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## Monetary and fiscal policies in times of large debt: Unity is strength

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## Monetary and Fiscal Policy in Times of Large Debt: Unity is Strength

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# Monetary and Fiscal Policy in Times of Large Debt: Unity is Strength

## Abstract

We build and estimate a novel TANK model with partially unfunded debt to study whether the record high debt-to-GDP ratio threatens US inflation stability. In response to business cycle shocks, the monetary authority controls inflation, and the fiscal authority stabilizes debt. The central bank accommodates unfunded fiscal shocks, causing persistent movements in inflation and real interest rates, leading to a fiscal theory of trend inflation. Fiscal trend inflation accounts for the bulk of inflation dynamics. The current situation is in line with historical experience. Unfunded shocks sustain the recovery and cause a temporary inflation increase that counteracts deationary non-policy shocks.

## Resume

Vi opstiller og estimerer en ny TANK-model med delvist ufinansieret gæld for at undersøge, om det rekordhøje forhold mellem gæld og BNP truer den stabile amerikanske inflation. Som reaktion på konjunkturchok styrer den pengepolitiske myndighed inflationen, og den finanspolitiske myndighed stabiliserer gælden. Når centralbanken reagerer på ufinansierede finanspolitiske stød, medfører det vedvarende udsving i inflation og realrenter. Det fører til en finanspolitisk teori om trendinflation. Finanspolitisk trendinflation tegner sig for hovedparten af inflationsdynamikkerne i modellen. Den nuværende situation i økonomien er i tråd med den historiske udvikling. Ufinansierede stød understøtter opsvinget og forårsager en midlertidig stigning i inflationen, hvilket modvirker deflationære ikke-politiske stød.

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## Key words

Monetary policy; Public finances; fiscal policy.

## JEL classification

E50; E62; E30.

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# Monetary and Fiscal Policies in Times of Large Debt: Unity is Strength\*

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## Abstract

We build and estimate a novel TANK model with partially unfunded debt to study whether the record high debt-to-GDP ratio threatens US inflation stability. In response to business cycle shocks, the monetary authority controls inflation, and the fiscal authority stabilizes debt. The central bank accommodates unfunded fiscal shocks, causing persistent movements in inflation and real interest rates, leading to a fiscal theory of trend inflation. Fiscal trend inflation accounts for the bulk of inflation dynamics. The current situation is in line with historical experience. Unfunded shocks sustain the recovery and cause a temporary inflation increase that counteracts deflationary non-policy shocks.

JEL Codes: E50, E62, E30.

Keywords: Unfunded fiscal spending, trend inflation, large public debt, inflation risk, shock-specific rule.

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# 1 Introduction

An important legacy of the COVID-19 pandemic is a record-high U.S. government debt. Even before the onset of the pandemic, the U.S. fiscal imbalance was significant by historical standards and required fiscal adjustments. In January 2020 –before the severity of the Pandemic recession was known– the Congressional Budget Office (CBO) estimated that, under current law, federal debt at the end of the decade would be higher as a percentage of GDP than at any time since 1946. If no fiscal adjustment is made, debt will continue to increase, and in 2050 it will reach the highest level ever recorded in the United States. The recent fiscal interventions meant to alleviate the consequences of the COVID19 pandemic and the recession caused by the various restrictive measures needed to contain the spreading of COVID-19 have contributed to exacerbate the already strained fiscal situation.

This dire situation has sparked a lively debate. Some prominent commentators and economists have argued that the current policies might lead to a return to the high and volatile inflation of the 1970s (Blanchard, 2021, and Summers, 2021). On the other hand, policymakers and other economists seem concerned about the risk that a fiscal adjustment could deeply affect the speed and strength of the recovery at a time in which monetary policy is constrained by the low interest rate environment (Powell, 2020).

In this paper, we show that this apparent policy trade-off admits a more favorable and likely outcome: A coordinated monetary and fiscal policy strategy meant to generate a temporary increase in inflation over the long-run 2% target. We show that this coordinated strategy achieves two important goals. First, it mitigates the persistent drag on economic activity due to expectations of future fiscal adjustments, which are needed to stabilize the large post-pandemic debt. Second, it corrects a deflationary bias that has characterized the past twenty years (Bianchi et al. 2021), allowing the central bank to remove the risk of deflation and move away from a low interest rate environment. We provide evidence that this coordinated strategy is not new in the US and is consistent with a novel *fiscal theory of trend inflation*. We finally offer an important caveat by pointing out that the coordinated strategy requires clear policy communication to avoid large swings in beliefs about the amount of debt that will be stabilized with inflation.

We build and estimate a state-of-the-art Two Agents New Keynesian (TANK) model with partially unfunded debt. The model features all the ingredients that have been proven successful in matching US business cycle dynamics, including a large set of business cycle shocks. With respect to these shocks, monetary policy satisfies the Taylor principle and the fiscal authority is in control of debt stabilization. Thus, in this respect, the model behaves as its counterparts extensively studied in the literature. However, the model also features

unfunded fiscal shocks. We model these as shocks to transfers that are not backed by future fiscal adjustments, implying that a share of the overall government debt is unfunded.<sup>1</sup> The central bank accommodates the increase in inflation necessary to stabilize the unfunded amount of debt. As a result, these shocks trigger persistent movements in inflation and a decline in real interest rates, leading to a fiscal theory of trend inflation.

As mentioned above, the model features a rich set of shocks, including a persistent shifter to the New Keynesian Phillips Curve (NKPC) that is meant to capture autonomous factors, such as globalization and demographic changes, that can affect inflation in the long run. Thus, it is an empirical question whether the unfunded shocks play an important role to explain the data. We show that fiscal trend inflation accounts for the bulk of inflation dynamics.

A persistent and partially unfunded increase in transfers in the mid-1960s, related to the introduction of the Great Society initiatives, accounts for the persistent increase in inflation during the Great Inflation. Symmetrically, the end of the Great Inflation is explained by a sharp revision in the amount of inflation that the Federal Reserve was going to tolerate to stabilize the portion of unfunded debt. In this respect, the aggressive increase in interest rates implemented by the Federal Reserve Chairman Paul Volcker can be interpreted as a strong signal of this policy change.

Starting from the 1990s, the amount of unfunded debt has been increasing sluggishly, counteracting a deflationary bias due to persistent non-policy shocks, arguably related to demographics and international trade. Finally, large part of the economic rebound at the end of 2020 is attributed to beliefs that a small fraction of the \$2.2 trillion fiscal package introduced in March to combat the consequences of the pandemic crisis will be partially unfunded. This change in beliefs is observed in the last quarter of 2020 when the Federal Reserve announced its new operating framework which contemplates the possibility of letting inflation overshoot its 2% target after the Pandemic recession.<sup>2</sup> This new monetary policy strategy affected the path of the federal funds rate, which we observe in the estimation.

We then study the implications of the model for the current situation, with a special focus on the risk of inflation posed by the large fiscal interventions implemented in response to the pandemic. We proceed in two steps. First, we produce a projection for inflation

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<sup>1</sup>We focus on shocks to transfers because historically government purchases (e.g., “G”) have been constantly declining as a fraction of GDP since WWII. Thus, government purchases do not seem to represent a problem for fiscal sustainability, while transfers have been increasing over the same period. Our results are robust to allowing both types of spending to be partially unfunded.

<sup>2</sup>On August 27, 2020, Federal Reserve Chairman Jerome Powell announced the new framework for the first time at the Jackson Hole Economic Symposium as follows: “Following periods when inflation has been running below 2 percent, appropriate monetary policy will likely aim to achieve inflation moderately above 2 percent for some time.”

using only data up to the end of 2020. The model predicts a modest inflation overshoot for the next four years, with a peak of 4%, and then a gradual return to the 2% target. We argue that this scenario depends on the presence of unfunded fiscal shocks in response to the pandemic. Absent these shocks, the model predicts a much weaker recovery and inflation to run persistently below the 2% target. The undershooting of inflation in this counterfactual scenario is driven by the long-lasting non-policy shocks that have been exerting progressively larger deflationary effects on the US economy since the early 2000s.

We then consider the effects of the recent American Rescue Plan Act (ARPA). We assign part of the increase in spending in the first quarter of 2021 to unfunded transfers based on the historical evidence. We then construct a conditional forecast and show that the model can replicate very closely the inflation figures for 2021Q1. The model now predicts a more robust, but still contained, overshoot of inflation.

Based on these results, we conclude that unfunded spending has played an important role in accounting for inflation dynamics. Thus, the current situation is not necessarily different from the historical experience for the United States. However, two qualifications have to be made regarding the present. First, the slowly widening deflationary bias is a critical challenge for policymakers that strive to avert long-run inflation expectations from progressively falling. Against this backdrop, an increase in fiscal trend inflation does not necessarily cause inflation expectations to become progressively de-anchored like in the 1960s-1970s. Rather, fiscal trend inflation can help policymakers to counteract these persistent deflationary forces and re-anchor inflation expectations to the desired level. In this respect, our results are in line with Chris Sims' remarks at the 2016 Jackson Hole meeting (Sims 2016) that policymakers should make clear that fiscal policy also aims at achieving a certain level of inflation.

Second, spending is now at an historical maximum. This implies that even small revisions in beliefs about the way it will be stabilized can lead to large swings in inflation. In the past, the amount of unfunded debt and fiscal trend inflation have evolved sluggishly, while the recent events led to a sudden acceleration. In this context, monetary policy can play an important role to coordinate and anchor beliefs. Specifically, the monetary authority can prevent swift changes in beliefs by setting a limit to the deviations of inflation from its long-run target. If such announcements are credible, they will coordinate and anchor beliefs on the share of unfunded spending. As discussed earlier, monetary policy seems to have already played this type of coordination role during the Volcker disinflation.

From the methodological standpoint, this paper develops a new class of models in which policymakers are allowed to react differently to different shocks. This allows us to combine the insights of the Fiscal Theory of Price Level in an otherwise standard TANK model. In this respect, the paper builds on Bianchi and Melosi (2019) who introduced the concept of

shock-specific rules as a way to resolve a conflict between the monetary and fiscal authorities in the presence of a high fiscal burden that the fiscal authority is reluctant or unable to stabilize. In that paper, we apply a shock specific rule to study the macroeconomic effects of introducing an emergency budget to mitigate a large recession. In this paper, we extend the notion of shock-specific rules to solve general equilibrium models in which monetary and fiscal authorities adopt state-dependent targets. This delivers a fiscal theory of trend inflation that is always at work, not only in response to exceptional events. We validate this theory conducting a structural estimation of the model.

This paper is connected to the vast literature on monetary-fiscal policy interaction (Sargent and Wallace 1981; Leeper 1991; Sims 1994; Woodford 1994, 1995, 2001; Cochrane 1998, 2001; Schmitt-Grohe and Uribe 2000, 2002; Bassetto 2002; Reis 2016; Bassetto and Sargent 2021, among many others). Aiyagari and Gertler (1985) study the implications of fiscal backing of government bonds for the propagation of shocks. They find that for debt to be irrelevant, the model needs to feature a considerable degree of accommodation with respect to the monetary authority. Leeper and Zhou (2013) find that inflation plays an important role in the optimal marginal financing of fiscal needs in models similar to the one used in our empirical analysis. Hall and Sargent (2011) show that historically most of US debt stabilization has been achieved through a combination of growth, revaluation effects, and low real interest rates. Davig and Leeper (2005) study the implications of regime changes in the policy mix in a calibrated NK model. Bianchi and Ilut (2017) estimate a model with regime changes in the monetary/fiscal policy mix and link the high inflation of the 1960s-1970s to a Fiscally-led regime. Bianchi and Melosi (2017) argue that the possibility of a return to such regime can explain the lack of deflation in the aftermath of the Great Recession. Mertens and Ravn (2014) study the fiscal multiplier at the zero lower bound.

With respect to models with regime changes in the monetary/fiscal policy mix, we move in a new and different direction. Monetary-led and Fiscally-led rules coexist in our model and the policy coordination is shock specific. Shocks to unfunded transfers are dealt with fiscally-led policies. With respect to all other shocks, the monetary authority controls inflation and the fiscal authority is responsible for debt stabilization. As a result, this new modeling approach delivers low-frequency movements in inflation linked to unfunded fiscal shocks, while at the same time preserving the typical propagation of the business cycle shocks employed in New Keynesian models.

Some scholars have recently advocated for *deficit monetization* or *helicopter money* to respond to the dreadful consequences of the pandemic recession (e.g., Galí 2019). In practice, the effects of unfunded transfer shocks are similar to those of deficit monetization, even if the equilibrium determination is different. While the model we used in our analysis is cashless,



we could assume that households derive utility from holding money and could expand the government budget constraint to include money growth. However, the mechanism through which prices rise would be the same as in the cashless model. Whatever happens to money in equilibrium is not necessary to pinpoint the source of inflation, which lies in the agreement between the fiscal and the monetary authorities about how to finance an existing fiscal burden.

The paper is organized as follows. Section 2 introduces the new class of models in which a Monetary-led regime and a Fiscally-led regime can coexist at the same time. In Section 3, we present a full-fledged, quantitative model. In Section 4 we discuss the estimation of the model parameters and in Section 5 we present the results. Section 6 concludes.

## 2 Fiscal Trend Inflation in DSGE Models

In this section, we introduce a new class of models in which a Monetary-led regime and a Fiscally-led regime can coexist at the same time. The propagation of shocks changes depending on the shock specific policy response. This allows us to introduce unfunded fiscal shocks in an otherwise standard model. We illustrate the logic of this new class of models with shock specific rules in the context of a simple Fisherian model (Leeper 1991 and Sims 1994 and 2016). Our focus is on fiscal trend inflation, but the method can be applied in other settings in which a researcher is interested in modeling shock specific policy responses.

**Standard model** The economy is populated by a continuum of infinitely many households and a government. The representative household has concave and twice continuously differentiable preferences over non-storable consumption goods and is endowed in each period with a constant quantity  $Y$  of these goods. The government issues one-period debt  $B_t$  to households who can trade them for consumption goods. The representative household chooses consumption and government bonds so as to maximize:

$$\max \sum_{t=0}^{\infty} \beta^t U(C_t),$$

subject to the flow budget constraint  $P_t C_t + Q_t B_t + P_t \tau_t = P_t Y + B_{t-1}$ , where  $\beta < 1$  is the households' discount factor,  $P_t$  denotes the price of consumption goods,  $\tau_t$  denotes real lump-sum net taxes, and  $Q_t = 1/R_t$  is the price of the one period government bond  $B_t$ , equal to the inverse of the gross interest rate  $R_t$ .

The government budget constraint reads as follows:

$$Q_t B_t + P_t \tau_t = B_{t-1}.$$

Since there are no government purchases, net taxes,  $\tau_t$ , coincide with the real primary surplus.

The central bank behaves according to the following monetary rule:

$$R_t/R = (\Pi_t/\Pi)^\phi,$$

where  $\Pi_t = P_t/P_{t-1}$  is the gross inflation rate at time  $t$ , variables without the time subscript denote the corresponding steady states, and the parameter  $\phi$  controls the strength with which the central bank reacts to movements of inflation from its target.

The fiscal authority moves lump-sum taxes according to the following fiscal rule:

$$\tau_t/\tau = (b_t/b)^\gamma e^{\epsilon_t},$$

where  $b_t = B_t/P_t$  denotes real debt,  $\tau$  and  $b$  are the steady-state values for taxes and real debt, respectively,  $\epsilon_t$  is a shock to lump-sum taxes, and the parameter  $\gamma$  determines how strongly the fiscal authority adjusts primary surpluses to fluctuations in debt.

Combining the households' Euler equation with the market clearing condition  $C_t = Y$  in every period leads to the Fisher equation:

$$Q_t = \beta (\mathbb{E}_t \Pi_{t+1})^{-1}.$$

**Linearized system of equations** We linearize the model equations around the deterministic steady state. Henceforth, hatted variables denote variables in log-deviation from their steady-state values. We obtain the following system of equations:

$$\hat{r}_t = \mathbb{E}_t \hat{\pi}_{t+1} \tag{1}$$

$$\hat{b}_t = \beta^{-1} \hat{b}_{t-1} - \hat{\tau}_t + b(\hat{r}_t - \beta^{-1} \hat{\pi}_t) \tag{2}$$

$$\hat{r}_t = \phi \hat{\pi}_t \tag{3}$$

$$\hat{\tau}_t = \gamma \hat{b}_{t-1} + \epsilon_t. \tag{4}$$

Plugging the monetary rule (3) into the Fisher equation (1) leads to the *monetary block*:

$$\mathbb{E}_t \hat{\pi}_{t+1} = \phi \hat{\pi}_t. \tag{5}$$

Combining the law of motion for real debt (2) with the fiscal rule (4) yields the *fiscal block*

$$\hat{b}_t = (\beta^{-1} - \gamma) \hat{b}_{t-1} - b(\beta^{-1} - \phi) \hat{\pi}_t - \epsilon_t. \quad (6)$$

**Existence and uniqueness of a solution** As shown by Leeper (1991), in this class of models there are two regions of the parameter space that deliver existence and uniqueness of a stationary solution. In the first region, monetary policy is active and responds more than one-to-one to deviations of inflation from its target ( $\phi > 1$ ). The fiscal authority implements the necessary fiscal adjustments to keep debt on a stable path ( $\gamma > \beta^{-1} - 1$ ). Fiscal policy is defined as passive because it passively accommodates the behavior of the monetary authority. We label this policy combination the Monetary-led policy mix. The distinctive feature of the Monetary-led policy mix is that the macroeconomy is completely insulated from the fiscal block and fiscal imbalances are irrelevant for inflation determination in equilibrium (*Monetary and Fiscal Dichotomy*). This is because debt stability is achieved with fiscal adjustments.<sup>3</sup> The first panel of Figure 1 illustrates this point by showing that inflation does not move in response to a negative shock to primary surpluses.

In the second region of the parameter space, labelled Fiscally-led policy mix, the fiscal authority is not committed to implementing the necessary fiscal adjustments. Monetary policy is now passive ( $\phi \leq 1$ ) because it passively accommodates the behavior of the active fiscal authority ( $\gamma \leq \beta^{-1} - 1$ ). Under the Fiscally-led policy mix, the macroeconomy is not insulated with respect to fiscal imbalances. In fact, inflation is determined by the need of stabilizing government debt. Consequently, fiscal imbalances affect inflation. The second panel of Figure 1 illustrates this point. Now a negative shock to primary surpluses leads to an increase in inflation. This increase in inflation is accommodated by the central bank and debt stability is preserved. This logic extends to richer models, in which fiscal imbalances will affect all macroeconomic variables, not just inflation.

**Shock specific rules and partially unfunded debt** We now extend the model to allow the Monetary-led and Fiscally-led policy mixes to coexist at the same time. In this new class of models, the dynamics typical of a Monetary-led policy mix coexist with the dynamics typical of a Fiscally-led policy mix. We focus on fiscal shocks, but the logic outlined below applies to all types of shocks that move the fiscal burden of the economy, as illustrated in the richer model considered in our empirical analysis. In what follows, we use the superscript  $M$  and  $F$  to denote policy parameters that imply a behavior in line with a Monetary-led policy mix and a Fiscally-led policy mix, respectively.

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<sup>3</sup>In richer models with distortionary taxation and government purchases, fiscal variables affect the macroeconomy, but through a different channel with respect to the one analyzed here.

We consider the following fiscal rule:

$$\tau_t/\tau = (b_{t-1}/b_{t-1}^F)^{\gamma^M} (b_{t-1}^F/b)^{\gamma^F} e^{\epsilon_t^M + \epsilon_t^F}, \quad (7)$$

where  $\epsilon_t^M$  and  $\epsilon_t^F$  denote funded and unfunded fiscal shocks, respectively. With respect to the amount of unfunded debt  $b_t^F$  accumulated as a result of the unfunded fiscal shocks, the fiscal authority is not committed to implement a large enough fiscal adjustment:  $\gamma^F < \beta^{-1} - 1$ . Instead, the fiscal authority is willing to fully stabilize deviations of debt from its unfunded component:  $\gamma^M > \beta^{-1} - 1$ . Thus, fiscal policy is passive with respect to the funded component of debt, while it is active with respect to the unfunded component of debt.

The new monetary rule is:

$$R_t/R = (\Pi_t/\Pi_t^F)^{\phi^M} (\Pi_t^F/\Pi)^{\phi^F}. \quad (8)$$

where  $\Pi_t^F$  denotes fiscal inflation, i.e., the amount of inflation that is tolerated by the central bank due to unfunded fiscal shocks. With respect to fiscal inflation, monetary policy is passive: the central bank reacts less than one-to-one,  $\phi^F \leq 1$ . Instead, the central bank is active in stabilizing inflation in deviations from fiscal inflation:  $\phi^M > 1$ .

Linearizing the fiscal rule in equation (7) we obtain:

$$\hat{\tau}_t = \gamma^M (\hat{b}_{t-1} - \hat{b}_{t-1}^F) + \gamma^F \hat{b}_{t-1}^F + \epsilon_t^M + \epsilon_t^F. \quad (9)$$

Given that  $\gamma^F < \beta^{-1} - 1$ , the expression above makes clear that the fiscal adjustments are not large enough to cover the entirety of the fiscal burden.

Linearizing the monetary rule, we obtain:

$$\hat{r}_t = \phi^M (\hat{\pi}_t - \hat{\pi}_t^F) + \phi^F \hat{\pi}_t^F. \quad (10)$$

If we further assume  $\phi^F = 0$ , we obtain a Taylor rule that is isomorphic to a rule with a time-varying target:  $\hat{r}_t = \phi^M (\hat{\pi}_t - \hat{\pi}_t^F)$ . Crucially, the time-varying target  $\hat{\pi}_t^F$  is not an additional shock, but it is instead tightly related to the amount of inflation tolerated by the central bank to stabilize a portion of the overall fiscal burden, leading to a fiscal theory of trend inflation.

Appendix A exploits the linearity of the model to prove that the components of debt and inflation in deviations from their corresponding targets,  $\hat{b}_t^M = \hat{b}_t - \hat{b}_t^F$  and  $\hat{\pi}_t^M = \hat{\pi}_t - \hat{\pi}_t^F$ , are *exactly* the amounts of debt and inflation that would arise if the Monetary-led policy mix were always in place and only funded shocks occurred. We can then interpret  $\hat{b}_t^M$  as the amount of funded debt that the fiscal authority is committed to stabilize through fiscal

adjustments. Analogously,  $\hat{\pi}_t^M$  corresponds to movements in inflation originating from shocks that the central bank does not accommodate and that are instead responsibility of the fiscal authority. We use the superscript  $M$  to emphasize that the Monetary-led policy mix applies with respect to these variables.

Using the fact that in the linearized model the total amount of debt is the sum of two components, funded and unfunded debt,  $\hat{b}_t = \hat{b}_t^M + \hat{b}_t^F$ , we can rewrite the fiscal rule as:

$$\hat{\tau}_t = \gamma^M \hat{b}_{t-1}^M + \gamma^F \hat{b}_{t-1}^F + \epsilon_t^M + \epsilon_t^F.$$

Similarly, exploiting the fact that in the linearized model  $\hat{\pi}_t = \hat{\pi}_t^M + \hat{\pi}_t^F$ , the monetary rule can be re-written as:

$$\hat{r}_t = \phi^M \hat{\pi}_t^M + \phi^F \hat{\pi}_t^F.$$

Thus, the linearized model allows two equivalent ways to interpret the policy rules. First, the policy rules can be interpreted as describing a situation in which policymakers react to time-varying targets that are driven by the need of stabilizing the amount of unfunded debt. Alternatively, the policy rules can be interpreted as shock specific rules in which policymakers react differently to the different components of the endogenous target variables depending on the shocks that generate the fluctuations.

Substituting the monetary rule (10) into the Fisherian equation (1) yields the monetary block of the model with partially unfunded debt:

$$\mathbb{E}_t \hat{\pi}_{t+1} = \phi^M (\hat{\pi}_t - \hat{\pi}_t^F) + \phi^F \hat{\pi}_t^F. \quad (11)$$

Similarly, plugging the monetary and fiscal rules in the law of motion of debt (2), yields the fiscal block:

$$\hat{b}_t = (\beta^{-1} - \gamma^M) \hat{b}_{t-1} + \gamma^M \hat{b}_{t-1}^F - b (\beta^{-1} - \phi^M) \hat{\pi}_t - b (\phi^M - \phi^F) \hat{\pi}_t^F - \epsilon_t^M - \epsilon_t^F, \quad (12)$$

where to simplify the exposition, and without loss of generality, we have assumed that the fiscal authority completely disregards the amount of unfunded debt:  $\gamma^F = 0$ .

To close the model, we need to characterize the dynamics of fiscal inflation,  $\hat{\pi}_t^F$ , and of the associated amount of unfunded debt,  $\hat{b}_t^F$ . To do so, we construct a shadow economy in which the Fiscally-led policy mix is always in place and only the shocks to unfunded spending  $\epsilon_t^F$  occur. The shadow economy keeps track of fiscal inflation and the amount of unfunded debt.

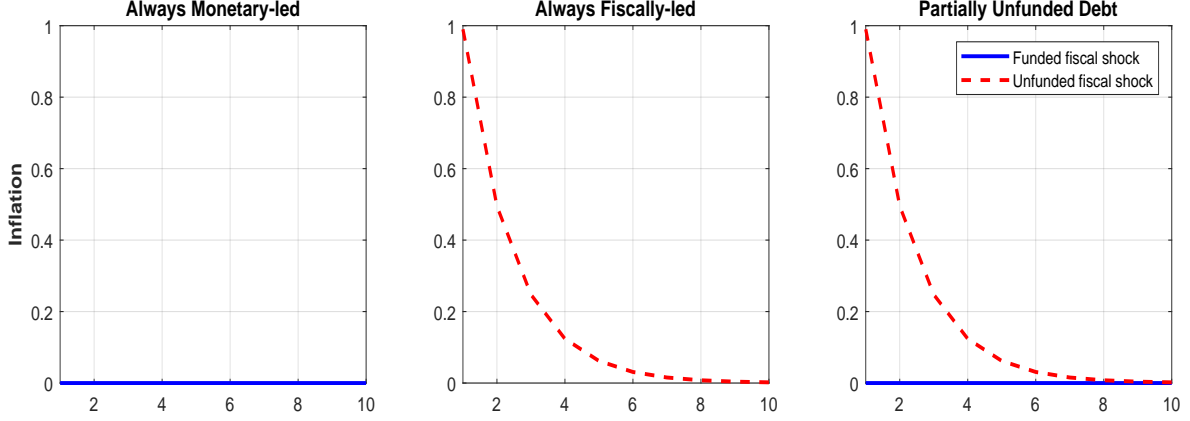


Figure 1: Impulse response of inflation to a fiscal shock. The discount factor  $\beta$  is set to 0.99 and the steady-state value of debt  $b$  to 1. In the model with partially unfunded debt, the monetary policy parameters are  $\phi^M = 2.0$  and  $\phi^F = 0.5$  and the fiscal policy parameters are  $\gamma^M = 0.2$  and  $\gamma^F = 0$ . The Always Monetary-led model is parameterized as follows:  $\phi = \phi^M$  and  $\gamma = \gamma^M$ . The Always Fiscally-led model is parameterized as follows:  $\phi = \phi^F$  and  $\gamma = \gamma^F$ . Fiscal shocks are i.i.d. with unit variance.

The monetary and fiscal blocks for the shadow economy are then:

$$\mathbb{E}_t \hat{\pi}_{t+1}^F = \phi^F \hat{\pi}_t^F, \quad (13)$$

$$\hat{b}_t^F = \beta^{-1} \hat{b}_{t-1}^F - b(\beta^{-1} - \phi^F) \hat{\pi}_t^F - \epsilon_t^F. \quad (14)$$

Note that the monetary and fiscal blocks for the shadow economy are isomorphic to those in equations (5) and (6) once the parameter restrictions for the Fiscally-led policy mix are imposed and the only fiscal shocks are the funded ones.

The set of equations (11), (12), (13), and (14) describe the model with partially unfunded debt. Since there are two non-predetermined variables ( $\hat{\pi}_t$  and  $\hat{\pi}_t^F$ ) and two non-stationary eigenvalues associated with equations (11) and (14), the model satisfies the Blanchard and Khan conditions and is thereby determinate – there exists a unique stable Rational Expectations equilibrium.

The third panel of Figure 1 presents the impulse responses in the model with partially unfunded debt.<sup>4</sup> In response to a funded spending shock (solid blue line), the economy with partially unfunded debt behaves exactly as in the left panel, where policymakers always follow the Monetary-led policy mix, and inflation is unaffected by the shock. In response to an unfunded spending shock, inflation increases. The economy with partially unfunded debt behaves exactly as in the middle panel, where policymakers always follow the Fiscally-led policy mix. The policy rules in the model with partially unfunded debt are shock-specific and policymakers respond differently depending on the nature of the fiscal shocks. Thus, the properties of the Monetary-led and Fiscally-led policy mix coexist in the model with

<sup>4</sup>In Appendix A, we also report the responses of debt, the interest rate, and the primary surplus.

partially unfunded debt.

The focus of this paper is on the effects of unfunded fiscal shocks. However, this new class of models can be used to study other forms of heterogeneity in policy responses. Furthermore, the results presented here extend to more complex models in which fiscal policy is distortionary and more shocks are present. In our empirical analysis, we consider a state-of-the-art TANK model with a rich set of shocks. With respect to the typical business cycle shocks, the economy behaves as in other TANK models studied in the literature. However, unfunded fiscal shocks lead to the dynamics typical of a Fiscally-led regime.

Finally, it is worth mentioning that we could also have solved the model by constructing a different (Monetary-led) shadow economy in which all public debt is funded, the central bank always follows the Taylor principle, but only the funded fiscal shocks occur. This duality in solving models with shock specific rules stems from the linearity of the model. Linearity implies that the two shadow economies are indeed additive sub-economies of the actual economy. This means that the sum of the inflation rates and the sum of debts in the two parallel economies are equal to their counterparts in the actual economy, as shown in Appendix A.

### 3 The Model

We build and estimate a state-of-the-art Two Agents New Keynesian (TANK) model with a rich fiscal block and partially unfunded debt. The model features all the ingredients that have been proven successful in matching US business cycle dynamics, including a large set of business cycle shocks. With respect to these shocks, monetary policy satisfies the Taylor principle and the fiscal authority is in control of debt stabilization. Thus, in this respect, the model behaves as its counterparts extensively studied in the literature (see among many others, Christiano et al. (2005) and Leeper et al. (2017)). However, the model also features unfunded fiscal shocks. These shocks to transfers are not backed by future fiscal adjustments, implying that a share of the overall government debt is unfunded. The central bank accommodates the increase in inflation necessary to stabilize the unfunded amount of debt. As a result, these shocks allow the model to generate persistent movements in inflation and a decline in real interest rates, leading to a fiscal theory of trend inflation. In what follows, we outline the model in detail.

### 3.1 The economy

The economy is populated by a unit measure of households, of which a fraction  $\mu$  are hand-to-mouth consumers. The remaining fraction,  $1 - \mu$ , are savers and we indicate them with an  $S$  superscript. The presence of hand-to-mouth households, together with distortionary taxation, breaks Ricardian equivalence and makes transfers relevant for a fraction of the population even under a Monetary-led policy mix.

**Savers** A household of optimizing saving agents, indexed by  $j$ , derives utility from the consumption of a composite good,  $C_t^{*S}(j)$ , which comprises private consumption  $C_t^S(j)$  and government consumption  $G_t$  such that  $C_t^{*S}(j) = C_t^S(j) + \alpha_G G_t$ . The parameter  $\alpha_G$  governs the substitutability between private and government consumption. When negative, the goods are complements; when positive, they are substitutes. External habits in consumption imply that utility is derived relative to the previous period value of aggregate savers' consumption of the composite good  $\theta C_{t-1}^{*S}$ , where  $\theta \in [0, 1]$  is the habit parameter. Saver households also derive disutility from the supply of differentiated labor services from all its members, indexed by  $l$ ,  $L_t^S(j) = \int_0^1 L_t^S(j, l) dl$ . The period utility function is given by  $\mathcal{U}_t^S(j) = u_t^b \left( \ln(C_t^{*S}(j) - \theta C_{t-1}^{*S}) - L_t^S(j)^{1+\xi} / (1 + \xi) \right)$ , where  $u_t^b$  is a discount factor shock and  $\xi$  is the Frisch elasticity of labor supply.

Households accumulate wealth in the form of physical capital  $\bar{K}_t^S$ . The stock of capital depreciates at rate  $\delta$  and accrues with investment  $I_t^S$ , net of adjustment costs. The law of motion for physical capital is:  $\bar{K}_t^S(j) = (1 - \delta) \bar{K}_{t-1}^S(j) + u_t^i [1 - s(I_t^S(j) / I_{t-1}^S(j))] I_t^S(j)$ , where  $u_t^i$  is a shock to the marginal efficiency of investment and  $s$  denotes an investment adjustment cost function that satisfies the properties  $s(e^\gamma) = s'(e^\gamma) = 0$  and  $s''(e^\gamma) \equiv s > 0$ , where  $\gamma$  is the steady-state growth rate of the economy.

Households derive income from renting effective capital  $K_t^S(j)$  to the intermediate firms. Effective capital is related to physical capital according to  $K_t^S(j) = \nu_t(j) \bar{K}_{t-1}^S(j)$ , where  $\nu_t(j)$  is the capital utilization rate. The cost of utilizing one unit of physical capital is given by the function  $\Psi(\nu_t(j))$ . Given the steady-state utilization rate  $\nu(j) = 1$ , the function  $\Psi$  satisfies the following properties:  $\Psi(1) = 0$ , and  $\frac{\Psi''(1)}{\Psi'(1)} = \frac{\psi}{1-\psi}$ , where  $\psi \in [0, 1)$ . We further denote the gross rental rate of capital as  $R_t^K$  and the tax rate on capital rental income as  $\tau_t^K$ .

The household can also save by purchasing two types of zero-coupon bonds which differ in their maturity. One-period bonds promising a nominal payoff  $B_{s,t}$  at time  $t + 1$  can be purchased at the present discounted value  $R_{b,t}^{-1} B_{s,t}$ , where the gross nominal interest rate  $R_{b,t}$  is related to the interest rate set by the Central Bank  $R_t$  through the equation  $R_{b,t} = u_t^{rp} R_t$ , and the wedge  $u_t^{rp}$  can be interpreted as a risk premium shock. Long-term government bond  $B_t$  with a maturity decaying at a constant rate  $\rho \in [0, 1]$  and duration  $(1 - \beta\rho)^{-1}$ , can be



purchased at price  $P_t^B$ .

Each period, the household receives after-tax nominal labor income, after-tax revenues from renting capital to the firms, lump-sum transfers from the government  $Z_t^S$  and dividends from the firms  $D_t$ . These resources can be spent to consume and to invest in physical capital and bonds. The nominal budget constraint for the saver household is:

$$\begin{aligned} & P_t (1 - \tau_t^C) C_t^S(j) + P_t I_t^S(j) + P_t^B B_t(j) + R_{b,t}^{-1} B_{s,t} \\ = & (1 + \rho P_t^B) B_{t-1}(j) + B_{s,t-1}(j) + (1 - \tau_t^L) \int_0^1 W_t(l) L_t^S(j, l) dl \\ & + (1 - \tau_t^K) R_t^k \nu_t(j) \bar{K}_{t-1}^S(j) - \psi(\nu_t) \bar{K}_{t-1}^S(j) + P_t Z_t^S(j) + D_t(j), \end{aligned} \quad (15)$$

where  $W_t(l)$  denotes the wage rate that applies to all household members, and  $\tau_t^C$  and  $\tau_t^L$  denote the tax rates on consumption and labor income, respectively. The household maximizes lifetime discounted utility  $\sum_{t=0}^{\infty} \beta^t \mathcal{U}_t^S(j)$  subject to the sequence of budget constraints in equation (15).

**Hand-to-mouth households** Every period, hand-to-mouth households consume all of their disposable, after-tax income, which comprises revenues from labor supply and government transfers. It is assumed that the hand-to-mouth households supply differentiated labor services, and set their wage to be equal to the average wage that is optimally chosen by the savers, as described below. Using the superscript  $N$  to indicate the non-saving, hand-to-mouth households, their budget constraint can be written as follows:

$$(1 + \tau_t^C) P_t C_t^N(j) = (1 - \tau_t^L) \int_0^1 W_t(l) L_t^N(j, l) dl + P_t Z_t^N(j),$$

where it is assumed that both savers and non-savers face the same tax rates on consumption and labor income.

**Final good producers** A perfectly competitive sector of final good firms produces the homogeneous good  $Y_t$  at time  $t$  by combining a unit measure of intermediate differentiated inputs using the technology  $Y_t = \left( \int_0^1 Y_t(i)^{\frac{1}{1+\eta_t^p+u_t^{NKPC}}} di \right)^{1+\eta_t^p+u_t^{NKPC}}$ , where  $\eta_t^p$  denotes an exogenous mark-up shock to the prices of intermediate goods and  $u_t^{NKPC}$  is a persistent shifter to the New Keynesian Phillips Curve (NKPC) that is meant to capture external forces such as globalization and demographic changes that can affect inflation in the long run. Profit maximization yields the demand function for intermediate goods  $Y_t(i) = Y_t (P_t(i) / P_t)^{-(1+\eta_t^p+u_t^{NKPC})/(\eta_t^p+u_t^{NKPC})}$ , where  $P_t(i)$  is the price of the differentiated

good  $i$  and  $P_t$  is the aggregate price of the final good.

**Intermediate good producers** Intermediate firms produce using the technology  $Y_t(i) = K_t(i)^\alpha (A_t L_t(i))^{1-\alpha} - A_t \Omega$ , where  $\Omega$  is a fixed cost of production that grows with the rate of labor-augmenting technological progress  $A_t$  and  $\alpha \in [0, 1]$  a parameter. It is assumed that technological progress  $A_t$  follows an exogenous process that is stationary in the growth rate. Specifically, we assume that  $u_t^a = (1 - \rho_a) \gamma + \rho_a u_{t-1}^a + \varepsilon_t^a$ , where  $u_t^a = \ln A_t - \ln A_{t-1}$  and  $\gamma$  is a drift parameter capturing the logarithm of the rate of technology growth in steady state. Intermediate firms rent capital and labor in perfectly competitive factor markets. It is assumed that  $L_t$  is a bundle of all the differentiated labor services supplied in the economy, which are aggregated into a homogeneous input by a labor agency, as described below. The nominal rental rate of capital is denoted by  $R_t^K$  and the wage rate by  $W_t$ . Cost minimization implies that all firms incur the same nominal marginal cost  $MC_t = (1 - \alpha)^{\alpha-1} \alpha^{-\alpha} (R_t^K)^\alpha W_t^{1-\alpha} A_t^{-1+\alpha}$ .

When setting prices, intermediate producers face frictions *à la* Calvo, i.e., at time  $t$  a firm  $i$  can optimally reset its price with probability  $\omega^p$ . Otherwise it adjusts the price with partial indexation to the previous period inflation rate according to the rule  $P_t(i) = (\Pi_{t-1})^{\chi_p} (\Pi)^{1-\chi_p} P_{t-1}(i)$ , where  $\chi_p \in [0, 1]$  is a parameter,  $\Pi_{t-1} = \frac{P_{t-1}}{P_{t-2}}$  and  $\Pi$  denotes the aggregate rate of inflation at steady state.

Intermediate producers that are allowed to reset their price maximize the expected discounted stream of nominal profits:

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \omega_p)^s \frac{\Lambda_{t+s}^S}{\Lambda_t^S} \left[ \left( \prod_{k=1}^s \Pi_{t+k-1}^{\chi_p} \Pi^{1-\chi_p} \right) P_t(i) Y_{t+s}(i) - MC_{t+s} Y_{t+s}(i) \right]$$

subject to the demand function of the final good sector, where  $\Lambda^S$  denotes the marginal utility of the savers.

**Wages** We assume that both savers and hand-to-mouth households are monopoly suppliers of a unit measure of differentiated labor service, indexed by  $l$ . Each period, a saver household gets an opportunity to optimally readjust the wage rate that applies to all of its workers,  $W_t(l)$ , with probability  $\omega_w$ . If the wage cannot be reoptimized, it will be increased at the geometric average of the steady-state rate of inflation  $\Pi$  and of last period inflation  $\Pi_{t-1}$ , according to the rule  $W_t(l) = W_{t-1}(l) (\Pi_{t-1} e^\gamma)^{\chi_w} (\Pi e^\gamma)^{1-\chi_w}$ , where  $\chi_w \in [0, 1]$  captures the degree of nominal wage indexation. It is assumed that the hand-to-mouth households set their wage to be equal to the average wage that is optimally chosen by the savers.

All households, including both savers and non-savers, sell their labor service to a repre-

sentative, competitive agency that transforms it into an aggregate labor input, according to the technology  $L_t = \left( \int_0^1 L_t(l)^{\frac{1}{1+\eta_t^w}} dl \right)^{1+\eta_t^w}$ , where  $\eta_t^w$  is an exogenous wage mark-up shock. The agency rents labor type  $L_t(l)$  at price  $W_t(l)$  and sells a homogeneous labor input to the intermediate producers at price  $W_t$ . The static profit maximization problem yields the demand function  $L_t(l) = L_t(W_t(l)/W_t)^{-(1+\eta_t^w)/\eta_t^w}$ .

**Monetary and Fiscal Policy** Assuming that one-period government bonds are in zero net supply, the government nominal budget constraint can be written as:

$$P_t^B B_t + \tau_t^K R_t^K K_t + \tau_t^L W_t L_t + \tau_t^C P_t C_t = (1 + \rho P_t^B) B_{t-1} + P_t G_t + P_t Z_t, \quad (16)$$

where  $C_t = \mu C_t^N + (1 - \mu) C_t^S$  denotes aggregate consumption and  $Z_t = \int_0^1 Z_t(j) dj = Z_t^S = Z_t^N$ , using the assumption that every household receives the same amount of transfers regardless of whether it is hand-to-mouth or saver. The budget constraint in equation (16) implies that the fiscal authority finances government expenditures, transfers, and the rollover of expiring long-term debt by raising taxes on consumption, labor and capital, and by issuing new long-term debt obligations.

We rescale the variables entering the fiscal rules by defining  $g_t = G_t/A_t$  and  $z_t = Z_t/A_t$ . In what follows, for each variable  $x_t$ , we use  $\hat{x}_t$  to denote the percentage deviation from its own steady state. Let  $s_{b,t} = \frac{P_t^B B_t}{P_t Y_t}$  be the debt-to-GDP ratio. As in the model presented in Section 2, the debt-to-GDP ratio in deviations from the steady state,  $\hat{s}_{b,t}$ , is the sum of two components, funded  $\hat{s}_{b,t}^M$  and unfunded  $\hat{s}_{b,t}^F$  debt. As before, we use superscripts  $M$  and  $F$  to emphasize that the Monetary-led policy mix applies to funded debt, while the Fiscally-led policy mix applies to unfunded debt. For the shocks, we use the superscripts only to label the two types of transfer shocks, while we assume that all other shocks only affect the funded portion of debt.

The fiscal authority adjusts government spending  $\hat{g}_t$ , transfers  $\hat{z}_t$ , and tax rates on capital income, labor income, and consumption  $\hat{\tau}_t^J$ ,  $J \in \{K, L, C\}$  as follows:

$$\hat{g}_t = \rho_G \hat{g}_{t-1} - (1 - \rho_G) \gamma_G \hat{s}_{b,t-1}^M + \zeta_{g,t}, \quad (17)$$

$$\hat{z}_t = \rho_Z \hat{z}_{t-1} - (1 - \rho_Z) \gamma_Z \hat{s}_{b,t-1}^M + \zeta_{z,t}^M + \zeta_{z,t}^F, \quad (18)$$

$$\hat{\tau}_t^J = \rho_J \hat{\tau}_{t-1}^J + (1 - \rho_J) \gamma_J \hat{s}_{b,t-1}^M \quad (19)$$

where  $\hat{s}_{b,t-1}^M = \hat{s}_{b,t-1} - \hat{s}_{b,t-1}^F$  denotes the portion of the debt-to-GDP ratio that the fiscal authority is committed to stabilize with fiscal adjustments. This commitment is captured by the values for the reaction parameters  $\gamma_G$ ,  $\gamma_Z$ , and  $\gamma_J > 0$  that are large enough to guarantee

that the portion  $\hat{s}_{b,t-1}^M$  of debt remains on a stable path. The fiscal authority does not make fiscal adjustments in response to the remaining, unfunded, portion of debt  $\hat{s}_{b,t-1}^F$ . The variables  $\zeta_{z,t}^M$  and  $\zeta_{z,t}^F$  denote shocks to transfers that are respectively funded and unfunded; the realizations of the unfunded shock, cumulated over time, determine the evolution of the shares of debt that is unfunded. The amount of funded debt is instead determined by the funded fiscal shocks and the business cycle shocks. The shocks  $\zeta_{g,t}$ ,  $\zeta_{z,t}^M$  and  $\zeta_{z,t}^F$  follow AR(1) Gaussian stochastic processes.

The central bank is fully committed to move the short-term interest rate  $\hat{R}_t$  in response to the movements of inflation originating from the typical business cycle shocks and the funded fiscal shocks, while it fully accommodates the movements in inflation necessary to stabilize the unfunded portion of debt. As explained in Section 2, this shock specific monetary policy rule can be captured by a standard Taylor rule in which the central bank reacts to deviations of inflation from the level of inflation needed to stabilize the unfunded share of debt. We call this level of inflation tolerated by the central bank, fiscal trend inflation,  $\hat{\pi}_t^F$ . It follows that the linearized monetary policy rule with an effective lower bound constraint (ELB) can be written as:

$$\hat{R}_t = \max \left[ -\ln \underline{R}, \rho_r \hat{R}_{t-1} + (1 - \rho_r) [\phi_\pi (\hat{\pi}_t - \hat{\pi}_t^F) + \phi_y \hat{y}_t] \right] + u_t^m. \quad (20)$$

where  $u_t^m$  is a monetary policy shock.

The parameter  $\phi_\pi > 1$  implies that the Taylor principle is satisfied and monetary policy is active when it comes to respond to deviations of inflation,  $\hat{\pi}_t$ , from fiscal trend inflation,  $\hat{\pi}_t^F$ , the level of inflation needed to stabilize the unfunded share of debt. Indeed, the variable  $\hat{\pi}_t^F$  measures the increase in trend inflation, relative to the central bank long-term target (and steady-state rate), that the central bank accommodates so as to stabilize the share of unfunded debt  $\hat{s}_{b,t-1}^F$ . The policy mix characterized by equations (17) - (20) therefore implies that monetary policy is active in response to deviations of inflation from fiscal trend inflation and passive (no response) with respect to the inflation needed to stabilize the share of unfunded debt in deviations from its long-term target. Concurrently, fiscal policy is passive with respect to its commitment in stabilizing the share of funded government debt  $\hat{s}_{b,t-1}^M$ , and active (no response) with respect to the unfunded share of debt. Thus, a monetary-led policy mix with respect to the typical business cycle shocks coexists with a fiscally-led policy mix with respect to the unfunded fiscal shocks.

The way fiscal trend inflation  $\hat{\pi}_t^F$  enters the Taylor rule is similar to a time-varying target or an inflation drift that are typically added to estimated medium-scale DSGE models to explain trend inflation in the data. (e.g., Christiano, Eichenbaum, and Evans 2005 and

Smets and Wouters 2007). However, while the inflation drift in these other models evolves exogeneously according to a close-to-random-walk process, the fiscal trend inflation in our model varies in response to the need to stabilize the share of unfunded debt, which is endogenous. Therefore, changes in trend inflation  $\hat{\pi}_t^F$  are made credible by the coordination between monetary and fiscal authorities regarding the stabilization of the existing public debt.

### 3.2 Solving the Model

The unit-root process followed by the labor-augmenting technology  $A_t$  implies that some variables are non-stationary. Hence, we first detrend the non-stationary variables and then we log-linearize the model equations around the steady-state equilibrium (transfers and primary surplus are linearized).<sup>5</sup>

To solve the model, we need to track the evolution of the shares of funded and unfunded debt and fiscal trend inflation. To do so, we construct a shadow economy that keeps track of the funded portion of debt and the associated evolution of the endogenous variables. This economy is characterized by the same set of equations as the actual economy except for the monetary and fiscal rules. Specifically, the rules in the shadow economy differ from those in the actual economy insofar as: i) the shocks to unfunded debt are shut down, implying that the entire debt in the shadow economy is funded; ii) the central bank always responds to inflation deviations from its fixed target (i.e., steady-state inflation). Denoting by the superscript  $M$  any variable that belongs to the parallel economy, the shadow rules for the monetary and fiscal authorities read as follows:

$$\hat{g}_t^M = \rho_G \hat{g}_{t-1}^M - (1 - \rho_G) \gamma_G \hat{s}_{b,t-1}^M + \zeta_{g,t}, \quad (21)$$

$$\hat{z}_t^M = \rho_Z \hat{z}_{t-1}^M - (1 - \rho_Z) \gamma_Z \hat{s}_{b,t-1}^M + \zeta_{z,t}^F, \quad (22)$$

$$\hat{\tau}_t^{J,M} = \rho_J \hat{\tau}_{t-1}^{J,M} + (1 - \rho_J) \gamma_J \hat{s}_{b,t-1}^M \quad (23)$$

$$\hat{R}_t^M = \max \left[ -\ln \underline{R}, \rho_r \hat{R}_{t-1}^M + (1 - \rho_r) [\phi_\pi \hat{\pi}_t^M + \phi_y \hat{y}_t^M] \right] + u_t^m, \quad (24)$$

where any difference with respect to the actual economy can be grasped by comparing equations (22) and (24) with (18) and (20), respectively. By construction, the shadow economy represents a counterfactual economy with no unfunded debt. Consequently, it allows us to track the evolution of the amount of funded debt that the fiscal authority is committed to stabilize with fiscal adjustments, i.e. the share of debt in the actual economy

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<sup>5</sup>The list of the log-linearized equations of the model is reported in Appendix B.

that is stabilized by a Monetary-led policy mix.

As explained earlier, in the actual economy the monetary authority responds to deviations of inflation,  $\hat{\pi}_t$ , from fiscal trend inflation, the level of inflation needed to stabilize the share of unfunded debt,  $\hat{\pi}_t^F$ . As shown for the stylized model in Section 2, this level of inflation is precisely the difference between the inflation rate in the actual economy and the inflation rate in the shadow economy; that is,  $\hat{\pi}_t^F = \hat{\pi}_t - \hat{\pi}_t^M$ . By plugging  $\hat{\pi}_t^M = \hat{\pi}_t - \hat{\pi}_t^F$  into the Monetary policy rule of the actual economy – equation (20) – we obtain

$$\hat{R}_t = \max \left[ -\ln \underline{R}, \rho_r \hat{R}_{t-1} + (1 - \rho_r) [\phi_\pi \hat{\pi}_t^M + \phi_y \hat{y}_t] \right] + u_t^m. \quad (25)$$

This rule implies that the monetary authority conducts active monetary policy only with respect to the inflation rate in the shadow economy.

With the level of debt entering the fiscal rules in the actual economy equal to the stock of debt in the shadow economy and the monetary rule of the actual economy expressed as a function of the inflation rate in the shadow economy, we can combine the policy functions characterizing the actual and the shadow economy to get a system of linear Rational Expectations equations. The model with partially unfunded debt can then be solved with standard solution algorithms and evaluated with a structural estimation.

## 4 Estimation

The model is estimated using Bayesian techniques. The posterior distribution is obtained combining the priors for the model parameters with the model’s likelihood function. The likelihood is evaluated with the Kalman filter. Section 4.1, introduces the data set used for estimation and presents the estimation strategy. The prior distributions for the model parameters are discussed in Section 4.2 and the posterior distributions for the parameters in Section 4.3.

### 4.1 Data and Estimation Strategy

The data set we use for estimation comprises ten variables for the U.S. economy observed at quarterly frequency over the period 1960:Q1 to 2020:Q4: real per-capita GDP growth; real per-capita consumption growth; real per-capita investment growth; a measure of the hours gap; the effective federal funds rate; the growth of average weekly earnings; price inflation based on the GDP deflator; the growth of real government transfers; the growth of government consumption and investment; the government debt-to-GDP ratio. Appendix C shows how these series are constructed.

To account for the federal funds rate being stuck at the effective lower bound from 2008:Q1 through 2015:Q3, we estimate the model over two subsamples: from 1960Q1 to 2007Q4 and then from 2008Q1 to 2020Q4. When estimating the model on the latter subsample, we add to the dataset the expectations for the federal funds rate one- through ten-quarters ahead, based on overnight index swaps.<sup>6</sup> Formally modeling the lower bound for the interest rate would raise substantially the computational challenge of our empirical exercise because it would introduce a non-linearity in the model, which requires using non-linear Monte Carlo filters to evaluate the likelihood (Fernandez-Villaverde and Rubio-Ramirez 2007). We adopt a simpler approach, following Campbell et al. (2012), who use data on market-based future federal funds rates to estimate the model after the fourth quarter of 2008. Agents’ expectations about the future interest rates are informed by the market forecasts, which enforce the effective lower bound in the model. Therefore, agents in the model are not surprised about not seeing negative interest rates in every period during the Great Recession and the Pandemic Recession.

## 4.2 Priors

To elicit the prior distributions for the model parameters, we follow the approach proposed by Del Negro and Schorfheide (2008). Some parameter values are fixed in estimation or implied by steady-state restrictions. We fix the discount factor  $\beta$  to the value of 0.99, so that the steady-state real interest rate is broadly consistent with its sample average. The quarterly rate of capital depreciation,  $\delta^K$ , is set to target an investment rate of 2.5%. The parameters governing the steady-state markups on wages and prices cannot be separately identified in estimation, so we set them to 0.14, following Leeper et al. (2017). The elasticity of output to capital in the production function  $\alpha$  is set to the standard value of 0.33. The parameter  $s_{ge}$ , capturing the ratio of government expenditures to GDP, is set to 0.11 following Leeper et al. (2017). Finally, the steady-state tax rates on labor, capital and consumption, denoted by the parameters  $\tau_L$ ,  $\tau_K$ , and  $\tau_C$ , are set to the values of 0.186, 0.218 and 0.023, respectively, also based on Leeper et al. (2017). The consumption tax rate  $\tau^C$  is assumed to be constant, so the parameters  $\gamma^C$  and  $\rho_C$  are set to zero.

The right panels in Tables 1 and 2 report the priors for the structural parameters and for the exogenous processes, respectively. The priors for both macroeconomic and fiscal variables are generally quite diffuse. We note that the decay rate of the maturity of long-term bonds,  $\rho$ , is set to 0.9593 to match an average duration of six years, as estimated by the Congressional Budget Office (2020). We follow Kaplan et al. (2014) and center the share of

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<sup>6</sup>We construct series of the market-expected federal funds rate in the same way as Campbell et al. (2017). See Appendix C for further details.

Prior and Posterior Distributions for Structural Parameters							
Param	Posterior Distribution				Prior Distribution		
	Mode	Median	5%	95%	Type	Mean	Std
$s_b$	2.1703	2.1834	2.0147	2.3497	N	1.8200	0.1000
$100\ln \mu$	0.4000	0.4001	0.3255	0.4925	N	0.5000	0.0500
$100\ln \Pi$	0.5402	0.5195	0.4267	0.6104	N	0.5000	0.0500
$\xi$	1.9704	1.9167	1.7493	2.1217	N	2.0000	0.2500
$\mu$	0.0771	0.0778	0.0652	0.0925	N	0.1100	0.0100
$\omega_w$	0.8041	0.8063	0.7861	0.8243	B	0.5000	0.1000
$\omega_p$	0.8663	0.8666	0.8375	0.8897	B	0.5000	0.1000
$\psi$	0.6596	0.6572	0.5755	0.7502	B	0.5000	0.1000
$s$	5.7144	5.5214	5.0185	5.9213	N	6.0000	0.5000
$\chi_w$	0.0372	0.0437	0.0164	0.0923	B	0.5000	0.2000
$\chi_p$	0.3117	0.2782	0.1279	0.4101	B	0.5000	0.2000
$\theta$	0.9106	0.9091	0.8985	0.9187	B	0.5000	0.2000
$\alpha^G$	-0.0455	-0.0396	-0.1832	0.0838	N	0.0000	0.1000
$\phi_y$	0.0012	0.0019	0.0001	0.0074	N	0.2500	0.1000
$\phi_\pi$	2.0577	2.0963	1.9462	2.2525	N	2.0000	0.1000
$\phi_{zy}$	0.0715	0.0439	0.0198	0.0719	G	0.1000	0.0500
$\gamma_G$	0.3800	0.3463	0.2218	0.4279	N	0.2500	0.1000
$\gamma_K$	0.0043	0.0064	0.0003	0.0335	N	0.2500	0.1000
$\gamma_L$	0.0163	0.0133	0.0009	0.0461	N	0.2500	0.1000
$\gamma_Z$	0.0017	0.0063	0.0003	0.0249	N	0.2500	0.1000
$\rho_r$	0.7250	0.7223	0.6650	0.7746	B	0.5000	0.1000
$\rho_G$	0.9637	0.9627	0.9340	0.9803	B	0.5000	0.1000

Table 1: Posterior modes, medians, 90% posterior credible sets, and prior moments for the structural parameters. The letters in the column with the heading "Prior Type" indicate the prior density function: N, G, and B stand for Normal, Gamma, and Beta, respectively. See Table 4 in Appendix D for a description of these parameters.

hand-to-mouth households  $\mu$  to 0.11, to match the share of poor hand-to-mouth consumers.

We note that the priors for the autocorrelation coefficients of both the funded and unfunded transfer shocks are tightly centered around a very persistent mean. We do so to fit the trend in transfers (see Figure 3 below). We also set the prior on the autocorrelation coefficient of the NKPC expectation shock so as to provide the model with a competing mechanism to explain trend inflation. Thus, the model allows, but does not require, trend inflation to be generated by the emission of unfunded debt. The autocorrelation coefficients of the fiscal rules ( $\rho_Z$ ,  $\rho_K$ ,  $\rho_L$ ) are set to 0.5 because it turns out that they are only weakly identified in the estimation.



Priors and Posteriors for the Exogenous Processes							
Param	Posterior Distribution				Prior Distribution		
	Mode	Median	5%	95%	Type	Mean	Std
$\rho_{eG}$	0.2868	0.3045	0.1506	0.3782	B	0.5000	0.1000
$\rho_{eZ}^F$	0.9954	0.9953	0.9933	0.9968	B	0.9950	0.0010
$\rho_{eZ}^U$	0.9958	0.9956	0.9937	0.9971	B	0.9950	0.0010
$\rho_a$	0.2987	0.2803	0.1711	0.3610	B	0.5000	0.1000
$\rho_b$	0.8237	0.8237	0.7774	0.8609	B	0.5000	0.1000
$\rho_m$	0.2407	0.2573	0.1692	0.3105	B	0.5000	0.1000
$\rho_i$	0.9205	0.9206	0.8990	0.9395	B	0.5000	0.1000
$\rho_{rp}$	0.9085	0.9062	0.8880	0.9220	B	0.5000	0.1000
$\rho_{\pi NKPC}$	0.9965	0.9966	0.9951	0.9977	B	0.9950	0.0010
$\sigma_G$	1.9046	1.9306	1.7416	2.1419	IG	0.5000	0.2000
$\sigma_Z^F$	2.9635	2.8922	2.6631	3.0924	IG	0.5000	0.2000
$\sigma_Z^U$	0.5166	0.5500	0.4194	0.7319	IG	0.1000	0.0500
$\sigma_a$	1.2113	1.1989	1.0895	1.3349	IG	0.5000	0.2000
$\sigma_b$	4.9850	4.9782	4.9214	4.9986	IG	0.2500	0.2000
$\sigma_m$	0.2375	0.2406	0.2154	0.2691	IG	0.5000	0.2000
$\sigma_i$	0.5192	0.5318	0.4734	0.5955	IG	0.5000	0.2000
$\sigma_w$	0.3487	0.3512	0.3156	0.3912	IG	0.5000	0.2000
$\sigma_p$	0.1625	0.1640	0.1427	0.1877	IG	0.5000	0.2000
$\sigma_{rp}$	0.3914	0.3990	0.3441	0.4586	IG	0.5000	0.2000
$\sigma_{\pi NKPC}$	1.3255	1.3763	1.2106	1.6382	IG	0.1000	0.0500
$\sigma_{GDP}^m$	0.4330	0.4352	0.3947	0.4831	IG	0.5000	0.2000
$\sigma_{by}^m$	0.3160	0.3032	0.2221	0.4217	IG	0.5000	0.2000

Table 2: Posterior modes, medians, 90% posterior credible sets, and prior moments for the structural parameters. The letters in the column with the heading "Prior Type" indicate the prior density function: N, G, and B stand for Normal, Gamma, and Beta, respectively. See Table 5 in Appendix D for a description of these parameters.

### 4.3 Posterior Estimates

The left panels of Tables 1 and 2 report the posterior estimates for the structural parameters and the exogenous processes, respectively, obtained over the sample period 1960Q1 to 2007Q4. The estimates obtained over the second subsample, 2008Q1 to 2020Q4, are reported in Appendix D. We note that the parameters governing the response of the tax instruments to debt, i.e.,  $\gamma_L$ ,  $\gamma_K$  and  $\gamma_C$  are positive but quantitatively small. The stabilization of the share of funded debt is therefore ensured by the relatively higher estimate of the parameter  $\gamma_G$ , which implies that debt stabilization is mostly achieved by changing government purchases rather than taxes. We estimate a relatively high degree of price and wage rigidities, consistent with the findings in Leeper et al. (2017), implying a relatively flat Phillips curve. Our estimate of the habit parameter lies towards the upper end of the multitude of estimates reported in the literature, but are smaller than those obtained by Leeper et al. (2017). The

output coefficient in the Taylor rule is close to zero, and smaller than typically obtained in the estimation of similar models, suggesting that changes in beliefs around the share of transfers that are funded cannibalize the interest rate response to output.

## 5 Results

In this section, we use the estimated TANK model with partially fiscally unfunded debt to analyze the role of fiscal trend inflation with respect to the historical evolution of inflation and to understand what role we should reasonably expect monetary and fiscal interactions will play in a large-debt environment. In Section 5.1, we show how shocks to funded and unfunded transfers propagate to the macroeconomy in the estimated model. The results in this section will help us to understand what drives the identification of the share of unfunded transfers in the data. In Section 5.2, we use the estimated model to investigate the historical dynamics of the unfunded share of transfers. In Section 5.3, we show that shocks to unfunded transfers explain the bulk of the dynamics of U.S. inflation in the postwar period. In Section 5.4 and in Section 5.5, we study the model predictions for inflation after the Pandemic Recession, which brought about a sizable increase in the U.S. fiscal imbalance.

### 5.1 Identification of Unfunded Transfer Shocks

In this subsection, we study how unfunded transfer shocks,  $\zeta_{z,t}^F$ , funded transfer shocks,  $\zeta_{z,t}^M$ , and shocks to long-run inflation expectations in the Phillips curve,  $u_t^{NKPC}$ , propagate through the economy. This analysis sheds light on how the three shocks are identified in the estimation. Figure 2 shows that the three shocks give rise to very different impulse responses for key macroeconomic variables, i.e. the inflation rate, the real interest rate, and the debt-to-GDP ratio.

The propagation of a funded transfer shock (dashed black line) produces only a modest impact on the macroeconomy, as the expansionary impulse of current transfers is offset by the expectations of higher taxes and/or a decrease in government spending in the future; qualitatively, inflation rises, following the positive stimulus to aggregate demand and real marginal costs. Concurrently, the debt-to-GDP ratio increases to fund the rise in transfers.

Unfunded transfer shocks, conversely, have a quantitatively strong expansionary effect on the macroeconomy (solid blue line). In sharp contrast with the propagation of funded transfer shocks, unfunded transfers lead to an fall in the real interest rate, as the fiscal and the monetary authorities coordinate to let inflation rise to stabilize the increase in transfers. This coordinated policies increase both inflation and inflation expectations. Note that 5 years

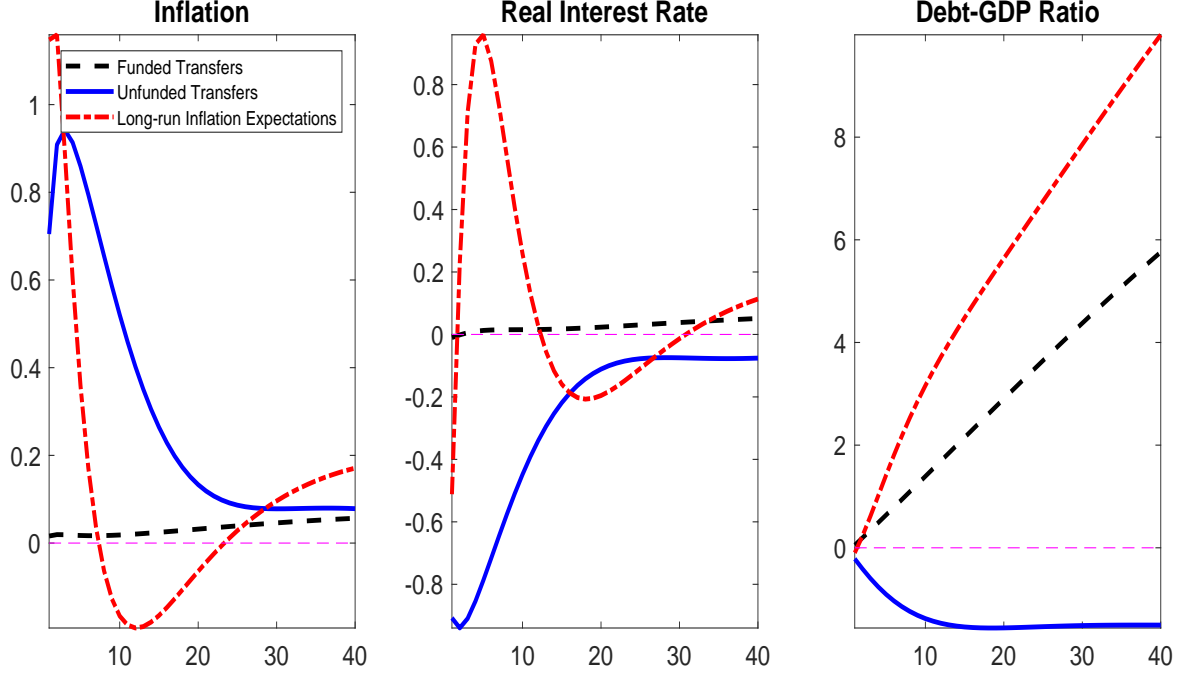


Figure 2: Impulse Response Function of Inflation (left), Real Interest Rate (middle), and Debt-To-GDP Ratio (right) to a one-standard deviation shock to the share of funded transfers (black dashed line), to the share of unfunded transfers (blue solid line), and to the persistent shifter of the Phillips curve (red dotted-dashed line).

after the shock, inflation remains above its long-run value. The lower real interest rate also stimulates aggregate production. Lower financing costs and higher GDP determine a fall in the debt-to-GDP ratio, despite the increase in spending. The opposite responses of the debt-to-GDP ratio produced by funded vs. unfunded transfers, together with the large effects on the macroeconomy of unfunded shocks that are absent in response to funded shocks, allow us to separately identify these two transfers shocks,  $\zeta_{z,t}^M$  and  $\zeta_{z,t}^F$  in the estimation.

A shock to long-run inflation expectations (dot-dashed red line) produces a temporary but short-lived rise in inflation. The real interest rate falls, but only on impact, and then rises persistently as the central bank reacts to the inflationary pressure. The rise in real rates and the associated contraction in aggregate production lead to an increase in the debt-to-GDP ratio. The opposite responses of the real interest rate and of the debt-to-GDP ratio following a shock to unfunded transfer shocks and a shock to long-run inflation expectations, provide identification for the two shocks,  $\zeta_{z,t}^F$  and  $u_t^{NKPC}$ .

## 5.2 Funded and Unfunded Transfers

Before using the model to infer the historical behavior of the share of funded and unfunded federal transfers, it is useful to take a look at how total per-capita real federal transfers

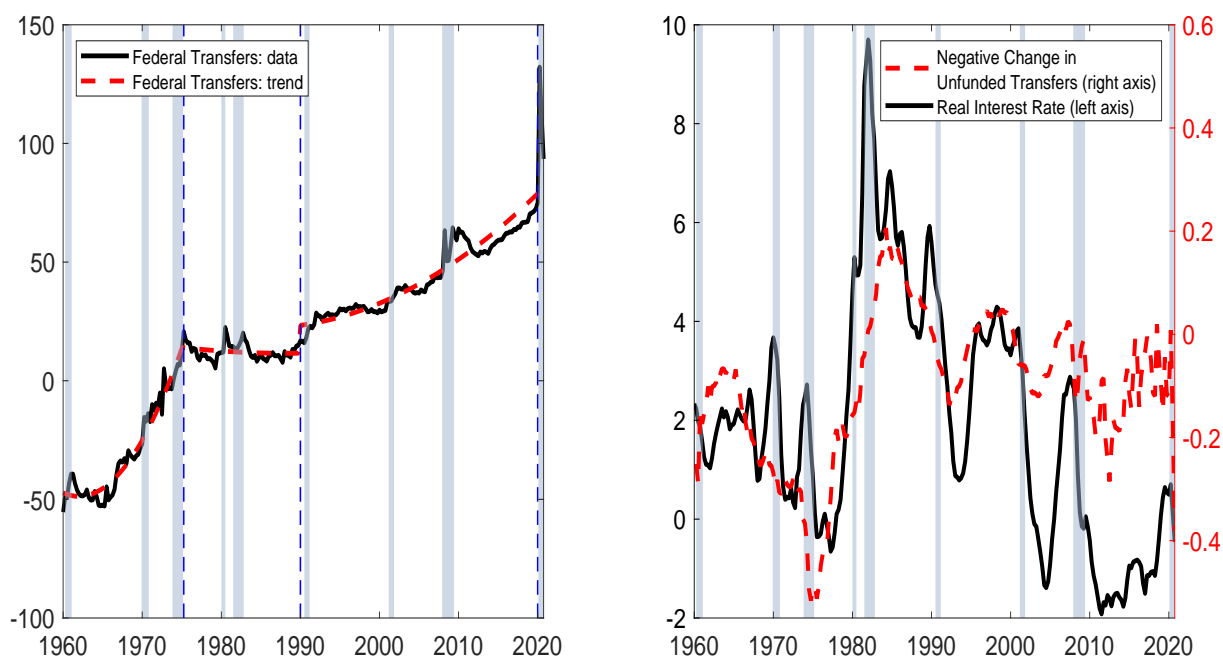


Figure 3: U.S. Real Federal Transfer Payments and Real Interest Rate. Left graph: Federal transfers in the data and its time trend measured by a quadratic polynomial fitted on each of the four time periods marked by the vertical blue bars. Right plot: The negative of the change in the share of unfunded transfers (black line) and the real interest rate (the red dashed line). The former is computed by taking the 3-year moving average of the changes in the share of unfunded transfers predicted by the model (smoothed estimates). The latter is computed by taking the one-year moving average in the real rate of interest predicted by the model (smoothed estimates).

have evolved over time. The left panel of Figure 3 plots the evolution of real U.S. government transfers from 1960Q1 to 2020Q4 (black line), which are observed in the structural estimation. The red dashed line in the figure corresponds to the time trend measured by a quadratic polynomial fitted on each of the four time periods marked by the vertical blue bars in the figure. The first period, spanning the 1960s and going up to the mid 1970s, was characterized by a sharp increase in real government transfers. These transfers reflect policy initiatives, initiated by President Johnson, aimed at reducing poverty levels. These initiatives were part of the Great Society program, which was aspiring to reduce racial injustice and crime and to improve the environment.

After President Johnson ended his second term in 1969, the level of transfers continued to increase during the Nixon's presidency (1969-1974). This is consistent with the fact that many of the welfare programs introduced in the 1960s shifted the long-term path of spending. But in the subsequent period, which starts from the mid 1970s and ends around 1990, the growth in transfers came to a halt, and their level remained broadly unchanged. Next, after 1990, the level of transfers started to increase again; the rate of growth has been rather stable throughout this third period, although smaller than the one observed between the

mid 1960s and the mid 1970s. Finally, the very last period, which captures the pandemic recession, has witnessed a large jump in the level of transfers, whose magnitude exceeds by far any increase observed over the estimation sample.

What portion of these changes can be attributed to funded and unfunded transfers? The right plot of Figure 3 shows the *negative* of the changes in the share of unfunded transfers based on the model estimates and its relation with the real interest rate. This plot illustrates how the structural estimation is able to attribute the observed changes in total transfers to the two components, funded and unfunded. The figure highlights that the real interest rate declines when the share of estimated unfunded transfers increases, and vice versa. This result is consistent with the impulse response functions shown in Figure 2, where the real interest rate responds negatively to increases in the share of unfunded transfers. Changing the share of unfunded transfers requires monetary and fiscal coordination. Specifically, monetary policy has to accommodate the movements in inflation resulting from this share of transfers, leading to fluctuations in the real interest rate.

Figure 4 presents the decomposition of the evolution of transfers into the shares that are estimated to be funded and unfunded, as depicted by the white and black bars, respectively. The four panels of this figure refer to the same four periods discussed above and highlighted in Figure 3. The red line in the right panel of Figure 4 plots the level of real transfers in deviations from the estimated trend  $A_t$ .

Changes in the share of unfunded transfers capture revisions of private expectations about the monetary and fiscal commitment to use fiscal instruments to repay the persistent flow of total transfers. For instance, total transfers may fall while the share of unfunded transfers rises (e.g., in 2020q4). In the estimation, the changes in the share of unfunded transfers are chiefly informed by the joint dynamics of inflation, real interest rate, and debt-to-GDP ratio, as shown in the previous section (Figure 2). Historical events like an exceptionally large recession, the creation of large welfare programs, the appointment of a new Chairman can be linked to the estimated movements in the share of unfunded transfers.

The panel in the top left corner shows that the increase in transfers occurred between the mid 1960s and the mid 1970s was partially funded, and partially unfunded. Specifically, the rise in the share of unfunded transfers over this period is substantial (black bars). In the following period, which ranges between the mid 1970s and 1990, the share of unfunded transfers exhibits a hump-shaped pattern, but by the end of this subsample, it ends up again hovering around the same value already reached by the mid-1970s.

As we will discuss in more detail below, the initial acceleration in the late 1960s and the subsequent slowdown in the second half of the 1970s play an important role in accounting for the rise and fall in inflation. The sharp rise in the real interest rate in the first half of the

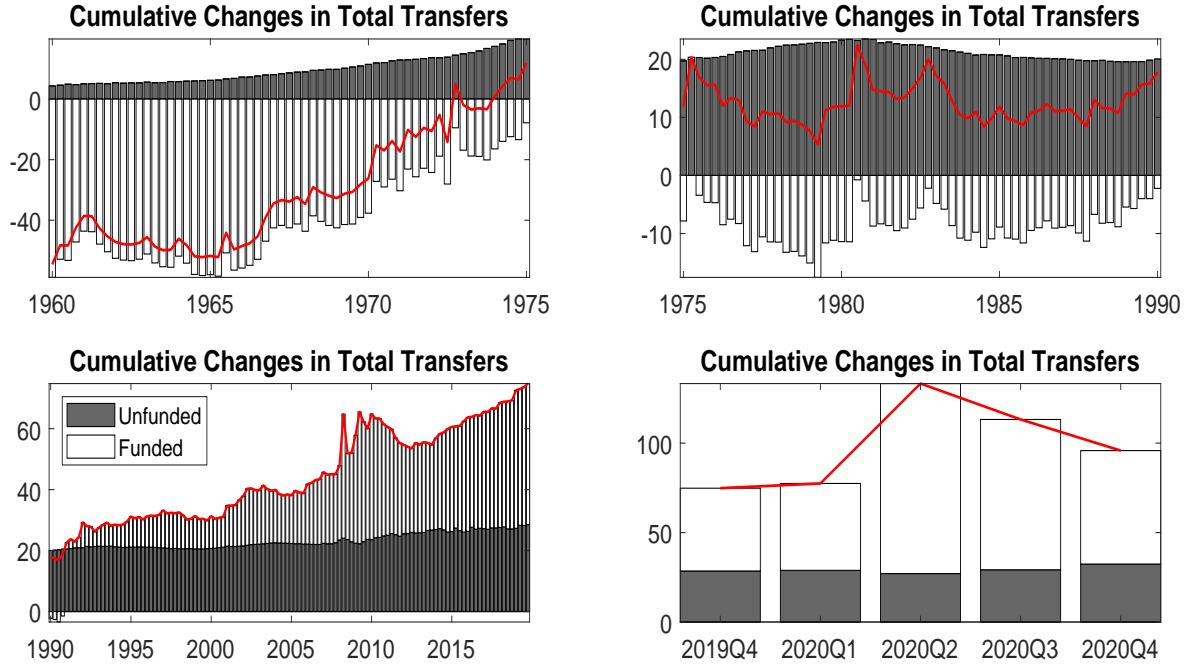


Figure 4: Estimated Decomposition of U.S. Government Transfers (red solid line) into their Funded and Unfunded Cumulative Components. These components isolate the cumulative effects of the historical realizations of funded or unfunded shocks on transfers. These realizations are estimated using the Smoother Kalman filter. Parameters are set at their posterior mode.

1980s –primarily due to Volcker’s aggressive monetary tightening – explains why a smaller fraction of transfers are interpreted as fiscally unfunded (the black bars). As we will see, this change in expectations implies a sharp reduction in the inflation rate that the central bank is expected to tolerate ( $\hat{\pi}_t^F$ ). Concurrently, the overall level of transfers (the red line) exhibits quite an erratic behavior, which mainly affects the funded share of transfers (the white bars). These movements are mostly due to a quite volatile economy, with two large recessions in the late 1970s and early 1980s.

The bottom left panel illustrates that the steady rise in transfers observed after 1990 was partially unfunded; the black bars rise steadily, albeit sluggishly, over this period. In the post-Millennial period, the model recovers a more rapid increase in the share of unfunded debt observed in the 2010s in light of a very accommodating monetary policy that engendered a decade-long negative real interest rate (Bianchi et al. 2016). As shown in the right panel of Figure 3, this pattern corresponds to a decline in the real interest rate, which is interpreted by the model as a sign that the central bank is willing to tolerate a higher amount of inflation.

Finally, the last panel of Figure 4 shows that total transfers were increased sharply by the federal government in the second quarter of 2020 in an attempt to combat the severe consequences of the pandemic crisis. However, the share of unfunded transfers slightly fell in that quarter and only increased in the fourth quarter of 2020. Interestingly, the increase

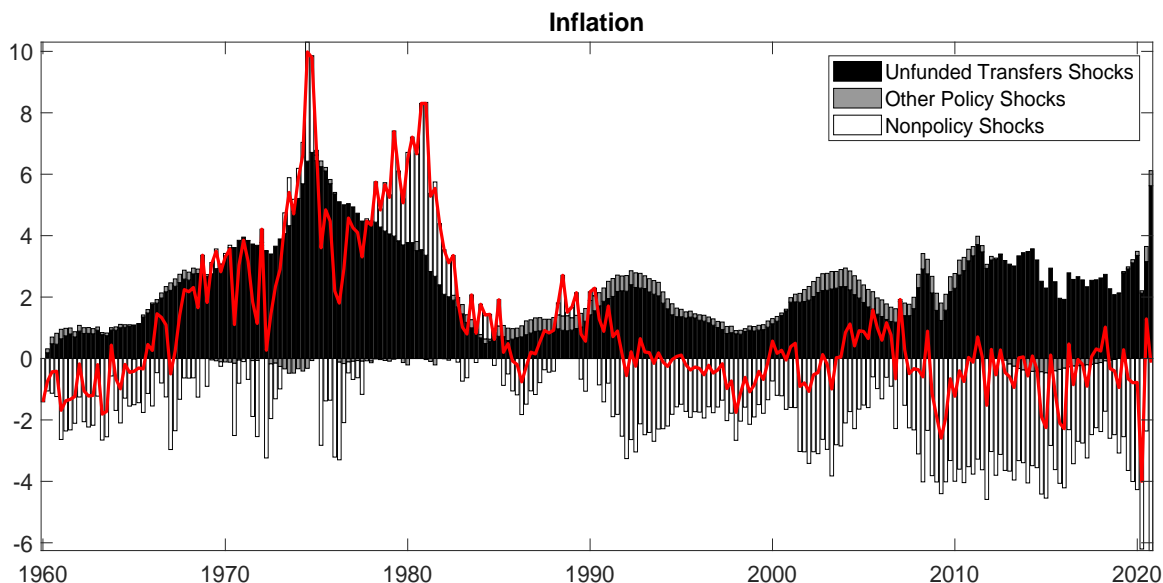


Figure 5: Drivers of Inflation. The red line is the inflation rate in the data in deviations from model’s steady-state inflation. The bars represent the cumulative effects of unfunded transfers shocks (black bars), other policy shocks (gray bars), and nonpolicy shocks (white bars) on inflation (in deviations from steady state). Other policy shocks include shocks to funded transfers and surprise and anticipated monetary policy shocks. Shocks are estimated using the Kalman smoother and setting the model parameters at their posterior mode.

in the expected share of unfunded transfers happened concomitantly with the introduction of new monetary framework in the last days of the third quarter of 2020 (August 27). The new framework contemplates the possibility for the Federal Reserve to let inflation overshoot its two-percent target after the Pandemic recession. This new monetary policy strategy is reflected in the change of the expected path of the future federal funds rate, which we observe in the estimation using overnight index swaps.

### 5.3 Drivers of Inflation and GDP growth

We now turn our attention to the analysis of the relation between unfunded transfer shocks and the the historical dynamics of inflation and GDP growth. Figure 5 provides a historical shock decomposition of inflation (red line). The black bars in the figure illustrate the level of inflation originating from unfunded transfer shocks, which we labelled fiscal trend inflation. The gray and white bars highlight the role of the other policy shocks and the non-policy shocks, respectively.<sup>7</sup>

The key result that emerges from Figure 5 is that fiscal trend inflation accounts for the bulk of inflation dynamics. The rise in trend inflation over the mid-1960s and up to the mid-1970s was accounted for, almost entirely, by the inflationary effects of the rise in unfunded

<sup>7</sup>Other policy shocks include the monetary policy shock, funded transfer shocks, and the other fiscal shocks. Non-policy shocks include all other shocks.

transfers that took place in that period, as illustrated in Figure 4. The inflationary effects of this rise in the level of transfers started to wither away during the late 1970s, while non-policy shocks were pushing up on the rate of inflation. Even though the share of unfunded transfers rose in the second half of the 1970s (upper right panel of Figure 4), the pace of this increase was too slow to sustain the high level of fiscal trend inflation caused by the large expansion in the share of unfunded transfers of the first half of the 1970s (upper left panel of Figure 4). As a result, fiscal trend inflation fell steadily in the second half of the 1970s, even if it remained elevated at the end of the 1970s, when unfunded transfers still explain about half of observed inflation.

The fall in inflation accelerated in the first half of the 1980s – mostly driven by a fall in fiscal trend inflation (the black bar), the amount of inflation tolerated by the central bank to stabilize the unfunded transfers. In the first five years of the 1980s, fiscal trend inflation declined by 3%, moving from 3.8% to 0.8% in deviations from the long-term inflation target. The sharp increase in the real interest rate due to the aggressive monetary tightening conducted by the Federal Reserve Chairman Volcker led to a large fall in fiscal trend inflation  $\hat{\pi}_t^F$ , in the first five years of the 1980s. Thus, as in Sims and Zha (2006), we document that a policy change occurred before the appointment of Fed Chairman Volcker in August 1979. However, we also find that the early 1980s led to an acceleration in the change of the policy environment, with the election of President Reagan who arguably provided the political backing for the actions of the Federal Reserve.<sup>8</sup> These results are also consistent with the evidence provided by Hazell et al. (2020). They argue that the Phillips curve has always been flat in the US and that the primary force behind the Volcker disinflation was a change in long-term inflation expectations triggered by the policy change, rather than high unemployment working through a steep Phillips curve.

Importantly, from about 1990 until the most recent years, fiscal inflation generated by the steady rise in the level of transfers contributes persistently to reflate the economy, offsetting the deflationary bias set off by non-policy shocks (the white bars consistently lying in the negative territory). The deflationary effects of these non-policy forces are persistent and keep dragging inflation down for a long period of time.

Figure 6 provides a similar historical shock decomposition for the series of GDP growth. A key take away from this figure is that unfunded transfers shocks have played an important role

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<sup>8</sup>Our results suggest an interesting interpretation for the finding of the seminal paper by Clarida et al. (2000) that the systematic response of the Federal Reserve to inflation was weaker in the 1970s than in the following decade, a result confirmed by subsequent work by Lubik and Schorfheide (2004), Fernandez-Villaverde and Rubio-Ramirez (2007b), and Bianchi (2013) in estimated DSGE models. In light of our results, the estimated coefficient of a standard Taylor rule could be interpreted as a weighted average of two different coefficients whose weights change depending on the type of shocks that determine movements in inflation.



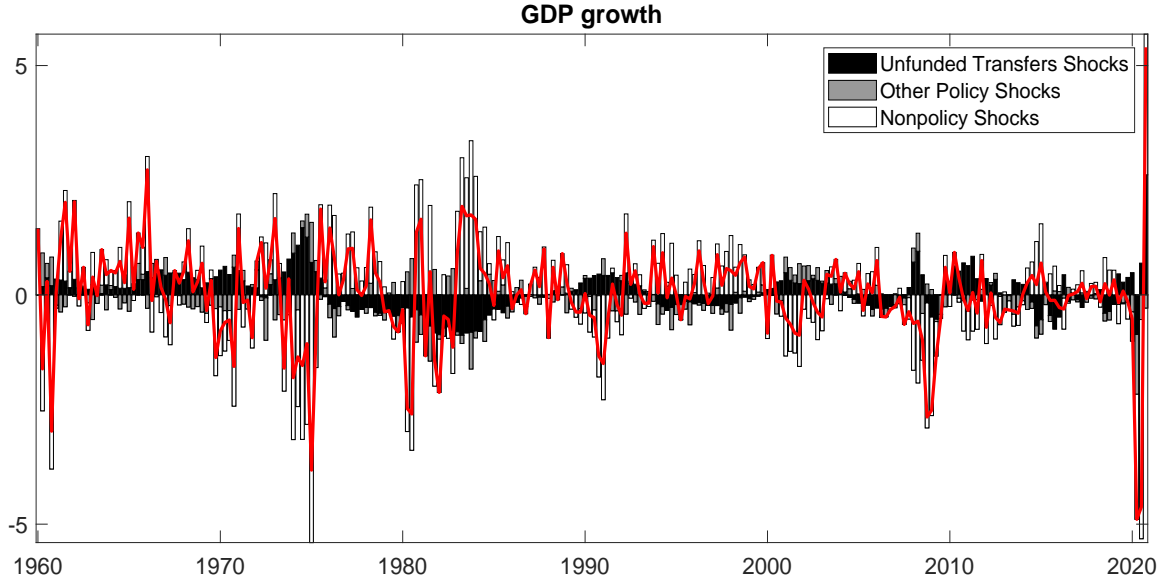


Figure 6: Drivers of GDP Growth. The red line is the annualized quarter-over-quarter growth rate of real percapita GDP in deviations from model’s steady-state inflation. The bars represent the cumulative effects of unfunded transfers shocks (black bars), other policy shocks (gray bars), and nonpolicy shocks (white bars) on GDP growth (in deviations from steady state). Other policy shocks include shocks to funded transfers and surprise and anticipated monetary policy shocks. Shocks are estimated using the Kalman smoother and setting the model parameters at their posterior mode.

in driving real activity, beyond their effects on inflation. Unfunded transfers are estimated to play an important role in counteracting the productivity slowdown of the late 1960s and mid-1970s, providing a positive contribution to growth. At the same time, their effects reverted in the 1980s, contributing negatively to growth.

Over the years of the Great Moderation (1984-2008), the contribution of unfunded transfers to fluctuations in growth becomes more modest, consistent with a more sluggish evolution. We see an important resurgence of their role during the 2008-2009 recession and even more so in correspondence of the pandemic recession. The model attributes a large portion of the rebound in economy activity to a robust increase in the amount of unfunded transfers implemented in response to the pandemic.

Interestingly, the change in the amount of unfunded transfers and the associated contribution to the rebound of the economy do not coincide with the increase in fiscal transfers, but rather with the announcement of the change of the policy strategy followed by the Federal Reserve. Thus, it is the coordination between monetary and fiscal authorities that triggers the large rebound of the economy. The increase in funded transfers alone has limited efficacy because it also generates an expectation of large tax increases in the future. This result holds despite the fact that we allow for hand-to-mouth consumers that immediately spend the transfers that they receive. Instead, an increase in unfunded transfers leads to a reflation of the economy, a decline in real interest rates, and an increase in real activity.

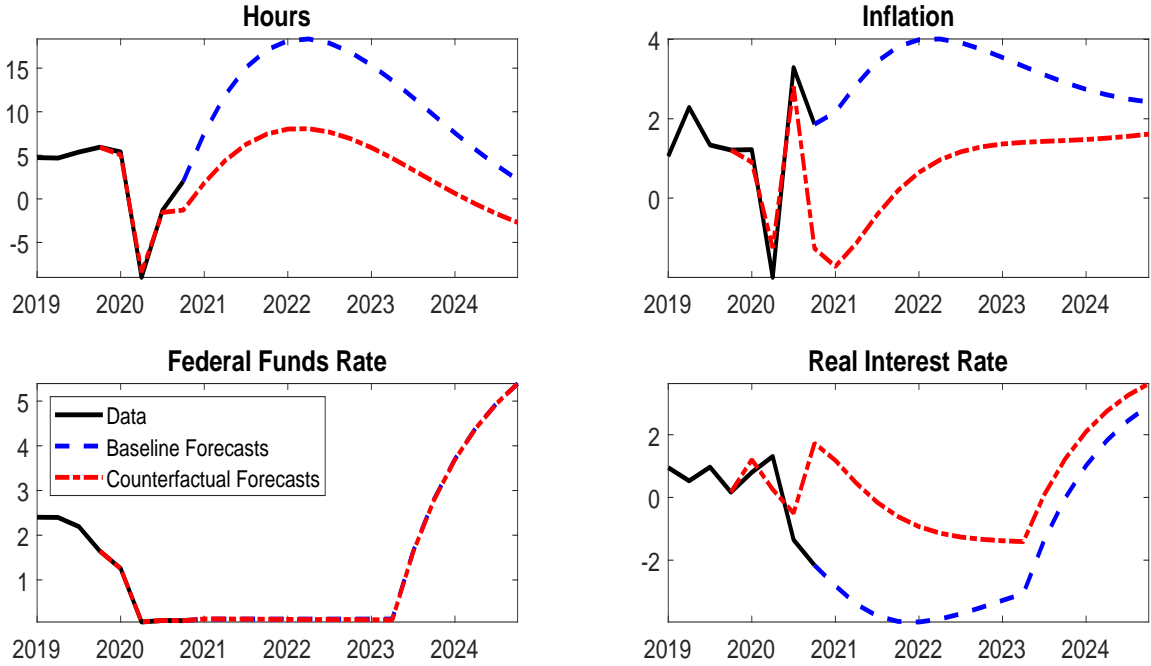


Figure 7: The Role of Unfunded Debt in The Past Recession. Model's forecast on hours worked, inflation, the Federal Funds rate, and the real interest rate, conditional on using filtered data up to 2020Q4 (dashed blue line). The counterfactual scenario is based on the assumption that all transfer shocks estimated during the Pandemic Recession (i.e., 2020Q1 through 2020Q4) are funded (dot-dashed red line). Shocks are estimated using the Kalman smoother. Model parameters are set at their posterior mode.

## 5.4 The Importance of Fiscal and Monetary Policy Coordination

In this section, we analyze the importance of monetary and fiscal policy coordination in the context of the pandemic recession. To do so, we isolate the role played by the expected share of unfunded fiscal transfers in contrasting deflationary pressure and providing a boost to real economic activity. Specifically, Figure 7 shows the model forecasts of some key macroeconomic aggregates, e.g., hours, inflation, the Federal Funds rate, and the real interest rate, conditional on using filtered data up to 2020Q4 (dashed blue line). To isolate the contribution of the recent unfunded transfer shocks, we contrast this baseline forecast with a counterfactual scenario in which we assume that all transfer shocks estimated during the Pandemic Recession (i.e., 2020Q1 through 2020Q4) are funded (dot-dashed red line).

A first key takeaway from the figure, is that the rise in inflation necessary to wear away the share of 2020 transfers that are unfunded is predicted to be moderate and relatively persistent. As noted earlier, this overshoot of inflation over the central bank's 2% target allows the central bank to regain space for future monetary policy in an environment of elevated ELB risk.

If all transfer shocks that occurred in 2020 had been funded, the rate of inflation would have temporarily fallen in negative territory, persistently undershooting the long-run target

of 2% throughout the forecast horizon. This deflationary bias is the consequence of the long-run drag on inflation played by a number of persistent non-policy shocks (including the almost-unit root shifter of the price Phillips curve,  $u_t^{NKPC}$ ) on inflation at the end of the sample. The highly persistent effect of these shocks on inflation is denoted by the white bars in Figure 5. The increase in the share of unfunded transfers in the fourth quarter of 2020 gives rise to inflationary pressure that dominates these long-lasting deflationary forces allowing the central bank to overshoot its target and reflate the economy.

The fall in real interest rates generated by the rise in inflation expectations provides persistent stimulus to the economy, boosting total hours up to 18% above their steady-state value. Furthermore, since part of the transfers paid by the government to help the economy weather the pandemic recession are expected to be unfunded, agents anticipate that future fiscal adjustments will be lighter in the baseline scenario, also contributing to the economic recovery that follows the pandemic recession.

The increase in inflation required to provide this stimulus is tolerated by the central bank, as it can be seen by noting that the central bank does not anticipate the lift-off of the interest rate. Indeed, the forecasts on the path of the federal funds rate shown in the bottom left plot of Figure 7 is no different in the baseline scenario and in the counterfactual scenario. This behavior of the nominal interest rate reflects the coordinated action of the monetary and fiscal authorities, aimed at stabilizing a fraction of the 2020 fiscal stimulus with inflation. From this point of view, the interaction of monetary and fiscal policies plays a key role in implementing the asymmetric policy strategy outlined by the Federal Reserve following its 2020 policy review (Bianchi et al. (2019)). The increase in the share of unfunded transfers makes the asymmetric strategy not only credible, but also necessary.<sup>9</sup>

## 5.5 ARPA Fiscal Stimulus and Macroeconomic Stability

Since the model is estimated using data up to 2020Q4, the baseline forecast in Figure 7 accounts for the fiscal stimulus introduced by President Trump’s administration, but not for the subsequent stimulus of President Biden, i.e., the American Rescue Plan Act (ARPA). This additional \$1.9 trillion stimulus has led influential commentators and scholars to express concern about price stability. (e.g., Blanchard, 2021, and Summers, 2021)

To investigate the macroeconomic implications of the ARPA stimulus, Figure 8 compares the baseline forecast illustrated in Figure 7 (the blue dashed line in both figures), with a revised forecast that accounts for the transfers observed in 2021Q1 (the red dot-dashed line). We emphasize that the dataset used to produce this revised forecast differs from the

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<sup>9</sup>This finding echoes the remarks by Sims (2016) at the 2016 Jackson Hole symposium: “[...] ]interest rate policy, tax policy, and expenditure policy, [...] jointly determine the price level.”

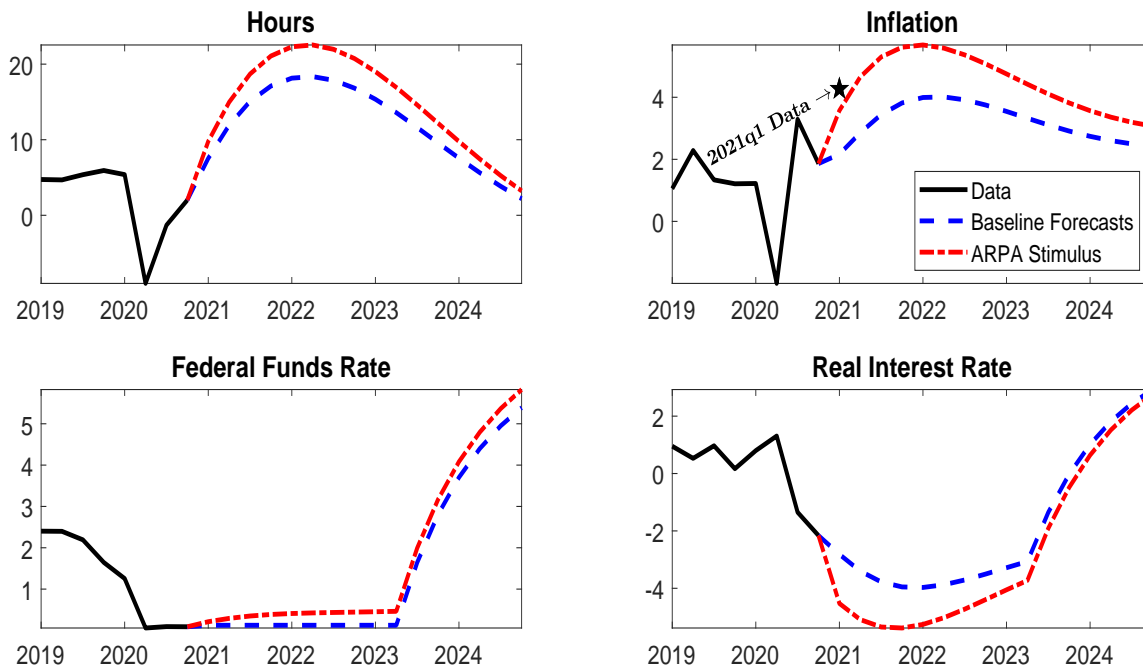


Figure 8: The Macroeconomic Effects of the ARPA Stimulus. Model's forecast on hours worked, inflation, the Federal Funds rate, and the real interest rate, conditional on using filtered data up to 2020Q4 (dashed blue line). The counterfactual scenario is conditional on filtered data up to 2020Q4 and on the federal transfers payment in the first quarter of 2021, which is when most of the ARPA stimulus checks were sent out (dot-dashed red line). Shocks are estimated using the Kalman smoother. Model parameters are set at their posterior mode.

dataset used to produce the baseline forecast only to the extent that it adds information on the transfers implemented in 2021Q1. In other words, we do not include observations for the other series of the model. ARPA stimulus mostly rested on providing additional transfers to households, which received a new stimulus check in their mailbox the last weeks of March 2021. Hence the difference between the two projections isolates the contribution of the ARPA stimulus predicted by our model, everything else being equal. Importantly, the increase in transfers observed in 2021Q1 is attributed by the filter to funded and unfunded shocks according to their historical pattern.<sup>10</sup>

Figure 8 shows that the ARPA stimulus produces a further increase in inflation, up from close to 4% to about 5% at its peak. Nevertheless, we do not find evidence that the central bank will lose control of inflation. After completing a larger overshoot, inflation retrenches to the central bank target in 2025. Our conditional forecast can replicate very well the inflation data for 2021Q1 (the star point in Figure 8), a data point that is not used in the filtering exercise.

Because of the larger increase in inflation, the real interest rate falls further, providing

<sup>10</sup>The decomposition of the transfers in 2021Q1 into the funded and unfunded components is shown in Figure 10 in Appendix F.

an even bigger boost to the economy in 2021 and over the following years, as shown by the forecasts on hours (upper left panel). This is despite an anticipated lift-off of the nominal interest rate, relative to the baseline case. This anticipated lift-off occurs because part of the increase in transfers due to the fiscal stimulus is fiscally funded and the central bank does not accommodate the (small) increase in inflation attributed to funded fiscal shocks (note that in the exercise we keep the anticipated MP shocks fixed).

In light of these results, we conclude that the current macroeconomic situation is in line with the historical experience for the United States, where unfunded spending has played an important role in accounting for inflation dynamics. In this sense, the current situation is not necessarily different from the past. In fact, the recent fiscal interventions might be necessary to help the recovery and move away from a low interest rate environment that limits the actions of the Federal Reserve. However, spending is now at an historical maximum. This implies that even small revisions in beliefs about the way it will be stabilized can lead to large swings in inflation. As seen in the historical analysis of Section 5.2, in this situation monetary policy can play an important role in coordinating and keeping expectations anchored. Historically, the actions of the Federal Reserve seem to have played a key role in inducing changes of agents' beliefs about the amount of unfunded spending. An implications of these findings is that clear communication of the acceptable overshoot of inflation is critical to coordinate these beliefs and anchor expectations.

## 6 Conclusions

A legacy of the COVID-19 recession is a more proactive fiscal policy and a large stock of public debt. In this paper, we have built a novel TANK model with unfunded fiscal shocks to study the risk of high inflation stemming from this historically high fiscal burden. In the model, the central bank accommodates unfunded fiscal shocks by allowing persistent increases in inflation, delivering a fiscal theory of trend inflation. Our empirical results show that fiscal trend inflation has played a major role in explaining movements in inflation. Thus, the current situation is not necessarily different from the past. The model predicts a moderate increase in inflation above the 2% inflation target when considering the recent ARPA shock. Inflation is then expected to revert to its long-term value over the next five years. However, the historically large amounts of spending and debt make the economy at risk of large swings in beliefs about the share of unfunded debt. Thus, clear coordination between the two authorities is of foremost importance. The central bank can play a key role in coordinating and anchoring beliefs by clearly communicating the acceptable inflation path going forward.

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## A Solving Economies with Partially Unfunded Debt

To prove that the system of equations (11), (12), (13), (14), are the correct policy functions of the model with partially unfunded debt, we have to show that the two following claims are true. First, the difference between the overall stock of debt and its unfunded share is funded, that is,  $\hat{b}_t - \hat{b}_t^F = \hat{b}_t^M$ . Second, the inflation rate the central bank strives to stabilize with active monetary policy in the actual economy is precisely the actual rate of inflation net of the inflation needed to stabilize the unfunded debt (i.e.,  $\pi_t^M = \pi_t - \pi_t^F$ ).

Both claims can be proved by constructing yet another parallel economy to pin down (i) the share of inflation the monetary authority has to control with active monetary policy,  $\pi_t^M$ , and (ii) the share of funded debt,  $\hat{b}_t^M$ , which is the share of debt the fiscal authority is responsible to repay by raising future surpluses. This parallel economy is as follows:

$$\mathbb{E}_t \hat{\pi}_t^M = \hat{r}_t^M, \quad (26)$$

$$\hat{b}_t^M = \beta^{-1} \hat{b}_{t-1}^M - \hat{r}_t^M - b\beta^{-1} \pi_t^M + b_{\hat{r}_t^M}, \quad (27)$$

$$\hat{r}_t^M = \phi^M \hat{\pi}_t^M, \quad (28)$$

$$\hat{\tau}_t^M = \gamma^M \hat{b}_t^M + \epsilon_t^M. \quad (29)$$

In this parallel economy, all fiscal shocks are funded,  $\epsilon_t^M$ , and the policy mix is monetary led ( $\phi_M > 1$  and  $\gamma > \beta^{-1} - 1$ ). The monetary and fiscal blocks are obtained as done for the other economies we studied in the main text.

$$\mathbb{E}_t \hat{\pi}_{t+1}^M = \phi^M \hat{\pi}_t^M, \quad (30)$$

$$\hat{b}_t^M = (\beta^{-1} - \gamma^M) \hat{b}_{t-1}^M - b(\beta^{-1} - \phi^M) \hat{\pi}_t^M - \epsilon_t^M. \quad (31)$$

It is convenient to prove the second claim first. We need to show that  $\hat{\pi}_t^M = \hat{\pi}_t - \hat{\pi}_t^F$ . If the equation is rolled one period forward and we apply the expectation operator on both sides, we obtain

$$\mathbb{E}_t \hat{\pi}_{t+1}^M = \mathbb{E}_t \hat{\pi}_{t+1} - \mathbb{E}_t \hat{\pi}_{t+1}^F. \quad (32)$$

The right-hand side can be pinned down using the monetary block of the actual economy (11) and that in the parallel economy in the main text – equation (13). After making these substitutions we obtain:

$$\mathbb{E}_t \hat{\pi}_{t+1}^M = \phi^M (\hat{\pi}_t - \hat{\pi}_t^F). \quad (33)$$

Substituting the monetary block of the parallel economy in equation (30) to replace the expectations on the left hand side proves the second claim.

Let us now turn to the first claim. This claim requires us to show that  $\hat{b}_t^M = \hat{b}_t - \hat{b}_t^F$ . Substituting the fiscal rule in equation (9) (with  $\gamma_F = 0$ ) and the monetary rule in equation (10) into the law of motion of debt in equation (2), yields:<sup>11</sup>

$$\hat{b}_t = (\beta^{-1} - \gamma^M) \hat{b}_{t-1} + \gamma^M \hat{b}_{t-1}^F - b(\beta^{-1} - \phi^M) \hat{\pi}_t - b(\phi^M - \phi^F) \hat{\pi}_t^F - \epsilon_t^M - \epsilon_t^F. \quad (34)$$

We plug the second claim,  $\hat{\pi}_t = \hat{\pi}_t^M + \hat{\pi}_t^F$ , into the above equation to obtain

$$\hat{b}_t = (\beta^{-1} - \gamma^M) \hat{b}_{t-1} + \gamma^M \hat{b}_{t-1}^F - b(\beta^{-1} - \phi^M) \hat{\pi}_t^M - b(\beta^{-1} - \phi^F) \hat{\pi}_t^F - \epsilon_t^M - \epsilon_t^F. \quad (35)$$

We now subtract  $\hat{b}_t^F$  from both sides of this equation and use equation (14) that defines the share of unfunded debt in equilibrium to get

$$\hat{b}_t - \hat{b}_t^F = (\beta^{-1} - \gamma^M) (\hat{b}_{t-1} - \hat{b}_{t-1}^F) - b(\beta^{-1} - \phi^M) \hat{\pi}_t^M - \epsilon_t^M. \quad (36)$$

We then subtract  $\hat{b}_t^M$  from both sides of this equation and use its definition from the fiscal block of this parallel economy – equation (31) to get

$$\hat{b}_t - \hat{b}_t^F - \hat{b}_t^M = (\beta^{-1} - \gamma^M) (\hat{b}_{t-1} - \hat{b}_{t-1}^F - \hat{b}_{t-1}^M). \quad (37)$$

The only way to satisfy the equation above is when the first claim  $\hat{b}_t^M = \hat{b}_t - \hat{b}_t^F$  is satisfied in every period, which is precisely the first claim.

Finally, Figure 9 shows the complete plot of the response of all the variables of the toy model to funded and unfunded fiscal shocks.

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<sup>11</sup>The fiscal rule could be equivalently expressed as  $\tau_t/\tau = (b_{t-1}/b_{t-1}^M)^{\gamma^F} (b_{t-1}^M/b)^{\gamma^M} e^{\epsilon_t^M + \epsilon_t^F}$ .

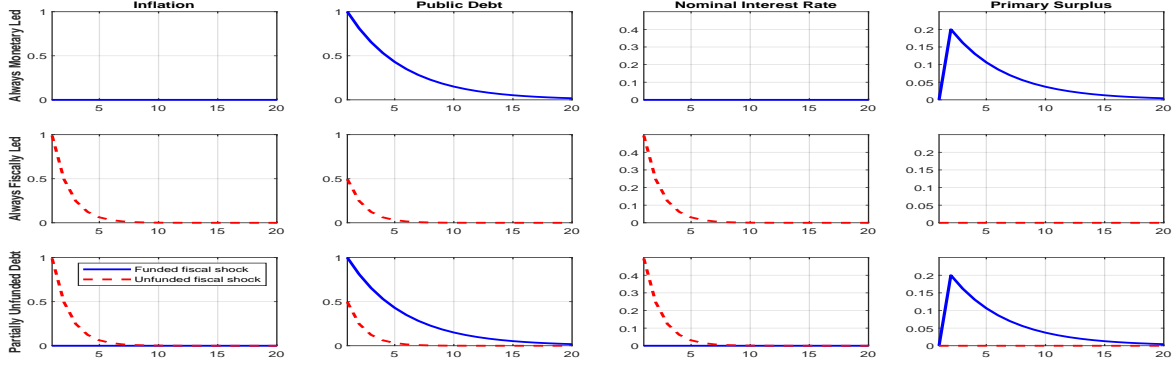


Figure 9: Response of Model Variables to a Fiscal Shock.

## B The Log-Linearized Model

This model features a trend in the state of labor-augmenting technological progress. In order to make the model stationary, we define the following variables:  $y_t = \frac{Y_t}{A_t}$ ,  $c_t^{*S} = \frac{C_t^{*S}}{A_t}$ ,  $c_t^S = \frac{C_t^S}{A_t}$ ,  $c_t^N = \frac{C_t^N}{A_t}$ ,  $k_t = \frac{K_t}{A_t}$ ,  $g_t = \frac{G_t}{A_t}$ ,  $z_t = \frac{Z_t}{A_t}$ ,  $b_t = \frac{P_t^B B_t}{P_t A_t}$ ,  $w_t = \frac{W_t}{P_t A_t}$ , and  $\lambda_t^S = \Lambda_t^S A_t$ . We list below the equations of the log-linear model, starting with those that characterize the actual-economy block.

Production function:

$$\hat{y}_t = \frac{y + \Omega}{y} \left[ \alpha \hat{k}_t + (1 - \alpha) \hat{L}_t \right]. \quad (38)$$

Capital-labor ratio:

$$\hat{r}_t^K - \hat{w}_t = \hat{L}_t - \hat{k}_t. \quad (39)$$

Marginal cost:

$$\widehat{mc}_t = \alpha \hat{r}_t^k + (1 - \alpha) \hat{w}_t. \quad (40)$$

Phillips curve:

$$\hat{\pi}_t = \frac{\beta}{1 + \chi_p \beta} \mathbb{E}_t \hat{\pi}_{t+1} + \frac{\chi_p}{1 + \chi_p \beta} \hat{\pi}_{t-1} + \kappa_p \widehat{mc}_t + \kappa_p \hat{\eta}_t^p + \kappa_p \hat{u}_t^{NKPC}, \quad (41)$$

where  $\kappa_p = [(1 - \beta \omega_p)(1 - \omega_p)] / [\omega_p(1 + \beta \chi_p)]$ .

Saver household's FOC for consumption:

$$\hat{\lambda}_t^S = \hat{u}_t^b - \frac{\theta}{e^\gamma - \theta} \hat{u}_t^a - \frac{e^\gamma}{e^\gamma - \theta} c_t^{*S} + \frac{\theta}{e^\gamma - \theta} c_{t-1}^{*S} - \frac{\tau^C}{1 + \tau^C} \hat{\tau}_t^C, \quad (42)$$

where  $\hat{u}_t^a = u_t^a - \gamma$ .

Public/private consumption in utility:

$$\hat{c}_t^* = \frac{c^S}{c^S + \alpha_G g} \hat{c}_t^S + \frac{\alpha_G g}{c^S + \alpha_G g} \hat{g}_t. \quad (43)$$

Euler equation:

$$\hat{\lambda}_t^S = \hat{R}_t + \mathbb{E}_t \hat{\lambda}_{t+1}^S - \mathbb{E}_t \hat{\pi}_{t+1} - \mathbb{E}_t \hat{u}_{t+1}^a + \hat{u}_t^{rp}. \quad (44)$$

Maturity structure of debt:

$$\hat{R}_t + \hat{P}_t^B = \frac{\rho}{R} \mathbb{E}_t \hat{P}_{t+1}^B + \hat{u}_t^{rp}. \quad (45)$$

Saver household's FOC for capacity utilization:

$$r_t^k - \frac{\tau^K}{1 - \tau^K} \hat{\tau}_t^K = \frac{\psi}{1 - \psi} \hat{\nu}_t. \quad (46)$$

Saver household's FOC for capital:

$$\hat{q}_t = \mathbb{E}_t \hat{\pi}_{t+1} - \hat{R}_t + \beta e^{-\gamma} (1 - \tau^K) r^k \mathbb{E}_t \hat{r}_{t+1}^k - \beta e^{-\gamma} \tau^K r^k \mathbb{E}_t \hat{\tau}_{t+1}^K + \beta e^{-\gamma} (1 - \delta) \mathbb{E}_t \hat{q}_{t+1} - \hat{u}_t^{rp}. \quad (47)$$

Saver household's FOC for investment:

$$\hat{i}_t + \frac{1}{1 + \beta} \hat{u}_t^a - \frac{1}{(1 + \beta) s e^{2\gamma}} \hat{q}_t - \hat{u}_t^i - \frac{\beta}{1 + \beta} \mathbb{E}_t \hat{i}_{t+1} - \frac{\beta}{1 + \beta} \mathbb{E}_t \hat{u}_{t+1}^a = \frac{1}{1 + \beta} \hat{i}_{t-1}. \quad (48)$$

Effective capital:

$$\hat{k}_t = \hat{\nu}_t + \hat{k}_{t-1} - \hat{u}_t^a. \quad (49)$$

Law of motion for capital:

$$\hat{k}_t = (1 - \delta) e^{-\gamma} (\hat{k}_{t-1} - \hat{u}_t^a) + [1 - (1 - \delta) e^{-\gamma}] [(1 + \beta) s e^{2\gamma} + \hat{i}_t]. \quad (50)$$

Hand-to-mouth household's budget constraint:

$$\tau^C c^N \hat{\tau}_t^C + (1 + \tau^C) c^N \hat{c}_t^N = (1 - \tau^L) w L (\hat{w}_t + \hat{L}_t) - \tau^L w L \hat{\tau}_t^L + z \hat{z}_t. \quad (51)$$

Wage equation:

$$\begin{aligned} \hat{w}_t = & \frac{1}{1 + \beta} \hat{w}_{t-1} + \frac{\beta}{1 + \beta} \mathbb{E}_t \hat{w}_{t+1} - \kappa_w \left[ \hat{w}_t - \xi \hat{L}_t + \hat{\lambda}_t^S - \frac{\tau^L}{1 - \tau^L} \hat{\tau}_t^L \right] \\ & + \frac{\chi^w}{1 + \beta} \hat{\pi}_{t-1} - \frac{1 + \beta \chi^w}{1 + \beta} \hat{\pi}_t + \frac{\beta}{1 + \beta} \mathbb{E}_t \hat{\pi}_{t+1} + \frac{\chi}{1 + \beta} \hat{u}_{t-1}^a - \frac{1 + \beta \chi - \rho_a \beta}{1 + \beta} \hat{u}_t^a + \kappa_w \hat{\pi}_t^w \end{aligned} \quad (52)$$

where  $\kappa_w \equiv [(1 - \beta\omega_w)(1 - \omega_w)] / \left[ \omega_w(1 + \beta) \left( 1 + \frac{(1+\eta^w)\xi}{\eta^w} \right) \right]$ .

Aggregate households' consumption

$$c\hat{c}_t = c^S(1 - \mu)\hat{c}_t^S + c^N\mu\hat{c}_t^N. \quad (53)$$

Aggregate resource constraint:

$$y\hat{y}_t = c\hat{c}_t + i\hat{i}_t + g\hat{g}_t + \psi'(1)k\hat{v}_t. \quad (54)$$

Government budget constraint:

$$\begin{aligned} & \frac{b}{y}\hat{b}_t + \tau^K r^k \frac{k}{y} \left[ \hat{\tau}_t^K + \hat{r}_t^k + \hat{k}_t \right] + \tau^L w \frac{L}{y} \left[ \hat{\tau}_t^L + \hat{w}_t + \hat{L}_t \right] + \tau^C \frac{c}{y} (\hat{\tau}_t^C + \hat{c}_t) \\ = & \frac{1}{\beta} \frac{b}{y} \left[ \hat{b}_{t-1} - \hat{\pi}_t - \hat{P}_{t-1}^B - \hat{u}_t^a \right] + \frac{b}{y} \frac{\rho}{e^\gamma} \hat{P}_t^B + \frac{g}{y} \hat{g}_t + \frac{z}{y} \hat{z}_t. \end{aligned} \quad (55)$$

Fiscal Rules

$$\hat{g}_t = \rho_G \hat{g}_{t-1} - (1 - \rho_G) \gamma_G \hat{s}_{b,t-1}^M + \zeta_{g,t} \quad (56)$$

$$\hat{z}_t = \phi_{zy} \hat{y}_t + \rho_Z \hat{z}_{t-1} - (1 - \rho_Z) \gamma_Z \hat{s}_{b,t-1}^M + \zeta_{z,t}^M + \zeta_{z,t}^F \quad (57)$$

$$\hat{\tau}_t^L = \rho_L \hat{\tau}_{t-1}^L + (1 - \rho_L) \gamma_L \hat{s}_{b,t-1}^M + \zeta_{\tau_L,t} \quad (58)$$

$$\hat{\tau}_t^K = \rho_K \hat{\tau}_{t-1}^K + (1 - \rho_K) \gamma_K \hat{s}_{b,t-1}^M + \zeta_{\tau_K,t} \quad (59)$$

Monetary Rule:

$$\hat{R}_t = \max \left( -\ln R_M, \rho_r \hat{R}_{t-1} + (1 - \rho_r) [\phi_\pi \hat{\pi}_t^M + \phi_y \hat{y}_t] \right) + \epsilon_{R,t} \quad (60)$$

The variables with the superscript  $M$  in equations (56) to (60) above belong to the shadow economy. In turn, the block of equations that characterize the shadow economy consists in an additional set of equations (38) to (55), where any variable that refers to the actual economy  $x_t$  is replaced by the same variable in the shadow economy  $x_t^M$ , plus the rule for the monetary authority

$$\hat{R}_t^M = \max \left( -\ln R, \rho_r \hat{R}_{t-1}^M + (1 - \rho_r) [\phi_\pi \hat{\pi}_t^M + \phi_y \hat{y}_t^M] \right) + \epsilon_{R,t} \quad (61)$$

and the rules for the fiscal authority,

$$\hat{g}_t^M = \rho_G \hat{g}_{t-1}^M - (1 - \rho_G) \gamma_G \tilde{b}_{b,t-1}^M + \zeta_{g,t} \quad (62)$$

$$\hat{z}_t^M = \phi_{zy}\hat{y}_t^M + \rho_Z\hat{z}_{t-1}^M - (1 - \rho_Z)\gamma_Z\tilde{b}_{t-1}^M + \zeta_{z,t}^M \quad (63)$$

$$\hat{\tau}_t^{LM} = \rho_L\hat{\tau}_{t-1}^{LM} + (1 - \rho_L)\gamma_L\tilde{b}_{t-1}^M + \zeta_{\tau_L,t} \quad (64)$$

$$\hat{\tau}_t^{KM} = \rho_K\hat{\tau}_{t-1}^{KM} + (1 - \rho_K)\gamma_K\tilde{b}_{t-1}^M + \zeta_{\tau_K,t}. \quad (65)$$

## C The Dataset

Real GDP growth is computed as the growth rate of nominal GDP (GDP), divided by the GDP deflator (JGDP). Real consumption growth is the growth rate of the sum of personal consumption expenditures in non durable goods (PCND) and services (PCESV), divided by their price indexes (DNDGRG3M086SBEA and DSERRG3M086SBEA, respectively). Real investment growth is the growth rate of the sum of gross private domestic investment (GP-DICTPI) and personal consumption expenditures in durable goods (PCDG), divided by the respective price deflators (GPDICTPI and DDURRG3M086SBEA), and scaled by the 16+ US civilian population (CNP16OV). We construct a measure of hours per capita by dividing total hours worked (PRS85006023) by population (CNP16OV). We then construct a measure of the hours gap by taking the difference of hours per capita from its trend, which is computed as a fourth degree polynomial. We compute a measure of hourly wages dividing wage compensation (A576RC1) by average weekly hours in the nonfarm business sector (PRS85006023). Based on this series, we create a nominal wage index, which we divide by an index of the GDP deflator (based on JGDP) and take growth rates. The debt to GDP ratio is constructed dividing the nominal market value of gross federal debt (MVGFD027MNFRBDAL) by nominal GDP (GDP). The growth of government consumption and investment expenditures is computed as follows. We add nominal federal government consumption expenditures (A957RC1Q027SBEA) to nominal gross government investment (A787RC1Q027SBEA), divide by the implicit price deflator (A822RD3Q086SBEA) and by an index of the U.S. population, with base 2012Q3 (CNP16OV) and finally take growth rates. The growth of real government transfers is computed as follows. We add government social benefits (B087RC1Q027SBEA) to other current transfer payments, which include grants-in-aid to state and local governments (FGSL), create an index with base 2012Q3, divide by an index of the U.S. population (CNP16OV) and an index of the GDP deflator (GDPDEF) with the same base year and finally take growth rates. Finally, inflation is computed as the rate of growth of the GDP deflator (JGDP) and the interest rate is given the Effective Federal Funds Rate (FEDFUNDS).

## D Second Sample Estimates

Prior and Posterior Distribution: Second sample							
Posterior Distribution					Prior Distribution		
Param	Mode	Median	5%	95%	Type	Mean	Std
$\sigma_G$	3.2021				IG	0.5000	0.2000
$\sigma_Z^F$	4.9982				IG	0.5000	0.2000
$\sigma_Z^U$	1.0214				IG	0.1000	0.0500
$\sigma_a$	3.7944				IG	0.5000	0.2000
$\sigma_b$	4.9975				IG	0.2500	0.2000
$\sigma_m$	0.1242				IG	0.5000	0.2000
$\sigma_i$	2.5281				IG	0.5000	0.2000
$\sigma_w$	0.6567				IG	0.5000	0.2000
$\sigma_p$	0.1630				IG	0.5000	0.2000
$\sigma_{rp}$	2.8727				IG	0.5000	0.2000
$\sigma_{\pi NKPC}$	4.9939				IG	0.1000	0.0500
$\sigma_{GDP}^m$	1.7952				IG	0.5000	0.2000
$\sigma_{by}^m$	4.9963				IG	0.5000	0.2000

Table 3: Priors and Posterior for post-2008 estimation.

## E Notations and Definition of the Model Parameters

Notation and Definition of Structural Parameters	
	Parameters
Debt to annualized GDP ratio	$s_b$
Steady-state growth rate	$100 \ln \mu$
Steady state inflation rate	$100 \ln \Pi$
Inverse Frisch elasticity	$\xi$
Share of hand-to-mouth households	$\mu$
Wage Calvo parameter	$\omega_w$
Price Calvo parameter	$\omega_p$
Capital utilization cost	$\psi$
Investment adjustment cost	$s$
Wage inflation indexing parameter	$\chi_w$
Price inflation indexing parameter	$\chi_p$
Habits in consumption	$\theta$
Substitutability of private vs. gov. consumption	$\alpha_G$
Taylor rule response to output	$\phi_y$
Taylor rule response to inflation	$\phi_\pi$
Transfers response to output	$\phi_{zy}$
Inverse Frisch elasticity	$\xi$
Government consumption response to debt	$\gamma_G$
Tax on capital response to debt	$\gamma_K$
Tax on labor response to debt	$\gamma_L$
Transfers response to debt	$\gamma_Z$
Serial correlation on interest rate in Taylor rule	$\rho_r$
Serial correlation on government consumption rule	$\rho_G$
Serial correlation on transfers rule	$\rho_Z$
Serial correlation on capital tax rule	$\rho_K$
Serial correlation on labor tax rule	$\rho_L$
Serial correlation on consumption tax rule	$\rho_C$

Table 4: Notations for the Model Parameters.



Notation and Definition of the Exogenous-Process Parameters	
	Parameters
AR coefficient on government consumption policy shocks	$\rho_{eG}$
AR coefficient on funded transfers' shocks	$\rho_{eZ}^F$
AR coefficient on unfunded transfers' shocks	$\rho_{eZ}^U$
AR coefficient on technology shocks	$\rho_a$
AR coefficient on preference shocks	$\rho_b$
AR coefficient on monetary policy shocks	$\rho_m$
AR coefficient on investment shocks	$\rho_i$
AR coefficient on risk premium shocks	$\rho_{rp}$
AR coefficient on inflation drift shocks	$\rho_{\pi^{NKPC}}$
Standard deviation government consumption shocks	$\sigma_G$
Standard deviation funded transfers' shocks	$\sigma_Z^F$
Standard deviation unfunded transfers' shocks	$\sigma_Z^U$
Standard deviation technology shocks	$\sigma_a$
Standard deviation preference shocks	$\sigma_b$
Standard deviation monetary policy shocks	$\sigma_m$
Standard deviation investment shocks	$\sigma_i$
Standard deviation wage markup shocks	$\sigma_w$
Standard deviation price markup shocks	$\sigma_p$
Standard deviation risk premium shocks	$\sigma_{rp}$
Standard deviation inflation drift shocks	$\sigma_{\pi^*}$
Measurement error on GDP	$\sigma_{GDP}^m$
Measurement error on debt to GDP ratio	$\sigma_{by}^m$

Table 5: Notations for the Model Parameters.

## F ARPA Stimulus Decomposition: Funded vs. Unfunded Transfers

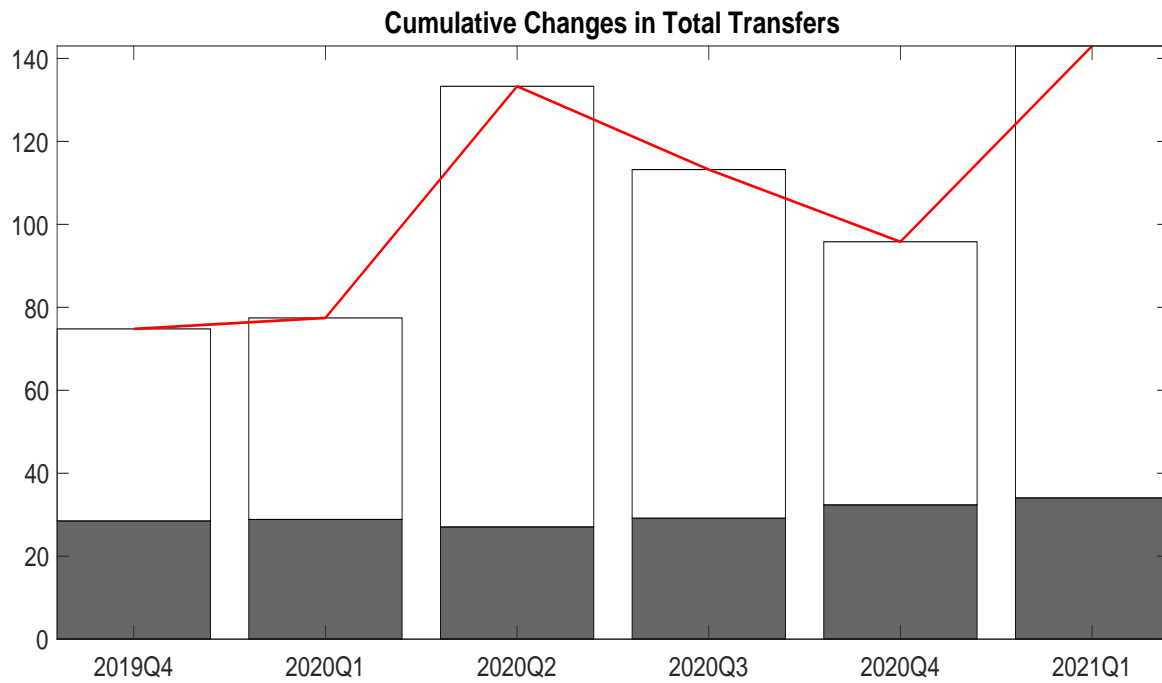


Figure 10: Decomposition of the ARPA stimuli: Funded vs. Unfunded Transfers.

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