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# Epictetusian Rationality\*

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## Abstract

According to Epictetus, mental freedom and happiness can be achieved by distinguishing between, on the one hand, things that are upon our control (our acts, opinions and desires), and, on the other hand, things that are not upon our control (our body, property, offices and reputation), and by wishing for nothing that is outside our control. This article proposes two accounts of Epictetus's precept: the I account of Epictetus's precept requires indifference between outcomes differing only on circumstances, whereas the IB account requires indifference between outcomes involving the best replies to circumstances. We study the implications of these precepts on the preference relation and on the existence of Epictetusian rationality. The I account implies that the preference relation satisfies independence of circumstances, whereas the IB account implies robustness to dominated alternatives. Unlike the IB account, the I account rules out (counter)adaptive preferences. Finally, when examining game-theoretical implications of Epictetusian rationality, we show that the two accounts of Epictetus's precept exclude the existence of prisoner's dilemmas.

*Keywords:* rationality, Epictetus, preferences, indifference, independence, adaptive preferences.

*JEL classification codes:* B11, D01, D10.

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# 1 Introduction

Whereas the interest of economists in ethical preferences has been recently renewed (see Roemer 2010, 2019, Alger and Weibull 2013, 2016), philosophers have had a long lasting interest in ethical behavior, i.e., in attitudes that individuals should adopt.<sup>1</sup> Although philosophers's theorizations about ethical behavior did not, generally, rely on the concept of preferences, their explorations can nonetheless serve as a starting point for revisiting the content and scope of ethical preferences.

An early attempt to conceptualize and justify some form of ethical behavior can be found in the works of the Stoic philosopher Epictetus (55AC – 135 AC), in particular in the *Manual* – also known as the *Encheiridion* or the *Handbook* –, which is a short compilation of Epictetus's practical precepts.<sup>2</sup> In a nutshell, the *Manual* compiles a short selection of Epictetus's ethical advices aimed at helping the layman in everyday life, in all – possibly highly adverse – circumstances.

According to Epictetus, mental freedom and happiness can be achieved by distinguishing between, on the one hand, things that are upon our control (our acts, opinions and desires), and, on the other hand, things that are not upon our control (our body, property, offices and reputation), and by wishing for nothing that is outside our control.

The distinction between things upon our control and things not upon our control appears at the very beginning of the *Manual* (I):

Of things some are in our power, and others are not. In our power are opinion, movement towards a thing, desire, aversion, turning from a thing; and in a word, whatever are our acts. Not in our power are the body, property, reputation, offices (magisterial power), and in a word, whatever are not our own acts. And the things in our power are by nature free, not subject to restraint or hindrance; but the things not in our power are weak, slavish, subject to restraint, in the power of others.

Epictetus's precept consists of a differentiated attitude towards these two classes of things: in order to have a good life, one should reorient one's desires and aversions away from the things outside one's control, and cultivate desires and aversions only for things that are upon control (*Manual*, II):

Remember that desire contains in it the profession (hope) of obtaining that which you desire; and the profession (hope) in aversion (turning from a thing) is that you will not fall into that which you attempt to avoid; and he who fails in his desire is unfortunate; and he who falls into that which he would avoid is unhappy. If then

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<sup>1</sup>An earlier economic analysis of ethical preferences through the lenses of Kant can be found in Laffont (1975).

<sup>2</sup>Epictetus is regarded as a major Stoic moralist. He served part of his life as a slave, and kept during his remaining life some bodily stigma of bad treatments due to his past master. Epictetus's lectures were recorded by his pupil Arrian. See Clark (1995).

you attempt to avoid only the things contrary to nature which are within your power you will not be involved in any of the things which you would avoid. But if you attempt to avoid disease, or death, or poverty, you will be unhappy.

Epictetus's precept is summarized as follows (*Manual*, II):

Take away then aversion from all things which are not in our power, and transfer it to the things contrary to nature which are in our power.

Whereas Stoicism is often presented as defending 'Life according to Nature' (Moore 1903, Sidgwick 1907), and, hence, a kind of overall indifference, Epictetus's precept is not about cultivating indifference with respect to *all* things.<sup>3</sup> Epictetus's Stoicism requires to cultivating indifference only with respect to the things that are not under control (events that are external and independent of one's will). Desires and aversions should not be developed for things that do not depend on us, but well for things that are under our control. From Epictetus's perspective, the good life is reached by being concerned only with things that are up to us, that is, only concerned with our proper self (see Long 2002).

Various aspects of Epictetus's precept can be questioned. First, one can criticize the distinction drawn by Epictetus between things upon our control and things not upon our control. The notion of control is complex, especially in a world of multiple interdependencies. This makes Epictetus's precept hard to apply in practice. Second, one can also question the logics of Epictetus's rationale. Moore (1903) criticized Stoicism in general on the ground that it is a metaphysical ethical doctrine, that is, a doctrine that deduces what should be done from metaphysical premises.<sup>4</sup> But even if one takes Epictetus's distinction for granted, this sole distinction does not imply anything regarding what a person *should* do. Epictetus's precept aims at achieving mental freedom and happiness, but there is no reason why one should adopt these goals.<sup>5</sup>

In this article, we will leave these issues aside, and focus only on one question: can an Epictetusan decision-maker be rational? The answer depends on how one interprets Epictetus's precept and on how one defines the concept of rationality. In this paper, rationality is defined as the existence of a preference ordering representing the interests of the person in a coherent manner.<sup>6</sup> But

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<sup>3</sup>See Moore (1903), p. 93 and Sidgwick (1907), p. 377. These two authors underlined the problems of circularity for an ethical doctrine recommending 'Life according to Nature': any act can always be interpreted as 'natural'.

<sup>4</sup>See Moore (1903), p. 161.

<sup>5</sup>Epictetus seems here to deduce a 'should' from an 'is', some form of naturalistic fallacy (Moore 1903).

<sup>6</sup>Hence, we define here rationality in a sense that differs from how Epictetus himself defined rationality. According to Epictetus, human beings are rational in the sense that they can use their impressions in a reflexive manner, in such a way as to determine whether their impressions are true or false. This feature of human beings was regarded by Epictetus as most promising, and making humans distinct from other animals. On Epictetus's views on rationality, see Graver (2009).

even under this definition of rationality, it is far from clear to see whether an Epictetian decision-maker can be rational. Can one be both Epictetian *and* rational? If yes, what form Epictetian rationality would take?

Although Epictetus's *Manual* and modern microeconomic theory belong to two distant domains of thought, one reason for studying the logical connections between these can be found in Sen's criticism of the concept of preferences (Sen 1976). Sen argued that preferences carry too many distinct functions in economic analysis, and encouraged economists to disentangle the various – positive and normative – dimensions of preferences. Formalizing Epictetus's precept about desires and aversions and studying the possibility of Epictetian rationality contribute to explore some normative aspects of preferences.

This article proposes two accounts of Epictetus's precept in the (acts, circumstances) space, where acts are under control, whereas circumstances are not. The indifference account (I account) of Epictetus's precept requires indifference between outcomes differing only on circumstances. The indifference between best replies account (IB account) requires indifference only between outcomes involving the best replies to circumstances. We study the implications of these precepts on the preference relation and on the existence of Epictetian rationality. We show that the I account of Epictetus's precept implies, under reflexivity, completeness and transitivity of the preference relation, that preferences satisfy independence of circumstances. This corollary is robust to weakening transitivity to Suzumura consistency (Suzumura 1976). The IB account of Epictetus's precept implies that, under reflexivity, completeness and transitivity, the preference relation satisfies robustness to dominated alternatives.

In sum, this paper does not only propose formal accounts of Epictetus's precept, but, also, explores the form of preferences that a decision-maker adopting Epictetus's precept could have. In the light of our results, the term 'Epictetian rationality' is not an oxymoron: if rationality is defined as the existence of a preference ordering, Epictetus's precept is compatible with some form of rationality. Its precise form depends on the account of Epictetus's precept: while the I account implies a preference ordering satisfying independence of circumstances, the IB account requires robustness to dominated alternatives.

Regarding the comparison of the two concepts of Epictetian rationality, our analysis provides some support for the I account over the IB account. The I account of Epictetian rationality exhibits two advantages over the IB account. First, when considering the implications of our results in the (desires, circumstances) space, it is shown that the I account excludes adaptive and counteradaptive preferences, unlike the IB account. Given the problems raised by adaptive preferences for normative analysis, this result supports the I account. Second, when considering game-theoretical contexts, we show that both the I account and the IB account of Epictetus's precept rule out the existence of prisoner's dilemmas. However, in games where all players agree on the fact that the outcome where everyone cooperates Pareto-dominates the outcome where no one cooperates, Epictetian players of the I type regard cooperation as a dominant strategy, something that does not necessarily occur under the IB account. Thus the I account of Epictetian rationality does, in that case, encourage

cooperation, something not always true under the IB account.

The present paper is in line with several branches of the literature. First, this is related to the increasingly large literature on ethical preferences (see Roemer 2010, 2019, Alger and Weibull 2013, 2016). Our analysis of Epictetusian rationality is related to the renewed attention paid by economists to ethical preferences, in particular Kantian preferences, i.e., preferences of persons seeking to choose the best generalizable acts, in line with Kant’s categorical imperative (Kant 1785).<sup>7</sup> The present article focuses not on generalizability, but on another key ethical dimension of preferences: *the exclusive focus on things under control* (*our* acts, *our* desires, *our* opinions), which implies a particular division of moral labour within the population. In line with the literature on ethical preferences, we examine also how preferences satisfying Epictetus’s precept allow to escape from the prisoner’s dilemma. Second, the present paper is linked to the economics literature focusing on the definition of ‘rationality’ (Sen 1969, 1976, Bossert and Suzumura, 2010). The contribution of this paper to that literature lies in its analysis of the conditions under which Epictetus’s precept can be part of human rationality, as defined by the existence of a preference ordering on outcomes. Third, this paper is also related to the literature in economic psychology studying adaptive preferences (Elster 1983, von Weizsäcker 2014), because we analyse the relations between several variants of Epictetusian rationality and the occurrence of (counter)adaptive preferences.

The paper is organized as follows. Section 2 presents the model. Section 3 proposes two distinct accounts of Epictetus’s precept. Their logical implications for the structure of preferences and for the possibility of Epictetusian rationality are explored in Section 4. Section 5 studies preferences over desires under Epictetus’s precept. Section 6 explores some implications of Epictetus’s precept in game-theoretical contexts. Concluding remarks are left to Section 7.

## 2 The model

A proper formalization of Epictetusian rationality must rely on Epictetus’s distinction between, on the one hand, things under the control of the person, and, on the other hand, things that are not under her control.<sup>8</sup> Let us define by  $O$  the (non-empty) set of alternative things under control of the decision-maker (e.g. acts, opinions, desires), and by  $N$  the (non-empty) set of alternative things outside control of the decision-maker (e.g. the body, property, offices and honours). Note that the sets  $O$  and  $N$  include elements of various natures, which would require separate analyses. Hence, for the sake of simplicity, we will focus here on *subsets* of  $O$  and  $N$ . Regarding things under control, we will focus on acts of the decision-maker, the set of all possible acts being  $A \subseteq O$ . As far as things outside control are concerned, we will focus on circumstances, the set of all circumstances being  $C \subseteq N$ . An outcome is here defined as a pair (act,

<sup>7</sup>See Laffont (1975). Recent works on Kantian preferences include Roemer (2010), Curry and Roemer (2012), Roemer (2019) and De Donder et al (2021).

<sup>8</sup>See the *Manual*, in particular Section I quoted above.

circumstance). The (non-empty) set of outcomes is  $Y = A \times C$ .<sup>9</sup>

In order to explore the relation between Epictetus's precept and rationality, this paper adopts a standard definition of rationality as the existence of a preference ordering defined on the set of outcomes. More formally, we will postulate the existence of a binary relation on the set of outcomes, and we will examine under which conditions this binary relation can be interpreted as a preference relation while satisfying Epictetus's ethical precept. Let  $R \subseteq Y \times Y$  be a (binary) relation on  $Y$ .

Following Bossert and Suzumura (2010, p. 32-33), we can define the symmetric factor, asymmetric factor and non-comparable factors of  $R$  as follows.

The symmetric factor  $I(R)$  of  $R$  is:

$$I(R) = \{(x, y) \in Y \times Y \mid (x, y) \in R \text{ and } (y, x) \in R\}$$

The asymmetric factor  $P(R)$  of  $R$  is:

$$P(R) = \{(x, y) \in Y \times Y \mid (x, y) \in R \text{ and } (y, x) \notin R\}$$

The non-comparable factor  $NC(R)$  of  $R$  is:

$$NC(R) = \{(x, y) \in Y \times Y \mid (x, y) \notin R \text{ and } (y, x) \notin R\}$$

Throughout this paper, we interpret  $R$  as a weak preference relation, while  $P(R)$  is interpreted as a strict preference relation,  $I(R)$  is interpreted as the indifference relation, and  $NC(R)$  is interpreted as the non-comparability relation.

In welfare economics, three fundamental properties of binary relations are reflexivity, completeness and transitivity. These are defined as follows.

Defining  $\Delta$  as the diagonal relation on  $Y$  by  $\Delta = \{(x, x) \mid x \in Y\}$ , reflexivity is defined by:

$$\Delta \subseteq R$$

Completeness is defined as follows:

$$\forall x, y \in Y \text{ such that } x \neq y, (x, y) \in R \text{ or } (y, x) \in R$$

Transitivity is defined as follows:

$$\forall x, y, z \in Y, [(x, y) \in R \text{ and } (y, z) \in R] \implies (x, z) \in R$$

When  $R$  satisfies reflexivity and transitivity,  $R$  is a quasi-ordering. When  $R$  is a quasi-ordering and satisfies, completeness,  $R$  is an ordering.

Finally, note that the transitive closure  $tc(R)$  of a binary relation  $R$  on  $Y$  is defined as follows (see Bossert and Suzumura, 2010, p. 34):

$$tc(R) = \left\{ (x, y) \mid \begin{array}{l} \exists K \in \mathbb{N} \text{ and } x^0, \dots, x^K \in Y \text{ such that:} \\ [x = x^0 \text{ and } (x^{k-1}, x^k) \in R \text{ for all } k \in \{1, \dots, K\} \text{ and } x^K = y] \end{array} \right\}$$

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<sup>9</sup>Section 5 will examine the implications of Epictetian rationality for the structure of desires over desires, by considering instead the (desires, circumstances) space.

### 3 Two accounts of Epictetus's precept

Epictetus's precept of 'wishing nothing that is outside our control' can be interpreted in several ways. This section proposes two distinct accounts of Epictetus's precept, which can be related to some passages of Epictetus's *Manual*.

Before defining these accounts formally, let us first present the attitude that Epictetus recommends when facing different life circumstances. From Epictetus's perspective, things that happen beyond our control are neither good nor bad, and do not concern us. These circumstances should be regarded merely as giving rise to opportunities to develop particular skills or abilities. From that perspective, there is neither good news nor bad news, but only circumstances that favour the flourishing some skills of the person (*Manual*, X):

On the occasion of every accident (event) that befalls you, remember to turn to yourself and inquire what power you have for turning it to use. If you see a fair man or a fair woman, you will find that the power to resist is temperance (continence). If labor (pain) be presented to you, you will find that it is endurance. If it be abusive words, you will find it to be patience. And if you have been thus formed to the (proper) habit, the appearances will not carry you along with them.

What matters is not the circumstances that prevail, but what the person can do of these circumstances. This view is also presented by Epictetus in a synthetic manner, by means of the handles metaphor (*Manual*, XLIII):

Everything has two handles, the one by which it may be borne, the other by which it may not. If your brother acts unjustly, do not lay hold of the act by that handle wherein he acts unjustly, for this is the handle which cannot be borne; but lay hold of the other, that he is your brother, that he was nurtured with you, and you will lay hold of the thing by that handle by which it can be borne.

From Epictetus's perspective, the good life requires to take things beyond our control by means of the 'good handle', that is, the 'good part' of these things. Taking the 'good part' of circumstances requires to use circumstances to make skills flourish. But the development of skills can only be carried out provided persons can free themselves from these circumstances. This is the reason why Epictetus recommends persons to free themselves from desires or aversion toward things that are not under their control (*Manual*, XIV):

He is the master of every man who has the power over the things which another person wishes or does not wish, the power to confer them on him or to take them away. Whoever then wishes to be free let him neither wish for anything nor avoid anything which depends on others: if he does not observe this rule, he must be a slave.



Epictetus underlines that things not in our power are neither good nor bad, and that individuals should feel indifferent by these things (*Manual*, XXXII):

For if it is any of the things which are not in our power, it is absolutely necessary that it must be neither good nor bad. Do not then bring to the diviner desire or aversion: if you do, you will approach him with fear. But having determined in your mind that everything which shall turn out (result) is indifferent, and does not concern you, and whatever it may be, for it will be in your power to use it well, and no man will hinder this, come then with confidence to the gods as your advisers.

According to Epictetus, persons should feel indifferent with respect to things outside their control, and should only try to make the best of these things once they occurred. Thus Epictetus's precept requires indifference with respect to circumstances. But there are - at least - two distinct ways to formalize indifference with respect to circumstances, and, hence, Epictetus's precept.

One way is to require *indifference between outcomes differing only on circumstances, everything else being equal*. This approach does justice to the general idea that circumstances are neither good nor bad. But another formalization consists of requiring indifference between the best outcomes prevailing under each set of circumstances. This second approach captures the idea that Epictetus's precept requires *individuals to do the best they can given the prevailing circumstances*. This second formalization is distinct from the previous one, but is more in line with the handles metaphor, the good life being defined as the most adequate behavior of the self given external constraints.

Let us now define these accounts formally. A first formulation of Epictetus's precept defines 'wishing nothing that is outside our control' as the requirement of indifference between outcomes that differ only on circumstances.

**Definition 1 (I account of Epictetus's precept)** *For all  $x, y \in Y$  such that  $x = (a, m)$  and  $y = (b, n)$  with  $a = b$  and  $m \neq n$ , we have:*

$$(x, y) \in I(R)$$

The I account of Epictetus's precept does justice to Epictetus's idea that things outside control are neither good nor bad, that is, that circumstances are neither good news, nor bad news. This view requires that the decision-maker is indifferent between outcomes that differ only on circumstances. Under the I account, 'wishing nothing that is outside control' is translated as 'being indifferent with respect to circumstances, everything else being equal'.

While the I account requires indifference between *all* outcomes differing only on circumstances, one may consider that this indifference requirement is too general, and that indifference should only concern the outcomes involving the *best* acts carried out under each set of circumstances. This alternative account of Epictetus's precept, which can be called indifference between best replies to circumstances (or IB account), can be defined formally as follows.

**Definition 2 (IB account of Epictetus’s precept)** *Suppose that, for a circumstance  $m$ , we have that:*

$$\exists w = (a, m) \in Y : (w, x) \in P(R) \text{ for all } x = (b, m) \in Y \text{ with } b \neq a$$

*Suppose that, for a circumstance  $n \neq m$ , we have that:*

$$\exists y = (c, n) \in Y : (y, z) \in P(R) \text{ for all } z = (d, n) \in Y \text{ with } d \neq c$$

*Then one has:*

$$(w, y) \in I(R)$$

The IB account requires not indifference between outcomes involving the same acts but different circumstances, but, only, between outcomes involving the best acts under different circumstances. This account of Epictetus’s precept is close to the handles metaphor: there is always a good way to take circumstances, and once this good way is followed, circumstances do not matter. The IB account does justice to that intuition, by requiring indifference between outcomes including the best replies to given circumstances.

To illustrate the distinction between these two accounts of Epictetus’s precept, let us consider the following example, involving two circumstances (either bodily comfort or discomfort), and two acts (either gardening or playing the flute), leading to four possible outcomes  $w$ ,  $x$ ,  $y$  and  $z$ .<sup>10</sup>

Acts	Circumstances	
	bodily comfort	bodily discomfort
gardening	$w = (\text{gardening, comfort})$	$x = (\text{gardening, discomfort})$
playing the flute	$y = (\text{flute, comfort})$	$z = (\text{flute, discomfort})$

Table 1: Four outcomes in the (acts, circumstances) space.

What does Epictetus’s precept tell us about preferences in this example? Epictetus recommends that persons adopt indifference with respect to circumstances, and only with respect to these.<sup>11</sup> From Epictetus’s perspective, things outside our control are neither good nor bad. Thus whether the person suffers from bodily discomfort or not is neither a good news nor a bad news for her. The reason is that each circumstance is favorable to the enhancement of some skills or abilities. Thus bodily discomfort should not be regarded as an inevitable obstacle making gardening or playing the flute less desirable. Actually, the core of Epictetian rationality is to highlight that *circumstances are no inevitable obstacle in life*: circumstances only invite the enhancement of alternative skills,

<sup>10</sup>Bodily discomfort is a circumstance that Epictetus has personally experienced, and that is mentioned several times in the *Manual*.

<sup>11</sup>Hence, Epictetus’s precept does not tell us, for instance, whether gardening should be preferred to playing the flute when there is no bodily discomfort.

but should not make persons better off or worse off. Only things under control can make persons better off or worse off.

One way to capture this view is to require that, for a given act, circumstances should not matter. The I account of Epictetus's precept requires thus indifference between outcomes  $w$  (gardening under bodily comfort) and  $x$  (gardening under bodily discomfort), and between outcomes  $y$  (playing the flute under bodily comfort) and  $z$  (playing the flute under bodily discomfort), that is,  $(w, x), (y, z) \in I(R)$ .

Another way of capturing Epictetus's views is to require that, for the best act carried out under each circumstance, circumstances should not matter. The IB account of Epictetus's precept requires indifference between the outcomes involving the best act for given circumstances. For instance, if gardening is preferred under bodily comfort, and playing the flute under bodily discomfort, the IB requirement implies indifference between the outcomes  $w$  (gardening under bodily comfort) and  $z$  (playing the flute under bodily discomfort):  $(w, z) \in I(R)$ .

## 4 Implications for Epictetian rationality

Let us now examine some implications of the two accounts of Epictetus's precept on the form of the preference relation  $R$ , and, hence, on the possibility - or impossibility - of Epictetian rationality. For that purpose, let us first define the following property of the relation  $R$ .

**Definition 3 (Independence of circumstances)** *The relation  $R$  on  $Y$  satisfies independence of circumstances if and only if, for any outcomes  $(a, n)$ ,  $(b, n)$ ,  $(a, m)$ ,  $(b, m) \in A \times C$  with  $a \neq b$ ,  $n \neq m$ , we have:*

$$((a, n), (b, n)) \in P(R) \iff ((a, m), (b, m)) \in P(R)$$

and

$$((a, n), (b, n)) \in I(R) \iff ((a, m), (b, m)) \in I(R)$$

Independence of circumstances imposes some structure on the symmetric and asymmetric factors of  $R$ . That property states that, when a decision-maker prefers strictly, for *given* circumstances, one act over another, the decision-maker must also exhibit the same ranking when these circumstances are replaced by *other* circumstances. Put it differently, independence of circumstances requires the robustness or invariance of the ranking of acts to circumstances.

We can now consider a first implication of the I account of Epictetus's precept on the form of Epictetian rationality.

**Proposition 1** *If the relation  $R$  on  $Y$  satisfies the I account of Epictetus's precept, and is complete, reflexive and transitive,  $R$  must satisfy independence of circumstances.*

**Proof.** See the Appendix. ■

Proposition 1 states that the I account of Epictetus's precept, if imposed on  $R$  together with standard requirements of reflexivity, completeness and transitivity, has strong implications on the structure of  $R$ :  $R$  must satisfy independence of circumstances. Thus, under the I account, the ranking of acts for given circumstances should be robust across all possible circumstances.

As such, Proposition 1 provides an important result for the existence of Epictetusian rationality. It suggests that an Epictetusian decision-maker (in the sense of the I account) can only be coherent provided he has preferences that satisfy independence of circumstances, that is, preferences over acts that are robust to changes of circumstances. Otherwise, the Epictetusian decision-maker would lack coherence. There would then be a conflict between Epictetus's precept and basic requirements of rationality, making Epictetusian rationality an oxymoron. Independence of circumstances is thus a logical corollary of the I account of Epictetus's precept and standard requirements of  $R$ .

In order to give the intuition behind that result, it is useful to turn back to the example of Table 1. As stated above, the I account of Epictetus's precept requires indifference between outcomes  $w$  and  $x$  and between outcomes  $y$  and  $z$ , that is,  $(w, x), (y, z) \in I(R)$ . What Proposition 1 tells us is that, if the relation  $R$  on  $Y = \{w, x, y, z\}$  satisfies the I account of Epictetus's precept, and is complete, reflexive and transitive, then  $R$  must also satisfy independence of circumstances, that is, we must have: either  $(w, y), (x, z) \in P(R)$  or  $(y, w), (z, x) \in P(R)$  or  $(w, y), (x, z) \in I(R)$ , but we cannot have  $(w, y), (z, x) \in P(R)$  or  $(y, w), (x, z) \in P(R)$ . Indeed, if independence of circumstances were not satisfied by  $R$ , there would be a preference cycle leading to a contradiction. For instance, if we had  $(w, x), (y, z) \in I(R)$  and  $(y, w), (x, z) \in P(R)$ , we would deduce the following:

$$\begin{aligned} (y, w) &\in R \text{ and } (w, x) \in R \implies (y, x) \in R \text{ (by transitivity)} \\ (y, x) &\in R \text{ and } (x, z) \in R \implies (y, z) \in R \text{ (by transitivity)} \end{aligned}$$

a contradiction with  $(x, z) \in P(R)$ .

When looking at Proposition 1, one may wonder whether this result is due to coherence requirements. Proposition 1 assumes that  $R$  satisfies a strong coherence requirement: transitivity. One may be curious to know whether the result of Proposition 1 still prevails once one weakens the coherence requirement on  $R$ . For that purpose, we will now consider replacing transitivity of  $R$  by either quasi-transitivity (Sen 1969) or Suzumura consistency (1976).

A relation  $R$  is quasi-transitive if and only if  $P(R)$  is transitive:

$$\forall x, y, z \in Y, (x, y) \in P(R) \text{ and } (y, z) \in P(R) \implies (x, z) \in P(R)$$

A relation  $R$  is Suzumura consistent if and only if

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Proposition 2 summarizes our results regarding the weakening of coherence requirement of the relation  $R$  under the I account of Epictetus's precept.

**Proposition 2** • *A relation  $R$  on  $Y$  that satisfies the I account of Epictetus's precept, is complete, reflexive and quasi-transitive does not necessarily satisfy independence of circumstances.*

- *A relation  $R$  on  $Y$  that satisfies the I account of Epictetus's precept, is complete, reflexive and Suzumura consistent must necessarily satisfy independence of circumstances.*

**Proof.** See the Appendix. ■

Proposition 2 suggests that weakening transitivity to quasi-transitivity does not preserve the result of Proposition 1, but that weakening transitivity to Suzumura consistency preserves that result: a relation  $R$  satisfying reflexivity, completeness, Suzumura consistency and the I account of Epictetus's precept must also satisfy independence of circumstances.

Propositions 1 and 2 suggest that, under the I account of Epictetus's precept, Epictetusian rationality can hardly be separated from independence of circumstances, that is, robustness of the ranking of acts to the prevailing circumstances. Independence of circumstances appears thus to be a central component of Epictetusian rationality under the I account of Epictetus's precept.

Let us now examine the implications of the IB account of Epictetus's precept. For that purpose, we need first define robustness to dominated alternatives.

**Definition 4 (Robustness to dominated alternatives)** *The relation  $R$  on  $Y$  satisfies robustness to dominated alternatives if and only if, for any outcomes  $(a, m)$ ,  $(b, m)$ ,  $(a, n)$ ,  $(b, n) \in Y$  with  $a \neq b$ ,  $n \neq m$ , we have:*

$$\begin{aligned} \{((a, m), (b, m)), ((a, n), (b, n))\} &\subset P(R) \iff \{((a, n), (b, m)), ((a, m), (b, n))\} \subset P(R) \\ \{((a, m), (b, m)), ((b, n), (a, n))\} &\subset P(R) \iff \{((a, m), (a, n)), ((b, n), (b, m))\} \subset P(R) \end{aligned}$$

Robustness to dominated alternatives states that, when comparing four outcomes involving two distinct acts and two distinct circumstances, if the asymmetric part of  $R$  includes two pairs of outcomes, the asymmetric part of  $R$  must also include the two pairs of outcomes obtained by shifting the dominated outcomes across the dominant outcomes. The property of robustness to dominated alternatives is close, but distinct from the double cancellation axiom.

**Proposition 3** *If the relation  $R$  on  $Y$  satisfies the IB account of Epictetus's precept, and is complete, reflexive and transitive,  $R$  must satisfy robustness to dominated alternatives.*

**Proof.** See the Appendix. ■

Proposition 3 suggests that if one adopts the IB account of Epictetus's precept, then, under standard completeness and coherence requirements of  $R$ , it must be the case that  $R$  satisfies robustness to dominated alternatives. Robustness to dominated alternatives is thus a key component of what Epictetusian rationality means under the IB account of Epictetus's precept.

Proposition 4 examines the robustness of that result to relaxing coherence requirements for the relation  $R$ .

**Proposition 4** • *A relation  $R$  on  $Y$  that satisfies the IB account of Epictetus's precept, is complete, reflexive and quasi-transitive does not necessarily satisfy robustness to dominated alternatives.*

- *A relation  $R$  on  $Y$  that satisfies the IB account of Epictetus's precept, is complete, reflexive and Suzumura consistent must necessarily satisfy robustness to dominated alternatives.*

**Proof.** See the Appendix. ■

The result of Proposition 3 is robust to weakening the transitivity requirement to Suzumura consistency. Thus robustness to dominated alternatives is a key component of Epictetusian rationality under the IB account.

In sum, this section highlights that the two accounts of Epictetus's precepts have different implications concerning the structure of the preference relation  $R$ , and, hence, concerning the form of Epictetusian rationality. The I account implies that  $R$  satisfies independence of circumstances, whereas the IB account implies that  $R$  satisfies robustness to dominated alternatives.

Which variant of Epictetusian rationality is the most appealing? In order to compare the merits of these two accounts of Epictetusian rationality, the next sections examine their respective implications in two general contexts: on the one hand, the context of preferences over desires (Section 5), and, on the other hand, the context of game-theoretical interactions (Section 6).

## 5 On desired desires

Up to now, our analysis formalized Epictetus's precept in the (acts, circumstances) space. However, Epictetus's precept extended beyond that domain, to cover all things that are either under control of the person, or, alternatively, out of her control. Among the things under control, *desires* occupy a central place in Epictetus's thought. In the *Manual*, Epictetus invites persons to reorient their desires only toward things they can control.<sup>12</sup> This focus of desires and aversions on things under control is an essential component of Epictetus's precept.

In order to explore the implications of Epictetus's precept for the structure of preferences on desires - or metapreferences - this section focuses on the space of outcomes in the (desires, circumstances) space. The (non empty) set of all desires is denoted by  $D$ , so that the (non empty) set of outcomes is here  $Z = D \times C$ . This section considers the implications of Epictetus's precept for the form of the binary relation  $R$  defined on  $Z \times Z$ .

It should be stressed that all results of the previous section, which were derived for a binary relation  $R$  defined on  $Y = A \times C$ , can also be shown to be true for a binary relation  $R$  defined on  $Z = D \times C$ . A relation  $R$  on  $Z$  that is complete, reflexive and transitive (or Suzumura consistent) and satisfies the I account of Epictetus's precept (defined on  $Z$ ) satisfies independence of

<sup>12</sup>See, in particular, sections VII, XXI, XL, XLII and L of the *Manual*.

circumstances. Moreover, a relation  $R$  on  $Z$  that is complete, reflexive and transitive (or Suzumura consistent) and satisfies the IB account of Epictetus's precept (defined on  $Z$ ) satisfies robustness to dominated alternatives.

To have an idea of the implications of these results, let us consider the example of Table 2. According to Epictetus, being elected to public office (or not) is a circumstance, and, hence, should be regarded as neither good nor bad. Thus whether the person is elected or not to a public office is neither a good news, nor a bad news, and should not make that person better off or worse off.

Desires	Circumstances	
	elected to public office	not elected to public office
desire for social recognition	$w = (\text{desire for recognition, office})$	$x = (\text{desire for recognition, no office})$
desire for tranquility	$y = (\text{desire for tranquility, office})$	$z = (\text{desire for tranquility, no office})$

Table 2: Four outcomes in the (desires, circumstances) space.

The I account of Epictetus's precept captures that intuition. Under the I account, the person who desires social recognition should be indifferent between being elected to a public office (outcome  $w$ ), and not being elected (outcome  $x$ ). That indifference can be interpreted as follows: from Epictetus's perspective, social recognition can be gained in many different manners, whether the person is elected or not to public office. There are plenty ways for the person to satisfy her desire for social recognition, and that person should thus be immunized against the particular circumstances that occur. Each set of circumstances - election or not - will provide distinct foundations for the enhancement of some particular skills required for achieving social recognition. But in no way can the election to public office be regarded as a necessary obstacle for social recognition. The core of Epictetusian rationality amounts to highlight that there exists no such inevitable obstacle. Similarly, whether a person who desires tranquility is elected or not to public office should not make her better off or worse off, because the skills to be developed to achieve tranquility differ across circumstances, and will make the person flourish in both cases. We thus have:  $(w, x), (y, z) \in I(R)$ .

From Proposition 1, we know also that  $R$ , if it is complete, reflexive and transitive, and satisfies the I account of Epictetus's precept, must also satisfy independence of circumstances. This implies, in the present case, that the ranking over desires should be invariant to the prevailing circumstances. Under the I account of Epictetus's precept, if the person prefers desiring social recognition over desiring tranquility when she is elected to public office, she should also prefer desiring social recognition over desiring tranquility when she is not elected to public office. Thus the fact of not being elected should not make the person adapt her desires towards desiring tranquility rather than social recognition.<sup>13</sup>

<sup>13</sup>The reason is that there exist various ways to achieve social recognition (under a public office or not), and that there is no reason why circumstances should make that quest for social recognition less desirable.

To examine the relation between Epictetus's precept and adaptive and counteradaptive preferences (Elster 1982), let us introduce the following definitions.

**Definition 5** • *Adaptive preferences take place when there is a variation in the ranking over desires when circumstances change, in the direction of desiring less things that are less reachable under the new circumstances.*

- *Counteradaptive preferences take place when there is a variation in the ranking over desires when circumstances change, in the direction of desiring more things that are less reachable under the new circumstances.*

In Table 2, adaptive preferences take place when the person prefers desiring social recognition over desiring tranquility when being elected to public office, and prefers desiring tranquility over desiring social recognition when not being elected. Counteradaptive preferences occur when the person prefers desiring social recognition over desiring tranquility when not being elected, but prefers desiring tranquility over desiring social recognition when being elected.

As stated in Proposition 5, the I account of Epictetus's precept, by implying, under mild richness and coherence conditions, that  $R$  satisfies independence of circumstances, rules out adaptive and counteradaptive preferences.

**Proposition 5** *When  $R$  is complete, reflexive, transitive and satisfies the I account of Epictetus's precept, there can be no (counter)adaptive preferences.*

**Proof.** By Proposition 1, the relation  $R$  satisfies independence of circumstances. This property requires, in the (desires, circumstances) space, that the ranking of outcomes involving different desires but the same circumstances should be invariant to the postulated circumstances. This invariance excludes adaptive and counteradaptive preferences. ■

Let us now consider the implications of the IB account of Epictetus's precept. The IB account of Epictetus's precept requires to feel indifferent only between the best replies to prevailing circumstances. If one assumes that desiring social reputation is more appropriate when the person is elected to public office, while desiring tranquility is more adequate when the person is not elected, we have that, under the IB account of Epictetus's precept, there should be indifference between desiring social recognition when being elected to public office, and desiring tranquility when not being elected. We thus have:  $(w, z) \in I(R)$ .

From Proposition 3, we know that if  $R$  is complete, reflexive, transitive and satisfies the IB account of Epictetus's precept,  $R$  must also satisfy robustness to dominated alternatives. That property states here that, if one desire is preferred to another desire under some circumstances, and if we also have that one desire is preferred over another under other circumstances, these desires should remain dominant when circumstances are inverted across outcomes. Unlike independence of circumstances, robustness to dominated alternatives does not rule out adaptive and counteradaptive preferences.

**Proposition 6** *When  $R$  is complete, reflexive, transitive and satisfies the IB account of Epictetus's precept, there can be (counter)adaptive preferences.*



**Proof.** See Table 2. The IB account is compatible with, for instance, the case of adaptive preferences where the person prefers desiring tranquility over desiring social recognition when not being elected to public office, and prefers desiring social recognition over desiring tranquility when being elected to public office. The IB account is also compatible with the case of counteradaptive preferences where the person prefers desiring tranquility over desiring social recognition when being elected to public office, and prefers desiring social recognition over desiring tranquility when not being elected to public office. ■

In sum, the relation between Epictetian rationality and adaptive or counteradaptive preferences depends on the adopted account of Epictetus’s precept. While the I account rules out adaptive and counteradaptive preferences (i.e., the intensity of desires varying across circumstances), the IB account allows for these adaptation of desires to circumstances. Given that the (counter)adaptive preferences phenomenon is usually regarded as disqualifying welfarist normative approaches, the fact that the I account of Epictetus’s precept rules out (counter)adaptive preferences can be interpreted as a strength of that account in comparison to the IB account of Epictetus’s precept.

## 6 Epictetian rationality and the Prisoner’s dilemma

Let us finally consider implications of Epictetian rationality in the context of interactions between humans. As stated by Epictetus, whereas the acts of a person should be the object of her desires, the acts of others are mere circumstances for her, with respect to which she should be indifferent.<sup>14</sup> But as previously discussed, there is no unique way to formalize Epictetus’s precept, and the induced indifference relations can take, here again, various forms.

To examine the implications of Epictetian rationality in game theory, let us consider a simple simultaneous cooperation game involving two players (A and B) and two strategies (does not cooperate or cooperates).

		player B	
player A		does not cooperate	cooperates
does not cooperate		$(a, w)$	$(b, x)$
cooperates		$(c, y)$	$(d, z)$

Table 3. A two-person cooperation game.

In Table 3, the letters in brackets indicate the pay-offs for each player under the associated outcome, the first letter corresponding, as usual, to the pay-off for player A, while the second letter indicates the pay-off for player B.

What are the implications of the I account and the IB account of Epictetus’s precept in this simple game?

<sup>14</sup>See, in particular, sections XI, XXI, XXVII, XXXIII and XLIV of the *Manual*.

**Proposition 7** *Under the I account and the IB account of Epictetus’s precept, Epictetian rationality rules out the existence of Prisoner’s dilemma.*

**Proof.** See the Appendix. ■

Proposition 7 states an important result for the implications of Epictetian rationality in a game-theoretical context. Epictetian rationality, whether one formalizes it under the I account or the IB account of Epictetus’s precept, rules out the existence of prisoner’s dilemmas. In other words, when individuals have Epictetian rationality, the situation where a Nash equilibrium is Pareto-dominated by a non-Nash equilibrium cannot arise.

What is the intuition behind that result? Consider first the I account. Suppose that the situation where every player cooperates Pareto-dominates the situation where each player does not cooperate. By the I account, it must also be the case that player A is indifferent between the situation where everyone cooperates and the situation where he cooperates and player B does not. Moreover, still by the I account, player A must be indifferent between the situation where no one cooperates and the situation where player A does not cooperate and player B cooperates. These two indifference relations, together with the assumption that cooperation for all Pareto-dominates non cooperation for all, imply that *cooperating must be a dominant strategy* (for player A and for player B). Hence the situation where no one cooperates cannot be a Nash equilibrium. Thus the I account of Epictetus’s precept rules out the case where a prisoner’s dilemma occurs. Consider now the IB account of Epictetus’s precept. Under all possible assumptions about the levels of pay-offs under the four outcomes of the game, the IB account of Epictetus’s precept implies indifference relations between outcomes involving the best reply of each player. These indifference relations tend, jointly with transitivity, to exclude the case where a Nash equilibrium can be Pareto-dominated by an outcome that is not a Nash equilibrium.

Proposition 7 states that Epictetian rationality rules out the existence of prisoner’s dilemmas. However, it does not tell us how Epictetian players play the cooperation game of Table 3. In order to study the implications of Epictetian rationality on the outcome of the cooperation game, let us suppose that each player knows that the outcome where all players cooperate Pareto-dominates the outcome where no player cooperates, and examine the implications of Epictetian rationality in that context. Proposition 8 summarizes our results.

**Proposition 8** *Consider the game of Table 3. Suppose that each player knows that the outcome of the game where all players cooperate Pareto-dominates the outcome where no player cooperates, that is:  $d > a$  and  $z > w$ . Then:*

- *cooperating is a dominant strategy for Epictetian players under the I account;*
- *cooperating is not necessarily a dominant strategy for Epictetian players under the IB account..*

**Proof.** See the Appendix. ■

Proposition 8 points to a major difference between the I account and the IB account of Epictetus’s precept. If the outcome where everyone cooperates Pareto-dominates the outcome where no one cooperates, cooperating is necessarily a dominant strategy for Epictetian players under the I account, but not necessarily for Epictetian players under the IB account.

The intuition is that, for Epictetian players of the I type, the common belief that the outcome where everyone cooperates Pareto-dominates the outcome where no one cooperates implies two indifference relations: one between the outcome where everyone cooperates and the outcome where they cooperate and the others do not, and, also, one between the outcome where no one cooperates and the outcome where they do not cooperate and others do. Together with the common belief, these two indifference relations imply that cooperation is a dominant strategy. But the same deduction cannot be made when considering Epictetian players under the IB account.

In sum, whereas both the I account and the IB account of Epictetus’s precept rule out the existence of prisoner’s dilemmas, the two variants of Epictetian rationality do not have the same implications regarding how players play cooperation games. If one assumes that the outcome where every player cooperates Pareto-dominates the outcome where no one cooperates, the I variant of Epictetian rationality implies that cooperating is the dominant strategy, and, hence, pushes towards cooperation. The same is not true under the IB variant of Epictetian rationality. The fact that the I account of Epictetus’s precept encourages cooperation in this context gives it an advantage over the IB account of Epictetus’s precept.

## 7 Conclusions

Epictetus highlighted that mental freedom and happiness are reached by distinguishing between things that are under control and things that are not under control, and by wishing nothing regarding the latter. We proposed two distinct accounts of Epictetus’s precept, which formalize Epictetus’s views as a requirement of indifference between all outcomes that differ only on circumstances (I account), or indifference only between outcomes involving the best reply to circumstances (IB account).

Our analysis suggests that Epictetus’s precept has clear implications on the structure of Epictetian rationality. Under the I account, a preference relation that is complete, reflexive and transitive (or Suzumura consistency) must also satisfy independence of circumstances. However, under the IB account, a preference relation that is complete, reflexive and transitive (or Suzumura consistent) must satisfy robustness to dominated alternatives. Thus our analyses, by exploring the implications of Epictetus’s precept on the structure of rationality, suggest that the term ‘Epictetian rationality’ is not an oxymoron, but has some implications on the structure of the preference ordering. Under the I account of Epictetus’s precept, Epictetian rationality requires independence of circumstances. Under the IB account, Epictetian rationality requires

robustness to dominated alternatives. Thus the precise form of Epictetian rationality varies with how one formalizes Epictetus’s ethical thought.

Regarding the comparison of the two variants of Epictetian rationality, our analyses give some advantage to the I account over the IB account. The advantage of the former is twofold. First, when exploring the implications of our results in the (desire, circumstances) space, we find that the I account excludes (counter)adaptive preferences, unlike the IB account. Given the problems caused by (counter)adaptive preferences for normative analysis, this result supports the I account. Second, both accounts of Epictetian rationality rule out the existence of prisoner’s dilemmas, but the I variant encourages cooperation more than the IB account. More precisely, when players agree on the fact that the outcome where everyone cooperates Pareto-dominates the outcome where no one cooperates, cooperation is a dominant strategy for Epictetian players of the I type, but not necessarily under the IB variant. This result provides the strongest case for Epictetian rationality under the I variant, especially at a time where the tragedy of commons is exemplified by climate change and other global disorders that emerge under the standard form of rationality. It should be stressed, however, that the avoidance of prisoner’s dilemmas arises also under other forms of rationality, such as Kantian rationality.

This leads us to a final remark. While the present paper casts some light on what Epictetus’s precept means, as well as on its implications for the possibility of Epictetian rationality, this does not provide an answer to the question: *should* a person adopt Epictetian rationality? As for Kantian rationality or any other type of rationality, this question remains without answer. Epictetian rationality is just one way of representing the world among many other frameworks, and the justification of this particular type of rationality goes beyond the scope of this paper.

## 8 References

- Alger, I., Weibull, J. (2013). Homo moralis - preference evolution under incomplete information and assortative matching. *Econometrica*, 81, 2269-2302.
- Alger, I., Weibull, J. (2016). Evolution and Kantian morality. *Games and Economic Behavior*, 98, 65-67.
- Bossert, W., Suzumura, K. (2010). *Consistency, Choice and Rationality*. Harvard University Press.
- Clark, S. (1995). Epictetus. In T. Honderich (ed.) *Oxford Companion to Philosophy*. Oxford University Press.
- Curry, P., Roemer, J. (2012). Evolutionary stability of Kantian optimization. *Hacienda Publica Espanola / Review of Public Economics*, 200, 131-148.
- De Donder, P., Llavador, H., Penczynski, S., Roemer, J., Velez, R. (2021). A game-theoretic analysis of childhood vaccination behavior: Nash versus Kant. TSE Working Papers, 21-1278.
- Elster, J. (1983). *Sour Grapes. Studies in the Subversion of Rationality*. Cambridge University Press.

- Epictetus, *Manual*. In: *A Selection from the Discourses of Epictetus With the Encheiridion*. Translated by George Long. The Gutenberg Project.
- Graver, M. (2009). Epictetus. *Stanford Encyclopedia of Philosophy*.
- Kant, E. (1785). *Groundworks for the Metaphysics of Morals*. Edited and translated by A. Wood (2002), Yale University Press.
- Laffont, J.J. (1975). Macroeconomic constraints, economic efficiency and ethics: an introduction to Kantian economics. *Economica*, 42, 430-437.
- Long, A.A. (2002). *Epictetus: A Stoic and Socratic Guide to Life*. Oxford University Press.
- Moore, G.E. (1903). *Principia Ethica*. Cambridge University Press.
- Roemer, J. (2010). Kantian equilibrium. *Scandinavian Journal of Economics*, 112: 1-24.
- Roemer, J. (2019). *How We Cooperate? A Theory of Kantian Optimization*. Yale University Press.
- Sen, A. (1969). Quasi-transitivity, rational choice and collective decisions. *Review of Economic Studies*, 36, 381-393.
- Sen, A. (1976). Rational fools: a critique of the behavioural foundations of economic theory. *Philosophy and Public Affairs*, 6, 317-344.
- Sidgwick, H. (1907). *The Methods of Ethics*. 7th edition. Hackett.
- Suzumura, K. (1976). Remarks on the theory of collective choices. *Economica*, 43, 381-390.
- von Weizsäcker, C. (2014). Adaptive preferences and institutional stability. *Journal of Institutional and Theoretical Economics*, 170, 27-36.

## 9 Appendix

### 9.1 Proof of Proposition 1

Let us prove this proposition by contradiction. For that purpose, let us take the following example with two acts ( $a$  and  $b$ ) and two circumstances ( $m$  and  $n$ ). The table below shows the four possible outcomes.

Acts	Circumstances	
	$m$	$n$
$a$	$(a, m)$	$(a, n)$
$b$	$(b, m)$	$(b, n)$

The account I of Epictetus's precept implies  $((a, m), (a, n)) \in I(R)$  as well as  $((b, m), (b, n)) \in I(R)$ . Suppose that independence of circumstances is not satisfied. For instance, one has  $((a, m), (b, m)) \in P(R)$  and  $((b, n), (a, n)) \in P(R)$ .

$((a, m), (b, m)) \in R$  and  $((a, m), (a, n)) \in R$  imply, by transitivity,

$$((a, n), (b, m)) \in R$$

$((a, n), (b, m)) \in R$  and  $((b, m), (b, n)) \in R$  imply, by transitivity,

$$((a, n), (b, n)) \in R$$

But  $((a, n), (b, n)) \in R$  contradicts  $((b, n), (a, n)) \in P(R)$ .

Similar contradictions would be reached under other violations of independence of circumstances. For instance, if one has  $((a, m), (b, m)) \in I(R)$  and  $((b, n), (a, n)) \in P(R)$ , one deduces, by means of transitivity, that  $((a, n), (b, n)) \in R$ , which contradicts  $((b, n), (a, n)) \in P(R)$ .

Finally, note that similar preference cycles and contradictions would be obtained in examples involving a larger number of acts and circumstances. Indeed, the I account of Epictetus's precept implies indifference relations between all outcomes with same acts and different circumstances, which leads to preference cycles and contradictions when the independence of circumstances is not satisfied by  $R$ .

## 9.2 Proof of Proposition 2

Let us first consider quasi-transitivity. Proposition 2 can be proved by finding a single example of relation  $R$  that satisfies account I, reflexivity, completeness, quasi transitivity and that violates independence of circumstances. Such an example is given by the following example,

Acts	Circumstances	
	$m$	$n$
$a$	$u = (a, m)$	$v = (a, n)$
$b$	$w = (b, m)$	$x = (b, n)$
$c$	$y = (c, m)$	$z = (c, n)$

Account I implies  $\{(u, v), (w, x), (y, z)\} \subset I(R)$ . We assume also a violation of independence of circumstances:  $\{(u, w), (x, v)\} \subset P(R)$ . We also assume that  $\{(w, y), (u, y), (z, x), (z, v)\} \subset P(R)$ .

We assume that  $R$  includes also other pairs, so that:

$$R = \left\{ \begin{array}{l} (u, u), (v, v), (w, w), (x, x), (y, y), (z, z), (u, v), (v, u), (w, x), (x, w), (y, z), (z, y), (y, v), (v, y), \\ (u, y), (z, v), (w, z), (z, w), (u, x), (x, u), (u, z), (z, u), (w, v), (v, w), (x, y), (y, x), (w, y), (z, x) \end{array} \right\}$$

$R$  satisfies reflexivity, since  $\{(u, u), (v, v), (w, w), (x, x), (y, y), (z, z)\} \subset R$ .

$R$  satisfied account I of Epictetus's precept by construction.

$R$  satisfies also completeness.

$R$  violates independence of circumstances by construction.

But  $R$  satisfies also quasi-transitivity. Indeed, we have:  $\{(u, w), (w, y), (u, y)\} \subset P(R)$  as well as  $\{(z, x), (x, v), (z, v)\} \subset P(R)$ , so that  $P(R)$  is transitive.

The second part of Proposition 2 can be proved by showing that a relation  $R$  that satisfies account I, reflexivity, completeness, and Suzumura consistency must necessarily satisfy independence of circumstances.

Let us prove this by contradiction and show that, when  $R$  that satisfies account I, reflexivity, completeness, and violates independence of circumstances, it violates Suzumura consistency.

Consider first the following 2x2 case.

Acts	Circumstances	
	$m$	$n$
$a$	$u = (a, m)$	$v = (a, n)$
$b$	$w = (b, m)$	$x = (b, n)$

Account I implies  $\{(u, v), (w, x)\} \subset I(R)$ . We assume also a violation of independence of circumstances:  $\{(u, w), (x, v)\} \subset P(R)$ .

Let us assume that  $R$  is reflexive, so that:  $\{(u, u), (v, v), (w, w), (x, x)\} \subset R$ .

Given the completeness of  $R$ , there are 9 possible cases:

Case 1:  $(u, x) \in P(R)$  and  $(v, w) \in P(R)$

Case 2:  $(u, x) \in P(R)$  and  $(w, v) \in P(R)$

Case 3:  $(u, x) \in P(R)$  and  $(v, w) \in I(R)$

Case 4:  $(x, u) \in P(R)$  and  $(v, w) \in P(R)$

Case 5:  $(x, u) \in P(R)$  and  $(w, v) \in P(R)$

Case 6:  $(x, u) \in P(R)$  and  $(v, w) \in I(R)$

Case 7:  $(u, x) \in I(R)$  and  $(v, w) \in P(R)$

Case 8:  $(u, x) \in I(R)$  and  $(w, v) \in P(R)$

Case 9:  $(u, x) \in I(R)$  and  $(v, w) \in I(R)$

Let us now show that in each case,  $R$  violates Suzumura consistency.

In case 1, the transitive closure of  $R$  is:

$$\begin{aligned}
tc(R) &= \left\{ \begin{array}{l} (u, u), (v, v), (w, w), (x, x), (u, v), (v, u), (w, x), (x, w), (u, w), (x, v), \\ (u, x), (v, w) \\ (v, x), (w, v), (x, u) \end{array} \right\} \\
&= R \cup \{(v, x), (w, v), (x, u)\} \supseteq R
\end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Here  $(v, w) \in P(R)$  and  $(w, v) \in tc(R)$ , thus a violation of Suzumura consistency.

In case 2, the transitive closure of  $R$  is:

$$\begin{aligned}
tc(R) &= \left\{ \begin{array}{l} (u, u), (v, v), (w, w), (x, x), (u, v), (v, u), (w, x), (x, w), (u, w), (x, v), \\ (u, x), (w, v) \\ (v, w), (v, x), (x, u) \end{array} \right\} \\
&= R \cup \{(v, x), (v, w), (x, u)\} \supseteq R
\end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Here  $(w, v) \in P(R)$  and  $(v, w) \in tc(R)$ , thus a violation of Suzumura consistency.

In case 3, the transitive closure of  $R$  is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{l} (u, u), (v, v), (w, w), (x, x), (u, v), (v, u), (w, x), (x, w), (u, w), (x, v), \\ (u, x), (w, v), (v, w) \\ (v, x), (x, u) \end{array} \right\} \\ &= R \cup \{(v, x), (x, u)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Here  $(u, x) \in P(R)$  and  $(x, u) \in tc(R)$ , thus a violation of Suzumura consistency.

In case 4, the transitive closure of  $R$  is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{l} (u, u), (v, v), (w, w), (x, x), (u, v), (v, u), (w, x), (x, w), (u, w), (x, v), \\ (x, u), (v, w) \\ (w, v), (u, x), (x, w) \end{array} \right\} \\ &= R \cup \{(w, v), (u, x), (x, w)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Here  $(v, w) \in P(R)$  and  $(w, v) \in tc(R)$ , thus a violation of Suzumura consistency.

In case 5, the transitive closure of  $R$  is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{l} (u, u), (v, v), (w, w), (x, x), (u, v), (v, u), (w, x), (x, w), (u, w), (x, v), \\ (x, u), (w, v) \\ (v, w), (w, u), (u, x) \end{array} \right\} \\ &= R \cup \{(v, w), (w, u), (u, x)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Here  $(w, v) \in P(R)$  and  $(v, w) \in tc(R)$ , thus a violation of Suzumura consistency.

In case 6, the transitive closure of  $R$  is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{l} (u, u), (v, v), (w, w), (x, x), (u, v), (v, u), (w, x), (x, w), (u, w), (x, v), \\ (x, u), (w, v), (v, w) \\ (w, u), (u, x) \end{array} \right\} \\ &= R \cup \{(w, u), (u, x)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$



Here  $(x, u) \in P(R)$  and  $(u, x) \in tc(R)$ , thus a violation of Suzumura consistency.

In case 7, the transitive closure of  $R$  is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{l} (u, u), (v, v), (w, w), (x, x), (u, v), (v, u), (w, x), (x, w), (u, w), (x, v), \\ (x, u), (u, x), (v, w) \\ (v, x), (w, u), (w, v) \end{array} \right\} \\ &= R \cup \{(v, x), (w, u), (w, v)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Here  $(v, w) \in P(R)$  and  $(w, v) \in tc(R)$ , thus a violation of Suzumura consistency.

In case 8, the transitive closure of  $R$  is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{l} (u, u), (v, v), (w, w), (x, x), (u, v), (v, u), (w, x), (x, w), (u, w), (x, v), \\ (x, u), (u, x), (w, v) \\ (v, w), (v, x), (w, u) \end{array} \right\} \\ &= R \cup \{(v, x), (w, u), (v, w)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Here  $(w, v) \in P(R)$  and  $(v, w) \in tc(R)$ , thus a violation of Suzumura consistency.

In case 9, the transitive closure of  $R$  is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{l} (u, u), (v, v), (w, w), (x, x), (u, v), (v, u), (w, x), (x, w), (u, w), (x, v), \\ (x, u), (u, x), (w, v), (v, w) \\ (v, x), (w, u) \end{array} \right\} \\ &= R \cup \{(v, x), (w, u)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Here  $(u, w) \in P(R)$  and  $(w, u) \in tc(R)$ , thus a violation of Suzumura consistency.

Together, these 9 cases cover all possible cases in the 2x2 example. In each of these cases, the relation  $R$  satisfying reflexivity, completeness, I account of Epictetus's precept and violating independence of circumstances also violate Suzumura consistency.

The previous proof can be extended to all cases involving more than 2 acts, and more than 2 circumstances. The reason why this proof can be extended lies in the fact that the I account of Epictetus's precept imposes indifference

between all outcomes differing only along circumstances. Thus, the addition of a number of circumstances does not make the preference cycle vanish, because the indifference relations across all circumstances just increase the width of the preference cycle. Hence, when independence of circumstances is violated, the I account, by requiring all these indifference relations, makes  $R$  violate Suzumura consistency. In a similar way, adding a number of acts does not prevent the occurrence of a preference cycle when independence of circumstances is violated. Thus the previous proof can be extended to cases of all dimensions concerning the number of acts and circumstances.

### 9.3 Proof of Proposition 3

Let us prove this proposition by contradiction. For that purpose, let us take the following example with two acts ( $a$  and  $b$ ) and two circumstances ( $m$  and  $n$ ). The table below shows the four possible outcomes.

Acts	Circumstances	
	$m$	$n$
$a$	$(a, m)$	$(a, n)$
$b$	$(b, m)$	$(b, n)$

Note that if  $P(R) = \emptyset$ , the property of robustness to dominated alternatives is trivially satisfied. Same thing if  $P(R) \neq \emptyset$  but  $P(R)$  includes a single pair of outcomes or two pairs of outcomes sharing the same circumstances.

Suppose now that  $\{((a, m), (b, m)), ((b, n), (a, n))\} \subset P(R)$ . The account IB of Epictetus's precept implies  $((a, m), (b, n)) \in I(R)$ . Suppose that robustness to dominated alternatives is not satisfied. For instance, one has  $((a, n), (a, m)) \in P(R)$ .

In that case, we have that  $((a, n), (a, m)) \in R$  and  $((a, m), (b, n)) \in R$ , so that, by transitivity:

$$((a, n), (b, n)) \in R$$

a contradiction with  $((b, n), (a, n)) \in P(R)$ .

Alternatively, if one had another violation of robustness to dominated alternatives, such as  $((b, m), (b, n)) \in P(R)$ , we would have that  $\{((b, m), (b, n)), ((b, n), (a, m))\} \subset R$ , which implies, by transitivity;

$$((b, m), (a, m)) \in R$$

a contradiction with  $((a, m), (b, m)) \in P(R)$ .

Finally, note that similar preference cycles and contradictions would be obtained in examples involving a larger number of acts and circumstances. Indeed, the IB account of Epictetus's precept implies indifference relations between all outcomes with the best act for given circumstances, which leads to preference cycles and contradictions when the robustness to dominated alternatives is not satisfied by  $R$ .

## 9.4 Proof of Proposition 4

Regarding the first bullet list item, this can be proved by showing an example of case where a preference relation  $R$  satisfying the IB account of Epictetus's precept, completeness, reflexivity and quasi-transitivity but violates robustness to dominated alternatives. For that purpose, let us take the following example with three acts ( $a$ ,  $b$  and  $c$ ) and two circumstances ( $m$  and  $n$ ). The table below shows the six possible outcomes.

Acts	Circumstances	
	$m$	$n$
$a$	$u = (a, m)$	$v = (a, n)$
$b$	$w = (b, m)$	$x = (b, n)$
$c$	$y = (c, m)$	$z = (c, n)$

Let us assume that  $\{(u, w), (w, y), (u, y), (v, x), (x, z), (v, z)\} \subset P(R)$ .

Account IB implies  $(u, v) \in I(R)$ .

We assume also a violation of robustness to dominated alternatives:  $(x, u) \in I(R)$ .

We assume that  $R$  includes also other pairs, so that:

$$R = \left\{ \begin{array}{l} (u, u), (v, v), (w, w), (x, x), (y, y), (z, z), (u, v), (v, u), \\ (u, w), (w, y), (u, y), (v, x), (x, z), (v, z), (u, z), (z, u), \\ (x, u), (u, x), (w, x), (x, w), (y, z), (z, y), (w, v), (v, w), (w, z), (z, w), \\ (y, v), (v, y), (y, x), (x, y) \end{array} \right\}$$

$R$  satisfies reflexivity, since  $\{(u, u), (v, v), (w, w), (x, x), (y, y), (z, z)\} \subset R$ .

$R$  satisfies the account IB of Epictetus's precept by construction.

$R$  satisfies also completeness.

$R$  violates robustness to dominated alternatives by construction.

But  $R$  satisfies also quasi-transitivity. Indeed, we have:  $\{(u, w), (w, y), (u, y)\} \subset P(R)$  as well as  $\{(z, x), (x, v), (z, v)\} \subset P(R)$ , so that  $P(R)$  is transitive.

The second part of Proposition 4 can be proved by showing that a relation  $R$  that satisfies the account IB, reflexivity, completeness, and Suzumura consistency must necessarily satisfy robustness to dominated alternatives.

Let us prove this by contradiction and show that, when  $R$  that satisfies account IB, reflexivity, completeness, and violates robustness to dominated alternatives, it violates Suzumura consistency.

Consider first the following 2x2 case.

Acts	Circumstances	
	$m$	$n$
$a$	$u = (a, m)$	$v = (a, n)$
$b$	$w = (b, m)$	$x = (b, n)$

Suppose that  $P(R) = \emptyset$ . Then robustness to dominated alternatives is trivially satisfied. Suppose that  $P(R)$  includes only one element, or elements only for a given circumstance. Here again, robustness to dominated alternatives would be trivially satisfied.

Thus the relevant examples concern cases where we have, for instance,  $\{(u, w), (v, x)\} \subset P(R)$ .

Account IB implies  $(u, v) \in I(R)$ .

Let us assume that  $R$  is reflexive, so that:  $\{(u, u), (v, v), (w, w), (x, x)\} \subset R$ .

Given the completeness of  $R$ , we also assume  $(w, x) \in I(R)$ , but a similar proof could be obtained under either  $(w, x) \in P(R)$  or  $(x, w) \in P(R)$ .

There are 8 possible cases concerning the violations of robustness to dominated alternatives:

Case 1:  $(x, u) \in P(R)$  and  $(v, w) \in P(R)$

Case 2:  $(x, u) \in P(R)$  and  $(w, v) \in P(R)$

Case 3:  $(x, u) \in P(R)$  and  $(v, w) \in I(R)$

Case 4:  $(u, x) \in P(R)$  and  $(w, v) \in P(R)$

Case 5:  $(u, x) \in P(R)$  and  $(w, v) \in I(R)$

Case 6:  $(u, x) \in I(R)$  and  $(v, w) \in I(R)$

Case 7:  $(u, x) \in I(R)$  and  $(v, w) \in P(R)$

Case 8:  $(u, x) \in I(R)$  and  $(w, v) \in P(R)$

Let us now show that in each case,  $R$  violates Suzumura consistency.

In case 1, the transitive closure of  $R$  is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{l} (u, u), (v, v), (w, w), (x, x), (u, v), (v, u), (w, x), (x, w), (u, w), (x, v), \\ (x, u), (v, w), \\ (w, v), (u, x) \end{array} \right\} \\ &= R \cup \{(w, v), (u, x)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Here  $(v, w) \in P(R)$  and  $(w, v) \in tc(R)$ , thus a violation of Suzumura consistency.

In case 2, the transitive closure of  $R$  is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{l} (u, u), (v, v), (w, w), (x, x), (u, v), (v, u), (w, x), (x, w), (u, w), (x, v), \\ (x, u), (w, v) \\ (v, w), (w, u) \end{array} \right\} \\ &= R \cup \{(v, w), (w, u)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Here  $(w, v) \in P(R)$  and  $(v, w) \in tc(R)$ , thus a violation of Suzumura consistency.

In case 3, the transitive closure of  $R$  is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{l} (u, u), (v, v), (w, w), (x, x), (u, v), (v, u), (w, x), (x, w), (u, w), (x, v), \\ (x, u), (w, v), (v, w) \\ (w, u) \end{array} \right\} \\ &= R \cup \{(w, u)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Here  $(u, w) \in P(R)$  and  $(w, u) \in tc(R)$ , thus a violation of Suzumura consistency.

In case 4, the transitive closure of  $R$  is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{l} (u, u), (v, v), (w, w), (x, x), (u, v), (v, u), (w, x), (x, w), (u, w), (x, v), \\ (u, x), (w, v) \\ (v, w), (v, x), (x, u) \end{array} \right\} \\ &= R \cup \{(v, w), (x, u), (v, x)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Here  $(w, v) \in P(R)$  and  $(v, w) \in tc(R)$ , thus a violation of Suzumura consistency.

In case 5, the transitive closure of  $R$  is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{l} (u, u), (v, v), (w, w), (x, x), (u, v), (v, u), (w, x), (x, w), (u, w), (x, v), \\ (u, x), (w, v), (v, w) \\ (v, x) \end{array} \right\} \\ &= R \cup \{(v, x)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Here  $(x, v) \in P(R)$  and  $(v, x) \in tc(R)$ , thus a violation of Suzumura consistency.

In case 6, the transitive closure of  $R$  is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{l} (u, u), (v, v), (w, w), (x, x), (u, v), (v, u), (w, x), (x, w), (u, w), (x, v), \\ (u, x), (x, u), (w, v), (v, w) \\ (w, u) \end{array} \right\} \\ &= R \cup \{(w, u)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Here  $(u, w) \in P(R)$  and  $(w, u) \in tc(R)$ , thus a violation of Suzumura consistency.

In case 7, the transitive closure of  $R$  is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{l} (u, u), (v, v), (w, w), (x, x), (u, v), (v, u), (w, x), (x, w), (u, w), (x, v), \\ (x, u), (u, x), (v, w) \\ (v, x) \end{array} \right\} \\ &= R \cup \{(v, x)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Here  $(x, v) \in P(R)$  and  $(v, x) \in tc(R)$ , thus a violation of Suzumura consistency.

In case 8, the transitive closure of  $R$  is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{l} (u, u), (v, v), (w, w), (x, x), (u, v), (v, u), (w, x), (x, w), (u, w), (x, v), \\ (x, u), (u, x), (w, v) \\ (v, w), (v, x), (w, u) \end{array} \right\} \\ &= R \cup \{(v, x), (w, u), (v, w)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Here  $(w, v) \in P(R)$  and  $(v, w) \in tc(R)$ , thus a violation of Suzumura consistency.

Together, these 8 cases cover all possible cases in the 2x2 example. In each of these cases, the relation  $R$  satisfying reflexivity, completeness, I account of Epictetus's precept and violating independence of circumstances also violate Suzumura consistency.

The previous proof can be extended to all cases involving more than 2 acts, and more than 2 circumstances. The reason why this proof can be extended lies in the fact that the IB account of Epictetus's precept imposes indifference between best replies to circumstances. Thus, the addition of a number of circumstances does not make the preference cycle vanish, because the indifference relations across all best replies to circumstances just increase the width of the preference cycle. Hence, when robustness to dominated alternatives is violated, the IB account, by requiring all indifference relations between best replies to circumstances, makes  $R$  violate Suzumura consistency. In a similar way, adding a number of acts does not prevent the occurrence of a preference cycle when robustness to dominated alternatives is violated. Thus the previous proof can be extended to cases of all dimensions concerning the number of acts and circumstances.

## 9.5 Proof of Proposition 7

		player B	
player A	does not cooperate	cooperates	
does not cooperate	$(a, w)$	$(b, x)$	
cooperates	$(c, y)$	$(d, z)$	

Table 3: A simple cooperation game.

A prisoner's dilemma consists of the existence of a Nash equilibrium that is Pareto-dominated by an outcome that is not a Nash equilibrium.

Let us show that the I account of Epictetus's precept rules out this case. Actually, if persons take the behavior of others as a circumstance, the I account implies:

$$\begin{aligned} a &= b \text{ and } c = d \\ w &= y \text{ and } x = z \end{aligned}$$

These equalities are incompatible with the existence of a prisoner's dilemma. Indeed, under these equalities, the Table 3 becomes:

		player B	
player A		does not cooperate	cooperates
does not cooperate		$(a, w)$	$(a, x)$
cooperates		$(c, w)$	$(c, x)$

Table 3: A simple cooperation game.

Given this matrix of pay-offs, it is hard to see how a Nash equilibrium could be Pareto-dominated by an outcome that is not a Nash equilibrium. Suppose, for instance, that no cooperation for all is a Nash equilibrium. Then  $a > c$  and  $w > x$ . These two inequalities prevent any other outcome of the game to Pareto-dominate the outcome where no one cooperates. The same conclusion would hold for any other Nash equilibrium given the pay-off matrix. Thus the I account of Epictetus's precept rules out the existence of prisoner's dilemmas.

Let us now consider the IB account of Epictetus's precept. Several cases should be distinguished.

Consider first the case where no cooperation is the best reply to cooperation or no cooperation. We then have:  $a > c$ ,  $b > d$  and  $w > x, y > z$  and the outcome of no cooperation is the Nash equilibrium. The IB account requires:  $a = b$  and  $w = y$ .

Under the IB account, the pay off matrix becomes:

		player B	
player A		does not cooperate	cooperates
does not cooperate		$(a, w)$	$(a, x)$
cooperates		$(c, w)$	$(d, z)$

Table 3: A simple cooperation game.

Given the inequalities  $a > c$  and  $w > x$ , the Nash equilibrium cannot be Pareto-dominated by the outcomes where one player cooperates and the other

does not. Moreover, given  $a = b > d$  and  $w = y > z$ , the outcome of cooperation for all cannot Pareto-dominate the Nash equilibrium. Thus no prisoner's dilemma exists.

Consider now the case where the best reply to no cooperation is no cooperation, and the best reply to cooperation is cooperation. We thus have:  $a > c$ ,  $w > x$ , as well as  $d > b$  and  $z > y$ . The IB account implies that:  $a = d$  and  $z = w$ . Hence the pay off matrix becomes:

		player B	
player A		does not cooperate	cooperates
does not cooperate		$(a, w)$	$(b, x)$
cooperates		$(c, y)$	$(a, w)$

Table 3: A simple cooperation game.

Given the pay off inequalities  $a > c$ ,  $w > x$ , the outcome where A cooperates and B does not cooperate cannot Pareto-dominate the outcome where no one cooperates. Similarly, the outcome where B cooperates and A does not cooperate cannot Pareto-dominate the outcome where no one cooperates. Moreover, the outcome where everyone cooperates does not Pareto-dominate the one where no one cooperates.

Consider now the case where not cooperating is the best reply to cooperating, and cooperating is the best reply to not cooperating. We thus have:  $c > a$ ,  $b > d$  as well as  $y > z$ ,  $x > w$ . From the IB account, we have then:  $c = b$  and  $y = x$ , and the pay off matrix becomes:

		player B	
player A		does not cooperate	cooperates
does not cooperate		$(a, w)$	$(b, x)$
cooperates		$(b, x)$	$(d, z)$

Table 3: A simple cooperation game.

Given  $c = b > a$ , the outcome where no one cooperates cannot Pareto-dominate the one where A cooperates and B does not cooperate. Moreover, given  $b > d$ , the outcome where everyone cooperates cannot Pareto-dominate the one where A cooperates and b does not cooperate. Finally, the outcome where A does not cooperate and B cooperates does not Pareto-dominate the one where A cooperates and B does not cooperate. Similarly, the outcome where A does not cooperate and b cooperates is not Pareto-dominated by the one where no one cooperates, since  $x > w$ . Moreover, the outcome where A does not cooperate and b cooperates is not Pareto-dominated by the one where everyone cooperates, since  $b > d$ . Thus, there can be no prisoner's dilemma under the IB account of Epictetus's precept.



## 9.6 Proof of Proposition 8

Let us consider Table 3 again, with  $d > a$  and  $z > w$ .

player A	player B	
	does not cooperate	cooperates
does not cooperate	$(a, w)$	$(b, x)$
cooperates	$(c, y)$	$(d, z)$

Table 3: A simple cooperation game.

Under the I account, for player A we must have  $a = b$  and  $c = d$ . Together with  $d > a$ , these inequalities imply  $c > a$  and  $d > b$ . Thus cooperation is a dominant strategy for player A. Concerning player B, the I account implies  $w = y$  and  $x = z$ . Together with  $z > w$ , these inequalities imply:  $x > w$  and  $z > y$ . Thus cooperation is a dominant strategy for player B. More generally, under the assumption  $d > a$ , cooperation is a dominant strategy under the I account.

Let us now consider the IB account. For player A, the inequality  $d > a$  rules out the case where cooperation is the best reply to cooperation and not cooperating is the best reply to not cooperating. Indeed, if that case were true, the IB account would imply  $d = a$ . Same result for player B. Suppose now that not cooperating is a dominant strategy. Then the IB account would imply that  $a = b$ . But since  $d > a$ , we would have also  $d > b$ , which contradicts the idea that not cooperating is a dominant strategy. A similar contradiction can be reached for player B. Consider now the case where cooperating is a dominant strategy. We then have  $c > a$  and  $d > b$ . The IB account implies  $c = d$ . These inequalities are compatible with  $d > a$ . Cooperation can also be a dominant strategy for player B. Consider now the case where cooperating is the best reply to not cooperating ( $c > a$ ), and not cooperating is the best reply to cooperating ( $b > d$ ). The IB account would then imply  $b = c$ . But then  $b = c$  and  $c > a$  imply  $b > a$ . Moreover,  $b > d$  and  $b = c$  imply  $c > d$ . These inequalities are compatible with  $d > a$ , so this case cannot be ruled out. Thus it is not necessarily the case that cooperation is a dominant strategy under the IB account.