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Partisan competition authorities, Cournot-oligopoly, and endogenous market structure

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Abstract

The so-called excess-entry theorem establishes conditions guaranteeing that more firms enter a homogeneous Cournot-oligopoly in equilibrium than a benevolent government prefers. We generalize the approach and analyze the behavior of a competition authority, which attaches different weights to the firms' and consumers' payoffs, with welfare-maximization constituting a special case. The greater the importance of consumers, the less likely entry restrictions are, whereas a greater relevance of firms makes a monopoly more probable. The nature of entry restrictions also depends on the competition authority's instruments. The essential insights continue to apply if firms are heterogeneous concerning costs and the timing of output choices.

KEYWORDS

competition authority, Cournot-oligopoly, excessive entry, monopoly, partisan objective

JEL CLASSIFICATION

D42, D43, D72, L12, L13, L51

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1 | INTRODUCTION

Normative analyses of oligopolies with endogenously determined market structures are widespread. In a homogeneous Cournot-oligopoly, there will be excessive entry in the presence of business stealing (Mankiw & Whinston, 1986; Perry, 1984; Suzumura & Kiyono, 1987; von Weizsäcker, 1980). If competition authorities employed this insight, there should be extensive entry restrictions. Although some contributions describe corresponding behavior for Japan (Ghosh & Morita, 2007a; Suzumura, 1995), empirical analyses provide no consistent picture (see Berry & Waldfoegel, 1999; Hsieh & Moretti, 2003; Maruyama, 2011; Onishi et al., 2018). Consequently, investigations building on the seminal contributions by von Weizsäcker (1980), Perry (1984), Mankiw and Whinston (1986), and Suzumura and Kiyono (1987) have not focused on the application of the excess-entry prediction to economic policy. Instead, many analyses consider exceptions to it. The policy implication resulting from the scrutiny of cases in which the excess-entry prediction does not apply, namely that competition authorities should foster entry and prevent mergers, appears to be more in line with everyday evidence. It is striking that virtually all investigations of the excess-entry theorem assume that a social planner maximizes welfare. Therefore, policy conclusions are based on the presumption of an unbiased regulator.

This article departs from this benchmark and assumes a regulatory agency, referred to as competition authority, which maximizes a weighted sum of profits and consumer surplus, with welfare-maximization constituting a special case.¹ We provide a positive analysis of such partisan competition authority and pay special attention to the relationship between the exogenous degree of its bias and the number of competitors. Moreover, we determine the circumstances which make entry regulations more or less likely. In our analysis, we distinguish between a setting in which competition authorities determine the number of firms only (“second-best”) and one in which they can regulate output as well (“first-best”). This helps to ascertain how the tools at a competition authority’s disposal affect its regulatory activities. In sum, the investigation can enhance our understanding of a competition authority’s behavior and rationalize the nature of entry and merger entry restrictions.

There are various reasons why a competition authority may be biased and attach different weights to the payoffs of firms and consumers. First, restricting the number of entrants constitutes a public good for firms (and public bad for consumers). Since small groups can overcome a free-rider problem more easily (Olson Jr, 1965), the impact of firms on the government’s payoff may exceed that of consumers (Hillman, 1989). Moreover, the amount spent on the relevant good is likely to constitute a small share of a consumer’s expenditure, suggesting that firms have a greater interest in regulatory actions (Motta & Ruta, 2012). These arguments are in line with Stigler’s (1971, p. 3) assumption “that, as a rule, regulation is acquired by the industry” and suggest that competition authorities may act more on behalf of firms than in the interest of consumers.

Second, and in contrast to the prior argument, Amir et al. (2019, 2022) contend that competition authorities may pursue a “populist” objective, consisting of the sum of welfare and consumer surplus. One can refer to the United States in support of this assumption, where antitrust law focuses on “consumer welfare.” The interpretation of this term extends from consumer surplus to welfare, that is, the sum of consumer surplus and profits (see, e.g., Orbach (2011) and the comprehensive discussion in Farrell and Katz (2006)). Furthermore, it is often assumed that the European Union’s merger policy also aims to maximize consumer surplus (Katsoulacos et al., 2016; Neven &

¹This assumption has already been employed by Baron and Myerson (1982) who analyze the regulation of a monopolist and assign a greater weight to consumer surplus than profits in the regulator’s objective, with welfare maximization as the limiting case.

Röller, 2005). Consequently, the weight of consumers in the objective of the competition agency is likely to exceed that of firms and may even be greater than captured by the populist objective.

Third, if consumption predominantly takes place domestically, whereas foreign ownership of firms is more common, this may affect the behavior of the domestic competition authority. As firms make profits if entry is constrained, the entire profit effect of such entry restriction is not realized domestically. In this case, competition authorities, which attach equal weights to domestic profits and consumer surplus, would effectively maximize an objective for which the weight of firms falls short of the weight of consumers. This argument is particularly relevant in a globalized world with substantial profit-shifting. Conversely, if there is extensive cross-border shopping, the competition authority may be biased toward firms. Thus, our analysis indicates how easily the assessment of mergers or entry restrictions by national and transnational competition authorities can diverge if national authorities ignore payoffs to firms or consumers residing in other countries.

Fourth, national parliaments may elect decision-making bodies, as is the case in Switzerland. Therefore, the selection of individuals who make up competition authorities and determine their objective can mirror electoral outcomes.

Fifth, if the competition authority is subject to lobbying, its objective could attach different weights to the payoffs of firms and consumers. A competition authority may also be corruptible and maximize bribes instead of welfare.

Finally, the stance taken by the competition authority may be overruled by a political decision body. Thus, the effective objective would consist of a weighted sum of the competition authority's and, for example, the government's objective, where the weights reflect the (ex-ante) probability that political interference occurs (see Motta & Ruta, 2012).

In sum, there are powerful arguments why the objective of a benevolent social planner does not adequately describe the competition authority's behavior. This view is compatible with the evidence that entry regulations are not welfare-enhancing. Gutiérrez and Philippon (2019) show for the United States that the recent decline in entry is primarily due not to an increase in its fixed costs but to lobbying and regulations. Djankov et al. (2002) analyze entry regulations for a cross-section of up to 85 countries and conclude that they are unlikely to reflect welfare-maximizing behavior.

The present study takes the seminal contributions by von Weizsäcker (1980), Mankiw and Whinston (1986), and Suzumura and Kiyono (1987) as its starting point. Firms are homogeneous and incur fixed and irreversible costs of market entry. Since output per firm declines in the number of Cournot-competitors, there is business stealing. The market equilibrium is characterized by excessive entry, that is, more than the welfare-maximizing number of firms take up production. We do not incorporate further distortions in our basic set-up, which may mitigate or reverse the business-stealing externality.²

For such a setting, we first show that a competition authority is less likely to restrict entry at all, the greater the relevance of the consumers' payoff in its objective is. This result is due to the well-established feature that aggregate output and consumer surplus rise with the number of firms, although entry reduces output and profits per firm. Conversely, a competition authority is more likely to establish a monopoly, the less important consumers are. This basic feature is independent of the instruments the competition authority is equipped with, that is, whether we consider a first-best or a second-best setting. Second, even modest deviations from the aim of

²Such distortion may arise in a vertical relationship (see de Pinto & Goerke, 2020; Ghosh & Morita, 2007a, 2007b; Marjit & Mukherjee, 2013; Mukherjee, 2009), such that input prices exceed the society's marginal production costs. Entry can also be insufficient if one firm, which surely enters the market, has a cost advantage (Mukherjee, 2012a; Mukherjee & Tsai, 2014) or in the presence of network effects (Gama & Samano, 2021).

welfare maximization can have dramatic regulatory consequences. In an illustrative numerical example, we show that if profits represent 40% of the regulatory authority's payoff, instead of 50% as in the case of welfare maximization, the competition authority will refrain from restricting entry, although doing so would raise welfare. Third, if a competition authority can determine entry *and* output per firm, entry is less likely to be restricted, and a monopoly is more likely to arise than if the authority can only regulate entry. This is because the preferred number of firms can be selected without having to take repercussions on the output choices into account. Fourth, if the competition authority solely regulates the number of firms, the existence of entry restrictions depends on marginal costs and not on entry costs and demand. The rationale is that since profits are zero in market equilibrium, limiting entry is beneficial for the competition authority if the consumers' payoff declines with the number of firms. Whether this is the case or not depends on marginal costs only.

The consequences of regulatory interventions in an oligopoly with an endogenously determined market structure by a partisan competition authority have not yet received much attention. Amir et al. (2019) and Goerke (2020) consider open economy extensions of the basic closed-economy, free-entry Cournot-oligopoly. Amir et al. (2019) build on the finding that moving from autarky to free trade raises either consumer surplus or welfare, but not both. They then show that the sum of welfare and consumer surplus, that is, the value of a populist objective that gives consumers twice the weight of firms, is always higher under free trade than autarky. Goerke (2020) investigates horizontal FDI in a multi-period setting and assumes that such activities undermine any entry restrictions. He, *inter alia*, shows that the government can object to FDI, despite a rise in welfare, since its payoff may decline if it attaches different weights to the payoffs of firms and consumers. Marjit and Mukherjee (2013, 2015) and Han et al. (2022) assume that the government evaluates entry by considering the effects on domestic profits and consumer surplus. While they do not explicitly study different weights in the government's objective, they effectively utilize the third argument put forward above.³ Chang et al. (2010) consider a setting in which a positive fraction of the good is consumed outside the jurisdiction, which is relevant for the definition of welfare. Accordingly, consumer surplus accruing in other jurisdictions reduces the optimal number of firms while not affecting entry in market equilibrium (see also Han et al., 2022). Analytically, the set-up by Chang et al. (2010) is comparable to a framework with domestic consumers only, whose weight in the welfare objective is less than that of firms. Finally, Amir and Burr (2015) analyze firms that pay a bribe-maximizing official a constant share of profits. They show that this official effectively ignores consumer surplus and favors a monopoly.

In sum, previous contributions considering different weights of firms and consumers in the government's or regulator's objective have assumed particular values and, therefore, not systematically investigated the effects of such biases on regulatory decisions concerning market entry. In order to such a systematic analysis, this article proceeds as follows: We describe our set-up in Section 2 and derive the market outcome in Section 3. In Section 4, we analyze the behavior of a competition authority that can solely control entry. In Section 5, the competition authority has a more comprehensive set of instruments and can also set output. In Section 6, we bring our findings together and illustrate them graphically. Section 7 analyzes modifications of the basic framework and allows for two types of heterogeneity, namely, a Stackelberg-setting and a world with cost differences. Section 8 briefly looks at further extensions and limitations of the analysis. The Appendix contains more elaborate computations and detailed derivations of some of the findings described in Section 7.

³The distinction is also employed in other contributions not directly related to the issue we consider. Barros and Cabral (1992, 1994) use the discrepancy between domestic and, thereby, welfare-enhancing profits and profits accruing abroad to investigate the welfare effects of foreign entry and merger in open economies, respectively.

2 | SET-UP

We investigate three settings that differ according to the actors determining economic activities. First, we consider a world without regulatory intervention. In the first stage, profit-maximizing firms decide on entry. They will take up production as long as it is (weakly) profitable. In stage two, the number of competitors, n , is fixed, and firms decide simultaneously about their respective output, taking as given the output decisions of other firms (Cournot–Nash behavior). When deciding on entry, each firm correctly anticipates the equilibrium number of competitors, n^* , and their output choices, where a “*” indicates choices and outcomes in market equilibrium. A firm’s output choice in (a symmetric) equilibrium is denoted by $q^*(n^*)$.

In the second setting, the competition authority decides on entry in the first stage. It will choose the number of firms, n^{sb} , that maximize its objective, V , to be specified below. Given entry, firms simultaneously decide on output, $q^{sb} = q^*(n^{sb})$, in stage two, again taking the choices of other firms as given and assuming symmetry. We refer to this setting as second-best, as indicated by the superscript “*sb*,” because firms choose output. We solve the model by backward induction when looking at the market equilibrium and the second-best situation.

In the third setting, the competition authority simultaneously determines the number of firms, n^{fb} , that take up production and each firm’s output level, q^{fb} . In the case of a welfare-maximizing competition authority, this set-up is referred to as first-best. We also adopt this labeling, irrespective of the competition authority’s objective, V , and utilize the superscript “*fb*.” As is often done in the analysis of the excess-entry prediction, we view the number of firms, n , as a continuous variable and assume that there is at least one producer. Moreover, we consider a static set-up in which entry is only feasible once, and exit is impossible.⁴

Throughout the analysis, demand is linear to facilitate the calculation of explicit solutions and the comparison with other contributions. Revenues of firm j , $j = 1, \dots, n$, equal the product of the price, $P(Q) = a - Q$, and output, q_j . The choke price is given by a , $a > 0$, and Q denotes aggregate output. It equals the sum of the firm j ’s production, q_j , and output of all other firms, Q_{-j} , $Q := q_j + Q_{-j}$. Each firm incurs quadratic production costs, $c(q_j) = c_0 q_j + 0.5c q_j^2$, $c_0 \geq 0$, $c > 0$, and fixed costs of entry, F , which are sunk and can generate economies of scale.

We denote profits of firm j by π_j .

$$\pi_j(q_j) = \left(a - (q_j + Q_{-j}) \right) q_j - c_0 q_j - \frac{c q_j^2}{2} - F. \quad (1)$$

If costs were linear, $c = 0 < c_0$, aggregate production costs would be lowest for the smallest feasible number of firms. Therefore, the first-best number of firms, n^{fb} , would be minimal. To make the decision problem of the competition authority an interesting one, costs have to be convex. For simplicity, we assume them to be quadratic (see von Weizsäcker, 1980). Moreover, the term $a - c_0$ has to be sufficiently large to ensure positive profits. To streamline notation, and without loss of generality, we set $c_0 = 0$.

The competition authority’s objective, V , is given by

⁴See, for example, Seade (1980) for the assumption that the number of firms can vary continuously. Amir and Lambson (2003) provide a dynamic Cournot framework with integer constraint and show that this extension does not fundamentally affect the excess-entry prediction.

$$V = \alpha \sum_{j=1}^n \pi_j + (1 - \alpha) \frac{Q^2}{2} = \alpha \sum_{j=1}^n \left((a - Q)q_j - \frac{cq_j^2}{2} \right) - \alpha nF + (1 - \alpha) \frac{Q^2}{2}, \quad (2)$$

where the parameter α , $0 < \alpha < 1$, measures its partisanship. If $\alpha > (<) 0.5$ holds, the competition authority is biased toward the interests of firms (consumers). If $\alpha = 0.5$, it weighs the pay-offs of firms and consumers equally and maximizes welfare.

If competition authorities do not maximize welfare, they may prefer a number of active firms that is too high to guarantee each producer non-negative profits. We assume below that the profit constraint does not bind, for example, because firms can be paid to enter the market. The required resources could be obtained by imposing a lump-sum tax on consumers, all potential entrants, or other firms in the economy.⁵ Therefore, the competition authority does not have to be able to determine the number of firms directly. To isolate the effects which result from partisan entry regulations and separate them from the consequences of budgetary needs, we disregard the mechanism that induces firms to enter the market and assume that the number of firms is a choice variable of the competition authority.

3 | MARKET EQUILIBRIUM

This section describes the market equilibrium and, thus, assumes that the competition authority plays no role. In stage two, the number of firms is given. Maximization of firm j 's profits with respect to output, q_j , yields:

$$\frac{\partial \pi_j}{\partial q_j} = a - Q - q_j - cq_j = 0. \quad (3)$$

As all firms behave equally, we omit the firm index j and utilize $Q = qn$. Since the derivative of Equation (3) with respect to $q = q_j$ is negative, the second-order condition is satisfied, and profit-maximizing output equals:

$$q^*(n) = \frac{a}{1 + c + n}. \quad (4)$$

There is business stealing because output per firm declines with their number, n . In stage one, firms enter the market as long as profits are non-negative.⁶

$$\pi(n^*) = (a - n^* q^*(n^*)) q^*(n^*) - \frac{c(q^*(n^*))^2}{2} - F = \frac{(2 + c)a^2}{2(1 + c + n^*)^2} - F = 0. \quad (5)$$

⁵Such lump-sum transfers are payoff-neutral if they do not affect behavior, and the weight of the tax payers in the competition authority's objective who finance the transfer equals the weight, α , of firms.

⁶The market equilibrium, described by Equations (3) and (5), is stable, if the determinant of the system of these equations is: positive. Given a linear demand schedule and quadratic costs, this condition is: fulfilled.

Solving Equation (5), the number of firms is (see von Weizsäcker, 1980):

$$n^* = a\sqrt{\frac{2+c}{2F}} - (1+c). \quad (6)$$

To ensure that at least one firm enters in market equilibrium, $n^* \geq 1$, the choke price, a , has to exceed a critical level. Thus, we base our analysis on:

Assumption A. $M := \frac{a^2}{2F} > 2 + c := M^{\text{Min}}$.

Combining Equations (4) and (6), we calculate aggregate production, Q^* , in market equilibrium as:

$$Q^* = n^*q^* = n^*\sqrt{\frac{2F}{2+c}} = a - \sqrt{\frac{2F}{2+c}}(1+c). \quad (7)$$

4 | ENTRY REGULATION

This section assumes that the competition authority determines the number of firms, such that a second-best outcome results. Alternatively, the competition authority may impose an entry tax $T > 0$ (pay a subsidy $T < 0$), which raises (reduces) a firm's fixed costs to $F + T$. A tax reduces a firm's willingness to enter the market while a subsidy enhances it.⁷ Therefore, from an analytical vantage point, a setting in which a competition authority can directly determine the number of firms, n , is equivalent to a modeling set-up in which it can tax or subsidize entry. For simplicity, we consider the former case.

Given identical firms and the assumption that firms choose output in a profit-maximizing manner in stage two, as captured by Equation (4), the competition authority's objective in stage one is:

$$V(n) = \alpha n \left((a - nq^*(n))q^*(n) - \frac{cq^*(n)^2}{2} - F \right) + (1-\alpha)\frac{(nq^*(n))^2}{2}. \quad (8)$$

Maximizing $V(n)$, using Equations (4) and (5), the definition of M , and collecting terms, we can express the first-order condition as:

$$\begin{aligned} \frac{dV(n)}{dn} &= \alpha\pi + \alpha n(P(Q(n)) - cq^*(n))\frac{dq^*}{dn} + Q(n)\left(q^*(n) + n\frac{dq^*}{dn}\right)(1-2\alpha) \\ &= \alpha\pi + \frac{na^2[(1+c)(1-2\alpha) - \alpha]}{(1+c+n)^3}, \end{aligned} \quad (9a)$$

⁷Suppose profits are given by $\pi = (a - Q)q - 0.5cq^2 - (F + T)$. The number of entrants in market equilibrium would then equal $a\sqrt{(2+c)/(2(F+T))} - (1+c)$ (cf. Equation (6)), and decline in T , such that the competition authority could employ such payment to determine the number of firms.

$$\begin{aligned}
 &= \frac{\alpha^2}{2(1+c+n)^3} [\alpha(2+c)(1+c) + n[2(1+c) - \alpha(4+3c)]] - \alpha F \\
 &= \alpha^2 \frac{\alpha(2+c)(1+c) + n[2(1+c) - \alpha(4+3c)] - \alpha \frac{(1+c+n)^3}{M}}{2(1+c+n)^3} = 0.
 \end{aligned} \tag{9b}$$

The second derivative of $V(n)$ is:

$$\frac{d^2V(n)}{dn^2} = \alpha^2 \frac{[2(1+c) - \alpha(4+3c)](1+c-2n) - (1+c)3\alpha(2+c)}{2(1+c+n)^4}. \tag{10}$$

The derivative in Equation (10) is negative for $(1+c)/(5+3c) < \alpha < (1+c)/(2+1.5c)$ and, therefore, for $\alpha = 1/3$ and $\alpha = 0.5$. However, its sign may become positive if the weight, α , of firms in the competition authority's objective is sufficiently low. Because the numerator of Equation (9b) is a third-order polynomial in n , and the number of firms must be non-negative, there can be at most two interior and meaningful solutions. For our subsequent analysis, we assume that an interior solution for the competition authority's maximization problem is unique, as is the case for the values of α we pay special attention to. Moreover, we will discuss a non-interior outcome for the second-best optimal number of firms, n^{sb} .

The expression in Equation (9a), evaluated at the market equilibrium ($\pi = 0$), is zero for a weight of firms in the competition authority's objective equal to:

$$\frac{1}{2} > \alpha^{L, sb} = \frac{1+c}{3+2c} > \frac{1}{3}. \tag{11}$$

If $\alpha = \alpha^{L, sb}$, competition authorities evaluate the increase in consumer surplus due to more competition so highly that they allow the same number of firms to enter, as is the case in market equilibrium. If $\alpha < \alpha^{L, sb}$, competition authorities prefer more active firms than n^* , and entry is insufficient (Ritz, 2018; see also Armstrong et al., 1994, p. 108). If $\alpha > \alpha^{L, sb}$ there are too many firms, and entry is restricted. Consequently, we obtain the standard excess-entry prediction for a welfare-maximizing ($\alpha = 0.5$) competition authority as a special case (cf. Amir et al., 2014; Mankiw & Whinston, 1986; Suzumura & Kiyono, 1987). Note that $\alpha^{L, sb}$ increases in the indicator of marginal costs, c , and exceeds $1/3$ since $c > 0$. Therefore, if the weight of the consumers' payoff in the competition authority's objective is at least twice as that of firms ($1 - \alpha \geq 2/3$), there will be no entry restriction. Put differently, competition authorities which pursue a populist objective given by $\alpha = 1/3$, always foster competition.

The expression in Equation (9b) is negative for $n = 1$, if the weight, α , is greater than:

$$\alpha^{H, sb} = \frac{2M(1+c)}{(2+c)^3 + M(2-c^2)}. \tag{12}$$

This implies that competition authorities restrict entry to one firm, that is, establish a monopoly, if the firm's weight in their objective is weakly greater than $\alpha^{H, sb}$. Although aggregate output and, thus, consumer surplus, rise in the number of firms, the reduction in profits

due to fewer competitors more than outweighs this effect if the importance of firms is sufficiently large.

Since $M = a^2/(2F) > 2 + c$, the critical value $\alpha^{H,sb}$ is surely greater than $1/3$ and surpasses $\alpha^{L,sb}$.⁸ Moreover, $\alpha^{H,sb}$ rises with the choke price, a , and declines with market entry costs, F . Fixed costs reduce both profits and the competition authority's gain from expanding the number of firms. If this gain declines, the competition authority prefers a smaller number of firms and the critical value of the weight of firms in its objective falls, which ensures that the preferred number is one. Thus, a monopoly results for a greater range of values of the firms' weight, α . A higher choke price, a , in contrast, enhances the gain from increasing the number of competitors by more than the competition authority's costs of doing so. Therefore, the critical value $\alpha^{H,sb}$ rises, and a monopoly is less likely to occur. Furthermore, $\alpha^{H,sb}$ varies with the indicator of marginal cost, c , in an ambiguous way because it is uncertain whether higher marginal costs have a stronger impact on marginal (aggregate) profits or marginal consumer surplus (cf. Equation 9b). Finally, the exact value of $\alpha^{H,sb}$ depends crucially on the assumption that the number of firms can vary continuously. In the presence of an integer constraint, $\alpha^{H,sb}$ would exceed the value defined in Equation (12), and the magnitude of the deviation, inter alia, hinges on the difference in the competition authority's payoffs (cf. Equation (8)) that result for one and two entrants, that is, $V(n = 1) - V(n = 2)$.

5 | ENTRY AND OUTPUT REGULATION

This section assumes that the competition authority has a set of instruments which allow it to regulate entry *and* determine each firm's production quantity. Given identical firms, the first-order conditions for a maximum of $V(n, q)$ (cf. Equations (2) and (8)) are:

$$\frac{\partial V}{\partial n} := V_n = q \left[\alpha \left(a - 2Q - cq + \frac{cq}{2} - \frac{F}{q} \right) + (1 - \alpha)Q \right] = \alpha\pi + (1 - 2\alpha)Qq = 0, \quad (13)$$

$$\frac{\partial V}{\partial q} := V_q = n[\alpha(a - 2Q - cq) + (1 - \alpha)Q] = n[\alpha(a - cq) - Q(3\alpha - 1)] = 0. \quad (14)$$

The second-order sufficiency conditions require $3\alpha > 1$.⁹ As in the second-best setting, we initially focus on an interior outcome. The comparison of Equations (13) and (14) shows that the first-order conditions necessitate (see von Weizsäcker, 1980 or Konishi, 1990):

$$q^{fb} = \sqrt{\frac{2F}{c}}. \quad (15)$$

Rewriting Equation (14) for $Q = qn^{fb}$, yields:

⁸If the monopolist is profitable, $F < 0.5a^2/(2 + c)$ holds. This restriction then ensures $\alpha^{H,sb} > 0.5$. If the competition authority can cover a firm's losses, the profit constraint does not bind and $\alpha^{H,sb} < 0.5$ becomes feasible. Furthermore, we have $\alpha^{H,sb} - \alpha^{L,sb} = \frac{[M - (2 + c)](1 + c)(2 + c)^2}{((2 + c)^3 + M(2 - c^2))(3 + 2c)} > 0$.

⁹They are $V_{nn} = q^2(1 - 3\alpha) < 0$, $V_{qq} = -n[\alpha c + 3\alpha - 1] < 0$, and $V_{nn}V_{qq} - (V_{qn})^2 = Qq\alpha c(1 - 3\alpha) < 0$.

$$n^{fb}(q^{fb}) = \frac{\alpha(a - cq^{fb})}{q^{fb}(3\alpha - 1)} = \frac{\alpha}{3\alpha - 1} [\sqrt{Mc} - c]. \quad (16)$$

The first-best number of firms declines in the weight, α , of the firms' payoff and is given by the expression in square brackets in Equation (16) if authorities maximize welfare ($\alpha = 3\alpha - 1 = 0.5$; cf. von Weizsäcker, 1980). The welfare-maximizing, first-best number of firms, $n^{fb}(\alpha = 0.5)$, may well be higher than the second-best number, $n^{sb}(\alpha = 0.5)$, if the indicator of marginal costs, c , is high enough (see Appendix A.1 for a derivation and intuition).

The production level per firm, q^{fb} , exceeds the equilibrium quantity ($q^{fb} > q^*(n^*)$) and is independent of α . The first-best quantity trades off the impact of fixed and variable costs and ensures that the price equals marginal costs, implying that production is efficient. The number of firms then determines how the surplus is divided between firms and consumers. In particular, profits per firm are zero for $\alpha = 0.5$ and rise with α .

Whether the market equilibrium results in more entry than desired by competition authorities depends on its bias. Since the first-best number of firms, $n^{fb}(\alpha)$, declines in α , whereas the equilibrium number is unaffected, entry is too high from the authority's perspective if it is (weakly) biased in favor of firms ($\alpha \geq 0.5$).¹⁰ Solving $n^* - n^{fb}(\alpha) = 0$ yields a critical value that ensures that competition authorities favor the equilibrium number of firms.

$$\alpha^{L,fb} = \frac{\sqrt{M}\sqrt{2+c} - (1+c)}{\sqrt{M}(3\sqrt{2+c} - \sqrt{c}) - (3+2c)}. \quad (17)$$

The critical value $\alpha^{L,fb}$ declines in the choke price, a , and rises with the fixed costs of entry, F , since $M = a^2/(2F)$, such that $\alpha^{L,fb} < 0.5$.¹¹ The effects of a variation in c are ambiguous for the same reasons as outlined above for $\alpha^{H,sb}$. If the actual weight of firms in the competition authority's objective falls below $\alpha^{L,fb}$, the authority prefers more firms to enter the market than do so in equilibrium. If $\alpha > \alpha^{L,fb}$, it will restrict entry.

Comparing the critical values for a first- and second-best setting, which induce competition authorities not to limit but rather to foster entry, we find:

$$\alpha^{L,fb} - \alpha^{L,sb} = \frac{\sqrt{M}(\sqrt{c}(1+c) - c\sqrt{2+c})}{(\sqrt{M}(3\sqrt{2+c} - \sqrt{c}) - (3+2c))(3+2c)} > 0. \quad (18)$$

Consequently, it is more likely that the actual value, α , is less than the lower critical value in a first-best setting, $\alpha^{L,fb}$, than in a second-best environment, $\alpha^{L,sb}$. Competition authorities may want to restrict entry because of business-stealing and since it reduces profits. As the first

¹⁰We have:

$$\begin{aligned} n^* - n^{fb}(\alpha = 0.5) &= \sqrt{M}(\sqrt{2+c} - \sqrt{c}) - 1 > \sqrt{2+c}(\sqrt{2+c} - \sqrt{c}) - 1 \\ &= 1 + c - \sqrt{2+c}\sqrt{c} = \sqrt{1+2c+c^2} - \sqrt{2c+c^2} > 0. \end{aligned}$$

¹¹Since $\alpha^{L,fb}$ declines in M , it is maximal for $M^{\min} = 2+c$. Replacing this expression for M in Equation (17) shows that $\alpha^{L,fb} \leq 1/(3+c - \sqrt{c(2+c)}) < 1/(3+c - \sqrt{(1+c)^2}) = 0.5$.

effect is absent in a first-best situation, the incentives to restrict entry at all are weaker if competition authorities can also determine quantities.

If the indicator of marginal costs is unity ($c = 1$), the critical value $\alpha^{L, sb}$ equals 0.4 and $\alpha^{L, fb}$ exceeds this value (cf. Equation (18)). A competition authority that attaches a weight of 60% to consumers in its objective, and 40% to firms, will not restrict entry, although this would enhance welfare. Therefore, one could argue that even a modest degree of partisanship results in rather dramatic regulatory consequences, irrespective of the instruments at hand.

If the weight α exceeds the value

$$\alpha^{H, fb} = \frac{1}{3 + c - \sqrt{M}\sqrt{c}} > \frac{1}{3}. \quad (19)$$

The competition authority allows only a single firm to enter and establishes a monopoly. The critical value $\alpha^{H, fb}$ rises with the choke price, a , and the measure of marginal costs, c , declines with the fixed costs, F , and reaches a value of unity for $M^{\text{Max}} = (2 + c)^2 / c$.¹²

The difference between $\alpha^{H, fb}$ and $\alpha^{H, sb}$ is positive for low values of $M = a^2 / (2F)$, then becomes negative and is zero for the maximum value M^{Max} (for the proof, see Appendix A.3). For $c = 1$, $\alpha^{H, sb} > \alpha^{H, fb}$ holds for about 90% of all possible values of M . Therefore, the range for the firms' weight in the competition authority's objective, which induces it to limit entry to one firm, is smaller if the competition authority can regulate entry only ("second-best") than if it can determine output per firm as well ("first-best"). Put differently, for a wide range of values of M , it is more likely that the competition authority does not establish a monopoly if the set of its instruments is restricted (to determining entry).

6 | SUMMARY

Figure 1 relates the number of firms preferred by the competition authority to the critical values of the firms' weight, α , in its objective, as derived above. The number of firms in the first-best situation, $n^{fb}(\alpha)$, falls in the firms' weight, α , at a declining rate (cf. Equation (16)). A higher value of α also reduces the preferred number of firms in a second-best setting (see Appendix A.4). The curvature of $n^{sb}(\alpha)$ cannot be readily established, and we have depicted it as linear for simplicity. The number of firms in market equilibrium, n^* , is independent of α .

A competition authority which can only determine the number of firms prefers a greater number than n^* if $\alpha < \alpha^{L, sb}$. This critical value exceeds 1/3, as indicated in Figure 1. If the competition authority can determine output and the number of firms, it prefers more than n^* producers for any $\alpha < \alpha^{L, fb} > \alpha^{L, sb}$, where $\alpha^{L, fb} < 0.5$.

If $\alpha = 0.5$, the first-best number is lower than the second-best welfare-maximizing number of firms in the case of linear costs because there is no trade-off between the number of firms

¹²The effects of a and F are obvious and the derivative with respect to c is:

$$\frac{\partial \alpha^{H, fb}}{\partial c} = \frac{\frac{1}{2\sqrt{c}}\sqrt{M} - 1}{(3 + c - \sqrt{cM})^2} > \frac{\frac{1}{2\sqrt{c}}\sqrt{2+c} - 1}{(3 + c - \sqrt{cM})^2} = \frac{\frac{\sqrt{4+4c+c^2}}{\sqrt{4c}} - 1}{(3 + c - \sqrt{cM})^2} > 0.$$

Moreover, the difference between $\alpha^{H, fb}$ and $\alpha^{L, fb}$ is positive (see Appendix A.2).

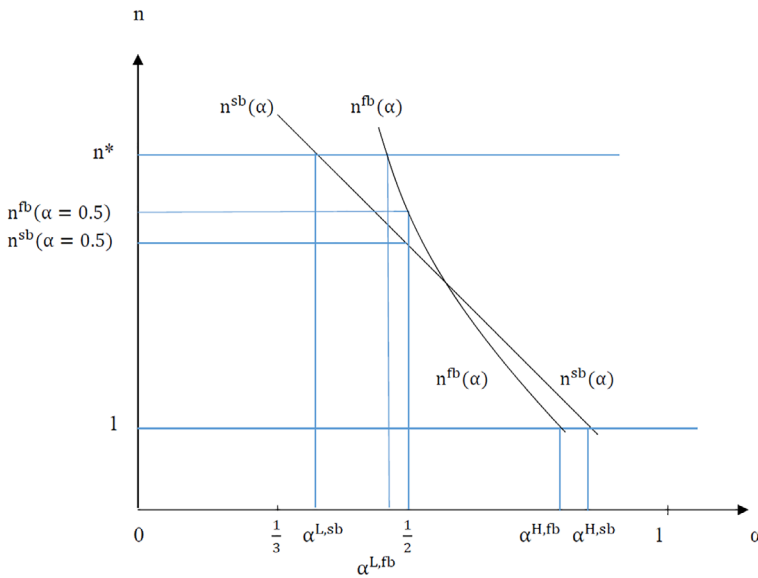


FIGURE 1 Graphical summary

and output if the competition authority can regulate both. However, there exists such trade-off in a second-best setting. Moreover, in a world with quadratic costs, marginal costs rise with output per firm, such that the first-best welfare-maximizing number of firms is likely greater than one, as depicted in Figure 1. It will also be larger than the second-best welfare-maximizing number if the indicator of marginal costs, c , is sufficiently high (see Appendix A.1).

If the weight of the firms' payoff in the competition authority's objective exceeds $\alpha^{H,fb}$, the first-best number of firms is one. We assume $\alpha^{H,fb} > 0.5$ in Figure 1. In addition, if M is sufficiently large, a competition authority which can only regulate entry prefers a monopoly for a higher value of the firms' weight than if it can determine entry and output, that is, $\alpha^{H,fb} < \alpha^{H,sb} < 1$ (see Section 5 and Appendix A.3).

From Figure 1, we obtain four main insights:

1. A competition authority which determines the number of firms *and* output is less likely to restrict entry at all than a regulatory agency which can limit entry only.
2. If a competition authority maximizes the sum of welfare and consumer surplus, implying that $\alpha = 1/3$ holds, it never restricts entry.
3. If the weight of firms in the competition authority's objective is sufficiently high, the authority establishes a monopoly, irrespective of the instruments at its disposal.
- 4a. A competition authority which can regulate entry *and* output is more likely to establish a monopoly than an authority which can restrict entry only if the consumers' maximal willingness to pay is sufficiently high relative to market entry costs.
- 4b. If the competition authority can regulate entry only, the likelihood of entry restrictions is independent of the choke price, a , and the fixed costs of entry, F .

To provide intuition for *Insight 1*, note that entry restrictions have a greater benefit for a competition authority that can affect the number of competitors than for a regulatory body that can also directly control output. This is because fewer entrants result in higher output per firm.

Since output per firm is too low in Cournot-oligopoly, the incentives to limit entry are stronger in a world where competition authorities can affect market outcomes solely via entry constraints. Because $1 - \alpha^{L,fb}$ and $1 - \alpha^{L,sb}$ define the range for which entry restrictions are imposed, and since $\alpha^{L,fb} > \alpha^{L,sb}$ holds, *Insight 1* suggests that a competition authority is less inclined to limit entry at all if its set of instruments is more comprehensive. This generates the empirically testable hypothesis that there are fewer entry restrictions in countries with more powerful competition authorities. Conversely, if competition authorities in different legal environments have the same regulatory instruments, the existence or absence of entry restrictions can provide information about the authorities' objectives.

Insight 2 indicates that competition authorities may not restrict entry because of a focus on consumer interests. Extending this insight to merger policy, we can deduce that populist competition authorities will be hesitant to allow take-overs, since reducing the number of competitors is likely to lower consumer surplus by more than it raises welfare.

Insight 3 shows that if competition authorities are sufficiently partisan and biased toward the interests of firms, they limit competition, irrespective of the instruments they have at their disposal. This is because the adverse consequences of restricting entry on aggregate output only have a small impact on the competition authority's payoff, which is dominated by the rise in profits resulting from less intensive competition.

According to *Insight 4a*, strict entry restrictions resulting in a monopoly are more likely if competition authorities regulate entry *and* a firm's behavior. This is the case because a more comprehensive set of instruments mitigates or eliminates the adverse production effects of limiting competition. Consequently, the gains from restraining entry are larger than if competition authorities can solely limit the number of competitors. *Insight 4a* suggests that welfare may increase even if a competition authority which is biased toward firms is endowed with greater power and allowed to affect output decisions as well.

Finally, higher fixed entry costs reduce profits and, *ceteris paribus*, make entry less attractive to competition authorities. However, the number of firms in market equilibrium also declines. In a second-best setting, the incentives to restrict entry due to higher fixed costs are the same as in market equilibrium. This is the case because firms decide on production levels. Therefore, the weight of firms in the competition authority's objective, which has to be exceeded to restrict entry, is independent of the fixed costs of entry, F , as *Insight 4b* clarifies. A similar line of argument applies to the choke price, a . Therefore, a competition authority which can only determine the number of firms restricts entry independently of consumers' (maximal) willingness to pay and the firms' fixed cost of entry. In a first-best setting, the gain from restricting entry due to higher fixed costs is less pronounced because the output is determined optimally. Therefore, the desired number of firms shrinks by less than in market equilibrium. Accordingly, the higher fixed costs of entry, F , the less likely entry restrictions are in a first-best world. The reverse argument applies to the choke price, a (see *Insight 4a*). The findings summarized in *Insight 4b* also generate empirically testable predictions, which differ according to the competition authority's regulatory power.

7 | FIRM HETEROGENEITY—TWO EXTENSIONS

The base model assumes that firms are identical. However, firm heterogeneity can affect both the number of firms in market equilibrium and the socially optimal extent of entry. Accordingly, insufficient entry may result in the presence of heterogeneous firms in a setting in which

there would be too many homogeneous oligopolists. To investigate to what extent differences between firms affect the findings summarized in Section 6, we consider two types of differences.¹³ One of them concerns the output market, and the other focuses on inputs. Section 7.1 assumes that one firm can determine its quantity prior to all other oligopolists. The existence of a Stackelberg-leader can deter entry and, thereby, raise welfare (Etro, 2007, 2008). Therefore, the question arises of whether sequential output choices also affect the competition authority's behavior. Section 7.2 reverts to a Cournot-setting and assumes that production costs differ across firms. When deciding on entry, firms do not know about the cost realization and face uncertainty. Since such cost uncertainty raises expected profits, it enhances entry in market equilibrium and a second-best setting (de Pinto & Goerke, 2022). Once more, a partisan competition authority's incentives to regulate entry may change.

7.1 | Stackelberg-setting

In a Stackelberg-world, the leader may have an incentive to raise its output level beyond the Cournot-quantity to reduce the followers' production. Consequently, the strength of the business-stealing externality changes, and there may no longer be excessive entry (Etro, 2007, 2008; Mukherjee, 2012b). Since sequential output choices are likely to affect output as well as profits, both the market outcome and the competition authority's choices will vary.

Suppose, therefore, that a leader, indexed by the subscript L , enters the market in stage one at costs F . In stage two, n_S followers enter the market, where the subscript S indicates the Stackelberg-setting. Each follower incurs the same fixed costs of entry, F . In stage three, the leader chooses its quantity, q_L . Finally, in stage four, the followers determine their respective output levels.¹⁴ The other ingredients of the model outlined in Section 2 remain unaffected.

We assume that entry by the leader is profitable and desirable from the competition authority's vantage point. Therefore, we can focus on the entry decisions of followers. Moreover, in market equilibrium entry is profitable for at least one follower. We consider the various settings in the same order as the main analysis.

7.1.1 | Market equilibrium

To ascertain the effects of sequential output choices in market equilibrium, we initially determine the followers' decisions. Profits of follower j can be expressed as

$$\pi_{sj} = \left(a - q_L - q_{sj} - Q_{Sj} \right) q_{sj} - c \frac{\left(q_{sj} \right)^2}{2}, \quad (20)$$

¹³I am very grateful to an anonymous referee for proposing these lines of enquiry.

¹⁴Since entry is a long-term choice, we suppose that the Stackelberg-leader chooses its output after the followers have decided on entry (see Mukherjee (2012b) and Chao et al. (2017) for comparable approaches in the analysis of entry decisions). In contrast, Etro (2007, 2008) and Cato and Matsumura (2019) assume that the Stackelberg-leader chooses its quantity before followers can enter. Ino and Matsumura (2012) denote the alternative settings as weakly and strongly persistent-leadership models.

where Q_{S-j} denotes the output of all followers other than firm j . When follower j decides about q_{Sj} , entry decisions and the output choice of the leader have already been made. Furthermore, for firm j the quantities produced by other followers are given. Assuming symmetric followers and, therefore, omitting the subscript j , we can calculate the output of a follower as:

$$q_S(n_S, q_L) = \frac{a - q_L}{1 + c + n_S}. \quad (21)$$

Given n_S followers, and anticipating their quantity responses, the leader chooses

$$q_L(n_S) = \frac{a(1+c)}{Z(n_S)}, \quad (22)$$

where we use $Z(n_S) := (2+c)(1+c+n_S) - 2n_S = (2+c)(1+c) + n_S c$ for notational convenience. The output of followers and aggregate output are:

$$q_S(n_S) = \frac{a}{1+c+n_S} \left[1 - \frac{1+c}{Z(n_S)} \right], \quad (23)$$

$$Q_S(n_S) = \frac{a}{1+c+n_S} \left[n_S + \frac{(1+c)^2}{Z(n_S)} \right] = a - (1+c)q_S(n_S). \quad (24)$$

The leader produces a higher quantity than a follower, and the aggregate quantity rises in the number of followers, n_S (see Appendix A.5).

The number of entrants which ensures that profits of followers are zero in market equilibrium is denoted by n_S^* . Substituting Equations (22) and (23) into Equation (20), we can calculate the output level, q_S^* , which ensures that exactly n_S^* followers enter. This output level, $q_S(n_S^*)$, is the same as selected by a Cournot-oligopolist, $q_S(n_S^*) = q^*$ (see Equation (7)). The same equivalence results for aggregate output:

$$Q_S(n_S^*) = Q_S^* = a - (1+c)q_S^* = a - (1+c)\sqrt{\frac{2F}{2+c}} = Q^*. \quad (25)$$

The number of followers in market equilibrium is (see Appendix A.5):

$$n_S^* = \sqrt{\frac{M(2+c)}{4} + \frac{(1+c)^2}{c^2} - \frac{(1+c)^2}{c} + \frac{\sqrt{M(2+c)}}{2}} < n^* - 1. \quad (26)$$

Therefore, the equilibrium number of firms in a Stackelberg-setting is lower than in a Cournot-oligopoly.¹⁵

¹⁵Etro (2008) derives the same result in a setting with quadratic costs in which the leader determines its output before followers decide on entry and can thereby affect the followers' entry decisions. This linkage does not exist in the present framework.

7.1.2 | Entry regulation

The first-order condition, which characterizes the number of firms a competition authority prefers if it can regulate entry, is somewhat involved. Therefore, comparing the number of firms in market equilibrium, $n_S^* + 1$, with the second-best number of followers, $n_S^{sb}(\alpha)$, yields no meaningful insights. This is due to the co-existence of a quadratic cost function and asymmetric firms. Therefore, we attempt to gain further insights by assuming linear costs, cq . This is feasible because the excess-entry prediction in a second-best setting results in the presence both of constant and increasing marginal costs (cf. Mankiw & Whinston, 1986; Suzumura & Kiyono, 1987; von Weizsäcker, 1980).

The analysis shows (see Appendix A.6), first, that the number of followers in market equilibrium exceeds the number preferred by a welfare-maximizing competition authority.¹⁶ Second, this implies that the weight of firms in the competition authority's objective, which induces the authority to choose the equilibrium number of entrants, is less than 0.5. It stems from a smaller range than $\alpha^{L, sb}$, but is higher than the respective value for a Cournot-setting with linear costs. Third, the competition authority prefers no entry of followers and, hence, a monopoly leader for a broad range of values of the firm's weight in its partisan objective. It exceeds the value which induces the competition authority to prefer the market equilibrium with at least one follower and, thus, mirrors the findings that $\alpha^{H, sb} > \alpha^{L, sb}$ for a Cournot-setting with quadratic costs (see Equations (10) and (11)). Moreover, the value of the firms' weight in the competition authority's objective, which ensures that it prefers a monopoly in a Stackelberg-setting with linear costs, is higher than the respective value for Cournot-competition with linear costs because the decline in profits when moving to a duopoly is smaller in a Stackelberg-world.

In summary, the findings for the Stackelberg-setting with linear production costs for a second-best world qualitatively resemble those derived for a Cournot-setting with either quadratic or linear costs. This suggests that the nature of competition in the output market does not fundamentally affect the analysis of a partisan competition authority if it can regulate entry.

7.1.3 | Entry and output regulation

In a first-best world, comparing the market equilibrium in a Stackelberg-setting with the competition authority's choice is straightforward in the presence of increasing marginal costs. This is the case because leader and followers face the same quadratic cost function. Therefore, aggregate profits are maximal for a given number of firms and any aggregate output if each firm produces the same quantity. Accordingly, there will be no distinction between leader and follower in the first-best setting, irrespective of the competition authority's objective, and the authority faces the same optimization problem as in a Cournot-setting.

¹⁶Mukherjee (2012b) considers a comparable analytical framework in which, however, the leader's marginal costs are lower than the followers' marginal costs. Mukherjee (2012b) shows that there is excessive entry if the cost difference is not too large. If the leader determines output prior to the followers' entry decision, the market equilibrium is characterized by entry deterrence, such that there is no follower (see Etro, 2007, 2008). Therefore, the sequence of decisions is important in a Stackelberg-framework (see also Ino & Matsumura, 2012).

The output level imposed by the competition authority and the number of followers it induces to enter are given by $q_S^{fb} = \sqrt{2F/c}$ (cf. Equation (14)) and $n_S^{fb} = n^{fb} - 1$ (see Equation (16)). The first best-number of firms of a welfare-maximizing competition authority falls short of the number of entrants in market equilibrium, implying that $n^{fb}(\alpha = 0.5) < n_S^* + 1$ holds. Thus, the critical value, $\alpha_S^{L,fb}$, which ensures $n_S^* - n^{fb}(\alpha) = 0$, will be less than 0.5, as is the case in a Cournot-setting.¹⁷ Finally, the value of the firms' weight in the competition authority's objective, which ensures that it prefers a monopoly in a first-best world, is given by Equation (18) and the same as in a Cournot-setting.

We can summarize that the main results concerning the behavior of a partisan competition authority facing a Cournot-oligopoly also hold in a Stackelberg-setting. While the exact values of the weight of firms, respectively consumers, in the competition authority's objective, which induce it to behave in a particular way (see, e.g., *Insight 2*), depend on the structure of the oligopolistic market, the essential features appear to be the same.

7.2 | Heterogeneous costs

Suppose next that production costs vary across firms. For simplicity, we assume that costs are either high or low with the same probability for a given quantity and equal $0.5c_i(q_{ji})$, $i = h, l$, $c_h = c + \varepsilon \geq c \geq c - \varepsilon = c_l > 0$. Since this is the only modification in comparison to the basic set-up, for $\varepsilon = 0$ the findings outlined below collapse to those of Sections 3–5.

If firms learn about the cost realization before entry, and there are sufficiently many potential low-cost entrants, no high-cost firm would enter the market in equilibrium. Moreover, the competition authority would only allow low-cost firms to produce. Therefore, cost differences can have an impact if neither a potential entrant nor the competition authority can observe the cost realization prior to the entry decision. If variable costs become known subsequent to having incurred the fixed costs, F , the entry decision occurs under uncertainty.

In our setting, uncertainty occurs only at the firm level but not in aggregate. In particular, we suppose that the ex-ante distribution of marginal production costs is also realized ex-post. Moreover, no entrant produces zero output. These assumptions imply that half of the firms face high costs ex-post, and the other 50% of entrants are characterized by low marginal production costs. This simplification enables us to focus on the impact of cost heterogeneity.¹⁸ Firms learn about their costs after entering the market and determine output knowing their cost realization. The entry decision in market equilibrium is based on

¹⁷An explicit value of $\alpha_S^{L,fb}$ can be obtained by setting equal Equations (26) and (16), and solving the equality for α . The resulting expression does not lend itself to an insightful analysis.

¹⁸Cost uncertainty in Cournot models with free entry has been looked at by Creane (2007), Deo and Corbett (2009), and de Pinto and Goerke (2022). None of these contributions compares a partisan regulator's choices in a first- and second-best setting. The present analysis builds on de Pinto and Goerke (2022) who focus on the impact of greater cost uncertainty, that is, the effects of an increase in the parameter, ε , for example, on the extent of excessive entry. They derive the main results for a setting with linear costs. However, de Pinto and Goerke (2022) also report the findings from the investigation of a model with quadratic production costs, akin to the one we look at here. Therefore, we can relate some of our findings to those reported in de Pinto and Goerke (2022). Moreover, de Pinto and Goerke (2022) establish a qualitative equivalence of a setting without aggregate uncertainty and no integer constraint relating to the number of firms and a model, which features aggregate uncertainty and contains an integer constraint.

expected profits, π^e . The competition authority maximizes the weighted sum of aggregate expected profits and consumer surplus. Except for output levels per firm, which we indicate by the subscripts h and l , for high and low costs, respectively, we denote all variables with the subscript d , signposting the existence of costs differences. The sequence of decisions is the same as in the base model of Section 2.

7.2.1 | Market equilibrium

Expected profits of firm j , facing marginal production costs $c_j q_{jv}$, are:

$$\pi_j^e = \frac{1}{2}(\pi_{jl} + \pi_{jh}) = \frac{1}{2} \left[(a - Q_d)(q_{jl} + q_{jh}) - (c + \varepsilon) \frac{(q_{jh})^2}{2} - (c - \varepsilon) \frac{(q_{jl})^2}{2} \right] - F. \quad (27)$$

For a given number of entrants, the firms' optimisation decisions in market equilibrium result in an aggregate output level, $Q_d^*(n_d)$ (see Appendix A.7 for the calculations):

$$Q_d^*(n_d) = \frac{n_d a (1 + c)}{(1 + n_d + c)(1 + c) - \varepsilon^2}. \quad (28)$$

Expected profits are:

$$\pi^e(n_d) = a^2 \frac{(2 + c)(1 + c)^2 - c\varepsilon^2}{2((1 + n_d + c)(1 + c) - \varepsilon^2)^2} - F. \quad (29)$$

Solving $\pi^e(n_d) = 0$, we obtain:

$$n_d^* = \frac{\sqrt{M} \sqrt{(2 + c)(1 + c)^2 - c\varepsilon^2 + \varepsilon^2}}{1 + c} - (1 + c). \quad (30)$$

The number of entrants in market equilibrium, n_d^* , rises with the indicator of cost heterogeneity, ε . Therefore, $n_d^* > n^*$ holds for $\varepsilon > 0$. Cost heterogeneity allows firms with low costs to expand production, whereas high-cost firms curtail output, relative to a setting in which costs are certain and equal to the average amount at a given output level. In consequence, expected profits rise, and entry becomes more attractive.¹⁹

¹⁹The mechanism at work has initially been explored in the analysis of price variability on firms' output choices. Such price or cost variability can be beneficial for firms and consumers because of the convexity of the profit function (see Waugh (1944), Oi (1961), and Massell (1969) for the seminal contributions on price variability).

7.2.2 | Entry regulation

Turning to the competition authority, we initially assume that it can only determine the number of firms. The first-order condition for a maximum of its objective, $V_d(n_d) = \alpha n_d \pi^e(n_d) + (1 - \alpha)0.5(Q_d(n_d))^2$, is²⁰:

$$\frac{dV_d}{dn_d} = \alpha \left[\pi^e(n_d) - \frac{n_d a^2 (1+c) ((2+c)(1+c)^2 - c\epsilon^2)}{((1+n_d+c)(1+c) - \epsilon^2)^3} \right] + (1-\alpha) \frac{n_d a^2 (1+c)^2 ((1+c)^2 - \epsilon^2)}{((1+n_d+c)(1+c) - \epsilon^2)^3} = 0. \quad (31)$$

If the first-order condition holds, the term in square brackets in Equation (31) is negative as the second summand is greater than zero. Therefore, the derivative in Equation (31) declines in the weight of firms, α , in the competition authority's objective.

The weight, $\alpha_d^{L, sb}$, of firms in the competition authority's objective such that it prefers the number of equilibrium entrants is:

$$\alpha_d^{L, sb} = \frac{(1+c)^2 - \epsilon^2}{(1+c)(3+2c) - \epsilon^2}. \quad (32)$$

Because $\alpha_d^{L, sb}$ declines with ϵ , it is smaller than the respective value in the absence of cost heterogeneity, that is, $\alpha_d^{L, sb} < \alpha^{L, sb} < 0.5$. Therefore, it can be argued that cost heterogeneity aggravates excess entry (see de Pinto & Goerke, 2022).

A competition authority which cannot affect quantity choices prefers a monopoly if the weight of firms in its objectives exceeds $\alpha_d^{H, sb}$.

$$\alpha_d^{H, sb} = \frac{2(1+c)^2 M ((1+c)^2 - \epsilon^2)}{M [(1+c)^3 (2-c^2) + \epsilon^2 (1+c) 2(1+2c) - c\epsilon^4] + ((2+c)(1+c) - \epsilon^2)^3}. \quad (33)$$

This value cannot be related directly to other critical values of the firms' weight.

7.2.3 | Entry and output regulation

We next consider a setting in which the competition authority chooses the number of firms and each firm's output. Since a firm knows its marginal costs when deciding on output, we assume

²⁰We assume that Equation (31) defines a maximum for V_d for all values of α of interest. The second-order condition is:

$$\frac{d^2 V_d(n_d)}{d(n_d)^2} = \alpha \frac{d\pi^e}{dn_d} - a^2 \frac{(1+c)((1+c)(1+c-2n_d) - \epsilon^2)}{((1+n_d+c)(1+c) - \epsilon^2)^4} (\alpha((2+c)(1+c)^2 - c\epsilon^2) - (1-\alpha)(1+c)((1+c)^2 - \epsilon^2)).$$

that the competition authority can likewise differentiate the output levels of high- and low-cost firms. Its objective is:

$$V_d(n_d, q_l, q_h) = \alpha n_d \left\{ \left(a - n_d \frac{(q_l + q_h)}{2} \right) \frac{(q_l + q_h)}{2} - \frac{(c + \varepsilon)(q_h)^2}{4} - \frac{(c - \varepsilon)(q_l)^2}{4} - F \right\} + \frac{(1 - \alpha)}{2} \left(n_d \frac{q_l + q_h}{2} \right)^2. \tag{34}$$

From the first-order conditions, we compute output per firm and their first-best number (see Appendix A.7).

$$q_l^{fb} = \sqrt{\frac{4F(c + \varepsilon)}{2c(c - \varepsilon)}} > q^{fb} > q_h^{fb} = \sqrt{\frac{4F(c - \varepsilon)}{2c(c + \varepsilon)}}, \tag{35}$$

$$n_d^{fb}(\alpha) = \frac{\alpha}{3\alpha - 1} \left(\sqrt{M \frac{c^2 - \varepsilon^2}{c}} - \frac{c^2 - \varepsilon^2}{c} \right). \tag{36}$$

The first fraction in Equation (36) declines in α and attains a value of unity for $\alpha = 1$. Therefore, the first-best number of firms will be greater than one if $\sqrt{M(c^2 - \varepsilon^2)/c} > (c^2 - \varepsilon^2)/c$ and $\sqrt{M(c^2 - \varepsilon^2)/c} > 1$. The derivative of n_d^{fb} with respect to ε is:

$$\frac{\partial n_d^{fb}}{\partial \varepsilon} = \frac{2\varepsilon\alpha}{(3\alpha - 1)c} \left(1 - \frac{1}{\sqrt{M \frac{c^2 - \varepsilon^2}{c}}} \right). \tag{37}$$

Given $n_d^{fb} \geq 1$, greater cost heterogeneity raises the first-best number of firms.

The weight in the competition authority's objective, $\alpha_d^{L,fb}$, which ensures that it prefers the equilibrium number of firms, n_d^* , can be computed in the same manner as in Equation (16):

$$\alpha_d^{L,fb} = \frac{n_d^*}{3n_d^* - n_d^{fb}}. \tag{38}$$

Substituting for n_d^* and n_d^{fb} in line with Equations (30) and (36) yields no further insights. In particular, an analytical proof that $\alpha_d^{L,fb} < 0.5$ holds for all potential values of the parameter measuring cost heterogeneity, ε , is not feasible without further restrictions.

A value of $\alpha \geq \alpha_d^{H,fb}$ induces the competition authority to prefer a monopoly:

$$\alpha_d^{H,fb} = \frac{1}{3 - \left(\sqrt{M \frac{c^2 - \varepsilon^2}{c}} - \frac{c^2 - \varepsilon^2}{c} \right)}. \tag{39}$$

This value of the firms' weight in the objective of the competition authority is greater than $1/3$ since $n_d^{fb} > 0$ requires that the term in brackets in the denominator of Equation (39) is positive.

We can conclude that a partisan competition authority faces fundamentally the same trade-off, which induces it to restrict the number of firms either to the market equilibrium outcome or to one, in the presence of cost heterogeneity, as in the absence of differences in marginal costs. The findings concerning excessive entry for a second-best setting, in which only the number of firms can be determined, are also qualitatively the same as in the absence of cost heterogeneity. How differences in marginal costs affect a partisan competition authority's incentives to establish a monopoly may depend on the extent of the cost heterogeneity. In sum, most but not all insights for the basic set-up summarized in Section 6 also apply in a word with cost heterogeneity.

8 | LIMITATIONS AND FURTHER EXTENSIONS

We have derived the results summarized in Section 6 under several, possibly restrictive assumptions. The analysis in Section 7 clarifies that modifications of the basic setting may sometimes require additional assumptions for the main results to hold. Moreover, not all insights stated in Section 6 can be derived for the modified set-ups. In this concluding section, we briefly discuss some further potentially important simplifications of our basic framework.

First, the demand schedule is linear, and the cost function is quadratic. These assumptions help to explicitly compute some outcomes and the critical values of the weight of firms, α , and consumers, $1 - \alpha$, in the competition authority's objective. Since the excess-entry result holds for a much broader class of demand schedules and cost functions (cf. Amir et al., 2014), our findings are unlikely to be affected qualitatively. The critical values computed for α and the numerical examples obviously depend on the simplifying assumptions.

Second, given entry, a firm can only decide on output. Especially in a long-run setting, it is likely that firms can undertake investments to alter production technologies and reduce marginal costs. The respective incentives depend on output levels and, therefore, the competition authority's objective if entry is regulated. While the excess-entry result can also arise in settings with innovation (Chao et al., 2017; Mukherjee, 2012a; Okuno-Fujiwara & Suzumura, 1993), this does not necessarily imply that our findings are robust to such extension. This is the case because there is a second channel of adjustment, which may influence entry in market equilibrium differently than in a regulated setting.

Third, we have analyzed a model in which the weight of firms and consumers in the competition authority's objectives is given exogenously. The value of α could also be determined endogenously and hinge on lobbying contributions (see Section 1). Equilibrium contributions would then depend on payoff levels and the means to overcome the public good character of regulation. They would, accordingly, vary with the exact specification of the demand schedule, cost functions, the mechanism used to aggregate firms' and consumers' preferences, and the treatment of lobbying contributions in the competition authority's objective.

In summary, the basic trade-off we have identified is likely to be important in more elaborate settings as well. Our investigation provides the first step for a more comprehensive examination of partisan competition authorities in oligopolistic markets.

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APPENDIX

A.1 | First-best versus second-best welfare-maximizing number of firms

To compare $n^{fb}(\alpha = 0.5) = \sqrt{Mc} - c$ (cf. (16)) and $n^{sb}(\alpha = 0.5)$, we evaluate Equation (9) at n^{fb} .

$$\begin{aligned} \frac{dV(n)}{dn} \Big|_{\alpha=0.5, n=n^{fb}} &= \frac{F}{(1+c+n^{fb})^3} \left\{ M[(2+c)(1+c) + n^{fb}c] - (1+c+n^{fb})^3 \right\} \\ &= \frac{F}{(1+c+n^{fb})^3} \left\{ M[(2+c)(1+c) + (\sqrt{Mc}-c)c] - (1+\sqrt{Mc})^3 \right\} \\ &= \frac{F}{(1+c+n^{fb})^3} \left\{ 2M - 1 - 3\sqrt{Mc} \right\}. \end{aligned} \quad (\text{A.1})$$

The expression in curly brackets is positive for $c \rightarrow 0$ because M weakly exceeds $M^{\text{Min}} = 2 + c$, and decreases in c . Moreover, it increases in M , since the derivative with respect to M ,

$$\frac{\partial(2M - 1 - 3\sqrt{Mc})}{\partial M} = 2 - \frac{3\sqrt{c}}{2\sqrt{M}} > 2 - \frac{3\sqrt{c}}{2\sqrt{M^{\text{Min}}}} = 2 - \frac{3\sqrt{c}}{2\sqrt{2+c}} > 0, \quad (\text{A.2})$$

is positive for $M = M^{\text{Min}} = 2 + c$ and rises in M .

Assuming $c = 1$ ($c = 2$), the derivative in Equation (A.1) is positive for any $M > (\sqrt{17} + 3)^2 / 16$ (for any $M > \sqrt{13/8} + \sqrt{9/8}$). Therefore, if c is relatively low ($c = 1$), the derivative in Equation (A.1) may well be positive, such that the second-best number of firms exceeds the first-best number. If, however, marginal costs are higher ($c = 2$), the reverse may be true.

The intuition for the uncertain relationship between n^{fb} and n^{sb} is as follows: The gains from restricting the number of production sites, in order to reduce market entry costs, F , become smaller the higher marginal costs are. Moreover, output per firm declines in the number of firms, n , if it is chosen by producers (cf. Equation (4)), whereas it does not vary with n in a first-best setting (cf. Equation (15)). Therefore, the increase in the indicator of marginal costs, c ,

ceteris paribus, has a lower impact via the convexity of the cost function if firms choose output than if the competition authority determines the production level, as well.

A.2 | Comparison of $\alpha^{H,jb}$ and $\alpha^{L,jb}$

From Equation (19) we know that $\alpha^{H,jb}$ rises in M . Using Equation (17), we see that $\alpha^{L,jb}$ falls in M .

$$\begin{aligned} \frac{\partial \alpha^{L,jb}}{\partial \sqrt{M}} &= \frac{(1+c)(3\sqrt{2+c}-\sqrt{c})-\sqrt{2+c}(3+2c)}{(\sqrt{M}(3\sqrt{2+c}-\sqrt{c})-(3+2c))^2} \\ &= \frac{c\sqrt{2+c}-(1+c)\sqrt{c}}{(\sqrt{M}(3\sqrt{2+c}-\sqrt{c})-3-2c)^2} = \frac{\sqrt{2c^2+c^3}-\sqrt{c+2c^2+c^3}}{(\sqrt{M}(3\sqrt{2+c}-\sqrt{c})-3-2c)^2} < 0. \end{aligned} \quad (\text{A.3})$$

Computing the difference between $\alpha^{H,jb}$ and $\alpha^{L,jb}$ at $M^{\text{Min}} = 2+c$, yields:

$$\begin{aligned} \alpha^{H,jb}(\sqrt{M} = \sqrt{2+c}) - \alpha^{L,jb}(\sqrt{M} = \sqrt{2+c}) \\ = \frac{1}{3+c-\sqrt{(2+c)c}} - \frac{1}{3(2+c)-\sqrt{(2+c)c}-(3+2c)} = 0. \end{aligned} \quad (\text{A.4})$$

Since the difference rises in M , it is positive for $M > M^{\text{Min}}$.

A.3 | Comparison of $\alpha^{H,sb}$ and $\alpha^{H,jb}$

$\alpha^{H,sb}$, as defined in Equation (12), is increasing and concave in M .

$$\frac{\partial \alpha^{H,sb}(M,c)}{\partial M} = \frac{2(1+c)(2+c)^3}{((2+c)^3 + M(2-c^2))^2} > 0 > \frac{\partial^2 \alpha^{H,sb}(M,c)}{\partial M^2}. \quad (\text{A.5})$$

The respective value of α for the first-best setting (cf. Equation (19)) rises in M .

$$\frac{\partial \alpha^{H,jb}(M,c)}{\partial M} = \frac{\sqrt{c}}{(3+c-\sqrt{Mc})^2 2\sqrt{M}} > 0. \quad (\text{A.6})$$

Additionally, $\alpha^{H,jb}$ is strictly concave in M if c is sufficiently low (i.e., for values of c close to zero) and strictly convex if c is large enough (e.g., for $c \geq 0.6$).

$$\frac{\partial^2 \alpha^{H,jb}(M,c)}{\partial M^2} = \frac{\sqrt{c}(3\sqrt{Mc}-3-c)}{(3+c-\sqrt{Mc})^3 4M^{1.5}} > \frac{\sqrt{c}(3\sqrt{2c+c^2}-3-c)}{(3+c-\sqrt{Mc})^3 4M^{1.5}}. \quad (\text{A.7})$$

The inequality in Equation (A.7) results because M in the numerator is replaced by $M^{\text{Min}} = 2+c$.

We next calculate the difference between the critical values $\alpha^{H,fb}$ and $\alpha^{H,sb}$.

$$\begin{aligned} \alpha^{H,fb}(M) - \alpha^{H,sb}(M) &= \frac{1}{3 + c - \sqrt{cM}} - \frac{2M(1+c)}{(2+c)^3 + M(2-c^2)} \\ &= \frac{(2+c)^3 + M(2-c^2) - 2M(1+c)(3+c) + 2M(1+c)\sqrt{cM}}{((2+c)^3 + M(2-c^2))(3+c - \sqrt{cM})} \\ &= \frac{(2+c)^3 - M(4+8c+3c^2) + 2M^{1.5}(1+c)\sqrt{c}}{((2+c)^3 + M(2-c^2))(3+c - \sqrt{cM})}. \end{aligned} \tag{A.8}$$

This sign of this difference depends on the sign of the numerator of Equation (A.8), labeled $\Delta(M)$, and is positive for $M^{\text{Min}} = 2 + c$.

$$\begin{aligned} \Delta(M)|_{M=M^{\text{Min}}} &= (2+c)^3 - (2+c)(4+8c+3c^2) + 2(2+c)^{1.5}(1+c)\sqrt{c} \\ &= (2+c)2[\sqrt{2+c}(1+c)\sqrt{c} - c(2+c)] \\ &= (2+c)2[\sqrt{2c+5c^2+4c^3+c^4} - \sqrt{4c^2+4c^3+c^4}] > 0. \end{aligned} \tag{A.9}$$

$\alpha^{H,fb}(M)$ and $\alpha^{H,fb}(M)$ attain their maximum of unity at $M = M^{\text{Max}}$, such that $\Delta(M^{\text{Max}}) = 0$.

$$\alpha^{H,fb}(M, c) = 1 \Rightarrow 2M(1+c) = (2+c)^3 + M(2-c^2) \Rightarrow M^{\text{Max}} = \frac{(2+c)^2}{c}, \tag{A.10}$$

$$\alpha^{H,fb}(M, c) = 1 \Rightarrow 3+c - \sqrt{c}\sqrt{M} = 1 \Rightarrow M^{\text{Max}} = \frac{(2+c)^2}{c}. \tag{A.11}$$

Moreover, the difference $\Delta(M)$ is increasing in M at $M = M^{\text{Max}}$.

$$\begin{aligned} \frac{d\Delta(M)}{dM}|_{M=M^{\text{Max}}} &= 3\sqrt{M^{\text{Max}}}(1+c)\sqrt{c} - (4+8c+3c^2) \\ &= 3(2+c)(1+c) - (4+8c+3c^2) = 2+c > 0. \end{aligned} \tag{A.12}$$

To summarize: For $M = M^{\text{Min}}$, we have $\alpha^{H,fb}(M^{\text{Min}}) - \alpha^{H,fb}(M^{\text{Min}}) > 0$. Furthermore, the critical values $\alpha^{H,fb}(M)$ and $\alpha^{H,fb}(M)$ increase in M , and $\alpha^{H,fb}(M)$ is strictly concave in M . In addition, we know that $\alpha^{H,fb}(M) = \alpha^{H,fb}(M)$ for $M^{\text{Max}} = (2+c)^2/c$, and that $\alpha^{H,fb}(M) - \alpha^{H,fb}(M) < 0$ for $M < M^{\text{Max}}$ and $\alpha^{H,fb}(M) - \alpha^{H,fb}(M) > 0$ for $M > M^{\text{Max}}$. Thus, the critical value in a second-best setting is higher than the critical value in a first-best framework for a range of values of M , $M^{\text{Min}} < M < M^{\text{Max}}$.

If we assume $c = 1$ and solve Equation (A.8) for $M^{\text{Min}} = 2 + c = 3$, $\alpha^{H,fb}(M^{\text{Min}}) - \alpha^{H,fb}(M^{\text{Min}}) = 0.441 - 0.4 > 0$. The difference becomes zero at about $M = 3.69$ ($\alpha^{H,fb}(3.69) \approx \alpha^{H,fb}(3.69) \approx 0.481$) and remains positive for all $M > 3.69$ and less than $M^{\text{Max}} = 9$. Therefore, $\alpha^{H,fbB}(M) < \alpha^{H,fb}(M)$ holds for 88.5% ($= [9 - 3.69]/6$) of the feasible values of M .

A.4 | Effect of weight α on the number of firms in second-best setting

The effect of α on the second-best number of firms, n^{sb} , defined in Equation (9), is given by:

$$\frac{dn^{sb}}{d\alpha} = -\frac{\frac{\partial^2 V(n)}{\partial n \partial \alpha}}{\frac{\partial^2 V(n)}{\partial n^2}}. \quad (\text{A.13})$$

Since the denominator of Equation (A.13) is negative if the second-order condition holds, the sign of the numerator determines the variation in n^{sb} . Its sign is equivalent to that of the derivative of the numerator of Equation (9b), denoted by Z , because the denominator of Equation (9b) is independent of α .

$$\begin{aligned} Z &= \frac{a^2}{2(1+c+n)^3} \left\{ (2+c)(1+c) - n(4+3c) - \frac{(1+c+n)^3}{M} \right\} \\ &< \frac{a^2}{2(1+c+n)^3} \left[2(1-2n) + 3c(1-n) + c^2 - \frac{(2+c)^3}{M^{\text{Max}}} \right] \\ &= \frac{a^2}{2(1+c+n)^3} [2(1-2n) + 3c(1-n) + c^2 - 2c - c^2] < 0. \end{aligned} \quad (\text{A.14})$$

The inequality sign in Equation (A.14) is due to the substitution of the last term in curly brackets by $2+c \leq 1+c+n$, and M in the denominator by the largest possible value $M^{\text{Max}} = (2+c)^2/c$.

A.5 | Stackelberg-extension with quadratic costs

A.5.1. | Output levels

Comparing Equations (22) and (23) clarifies that the leader produces a higher quantity than a follower.

$$q_L(n_S) - q_S(n_S) = \frac{a(1+c)}{Z(n_S)} - \frac{a}{1+c+n_S} \left[1 - \frac{1+c}{Z(n_S)} \right] = \frac{an_S}{Z(n_S)(1+c+n_S)} > 0. \quad (\text{A.15})$$

The aggregate quantity, defined in Equation (24), rises in the number of followers, n_S .

$$\begin{aligned} \frac{dQ_S(n_S)}{dn_S} &= \frac{-a}{(1+c+n_S)^2} \left[n_S + \frac{(1+c)^2}{Z(n_S)} \right] + \frac{a}{1+c+n_S} \left[1 - \frac{c(1+c)^2}{(Z(n_S))^2} \right] \\ &= \frac{a(1+c) [(n_S)^2 c^2 + (1+c)^2 (2+c(2+c+2n_S))] }{(1+c+n_S)^2 (Z(n_S))^2} > 0. \end{aligned} \quad (\text{A.16})$$

A.5.2. | Number of followers

We compute the number of followers in market equilibrium by combining Equations (7) and (23).

$$q_S(n_S) = \frac{a}{1+c+n_S} \left[1 - \frac{1+c}{Z(n_S)} \right] = \sqrt{\frac{2F}{2+c}} = q_S^*. \quad (\text{A.17})$$

Substituting for $Z(n_S)$ and using $M := a^2/2F > 2+c$, we rewrite Equation (A.17) as:

$$(1+c)^2 + cn_S = \frac{(1+c+n_S)((2+c)(1+c) + cn_S)}{\sqrt{M}\sqrt{2+c}}. \quad (\text{A.18})$$

Simplification of Equation (A.18) yields:

$$\begin{aligned} & \frac{(1+c)^2}{c} \left(\sqrt{M}\sqrt{2+c} - (2+c) \right) + \left(\frac{(1+c)^2}{c} - \frac{\sqrt{M}\sqrt{2+c}}{2} \right)^2 \\ &= (n_S)^2 + 2n_S \left(\frac{(1+c)^2}{c} - \frac{\sqrt{M}\sqrt{2+c}}{2} \right) + \left(\frac{(1+c)^2}{c} - \frac{\sqrt{M}\sqrt{2+c}}{2} \right)^2. \end{aligned} \quad (\text{A.19})$$

Applying the binomial formula to the right-hand side of Equation (A.19) and simplifying its left-hand side, we arrive at:

$$n_S + \frac{(1+c)^2}{c} - \sqrt{\frac{M(2+c)}{4}} = \mp \sqrt{\frac{M(2+c)}{4} + \frac{(1+c)^2}{c^2}}. \quad (\text{A.20})$$

Since the expression under the square root on the right-hand side of Equation (A.20) exceeds the term in magnitude, which is deducted on the left-hand side, a positive number of followers requires the right-hand side of Equation (A.20) to be positive. Solving Equation (A.20), we obtain Equation (26).

To compare the number of firms in market equilibrium in a Stackelberg-setting, $n_S^* + 1$, with the number of entrants in a Cournot-world, we deduct Equation (6) from Equation (26):

$$\begin{aligned} n_S^* + 1 - n^* &= \sqrt{\frac{M(2+c)}{4} + \frac{(1+c)^2}{c^2}} - \frac{(1+c)^2}{c} + \frac{\sqrt{M}\sqrt{2+c}}{2} + 1 - \sqrt{M}\sqrt{2+c} + (1+c) \\ &= \sqrt{\frac{M(2+c)}{4} + \frac{1+2c+c^2}{c^2}} - \sqrt{\left(\frac{\sqrt{M}\sqrt{2+c}}{2} + \frac{1}{c} \right)^2} \\ &= \sqrt{\frac{M(2+c)}{4} + \frac{1}{c^2} + \frac{2+c}{c}} - \sqrt{\frac{M(2+c)}{4} + \frac{\sqrt{M}\sqrt{2+c}}{c} + \frac{1}{c^2}} < 0. \end{aligned} \quad (\text{A.21})$$

Since $M > 2 + c$, there is more entry in a Cournot-oligopoly in market equilibrium than in a Stackelberg-setting, and we have $n_S^* < n^* - 1$.

A.5.3. | Entry in a first-best setting

To compare the number of firms in market equilibrium, n_S^* , to the first-best number, $n_S^{fb}(\alpha=0.5)$, we, first, utilize the feature that output of each follower is independent of their number in market equilibrium. Second, we compare aggregate output in market equilibrium to the hypothetical output that leader and followers would produce if the number of followers equaled $n^{fb}(\alpha=0.5) - 1$. If $Q_S(n^{fb}(\alpha=0.5) - 1) < Q_S^*$, the equilibrium number of followers exceeds $n^{fb}(\alpha=0.5) - 1$, as aggregate output rises in their number (see Equation (A.16)).

Aggregate output at $q_S = \sqrt{2F/(2+c)}$, assuming that there are $n^{fb}(\alpha=0.5) - 1$ followers, and using q_L from Equation (22), is given by:

$$\begin{aligned} Q_S(n^{fb}(\alpha=0.5) - 1) &= (n^{fb}(\alpha=0.5) - 1) \sqrt{\frac{2F}{2+c} + \frac{a(1+c)}{Z(n^{fb}(\alpha=0.5) - 1)}} \\ &= \left(\frac{a}{\sqrt{2F}} \sqrt{c} - (1+c) \right) \sqrt{\frac{2F}{2+c} + \frac{a(1+c)}{Z(n^{fb}(\alpha=0.5) - 1)}} \\ &= a \sqrt{\frac{c}{2+c} - (1+c)} \sqrt{\frac{2F}{2+c} + \frac{a(1+c)}{Z(n^{fb}(\alpha=0.5) - 1)}}. \end{aligned} \quad (\text{A.22})$$

Using Equation (24), and $A_S := c(n^{fb}(\alpha=0.5) - 1)(\sqrt{c} - \sqrt{2+c}) < 0$, we have:

$$\begin{aligned} &Q_S(n^{fb}(\alpha=0.5) - 1) - Q_S(n_S^*) \\ &= \frac{a}{Z(n^{fb}(\alpha=0.5) - 1)\sqrt{2+c}} \left[(1+c)\sqrt{2+c} - (\sqrt{2+c} - \sqrt{c})Z(n^{fb}(\alpha=0.5) - 1) \right] \\ &= \frac{a}{Z(n^{fb}(\alpha=0.5) - 1)\sqrt{2+c}} \left[\sqrt{c}(2+c)(1+c) + \sqrt{c}c(n^{fb}(\alpha=0.5) - 1) + (1+c)\sqrt{2+c} \right. \\ &\quad \left. - c(n^{fb}(\alpha=0.5) - 1)\sqrt{2+c} - \sqrt{2+c}(2+c)(1+c) \right] \\ &= \frac{a}{Z(n^{fb}(\alpha=0.5) - 1)\sqrt{2+c}} \left[\frac{(1+c)}{\sqrt{c}} \left[c(2+c) - \sqrt{c}\sqrt{2+c}(1+c) \right] + A_S \right] \\ &= \frac{a}{Z(n^{fb}(\alpha=0.5) - 1)\sqrt{2+c}} \left[\frac{(1+c)}{\sqrt{c}} \left[(\sqrt{2c+c^2})^2 - (1+c)\sqrt{2c+c^2} \right] + A_S \right] \\ &\quad \left[\underbrace{\sqrt{2c+c^2} - \sqrt{1+2c+c^2}}_{(-)} + \underbrace{\frac{A_S}{(1+c)\sqrt{2+c}}}_{(-)} \right] \\ &= a(1+c) \frac{\underbrace{\underbrace{\sqrt{2c+c^2} - \sqrt{1+2c+c^2}}_{(-)} + \underbrace{\frac{A_S}{(1+c)\sqrt{2+c}}}_{(-)}}_{Z(n^{fb}(\alpha=0.5) - 1)} < 0. \end{aligned} \quad (\text{A.23})$$

Therefore, output would be lower if there were $n^{fb}(\alpha=0.5) - 1$ followers than the output in market equilibrium. In consequence, $n^{fb}(\alpha=0.5) < n_S^* + 1$ holds.

A.6 | Second-best Stackelberg-setting with linear costs

To solve the Stackelberg-model with linear costs, we proceed in the same manner as for the setting with quadratic costs. For simplicity, we use the same notation.

Profits, π_{sj} , of follower j read:

$$\pi_{sj} = (a - q_{sj} - Q_{S,j} - q_L - c)q_{sj} - F. \quad (\text{A.24})$$

Assuming symmetry, each follower produces:

$$q_S(n_S) = \frac{a - c - q_L}{1 + n_S}. \quad (\text{A.25})$$

Maximization of the leader's profits, $\pi_L = (a - n_S q_S(q_L) - q_L - c)q_L - F$, results in an output level, which is independent of the number of followers.

$$q_L^* = \frac{a - c}{2}. \quad (\text{A.26})$$

Therefore, the aggregate quantity equals:

$$Q_S(n_S) = q_L^* + n_S \frac{a - c}{2(1 + n_S)} = \frac{(a - c)(1 + 2n_S)}{2(1 + n_S)}. \quad (\text{A.27})$$

Employing the above information, we can calculate the profit levels, consumer surplus and the competition authority's objective, V , as functions of the number of firms, n_S , as well as the number of followers, n_S^* , in market equilibrium.

$$\pi_S(n_S) = (a - Q_S(n_S) - c)q_S(n_S) - F = \frac{(a - c)^2}{4(1 + n_S)^2} - F, \quad (\text{A.28})$$

$$\pi_S(n_S) = 0 \Rightarrow n_S^* = \frac{a - c}{2\sqrt{F}} - 1, \quad (\text{A.29})$$

$$\pi_L(n_S) = (a - Q_S(n_S) - c)q_L^* - F = \frac{(a - c)^2}{4(1 + n_S)} - F, \quad (\text{A.30})$$

$$\frac{(Q_S(n_S))^2}{2} = \frac{(a - c)^2(1 + 2n_S)^2}{8(1 + n_S)^2}, \quad (\text{A.31})$$

$$\begin{aligned} V(n_S) &= \alpha[n_S \pi_S(n_S) + \pi_L(n_S)] + (1 - \alpha) \frac{(Q_S(n_S))^2}{2} \\ &= \alpha \left[\frac{(a - c)^2(1 - 4(n_S)^2)}{8(1 + n_S)^2} - (1 + n_S)F \right] + \frac{(a - c)^2(1 + 2n_S)^2}{8(1 + n_S)^2}. \end{aligned} \quad (\text{A.32})$$

The derivatives of $V(n_S)$ are:

$$\frac{\partial V(n_S)}{\partial n_S} = -\alpha \left[\frac{(a-c)^2(1+4n_S)}{4(1+n_S)^3} + F \right] + \frac{(a-c)^2(1+2n_S)}{4(1+n_S)^3}, \quad (\text{A.33})$$

$$\frac{\partial^2 V(n_S)}{\partial n_S^2} = \alpha \frac{(a-c)^2(1-8n_S)}{4(1+n_S)^8} - \frac{(a-c)^2(1+4n_S)}{4(1+n_S)^4} < 0. \quad (\text{A.34})$$

Setting the first-order derivative in Equation (A.33) equal to zero, evaluating it at the zero-profit level of followers, and solving the resulting expression for α , we obtain:

$$0.4 < \alpha^{L, sb} = \frac{1+2n_S^*}{2+5n_S^*} = \frac{2\frac{a-c}{2\sqrt{F}}-1}{5\frac{a-c}{2\sqrt{F}}-4} < \frac{3}{7}. \quad (\text{A.35})$$

The upper bound is obtained by noting that $\alpha^{L, sb}$ declines in n_S^* and assuming $n_S^* = 1$. Since $\alpha^{L, sb} < 0.5$, there is excessive entry. Evaluating the derivative in Equation (A.33) at $n_S = 0$, we find that the resulting expression is negative for any α exceeding

$$\alpha^{H, sb} = \frac{(a-c)^2}{(a-c)^2 + 4F}. \quad (\text{A.36})$$

Since a leader is profitable if $(a-c)^2 > 4F$ (see Equation (A.30)), $\alpha^{H, sb} > 0.5$ under this restriction.

A.7 | Cost heterogeneity

The first-order condition for a maximum of $\pi_{ji}(q_{ji}) = (a - Q_d)q_{ji} - 0.5c_i(q_{ji})^2 - F$ is:

$$\frac{\partial \pi_{ji}}{\partial q_{ji}} = -q_{ji} + a - Q_d - c_i q_{ji} = 0. \quad (\text{A.37})$$

Because all firms that face the same costs behave identically, we can solve Equation (A.37) for the reaction functions of firms of each type:

$$q_l(q_h, n_d) = \frac{2a - n_d q_h}{2 + n_d + 2(c - \varepsilon)} \quad \text{and} \quad q_h(q_l, n_d) = \frac{2a - n_d q_l}{2 + n_d + 2(c + \varepsilon)}. \quad (\text{A.38})$$

Combining these equations, equilibrium outputs by both types of firms and in aggregate can be computed as functions of the number of firms, n_d :

$$q_l^*(n_d) = \frac{a(1+c+\varepsilon)}{(1+n_d+c)(1+c)-\varepsilon^2}, \quad (\text{A.39})$$

$$q_h^*(n_d) = \frac{a(1+c-\varepsilon)}{(1+n_d+c)(1+c)-\varepsilon^2}, \quad (\text{A.40})$$

$$Q_d^*(n_d) = \frac{n_d}{2}(q_l^*(n_d) + q_h^*(n_d)) = \frac{n_d a(1+c)}{(1+n_d+c)(1+c)-\varepsilon^2}. \quad (\text{A.41})$$

Using Equation (27), as well as Equations (A.39)–(A.41), expected profits can be computed as in Equation (29).

The first-order conditions for a maximum of V_d in a first-best setting are:

$$\begin{aligned} \frac{\partial V_d}{\partial n_d} = 0, \quad \frac{\partial V_d}{\partial n_d} = 0 &\Rightarrow V_{n_d} := \alpha \\ \left(a - 2Q_d - \frac{(c-\varepsilon)q_l^2}{2(q_l+q_h)} - \frac{(c+\varepsilon)q_h^2}{2(q_l+q_h)} - \frac{2F}{q_l+q_h} \right) + (1-\alpha) & \\ Q_d = 0, & \end{aligned} \quad (\text{A.42})$$

$$\frac{\partial V_d}{\partial q_l} = 0 \Rightarrow V_{q_l} := \alpha(a - 2Q_d - (c-\varepsilon)q_l) + (1-\alpha)Q_d = 0, \quad (\text{A.43})$$

$$\frac{\partial V_d}{\partial q_h} = 0 \Rightarrow V_{q_h} := \alpha(a - 2Q_d - (c+\varepsilon)q_h) + (1-\alpha)Q_d = 0. \quad (\text{A.44})$$

The second-order derivatives are:

$$V_{n_d n_d} = V_{q_l n_d} = V_{q_h n_d} = (1-3\alpha) \frac{q_l + q_h}{2}, \quad (\text{A.45})$$

$$V_{n_d q_l} = \alpha \left(-n_d - \frac{(c-\varepsilon)q_l(2q_h+q_l)}{2(q_l+q_h)^2} + \frac{(c-\varepsilon)q_h^2}{2(q_l+q_h)^2} + \frac{2F}{(q_l+q_h)^2} \right) + (1-\alpha)n_d, \quad (\text{A.46})$$

$$V_{n_d q_h} = \alpha \left(-n_d + \frac{(c-\varepsilon)q_l^2 - (c+\varepsilon)q_h(2q_l+q_h)}{2(q_l+q_h)^2} + \frac{2F}{(q_l+q_h)^2} \right) + (1-\alpha)n_d, \quad (\text{A.47})$$

$$V_{q_l q_l} = 0.5n_d(1-3\alpha) - \alpha(c-\varepsilon), \quad (\text{A.48})$$

$$V_{q_l q_h} = V_{q_h q_l} = 0.5n_d(1-3\alpha), \quad (\text{A.49})$$

$$V_{q_h q_h} = 0.5n_d(1-3\alpha) - \alpha(c+\varepsilon). \quad (\text{A.50})$$

We assume that the first-order conditions characterize a maximum of V_d . Combining Equations (A.43) and (A.44) demonstrates that the competition authority chooses output levels so that marginal costs are equalized across both types of firms.

$$(c - \varepsilon)q_l = (c + \varepsilon)q_h. \quad (\text{A.51})$$

Combining Equations (A.42) and (A.43) yields:

$$(c - \varepsilon)q_l = \frac{(c - \varepsilon)q_l^2}{2(q_l + q_h)} + \frac{(c + \varepsilon)q_h^2}{2(q_l + q_h)} + \frac{2F}{q_l + q_h}. \quad (\text{A.52})$$

Substituting in Equation (A.52) in accordance with Equation (A.51), we obtain:

$$q_l^{fb} = \sqrt{\frac{4F(c + \varepsilon)}{2c(c - \varepsilon)}} > q^{fb} > q_h^{fb} = \sqrt{\frac{4F(c - \varepsilon)}{2c(c + \varepsilon)}}. \quad (\text{A.53})$$

To calculate the number of firms, we substitute Equation (A.53) into Equation (A.44) and utilize the feature that $Q_d = 0.5n_d(q_l + q_h) = 0.5n_d((c + \varepsilon)q_h / (c - \varepsilon) + q_h)$. This yields Equation (36).