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Trade Liberalization and Wage Inequality: New Insights from a Dynamic Trade Model with Heterogeneous Firms and
Comparative Advantage^{*}

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We develop a dynamic general equilibrium trade model with comparative advantage, heterogeneous firms, heterogeneous workers and endogenous firm entry to study wage inequality during the adjustment after trade liberalization and potential policy responses to reduce wage inequality. In the short run, inter-sectoral wage inequality is high but then recedes as more and more workers move to the expanding exporting sector. The skill premium does not change much in the short run but increases substantially in the medium and long run. Training subsidies are more powerful than sector-migration subsidies in reducing the wage inequality induced by trade liberalization.

Keywords: trade liberalization; wage inequality; adjustment dynamics

JEL Classification: E24, F11, F16, J31, J62

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1 Introduction

Among economists it is widely accepted that trade liberalization is beneficial and that free trade is (at least partially) responsible for the prosperity in developed countries. This is in stark contrast to the strong headwinds that trade liberalization is currently facing in the popular debate. The rise of populist right-wing parties in Europe, the British vote to leave the European Union and the election of Donald Trump to be the 45th US president are recent manifestations of this trend. The main popular concern is that the gains from trade liberalization are very unevenly distributed and that this leads to the destruction of jobs and the displacement of workers. To be able to defend the benefits of free trade, one needs to understand the problems it creates.

This paper contributes to this understanding by developing a dynamic model with heterogeneous firms and heterogeneous workers that allows to study the effects of trade liberalization on wages and wage inequality both in the short run and in the long run. We find that trade liberalization increases inequality both in the short run and in the long run but the sources behind these developments differ: short run inequality is driven by sectoral wage differences, while long run inequality is driven by an increase in the skill premium. These developments depend crucially on the mobility of workers across sectors and skill classes. We use the model to analyze accompanying economic policy and find that training subsidies are more powerful than sector-migration subsidies in reducing wage inequality.

Wage inequality in developed countries has been rising already for quite some time and the sources of this trend have been subject to a lively debate in the economic literature. Until recently the dispute seemed to be settled in favor of skill-biased technological change as being the main contributor to rising wage inequality (see Katz and Autor (1999)). However, while traditionally the trade of a developed country was mainly with other developed countries, the recent enormous rise in trade with low-income countries (most notably China and India) has brought a shift in the structure of trade. This shift is associated with fears that unskilled workers from developed countries might lose out from competition with workers from developing countries.

And indeed, Autor et al. (2013) show that in the United States (U.S.) increased trade with China goes hand in hand with a decrease in the share of manufacturing employment and that local labor markets that are exposed to Chinese imports suffer higher unemployment and lower wages. In a similar vein, Ebenstein et al. (2014) find that import competition is associated with wage declines. Pierce and Schott (2016) identify a direct causal link between the sharp drop in U.S. manufacturing employment after 2001 and the elimination of trade policy uncertainty that resulted from the granting of permanent normal trade relations to China in late 2000. Industries that experienced the sharpest reduction in tariff threats experienced greater employment loss due to suppressed job creation, exaggerated job destruction and a substitution away from unskilled workers. For Germany, Dauth et al. (2014) document that increased trade has led to lower employment in import-competing sectors.

These recent empirical studies concentrate on inter-sectoral comparisons, i.e., how does a worker fare in the import-competing sector relative to other sectors. Therefore, the main source of inequality is due to the wage differential between workers employed in different sectors. Another potential effect of trade liberalization is that it increases the demand for skilled workers and thereby the skill premium, the wage differential between skilled and unskilled workers. This source of wage inequality has been less prominent in the China-trade-literature, probably because these effects are harder to identify and take a longer time to materialize. Here the look through a modeling-lens can help to clarify the picture.

A comprehensive study of the effects of trade liberalization on wage inequality should, in our view, contain the following features: i) comparative advantage to study the tension between shrinking, comparative disadvantage sectors and expanding, comparative advantage sectors; ii) skilled and unskilled workers to study changes in the skill premium; iii) adjustment

dynamics, because the structure of the economy is unlikely to change over night iv) adjustment costs of workers, because it takes time and resources to switch sectors or to train; v) firm heterogeneity, endogenous firm entry and selection into export markets, because these features have been shown to be important ingredients of international trade models.

In this paper we present a model that takes account of each aspect. It is based on the model of Bernard et al. (2007) (BRS henceforth) which consists of two countries, two factors and two sectors, introducing comparative advantage into the heterogeneous firm model of Melitz (2003). Thus, it offers a framework that is rich enough to capture points i), ii) and v) above. However, the BRS analysis is restricted to the steady state and thus ignores dynamic adjustment. In our view this is a serious shortcoming when it comes to the analysis of trade with China which we know leads to the dislocation of workers. The associated adjustment costs and implications for wage inequality are key factors behind the widespread disillusionment with free international trade. The modelling of these adjustment costs is thus crucial for the understanding of these concerns and the policy measures that are aimed at facilitating the adjustment process. Further advantages of a formal dynamic modelling of the adjustment process are that it allows a) for non-linear adjustment paths, b) to analyze the role of mobility frictions, and c) to understand the relative importance of various sources of wage inequality in the short versus medium to long run. We contribute to the literature by developing a dynamic version of BRS that allows us to take account of all these aspects.

As is standard in the literature, we model trade liberalization as a decrease in the costs of trade. This leads to a shift in production. Each country specializes production in the sector where it has its comparative advantage. The rich country, being endowed with more skilled labor, specializes in the production of the skill-intensive good. This leads to a reallocation of firms and workers from the unskilled-intensive sector to the skill-intensive sector.

In our model, newly entering firms need to pay a sunk entry cost in order to enter either of two sectors (one skill-intensive, one unskilled-intensive). Upon entering they draw their productivity from a Pareto distribution. In contrast to Melitz (2003), but in line with Ghironi and Melitz (2005) (GM henceforth), firms do not have to pay fixed production costs, and therefore all newly entering firms take up production. However, firms have to pay a fixed cost of exporting if they want to serve the foreign market. This results in selection into export markets, as in Melitz (2003), i.e., only the most productive firms take up exporting. Each firm is subject to an exogenous rate of exit. This gives rise to non-trivial but tractable adjustment dynamics after trade liberalization, because existing firms keep operating and are stuck in their sector, while newly entering firms are more flexible.¹ Thus, the reallocation of firms from one sector to the other takes place via the exit of old firms. They are replaced by newly entering firms which tend to prefer the expanding sector over the shrinking sector.

Workers can be either skilled or unskilled and can be employed in either of the two sectors. Concerning the mobility of workers we distinguish various adjustment mechanisms: i) workers retire at an exogenous rate and get replaced by newly entering workers who are more flexible in their choice of sector; ii) incumbent workers might or might not be allowed to switch sectors after paying a randomly distributed sector migration cost; iii) newly entering workers might or might not be allowed to become skilled after paying a randomly distributed training cost. By simulating various combinations of these mobility assumptions we are able to highlight the role of labor adjustment costs.

In our analysis we focus on the effects of trade liberalization on wage inequality in the rich country. We mainly concentrate on two measures of wage inequality: i) inter-sectoral wage inequality, i.e., the wage differential between workers who are in the same skill class but in different sectors and ii) the skill premium, i.e., the wage differential between skilled and

¹Burstein and Melitz (2012) show that positive fixed costs of domestic production would eliminate all transitional dynamics in GM. This is not the case in our model due to the slow adjustment of workers. We nevertheless prefer to use the GM assumption that fixed costs of domestic production are zero, due to tractability and the numerical problems discussed by Chaney (2005). In section 7 we discuss the role of firm adjustment in more detail.

unskilled workers.² The effects of trade liberalization on wage inequality depend importantly on the assumption whether the supply of skilled workers is endogenous or exogenous.

If we follow the standard practice in the trade literature and assume fixed endowments with skilled and unskilled workers (as, e.g., in BRS), we find that income inequality strongly increases after trade liberalization. In the short run, this is driven by a rise in inter-sectoral wage inequality. In the medium to long run, inequality rises due to a rising skill premium. The two inequality measures have starkly different dynamics: the skill premium reacts only slowly while inter-sectoral wage inequality jumps up on impact and then slowly recedes. The reason is that the reallocation of workers reduces inter-sectoral wage inequality, while it is necessary for changes in the skill premium. Inter-sectoral wage inequality can exist only due to mobility restrictions of workers. In the short run these restrictions are substantial but over time the wage differential will induce more and more workers to choose the exporting sector over the import-competing sector. In this way the supply of workers in the exporting sector increases until the wage-differential has abated. In contrast, the skill premium is determined by the relative productivity of both types of workers which in turn depends solely on the relative share of both in the production of each sector. Due to the mobility restrictions these input-shares cannot change much in the short run. However, as workers (both skilled and unskilled) migrate to the exporting sector, their relative input shares within each sector will also change. Then the increased demand for the exporting good will result in more beneficial input shares for the skilled workers and thus a higher skill premium. The migration of workers across sectors interacts with the migration of firms across sectors. Thus, the latter does not have a direct impact on the skill premium, but an indirect one via its impact on the migration of workers.

This suggests that the inter-sectoral wage inequality identified by the recent empirical literature is only a temporary phenomenon. However, this does not imply that trade liberalization does not have long run effects on wage inequality, because the skill premium is expected to increase. This discussion demonstrates that it is crucial to use a dynamic model in order to be able to distinguish between short-run and long-run effects. In the long run, wage differentials between sectors are expected to vanish but in the short run they are an important source of wage inequality. This short run effect is completely ignored when analyzing steady state outcomes only, while the effect of the increased skill premium is exaggerated since it takes a long time to manifest.

The effects of trade liberalization on wage inequality are considerably different, when we relax the assumption of fixed endowments with skilled and unskilled workers by allowing newly entering workers to train and become skilled workers. Under fixed endowments with skilled and unskilled workers, the overall supply of skilled workers cannot react to the increased demand for skilled workers that comes along with trade liberalization. Thus, the wage of skilled workers has to go up a lot relative to the wage of unskilled workers. In contrast, with worker training the supply of skilled workers increases in response to trade liberalization, which has a dampening effect on the skill premium and thus overall wage inequality. This suggests that the common assumption of fixed endowments with skilled and unskilled workers is not an innocuous assumption, but instead crucial for the effects of trade liberalization on wage inequality.

Having established the sources of wage inequality, the next natural step is to analyze the consequences of economic policy aiming to reduce inequality in the immediate aftermath of trade liberalization and to smooth the adjustment process. We concentrate our analysis on the effects of sector-migration subsidies and training subsidies. We show that the former are successful in counteracting inter-sectoral wage inequality, but can do nothing to distribute the gains from trade more equally between skilled and unskilled workers. However, this can be accomplished by training subsidies which are therefore the

²A recent literature analyzes the effects of trade liberalization on unemployment (see, e.g., Egger and Kreickemeier (2009), Felbermayr et al. (2011), Helpman and Itzhoki (2010), Helpman et al. (2010) or Larch and Lechthaler (2011)) and stresses within-group wage inequality as a contributor to overall wage inequality (see, e.g., Helpman et al. (2010)). Given the already complicated structure of our model we concentrate on just two measures of wage inequality and leave the analysis of unemployment and within-group wage inequality for future research.

more powerful instrument to reduce the wage inequality ensuing from trade liberalization.

The rest of the paper is structured as follows. Section 2 includes a brief literature review. Section 3 describes the theoretical model. Section 4 describes the parametrization. In section 5 we describe our simulations of trade liberalization, while section 6 discusses possible policy responses. Section 7 discusses the role of firm adjustment. Section 8 provides further robustness. Finally, section 9 concludes.

2 Literature review

The recent empirical literature on trade with China and inequality was already discussed in the introduction. A related empirical literature documents the link between growing trade with developing countries and the shift in the task structure of work in many advanced economies toward non-routine interactive and analytical activity, and away from routine cognitive and manual labor. Several papers show that trade with developing countries is associated with an increased demand for non-routine task jobs and higher wage inequality between workers doing non-routine task jobs and workers doing routine task jobs. Ebenstein et al. (2014) show that the wages of domestic workers performing non-routine tasks in US multinational enterprises (MNEs) are less affected by trade with subsidiaries in developing economies than the wages of workers that perform routine tasks. Similarly, Becker et al. (2013) use micro-data on workers and trade in German MNEs to show that the ratio of non-routine-to-routine workers increases through related-party trade with developing economies. Mion et al. (2010) show that trade with less developed economies leads to skill upgrading within Belgian firms and industries. Kemeny and Rigby (2012) look beyond the decisions of multinational corporations and consider the broader impact of import competition on the task structure, and show that import competition from less developed economies is associated with sector-specific increases in the demand for non-routine tasks.

Closely related to this literature is the evidence for a polarization of the labor market since the 1990s, with the relative employment shares of low- and high-wage jobs increasing in tandem at the expense of middle-wage jobs. Goos et al. (2009), for instance, show for 16 European countries that the employment shares of managers, professionals and low-paid personal services workers increased between 1993 and 2006 at the expense of the employment shares of middling manufacturing and routine office workers. However, the employment shares increased much faster at the top than at the bottom of the wage distribution. See also Autor et al. (2003) and Autor et al. (2008) for the US evidence and Spitz-Oener (2006) for the German evidence. Both these strands of the empirical literature document a larger role for skilled, non-routine jobs which is very well in line with our theoretical results of an increased skill premium, although we do not model tasks explicitly. Another study that is in line with this result is Comite et al. (2018), who find that trade liberalization leads to an increase in the skill premium using data from World Input-Output tables and EU KLEMS data.

Concerning the theoretical literature, the introduction of models examining the role of firm heterogeneity in international trade (Melitz (2003)), has induced an innovative literature analyzing the labor market implications of trade liberalization in the context of heterogeneous firms, heterogeneous workers, and a variety of labor market frictions. With heterogeneous firms and heterogeneous workers in an industry, labor market equilibria and the labor adjustment process following trade liberalization depend on the mechanisms that match workers and firms. With heterogeneous firms, trade liberalization leads to a change in the distribution of firms serving the domestic and foreign market. With labor market frictions, ex ante identical workers may earn different wages and experience differential wage changes after trade liberalization because the change in the distribution of firms might also change the distribution of the wages that they pay. Studies with this type of models focus on the relationship between trade and within-group wage inequality. Some recent examples include Coşar et

al. (2016), Krishna et al. (2012), Almeida and Poole (2013), Helpman et al. (2012) and Itskhoki and Helpman (2014). This literature ignores the role of worker heterogeneity and the supply side of the labor market on wages.

More closely related to our paper is a recent but still small literature that focuses on between-sector adjustment. Kambourov (2009) models the sectoral reallocation after trade liberalization and highlights the importance of labor market institutions (firing costs) for this adjustment, arguing that trade liberalization should be accompanied by labor market reforms. Artuç et al. (2010), Dix-Carneiro (2014), and Coşar (2013) estimate small-open economy models to quantify the large adjustment costs that workers incur when moving between shrinking import-competing sectors and expanding exporting sectors. Kambourov (2009) and Coşar (2013) attribute these costs to sector-specific human capital that is imperfectly transferable across sectors. All of these papers use small open economy models which allows them to be estimated or calibrated in a more serious fashion. This is certainly an advantage but also implies that the terms-of-trade are exogenous and that their analysis is restricted to unilateral trade liberalization. We contribute to this literature by providing a dynamic model that does not rely on the assumption of a small open economy but instead features two large countries. This implies that the terms-of-trade are endogenous and allows us to model bilateral trade liberalization. In section 8 we demonstrate that this distinction matters: There are substantial quantitative differences between bilateral and unilateral trade liberalization, especially when the real exchange rate is exogenous. The benefits of international trade are substantially overstated in the case of an exogenous real exchange rate. This is because in a general equilibrium setting unilateral trade liberalization of imports is accompanied by a real exchange rate depreciation that splits the benefits of freer trade more equally across countries, whereas under an exogenous real exchange rate the benefits are more lopsided towards the liberalizing country.

3 Theoretical model

Our model economy consists of two countries, Home (H) and Foreign (F). Each country produces two goods, good 1 and good 2. The production of each good requires two inputs, skilled and unskilled labor. The sector that produces good 1 is skill-intensive, i.e., the production of good 1 requires relatively more skilled labor than the production of good 2. Country H has a comparative advantage in the production of the skill-intensive good due to a cheaper training technology. In the long run, all factors of production are assumed to be perfectly mobile between sectors but not across countries. In the short run, however, workers are imperfectly mobile both across sectors and across skill-classes. We discuss various scenarios with different degrees of short run mobility.

The following sequence of events takes place within a period of the model. At the beginning a constant share of incumbent workers retires and gets replaced by newly entering workers. Upon entering these workers choose their sector, and whether they want to invest in training to become skilled. At the same time incumbent workers can decide to switch sectors, subject to a sector-migration cost. Newly entering workers and switching workers immediately become active in the sector of their choice. Then production and investment in new firms take place. Newly entering firms become active with a one-period lag. Finally, consumption takes place and the next period starts. In the following section we describe all the decision problems in H; equivalent equations hold for F.

3.1 Households

In our model there are four types of workers, skilled workers in sector 1, skilled workers in sector 2 and likewise for unskilled workers. The utility of a skilled worker in sector i is given by:

$$E_t \left\{ \sum_{k=0}^{\infty} \gamma^k (1-s)^k [\log(C_{it+k}^s) - Cost_{t+k}] \right\}, \quad (1)$$

where C_{it+k}^s is the aggregate consumption bundle, γ is the subjective discount factor, s is the retirement rate, and the term $Cost_{t+k}$ summarizes the (potential) disutility from migration and training (see, e.g., Dix-Carneiro (2014)). A similar equation holds for unskilled workers. We model workers as rule-of-thumb consumers or credit-constrained consumers, i.e., they consume all their income, and can neither borrow nor lend. Thus consumption is

$$C_{it}^s = w_{it}^s + \Pi_t, \quad (2)$$

where w_{it}^s is the wage income of the workers and Π_t are the transfers of a mutual fund to be described further below.

We assume that workers are credit-constrained because that allows for simple aggregation. If workers were allowed to save and to switch sectors/skill classes, then the bond level of workers would depend on the employment history of the worker. If a worker changes her sector of employment, then her incentives to save change. Thus, her desired savings would differ from the savings of workers employed in her old sector. But her current bond holdings are determined by her old sector and, thus, are different from the bond holdings of workers in her new sector. In the transition, savings histories of workers who switch would depend on the time of the switch. This implies the necessity to keep track of the whole employment history of workers.

To avoid this problem, the macro-literature often assumes that workers pool their income within large households (see, e.g., Andolfatto (1996)). Then the consumption of a worker no longer depends on her wage earnings and the whole economy can be characterized by one representative household. However, since the focus of our analysis is precisely on wage inequality and its welfare implications, we prefer the assumption of credit-constrained workers. Nevertheless, in the working paper version of this paper we provide a model in which workers are allowed to save but cannot switch across sectors. Results do not differ significantly.

The composition of the aggregate consumption bundle is the same for all workers; only the quantity of consumed goods differs across workers. Therefore, in the following description we omit the indices for workers to avoid cumbersome notation. The aggregate consumption bundle C_t is a Cobb-Douglas composite of the goods produced in the two sectors:

$$C_t = C_{1t}^\alpha C_{2t}^{1-\alpha}, \quad (3)$$

where α is the share of good 1 in the consumption bundle for both H and F. We can obtain relative demand functions for each good from the expenditure minimization problem of a household. The implied demand functions are:

$$C_{1t} = \alpha \frac{P_t}{P_{1t}} C_t \quad \text{and} \quad C_{2t} = (1-\alpha) \frac{P_t}{P_{2t}} C_t, \quad (4)$$

where $P_t = \left(\frac{P_{1t}}{\alpha}\right)^\alpha \left(\frac{P_{2t}}{1-\alpha}\right)^{1-\alpha}$ is the price index that buys one unit of the aggregate consumption bundle C_t . The demand for each sector depends on its price relative to the price index (and thus relative to the price of the other sector) and its importance in the utility function of consumers.

Goods 1 and 2 are consumption bundles defined over a continuum of varieties Ω_i :

$$C_{it} = \left[\int_{\omega \in \Omega_i} c_{it}(\omega)^{\frac{\theta-1}{\theta}} d\omega \right]^{\frac{\theta}{\theta-1}}, \quad (5)$$

where $\theta > 1$ is the elasticity of substitution between varieties. Each variety is produced and sold by a single firm which acts as a monopolistic competitor. Furthermore, varieties are internationally traded. Thus a variety can either be produced at home or imported. At any given time, only a subset of varieties $\Omega_{it} \in \Omega_i$ is available in each sector. The consumption based price index for each sector is $P_{it} = \left[\int_{\omega \in \Omega_i} p_{it}(\omega)^{1-\theta} d\omega \right]^{\frac{1}{1-\theta}}$ and the household demand for each variety is $c_{it} = \left(\frac{p_{it}}{P_{it}} \right)^{-\theta} C_{it}$, depending on the price of the variety and total demand for the sector-good. It is useful to redefine the demand functions in terms of aggregate consumption units. To do so, let us define $\rho_{it} \equiv \frac{p_{it}}{P_t}$ and $\psi_{it} \equiv \frac{P_{it}}{P_t}$ as the relative prices for individual varieties and for the sector bundles, respectively. Then, we can rewrite the demand functions for varieties and sector bundles as $c_{it} = \left(\frac{\rho_{it}}{\psi_{it}} \right)^{-\theta} C_{it}$ and $C_{it} = \alpha \psi_{it}^{-1} C_t$, respectively.

3.2 Labor supply

In the long run, workers are perfectly mobile between sectors. However, in the short run, adjustment of workers will be slowed by adjustment costs: each worker has to pay a random, idiosyncratic sector migration cost in order to be able to switch sectors. We also assume that workers retire at rate s and are replaced by newly entering workers. These newly entering workers are free in their choice of sector and, thereby, also contribute to the reallocation of workers. Thus, even if the sector migration cost was so large that none of the incumbents would decide to switch sectors, the constant flow of more mobile new entrants would assure full adjustment of labor in the long run. Newly entering workers are also free to invest in human capital, subject to a training cost. That decision binds for the rest of their working life so incumbent unskilled workers are not able to become skilled. In the following we will first describe the sector migration decision of incumbent workers and then proceed with the sector and training decision of newly entering workers.

3.2.1 Incumbent workers

Skilled workers can move between sectors but doing so implies a non-negative idiosyncratic sector migration cost, measured in disutility,³ which is represented by an idiosyncratic ε_t^s , drawn each period from a random distribution with the cumulative distribution function $F(\varepsilon^s)$ and support on $[\varepsilon_{\min}^s, \infty)$ (implying that $F(\varepsilon_{\min}^s) = 0$). Unskilled workers can also move between sectors but they draw their sector migration cost ε_t^l from a different distribution $H(\varepsilon^l)$. Since skilled and unskilled workers face symmetric mobility decisions, it suffices to describe the decision of skilled workers. Analogous equations hold for unskilled workers.

A skilled worker decides to migrate from sector j to sector i whenever the corresponding gain in value is higher than the cost of switching sectors, i.e., if:

$$V_{it}^s - V_{jt}^s > \varepsilon_t^s, \quad (6)$$

where V_{it}^s is the value of a skilled worker in sector i , defined further below. Vice versa, a worker in sector i will migrate to sector j if $V_{jt}^s - V_{it}^s > \varepsilon_t^s$. Equation 6 defines a threshold, $\bar{\varepsilon}_t^s$, for which a worker in sector j is indifferent between switching and not switching sectors,

$$\bar{\varepsilon}_t^s = V_{it}^s - V_{jt}^s \quad (7)$$

³As in Dix-Carneiro (2014) we assume that the sector migration cost is paid in terms of utility, which has the benefit that the sector migration cost need not be traded in the market.

and the probability of her switching from sector j to sector i is⁴

$$\eta_{jit}^s = F(\max(\bar{\varepsilon}_t^s, \varepsilon_{\min}^s)). \quad (8)$$

We assume that moving costs are non-negative, i.e., $\varepsilon_{\min}^s \geq 0$. This implies that workers are only moving in one direction. Thus we are restricting ourselves to net sector migration flows which are relevant for reallocation and wage inequality.⁵

The crucial part of equation 7 is the worker's value of being employed in a specific sector, defined as:

$$V_{it}^s = \log(w_{it}^s + \Pi_t) + \gamma(1-s) \left[(1 - \eta_{ijt+1}^s) V_{it+1}^s + \int_{\varepsilon_{\min}^s}^{\max(-\varepsilon_{t+1}^s, \varepsilon_{\min}^s)} (V_{jt+1}^s - \varepsilon_{t+1}^s) dF(\varepsilon_{t+1}^s) \right]. \quad (9)$$

The worker's value is a function of contemporaneous utility and the expected discounted future value, adjusted for the probability of survival. The worker stays in the sector with probability $(1 - \eta_{ijt+1}^s)$. With probability η_{ijt+1}^s she migrates to the other sector and has to pay the migration cost ε_{t+1}^s . The integral provides the expected value of the migration cost.

3.2.2 Newly entering workers

As described above, workers are retiring at the exogenous rate s . In order to keep the working population constant, we assume that each period the retiring workers are replaced by newly entering workers We_t . Newly entering workers are unskilled. They do not only choose their sector but also whether they want to pay an idiosyncratic training cost to become skilled. We assume that workers first decide about training and then about the sector.⁶ We will start with the description of the latter decision.

Newly entering workers are not attached yet to a specific sector and are, therefore, more flexible in their choices than incumbents. We assume that the main factor influencing the choice of sector is the wage differential. Naturally, workers tend to prefer the sector that pays the higher wage.⁷ However, due to numerical reasons we assume that newly entering workers need to bear an entry cost for each sector that differs across sectors and across workers.⁸

What is relevant for the sector choice is not the absolute value of the entry cost, but the difference of the entry cost between the two sectors. Therefore, we do not model the entry cost explicitly, but only the differential across sectors. We denote this differential by ε^{Se} , with a positive number meaning that the worker can enter sector 1 relatively cheaply and a negative number meaning that the worker can enter sector 2 relatively cheaply. Every newly entering worker draws her relative entry cost from a random distribution with cumulative distribution function $G(\varepsilon^{Se})$ and support on $(-\infty, \infty)$ (unskilled workers draw their relative entry cost ε^{Le} from the random distribution $G(\varepsilon^{Le})$). We will parameterize the random distribution such that it has a negligible effect on the choice of sector, but it simplifies numerical simulations and implies a smooth transition to the new steady state.

Thus, an entering worker will choose to enter sector 1 if:

$$V_{1t}^s + \varepsilon_t^{Se} > V_{2t}^s. \quad (10)$$

⁴Note that the probability of switch from sector i to sector j is $\eta_{ijt}^s = F(\max(-\bar{\varepsilon}_t^s, \varepsilon_{\min}^s))$.

⁵Allowing for negative sector migration costs would imply positive gross flows across sectors, which are relevant empirically. However, for wage inequality only the relative supply of workers and thus net flows are relevant.

⁶Reversing the timing assumption would not have any implications for our results.

⁷This assumption is in line with empirical evidence in Ryoo and Rosen (2004) who find that the fraction of college graduates who are engineers is closely related to a measure of relative earnings prospects in engineering.

⁸Without this sector entry costs the choice of sector would not be well defined in the steady state, because workers are indifferent between the two sectors in the absence of wage differentials. Additionally, there would be no mechanism assuring that the steady state is hit, potentially implying overshooting and oscillatory dynamics.

Equation 10 defines a threshold value $\overline{\varepsilon_t^{Se}}$, for which a worker is indifferent between both sectors:

$$\overline{\varepsilon_t^{Se}} = V_{2t}^s - V_{1t}^s, \quad (11)$$

and the share of the newly entering skilled workers that choose sector 1 is:

$$\frac{Se_{1t}}{Se_{1t} + Se_{2t}} = 1 - G(\overline{\varepsilon_t^{Se}}), \quad (12)$$

where Se_{1t} is the number of skilled workers entering sector 1 and Se_{2t} is the number of skilled workers entering sector 2.

To describe the training decision of newly entering workers that takes place before the sector preference is known, we need to define the ex-ante value of a worker, i.e., the expected value of a worker before she has chosen a sector. For skilled workers this value is given by:⁹

$$V_t^s = (1 - G(\overline{\varepsilon_t^{Se}}))V_{1t}^s + G(\overline{\varepsilon_t^{Se}})V_{2t}^s. \quad (13)$$

A similar equation holds for unskilled workers. To become skilled a worker needs to pay a training cost ε^T that is drawn from a random distribution with the cumulative distribution function $\Gamma(\varepsilon^T)$ and support on $[\varepsilon_{\min}^T, \infty)$ (implying $\Gamma(\varepsilon_{\min}^T) = 0$, such that no workers would become skilled if the wage premium was not large enough). An entering worker decides to train if the value of being skilled is high enough to justify the training cost, i.e., if:

$$V_t^s - \varepsilon_t^T > V_t^l. \quad (14)$$

Equation 14 defines a threshold $\overline{\varepsilon_t^T}$ for which a worker is indifferent between training and not training:

$$\overline{\varepsilon_t^T} = V_t^s - V_t^l, \quad (15)$$

so that the probability of training is:

$$\eta_t^T = \Gamma \left[\max(\overline{\varepsilon_t^T}, \varepsilon_{\min}^T) \right]. \quad (16)$$

Thus a share η^T of all newly entering workers is skilled:

$$\frac{Se_t}{We_t} = \eta_t^T, \quad (17)$$

and the remainder is unskilled:

$$Le_t = We_t - Se_t. \quad (18)$$

3.2.3 Law of motion

Having characterized the migration decision of workers, and the sector and training choice of newly entering workers, we can now write down the law of motion for workers in specific sectors. The number of skilled workers in sector i at the end of period t equals the number of incumbents who did not switch sectors, the number of workers who switched from sector j to sector i and the new entrants, taking account of the retirement rate, such that:

$$S_{it} = (1 - s) \left((1 - \eta_{ijt}^s) S_{it-1} + \eta_{jit}^s S_{jt-1} \right) + Se_{it}. \quad (19)$$

⁹Note that the expected value of the sector-preference is zero and therefore drops out of this equation.

Equivalent equations hold for unskilled workers.

3.3 Production

There are two sectors of production in each country. A continuum of firms with heterogeneous productivity operates in each sector. To avoid cumbersome notation, we omit a firm-specific index in the following description of production. The production technology is assumed to be Cobb-Douglas in the two inputs of production:

$$Y_{it} = z_i S_{it}^{\beta_i} L_{it}^{(1-\beta_i)}, \quad (20)$$

where z_i is firm-specific productivity, while S_{it} and L_{it} is the amount of skilled and unskilled labor used by a firm. β_i is the share of skilled labor required to produce one unit of output Y_{it} in sector i . Sector 1 is assumed to be skill-intensive and sector 2 unskilled-intensive which implies that $1 > \beta_1 > \beta_2 > 0$. The labor market is assumed to be perfectly competitive implying that the real wage of both skilled and unskilled workers equals the values of their marginal products of labor. In addition, workers are perfectly mobile across all firms in a specific sector which implies that all firms within a sector pay the same wage. Consequently, relative labor demand can be described by the following condition:

$$\frac{w_{it}^s}{w_{it}^l} = \frac{\beta_i}{(1-\beta_i)} \frac{L_{it}}{S_{it}}, \quad (21)$$

which says that the ratio of the skilled real wage w_{it}^s to the unskilled real wage w_{it}^l for sector i is equal to the ratio of the marginal contribution of each factor into producing one additional unit of output. Note that this condition implies that relative demand for labor is the same across firms within a sector. Since relative demand for labor is independent of firm-specific productivity, equation 21 also holds at the sector level, i.e., relative labor demand per sector is entirely determined by the relative wages paid by firms in that sector. This condition is valid for both sectors.

Firms are heterogeneous in terms of their productivity z_i . The productivity differences across firms translate into differences in the marginal cost of production. Measured in the units of the aggregate consumption bundle, the marginal cost of production is $\frac{(w_{it}^s)^{\beta_i} (w_{it}^l)^{1-\beta_i}}{z_i}$.

Prior to entry, firms are identical and face a sunk entry cost f_{et} , which is produced by skilled and unskilled labor, equal to $f_{et} (w_{it}^s)^{\beta_i} (w_{it}^l)^{1-\beta_i}$ units of aggregate H consumption. Note that entry costs can differ between sectors due to different factor intensities and due to inter-sectoral wage differentials. Upon entry firms draw their productivity level z_i from a common distribution $G(z_i)$ with support on $[z_{min}, \infty)$. This firm productivity remains fixed thereafter. As in GM there are no fixed costs of production, so that all firms produce each period until they are hit by an exit shock, which occurs with probability $\delta \epsilon(0, 1)$ each period. This exit shock is independent of the firm's productivity level, so $G(z)$ also represents the productivity distribution of all producing firms.

Exporting goods to F is costly and involves both an iceberg trade cost $\tau_t \geq 1$ as well as a fixed cost f_{xt} , again measured in units of effective skilled and unskilled labor.¹⁰ In real terms, these costs are $f_{xt} (w_{it}^s)^{\beta_i} (w_{it}^l)^{1-\beta_i}$. The fixed cost of exporting implies that not all firms find it profitable to export, only firms above a certain productivity threshold.

All firms face a residual demand curve with constant elasticity in both H and F. They are monopolistically competitive and set prices as a proportional markup $\frac{\theta}{\theta-1}$ over marginal cost. A higher elasticity of substitution implies that consumers

¹⁰The iceberg trade costs are proportional to the value of the exported product and represent a number of different barriers to trade. These include both trade barriers that can be influenced by policy, like restrictive product standards or slow processing of imports at the border, and trade barriers that cannot be influenced by policy, like the costs of transportation. We follow the standard practice in the literature and model trade liberalization as a decrease in the iceberg trade cost.

can more easily find substitutes for a given variety implying stronger competition and lower markups. Let $p_{d,it}(z_i)$ and $p_{x,it}(z_i)$ denote the nominal domestic and export prices of a H firm in sector i . We assume that the export prices are denominated in the currency of the export market. Prices in real terms, relative to the price index in the destination market are then given by:

$$\rho_{d,it}(z_i) = \frac{p_{d,it}(z_i)}{P_t} = \frac{\theta}{\theta - 1} \frac{(w_{it}^s)^{\beta_i} (w_{it}^l)^{1-\beta_i}}{z_i}, \rho_{x,it}(z_i) = \frac{p_{x,it}(z_i)}{P_t^*} = \frac{1}{Q_t} \tau_t \rho_{d,it}(z_i), \quad (22)$$

where $Q_t \equiv \frac{P_t^*}{P_t}$ is the real exchange rate. Profits, expressed in units of the aggregate consumption bundle of the firm's location are $d_{it}(z_i) = d_{d,it}(z_i) + d_{x,it}(z_i)$, where

$$d_{d,it}(z_i) = \frac{1}{\theta} \left(\frac{\rho_{d,it}(z_i)}{\psi_{it}} \right)^{1-\theta} \alpha_i C_t \quad (23)$$

$$d_{x,it}(z_i) = \begin{cases} \frac{Q_t}{\theta} \left(\frac{\rho_{x,it}(z_i)}{\psi_{it}^*} \right)^{1-\theta} \alpha_i^* C_t^* - f_{xt} (w_{it}^s)^{\beta_i} (w_{it}^l)^{1-\beta_i}, & \text{if firm } z \text{ exports} \\ 0 & \text{otherwise.} \end{cases} \quad (24)$$

A firm will export if and only if it earns non-negative profits from doing so. For H firms, this will be the case if their productivity draw z_i is above some cutoff level $z_{x,it} = \inf\{z_i : d_{x,it} > 0\}$. We assume that the lower bound productivity z_{min} is identical for both sectors and low enough relative to the fixed costs of exporting so that $z_{x,it}$ is above z_{min} . Firms with productivity between z_{min} and $z_{x,it}$, serve only their domestic market.

3.3.1 Firm Averages

In every period a mass $N_{d,it}$ of firms produces in sector i of country H. These firms have a distribution of productivity levels over $[z_{min}, \infty)$ given by $G(z)$, which is identical for both sectors and both countries. The number of exporters is $N_{x,it} = [1 - G(z_{x,it})] N_{d,it}$. It is useful to define two average productivity levels, an average $\tilde{z}_{d,it}$ for all producing firms in sector i of country H and an average $\tilde{z}_{x,it}$ for all exporters in sector i of country H:

$$\tilde{z}_{d,it} = \left[\int_{z_{min}}^{\infty} z^{\theta-1} dG(z) \right]^{\frac{1}{(\theta-1)}}, \quad \tilde{z}_{x,it} = \left[\int_{z_{x,it}}^{\infty} z^{\theta-1} dG(z) \right]^{\frac{1}{(\theta-1)}}.$$

As in Melitz (2003), these average productivity levels summarize all the necessary information about the productivity distributions of firms.

We can redefine all the prices and profits in terms of these average productivity levels. The average nominal price of H firms in the domestic market is $\tilde{p}_{d,it} = p_{d,it}(\tilde{z}_{d,it})$ and in the foreign market is $\tilde{p}_{x,it} = p_{x,it}(\tilde{z}_{x,it})$. The price index for sector i in H reflects prices for the $N_{d,it}$ home firms and F's exporters to H. Then, the price index for sector i in H can be written as $P_{it}^{1-\theta} = [N_{d,it} (\tilde{p}_{d,it})^{1-\theta} + N_{x,it}^* (\tilde{p}_{x,it})^{1-\theta}]$. Written in real terms of aggregate consumption units this becomes $\tilde{\psi}_{it}^{1-\theta} = [N_{d,it} (\tilde{\rho}_{d,it})^{1-\theta} + N_{x,it}^* (\tilde{\rho}_{x,it})^{1-\theta}]$, where $\tilde{\rho}_{d,it} = \rho_{d,it}(\tilde{z}_{d,it})$ and $\tilde{\rho}_{x,it} = \rho_{x,it}(\tilde{z}_{x,it})$ are the average relative prices of H's producers and F's exporters.

Similarly we can define $\tilde{d}_{d,it} = d_{d,it}(\tilde{z}_{d,it})$ and $\tilde{d}_{x,it} = d_{x,it}(\tilde{z}_{x,it})$ such that $\tilde{d}_{it} = \tilde{d}_{d,it} + [1 - G(z_{x,it})] \tilde{d}_{x,it}$ are average total profits of H firms in sector i .

3.3.2 Firm Entry and Exit

The firms are owned by a mutual fund that invests in new firms, collects all the profits, and distributes any surplus in a lump-sum fashion to the households.¹¹ The mutual fund acts on behalf of the whole population and therefore uses the stochastic discount factor $\gamma^{s-t} (C_s/C_t)^{-1}$ to discount between periods s and t .

In every period there is an unbounded mass of prospective new firms in both sectors and both countries. We assume that entrants at time t only start producing at time $t+1$, which introduces a one-period time-to-build lag in the model. The exogenous exit shock occurs at the end of each period, after entry and production. Thus, a proportion δ of new entrants will never produce. The mutual fund is forward looking and computes the expected post-entry value of a firm entering sector i in period t as the present discounted value of its expected stream of profits $\{\tilde{d}_{is}\}_{s=t+1}^{\infty}$,

$$\tilde{v}_{it} = E_t \sum_{s=t+1}^{\infty} \left[\gamma^{s-t} (1-\delta)^{s-t} \left(\frac{C_s}{C_t} \right)^{-1} \tilde{d}_{is} \right]. \quad (25)$$

This also corresponds to the average value of incumbent firms after production has occurred. The fund discounts future profits using the aggregate stochastic discount factor adjusted for the probability of firm survival $1-\delta$. Note that equation 25 can be written in recursive form as:

$$\tilde{v}_{it} = \gamma(1-\delta)E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-1} (\tilde{v}_{it+1} + \tilde{d}_{it+1}) \right]. \quad (26)$$

The value of a firm today equals the discounted sum of its profits next period and its value next period. Entry occurs until the average firm value is equal to the entry cost:

$$\tilde{v}_{it} = f_{et} (w_{it}^s)^{\beta_i} (w_{it}^l)^{1-\beta_i}. \quad (27)$$

If the value of a firm were larger than the cost of entry, new firms would be created, increasing competition and thus lowering the profits of all firms and thus the value of a firm. This process continues until the value of a firm equals the cost of entry. The surplus of the mutual fund, given by firm profits minus investment in new firms, is distributed in a lump-sum fashion to the households:

$$\Pi_t ENDOW = \tilde{d}_{1t} N_{d,1t} + \tilde{d}_{2t} N_{d,2t} - \tilde{v}_{1t} N_{e,1t} - \tilde{v}_{2t} N_{e,2t} \quad (28)$$

where $ENDOW = L_t + S_t$ is the total working population. Finally, the number of firms active today equals the number of surviving firms active in the previous period plus the number of surviving new entrants:

$$N_{d,it} = (1-\delta)(N_{d,it-1} + N_{e,t-1}). \quad (29)$$

3.3.3 Productivity Distribution of Firms

Productivity z follows a Pareto distribution with lower bound z_{min} and shape parameter $k > \theta - 1$: $G(z) = 1 - \left(\frac{z_{min}}{z} \right)^k$. Let $\nu = \left\{ \frac{k}{[k-(\theta-1)]} \right\}^{\frac{1}{\theta-1}}$, then average productivities are

$$\tilde{z}_{d,it} = \nu z_{min} \text{ and } \tilde{z}_{x,it} = \nu z_{x,it}. \quad (30)$$

¹¹In contrast to Melitz (2003) it is not the households that invest themselves in new firms due to the credit constraints we are assuming.

The share of exporting firms in sector i in H is

$$\frac{N_{x,it}}{N_{d,it}} = 1 - G(z_{x,it}) = \left(\frac{\nu z_{min}}{\tilde{z}_{x,it}} \right)^k. \quad (31)$$

Together with the zero export profit condition for the cutoff firm, $d_{x,it}(z_{x,it}) = 0$, this implies that average export profits must satisfy

$$\tilde{d}_{x,it} = (\theta - 1) \left(\frac{\nu^{\theta-1}}{k} \right) f_{xt} (w_{it}^s)^{\beta_i} (w_{it}^l)^{1-\beta_i}. \quad (32)$$

3.4 Market Clearing Conditions, Aggregate Accounting and Trade

Market clearing requires that total production in each sector must equal total income so that:

$$N_{d,it} \left(\frac{\tilde{\rho}_{d,it}}{\tilde{\psi}_{it}} \right)^{1-\theta} \alpha_i C_t + Q_t N_{x,it} \left(\frac{\tilde{\rho}_{x,it}}{\tilde{\psi}_{it}^*} \right)^{1-\theta} \alpha_i C_t^* + \tilde{v}_{it} N_{e,it} = w_{it}^s S_{it} + w_{it}^l L_{it} + \tilde{d}_{it} N_{d,it}. \quad (33)$$

Total production of the sector (on the left hand side) includes the production of the aggregate consumption bundle (both for the domestic market and the foreign market) and the production of new firms. Total income generated by the sector (on the right hand side) includes wage earnings and profits.

Trade is balanced at any time so that the value of H exports must equal the value of F exports such that:¹²

$$\begin{aligned} Q_t N_{x,1t} \left(\frac{\tilde{\rho}_{x,1t}}{\tilde{\psi}_{1t}^*} \right)^{1-\theta} \alpha C_t^* + Q_t N_{x,2t} \left(\frac{\tilde{\rho}_{x,2t}}{\tilde{\psi}_{2t}^*} \right)^{1-\theta} (1-\alpha) C_t^* = \\ N_{x,1t}^* \left(\frac{\tilde{\rho}_{x,1t}^*}{\tilde{\psi}_{1t}} \right)^{1-\theta} \alpha C_t + N_{x,2t}^* \left(\frac{\tilde{\rho}_{x,2t}^*}{\tilde{\psi}_{2t}} \right)^{1-\theta} (1-\alpha) C_t. \end{aligned} \quad (34)$$

3.5 Summary

Table 1 summarizes the equilibrium conditions of the model. The equations in the table constitute a system of 119 equations in 119 endogenous variables. The system includes 19 sector-specific equations for sector $i = 1, 2$ for H , with an equivalent set holding for F , making 76 sector-specific equations in total, and 20 non-sector specific equations for H and equivalent for F , 41 non-sector specific equations in total including the balanced trade condition. The sector-specific endogenous variables for H include: $\tilde{d}_{it}, \tilde{d}_{d,it}, \tilde{d}_{x,it}, N_{e,it}, N_{d,it}, N_{x,it}, z_{x,it}, \tilde{v}_{it}, \tilde{\rho}_{d,it}, \tilde{\rho}_{x,it}, \tilde{\psi}_{it}, w_{it}^s, w_{it}^l, C_{it}^s, C_{it}^l, V_{it}^s, V_{it}^l, S_{it}, L_{it}, Se_{it}, Le_{it}, \eta_{ijt}^s, \eta_{ijt}^l$, 23 for each sector and 46 in total for H , with another 46 equivalent variables describing F . The non-sector specific variables for H are 13 and include: $C_t, \Pi_t, \bar{\varepsilon}_t^s, \bar{\varepsilon}_t^l, \bar{\varepsilon}_t^{Se}, \bar{\varepsilon}_t^{Le}, \bar{\varepsilon}_t^T, \eta_t^T, V_t^s, V_t^l, Se_t, Le_t, We_t$, with equivalent 13 relevant for F . Thus, we have 59 variables for H and 59 for F , and adding the real exchange rate Q_t makes a total of 119. The model also features two exogenous policy variables: the iceberg trade costs, τ_t and τ_t^* , which we use to simulate changes in trade policy.

Note that the model summarized in table 1 features worker training upon entry and worker sector-switching but only for skilled workers. It nests a simpler version of our model without worker training where the worker training equations in the last section of the table no longer hold, which results in a total of 14 equations dropping out of the model. Without training 7 variables are no longer determined/endogenous: $\bar{\varepsilon}_t^T, \eta_t^T, V_t^s, V_t^l, S_t, L_t, We_t$ for H and an equivalent 7 for F . Finally, in order to turn off sector switching for skilled workers, we can simply set $\bar{\varepsilon}_{ijt}^s$ and η_{ijt}^s equal to zero in both countries.

Throughout the text we have imposed the following parameter restrictions on our model: $0 < \beta_2 < \beta_1 < 1$, $0 < \alpha < 1$,

¹²Under the income-pooling assumption, we ran simulations allowing for trade in international bonds and unbalanced trade but the movements in the trade balance were negligible.

$0 < \gamma < 1, 0 < s < 1, 0 < \delta < 1, f_e > 0, f_e^* > 0, f_x > 0, f_x^* > 0, k > 0, z_{min} > 0, \theta > 1, k > \theta - 1, 0 < S < ENDOW, 0 < L < ENDOW, 0 < S^* < ENDOW^*, 0 < L^* < ENDOW^*$. They are necessary to hold for the model to have a unique and stable solution. These parameter restrictions are also necessary to ensure the existence and uniqueness of our steady state. Section A in the appendix gives a formal proof that our model has a unique steady state and discusses the implications of different parameter restrictions. Two more necessary conditions are that the probability distributions for the sector migration cost $F(\varepsilon_t^s)$ and for the training cost $\Gamma(\varepsilon_t^T)$ be only defined for non-negative values and give zero probability to negative values, like the Pareto or the Exponential distribution, for example.

	Model Summary	Equation #
Sector-specific equations		
Price index	$\tilde{\psi}_{it}^{1-\theta} = [N_{d,it} (\tilde{\rho}_{d,it})^{1-\theta} + N_{x,it}^* (\tilde{\rho}_{x,it}^*)^{1-\theta}]$	1
Domestic price	$\tilde{\rho}_{d,it} = \frac{\theta}{\theta-1} \frac{(w_{it}^s)^{\beta_i} (w_{it}^l)^{1-\beta_i}}{\tilde{z}_d}$	2
Export price	$\tilde{\rho}_{x,it} = \frac{\tau_t}{Q_t} \frac{\theta}{\theta-1} \frac{(w_{it}^s)^{\beta_i} (w_{it}^l)^{1-\beta_i}}{\tilde{z}_{xi}}$	3
Average profit	$\tilde{d}_{it} = \tilde{d}_{d,it} + \frac{N_{x,it}}{N_{d,it}} \tilde{d}_{x,it}$	4
Average domestic profit	$\tilde{d}_{d,it} = \frac{1}{\theta} \left(\frac{\tilde{\rho}_{d,it}}{\tilde{\psi}_{it}} \right)^{1-\theta} \alpha_i C_t$	5
Average export profit	$\tilde{d}_{x,it} = \frac{Q_t}{\theta} \left(\frac{\tilde{\rho}_{x,it}}{\tilde{\psi}_{it}^*} \right)^{1-\theta} \alpha_i C_t^* - f_{xt} (w_{it}^s)^{\beta_i} (w_{it}^l)^{1-\beta_i}$	6
Free entry	$\tilde{v}_{it} = f_{et} (w_{it}^s)^{\beta_i} (w_{it}^l)^{1-\beta_i}$	7
Export cut-off	$\tilde{d}_{x,it} = \left(\frac{\theta-1}{k-(\theta-1)} \right) f_{xt} (w_{it}^s)^{\beta_i} (w_{it}^l)^{1-\beta_i}$	8
Share of exporting firms	$\frac{N_{x,it}}{N_{d,it}} = \left(\frac{\tilde{z}_d}{\tilde{z}_{x,it}} \right)^k$	9
Number of firms	$N_{d,it} = (1-\delta)(N_{d,it-1} + N_{e,t-1})$	10
Average firm value	$\tilde{v}_{it} = \gamma(1-\delta)E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-1} \left(\tilde{v}_{it+1} + \tilde{d}_{it+1} \right) \right]$	11
Sector accounting	$N_{d,it} \left(\frac{\tilde{\rho}_{d,it}}{\tilde{\psi}_{it}} \right)^{1-\theta} \alpha_i C_t + Q_t N_{x,it} \left(\frac{\tilde{\rho}_{x,it}}{\tilde{\psi}_{it}^*} \right)^{1-\theta} \alpha_i C_t^* + \tilde{v}_{it} N_{e,it} = w_{it}^s S_{it} + w_{it}^l L_{it} + \tilde{d}_{it} N_{d,it}$	12
Relative demand for labor	$\frac{w_{it}^s}{w_{it}^l} = \frac{\beta_i}{(1-\beta_i)} \frac{L_{it}}{S_{it}}$	13
Household budget constraints	$C_{it}^s = w_{it}^s + \Pi_t$	14
	$C_{it}^l = w_{it}^l + \Pi_t$	15
Worker value functions	$V_{it}^s = \log(C_{it}^s) + \gamma(1-s) \left[(1-\eta_{ijt+1}^s) V_{it+1}^s + \int_{\varepsilon_{\min}^s}^{max(-\varepsilon_{t+1}^s, \varepsilon_{\min}^s)} (V_{jt+1}^s - \varepsilon_{t+1}^s) dF(\varepsilon_{t+1}^s) \right]$	16

	$V_{it}^l = \log(C_{it}^l) + \gamma(1-s) \left[(1 - \eta_{ijt+1}^l) V_{it+1}^l + \int_{\varepsilon_{\min}^l}^{\max(-\varepsilon_{t+1}^l, \varepsilon_{\min}^l)} (V_{jt+1}^l - \varepsilon_{t+1}^l) dF(\varepsilon_{t+1}^l) \right]$	17
Worker flows	$S_{it} = (1-s) \left((1 - \eta_{ijt}^s) S_{it-1} + \eta_{jit}^s S_{jt-1} \right) + Se_{it}$	18
	$L_{it} = (1-s) \left((1 - \eta_{ijt}^l) L_{it-1} + \eta_{jit}^l L_{jt-1} \right) + Le_{it}$	19
Non-sector-specific equations		
Aggregate price index	$\left(\frac{\tilde{\psi}_{1t}}{\alpha} \right)^\alpha \left(\frac{\tilde{\psi}_{2t}}{1-\alpha} \right)^{1-\alpha} = 1$	20
Balanced trade	$Q_t N_{x,1t} \left(\frac{\tilde{\rho}_{x,1t}}{\tilde{\psi}_{1t}^*} \right)^{1-\theta} \alpha C_t^* + Q_t N_{x,2t} \left(\frac{\tilde{\rho}_{x,2t}}{\tilde{\psi}_{2t}^*} \right)^{1-\theta} (1-\alpha) C_t^* =$ $N_{x,1t}^* \left(\frac{\tilde{\rho}_{x,1t}}{\tilde{\psi}_{1t}^*} \right)^{1-\theta} \alpha C_t + N_{x,2t}^* \left(\frac{\tilde{\rho}_{x,2t}}{\tilde{\psi}_{2t}^*} \right)^{1-\theta} (1-\alpha) C_t$	21
Firm transfers	$\Pi_t ENDOW = \tilde{d}_{1t} N_{d,1t} + \tilde{d}_{2t} N_{d,2t} - \tilde{v}_{1t} N_{e,1t} - \tilde{v}_{2t} N_{e,2t}$	22
Sector switching decision for incumbent workers	$\bar{\varepsilon}_t^s = V_{it}^s - V_{jt}^s$	23
	$\bar{\varepsilon}_t^l = 0$	24
	$\eta_{21t}^s = F(\max(\bar{\varepsilon}_t^s, \varepsilon_{\min}^s)) \text{ and } \eta_{12t}^s = F(\max(-\bar{\varepsilon}_t^s, \varepsilon_{\min}^s))$	25
	$\eta_{12t}^l = \eta_{21t}^l = 0$	26
Worker entry per sector	$\bar{\varepsilon}_t^{Se} = V_{2t}^s - V_{1t}^s$	27
	$\frac{Se_{1t}}{Se_t} = 1 - G(\bar{\varepsilon}_t^{Se})$	28
	$\bar{\varepsilon}_t^{Le} = V_{2t}^l - V_{1t}^l$	29
	$\frac{Le_{1t}}{Le_t} = 1 - G(\bar{\varepsilon}_t^{Le})$	30
	$Se_t = Se_{1t} + Se_{2t}$	31
	$Le_t = Le_{1t} + Le_{2t}$	32
Worker training	$V_t^s = \frac{Se_{1t}}{Se_t} V_{1t}^s + \frac{Se_{2t}}{Se_t} V_{2t}^s$	33
	$V_t^l = \frac{Le_{1t}}{Le_t} V_{1t}^l + \frac{Le_{2t}}{Le_t} V_{2t}^l$	34
	$\bar{\varepsilon}_t^T = V_t^s - V_t^l$	35
	$\eta_t^T = \Gamma[\max(\bar{\varepsilon}_t^T, \varepsilon_{\min}^T)]$	36
	$\frac{Se_t}{We_t} = \eta_t^T$	37
	$Le_t = We_t - Se_t$	38
	$We_t = sENDOW$	39

The equations above hold for sector $i = 1, 2$ for country H. An equivalent set of equations holds for country F, excluding the balanced trade condition. Variables with an asterisk pertain to country F. Note that due to the absence of fixed costs of production $\tilde{z}_d = (\frac{k}{k-(\theta-1)})^{\frac{1}{\theta-1}} z_{min}$ is given exogenously.

3.6 Measures for wage inequality

In order to analyze the effect of trade liberalization on wage inequality, we define a number of wage inequality measures. First, we define two measures of wage inequality across sectors. They measure the relative percentage difference across sectoral wages for skilled and unskilled workers

$$\begin{aligned} IndexS_t &= \left(\frac{w_{1t}^s}{w_{2t}^s} - 1 \right) 100, \\ IndexL_t &= \left(\frac{w_{1t}^l}{w_{2t}^l} - 1 \right) 100. \end{aligned}$$

Note that these indices are zero at the steady state, due to long run mobility across sectors. It is one of the advantages of our dynamic model that it can capture temporary increases in inter-industry inequality.

To measure wage inequality across the skill classes we define a skill premium for each sector and an average skill premium. The skill premium for sector i is defined as the percentage difference between the wage of skilled and unskilled workers

$$Skill_{it} = \left(\frac{w_{it}^s}{w_{it}^l} - 1 \right) 100.$$

To define the average skill premium for each country, we use the average wage of skilled workers, $w_t^s = \frac{S_{1t}}{S_t} w_{1t}^s + \frac{S_{2t}}{S_t} w_{2t}^s$, and the average wage of unskilled workers, $w_t^l = \frac{L_{1t}}{L_t} w_{1t}^l + \frac{L_{2t}}{L_t} w_{2t}^l$ to obtain

$$Skill_t = \left(\frac{w_t^s}{w_t^l} - 1 \right) 100.$$

Note that the average wage in country H is $w_t = \frac{S_{1t}}{S_t + L_t} w_{1t}^s + \frac{S_{2t}}{S_t + L_t} w_{2t}^s + \frac{L_{1t}}{S_t + L_t} w_{1t}^l + \frac{L_{2t}}{S_t + L_t} w_{2t}^l$.

Finally, we measure aggregate wage inequality for each country by constructing a theoretical Gini index, which is a standard measure of inequality. The Gini index measures the extent to which the distribution of wages among the different groups of workers within each country deviates from a perfectly equal distribution. A Gini index of 0 means perfect equality, while an index of 1 means perfect inequality. The Gini coefficient is defined as half the relative mean difference of a wage distribution. The Gini coefficient for country H is

$$\begin{aligned} Gini_t &= \frac{1}{2w_t} \frac{1}{(S_t + L_t)^2} (2S_{1t}S_{2t} |w_{1t}^s - w_{2t}^s| + 2L_{1t}L_{2t} |w_{1t}^l - w_{2t}^l| \\ &\quad + 2S_{1t}L_{1t} |w_{1t}^s - w_{1t}^l| + 2S_{2t}L_{2t} |w_{2t}^s - w_{2t}^l| + 2S_{1t}L_{2t} |w_{1t}^s - w_{2t}^l| + 2S_{2t}L_{1t} |w_{2t}^s - w_{1t}^l|). \end{aligned} \quad (35)$$

4 Parametrization

This section describes the parametrization of the model that we use for the numerical simulations. In most aspects we follow GM and BRS. We interpret each period as a quarter and, set the household discount rate γ to 0.99, the standard choice for quarterly business cycle models. We set the elasticity of substitution between varieties to $\theta = 3.8$, based on the estimates from plant-level U.S. manufacturing data in Bernard et al. (2003). We set the parameters of the Pareto distribution to $z_{\min} = 1$ and $k = 3.4$, respectively. This choice satisfies the condition for finite variance of log productivity: $k > \theta - 1$.

Changing the sunk cost of firm entry f_e only re-scales the mass of firms in an industry. Thus, without loss of generality we can normalize it so that $f_e = 1$. Similarly to GM, we set the fixed cost of exporting f_x to 23.5 percent of the per-period, amortized flow value of the sunk entry costs, $[1 - \gamma(1 - \delta)] / [\gamma(1 - \delta)] f_e$. We set the size of the exogenous firm exit probability to $\delta = 0.025$, to match the level of 10 percent job destruction per year in the US. These choices of parameter values are based on GM.

Iceberg trade costs are calibrated to deliver a share of US trade with China to US production of 6% before liberalization and 27% after liberalization. Specifically, these shares correspond to the average share of US-China manufacturing trade in manufacturing value added for the US over the pre-liberalization period of 1996-2000 before China's accession to the WTO in 2001, and for the post liberalization period 2011-2015. Assuming symmetric trade costs, this share implies trade costs of $\tau = \tau^* = 2.5$ before liberalization and $\tau = \tau^* = 1.56$ after liberalization.

To focus on comparative advantage, we assume that all industry parameters are the same across industries and countries except factor intensity (β_i). We calibrate (β_i) based on our own calculations using the NBER-CES Manufacturing Industry Database,¹³ which provides annual industry-level data from 1958-2009 on output, employment, payroll and other input costs, investment, capital stocks, TFP, and various industry-specific price indexes. We aggregate the data set to feature 19 3-digit NAICS industries and then classify these industries based on their revealed comparative advantage. In order to distinguish between comparative advantage and comparative disadvantage sectors, we use a measure of revealed comparative advantage (RCA) which takes account of exports and imports at the sector level. *RCA* is defined as the ratio of the export share of the sector in total manufacturing exports over the import share of the sector in total manufacturing imports ($RCA = \frac{Ex_{it}/Ex_{manuf,t}}{Im_{it}/Im_{manuf,t}}$), with $RCA > 1$ referring to comparative advantage sectors and $RCA < 1$ referring to comparative disadvantage sectors.¹⁴ In order to calibrate factor intensities of each sector, we calculate the wage share of production workers in the total payroll for comparative advantage sectors and comparative disadvantage sectors, where production workers are defined as blue-collar, unskilled workers. We take the period average from 1980 and 2009 because the US fully resumed diplomatic and commercial relations with China only in 1979 and 2009 is the most recent year in the dataset. We find that the implied wage share for skilled workers in comparative advantage sectors is $\beta_1 = 0.45$ and in comparative disadvantage sectors is $\beta_2 = 0.32$. Similarly, we calculate the average share of comparative advantage sectors in total sector revenue to be 0.627 for 1980-2009. We use it to calibrate $\alpha = 0.6$.

Given the definition of skilled workers and unskilled workers in the NBER-CES data, we calibrate the parameters of the training cost function in the training model and the endowments in no-training model based on the ratio of production workers to managers in figure 4 in Ebenstein et al. (2011). For the US this ratio is 4 to 1 in 1990 and 3 to 1 in 2005. For China the ratio is 8 to 1 in 1990 and 11 to 1 in 2005. Taking the average over the two available years and for a total population of 2000 workers these ratios imply for the no-training scenario that $S = 444$ and $L = 1356$ for the Home country and $S^* = 191$

¹³The data can be accessed at <http://www.nber.org/nberces/>.

¹⁴We prefer this measure over the more traditional measure of revealed comparative advantage introduced by Balassa because it not only takes account of the export structure of the sector but also of its import structure. In addition, data on US exports and imports at the NAICS industry classification is readily available at Peter Schott's website. In contrast the Balassa measure of RCA requires data on world export shares at the NAICS level which has to be converted from other industry classification systems.

and $L^* = 1809$ for the Foreign country. These endowments imply that the US has a higher relative endowment of skilled workers than China and thus a comparative advantage in producing skill-intensive goods. In the training scenario only the total endowment with labor is fixed at $ENDOW = S_t + L_t = 2000$ and $ENDOW^* = S_t^* + L_t^* = 2000$, while the share of skilled and unskilled workers is determined endogenously. The training cost follows an exponential distribution with a parameter $scaleT = 0.0047$ for H and $scaleT^* = 0.00086688$ for F.¹⁵ The parameters were set so that the pre-liberalization steady state training probability in H and F match the shares of skilled workers in the labor force of each country, such that $\eta_T = 0.222$ for H and $\eta_T = 0.0955$ for F. This ensures that the pre-liberalization steady state is the same in the model with and without training.

Concerning the migration of incumbent workers across sectors we follow the evidence in Wacziarg and Wallack (2004) , Autor et al. (2014) and Dix-Carneiro (2014). Wacziarg and Wallack (2004) show that there is little inter-sectoral mobility of workers. They do not distinguish between skilled and unskilled workers but find that a liberalizing country experiences an increase of yearly inter-sectoral job reallocation from 1.1% to 1.5% within five years after reforms. Autor et al. (2014) show that in the US unskilled workers are very immobile across sectors while skilled workers are mobile to a certain extent. Using Brazilian data Dix-Carneiro (2014) also shows that less educated workers face higher sectoral mobility costs than more educated workers. Thus, for most of our analysis we assume that unskilled workers face such high migration cost that they prefer to not switch sectors. For the skilled workers we assume that the migration cost follows an exponential distribution with scale parameter $scaleS = 0.003$. This implies that the probability for a skilled worker to switch sectors is 0.1% per quarter on average within five years after trade liberalization (matching the 0.4% increase in yearly reallocation reported by Wacziarg and Wallack (2004)).

Finally, we assume that the entering worker's relative sector entry cost follows a normal distribution. The mean of the distribution is set such that the steady state replicates a steady state with full mobility of workers across sectors, yielding equalization of wages across sectors. The standard deviation is set to $sd = 0.1$, ensuring a very narrow distribution so that the entry decision of a worker is mostly determined by sectoral wage differentials instead of fixed entry costs. This assumption is in line with empirical evidence in Ryoo and Rosen (2004) who find that the fraction of college graduates who are engineers is closely related to a measure of relative earnings prospects in engineering. We assume that workers retire at a rate of 2% per year which corresponds to a $s = 0.5\%$ per quarter.

5 Trade liberalization scenarios

In this section we describe the dynamic adjustment after a symmetric trade liberalization shock. We assume that the iceberg trade costs decrease for both sectors and for both countries from 2.5 to 1.56.¹⁶ Naturally, the length of adjustment depends on the ability of workers to move between sectors. In the long run workers are fully mobile so that they earn the same wage in both sectors. In the short and medium run, however, adjustment costs can lead to wage differentials between sectors. This effect can only be captured by using a dynamic model that can distinguish between the short run and the long run. Furthermore, our dynamic model allows for the analysis of adjustment frictions and the policies aimed at ameliorating them.

We distinguish three different scenarios: i) the first scenario features the slowest adjustment. Here we make the assumption that all incumbent workers cannot switch sectors due to sector-specific skills and that the total number of

¹⁵We choose the exponential distribution because it has only one parameter, the scale parameter, and its minimum is always zero, which provides simple analytic expressions for integrals.

¹⁶We obtain the steady states of the model with numerical methods in MATHEMATICA and use DYNARE in order to simulate the transition path of the economy from the pre- to the post-liberalization steady state.

skilled and unskilled workers is fixed.¹⁷ However, there are still workers who retire and get replaced by newly entering workers. These workers are more flexible because they have not invested in their skills yet. We do not consider this scenario as the most realistic one, but it helps to explain the workings of the model and to highlight the role of mobility assumptions by serving as a benchmark to the other scenarios. ii) The second scenario is identical to the first scenario except that we assume that incumbent skilled workers can switch sectors. We restrict this ability to skilled workers, because this is in line with the evidence in Autor et al. (2014). iii) In the third scenario, we finally illustrate our benchmark model in which newly entering workers choose their skill class. In our view this is the most realistic scenario, especially in the long run. However, in the trade literature it is often assumed that endowments with skilled and unskilled labor are fixed (like in BRS), and so scenario 2 is also a useful reference point.

In the following we concentrate on the analysis of the effects of trade liberalization on H, the country with higher endowment of skilled labor.

5.1 Scenario 1: No training, no switching

The solid black line in figures 1 (focusing on intersectoral reallocation) and 2 (focusing on wage inequality) shows the dynamic adjustment of selected variables for the first scenario, where only newly entering workers can choose in which sector to work, and where the endowments with skilled and unskilled workers are exogenously fixed. We use this scenario to explain the mechanisms of the model in more detail. The scenario will also serve as a benchmark against the other scenarios to highlight the role of mobility across sectors and skill classes. In these figures, and all the following ones, the number of quarters is at the horizontal axis. The decrease in trade costs happens in period 1. The vertical axis shows the percentage deviation of a specific variable from the pre-liberalization steady state.¹⁸

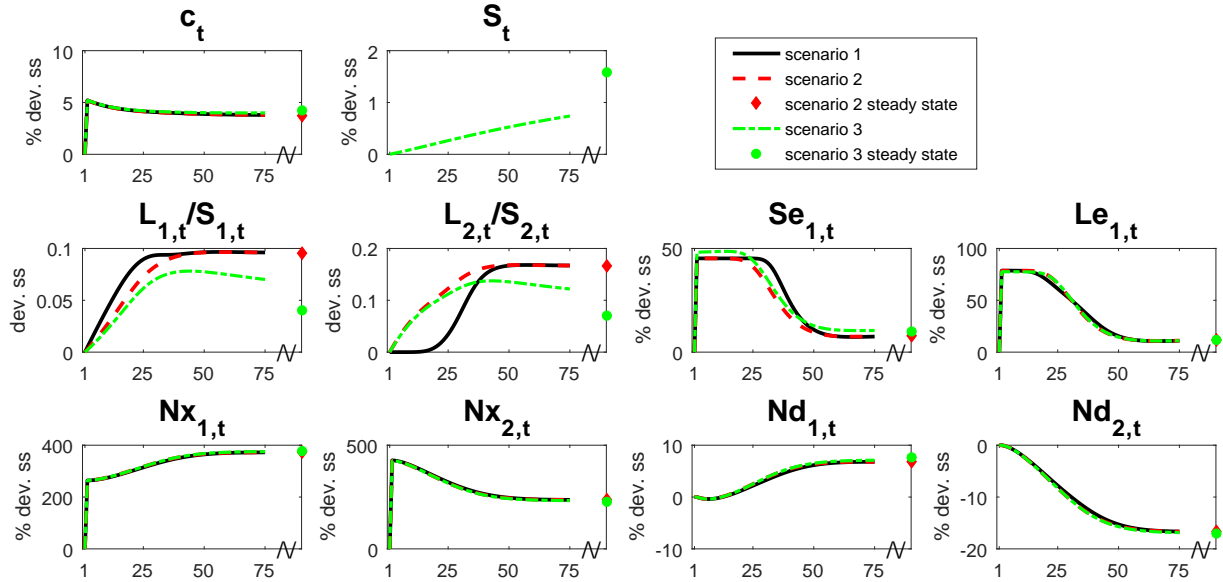


Figure 1: Dynamic adjustment in country H in response to a decrease in trade costs: Re-allocation.

Inter-sectoral reallocation. Our model features both intra-industry trade and inter-industry trade. Due to the CES-structure of sector goods consumers enjoy a large variety of goods, implying that intermediate goods within a sector

¹⁷Technically equation 17 becomes $\frac{Se_t}{Se_t + Le_t} = \frac{Se_0}{Se_0 + Le_0}$, so that the share of skilled workers remains constant.

¹⁸Some variables such as the index for inter-sectoral wage inequality are reported as absolute deviations from their pre-liberalization steady state value rather than percent deviations because they are zero at the pre-liberalization steady state.

are traded internationally. This is the intra-industry trade that is central to modern trade models. At the same time, international trade allows each country to consume more of a given sector-good than it produces (if in return it produces more than it consumes in the other sector). This is the inter-industry trade that is central to more traditional trade models. International trade allows countries to exploit comparative advantages and, thus, each country specializes in the production of the good where it has a comparative advantage, i.e., country H specializes in the production of the skill-intensive good and country F in the production of the unskilled-intensive good.

Trade costs reduce international trade and thus some of the gains from international specialization remain unexploited. To the contrary a reduction in trade costs increases international trade and fosters international specialization. Both countries concentrate their production even more in their respective exporting sectors, so that production in the import-competing sector goes down, while production in the exporting sector goes up. As figure 1 demonstrates this shift in production implies that it is more attractive for new firms to enter the exporting sector rather than the import-competing sector, so that the number of firms in the exporting sector goes up, while the number of firms in the import-competing sector goes down. This development is in line with the empirical evidence in Bernard et al. (2006), who show that plants that are exposed to imports from low-wage countries (our import sector) have a lower survival rate and a lower growth rate and tend to switch to industries with lower import-exposure and greater skill-intensity (our exporting sector).¹⁹

The shift in sectoral production also implies a shift in the sectoral demand for workers, increasing the wages of both skilled and unskilled workers in the exporting sector relative to their wages in the import-competing sector (see figure 2). This in turn induces an increase in the number of workers in the exporting sector at the cost of employment in the import-competing sector, but the adjustment is not immediate because all active workers are stuck in the sector where they have acquired their skills. In contrast, newly entering workers are very responsive to wage differentials. In the initial periods after trade liberalization all newly entering workers choose the expanding exporting sector. Only later, when the wage differentials between sectors have decreased sufficiently, again some of the newly entering workers choose the import-competing sector. In the new steady state, of course, the share of newly entering workers that chooses the exporting sector is permanently higher, because the number of workers in the exporting sector is also permanently higher (which implies that more workers are exiting the sector and thus for the number of workers to be stationary, more workers need to enter the sector).

This reallocation of workers is in line with recently presented empirical evidence both qualitatively and quantitatively. Pierce and Schott (2016) show that the elimination of trade policy uncertainty that resulted from the granting of permanent normal trade relations to China in late 2000 led to sharp employment losses in industries that experienced the sharpest reductions in tariff threats (import-competing sectors). Importantly, they also find that U.S. firms eventually adjusted to the new status quo by reallocating resources to more capital- and skill-intensive production, reflecting U.S. comparative advantage industries. Similarly, Autor et al. (2013) show that sectors that are exposed to Chinese imports (the import-competing sectors in our model) experienced substantial declines in employment.²⁰ According to tables 2 and A2 in their paper an increase in import-exposure by 92.6% yields a drop in employment by 0.75 percentage points over a decade. In our model import-exposure of the import-competing sector increases by 556.3% over a decade. Thus, extrapolating the empirical results of Autor et al. (2013) would predict a drop in employment by 4.5 percentage points. This is very close to the prediction of our model, 4.8 percentage points (over a decade).

For firms the reduction of trade costs makes exports cheaper and thus increases the profits that can be gained from exporting. This has two separate implications. On the one hand, existing exporters increase their sales on the foreign

¹⁹In our model firms only produce one product and cannot actively switch sectors but the implications are the same.

²⁰Dauth et al. (2014) show that the same is true for Germany.

market (intensive margin of trade). On the other hand, the share of exporting firms increases because more firms are able to finance the fixed cost of exporting (extensive margin of trade). This has counteracting effects on average productivity and thus welfare. Since only the most productive firms export, the expansion of their production and the ensuing partial crowding out of domestic producers with higher marginal costs raises productivity. However, the expansion in the extensive margin of trade implies that that now also firms with relatively lower productivity take up exporting, so that the average productivity of exporters decreases. The Melitz model features a third effect that is absent from our framework: a fixed cost of production implies that only firms above a certain productivity threshold are active. Trade liberalization raises this threshold so that low-productivity firms are driven out of the market. This raises the average productivity of active firms and thus welfare. This third effect is absent in our dynamic model, because (in line with GM) we had to abstract from fixed costs of production.²¹

The number of exporting firms jumps up immediately, because the decision to export is not associated with any sunk investment costs, so that active firms can react immediately to the drop in transport costs. Note, however, that the number of exporting firms in the import-competing sector overshoots, while the number of exporting firms in the exporting sector continues to rise slowly but persistently. The reason for this development lies in the slow adjustment in the total number of active firms: the number of firms in the import-competing sector declines and with it the number of exporting firms in that sector, and vice versa for the exporting sector.

Wage inequality. The focus of our analysis is on wage inequality. Figure 2 reports real wages as well as various measures of wage inequality as percentage (-point) deviations relative to the pre-liberalization steady state. The wages that we present in this and all the following figures are welfare-based real wages, i.e., nominal wages divided by the welfare-based price index.²² The welfare-based price index summarizes information on average prices and the number of available varieties. Thus, it can change due to changes in prices or due to changes in the number of varieties. Using a data-consistent price index to calculate data-consistent real wages has only quantitative effects. Also note, that all our measures of wage inequality are based on the ratio of wages. Thus, they are unaffected by the choice of the price index, since the price index cancels out. Results for data-consistent real wages are available upon request.

Due to restricted mobility in the short run, our model allows for wage inequality along two dimensions: i) a wage differential between the two sectors (see *IndexS* and *IndexL*); and ii) a wage differential between skilled and unskilled workers (the skill premium, see *Skill*). The first of the two wage differentials is due to mobility restrictions in the short run and will go away in the long run. The second exists even in the long run because otherwise workers would not have an incentive to invest in skills.

²¹Introducing fixed costs of production into our dynamic model is not tractable but it is possible to compute the steady states of such a model. Similarly to BRS, we assume that the fixed cost of production equals the fixed export cost. All other parameters remain identical to our benchmark model except for the trade costs which we re-calibrate to ensure that the share of international trade to GDP is the same as in the benchmark model (6% before and 27% after liberalization). For the model with worker training this implies a decline in trade costs from 2.586 to 1.663. Then, the share of exporting firms rises by 10.4 percentage points vs. 9.5 percentage points without fixed costs, the skill premium rises by 6.5 percentage points vs. 3 percentage points without fixed costs, and the Gini index rises by 4.3% vs. 2.8% without fixed costs. Thus, the model with fixed costs of domestic production yields even larger changes in inequality in the long run.

²²See GM for a discussion of welfare-based price indices.

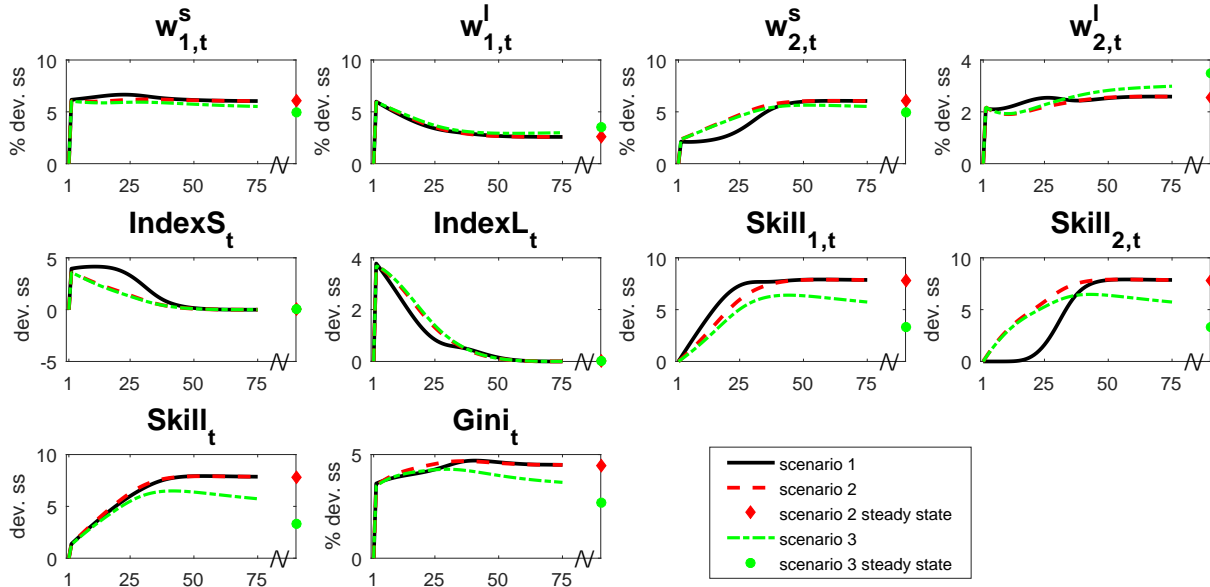


Figure 2: Dynamic adjustment in country H in response to a decrease in trade costs: Inequality.

The drop in transport costs increases demand in the exporting sector and, thus, raises the price in the exporting sector relative to the import-competing sector. This has an immediate impact on wages, which rise in line with the prices in the exporting sector relative to the import-competing sector. This is, of course, not only true for skilled workers but also for unskilled workers - both earn now higher wages in the exporting sector than in the import-competing sector, while they were earning the same wage in both sectors in the pre-liberalization steady state. On impact the inter-sectoral wage differential rises by almost 5% for skilled workers and by almost 4% for unskilled workers. This enhanced 'wage polarization' is again in line with empirical evidence, e.g., in Autor et al. (2013) or Ebenstein et al. (2014).

This wage differential implies that newly entering workers prefer the exporting sector, raising the supply of both skilled and unskilled workers in the exporting sector. This diminishes the inter-sectoral wage differential over time. In the new steady state workers again earn the same wage in both sectors, so that the distribution of workers across sectors can be stationary. Thus, trade liberalization brings along a temporary increase in wage inequality between the two sectors, which vanishes in the long run. According to our simulations it takes about 50 periods (12 and half years) for inter-sectoral wage inequality to abate.

While the wage differential across sectors peaks on impact and then slowly recedes over time, the development of the skill premium is the exact opposite. The wage differential between skilled and unskilled workers within one sector is solely determined by the relative productivity of both kinds of labor, which in turn is determined by their relative input shares. In other words, the skill premium in both sectors can only change when the relative input of skilled and unskilled workers changes. As a result, in the short run the skill premium does not change much because the supply of workers is slow to adjust. In the medium and long run, the increased demand for the skill-intensive exporting good increases the demand for skilled labor and, thus, increases the skill premium. In the process of moving workers from the import-competing sector to the exporting sector, the ratio of unskilled to skilled workers rises in both sectors (see figure 1), and with it the relative marginal product of skilled workers. Note that this adjustment is much faster in the exporting sector than in the import-competing sector, where the skill premium stays flat for more than 10 periods. Overall the skill premium increases by about 8 percentage points over ten years. In comparison, the US skill premium increased from 2000 to 2010 by 5 percentage points (based on data from Balleer and Van Rens (2013)). Thus, our model without worker training would predict a stronger increase than what we see in the data. Note, however, that the increase in the skill premium in the model with training is

much smaller.

Next we turn to the discussion of overall wage inequality which we measure by using the Gini coefficient, as defined in equation 35. Figure 2 illustrates that the Gini coefficient jumps up on impact by approximately 4% and then continues rising at a very slow pace until it settles at a new plateau at which the Gini is almost 5% higher than in the pre-liberalization steady state. In comparison, over the period 2000-2010 the Gini coefficient in the US increased by 6% based on OECD data,²³ just slightly more than the increase predicted by the model. In the short run the Gini coefficient increases mainly through the first effect, the increase in inter-sectoral wage dispersion for each skill-class. With the movement of workers from the import-competing sector to the exporting sector, the wage inequality from this source decreases, but the skill premium increases. The skill premium effect is quantitatively more important, but only slightly so that the bulk of the increase in the Gini happens in the very short run. Nevertheless, the increase in inequality stretches over a long period of time. Note that it is an advantage of our dynamic framework that we can quantify the relative importance of inter-sectoral wage inequality and the skill premium over time.

Another interesting feature can be found in the disaggregated data of wages. The wages of unskilled workers in the exporting sector are overshooting quite substantially. On impact their real wage increases by more than 5%, while in the long run the increase is only about 2%. This is so because trade liberalization leads to an immediate drop in the price index. Remember that the price index depends on the average price of domestic and foreign goods and on the number of varieties available for consumption. A decrease in trade costs directly reduces the price of foreign goods and increases the number of foreign goods available in H. Both effects lead to an immediate drop in the price index which causes an immediate increase in the real wage of all workers. However, the ensuing reallocation of workers and firms favors the skilled workers so that the wage of unskilled workers in the exporting sector drops during the adjustment period.

A note of caution is expedient here. Being a 'real' model, our model can only be used to make inference about real wages. Thus, our model mixes the effects of trade liberalization on nominal wages and on nominal prices. The real wage can rise because the nominal wage rises or because the nominal price drops. The real wage can rise even when the nominal wage drops, if the ensuing drop in nominal prices is even larger. This might explain why our model is not able to replicate the result in Autor et al. (2014) that unskilled wages in the import-competing sector decrease in the short run (they are focusing on nominal wages). Note, however, that this does not matter for our measures of wage inequality, because the same price index applies to all wages and thus cancels out when taking the ratio of two wages. Finally, let us stress that from a welfare point of view, real wages are of course the appropriate measure. Even if some workers would suffer nominal wage cuts, if their real wage goes up, their welfare goes up, because they can afford to buy more products. Nevertheless, we consider the modelling of nominal and real wages separately as a fruitful avenue for future research.

5.2 Scenario 2: No training, active switching of skilled workers

So far we have assumed that only workers newly entering the labor market can choose in which sector they want to work. We will now relax this assumption by allowing migration across sectors along the lines described in section 3.2.1.

In specifying the mobility assumptions we follow Autor et al. (2014). This is the empirical study most closely related to our analysis, since it also deals with the effects of trade of a large developing country (China) with a large developed country (the US). Autor et al. (2014) find that unskilled workers are very immobile across sectors. Skilled workers are much more mobile across sectors, but even their mobility is restricted. They also find that skilled workers gain higher wages when

²³The Gini coefficient before taxes and transfers increased by 6% from 0.44 to 0.468, while the Gini after taxes and transfers increased by 8% from 0.347 to 0.375

they migrate from an import-competing sector to another sector. Following their results we assume that skilled workers are mobile across sectors but face considerable migration costs, while unskilled workers are immobile across sectors.

The red dashed line in figures 1 and 2 shows the results. As in scenario 1 workers in the comparative advantage sector benefit relative to the workers in the comparative disadvantage sector. Both measures of sectoral wage inequality rise. Note, however, that there are now important differences between the skill classes. The inter-sectoral wage differential among unskilled workers is more persistent, while the inter-sectoral wage differential among skilled workers recedes much faster now. The reason is that in this scenario skilled workers are more sensitive to wage differentials because of their higher mobility, which puts upwards pressure on the wage of skilled workers in the import-competing sector. This also has implications for the unskilled workers in the import-competing sector who become relatively less productive and thus experience lower wages.

The faster reallocation of skilled workers has also implications for the skill premium. Due to the reduced productivity of unskilled workers in the import-competing sector the skill premium there increases much faster. On the other hand, the skill premium in the exporting sector increases more slowly. Since these effects are partly counteracting each other the change in the development of the Gini coefficient is only minor.

5.3 Scenario 3: Training, active switching of skilled workers

In BRS and in our first two scenarios it is assumed that the endowments of skilled and unskilled workers are fixed. Although workers are mobile between the two sectors, they are not mobile between skill classes. In this section we relax this assumption by simulating the full model described in section 3, including the endogenous training decision in section 3.2.2. The assumptions concerning inter-sectoral migration we are using in this scenario are equivalent to scenario 2: skilled workers can switch sectors, while unskilled workers cannot.

The dash-dot green line in figures 1 and 2 illustrates the results. In the initial periods after trade liberalization the differences are not too big. The build-up in the number of skilled workers in the exporting sector is a bit faster but not by too much (this is apparent in $L_{1,t}/S_{1,t}$ which is slightly lower initially). Similarly, the reduction in the number of skilled workers in the import-competing sector is faster only very little. As a consequence, the wage trajectories and our measures of wage inequality are not affected by much either. For inter-sectoral wage inequality it is obviously (and plausibly) the mobility of workers that is much more important.

However, this drastically changes in the medium to long run. The total number of skilled workers increases because trade liberalization increases the demand for skilled workers and thereby the incentives to invest in training. In the medium to long run this materializes in lower wages for skilled workers (because of higher supply) and higher wages for unskilled workers (because of lower supply). As a consequence, the skill premium in both sectors is considerably reduced, relative to the no-training scenarios. Note also that the development of the skill premium exhibits a hump-shaped response with strong overshooting in the medium term. It takes around ten years for the skill premium to reach its maximum, at which point it is more than 6 percentage points higher than in the pre-liberalization steady state. At this point sectoral reallocation is almost accomplished and the enhanced investments in training start to kick in so that the skill premium starts to decline again, until it is only 3.3 percentage points higher in the new steady state. Similarly, the increase in the Gini coefficient peaks after 30 periods and converges to a much lower level than in the no-training scenarios. It requires a fully dynamic model to capture this kind of development.

To summarize, allowing for training, the higher demand for skilled workers due to trade liberalization is channeled into both higher supply of skilled workers and higher wages of skilled workers, as one would expect. The first channel is ruled

Table 2: Welfare Table (consumption equivalents)

		Skilled Export Sector	Skilled Import Sector	Unskilled Export Sector	Unskilled Import Sector	Average of Consumption Equivalents
Scenario 1	Period 1	6.14	4.65	3.34	2.61	4.04
	Period 10	6.09	4.97	2.97	2.56	3.93
	Post-lib SS	5.83	5.83	2.47	2.47	3.77
Scenario 2	Period 1	5.98	5.11	3.38	2.52	4.04
	Period 10	5.95	5.39	3.00	2.47	3.93
	Post-lib SS	5.83	5.83	2.47	2.47	3.77
Scenario 3	Period 1	5.59	4.75	3.68	2.80	4.08
	Period 10	5.50	4.98	3.34	2.79	3.96
	Post-lib SS	4.82	4.82	3.33	3.33	3.91

out by assumption in the no-training scenarios and therefore all of the adjustment is channeled into the skill premium. Thus, ignoring training possibilities leads to exaggerated estimates of the effects of trade liberalization on the skill premium and overall wage inequality.

5.4 Welfare analysis

Before turning to the potential policy responses that could accompany trade liberalization, this section provides a more thorough welfare analysis than just looking at wages and wage inequality. To do so, table 2 illustrates by how much the steady state consumption (pre-liberalization) of a specific worker group would have to change to yield the same increase in welfare as the modelled liberalization of international trade.

We present these steady state consumption equivalents not only for currently active workers, but also for workers entering the labor force at different points in time. So, e.g., the 4.97 in the second row and second column illustrates that a skilled worker, who enters the labor force in period 10 after the liberalization of international trade, and who chooses to work in the import-competing sector, gains so much from increased international trade that he would need an increase in steady state consumption of 4.97% to be as well off under restricted trade. Put differently, in the absence of trade liberalization he would need an increase in steady state consumption of 4.97% to be indifferent between liberalizing trade or not.

Several interesting insights strike out from table 2:

- The gains from international trade in our model are substantial. On average the steady state consumption equivalent is around 4%.
- Skilled workers gain substantially more than unskilled workers. E.g., for a worker that enters the work force in the new steady state as a skilled worker the consumption equivalent is almost 2.4 times higher than that for an unskilled worker entering at the same time.
- In the scenario with endogenous training (scenario 3), skilled workers still gain more than unskilled workers but the difference is much smaller than without endogenous training. In scenario 3 the consumption equivalent of a skilled worker entering in the new steady state is 'only' 1.45 times higher than the consumption equivalent of an unskilled worker (compared to the 2.4 without training).
- The inter-sectoral differences are also substantial and quite persistent. For scenario 1 the consumption equivalent of skilled workers on impact of the trade shock is 1.32 times higher in the exporting sector than in the import-competing sector and for the unskilled workers it is 1.28 times higher. Ten periods into the adjustment these factors are still 1.23 and 1.16.

- The mobility of skilled workers across sectors substantially decreases the differences among skilled workers and slightly increases them for unskilled workers: In scenario 2, on impact the consumption equivalent of skilled workers in the exporting sector is 1.17 times the consumption equivalent of workers in the import-competing sector vs. 1.32 in scenario 1. For the unskilled workers this ratio is 1.34 vs. 1.28.
- There is substantial time variation for some worker groups and the gains from trade might be both increasing or decreasing for workers entering the work force later. E.g., on impact the consumption equivalent of skilled workers in the import-competing sector is 20% lower than for skilled workers that enter the same sector in the new steady state. For skilled workers entering the sector ten periods into the adjustment the consumption equivalent is still 15% lower. In contrast, on impact the consumption equivalent of unskilled workers in the exporting sector is 35% higher than for unskilled workers that enter the same sector in the new steady state.
- Finally, the last column reveals that the mobility of workers (both across sectors and skill classes), while being crucial for the distribution of the gains from international trade, has almost no relevance for the average gains from international trade.

To summarize, table 2 illustrates again the diverse effects of trade liberalization that depend on the skill-class of a worker, her sector of employment, the time at which she enters the workforce and the mobility of workers. How these interdependencies evolve over time can only be analyzed by using a dynamic model.

6 Policy response

In section 5 we have established the result that trade liberalization induces worker reallocation along two margins: i) workers migrate from the import-competing sector to the exporting sector; and ii) more workers invest in training to become skilled workers. However, we have also shown that this reallocation can take a long time and implies changes in the distribution of wages and income.

In this section we want to take a step further and analyze two prominent policy-interventions that are meant to speed up the adjustment process, reduce wage inequality and support workers that have to adjust: sector-migration subsidies and training subsidies. In both cases we concentrate on the benchmark version of the model, in which skill formation is endogenous and skilled workers can migrate across sectors. For a detailed analysis of a variety of policy instruments that could reduce wage inequality and smooth the adjustment to trade liberalization in the context of our framework, we refer the reader to Lechthaler and Mileva (2014).

6.1 Sector-migration subsidy

According to the empirical evidence cited above, unskilled workers are much more immobile across sectors than skilled workers. Our analysis demonstrates that unskilled workers also benefit less from trade liberalization than skilled workers. The immobility of unskilled workers across sectors was shown to lead to a significant increase in sectoral wage inequality, as unskilled workers in the exporting sector benefited from increased exports while unskilled workers in the import-competing sector were stuck in a shrinking sector. For these reasons we analyze here a sector-migration subsidy that is targeted towards unskilled workers in the import-competing sector.

To be able to simulate a sector-migration subsidy for unskilled workers, we assume that the migration cost of unskilled workers follows the same distribution as the migration cost of skilled workers but on top of that unskilled workers have to

pay a fixed amount that is large enough to prevent them from migrating in the absence of migration subsidies. Thus this model nests the model used above under the condition that sector-migration subsidies are zero.

Figure 3 illustrates a policy scenario in which the sector-migration subsidy covers all the sector-migration cost of unskilled workers for the first 20 periods after the drop in trade costs. We assume that the cost of this subsidy is financed by a proportional wage tax such that the government's budget is balanced at each point in time. On impact this tax is around 7% but then quickly recedes, because under free sector-migration obviously the bulk of adjustment takes place immediately.²⁴ As intended the subsidy leads to faster migration of unskilled workers from the import-competing sector to the exporting sector. Although compared to the total reallocation of unskilled workers from one steady state to the other the additional migration seems minor, it is sufficient to avoid the increase in inter-sectoral wage inequality.

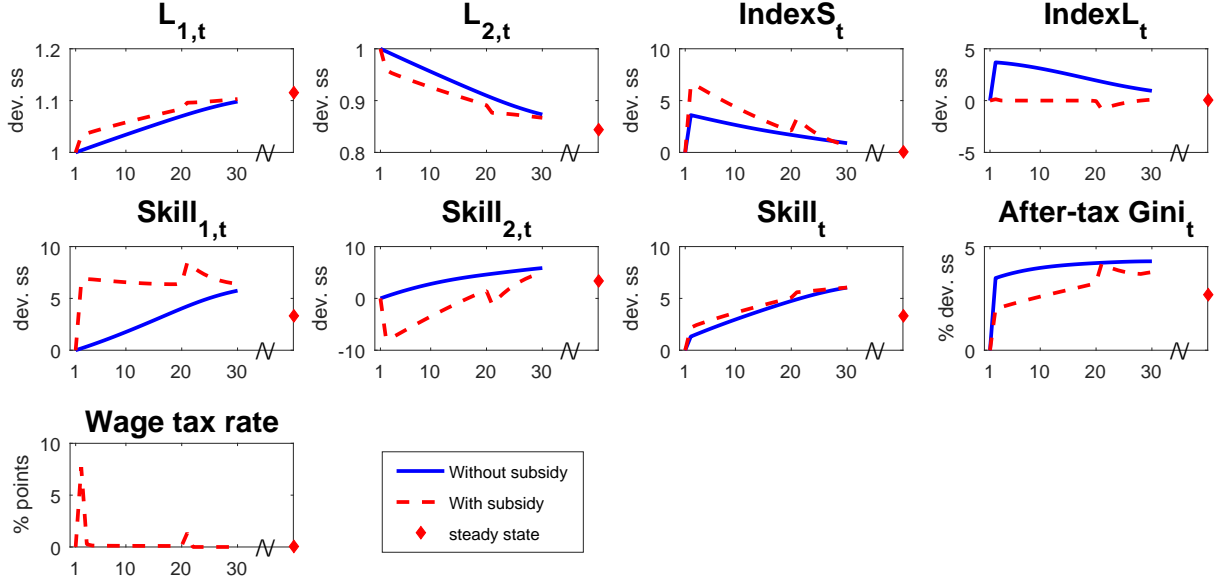


Figure 3: Sector-migration subsidy

Note that, although the subsidy is targeted towards unskilled workers only, it also has important implications for skilled workers. The faster migration of unskilled workers to the exporting sector makes the skilled workers in that sector more productive, relative to the benchmark. Of course, the opposite is true for skilled workers in the import-competing sector who become less productive. Consequently, the inter-sectoral wage inequality among skilled workers increases and the skill premium in the exporting sector increases, relative to benchmark, while the skill premium in the import-competing sector decreases.²⁵ Overall, we see a considerable decline in the Gini coefficient during the period in which the subsidy is in place. The effect is largest on impact when the increase in the Gini is reduced from 3.5% to 2% relative to pre-liberalization.

6.2 Training subsidy

As illustrated in section 5, trade liberalization increases the demand for skilled workers in the rich country, leading to an increase in the skill premium and enhanced investment in skills. Subsidizing investment in skills can speed up this process and at the same time slow down the increase in the skill premium.

²⁴ Although there is a further blip just before the subsidy runs out.

²⁵ A likewise migration subsidy to skilled workers would have the opposite effects, but the effects would be quantitatively smaller since skilled workers are already more mobile to begin with.

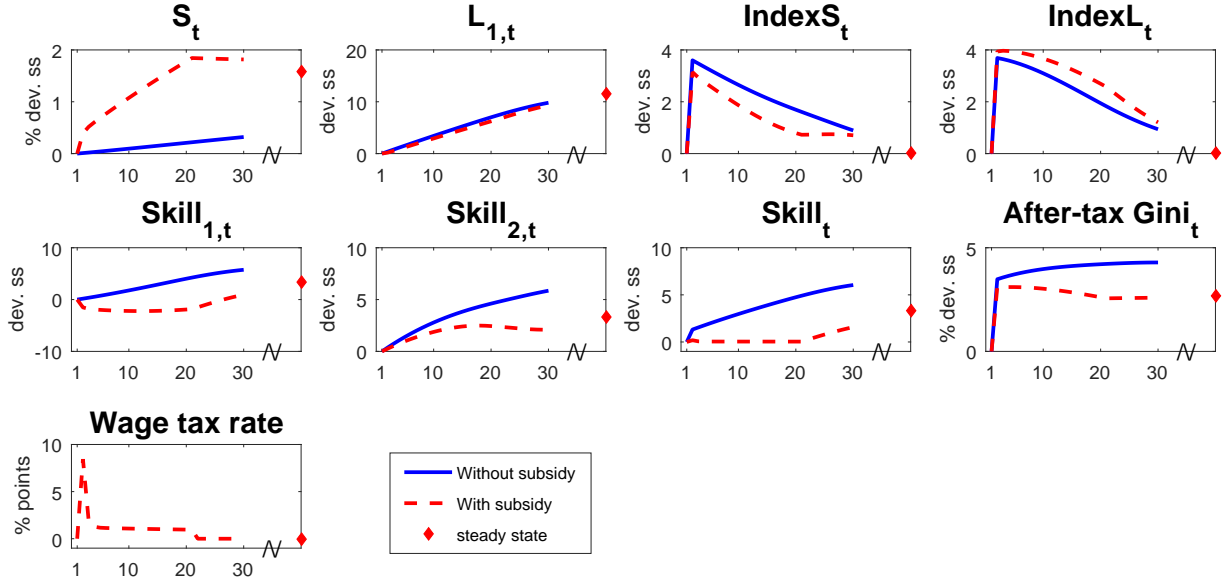


Figure 4: Training subsidy

Figure 4 illustrates the extreme case of a training subsidy that avoids any increase in the (aggregate) skill premium during the first 20 periods after the drop in trade barriers.²⁶ The subsidy considerably speeds up the increase in the number of skilled workers. Since the newly trained workers tend to prefer the exporting sector, this policy also speeds up the reallocation of skilled workers across sectors, reducing inter-sectoral wage inequality among skilled workers. Because of the faster reallocation of skilled workers, unskilled workers in the exporting sector become more productive, while the opposite is true for the unskilled workers in the import-competing sector. Furthermore, the reallocation of unskilled workers is slowed down a bit because fewer newly entering workers are unskilled, relative to the benchmark. Both effects tend to increase inter-sectoral wage inequality among unskilled workers.

The fact that newly entering skilled workers tend to prefer the exporting sector also implies that the skill premium in the exporting sector even goes down temporarily, while it goes up in the import-competing sector where the additional supply of skilled workers is relatively smaller. Overall wage inequality still increases in the short run due to the increase in inter-sectoral wage inequality but then falls as inter-sectoral wage inequality recedes and the skill premium is still flat. So overall this policy seems to be a success, the adjustment process is sped up and wage inequality reduced.²⁷

7 The role of firm entry and firm heterogeneity

7.1 Firm dynamics

In GM all the dynamics arise from the slow adjustment of firms. As noted in Burstein and Melitz (2012), the model would not yield any transitional dynamics if domestic firms had to pay fixed costs as well, because unproductive firms would drop out of the market immediately. This is different in our model, because sector migration costs and training costs give rise to slow labor market adjustment. Thus, even without the slow adjustment of firms, our model yields transitional dynamics.

To demonstrate this we shut off firm dynamics completely, by making the number of domestic firms ($N_{d,i}$), the number of newly entering firms ($N_{e,i}$) and the number of exporting firms ($N_{x,i}$) exogenous variables during the transition. We

²⁶Again we assume that the subsidy is financed by a proportional wage tax.

²⁷Note, however, that this policy reduces the welfare of skilled workers, since the increase in the number of skilled workers puts downwards pressure on their wage (not depicted). Their wage still goes up, but by less than in the benchmark without policy response.

assume that these variables immediately jump to their new steady state levels. The result is demonstrated by the dashed line in figure 5.²⁸ Since none of the parameters are changed, the initial and final steady states are the same as in our baseline simulations; only the transition is affected.

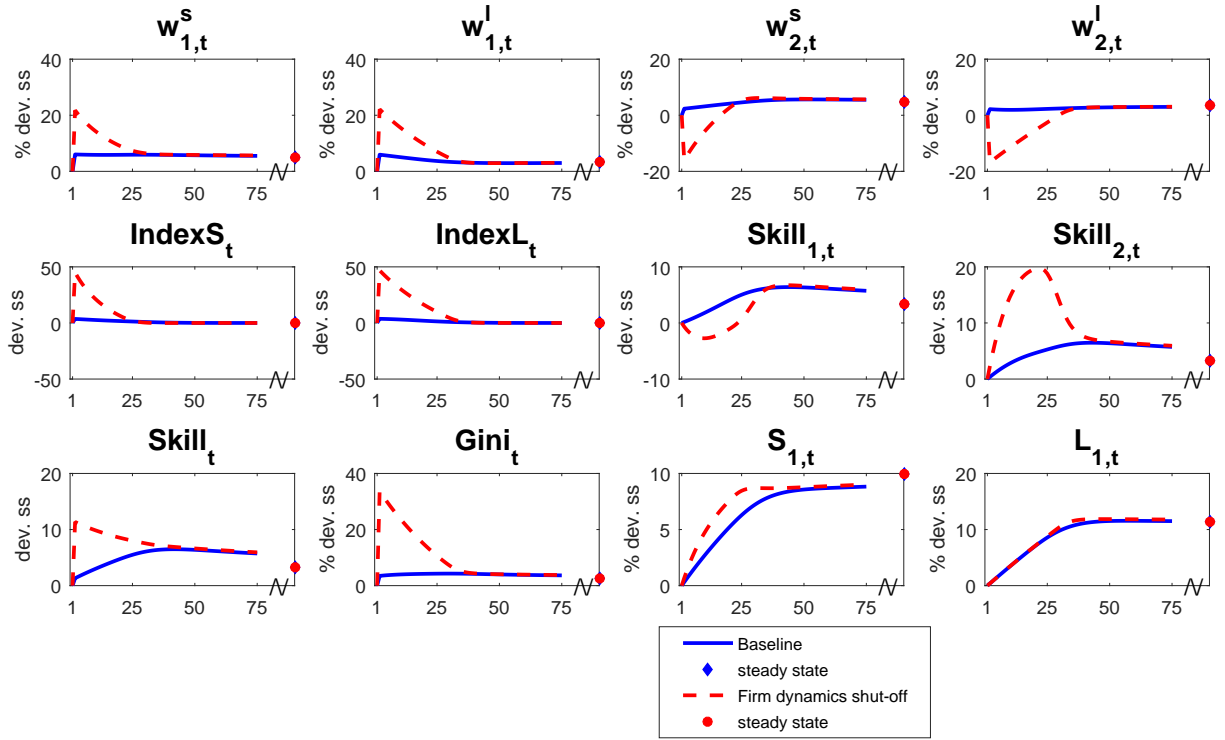


Figure 5: Trade Liberalization With Firm Dynamics Shut-Off with Training

The dashed line in figure 5 shows that the speed of reallocation of firms across sectors matters a great deal. Due to the instantaneous adjustment of firms, the number of firms in the import-competing sector drops much faster than in our baseline scenarios. This implies that the demand for labor in the exporting sector increases much faster, while demand for labor in the import competing sector falls much faster. As a consequence the wages of workers employed in the exporting sector (both skilled and unskilled) increase by much more on impact and substantially overshoot their long-run equilibrium. Conversely, the wages of workers employed in the import-competing sector are dropping substantially on impact, while they were increasing in the baseline. Naturally, this considerably increases inter-sectoral wage inequality.

The faster reallocation of firms across sectors also induces the more mobile skilled workers to migrate more quickly to the expanding export sector. This in turn raises the productivity of unskilled workers in the exporting sector due to their complementarity in the production function. As a result the skill premium in the exporting sector goes down a bit temporarily. The opposite is true for the unskilled workers in the import-competing sector, so that in the short run the skill premium in that sector increases much more and much faster. In aggregate the latter effect dominates so that the aggregate skill premium also increases by more. Finally, since inter-sectoral inequality increases by much more and the skill premium increases by much more, overall wage inequality also increases by much more.

We conclude that the speed of firm reallocation matters a great deal for workers and wage inequality. The slow reallocation of firms in our baseline model reduces the pressure on worker reallocation so that the effects on wage inequality, especially inter-sectoral wage inequality, are much subdued.

²⁸Again we restrict the analysis to our benchmark model. Results concerning the other scenarios are available upon request.

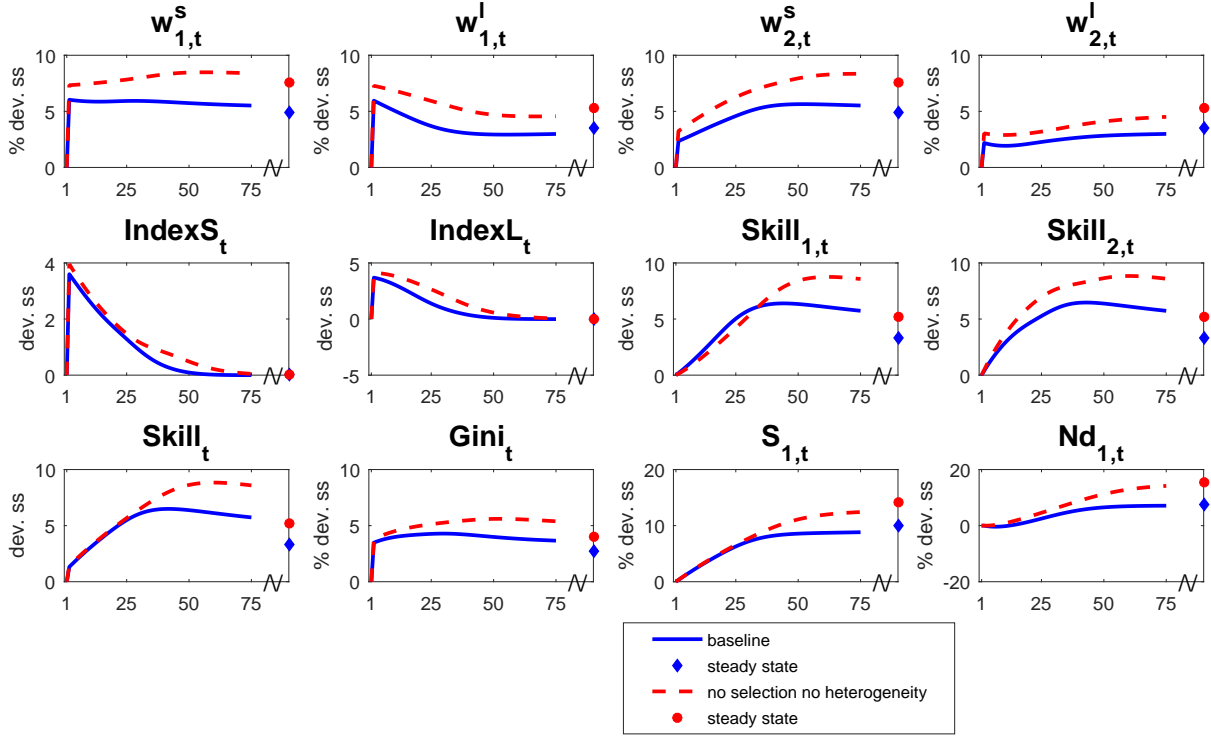


Figure 6: Trade Liberalization Without Selection Into Export Markets with Training

7.2 Selection into export markets and firm heterogeneity

In this section we analyze the role of selection into export markets and of firm heterogeneity. In contrast to Melitz (2003), in our model the two are indistinguishable because we do not have selection into the domestic market. Shutting off selection into export markets in our model implies that both the average productivity of domestic firms and the average productivity of exporting firms are fixed. This makes firm heterogeneity irrelevant because the model is isomorphic to one in which only one type of firm exists with productivity equal to the average of the productivity distribution of the heterogeneous firms model.

To study the role of selection into export markets and firm heterogeneity we thus set the fixed cost of exporting equal to zero. This implies that all active firms take up exporting, i.e., the share of exporting firms is always equal to one, and that firm heterogeneity becomes irrelevant. The result is illustrated in figure 6.

In our benchmark trade liberalization leads to a sudden and strong increase in the number (and share) of exporting firms. In contrast, shutting off selection in export markets implies that the share of exporting firms cannot change because it is already at 1. This implies that the reallocation of workers happens more gradually, so that inter-sectoral wage inequality subsides more slowly. As a result, in the short run the Gini coefficient is higher but only slightly.

In the long run, the total number of firms in the exporting sector increases by more when selection into export markets is shut off, relative to the benchmark with selection. The reason for this is that trade liberalization leads to stronger export demand in the exporting sector. In the benchmark this stronger export demand is partly met by a larger share of exporting firms. In the model without selection this is not possible since the share of exporting firms is always 1. Therefore, the number of firms must increase by more to satisfy the increase in demand. In sum, part of the expansion in production is done by less efficient firms which leads to a higher demand for skilled workers (the relatively more important factor in sector 1). This leads to higher skilled wages, a higher skill premium and higher overall wage inequality in the long run.

8 Further robustness

This section provides three further robustness checks, one in which trade liberalization is modelled as a decrease in the fixed cost of exporting, one where we use a utility function with a higher coefficient of relative risk aversion and one where we compare bilateral and unilateral liberalization.

8.1 Fixed costs of exporting

So far we have modeled trade liberalization as a decrease in the iceberg trade cost that is proportional to the value of exports. This is a commonly followed approach but trade liberalization could also be modeled as a decrease in the fixed cost of exporting. Figure 7 compares the two different approaches of modelling trade liberalization and shows that both have almost identical effects with one expected difference, the number of exporting firms increases much faster when the fixed cost of exporting is lowered than when the iceberg trade cost is lowered.²⁹

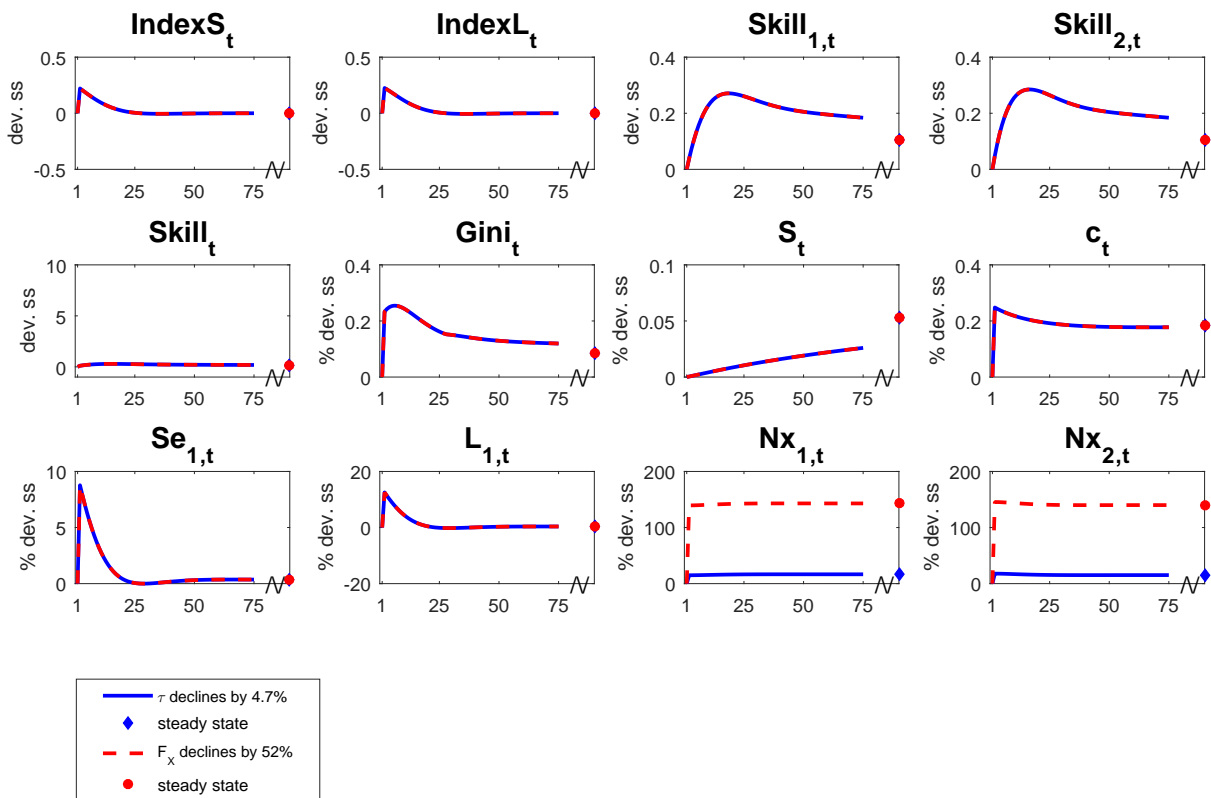


Figure 7: Comparison between a decline in trade cost and fixed cost of exporting with training.

8.2 Utility function

So far we have assumed log-utility of consumption. We now relax this assumption by assuming that utility from consumption is given by $C^{1-\sigma}/(1-\sigma)$, where σ is the coefficient of relative risk aversion. Specifically, figure 8 illustrates the case of $\sigma = 2$, apart from log-utility the most commonly used value in the macro literature, and compare it to our benchmark case.

A higher coefficient of relative risk aversion makes the utility function flatter (as long as $C > 1$ which is the case here). A flatter utility curve implies that a given gain in income yields a lower increase in utility. This in turn implies that the incentives to invest in training and to migrate across sectors are lower. On the one hand, this implies that the share of

²⁹For this exercise we simulate a decrease in the fixed cost of exporting by 52% and a decrease in the iceberg trade cost by 4.7%. Both imply an increase in the share of trade in GDP by 1%.

skilled workers in the post-liberalization steady state is lower, so that the skill premium, and with it overall wage inequality, is considerably higher. On the other hand, it slows down the reallocation of workers across sectors, implying a much more persistent increase in inter-sectoral wage inequality.

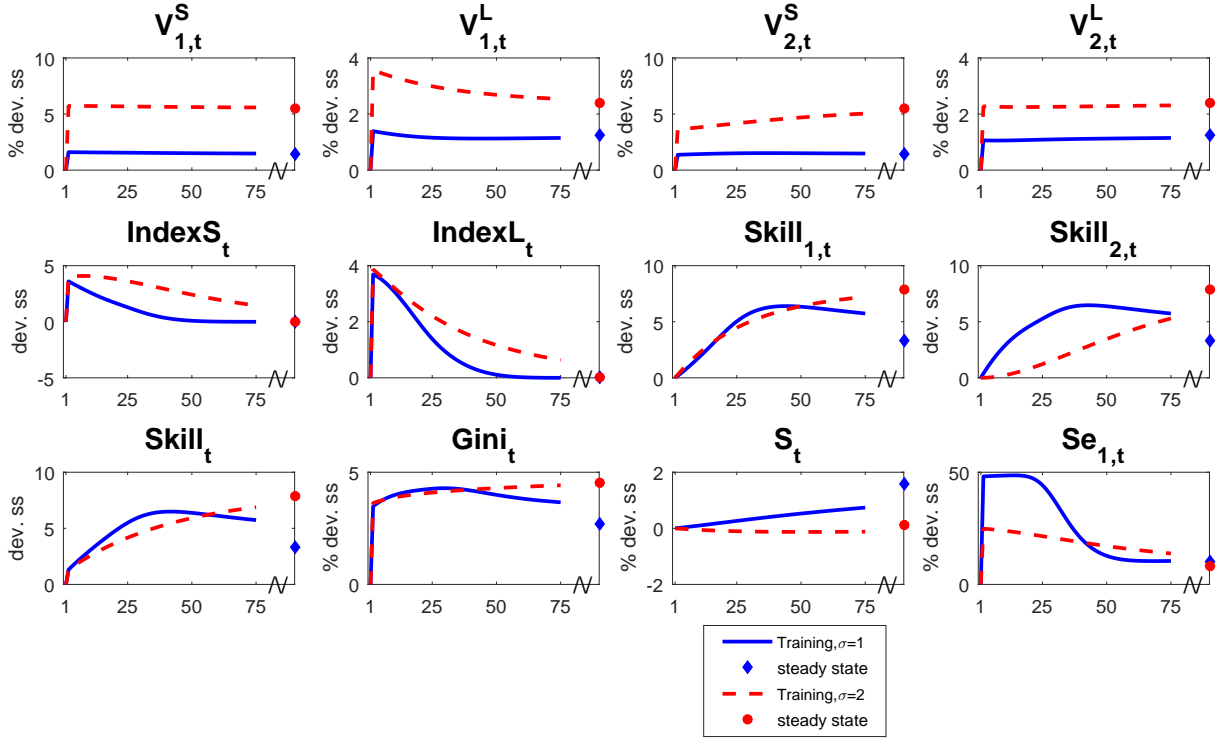


Figure 8: Trade liberalization with alternative utility function with training

8.3 Unilateral vs. bilateral trade liberalization

We have argued that one advantage of our approach relative to the existing literature is that our model is able to capture bilateral trade liberalization and endogenous adjustment in international prices. To demonstrate how these assumptions matter, figure 9 compares two further scenarios with our benchmark scenario, unilateral trade liberalization with and without fixed real exchange rate.

The dashed red line illustrates the case of unilateral trade liberalization, where the home country reduces trade barriers on foreign imports, whereas the trade barriers of the foreign country remain the same. Otherwise the model and simulations remain unchanged, including the endogenously determined real exchange rate. It can be seen that unilateral trade liberalization poses a smaller 'shock' to the economy. There is less reallocation necessary and therefore short-run inter-sectoral wage inequality increases by less. In the longer run, the smaller reallocation towards the exporting sector implies less reliance on skilled workers and thus a lower increase in the skill premium. These effects translate into a much smaller increase in overall wage inequality both in the short run and in the long run.

This demonstrates that the effects of unilateral trade liberalization are quite different from bilateral trade liberalization. However, our model differs from related papers in the literature also in that it captures general equilibrium effects while the related literature typically models small open economies. To demonstrate the relevance of general equilibrium effects, the green dot-dashed line in figure 9 illustrates the case of a unilateral trade liberalization scenario in which the real exchange rate between home and foreign is set exogenously and thus does not change with trade liberalization.³⁰ The main effect of keeping the real exchange rate fixed is visible in the huge difference in the response of aggregate consumption relative to

³⁰This is of course different from a small open economy but serves to demonstrate the relevance of adjustability in international prices.

the unilateral liberalization with endogenous real exchange rate. The different response of consumption demonstrates that the benefits of international trade are largely overstated in the case of an exogenous real exchange rate. The reason is that in the general equilibrium model unilateral trade liberalization is accompanied by a real exchange rate depreciation that splits the benefits of freer trade more equally across countries, whereas under an exogenous real exchange rate the benefits are more lopsided towards the home country.

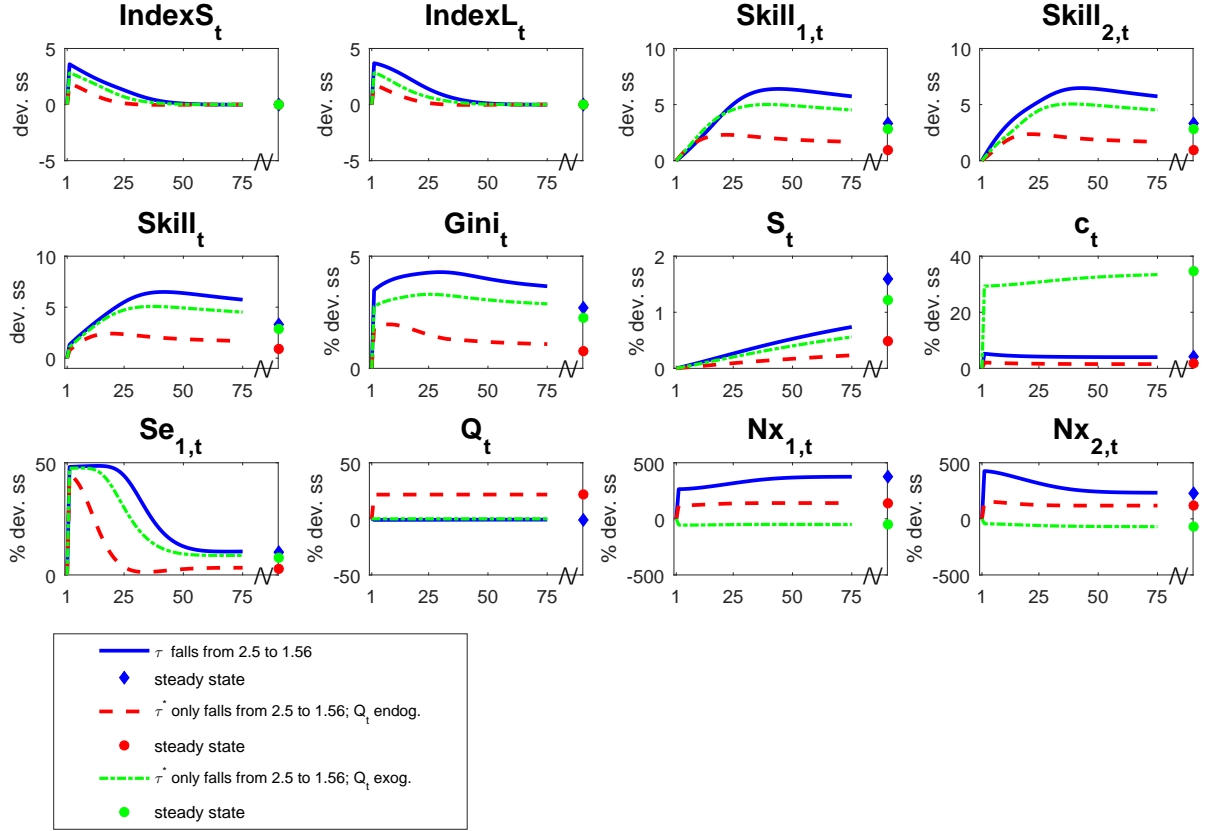


Figure 9: Comparison between bilateral decline in trade costs and unilateral decline in import trade costs, with endogenous real exchange rate Q_t and exogenous real exchange rate Q_t .

9 Conclusion

We build a two-country, two-factor, two-sector dynamic general equilibrium trade model with labor mobility costs in order to analyze the transitional dynamic effects of permanent trade liberalization. Our analysis concentrates on the change of wage inequality that occurs in developed countries from increased trade with developing countries. The advantage of our analysis is that we use a general equilibrium model of two large countries, while other recent dynamic papers use small open economy models. This implies that we can analyze bilateral trade liberalizations which are more appropriate scenarios for large nations and thus provide a more thorough analysis of the winners and losers throughout the adjustment.

We distinguish two potential sources of wage inequality, the wage differential between workers who are in the same skill class but in different sectors (comparative advantage versus comparative disadvantage sectors) and the skill premium, i.e., the wage differential between skilled and unskilled workers. In the short run, inter-sectoral wage inequality increases because the demand for the good produced by the exporting sector increases. In the medium run, inter-sectoral wage inequality recedes because workers move from the import-competing sector to the exporting sector. In contrast, the skill premium does not change much in the short run but only increases in the medium and then remains elevated. This permanently

increases overall wage inequality.

Another contribution of our paper is that we make the supply of skilled and unskilled labors endogenous as we allow newly entering workers to train to become skilled workers. This has important implications for the long run effects of trade liberalization. The long run effects of trade liberalization on wage inequality are considerably reduced because more of the adjustment is accomplished via quantities (more skilled workers) and less via wages.

Furthermore, we analyze whether sector-migration subsidies and training subsidies to workers can reduce inequality after trade liberalization and smooth the adjustment process. We find that a sector-migration subsidy to unskilled workers can reduce inter-sectoral wage inequality among unskilled workers but worsens inter-sectoral wage inequality among skilled workers and is unable to offset the increase in the skill premium. Training subsidies significantly increase the supply of skilled workers, and reduce the skill premium and thus overall wage inequality. They are, therefore, the more powerful instrument to reduce wage inequality and to smooth the adjustment after trade liberalization.

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A Proof of existence and uniqueness of the steady state

We start the proof by focusing on the steady state of the simplest version of our framework where workers do not have access to training and discuss the equilibrium under training and out-of-steady-state at the end. The following table contains the list of equations that define the steady state under no training. Asterisk denotes variables pertaining to F. For simplicity we have omitted the tilde sign on top of firm average productivities, prices and profits. Note that due to the absence of fixed costs of production $z_d = (\frac{k}{k-(\theta-1)})^{\frac{1}{\theta-1}} z_{min}$ is given. Equations A1-A19 hold for the Home country for each sector i where $i = 1$ is the skill intensive sector and $i = 2$ is the unskilled intensive sector. This results in a total of 38 sector specific equations for H. Aggregate equations (A20-A24) hold for H, which results in a total of 43 equations for H. Equivalent equations hold for F. The equations for both countries in addition to the balanced trade condition (A25) form a system of 87 equations that uniquely identifies the world steady state equilibrium, described by a vector of the following variables: $N_{xi}, N_{ei}, N_{di}, z_{xi}, d_{di}, d_{xi}, v_i, d_i, \rho_{d,i}, \rho_{xi}, \psi_i, S_i, L_i, w_i^s, w_i^l, v_i^s, v_i^l, S_{ei}, L_{ei}, c_i^s, c_i^l$ for each sector in H (42 in total for both sectors) and the equivalent variables in F. There remain the aggregate variables which are: $C, Q,$ and C^* for a total of 87 variables that describe the world economy.

The subsequent discussion of the existence and uniqueness of the steady state equilibrium is similar to the proof of proposition 3 in the appendix of Bernard et al. (2007). We focus our discussion on the H economy. Equivalent considerations hold for the F country.

In the long run we have assumed that workers are perfectly mobile across sectors (equations A20-A21). This implies that the wages of skilled and unskilled workers are equalized across sectors, i.e., $w_1^s = w_2^s \equiv w^s$ and $w_1^l = w_2^l \equiv w^l$. The same is true for consumption of workers across sectors such that $c_1^s = c_2^s \equiv c^s$ and $c_1^l = c_2^l \equiv c^l$.

Following Bernard et al. (2007) we suppose for the moment that the equilibrium wage vector is known. Defining factor intensities in the skill-intensive sector as $\lambda_1^s = S_1/S$ and $\lambda_1^l = L_1/L$, the factor-clearing equations A22 and A23 can be reformulated as

$$\begin{aligned} \frac{L}{S} &= \frac{L_1 + L_2}{S} \\ &= \frac{L_1}{S_1} \frac{S_1}{S} + \frac{L_2}{S_2} \frac{S_2}{S} \\ &= \frac{L_1}{S_1} \lambda_1^s + \frac{L_2}{S_2} (1 - \lambda_1^s) \end{aligned}$$

and

$$\begin{aligned} \frac{S}{L} &= \frac{S_1 + S_2}{L} \\ &= \frac{S_1}{L_1} \frac{L_1}{L} + \frac{S_2}{L_2} \frac{L_2}{L} \\ &= \frac{S_1}{L_1} \lambda_1^l + \frac{S_2}{L_2} (1 - \lambda_1^l) \end{aligned}$$

Table 3: Steady state equations

Equation number	Equation	Equation description
A1	$N_{di}(\rho_{di})^{1-\theta} + N_{xi}^*(\rho_{xi}^*)^{1-\theta} = (\psi_i)^{1-\theta}$	Sector price index equation
A2	$\rho_{di} = \frac{\theta}{\theta-1} (w_i^s)^{\beta_i} (w_i^l)^{1-\beta_i} \frac{1}{z_d}$	Average price of domestic producer
A3	$\rho_{xi} = \frac{\tau}{Q} \frac{\theta}{\theta-1} (w_i^s)^{\beta_i} (w_i^l)^{1-\beta_i} \frac{1}{z_{xi}}$	Average price of export producers
A4	$d_i = d_{di} + \frac{N_{xi}}{N_{di}} d_{xi}$	Average total profit
A5	$d_{di} = \frac{1}{\theta} \left(\frac{\rho_{di}}{\psi_i} \right)^{1-\theta} \alpha_i C$	Average profit of domestic producers
A6	$d_{xi} = \frac{Q}{\theta} \left(\frac{\rho_{xi}}{\psi_i^*} \right)^{1-\theta} \alpha_i C^* - f_x (w_i^s)^{\beta_i} (w_i^l)^{1-\beta_i}$	Average profit of export producers
A7	$v_i = f_e (w_i^s)^{\beta_i} (w_i^l)^{1-\beta_i}$	Free entry condition
A8	$d_{xi} = f_x (w_i^s)^{\beta_i} (w_i^l)^{1-\beta_i} \frac{\theta-1}{k-(\theta-1)}$	Cut off export condition
A9	$\frac{N_{xi}}{N_{di}} = \left(\frac{z_d}{z_{xi}} \right)^k$	Percent of exporters
A10	$\delta N_{di} = (1-\delta) N_{ei}$	Number of domestic producers
A11	$v_i = \gamma(1-\delta)(v_i + d_i)$	Recursive form of firm value
A12	$\frac{w_i^s}{w_i^l} = \frac{\beta_i}{(1-\beta_i)} \frac{L_i}{S_i}$	Relative labor demand
A13	$w_i^s S_i + w_i^l L_i + d_i N_{di} = v_i N_{ei} + N_{di} \left(\frac{\rho_{di}}{\psi_i} \right)^{1-\theta} \alpha_i C + N_{xi} Q \left(\frac{\rho_{xi}}{\psi_i^*} \right)^{1-\theta} \alpha_i C^*$	Total payments to labor and households equal total production of firms
A14	$S e_i = s S_i$	Skilled worker entry condition
A15	$L e_i = s L_i$	Skilled worker entry condition
A16	$v_i^s = \log(c_i^s) + \gamma(1-s)v_i^s$	Value function of skilled worker
A17	$v_i^l = \log(c_i^l) + \gamma(1-s)v_i^l$	Value function of unskilled worker
A18	$c_i^s = w_i^s + \frac{(d_1 N_{d1} - v_1 N_{e1} + d_2 N_{d2} - v_2 N_{e2})}{S+L}$	Skilled worker budget constraint
A19	$c_i^l = w_i^l + \frac{(d_1 N_{d1} - v_1 N_{e1} + d_2 N_{d2} - v_2 N_{e2})}{S+L}$	Unskilled worker budget constraint
A20	$v_1^s = v_2^s$	Perfect skilled labor mobility across sectors
A21	$v_1^l = v_2^l$	Perfect unskilled labor mobility across sectors
A22	$S = S_1 + S_2$	Labor market clearing for skilled labor
A23	$L = L_1 + L_2$	Labor market clearing for unskilled labor
A24	$\left(\frac{\psi_1}{\alpha} \right)^\alpha \left(\frac{\psi_2}{1-\alpha} \right)^{1-\alpha} = 1$	Aggregate price index
A25	$Q N_{x1} \left(\frac{\rho_{x1}}{\psi_1^*} \right)^{1-\theta} \alpha C^* + Q N_{x2} \left(\frac{\rho_{x2}}{\psi_2^*} \right)^{1-\theta} (1-\alpha) C^* = N_{x1}^* \left(\frac{\rho_{x1}}{\psi_1} \right)^{1-\theta} \alpha C + N_{x2}^* \left(\frac{\rho_{x2}}{\psi_2} \right)^{1-\theta} (1-\alpha) C$	Balanced trade

Using equation A12 for both sectors these become

$$\frac{L}{S} = \frac{(1 - \beta_1)}{\beta_1} \frac{w^s}{w^l} \lambda_1^s + \frac{(1 - \beta_2)}{\beta_2} \frac{w^s}{w^l} (1 - \lambda_1^s)$$

$$\frac{S}{L} = \frac{\beta_1}{(1 - \beta_1)} \frac{w^l}{w^s} \lambda_1^l + \frac{\beta_2}{(1 - \beta_2)} \frac{w^l}{w^s} (1 - \lambda_1^l)$$

Given wages and exogenous endowments these two equations uniquely define the factor intensities in the skill-intensive sector λ_1^s and λ_1^l , and thereby S_1 and L_1 . Then the factor-clearing conditions uniquely define S_2 and L_2 . In particular, $\lambda_1^s = -\frac{\beta_1(\beta_2 w^l L - (1 - \beta_2) w^s S)}{(\beta_1 - \beta_2) w^s S}$ and $\lambda_1^l = \frac{(\beta_1 - 1)((\beta_2 - 1) w^s S + \beta_2 w^l L)}{(\beta_1 - \beta_2) w^l L}$. An important restriction for the existence of the steady state is that $\beta_1 \neq \beta_2$ since otherwise the share of workers in the skill intensive sector is not well defined. Note that this restriction is satisfied as we assume that $\beta_1 > \beta_2$. Similarly, other restrictions necessary for existence are that $L > 0$ and $S > 0$. The entry conditions for workers in each sector (equations A14 and A15) deliver the number of skilled and unskilled workers entering each sector at the steady state Se_i and Le_i as a function of sector employment (S_i and L_i). Note that another important parameter restriction for the existence of positive worker entry at the steady state is $s > 0$.

The free entry condition (equation A7) pins down the average value of the firm v_i as a function of the wages and model parameters. For firms to have positive value, we require that $f_e > 0$. Otherwise we have unlimited firm entry. Combining the recursive form of firm value (equation A11) with the free entry condition (equation A7) yields

$$f_e (w^s)^{\beta_i} (w^l)^{1 - \beta_i} \frac{1 - \gamma (1 - \delta)}{\gamma (1 - \delta)} = d_i$$

which pins down average profits for each sector. Here we see other important parameter restrictions for the existence of positive firm profits at the steady state: $\gamma > 0$ and $\delta < 1$. Note that given a positive skill premium at the steady state, $\frac{w^s}{w^l} > 1$, and our assumption about factor intensities $1 > \beta_1 > \beta_2 > 0$, the average profits and firm value in sector 1 will be higher than the average profits and firm value in sector 2.

We can use equation A8 to derive the average profits for firms that export d_{xi} . Note that $k > 1 - \theta$ is another necessary restriction for the existence of the steady state export profit. In addition, note that only when $f_x > 0$, only a fraction of the existing firms export. If, $f_x = 0$, then all existing firms will be exporters.

Next, it is useful to define average domestic revenue $r_{di} = \left(\frac{\rho_{d,i}}{\psi_i}\right)^{1 - \theta} \alpha_i C$, average export revenue for H $r_{xi} = \left(\frac{\rho_{xi}}{\psi_i^*}\right)^{1 - \theta} \alpha_i C^*$, and F $r_{xi}^* = \left(\frac{\rho_{xi}^*}{\psi_i}\right)^{1 - \theta} \alpha_i C$. Using this definition for average domestic revenue in equation A4 and the fraction of exporters in A9 yields $d_i = \frac{r_{di}}{\theta} + \left(\frac{z_d}{z_{xi}}\right)^k d_{xi}$. Note that if $k = 0$, all domestic producers export and we require that $k > 0$ for only a fraction of the firms to export. The definitions of domestic and export revenues together with the pricing equations A2 and A3 imply that $\frac{r_{di}}{r_{xi}^*} = \frac{(\rho_{d,i})^{1 - \theta}}{(\rho_{xi}^*)^{1 - \theta}} = \frac{((w^s)^{\beta_i} (w^l)^{1 - \beta_i})^{1 - \theta}}{(Q \tau (w^{s*})^{\beta_i} (w^{l*})^{1 - \beta_i})^{1 - \theta}} \left(\frac{z_d}{z_{xi}^*}\right)^{\theta - 1}$ and $\frac{r_{di}}{r_{xi}} = \frac{(\rho_{d,i}^*)^{1 - \theta}}{(\rho_{xi})^{1 - \theta}} = \frac{((w^{s*})^{\beta_i} (w^{l*})^{1 - \beta_i})^{1 - \theta}}{\left(\frac{\tau}{Q} (w^s)^{\beta_i} (w^l)^{1 - \beta_i}\right)^{1 - \theta}} \left(\frac{z_d}{z_{xi}}\right)^{\theta - 1}$. Next, equations A6 and A8 imply that $r_{xi} = f_x (w^s)^{\beta_i} (w^l)^{1 - \beta_i} \frac{k}{k - (\theta - 1)} \frac{\theta}{Q}$ and $r_{xi}^* = f_x^* (w^{s*})^{\beta_i} (w^{l*})^{1 - \beta_i} \frac{k}{k - (\theta - 1)} \theta Q$. Thus, domestic revenues are only a function of the export cutoffs, wages, the real exchange rate and parameters,

$$r_{di} = \left[\frac{((w^s)^{\beta_i} (w^l)^{1 - \beta_i})^{1 - \theta}}{(Q \tau (w^{s*})^{\beta_i} (w^{l*})^{1 - \beta_i})^{1 - \theta}} \left(\frac{z_d}{z_{xi}^*}\right)^{\theta - 1} \right] \left[f_x^* (w^{s*})^{\beta_i} (w^{l*})^{1 - \beta_i} \frac{k}{k - (\theta - 1)} \right] \theta Q$$

and

$$r_{di}^* = \left[\frac{((w^{s*})^{\beta_i} (w^{l*})^{1-\beta_i})^{1-\theta}}{\left(\frac{\tau}{Q}(w^{s*})^{\beta_i} (w^{l*})^{1-\beta_i}\right)^{1-\theta}} \left(\frac{z_d}{z_{xi}}\right)^{\theta-1} \right] \left[f_x (w^s)^{\beta_i} (w^l)^{1-\beta_i} \frac{k}{k - (\theta - 1)} \right] \frac{\theta}{Q}.$$

Substituting these equations for domestic revenue in equation A4 delivers

$$d_i = \frac{1}{\theta} \left[\frac{((w^s)^{\beta_i} (w^l)^{1-\beta_i})^{1-\theta}}{(Q\tau(w^{s*})^{\beta_i} (w^{l*})^{1-\beta_i})^{1-\theta}} \left(\frac{z_d}{z_{xi}^*}\right)^{\theta-1} \right] \left[f_x^* (w^{s*})^{\beta_i} (w^{l*})^{1-\beta_i} \frac{k}{k - (\theta - 1)} \right] \theta Q + \left(\frac{z_d}{z_{xi}}\right)^k d_{xi}$$

and

$$d_{xi}^* = \frac{1}{\theta} \left[\frac{((w^{s*})^{\beta_i} (w^{l*})^{1-\beta_i})^{1-\theta}}{\left(\frac{\tau}{Q}(w^s)^{\beta_i} (w^l)^{1-\beta_i}\right)^{1-\theta}} \left(\frac{z_d}{z_{xi}}\right)^{\theta-1} \right] \left[f_x (w^s)^{\beta_i} (w^l)^{1-\beta_i} \frac{k}{k - (\theta - 1)} \right] \frac{\theta}{Q} + \left(\frac{z_d}{z_{xi}^*}\right)^k d_{xi}^*.$$

Note that from the latter condition and A8, we obtain, $\left(\frac{z_d}{z_{xi}^*}\right) = \left\{ \frac{d_{xi}^*}{f_x^* (w^{s*})^{\beta_i} (w^{l*})^{1-\beta_i} \frac{k}{k - (\theta - 1)}} - \frac{1}{\theta} \frac{1}{f_x^* (w^{s*})^{\beta_i} (w^{l*})^{1-\beta_i} \frac{k}{k - (\theta - 1)}} \right\}^{\frac{1}{k}}$, and substituting for $\left(\frac{z_d}{z_{xi}^*}\right)$ into the first condition, we obtain an equation only in terms of the export cutoff:

$$A_1 \left(\frac{z_d}{z_{xi}}\right)^k + A_2 A_3^{1-\theta} \left[\frac{A_4 - \frac{\theta}{Q w^{l*}} A_5 (A_6 z_{xi})^{1-\theta}}{f_x (\theta - 1)} \right]^{\left(\frac{\theta-1}{k}\right)} = d_i,$$

where $A_1 = \frac{(\theta-1)}{k+1-\theta}$, $A_2 = \frac{Q(f_x^*(k+1-\theta) + f_x(\theta-1))(w^{s*})^{\beta_i} (w^{l*})^{1-\beta_i}}{k+1-\theta}$, $A_3 = \frac{w^l \left(\frac{w^{l*}}{w^{s*}} \frac{w^s}{w^l}\right)^{\beta_i}}{Q\tau w^{l*}}$, $A_4 = \frac{f_e(1-(1-\delta)\gamma)(k+1-\theta)}{(1-\delta)\gamma}$, $A_5 = (\theta - 1) \left(\frac{w^{l*}}{w^{s*}}\right)^{\beta_i} + f_x(k+1-\theta) w^l \left(\frac{w^{l*}}{w^{s*}} \frac{w^s}{w^l}\right)^{\beta_i}$, $A_6 = \frac{Q}{\tau w^l z_{min}} \left(\frac{k}{1+k-\theta}\right)^{\frac{1}{1-\theta}} w^{l*} \left(\frac{w^l}{w^s} \frac{w^{s*}}{w^{l*}}\right)^{\beta_i}$. Note that $A_1 > 0$, $A_2 > 0$ as long as $\theta > 1$ and $f_x \geq f_x^*$, $A_3 > 0$, $A_4 > 0$, $A_5 > 0$, $A_6 > 0$ under the parameter restrictions discussed so far. Given positive wages and a positive real exchange rate, the left-side is a hyperbola for $z_{xi} > 0$ which guarantees existence and uniqueness for z_{xi} . Ghironi and Melitz (2005, TA) employ a similar strategy to prove uniqueness and existence of the steady state.

Now that we have obtained the export cutoffs equation, A9 allows us to pin down the fraction of exporting firms. We have also obtained average domestic revenue r_{di} and profits d_{di} .

Substituting for the N_{ei} from equation A10, and for average domestic and export revenue allows to write equation A13 as: $w_i^s S_i + w_i^l L_i = N_{di} \left(v_i \frac{\delta}{(1-\delta)} - d_i + r_{di} + \frac{N_{xi}}{N_{di}} Q r_{xi} \right)$, which allows us to pin down the number of producing firms N_{di} . Then equations A9 and A10 deliver the number of exporters as $N_{xi} = \left(\frac{z_d}{z_{xi}}\right)^k N_{di}$ and new entrants $N_{ei} = \frac{\delta N_{di}}{(1-\delta)}$. Note that to obtain positive firm entry at the steady state, we require that $0 < \delta < 1$.

The domestic and export prices are obtained from equations A2 and A3 as a function of wages and Q and equation A1 pins down the sector price index ψ_i . Note that equation A2 implies another important restriction for the existence of a positive steady state domestic price, namely $\theta > 1$ and $z_d > 0$ (which holds as long as $z_{min} > 0$ and $k > 1 - \theta$). In addition, we can write equation A1 as $N_{di} r_{di} + N_{xi}^* r_{xi}^* = \alpha_i C$ and obtain total revenue C in each country. Note that $1 > \alpha > 0$ in order to have positive demand in both sectors.

Finally, we can use equation A13 for each sector in each country to pin down the wage vector and the balanced trade condition to pin down the real exchange rate as a function of relative exports: $Q = \frac{N_{x1}^* r_{x1}^* + N_{x2}^* r_{x2}^*}{N_{x1} r_{x1} + N_{x2} r_{x2}}$.

This concludes the proof of equilibrium in the steady state. Out of steady state additional equations are required to pin down the allocation of workers across sectors, the value functions of the workers (9), the cutoff values and rates of reallocation of incumbents (7, 8), the cutoff values and shares of entry of newly entering workers (11, 12), and the law of motion (19). These equations uniquely pin down the allocation of workers out of steady state, since the rates of reallocation

are strictly increasing in the wage differential, while the marginal cost of reallocation is strictly increasing in the rates of reallocation. Put differently, wage differentials motivate workers to switch sectors, while migration costs reduce the incentives to switch. In equilibrium, both aspects balance and workers flows are uniquely pinned down.

Extending the model to incorporate training involves adding equations to ensure that the steady state supply of skilled and unskilled workers is identified. To assess the relative value of skilled versus unskilled entry, we define the average value of a skilled, and unskilled workers as $V^s = \frac{Se_1}{Se} V_1^s + \frac{Se_2}{Se} V_2^s$ and $V^l = \frac{Le_1}{Le} V_1^l + \frac{Le_2}{Le} V_2^l$, where $Se = Se_1 + Se_2$ and $Le = Le_1 + Le_2$ are the total skilled and unskilled workers entering the labor force. Note that equations A14 and A15 imply that for a stable steady state $Se = sS$, $Le = sL$ and for total worker entry $We = Se + Le = sENDOW$. Then, the level of the threshold training cost where a worker is indifferent between entering as skilled versus unskilled is $\bar{\varepsilon}^T = V^s - V^l$. In order to ensure a positive and unique probability of training at the steady state, we have to assume a probability distribution for the training cost $\Gamma(\varepsilon_t^T)$ that is only defined for non-negative values and gives zero probability to negative values, like the exponential distribution. Then, the probability of training is pinned down as $\eta^T = \Gamma(\bar{\varepsilon}^T) = 1 - \exp(-scaleT\bar{\varepsilon}^T)$ and the share of skilled workers is obtained by $\frac{S}{ENDOW} = \frac{Se}{We} = \eta^T$.