be difficult from a practical point of view as customers may not accept worsening of pricing conditions. Moreover, introducing new prices may involve menu and other transaction costs for the platform. In addition, the speed of customer base development under the two period strategy was not the highest possible. The momentary demand in the second sub-period was less than one. In this subsection we introduce a strategy, which guarantees the highest growth of customer base without a need of changing fees. A *two-part tariff strategy* adopts a single pricing scheme in the entire time horizon and automatically adjusts charges for the users: from negative in the beginning, to positive and constantly increasing later on.

Consider the following two-part tariff: $(18, 18, -9, -9)$. It sets negative membership fee combined with positive interaction fee. We can check that (i) a pair $(N_B, N_S) = (1, 1)$ constitutes an equilibrium and (ii) momentary platform profit constantly equals one. For $b_i \geq 10$ and $B_i \geq -1$ we get:

$$1 \geq N_i^D(N_j) = Pr((b_i - 18)N_j - (-9) + B_i \geq 0) \geq Pr(-8N_j + 8 \geq 0) = Pr(1 \geq N_j) = 1.$$

Hence, the total number of users in the platform is characterized by the system (9), similarly to the initial period of the two-period strategy from section 4.1.1. We note that a momentary platform's profit is negative for small values of t . First, the platform has to incur losses by subsidizing users on the early stage of development in order to generate positive profit later. The total platform's profit from the two-part tariff strategy is equal to 54.363.

We compare the effects of using the two-part tariff, and respectively the two-period pricing strategy in Fig.7. The two-part tariff guarantees that demand on each side is constantly equal to 1 and consequently speed of customer base development is the highest possible. However the maximum speed of development comes at a cost of lower temporary platform's profit per user. On the other hand, the two-period pricing strategy maintains a maximum momentary demand only in the initial period. Despite lower pace of growth, the total platform's profits are higher than in the case of the two-part tariff (61.907 vis-a-vis 54.363). These two examples suggest that an unregulated monopoly platform may have an incentive to restrict its rate of growth in order to earn higher profits, before reaching a full coverage equilibrium.

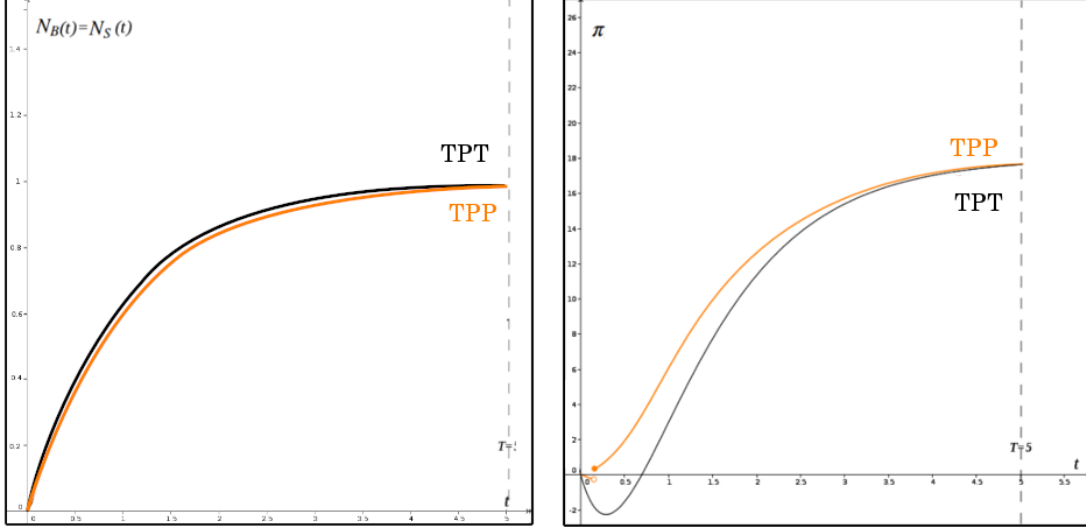


Figure 7: Number of users on specified side N_B, N_S and the momentary platform's profit π for a two-part tariff (TPT) and two-period pricing (TPP) strategies.

One more thing is noteworthy about two-part tariffs. In a static model, the tariff $(18, 18, -9, -9)$ and the benchmark $(9, 9, 0, 0)$ are totally equivalent in a full market equilibrium $(1, 1)$. This is because membership fees can be 'averaged into' interaction fees (Tirole & Rochet, 2006). In dynamic models these two strategies bring different effects. The two-part tariff ensures sustainable convergence to $(1, 1)$ while the benchmark keeps the platform in the initial equilibrium in $(0, 0)$.

4.1.3 Continuous price adjustment

The particular two-part tariff strategy from section 4.1.2 guarantees a maximal momentary demand on both sides and consequently the highest possible speed of users growth. The two-period pricing strategy allows for a relatively quick resignation from subsidies. Let us introduce a *continuous price adjustment strategy*, which connects in a way main advantages of the two previous strategies. It maintains maximal speed of user base development while reducing subsidization costs, compared to the two-part tariff:

$$(p_B, p_S, P_B, P_S) = \begin{cases} (0, 0, -1, -1) & \text{for } t \in [0, 0.15], \\ (10 - \frac{1}{1-e^{-t}}, 10 - \frac{1}{1-e^{-t}}, 0, 0) & \text{for } t \in [0.15, 5]. \end{cases} \quad (11)$$

In the first period, this strategy is identical to the two-period pricing strategy. It offers a subsidy to incentivize early users to join a platform. After the initial period the platform continuously adjusts variable fees to the level, which guarantees a maximal momentary demand on both sides. We already showed in subsection 4.1.1 that for the pricing scheme $(0, 0, -1, -1)$ the demand is equal to one for $t \in [0, 0.15]$. For the second sub-period we have:

$$1 \geq N_i^D = Pr((b_i - (10 - \frac{1}{1-e^{-t}}))(1 - e^{-t}) + B_i \geq 0) \geq Pr((10 - 10 + \frac{1}{1-e^{-t}})(1 - e^{-t}) - 1 \geq 0) \geq Pr(1 - 1 \geq 0) = 1$$

The solution of the system (6) on $t \in [0, 5]$ is the pair of functions $N_B = N_S(t) = 1 - e^{-t}$.⁵ Momentary profits can be expressed as $\pi = 2(10 - \frac{1}{1-e^{-t}})(1 - e^{-t})^2$ and are positive only in the second sub-period. The total platform's profits from using continuous price adjustment strategy in the entire time horizon equals to 62.259. This value is higher than for both previously discussed strategies. The momentary platform's profits from all three strategies are presented in Fig.8.

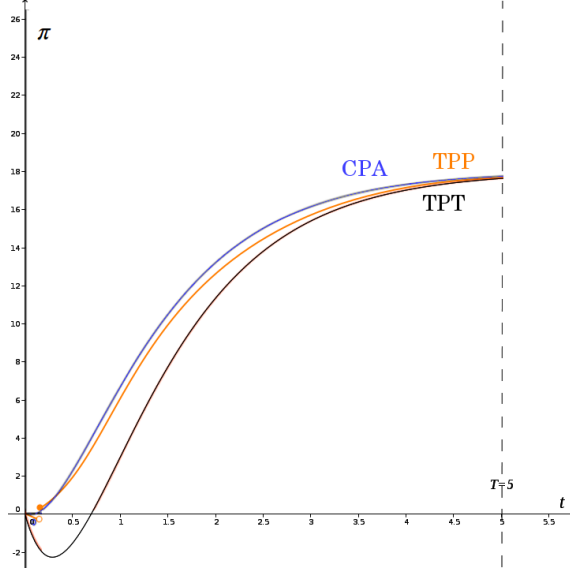


Figure 8: Momentary profits from a two-period pricing (TPP), two-part tariff (TPT) and a continuous price adjustment (CPA) strategies.

4.1.4 Asymmetric two-part tariff

All three strategies considered before set the same price levels for both sides of the platform. The results of [Tirole and Rochet \(2006\)](#) related to optimal pricing suggest that not only the overall level but also the structure of prices across sides matters. [Evans and Schmalensee \(2010\)](#) discuss asymmetric subsidy targeted at one side only, in the context of the failure-to-launch problem. The effects of asymmetric pricing can also be analyzed in the present model. To illustrate this, we introduce asymmetry between buyers and sellers into the two-part tariff from subsection (4.1.2). More specifically, in the entire time horizon the platform uses a single price scheme: $(p_B, p_S, P_B, P_S) = (18, 9, -9, 0)$ which ensures that the point $(N_B, N_S) = (1, 1)$ is an equilibrium. Let us compare profits of the asymmetric two-part tariff and the symmetric two-part tariff considered before. We check that a temporary demand of the subsidized B -side constantly equals one:

$$N_B^D = Pr((b_B - 18)N_S + 9 + B_B \geq 0) \geq Pr((10 - 18)N_S + 9 - 1 \geq 0) = Pr(-8N_S + 8 \geq 0) = 1.$$

Hence, according to (6) the membership on the B -side will evolve according to $N_B = 1 - e^{-t}$. Using this formula, the equality $1 - e^{-\ln(\frac{11}{10})} = \frac{1}{11}$ and the previously calculated momentary demand (10), we obtain the corresponding formula for a number of S -side users:

$$N_S^D(t) = \begin{cases} 6(1 - e^{-t}) & \text{for } t < \ln(\frac{11}{10}), \\ 1 - \frac{(e^{-t})^2}{20(1-e^{-t})} & \text{for } t \geq \ln(\frac{11}{10}). \end{cases}$$

The above solution of the problem $\dot{N}_S = N_S^D - N_S, N_S(0) = 0$ is unique.⁶ We apply the Runge-Kutta algorithm to approximate values of N_S in time. The evolution of sellers' side for asymmetric and symmetric two-part tariffs is compared in Fig.9.

⁵The uniqueness of that solution results from the Picard-Lindelöf theorem, because of Lipschitz continuity of the functions on the right-hand side.

⁶By virtue of the Picard-Lindelöf theorem the function N_S^D does not depend on N_B but only on t , and a component N_S is linear. It is worth pointing out that function N_S^D is continuous in the point $\ln(\frac{11}{10})$.

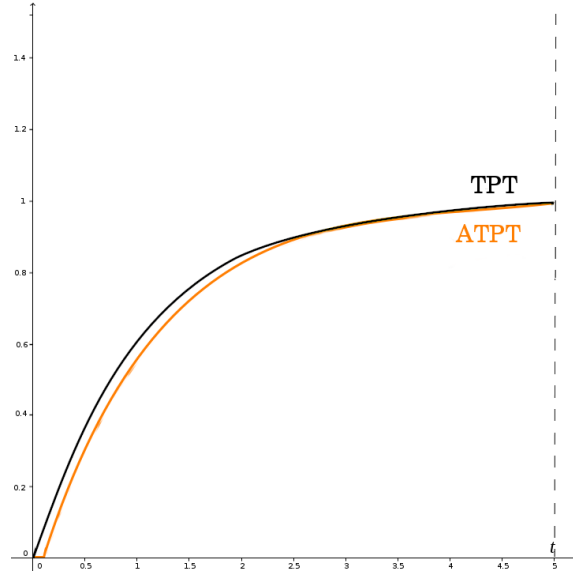


Figure 9: Number of sellers $N_S(t)$ for symmetric (TPT) and asymmetric (ATPT) two-part tariffs, approximated by the 4th order Runge-Kutta algorithm.

When only one side is subsidized, the users from the other side are joining platform at a slower pace than in the case of symmetric subsidies. Hence, lower level of total subsidy implies a slower customer base development. Depending on the magnitudes of these two effects asymmetric strategy can be more or less profitable than the otherwise identical symmetric one. In our example, the asymmetric pricing strategy generates greater profits, as can be seen in Fig.10. Both strategies converge to a full coverage equilibrium $(N_B, N_S) = (1, 1)$ and generate the same level of momentary profits in that point. There is however a difference in convergence paths, which translates into total profits. Approximated total profit equals 56.4823 for the asymmetric strategy and 54.363 for the symmetric one. The current example shows that it might be more profitable in a dynamic setting to apply asymmetric subsidies to initiate platform development.

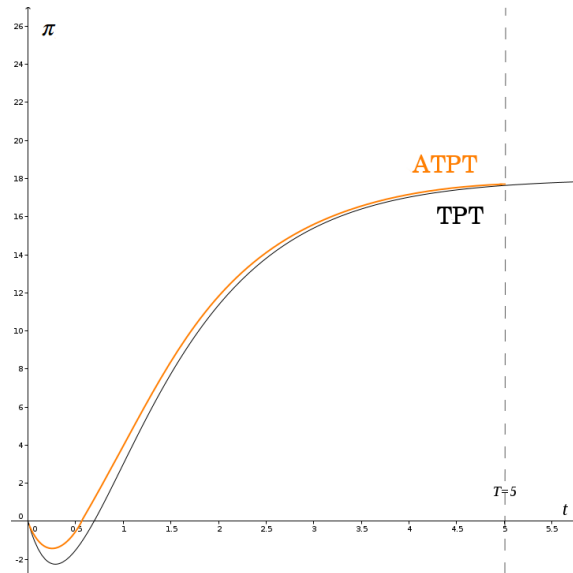


Figure 10: Momentary platform's profits π for symmetric (TPT) and asymmetric (ATPT) two-part tariffs.

4.2 Welfare analysis

In all considered situations, a platform was incurring initially some losses in order to initiate the development of its customer base and earn profits later. In this section we look at the welfare effects of different dynamic pricing strategies. We start from defining a momentary social welfare:

$$\pi(t) + CS(t) = \pi(t) + \int_{\mathcal{N}_B(t)} U_B(\xi, t) d\xi + \int_{\mathcal{N}_S(t)} U_S(\xi, t) d\xi \quad (12)$$

In the above formula $\mathcal{N}_i(t)$ denotes a set of platform users in a given moment t and $U_i(\xi, t) = (b_i(\xi_1) - p_i(t))N_j(t) + B_i(\xi_2) - P_i(t)$ is a utility function defined in expression (1). For the sake of simplicity we assume that the current set of users on the platform is representative for a set of agents, who can get positive utility from participation. This assumption guarantees that conditional expected values of the random variables b_i, B_i for users on-board are equal to the corresponding expected values of these variables for new users, who joined a platform in a given moment.

The total social welfare is an integral of the discounted momentary social welfare over time:

$$W = \int_0^T (\pi(t) + \int_{\mathcal{N}_B(t)} U_B(\xi, t) d\xi + \int_{\mathcal{N}_S(t)} U_S(\xi, t) d\xi) e^{-rt} dt = \Pi + \int_0^T (\int_{\mathcal{N}_B(t)} U_B(\xi, t) d\xi + \int_{\mathcal{N}_S(t)} U_S(\xi, t) d\xi) e^{-rt} dt. \quad (13)$$

The second and third components of the above formula represent consumer surplus CS on both sides. Let a momentary total costs be denoted by $tc(t)$ and the total cost in the entire time horizon be denoted by TC . With simple arithmetics we can eliminate from the social welfare expression cash flows between platform and users.

$$CS = \int_{\mathcal{N}_B(t)} U_B(\xi, t) d\xi + \int_{\mathcal{N}_S(t)} U_S(\xi, t) d\xi = \sum_{i \in \{B, S\}} \int_{\mathcal{N}_i(t)} ((b_i(\xi_1) - p_i)N_S(t) + B_i(\xi_2) - P_i(t)) d(\xi_1, \xi_2) = \quad (14)$$

$$= -p_B(t)N_B N_S - P_B(t)N_B - p_S(t)N_B N_S - P_S(t)N_S + \sum_{i \in \{B, S\}} \int_{\mathcal{N}_i(t)} (b_i(\xi_1)N_S(t) + B_i(\xi_2)) d(\xi_1, \xi_2) = \quad (15)$$

$$= -(\pi(t) + tc(t)) + \int_{\mathcal{N}_B(t)} (b_B(\xi_1)N_S(t) + B_B(\xi_2)) d(\xi_1, \xi_2) + \int_{\mathcal{N}_S(t)} (b_S(\xi_1)N_B(t) + B_S(\xi_2)) d(\xi_1, \xi_2) \quad (16)$$

After substitution into the total social welfare formula (13) we obtain the following equation:

$$W = \int_0^T (\int_{\mathcal{N}_B(t)} (b_B(\xi_1)N_S(t) + B_B(\xi_2)) d(\xi_1, \xi_2) + \int_{\mathcal{N}_S(t)} (b_S(\xi_1)N_B(t) + B_S(\xi_2)) d(\xi_1, \xi_2)) e^{-rt} dt - TC. \quad (17)$$

The new formula for social welfare is intuitive. The revenue received by a platform comes from user fees. Hence, any additional unit of revenue received by the platform has to be subtracted from the users as their expenditure. To sum up, social welfare equals the discounted aggregated users' gross utility net of the total costs of the platform.

All considered strategies, except of the static benchmark (9,9,0,0) generate positive social welfare. Every user on the platform receives a positive net utility.⁷ As a result, consumer surplus is positive as well. Let us consider the value of the social welfare when the platform uses strategy which maximizes the speed of customer base development, i.e. guarantees that momentary demand functions N_B^D, N_S^D constantly equal one. Using the assumption about expected values, we can

⁷This is because in the entire time horizon the following inequality holds: $0 < N_i < N_i^D$.

substitute the integrals of b_i and B_B into the formula (17):

$$W = \int_0^T (N_B(t)N_S(t)\mathbb{E}(b_B|\mathcal{N}_B) + N_B(t)\mathbb{E}(B_B|\mathcal{N}_B) + N_B(t)N_S(t)\mathbb{E}(b_S) + N_S(t)\mathbb{E}(B_S))e^{-rt}dt - TS$$

Because $N_i^D \equiv 1$ for every t from the time horizon, all users obtain a positive utility from participation. The expected values of b_i and B_i for users on board are simply equal to expected values in the entire population:

$$W = \int_0^T (N_B(t)N_S(t)(\mathbb{E}(b_B)-c) + N_B(t)(\mathbb{E}(B_B)-C_B) + N_B(t)N_S(t)\mathbb{E}(b_S) + N_S(t)(\mathbb{E}(B_S)-C_S))e^{-rt}dt.$$

The integrand from the above formula is an increasing function of $N_B(t)$ and $N_S(t)$. Consequently, faster growth of user base increases social welfare. The maximum social welfare level will be reached when the number of users increases as fast as possible, i.e. demand on both sides is equal to one. Intuitively, more users generate more interactions, which in turn generates a higher momentary utility.

In what follows we calculate the total welfare for the two strategies, which guarantee the highest momentary social welfare level in the entire time horizon - the two-part tariff from section 4.1.2 and the continuous price adjustment strategy from section 4.1.3. After substituting parameters specified before: $r = 0$, $\mathbb{E}(B_i) = -0.5$, $\mathbb{E}(b_i) = 15$, $N_i(t) = 1 - e^{-t}$ into the social welfare formula we obtain:

$$W = \int_0^T (30(1 - e^{-t})^2 - (1 - e^{-t}))dt \approx 101.40.$$

Both strategies generate the same level of a total social welfare, however its distribution between the platform and users is different. The aggregate platform profit is equal to 54.363 for the two-part tariff and 62.259 for the continuous price adjustment strategy. The corresponding customer surplus $CS = W - \Pi$ equals to 47.037 and 39.141 respectively.

4.2.1 Regulated platform

In this last section we ask a question about the strategy for the social planner who would be interested in maximizing the well-being of platform's users? We are looking for a strategy that maximizes a social welfare and passes all profits to consumers. Let us consider a modified two-part tariff with a price scheme (18, 18, -15.782, -15.782) used in the entire time horizon. This strategy resembles the symmetric two-part tariff considered in section (4.1.2) but has higher levels of fixed subsidies. The proposed strategy allows the platform to reach a full market equilibrium and it is socially optimal.⁸ At the same time it takes away all profits from the platform:

$$\Pi = \int_0^5 \pi e^{-0t} dt = \int_0^5 36(-e^{-t} + 1)^2 - 31.564(-e^{-t} + 1)dt \approx 0$$

The temporary platform's profit is shown in Fig.11. The membership subsidies are adjusted such that they transfer out all profits earned by the platform from interactions. It should be noted that the zero total profit condition is met exactly at the end-point of the specified time horizon [0, 5]. The platform is allowed to make a positive momentary profit to regain losses incurred in the initial stage.

5 Discussion and conclusions

This paper introduced a continuous-time dynamic model of a two-sided monopolistic platform that builds on the static model of [Tirole and Rochet \(2006\)](#). The form of individual utility and externalities is the same as in their model. There are however important points of departure as well.

⁸It induces the highest possible momentary demand because it is even more generous than the socially optimal two-part tariff considered in section 4.1.2.

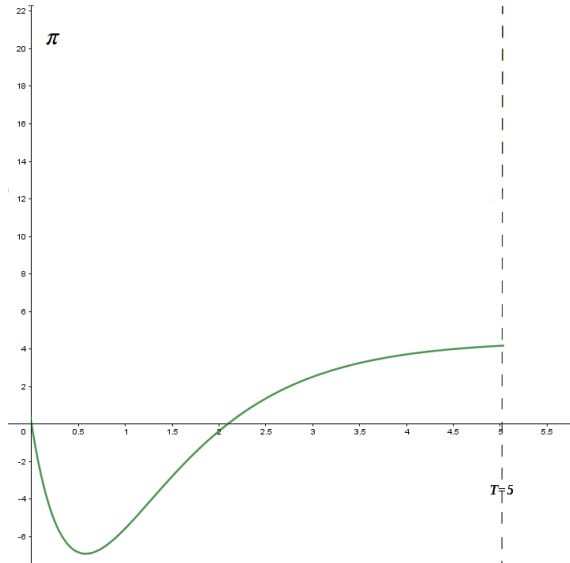


Figure 11: Momentary platform’s profits π for socially optimal regulated prices.

We account for a richer consumer heterogeneity and allow a platform to use more complex pricing schemes, composed of fixed membership fees and interaction fees. In static models membership fees are usually ignored because at the equilibrium, they can be ‘averaged’ into the interaction fees paid by the participants. In a dynamic framework fixed fees turn out to be a very useful instrument for provision of subsidies to early adopters. We showed that with a single two-part tariff, a platform can manage a complete evolution of the market: incentivize users in the initial period and then make a positive profit from interactions. There is no need to change a price scheme during operation in order to make the business sustainable. On the other extreme, we considered ‘truly’ dynamic pricing strategies, including the one with continuous price adjustments.

In a dynamic framework the focus is not on the optimal pricing in a particular equilibrium point, but on the equilibrium selection and convergence. Therefore, dynamic approach is particularly relevant for the analysis of a start-up platform, which has to resolve the chicken-and-egg problem or an established platform facing a failure-to-grow. We demonstrate how a platform can move between different equilibrium points by adopting dynamic pricing. In our model users are myopic and decide whether to join the platform based on a momentary net utility received. Participation constraint changes continuously with platform’s size and prices. We assume that the cost of reversing participation decision is negligible, so there is no risk of lock-in that could affect user beliefs. We do not provide a profit-maximizing solution for the convergence to the proffered point. Finding optimal pricing strategies in a general dynamic model is a complex tasks and requires simplifying assumptions about a class of pricing functions and a form of users heterogeneity. Hence, we deliberately focused on selected examples of different pricing strategies with an aim to demonstrate their impact on platform’s profits and social welfare in comparison to the static benchmark solution. The economic effects of these pricing strategies are summarized in Tab.1. We can derive few main lessons from these examples.

First, for a given price scheme, multiple equilibria can occur even in relatively simple market conditions. These equilibria may correspond to an empty and fully covered market, as well as any intermediate level of users’ participation. Multiplicity of equilibria implies that any static model essentially has no predictive power. There is no guarantee that the optimal fixed prices will lead to the most preferred outcome from the platform’s perspective. The added value of a dynamic approach is that it can determine which equilibrium will be achieved for particular initial conditions, how much time it will take and what adjustments in fees are required.

Second, in order to control the membership evolution, a platform has to recognize that each equilibrium has its own gravity zone, technically corresponding to a basin of attraction on a phase portrait. In the simplest case, a dynamic price may be composed of a single pricing scheme held

constantly. Such pricing strategies yield time-invariant set of equilibria. The platform is however not bound to keep constant prices over time. For every change of momentary prices, the set of equilibrium points and the corresponding phase portrait change. This introduces additional complexity into the price optimization problem. Optimal convergence path involves finding not only the preferred equilibrium, but also the speed of user base growth. In a dynamic framework, like the one proposed in this paper, a platform may go beyond trial and error in testing dynamic prices.

Third, the simplest type of pricing strategy with a constant pricing scheme, such as for example the two-part tariff, may be sufficient to reach a full market equilibrium and maintain business sustainability for the platform. Sometimes, two-part tariffs guarantee socially optimal outcomes. A necessary condition for maximization of welfare in a dynamic setting is to keep momentary demand constantly at a maximal level (one). This condition translates to the quickest user base development and technically can be achieved only if all potential users receive positive utility from entering. Pricing strategies that yield maximal momentary demand can be considered as a dynamic variant of insulation tariffs (Weyl, 2010). In static models it does not make sense to use fixed fees in the equilibrium, because they can be 'averaged into' the interaction fees. In dynamic models, fixed fees play a very important role in convergence to the equilibrium. Membership fees allow for initial subsidization without a need to adjust the price scheme on a later stage. On the other hand, our benchmark strategy composed only of interaction fees was incapable of solving a failure-to-launch problem.

Fourth, a platform can use time-varying pricing strategies and even adjust prices continuously. Continuous price adjustment strategy might have a practical relevance in algorithmic trading for example. Intuitively, if the menu costs are negligible, strategies that introduce inter-temporal price discrimination are more profitable compared to their time-invariant counterparts. The continuous adjustment strategy ensures that a platform minimizes the cost of subsidies while keeping a momentary demand at a maximal level.

Lastly, our examples point to a clear trade-off between profitability of the platform and the level of social welfare determined, as shown in Tab. 1. A platform will prefer an organic growth at a slower pace, while from the welfare perspective, it is desirable if users enjoy sufficient subsidies in order to activate full strength of network externalities as soon as possible.

Dynamic models cast a light on the evolution of multi-sided markets. With regards to the proposed framework the recommended directions of future research would be to: (i) explore the proposed model with more examples of pricing strategies under different conditions (ii) perform dynamic optimization for simple special cases of preference heterogeneity and (iii) introduce competing platforms to study splitting vs tipping equilibria.

Table 1: Comparison of the outcomes of different pricing strategies, for initial conditions (0,0)

strategy	parameters (p_B, p_S, P_B, P_S)	equilibrium ($N_B; N_S$)	market development speed	social welfare	platform profit
Two-period pricing	$(0, 0, -1, -1)$ for $t \in [0, 0.15)$, $(9, 9, 0, 0)$ for $t \in [0.15, 5]$	(1,1)	Less than maximal	Less than maximal	61.91
Two-part tariff	$(18, 18, -9, -9)$ for $t \in [0, 5]$	(1,1)	Maximal on both sides	Maximal: 101.40	54.36
Continuous adjustment	$(0, 0, -1, -1)$ for $t \in [0, 0.15)$, $(10 - \frac{1}{1-e^{-t}}, 10 - \frac{1}{1-e^{-t}}, 0, 0)$ for $t \in [0.15, 5]$	(1,1)	Maximal on both sides	Maximal: 101.40	62.26
Two-part asymmetric	$(18, 9, -9, 0)$ for $t \in [0, 5]$	(1,1)	Maximal on B-side; less than maximal on S-side	Less than maximal	56.48
Public platform	$(18, 18, -15.782, -15.782)$ for $t \in [0, 5]$	(1,1)	Maximal on both sides	Maximal: 101.40	0
Benchmark	$(9, 9, 0, 0)$ for $t \in [0, 5]$	(0,0)	zero	0	0

Appendix

A - Proof of the existence theorem

The thesis of the theorem 1 is equivalent to the existence of solution $(N_B, N_S) \in [0, 1] \times [0, 1]$ of the system (4) for each price scheme $(p_B, p_S, P_B, P_S) \in \mathbb{R}^4$. For any specified price scheme (p_B, p_S, P_B, P_S) demand functions $N_S^D(N_B), N_B^D(N_S)$ are well defined. The graphs of these functions can be presented on a joint plot as curves⁹:

$$I = \{(N_B, N_S) \in \mathbb{R}^2 : N_B = N_B^D(N_S)\},$$

$$II = \{(N_B, N_S) \in \mathbb{R}^2 : N_S = N_S^D(N_B)\}.$$

The intersection of both curves gives a set of solutions for the considered system. Each solution from this set determines an equilibrium for a specified price scheme if it has both coordinates from $[0, 1]$. Hence, to prove the theorem it is sufficient to show that the intersection of both curves and a unit cube is non-empty.

Let us point out, that the curve I is included in $([0, 1] \times \mathbb{R}) \subset \mathbb{R}^2$, and analogously the curve II is included in $(\mathbb{R} \times [0, 1]) \subset \mathbb{R}^2$. In consequence, the intersection of both curves is included in the intersection of those two sets, which is $[0, 1] \times [0, 1]$, because:

$$([0, 1] \times \mathbb{R}) \cap (\mathbb{R} \times [0, 1]) = [0, 1] \times [0, 1].$$

Therefore, it is sufficient to prove that the curves I, II have non-empty intersections i.e. $I \cap II \neq \emptyset$.

Let $\phi : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the transformation of a plane determined by the following formula:

$$\phi(x, y) = (x - N_B^D(y), y). \quad (18)$$

The image of the curve I in the transformation ϕ is the vertical axis of a standard coordinate system:

$$\phi(I) = \phi(\{(N_B, N_S) \in \mathbb{R}^2 : N_B = N_B^D(N_S)\}) = \{(N_B - N_B, N_S) \in \mathbb{R}^2\} = 0 \times \mathbb{R}.$$

The next step of the proof is to show that ϕ is continuous. Let (x, y) be any specified point from a two-dimensional plane. We will show that for every $\epsilon > 0$ there exists $\delta > 0$ such that, for all (x', y') satisfying $\|(x, y) - (x', y')\| < \delta$ we have $\|\phi(x, y) - \phi(x', y')\| < \epsilon$, where $\| \cdot \|$ is the standard euclidean norm.

$$\|\phi(x, y) - \phi(x', y')\| = \|(x - N_B^D(y), y) - (x' - N_B^D(y'), y')\| = \|(x - N_B^D(y) - x' + N_B^D(y'), y - y')\|$$

After applying twice the triangle inequality we get:

$$\begin{aligned} & \|(x - N_B^D(y) - x' + N_B^D(y'), y - y')\| \leq \\ & \leq \|(x - N_B^D(y) - x' + N_B^D(y')\| + |y - y'| \leq \\ & \leq |x - x'| + |N_B^D(y) - N_B^D(y')| + |y - y'|. \end{aligned}$$

Since $N_B^D(x)$ is a continuous function, we can find $\delta_1 > 0$, such that for all y' satisfying $|y - y'| < \delta_1$ there is $|N_B^D(y) - N_B^D(y')| < \frac{\epsilon}{3}$. Obviously, every $0 < \delta < \delta_1$ satisfies this condition, therefore we can take $\delta < \frac{\epsilon}{3}$. For all x' such that $|x - x'| < \delta$ we have:

$$|x - x'| + |N_B^D(y) - N_B^D(y')| + |y - y'| < 2\delta + \frac{\epsilon}{3} < \epsilon.$$

If $\|(x, y) - (x', y')\| < \delta$ then we have $|x - x'| < \delta$ and $|y - y'| < \delta$. This implies that $\|\phi(x, y) - \phi(x', y')\| < \epsilon$ and the function $\phi(x, y)$ is continuous.

The curve II is an image of the continuous transformation $\psi(x) : x \mapsto N_S^D(x)$, what implies that $\phi(II) = \phi(\psi(\mathbb{R}))$ is a curve as well, because the composition of a continuous functions $\phi \circ \psi$

⁹Let us clarify that we use a word 'curve' for an image of an interval (not necessarily bounded) produced by continuous transformation.

maintains continuity. Let us denote the standard projection of a plane into the axis N_B by Γ_{N_B} . The projection is continuous, so in consequence, the image of $\phi(II)$ is the curve included in the horizontal axis N_B . This implies that $\phi(II)$ is an interval (or a line if not bounded).

The last step is to prove that $0 \in \Gamma(\phi(II))$. Let us consider points $(x_1, y_1) = (2, N_S^D(2)) \in II$ and $(x_2, y_2) = (-2, N_S^D(-2)) \in II$. We have:

$$\|\Gamma(\phi(x, y)) - \Gamma(x, y)\| = \|\phi(x, 0) - (x, 0)\| = \|x - N_B^D(y) - x\| = \|N_B^D(y)\| \leq 1,$$

so for $k \in \{1, 2\}$:

$$\|\Gamma(\phi(x_k, y_k)) - (x_k, y_k)\| \leq \|\Gamma(\phi(x_k, y_k)) - (x_k, 0)\| = \|\Gamma(\phi(x_k, y_k)) - \Gamma(x_k, y_k)\| \leq 1,$$

what implies that $\Gamma(\phi(x_1, y_1)) \geq 1$ and $\Gamma(\phi(x_2, y_2)) \leq -1$.

$\Gamma(\phi(II))$ is an interval, including points $\Gamma(\phi(x_1, y_1)), \Gamma(\phi(x_2, y_2))$, so it includes 0 as well. Therefore $\phi^{-1}(\Gamma^{-1}(0)) \cap II \neq \emptyset$.

Let us consider $(N_B^*, N_S^*) \in \phi^{-1}(\Gamma^{-1}(0)) \cap II$. The condition $\phi(N_B^*, N_S^*) \in \Gamma^{-1}(0)$ is equivalent to the condition $N_B^* - N_B^D(N_S^*) = 0$. Consequently, $(N_B^*, N_S^*) \in I$. The considered point (N_B^*, N_S^*) is included in I and II as well, so it is included in their intersection. In consequence, this point is the solution of the considered system, i.e. Nash equilibrium in the strategic game $G_{(p_B, p_S, P_B, P_S)}$. \square

B - Runge-Kutta method (4th order)

Let us introduce the Runge-Kutta method. This algorithm has been used in the paper to approximate solutions of Cauchy problems, composed of the ordinary differential equation and initial condition. [Palczewski \(2004\)](#) argues that for 4th order Runge-Kutta scheme, the cost-to-accuracy ratio is the best possible.

Let ordinary differential equation have a form $x = f(t, x)$. The interval, on which the solution is approximated will be denoted by $[t_0, t_{end}]$. Let the initial condition be specified by $x(t_0) = x_0$. For an established number of steps n the algorithm splits the interval $[t_0, t_{end}]$ into n sub-intervals with a length equal to $h = \frac{t_{end} - t_0}{n}$ and calculates approximated value of the solution in the end point of each.

Let us present the scheme in C language, with the assumption that we have already declared values of variables n, x_0, t_0, t_end and function $f(t, x)$:

```
double x[], t[], h, K1, K2, K3, K4, K5;
x[0]=x_0;
t[0]=t_0;
h=(t_end-t_0)/n;

for(k=0; k<n + 1; k++){
    K1 = f(t[k], x[k]);
    K2 = f(t[k] + h/2, x[k] + (h/2)*K1);
    K3 = f(t[k] + h/2, x[k] + (h/2)*K2);
    K4 = f(t[k] + h, x[k] + h*K3);

    x[k+1] = x[k] + (h/6) * (K1 + 2*K2 + 2*K3 + K4);
    t[k+1] = t[k] + h;
}
```

The resulting table $x[]$ contains approximated values of the solution $x(t)$ in points of form $t_k = t_0 + kh, k \in \mathbb{Z}$, specified in $t[]$. In the literature another variants of this scheme can be found, with different values of constants $K1, K2, K3, K4, K5$.

References

- Armstrong, M. (2006). Competition in two-sided markets. *The RAND Journal of Economics*, 37(3), 668–691.
- Cabral, L. (2019). Towards a theory of platform dynamics. *Journal of Economics and Management Strategy*, 28(1), 60–72. doi: 10.1111/jems.12312
- Caillaud, B., & Jullien, B. (2001). Competing cybermediaries. *European Economic Review*, 45(4-6), 797–808.
- Caillaud, B., & Jullien, B. (2003). Chicken & Egg: Competition among Intermediation Service Providers. *The RAND Journal of Economics*, 34(2), 309. doi: 10.2307/1593720
- Evans, D. S. (2003). Some empirical aspects of multi-sided platform industries. *Review of Network Economics*, 2(3).
- Evans, D. S., & Schmalensee, R. (2010). Failure to launch: Critical mass in platform businesses. *Review of Network Economics*, 9(4).
- Hagiu, A. (2006). Pricing and commitment by two-sided platforms. *The RAND Journal of Economics*, 37(3), 720–737.
- Palczewski, A. (2004). *Równania różniczkowe zwyczajne: teoria i metody numeryczne z wykorzystaniem komputerowego systemu obliczeń symbolicznych [eng: Ordinary differential equations: Theory and numerical methods]*. Wydawnictwa Naukowo-Techniczne.
- Rochet, J.-C., & Tirole, J. (2003). Platform competition in two-sided markets. *Journal of the European Economic Association*, 1(4), 990–1029.
- Tirole, J., & Rochet, J. C. (2006). Two Sided Markets: A Progress Report. *RAND Journal of Economics*, 37(3), 645–667. doi: 10.1080/0092623X.2014.966397
- Weyl, E. G. (2010). A price theory of multi-sided platforms. *American Economic Review*, 100(4), 1642–1672.
- White, A., & Weyl, E. G. (2016). Insulated platform competition. *Available at SSRN 1694317*.

List of figures and tables

Figure 1. Demand functions, such that three equilibria exist	10
Figure 2. Demand functions, such that infinite equilibria exist	11
Figure 3. Phase portrait with stationary points.....	12
Figure 4. Phase portrait and equilibria for the benchmark unit prices	13
Figure 5. Demand functions as filled parts of rectangles.....	14
Figure 6. Demand functions for a two-period pricing strategy.....	15
Figure 7. Number of users and momentary profits for a two-part tariff and two-period pricing strategies... ..	16
Figure 8. Momentary profits from a two-period pricing, two-part tariff and continuous price adjustment strategies.....	17
Figure 9. Number of sellers for symmetric and assymtric two-part tariffs.....	18
Figure 10. Momentary profits for symmetric and assymmetric two-part tariffs.....	18
Figure 11. Momentary profits for socially optimal regulated prices.....	21
Table 1. Comparison of the outcomes of different pricing strategies	23

GETTING IN TOUCH WITH THE EU

In person

All over the European Union there are hundreds of Europe Direct information centres. You can find the address of the centre nearest you at: https://europa.eu/european-union/contact_en

On the phone or by email

Europe Direct is a service that answers your questions about the European Union. You can contact this service:

- by freephone: 00 800 6 7 8 9 10 11 (certain operators may charge for these calls),
- at the following standard number: +32 22999696, or
- by electronic mail via: https://europa.eu/european-union/contact_en

FINDING INFORMATION ABOUT THE EU

Online

Information about the European Union in all the official languages of the EU is available on the Europa website at: https://europa.eu/european-union/index_en

EU publications

You can download or order free and priced EU publications from EU Bookshop at: <https://publications.europa.eu/en/publications>. Multiple copies of free publications may be obtained by contacting Europe Direct or your local information centre (see https://europa.eu/european-union/contact_en).



The European Commission's science and knowledge service

Joint Research Centre

JRC Mission

As the science and knowledge service of the European Commission, the Joint Research Centre's mission is to support EU policies with independent evidence throughout the whole policy cycle.



EU Science Hub
ec.europa.eu/jrc



@EU_ScienceHub



EU Science Hub - Joint Research Centre



EU Science, Research and Innovation



EU Science Hub