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**Working Paper**

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Working Paper, No. 218

**Provided in Cooperation with:**

Oesterreichische Nationalbank (OeNB), Vienna

*Suggested Citation:* Niemann, Stefan; Pichler, Paul (2017) : Optimal fiscal policy and sovereign debt crises, Working Paper, No. 218, Oesterreichische Nationalbank (OeNB), Vienna

This Version is available at:

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## WORKING PAPER 218

Optimal fiscal policy and sovereign debt crises

Stefan Niemann and Paul Pichler

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**Publisher and editor** Oesterreichische Nationalbank  
Otto-Wagner-Platz 3, 1090 Vienna, Austria  
PO Box 61, 1011 Vienna, Austria  
[www.oenb.at](http://www.oenb.at)  
[oenb.info@oenb.at](mailto:oenb.info@oenb.at)  
Phone (+43-1) 40420-6666  
Fax (+43-1) 40420-046698

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of the Working Papers** Doris Ritzberger-Grünwald, Ernest Gnan, Martin Summer

**Coordinating editor** Martin Summer

**Design** Communications and Publications Division

**DVR 0031577**

**ISSN 2310-5321 (Print)**  
**ISSN 2310-533X (Online)**

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# Optimal fiscal policy and sovereign debt crises\*

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December 2017

## Abstract

This paper studies how sovereign risk – both fundamental and self-fulfilling – shapes the cyclical behavior of optimal fiscal policy. We develop a model with endogenous default costs where market sentiment can induce belief-driven debt rollover crises. Optimal taxes and public spending are generally procyclical, but the incidence of rollover risk gives rise to infrequent episodes of severely countercyclical fiscal activity. These endogenous regime changes are associated with pronounced countercyclical changes in the level of debt. Debt buildups are triggered already by relatively mild recessions, but successful fiscal consolidations occur only in exceptionally good times.

*JEL classification:* E62, F34, H63

*Keywords:* fiscal policy; sovereign debt; rollover crises; regime switches

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\*Earlier versions of this paper were presented at the RES 2016 Annual Conference (Sussex) and CEF2016 (Bordeaux). We would like to thank seminar audiences as well as James Costain, Morten Ravn and Almuth Scholl for helpful discussions. The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Oesterreichische Nationalbank or the European System of Central Banks.

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## Non-technical Summary

We study how the presence of sovereign risk - both fundamental solvency risk and sentiment-driven liquidity risk - affects the cyclical behavior of optimal fiscal policy and the pattern of government debt issuance. We develop a dynamic economic model with distortionary taxation, government spending and endogenous costs of government default arising from disruptions to the import of intermediate inputs. Market sentiment can give rise to self-fulfilling debt rollover crises when international lenders lose confidence in the government's willingness to honor its liabilities.

Optimal taxes and public spending are generally *procyclical* in our environment, which is well in line with empirical evidence from emerging economies where fiscal policy often does not contribute to smoothing business cycle fluctuations over time but rather exacerbates underlying boom-bust cycles. However, the incidence of rollover risk gives rise to occasional episodes of severely countercyclical fiscal activity. These episodes occur when the economy enters or leaves the critical region of indebtedness where self-fulfilling rollover crises are possible.

The government's optimal fiscal policy during recessions can then be expansionary. In order to avoid an excessive depression in output and consumption, the government optimally exploits its fiscal space even though this policy ultimately entails a discontinuous increase in sovereign risk and future borrowing costs. By contrast, if the level of government debt is initially very high, a sustained economic boom can induce the government to adopt a severely contractionary fiscal stance. It optimally reduces public spending and raises taxes in an effort to bring down debt to the point where rollover risk is eliminated. This policy lowers the default premia charged by international creditors and hence significantly reduces borrowing costs, which provides scope for future governments to reduce distortionary taxes.

In addition to prescribing large and fast-paced debt dynamics, our results also uncover a fundamental asymmetry underlying the optimal fiscal policy: Large fiscal expansions are triggered already by relatively mild but sustained recessions, whereas large fiscal contractions are triggered only by exceptional boom episodes.

# 1 Introduction

A well-established stylized fact in international macroeconomics is the procyclical pattern of fiscal policy in emerging economies. Different from advanced economies, fiscal policy in these countries does not contribute to smoothing business cycle fluctuations over time but rather exacerbates underlying boom-bust cycles. A common explanation for this phenomenon is that governments in emerging countries have limited access to international borrowing, and particularly so in bad times. Hence, in order to confront concerns about the sustainability of their public finances and to retain financial market access, they often impose contractionary fiscal measures even during severe recessions (cf. Vegh and Vuletin, 2015; Born, Mueller, and Pfeifer, 2015). This fiscal procyclicality can be rationalized insofar as austerity in the form of spending cuts or tax hikes helps to create better borrowing conditions as reflected by reduced sovereign spreads. Importantly, however, these spreads are not only determined by fundamentals but also subject to market sentiment (Calvo, 1988; Cole and Kehoe, 2000).<sup>1</sup>

In this paper we study how fundamental sovereign risk and market sentiment shape the pattern of optimal debt issuance and the cyclical behavior of optimal fiscal policy. We develop a model with distortionary taxation, government spending and endogenous default costs driven by disruptions to the import of intermediate inputs (cf. Mendoza and Yue, 2012). Market sentiment can give rise to belief-driven debt rollover crises when international lenders lose confidence in the government's willingness to honor its liabilities and hence refuse rolling over their debts (cf. Cole and Kehoe, 2000). When debt is sufficiently high and repayment of the maturing liabilities is hence sufficiently costly, the inability to issue new debt gives the government an incentive to default, thus making the investors' initial loss of confidence a self-fulfilling prophecy. Debt levels for which these self-fulfilling dynamics are possible fall into the so-called *crisis zone*.<sup>2</sup>

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<sup>1</sup>For example, a narrative advanced in the context of the recent European debt crisis has been that a (sudden) loss of confidence in some countries' capacity to serve their debt triggered a collapse of bond prices; this led to the adoption of severe austerity measures, which further contracted economic activity. Similarly, the Mexican debt crisis of 1994/95 saw the Mexican government unable to roll over its debt because of international investors' pessimistic beliefs and was resolved only through an international rescue package (cf. Cole and Kehoe, 1996).

<sup>2</sup>In addressing self-fulfilling expectations as a source of macroeconomic instability, our multiple-equilibria approach is related to, but conceptually distinct from, work on local (in)determinacy. For important applications dealing with the interaction of sovereign risk and fiscal policy, see e.g. Corsetti, Kuester, Meier, and Mueller (2013, 2014).

At the heart of the fiscal policy problem is a trade-off between the government's motives to frontload and to smooth consumption: Increasing current consumption comes at the cost of accumulating more debt and thus higher exposure to default risk, which makes future consumption more volatile. To what extent this happens actually depends on the sources of sovereign risk, and this is reflected in the pattern of optimal fiscal policy. We show that, while optimal taxes and public spending are generally procyclical,<sup>3</sup> the incidence of rollover risk gives rise to occasional episodes of severely countercyclical fiscal activity. These episodes are manifestations of *endogenous regime switches* when the economy enters or leaves the crisis zone. Below the crisis zone, the government's optimal fiscal policy during recessions is then expansionary. It exploits its fiscal space in order to avoid an excessive depression in output and consumption, even though this policy ultimately entails a transition into the crisis zone and hence a discontinuous increase in sovereign risk and future borrowing costs. By contrast, within the crisis zone, a sustained boom can induce the government to adopt a severely contractionary fiscal stance. It reduces public spending and raises taxes in an effort to bring down debt to the point where rollover risk is completely eliminated. This policy lowers the default premia charged by international creditors and hence significantly reduces borrowing costs, which provides space for future governments to reduce distortionary taxes. Fiscal austerity, if initiated during booms, can thus be expansionary and facilitate consumption smoothing. The traditional rationale for countercyclical fiscal policy familiar from advanced economies is therefore at work also in our framework where fiscal policy is constrained by both solvency considerations and market sentiment.

In addition to prescribing large and fast-paced debt dynamics during regime switches, our results also uncover a fundamental asymmetry underlying the optimal fiscal policy: Large fiscal expansions are triggered already by relatively mild (but sustained) recessions, whereas large fiscal contractions are triggered only by exceptional boom episodes, that is, when output is significantly above trend. Our normative results thus also facilitate a new perspective at empirically observed debt dynamics. Absent rollover risk, Eaton-Gersovitz-type models prescribe a procyclical debt policy, so that debt rises (falls) in good (bad) times. The empirical evidence, however, indicates that *large swings* in public debt display a different, countercyclical pattern

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<sup>3</sup>Throughout, we refer to procyclical fiscal policy as implying higher (lower) public spending and lower (higher) tax rates in good (bad) times.

(Abbas, Belhocine, El-Ganainy, and Horton, 2011).<sup>4</sup> The endogenous regime switches in our model are consistent with this and also with the finding that debt-consolidating fiscal austerity programs pay off particularly if initial economic conditions are benign (Born, Mueller, and Pfeifer, 2015).

Our paper also connects to an expanding literature investigating the role and properties of fiscal policy in environments subject to sovereign risk. This literature has evolved into three main directions. A first branch documents the empirical regularities of fiscal policy in emerging economies and contrasts them to those observed in developed economies. A common theme is that both public spending and taxes are procyclical in the former but much less so in the latter group of countries (see e.g. Gavin and Perotti, 1997; Talvi and Vegh, 2005; Kaminsky, Reinhart, and Vegh, 2005; Frankel, Vegh, and Vuletin, 2013; Vegh and Vuletin, 2015). The explanations advanced to rationalize this pattern include weak political institutions and tax enforcement, incomplete markets and borrowing constraints (see e.g. Tornell and Lane, 1999; Cuadra, Sanchez, and Sapriza, 2010; Ilzetzki, 2011; Bauducco and Caprioli, 2014). Relatedly, a second strand employs models of sovereign default to examine the relationship between fiscal rules or restrictions, bailouts and conditionality imposed by international financial institutions (see e.g. Juessen and Schabert, 2013; Goncalves and Guimaraes, 2015; Hatchondo, Martinez, and Roch, 2015; Fink and Scholl, 2016; Arellano and Bai, 2016). Finally, a third branch integrates fiscal policy into dynamic models of endogenous sovereign default from the perspective of optimal taxation with or without commitment (see e.g. Adam and Grill, 2013; Pouzo and Presno, 2015; Niemann and Pichler, 2016).

Most closely related to our work are the papers by Cuadra, Sanchez, and Sapriza (2010) and Cole and Kehoe (2000). Like us, Cuadra, Sanchez, and Sapriza (2010) consider a small open economy model where not only the repayment of debt but also taxes and public spending are endogenously chosen by the government in a time-consistent fashion. They consider only fundamental default risk and obtain fiscal procyclicality in a framework with exogenous default costs as in Arellano (2008). By contrast, our approach allows us to examine the interaction between fiscal policy and endogenous default costs and accommodates self-fulfilling debt crises, which we show to have important consequences for optimal fiscal policies.<sup>5</sup> Our detailed account

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<sup>4</sup>Section 5.4 below provides further details on this point.

<sup>5</sup>In Gomez-Oliveros Duran, Niemann, and Pichler (2016), we analyze the role of fiscal policy in shaping the



of fiscal policy and default costs also differentiates our work from Cole and Kehoe (2000) from whom we borrow the foundation for rollover risk. While distortionary, taxation in their model is limited to a time-invariant income tax; likewise, productivity is not stochastic and, following default, subject to a permanent exogenous reduction. Instead, our paper integrates flexible and optimally determined fiscal policy into a fully-fledged stochastic environment, which facilitates the quantitative assessment of its properties in normal times and during crisis and transition episodes. This has important implications. Whereas Cole and Kehoe (2000) establish that, generically, the optimal policy is to escape the crisis zone by decumulating debt, our model of optimal fiscal policy rationalizes persistent debt positions within the crisis zone as the generic outcome, which is broken only through rare regime changes following exceptional productivity dynamics. Finally, in a variation of the setup with time-invariant taxation in Cole and Kehoe (2000), Conesa and Kehoe (2015) obtain debt dynamics driven by the government’s motive in recessions to ‘gamble for resurrection’, which may result in a transition into the crisis zone. This is similar to our model, which, however, features empirically plausible productivity dynamics and thus also transitions out of the crisis zone, and in addition details the dynamic pattern of (variable) taxation and spending during these regime switches.

The rest of this paper is organized as follows. Section 2 presents our formal model environment, and Section 3 presents conditions characterizing optimal fiscal policy. Section 4 calibrates the model to data for the Mexican economy. Section 5 presents the results of our quantitative exercise. Section 6 concludes.

## 2 The model

Our model extends the framework by Mendoza and Yue (2012) by introducing fiscal policy, in the form of public spending and a linear consumption tax, as well as the possibility of self-fulfilling rollover crises (cf. Cole and Kehoe, 2000). We consider a small open economy populated by households, firms and a sovereign government which borrows from foreign lenders. Production is organized in two sectors, a sector  $f$  of final goods producers and a sector  $m$  of intermediate goods producers. Time is discrete,  $t = 0, 1, 2, \dots$

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output costs of default in greater detail.

## 2.1 Private sector

**Households.** Households choose consumption,  $c_t$ , and labor supply,  $L_t$ , to maximize a time-separable utility function,

$$E \sum_{t=0}^{\infty} \beta^t u(c_t - v(L_t), g_t), \quad (1)$$

where  $g_t$  denotes valued public expenditure and  $\beta \in (0, 1)$  is a discount factor. The period utility function  $u(\cdot)$  has standard properties and is separable in its two arguments; the first term complies with the specification in Greenwood, Hercowitz, and Huffman (1988) which removes the wealth effect on labor supply. Households take as given the wage rate  $w_t$ , firm profits  $\pi_t^f$  and  $\pi_t^m$  in the two sectors, and the consumption tax  $\tau_t$ . Since households do not participate in intertemporal asset markets,<sup>6</sup> their problem consists of maximizing (1) subject to the budget constraint

$$c_t = \frac{1}{1 + \tau_t} \left[ w_t L_t + \pi_t^f + \pi_t^m \right]. \quad (2)$$

The associated optimality condition for labor supply is

$$v'(L_t) = \frac{w_t}{1 + \tau_t}. \quad (3)$$

**Final goods producers.** Competitive firms in the  $f$  sector combine labor  $L_t^f$ , intermediate goods  $M_t$  and a time-invariant capital stock  $k$  to produce final goods. The production function is Cobb-Douglas and subject to productivity shocks  $\varepsilon_t$ ,

$$y_t = \varepsilon_t \left( M(m_t^d, m_t^*) \right)^{\alpha_M} \left( L_t^f \right)^{\alpha_L} k^{\alpha_k}, \quad (4)$$

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<sup>6</sup>The assumption that households cannot borrow directly on international financial markets is widely adopted in the sovereign debt literature. Lahiri, Singh, and Vegh (2007) point out that, in general, only a fraction of households have access to asset markets, and that even in the US the degree of asset market segmentation is remarkably high. Reinhart, Rogoff, and Savastano (2003, 2014) report that, historically, private foreign borrowing amounts to less than 10% of total foreign borrowing in more than two thirds of developing countries. Cuadra, Sanchez, and Sapriza (2010) provide some further discussion. While their baseline model does not allow for private borrowing on international markets, they also study an alternative model with private borrowing and find that their baseline results remain qualitatively unaffected.

where  $\alpha_M, \alpha_L, \alpha_k \in (0, 1)$  and  $\alpha_M + \alpha_L + \alpha_k = 1$ . The mix of intermediate inputs is determined by a CES Armington aggregator combining domestic inputs  $m_t^d$  and imported inputs  $m_t^*$ ,

$$M_t = [\lambda (m_t^d)^\mu + (1 - \lambda) (m_t^*)^\mu]^\frac{1}{\mu} \quad (5)$$

with weight  $\lambda \in [0, 1)$ , implying an elasticity of substitution of  $\eta_{m^d, m^*} = \left| \frac{1}{\mu - 1} \right|$ . Imported inputs, in turn, are given by a Dixit-Stiglitz aggregator combining a continuum of differentiated varieties  $m_j^*$ ,  $j \in [0, 1]$ ,

$$m_t^* = \left[ \int_{j \in [0, 1]} (m_{jt}^*)^\nu dj \right]^\frac{1}{\nu}, \quad (6)$$

where  $\nu \in (0, 1)$  so that there is a finite elasticity of substitution of  $\eta_{m_j^*} = \left| \frac{1}{\nu - 1} \right|$  across imported input varieties.

A subset  $\Omega$  of the imported input varieties, defined by the interval  $[0, \theta]$  with  $\theta \in (0, 1)$ , must be financed in advance via working capital loans  $\kappa_t$ . The timing protocol is such that these within-period loans are contracted after uncertainty about productivity and the government's repayment decision concerning its current debt service has been resolved. As Mendoza and Yue (2012) we assume that the availability of working capital loans to firms conditions on the government's access to international financial markets. Following a sovereign default, working capital loans become unavailable throughout the period of market exclusion. When the government repays, firms can contract loans at the risk-free world interest rate  $r_t^*$ ; in this case, their demand for working capital is

$$\frac{\kappa_t}{1 + r_t^*} \geq \int_0^\theta p_j^* m_{jt}^* dj, \quad (7)$$

where  $p_j^*$  denotes the exogenous, time-invariant price of the imported input variety  $j \in [0, 1]$ .

Final goods producers choose factor demands to maximize profits, taking  $w_t$ ,  $r_t^*$ ,  $p_j^*$  and  $p_t^m$ ,

the price of domestic inputs, as given. Profits in the final goods sector are given by

$$\begin{aligned}\pi_t^f &= \varepsilon_t (M_t)^{\alpha_M} (L_t^f)^{\alpha_L} k^{\alpha_k} - r_t^* \int_0^\theta p_j^* m_{jt}^* dj - \int_0^1 p_j^* m_{jt}^* dj - p_t^m m_t^d - w_t L_t^f \\ &= \varepsilon_t (M(m_t^d, m_t^*))^{\alpha_M} (L_t^f)^{\alpha_L} k^{\alpha_k} - P^*(r_t^*) m_t^* - p_t^m m_t^d - w_t L_t^f,\end{aligned}\quad (8)$$

where  $M(m_t^d, m_t^*)$  is given by (5) and  $P^*(r_t^*) = \left[ \int_0^\theta (p_j^*(1+r_t^*))^{\frac{\nu}{\nu-1}} dj + \int_\theta^1 (p_j^*)^{\frac{\nu}{\nu-1}} dj \right]^{\frac{\nu-1}{\nu}}$  is the price index for imported inputs resulting from CES aggregation. The first-order conditions associated with final goods firms' profit maximization problem are then given by

$$\alpha_M \varepsilon_t (M(m_t^d, m_t^*))^{\alpha_M - \mu} (L_t^f)^{\alpha_L} k^{\alpha_k} (1 - \lambda) (m_t^*)^{\mu-1} = P^*(r_t^*), \quad (9)$$

$$\alpha_M \varepsilon_t (M(m_t^d, m_t^*))^{\alpha_M - \mu} (L_t^f)^{\alpha_L} k^{\alpha_k} \lambda (m_t^d)^{\mu-1} = p_t^m, \quad (10)$$

$$\alpha_L \varepsilon_t (M(m_t^d, m_t^*))^{\alpha_M} (L_t^f)^{\alpha_L - 1} k^{\alpha_k} = w_t, \quad (11)$$

and

$$m_{jt}^* = \left( \frac{p_j^*}{P^*(r_t^*)} \right)^{-\frac{1}{1-\nu}} m_t^*, \quad \text{for } j \in [\theta, 1], \quad (12)$$

$$m_{jt}^* = \left( \frac{p_j^*(1+r_t^*)}{P^*(r_t^*)} \right)^{-\frac{1}{1-\nu}} m_t^* \quad \text{for } j \in [0, \theta]. \quad (13)$$

The expressions in (8), (9), (12) and (13) implicitly assumed the government and firms have access to capital markets. When the country is in default, the relevant price index for imported inputs becomes  $P_{aut}^* = \left[ \int_\theta^1 (p_j^*)^{\frac{\nu}{\nu-1}} dj \right]^{\frac{\nu-1}{\nu}}$  and the analogs of (12) and (13) are

$$m_{jt}^* = \left( \frac{p_j^*}{P_{aut}^*} \right)^{-\frac{1}{1-\nu}} m_t^*, \quad \text{for } j \in [\theta, 1], \quad (14)$$

$$m_{jt}^* = 0 \quad \text{for } j \in [0, \theta]. \quad (15)$$

**Intermediate goods producers.** Competitive firms in the  $m$  sector use labor  $L_t^m$  to produce intermediate goods according to the production function

$$m_t^d = A(L_t^m)^\gamma, \quad (16)$$

where  $A > 0$  and  $\gamma \in [0, 1]$ . Taking  $p_t^m$  and  $w_t$  as given, intermediate firms maximize profits,

$$\pi_t^m = p_t^m A(L_t^m)^\gamma - w_t L_t^m. \quad (17)$$

The associated optimality condition for labor demand is given by

$$\gamma p_t^m A(L_t^m)^{\gamma-1} = w_t. \quad (18)$$

Finally, notice that labor market clearing requires

$$L_t = L_t^f + L_t^m, \quad (19)$$

and that GDP, the value of the output of final goods net of the costs of imported inputs, is given by

$$gdp_t = y_t - P_t^* m_t^*. \quad (20)$$

## 2.2 Sovereign government and foreign lenders

The sovereign government implements consumption taxes  $\tau_t$ , provides public spending  $g_t$  and can borrow and lend in international credit markets. Let  $b_{t+1}$  denote the amount of debt issued in period  $t$ . Financial markets are incomplete because the government can issue only one-period, non-state-contingent discount bonds. The government cannot commit to repay its debt. Each period, conditional on being in good credit standing, the government chooses between honoring its outstanding foreign debt or defaulting on it. The government's inability to commit is the reason why borrowing from international creditors is also subject to self-fulfilling rollover risk: The fear of a future default may prompt lenders not to extend new credit, which in turn may induce an immediate default by the government. There is thus scope for *fundamental defaults*, driven by the government's willingness to repay, and *self-fulfilling rollover crises*, driven by a coordination failure between the government and its foreign lenders.

Irrespective of its ultimate source, the consequences of debt repudiation are always the same. Default wipes out the entirety of the government's outstanding debt at the cost of financial

autarky, that is, exclusion from international credit markets. When in bad credit standing, the government may regain access to international credit markets in the next period with an exogenous probability  $\phi$ . The government's default decision therefore trades off the direct costs of the resource transfer to international lenders associated with the repayment of the its non-contingent debt against the costs of temporary exclusion from credit markets given by the foregone benefits of consumption smoothing and the output loss in autarky due to the non-availability of working capital to firms.

The government's intertemporal problem can be expressed in recursive form. The fundamental state variables are the government's inherited bond position  $b$  and the current productivity level  $\varepsilon$ . In order to examine the implications of rollover risk, we follow Chatterjee and Eyigungor (2012) and consider the following static coordination game played at the beginning of each period between the sovereign government and foreign lenders.<sup>7</sup> The government faces some maturing debt  $b$  and, conditional on redeeming its current liabilities, seeks to issue new bonds. Table 1 details the relevant payoff matrix, whereby columns correspond to the sovereign government's strategies, and rows correspond to the lenders' strategies. If lenders extend new

Table 1: Payoff matrix for static coordination game

	<i>repay</i>	<i>default</i>
<i>rollover</i>	$0, \mathcal{V}^{nd}(b, \varepsilon)$	$-r^*/(1+r^*)\Delta, \mathcal{V}^d(\varepsilon)$
<i>crisis</i>	$0, \mathcal{V}^c(b, \varepsilon)$	$0, \mathcal{V}^d(\varepsilon)$

credit (*rollover*) and the government repays its maturing debt (*repay*), the former earn a net return of zero (that is, in expectation lenders earn the return  $r^*$  which is also the opportunity cost of their funds), and the latter receives the payoff under repayment and new borrowing, denoted by  $\mathcal{V}^{nd}(b, \varepsilon)$ . If lenders extend new credit and the government defaults (*default*), we assume that the new borrowing is returned to the lenders without it earning any interest; the (discounted) loss of interest earnings then is  $r^*/(1+r^*)\Delta$ , where  $\Delta$  is the amount of new lending. If lenders fail to provide new credit (*crisis*), their payoff is zero irrespective of the government's behavior. In this case, if the government repays but cannot borrow, it receives  $\mathcal{V}^c(b, \varepsilon) \leq \mathcal{V}^{nd}(b, \varepsilon)$ .<sup>8</sup> And if the government defaults, its payoff does not depend on the level

<sup>7</sup>For simplicity, and as in Chatterjee and Eyigungor (2012), we abstract from the coordination problem between multiple lenders and assume they act in a coordinated fashion. In Cole and Kehoe (2000), the game is played between the sovereign and many lenders acting independently.

<sup>8</sup>The weak inequality holds because the policy when fresh borrowing is allowed can always replicate the

of maturing debt and is given by  $\mathcal{V}^d(\varepsilon)$ .

We assume indifference on the side of the government or lenders is always resolved in favor of repayment and extending new credit, respectively. Depending on the value of  $\mathcal{V}^d(\varepsilon)$ , the game then has the following set of Nash equilibria. If  $\mathcal{V}^d(\varepsilon) \leq \mathcal{V}^c(b, \varepsilon) \leq \mathcal{V}^{nd}(b, \varepsilon)$ , the unique equilibrium is *(rollover, repay)*; if  $\mathcal{V}^c(b, \varepsilon) \leq \mathcal{V}^{nd}(b, \varepsilon) < \mathcal{V}^d(\varepsilon)$ , the unique equilibrium is *(crisis, default)*; and if  $\mathcal{V}^c(b, \varepsilon) < \mathcal{V}^d(\varepsilon) \leq \mathcal{V}^{nd}(b, \varepsilon)$ , both *(rollover, repay)* and *(crisis, default)* are equilibria. In the last case, we assume that the equilibrium is selected depending on the realization of a sunspot variable, denoted  $\xi$ . If  $\xi = 0$ , the *(rollover, repay)* equilibrium is selected, and if  $\xi = 1$ , the *(crisis, default)* equilibrium is selected. The latter case corresponds to a self-fulfilling rollover crisis where lenders refuse to lend because they believe that the sovereign will default, and the sovereign defaults because it believes that the lenders will refuse to lend. We assume the sunspot variable  $\xi$  is i.i.d. and takes value one with probability  $p$ .

Let  $\mathcal{V}(b, \varepsilon, \xi)$  denote the value function of the government, which depends on the fundamental states  $(b, \varepsilon)$  and the sunspot variable  $\xi$ . If the government has access to credit and does not default, it can issue new debt and finance expenditures subject to the following constraint,

$$g = \tau c + q(b', \varepsilon)b' - b, \quad (21)$$

where  $q(b', \varepsilon)$  is the bond pricing function, which is taken as given by the government. When implementing its policy, the government also needs to take into account the private sector's response given by the set of optimality conditions

$$\mathcal{E}^{nd} = \{(2), (3), (4), (7), (9), (10), (11), (12), (13), (16), (18), (19)\}.$$

Thus, the government problem conditional on repayment is

$$\mathcal{V}^{nd}(b, \varepsilon) = \max_{\tau, g, b'} \{u(c - v(L), g) + \beta E [(1 - p)\mathcal{V}(b', \varepsilon', 0) + p\mathcal{V}(b', \varepsilon', 1)]\} \quad (22)$$

subject to (21) and  $\mathcal{E}^{nd}$ . By contrast, if the government defaults, it is temporarily excluded 

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policy without borrowing, or even do better.

from international credit markets and faces the constraint

$$g = \tau c, \quad (23)$$

and the private-sector optimality conditions  $\mathcal{E}^d$ , where, relative to  $\mathcal{E}^{nd}$ ,  $\kappa = 0$ , the import price index is  $P_{aut}^*$  and (12) and (13) are replaced by (14) and (15). The problem of a defaulting government then is

$$\mathcal{V}^d(\varepsilon) = \max_{\tau, g} \{u(c - v(L), g) + \beta E [(1 - \phi)\mathcal{V}^d(\varepsilon') + \phi[(1 - p)\mathcal{V}(0, \varepsilon', 0) + p\mathcal{V}(0, \varepsilon', 1)]]\} \quad (24)$$

subject to (23) and  $\mathcal{E}^d$ . And similarly, the value function when international lenders refuse to extend new credit is given by

$$\mathcal{V}^c(b, \varepsilon) = \max_{\tau, g} \{u(c - v(L), g) + \beta E [(1 - p)\mathcal{V}(0, \varepsilon', 0) + p\mathcal{V}(0, \varepsilon', 1)]\} \quad (25)$$

subject to  $g = \tau c - b$  and  $\mathcal{E}^d$ .<sup>9</sup> Given  $\mathcal{V}^{nd}(b, \varepsilon)$ ,  $\mathcal{V}^d(\varepsilon)$  and  $\mathcal{V}^c(b, \varepsilon)$ , the government's value function is determined as

$$\mathcal{V}(b, \varepsilon, \xi) = \begin{cases} \mathcal{V}^{nd}(b, \varepsilon), & \text{if } \mathcal{V}^d(\varepsilon) \leq \mathcal{V}^c(b, \varepsilon) \text{ and } \xi \in \{0, 1\}, \\ \mathcal{V}^d(\varepsilon), & \text{if } \mathcal{V}^{nd}(b, \varepsilon) < \mathcal{V}^d(\varepsilon) \text{ and } \xi \in \{0, 1\}, \\ \mathcal{V}^{nd}(b, \varepsilon), & \text{if } \mathcal{V}^c(b, \varepsilon) < \mathcal{V}^d(\varepsilon) \leq \mathcal{V}^{nd}(b, \varepsilon) \text{ and } \xi = 0, \\ \mathcal{V}^d(\varepsilon), & \text{if } \mathcal{V}^c(b, \varepsilon) < \mathcal{V}^d(\varepsilon) \leq \mathcal{V}^{nd}(b, \varepsilon) \text{ and } \xi = 1. \end{cases} \quad (26)$$

The optimal default policy under rollover risk can be characterized with reference to (26) as

$$\mathcal{D}(b, \varepsilon, \xi) = \begin{cases} 0, & \text{if } \mathcal{V}(b, \varepsilon, \xi) = \mathcal{V}^{nd}(b, \varepsilon), \\ 1, & \text{if } \mathcal{V}(b, \varepsilon, \xi) = \mathcal{V}^d(\varepsilon). \end{cases} \quad (27)$$

This determines a default set, the set of productivity realizations such that, given the sunspot

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<sup>9</sup>Implicit in this formulation is the assumption that final goods firms do not have access to working capital loans when lenders refuse to buy government bonds. This parallels our earlier assumptions about the consequences of sovereign default.



and current debt, default is optimal,

$$\Gamma(b, \xi) = \{\varepsilon : \mathcal{D}(b, \varepsilon, \xi) = 1\}. \quad (28)$$

Moreover, given current productivity  $\varepsilon$  and some debt policy  $b'$ , the probability of default in the next period can be inferred from the default set and the transition process  $z(\varepsilon'|\varepsilon)$  for productivity as

$$\rho(b', \varepsilon) = (1 - p) \int_{\Gamma(b', 0)} dz(\varepsilon'|\varepsilon) + p \int_{\Gamma(b', 1)} dz(\varepsilon'|\varepsilon). \quad (29)$$

Bond prices are determined by international lenders, who are risk-neutral and have complete information. Facing an opportunity cost of funds equal to  $r^*$ , they invest in one-period sovereign bonds and in within-period private working capital loans. Competition implies that lenders expect zero profits and that the returns on sovereign debt and the world's risk-free asset are fully arbitrated, that is,

$$q(b', \varepsilon) = \frac{1 - \rho(b', \varepsilon)}{1 + r^*}. \quad (30)$$

Accordingly, the equilibrium bond price  $q(b', \varepsilon)$  reflects the risk of sovereign default.

**Equilibrium.** In equilibrium, private-sector allocations are optimal, given the policies implemented by the government; the government's debt, default and fiscal policy are optimal subject to the relevant implementability constraints and the bond pricing function  $q(b', \varepsilon)$ ; and foreign lenders are optimizing.

### 3 Optimal fiscal policy

We now examine the properties of optimal fiscal policy. A first result is that the monotonicity of the optimal (discretionary) default policy, which is familiar from models following Eaton and Gersovitz (1981), is preserved also in the presence of optimal fiscal policy.

**Proposition 1.** *Given a productivity shock  $\varepsilon$  and a pair of debt positions  $b$  and  $\tilde{b}$  such that  $b > \tilde{b} \geq 0$ , if default is optimal for  $\tilde{b}$ , then it is optimal also for  $b$ . That is,  $\Gamma(\tilde{b}, \xi) \subseteq \Gamma(b, \xi)$ .*

Since bond prices compensate for default risk, it follows that, for given current productivity, they must also be monotonic in the amount of debt issued. Indeed, the government's lack of commitment matters only if bond prices react to debt. This is because both taxation and public spending affect only the static equilibrium conditions. The following proposition characterizes the government's optimal tax and spending policy.

**Proposition 2.** *The government's optimal tax policy satisfies*

$$u_c \left\{ -\frac{\partial \frac{1}{1+\tau}}{\partial \tau} gdp \right\} = u_g \left\{ -\frac{\partial \frac{1}{1+\tau}}{\partial \tau} gdp + \frac{\tau}{1+\tau} \frac{\partial gdp}{\partial \tau} \right\},$$

which implies underprovision of public spending,  $u_c < u_g$ .

This condition has an interpretation in terms of marginal benefits and marginal costs of changing the tax rate. Variations in the government's tax policy are then seen to have two effects: a direct reallocation effect ( $-\frac{\partial \frac{1}{1+\tau}}{\partial \tau} gdp > 0$ ), and a budgetary effect ( $\frac{\tau}{1+\tau} \frac{\partial gdp}{\partial \tau} < 0$ ). In detail, for given GDP, an increase in the tax rate allows to reallocate resources from private to public consumption. However, this causes tax distortions which work to reduce GDP, the relevant tax base for the consumption tax, and thus has negative implications for the government's budget. In conjunction, these effects imply that the optimal fiscal policy limits distortions by keeping public expenditure below its first-best level.

To build intuition, it is also useful to consider the generalized Euler equation characterizing the government's optimal debt policy.<sup>10</sup>

**Proposition 3.** *The government's optimal debt policy satisfies*

$$u_g \left\{ q + \frac{\partial q}{\partial b'} b' \right\} = \beta E_{\xi'} E_{\varepsilon' \notin \Gamma(b', \xi')} u'_g.$$

An immediate implication of the generalized Euler equation is that increasing debt can only be optimal for the government if this generates additional resources, that is,  $q + \frac{\partial q}{\partial b'} b' > 0$ . So the optimal debt policy can never be subject to a debt Laffer curve. More generally, the generalized

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<sup>10</sup>Technically, this presumes differentiability of the bond pricing function  $q(b', \varepsilon)$  and the value function  $\mathcal{V}^{nd}(b, \varepsilon)$ . Notice therefore that the generalized Euler equation is presented merely to illustrate the intertemporal policy trade-off facing the government. We do not claim or prove differentiability, and also our numerical approach and quantitative results do not rest on it.

Euler equation balances today's marginal benefit of additional borrowing against the discounted marginal cost of higher debt tomorrow. The marginal benefit values with the marginal utility of public spending the additional resources available to the government from issuing an extra unit of debt. The marginal cost of higher inherited liabilities is to reduce the resources available for future public spending. Notice, however, that the possibility of a future default implies that the government takes this cost into account only for states which actually induce repayment.<sup>11</sup> Since the government will exercise its default option exactly in times when repayment would be associated with an excessively high marginal utility from public spending, this conditioning implies  $E_{\xi'} E_{\varepsilon' \notin \Gamma(b', \xi')} u'_g \leq E_{\xi'} E_{\varepsilon'} u'_g$ , where  $E_{\xi'} E_{\varepsilon'} u'_g$  is the marginal utility under the suboptimal policy dictating repayment in all states  $(\varepsilon, \xi)$ . How the government resolves the intertemporal trade-off underlying its debt policy therefore depends on the strength of its effective frontloading motive, as captured by its discount factor  $\beta$  and the reduction in the expected future marginal utility via the option value of default.

## 4 Calibration

We now study the model's quantitative implications in a calibrated environment. We target data for Mexico for two reasons. First, given our emphasis on fiscal policy, we can rely on time series data for public finances, whose availability and quality is better in Mexico compared to many other emerging economies including Argentina. Second, the Mexican debt crisis of 1994-1995 is widely interpreted as driven by self-fulfilling dynamics of the sort considered in our model. Notice also that, at that time, the maturity structure of sovereign debt was very low, with the majority of bonds having a maturity of just 91 days and overall maturity averaging at barely 200 days (Cole and Kehoe, 1996). Our quarterly model with one-period debt is thus a good approximation.

Similar to Cuadra, Sanchez, and Saprizza (2010), the period utility function in (1) is assumed

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<sup>11</sup>Key to this result is the fact that the level of maturing debt becomes irrelevant in the event of default. It therefore also does not matter whether a default occurs for fundamental or self-fulfilling reasons.

to take the following form,

$$u(c - v(L), g) = \pi \left( \frac{(c - \frac{L\omega}{\omega})^{1-\sigma}}{1-\sigma} \right) + (1 - \pi) \left( \frac{g^{1-\sigma}}{1-\sigma} \right). \quad (31)$$

Accordingly, the contributions of public expenditures and the consumption-leisure composite to utility are subject to the same intertemporal elasticity of substitution  $\frac{1}{\sigma}$  and are aggregated with relative weights  $(1 - \pi)$  and  $\pi$ . Labor supply is characterized by a constant Frisch elasticity of  $\frac{1}{\omega-1}$ . The benchmark parameterization for our quarterly model is summarized in Table 2. Parameters above the line are calibrated. The curvature parameters are set to  $\sigma = 2$  and  $\omega = 1.455$ , corresponding to a Frisch elasticity of  $\frac{1}{\omega-1} = 2.2$ . These are standard values in quantitative studies of sovereign default and international real business cycles (Mendoza, 1991; Neumeyer and Perri, 2005; Cuadra, Sanchez, and Sapriza, 2010; Mendoza and Yue, 2012). Likewise, the quarterly risk-free interest rate  $r^*$  is set to 1%. Given the other parameter values assigned in (31), the preference weight is obtained as  $\pi = 0.9$ , which generates a ratio of public to private consumption of 20%, the mean value observed in Mexico (1980-2009). The probability of reentry after default is set at  $\phi = 0.083$ ; the implied average exclusion period of three years is consistent with relevant empirical estimates (Dias and Richmond, 2009; Gelos, Sahay, and Sandleris, 2011; Cruces and Trebesch, 2013).

Our calibration of the parameters relating to the aggregation of intermediate inputs in (5) and (6) follows Mendoza and Yue (2012) who recur on Mexican data (1988-2004) and infer  $\lambda = 0.62$  and  $\mu = 0.65$  from optimality conditions (9) and (10).<sup>12</sup> There is thus a small bias in favor of domestic relative to imported inputs, and the elasticity of substitution between them is  $\eta_{m^d, m^*} = \frac{1}{1-\mu} = 2.86$ . The parameter  $\nu$  is pinned down by the elasticity of substitution across imported varieties, which – building on evidence reported in Gopinath and Neiman (2014) – is set at  $\eta_{m_j^*} = \frac{1}{1-\nu} = 2.44$  as in Mendoza and Yue (2012); this implies  $\nu = 0.59$ . Finally, the target for  $\theta$  is the share of working capital financing in GDP. As Schmitt-Grohe and Uribe (2007) and Mendoza and Yue (2012), we proxy working capital by the fraction of M1 held by firms, whereby we rely on an estimate for the US showing that firms hold about two-thirds of M1. On the basis of this strategy, our estimate for the importance of working capital results

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<sup>12</sup>These two values allow their model to match the average ratios in Mexican data of imported to domestic inputs at current and constant prices, which are 18% and 15.7%, respectively.

Table 2: Parameter values

Parameter		Value	Source / Target statistics
Inverse Frisch elasticity	$\omega$	1.455	Standard value
Inverse of intertemporal elasticity of consumption	$\sigma$	2	Standard value
Risk-free rate	$r^*$	0.01	Standard value
Reentry probability	$\phi$	0.083	Standard value
Armington weight of domestic inputs	$\lambda$	0.62	Mendoza and Yue (2012)
Armington curvature	$\mu$	0.65	Mendoza and Yue (2012)
Dixit-Stiglitz curvature	$\nu$	0.59	Mendoza and Yue (2012)
Working capital to GDP ratio	$\theta$	0.7	Mendoza and Yue (2012)
International goods share in gross output of final goods	$\alpha_M$	0.43	Mendoza and Yue (2012)
Labor share in gross output of final goods	$\alpha_L$	0.26	Labor income share in GDP (0.46)
Labor share in production of intermediate goods	$\gamma$	0.46	Labor income share in GDP (0.46)
Private cons. weight in utility	$\pi$	0.9	Public-private consumption ratio (0.2)
Intermed. sector productivity	$A$	0.18	Quarterly output drop in default (5%)
Autocorrelation of TFP shock	$\rho_\varepsilon$	0.98	GDP autocorrelation (0.84)
Standard dev. of TFP shock	$\sigma_\varepsilon$	0.01	GDP standard dev. (0.024)
Discount factor	$\beta$	0.94	Priv. consumption standard dev. (0.029)
Sunspot probability	$p$	0.01	Average annual spread (4.27%)

in  $\theta = 0.7$ . The share of intermediate goods in gross output is set to  $\alpha_M = 0.43$  as in Mendoza and Yue (2012). We assume identical labor shares in the final ( $f$ ) and intermediate ( $m$ ) goods sectors. We set  $\gamma = 0.46$ , which, according to OECD's Unit Labour Costs Annual Indicators data, is the average labor income share in Mexico during the period 1980-2009. A labor income share in value added of the  $f$  sector of  $\frac{\alpha_L}{1-\alpha_M} = \gamma = 0.46$  then implies  $\alpha_L = 0.26$ . Under constant returns to scale in the  $f$  sector, we finally have  $\alpha_k = 1 - \alpha_M - \alpha_L = 0.31$ .

Parameters below the line are set with SMM, similar to Mendoza and Yue (2012). Productivity shocks in final goods production are assumed to follow an AR(1) process,

$$\ln \varepsilon' = \rho_\varepsilon \ln \varepsilon + \varepsilon', \quad (32)$$

with  $E(\varepsilon) = 0$  and  $E(\varepsilon^2) = \sigma_\varepsilon$ . This productivity process is approximated by a discrete first-order Markov chain with 25 values using the procedure in Tauchen (1986). The parameters  $\rho_\varepsilon$  and  $\sigma_\varepsilon$  are then chosen to target the autocorrelation and standard deviation of quarterly GDP in Mexico (1980-2009, H-P filtered, OECD Quarterly National Accounts). The data moments are 0.84 and 2.4%, and the resulting parameter values are  $\rho_\varepsilon = 0.98$  and  $\sigma_\varepsilon = 0.01$ . The remaining parameters  $A$ ,  $\beta$  and  $p$  are chosen to target the decline in output at default, which is approximately 5% in the data; the standard deviation of HP-detrended private consumption, which is approximately 3%; and the average annualized interest rate spread, which equals 4.27% in the EMBI data provided by Cuadra, Sanchez, and Sapriza (2010). Given these targets, the SMM procedure yields  $A = 0.18$ ,  $\beta = 0.94$  and  $p = 0.01$ .

Some comments are in order. While our calibration of the discount factor at  $\beta = 0.94$  is quite high in comparison to much of the sovereign default literature (e.g. Mendoza and Yue, 2012 calibrate  $\beta = 0.88$ ), it remains low relative to the values typically assigned in quarterly business cycle models. This has two consequences for the simulation of our model and its quantitative interpretation. On the one hand, since the sovereign government is relatively patient, its frontloading motive is weaker than in most comparable papers concerned with sovereign default. Hence, the accumulation of debt remains moderate and the model generates relatively few fundamental default episodes. In order to match the empirically observed interest rate spreads, rollover risk driven by sunspots must fill the gap. We see this as an interesting and empirically

relevant feature of our model, which has important implications for the optimal conduct of fiscal policy. On the other hand, annual discount rates implied by  $\beta$  are still very high at around 28%. This seems hard to justify simply on the grounds of politico-economic distortions. The meaningful analysis of the trade-offs shaping optimal fiscal policy hence arguably calls for consideration of environments with a less dominant frontloading motive. Therefore, we complement our baseline scenario with  $\beta = 0.94$  by alternative parameterizations with higher values for the discount factor at  $\beta = 0.96$  and  $\beta = 0.98$ .

## 5 Results

In this section we present the results of our quantitative analysis. We start by establishing the existence of a *crisis zone*, that is, a region of the state space where self-fulfilling debt crises are possible, and examine the implications for the pricing of sovereign debt. We then move on to discuss the business cycle implications of our model and show that optimal fiscal policy is generally procyclical. Finally, we show that there are infrequent episodes of severely countercyclical fiscal policy, which occur when the economy moves into our out of the crisis zone, and we examine in detail the economic conditions underlying these *regime shift* events.

### 5.1 Crisis zone and bond prices

As in Cole and Kehoe (2000), the state space  $(b, \varepsilon)$  in our model can be partitioned into three zones: the *safe zone* where the government always prefers to honor its maturing debt, the *default zone* where the government always prefers default, and the *crisis zone* where there is scope for self-fulfilling debt crises. Figure 1 visualizes the different zones for the case when  $\beta = 0.94$ . The crisis threshold moves from about 4.5% to about 5.3% of annual GDP when TFP ranges within two standard deviations of its mean. When debt exceeds this threshold, self-fulfilling sovereign debt crises may occur depending on the realization of the sunspot variable  $\xi$ . The default threshold is in the order of magnitude of 9% of annual GDP. Interestingly, the default threshold expressed in terms of a debt-to-GDP ratio is (slightly) non-monotonic in TFP. This is because in low TFP states the direct effect of increasing productivity on GDP is stronger than its effect on sustainable debt. Maybe more importantly, Figure 1 illustrates that the crisis

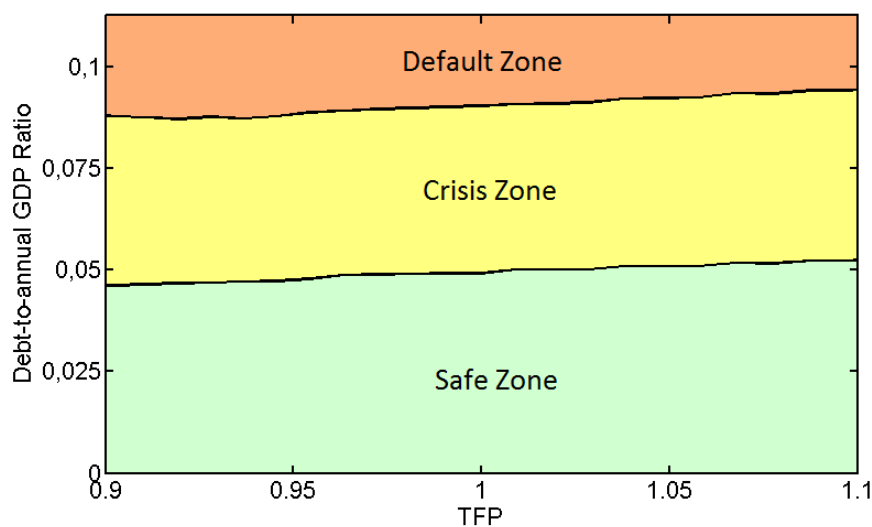


Figure 1: Crisis and default zones under optimal fiscal policy ( $\beta = 0.94$ )

zone in our model is pervasive, so that rollover risk is a concern over a large part of the state space. This has important implications for the behavior of bond prices. Figure 2 plots the bond pricing function (30) against the level of debt when  $\varepsilon = 1$ . As seen, there are two regions where

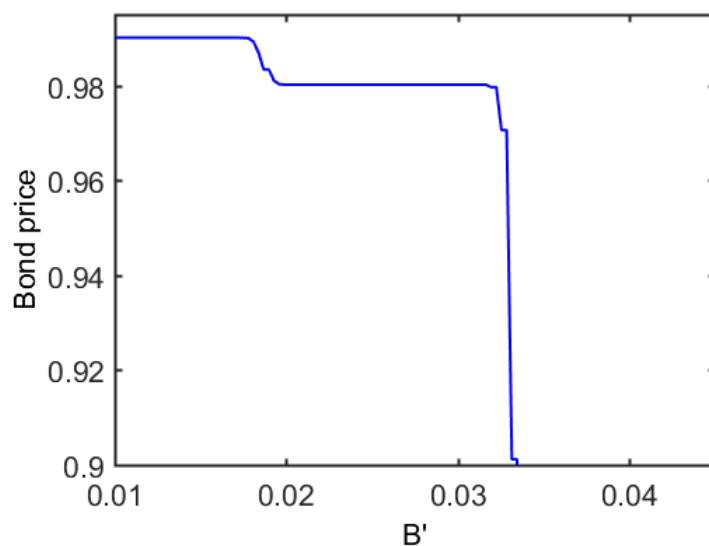


Figure 2: Bond pricing function

bond prices display a strong sensitivity to the amount of debt issued. The first drop in bond prices occurs when the economy approaches the crisis zone and default risk rises from zero to



$p$ . The second drop arises due to the surge in default risk when the country approaches the default zone.<sup>13</sup> This begs the question of how rollover risk is addressed by optimal fiscal policy. Underlying the properties of optimal fiscal policy is the trade-off between the government's motives to frontload and to smooth consumption. The answer to the above question therefore critically depends on the relative importance of the government's frontloading motive, which is parameterized by the discount factor  $\beta$ .

## 5.2 Fiscal procyclicality

Table 3 presents business cycle statistics generated from simulations of our model and compares them to quarterly data from Mexico (1980-2009). For the benchmark model with  $\beta = 0.94$ , the average level of debt over GDP is around 8%, which is well within the crisis zone. Table 3 shows further that our benchmark model reproduces several salient features of the Mexican business cycle: both private and public consumption are more volatile than GDP, whereas taxes, interest rate spreads and the trade balance are less volatile. Note that the relative volatility of spreads is significantly lower than in the data. This is because default risk varies very little over the business cycle. In particular, fundamental default risk is very low, while rollover risk does not vary with the level of debt within the crisis zone.<sup>14</sup>

The pattern of correlations implied by our model shows that default risk is countercyclical. This induces a procyclical debt policy, which manifests itself in a countercyclical trade balance. Despite this, spreads are countercyclical and positively correlated with the trade balance. Key to this correlation pattern is the convexity of the default costs in the underlying productivity state. Following Mendoza and Yue (2012), our model generates this structure endogenously. Turning to the cyclical properties of the other fiscal instruments, we see that public spending displays a tight positive correlation with GDP, whereas tax rates display a negative correlation. This pattern is robust to changes in the discount factor  $\beta$ . Fiscal policy in our model is thus

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<sup>13</sup>Similar two-step bond pricing functions can arise also in the context of default models with political risk (Hatchondo, Martinez, and Saprizza, 2009; Scholl, 2017), when the hazard of turnover to a more default-prone government induces bond prices akin to those depicted in Figure 2.

<sup>14</sup>Recall that our calibration implies that fundamental default risk contributes only 0.27 percentage points to the average annual interest rate spread of 4.27%. Nevertheless, it is the threat of fundamental default as opposed to rollover risk that acts as the key constraint on fiscal policy; see Appendix A.2.

Table 3: Business cycle statistics

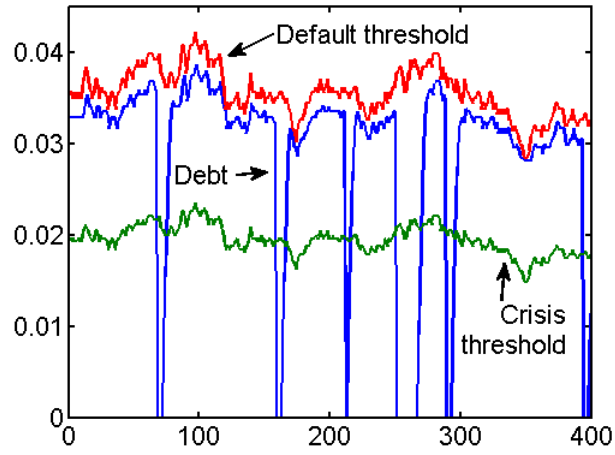
Statistic	(I)	(II)	(III)	(IV)
	Data	$\beta = 0.94$	$\beta = 0.96$	$\beta = 0.98$
<i>Standard deviation relative to standard deviation of GDP</i>				
$C$	1.20	1.28	1.30	1.69
$G$	1.08	1.39	1.44	2.01
$\tau$	0.24	0.54	0.67	1.29
$R_s$	0.82	0.04	0.09	0.01
$TB$	0.65	0.47	0.35	0.22
<i>Correlation with GDP</i>				
$C$	0.93	0.95	0.92	0.86
$G$	0.38	0.92	0.87	0.81
$\tau$	-0.33	-0.46	-0.35	-0.48
$R_s$	-0.64	-0.21	-0.23	-0.42
$TB$	-0.68	-0.47	-0.33	-0.31
$D$	-0.11	-0.13	-0.13	-0.17
<i>Correlation with spread</i>				
$C$	-0.56	-0.22	-0.21	-0.43
$G$	-0.16	-0.23	-0.20	-0.45
$TB$	0.65	0.18	0.03	0.32
Mean debt-to-annual GDP	29%	7.6%	6.8%	4.8%
Mean annual spread	4.27%	4.27%	3.25%	0.02%
Time spent in autarky	-	10.71%	8.28%	0.04%

Note: The statistics reported are computed as averages over  $N = 2000$  simulations of length  $T = 500$ , with the first 100 observations truncated. The simulated series are logged and filtered. The empirical measure for taxes is based on data for 1980-2007 and taken from Cuadra, Sanchez, and Sapriza (2010).

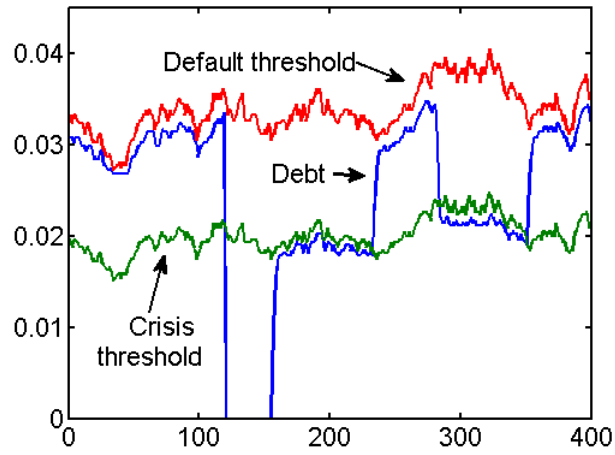
generally procyclical.<sup>15</sup>

A closer look reveals, however, that the presence of rollover risk gives rise to infrequent episodes of strongly countercyclical fiscal policy. Figure 3 shows the debt positions generated by our model specifications, together with the relevant crisis and default thresholds. The first panel considers the case when  $\beta = 0.94$  so that the government's frontloading motive is very pronounced. As seen, the simulated debt positions are generally close to the default threshold. There is a tight comovement between the two time series, which illustrates again the procyclical nature of both the government's borrowing and its incentives to repay. However,

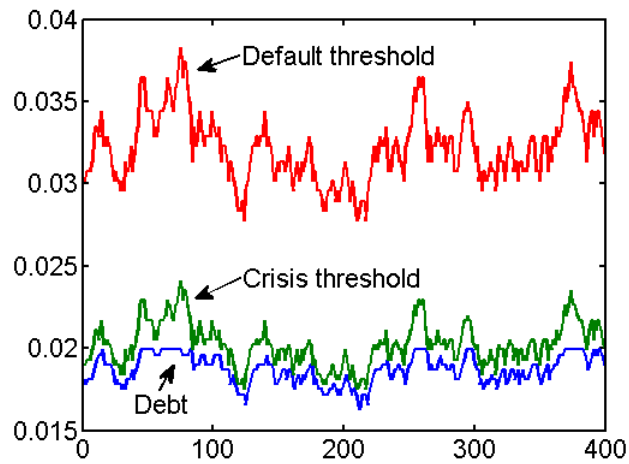
<sup>15</sup>In this respect, our findings confirm the results in Cuadra, Sanchez, and Sapriza (2010), though for an environment with rollover risk and endogenous default costs. Notice also that variations in  $\beta$  actually have a non-monotonic effect on the degree of procyclicality in taxation; this can be explained through the changing incidence of market exclusion.



(a)  $\beta = 0.94$



(b)  $\beta = 0.96$



(c)  $\beta = 0.98$

Figure 3: Debt, crisis threshold and default threshold for varying discount factor

in rare instances, self-fulfilling rollover crises or strong contractions in productivity and GDP trigger episodes of sovereign default, which lead to periods of market exclusion. But once the government regains access to international capital markets, debt positions are fast to reach the vicinity of the default threshold again. Moreover, under the optimal fiscal policy, there is no attempt to escape the crisis zone.

By contrast, the last panel examines the situation for  $\beta = 0.98$ , implying a relatively weak frontloading motive. Debt positions are again procyclical but now remain below the crisis zone. This is because default risk and bond prices would jump discontinuously if the country were to enter the crisis zone. The government thus limits its borrowing and eliminates default risk almost completely.<sup>16</sup>

The most interesting dynamics arise in the intermediate case when  $\beta = 0.96$ . Over the simulation, debt positions actually range from the safe zone to the crisis zone and even the default zone. Hence, we observe sovereign defaults as well as endogenous regime switches where the government enters or escapes the crisis zone. Notably, though, under an intermediate degree of impatience, debt displays considerable persistence both in the safe zone and in the crisis zone: Below the crisis threshold, the government will *normally* limit borrowing in order to prevent rollover risk – similar to panel (c). But above the crisis threshold, it will *normally* not be willing to shoulder the cost of decumulating debt; instead, since the degree of rollover risk does not vary within the crisis zone, debt levels converge close to the default threshold – similar to panel (a). Occasionally, however, the government finds it worthwhile to engineer a transition into or out of the crisis zone. The next section looks at the economic conditions triggering these transition events and the associated dynamics of fiscal policy in greater detail.

### 5.3 Countercyclical fiscal expansions and contractions

It is instructive to assess the transition dynamics through the lense of the optimality condition presented in Proposition 3. Presuming differentiability (again, this is just done to ease interpretation) and defining  $\epsilon_q(b') = \frac{\partial q}{\partial b'} \frac{b'}{q} \leq 0$  as the elasticity of bond prices with respect to debt

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<sup>16</sup>Some minimal default risk remains due to the small hazard of a transition into the crisis zone.

issuance, this condition can be rewritten as

$$u_g \{1 + \epsilon_q(b')\} = \frac{\beta}{q} E_{\xi'} E_{\varepsilon' \notin \Gamma(b', \xi')} u'_g. \quad (33)$$

Accordingly, the intertemporal smoothing of the marginal utility from public spending is complicated by three potential factors. On the right-hand side of (33), both the government's relative impatience,  $\beta/q \approx \beta(1 + r^*) < 1$ , and the insurance value implied by its future default option,  $E_{\xi'} E_{\varepsilon' \notin \Gamma(b', \xi')} u'_g \leq E_{\xi'} E_{\varepsilon'} u'_g$ , provide incentives for higher current spending (lower  $u_g$ ). Such policy comes at the cost of increased debt and consequently lower bond prices,  $\epsilon_q(b') \leq 0$ . Starting in the safe zone, these forces can dominate the government's precautionary motive and instead make it worthwhile to risk a transition into the crisis zone. Different from the default frontier, bond prices at the crisis threshold are locally very sensitive to the level of debt, but then flatten out and do not fall to zero (see Figure 2). This is because the crisis risk is small ( $p = 0.01$ ) and invariant to the level of debt. As a consequence, the effect on bond prices remains limited so that the joint effect of impatience and the option value of default can render a probabilistic (namely after a sufficiently bad productivity shock) transition into the crisis zone optimal. A similar logic applies to transitions out of the crisis zone, although the government's relative impatience makes them less likely. In this light, Figures 4 and 5 illustrate the model's prescriptions for macroeconomic dynamics, and in particular fiscal policy, during such transition episodes. They present simulated time-series data for an event window of eight quarters before and after transition events recorded in the model simulations.<sup>17</sup>

Figure 4 shows that transitions *into* the crisis zone are driven by a sequence of bad productivity draws. In response to that, and despite fiscal effort in the form of increased taxation and cuts in public spending to contain an excessive accumulation of debt, the debt-to-GDP ratio increases and the crisis threshold declines, so that the economy approaches the crisis zone. At some point (normalized as  $t = 0$ ), staying out of the crisis zone becomes too costly as formalized by (33). The government then terminates its procyclical policy and implements a large debt-financed fiscal expansion, reducing taxes and increasing public spending significantly. This policy has a positive effect on contemporaneous output, but comes at the cost of pushing the

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<sup>17</sup>Since endogenous transitions are most likely to occur when the government's frontloading motive is intermediate, the Figures examine the case when  $\beta = 0.96$ .

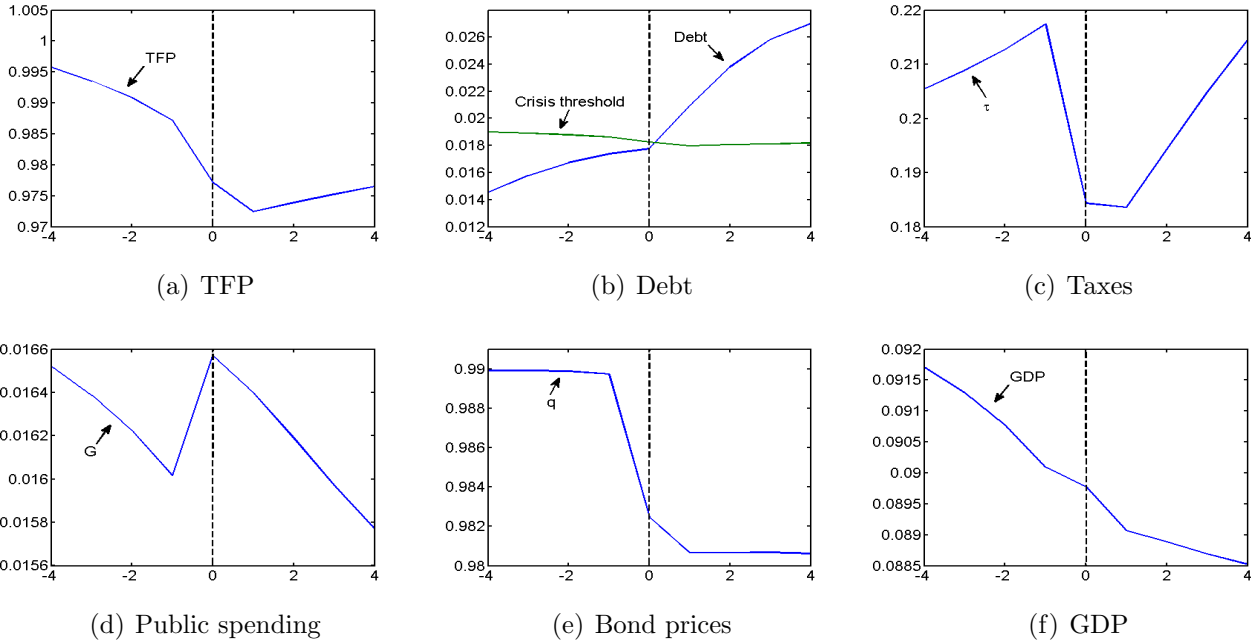


Figure 4: Fiscal policy when entering the crisis zone ( $\beta = 0.96$ )

economy into the crisis zone as reflected by depressed bond prices. As productivity recovers, the economy moves to its new normal within the crisis zone, with a significantly higher level of debt and, accordingly, a lower level of government spending and higher tax rates compared to the initial situation.

Figure 5 examines fiscal policy when the economy *leaves* the crisis zone. The vertical dashed lines indicate two points in time, the actual transition event from the crisis zone to the safe zone (normalized as  $t = 0$ ) and the reversal in debt-to-GDP positions when this transition is initiated (on average a year earlier in  $t = -4$ ). During booms, public spending and the level of debt initially rise in a procyclical fashion. Given its frontloading motive and the procyclical evolution of the default frontier, the government makes use of its fiscal space and issues additional debt. Eventually, however, the accumulated debt burden is so high that the implied decrease in expected future consumption is sufficiently strong to induce a reversal in the government's trade-off between frontloading and smoothing consumption. The government then finds it optimal to implement a severe fiscal contraction in order to escape the crisis zone and to create fiscal space for the future. The associated fiscal tightening involves both a substantial hike in tax rates and cuts in public spending. This induces a fast consolidation of

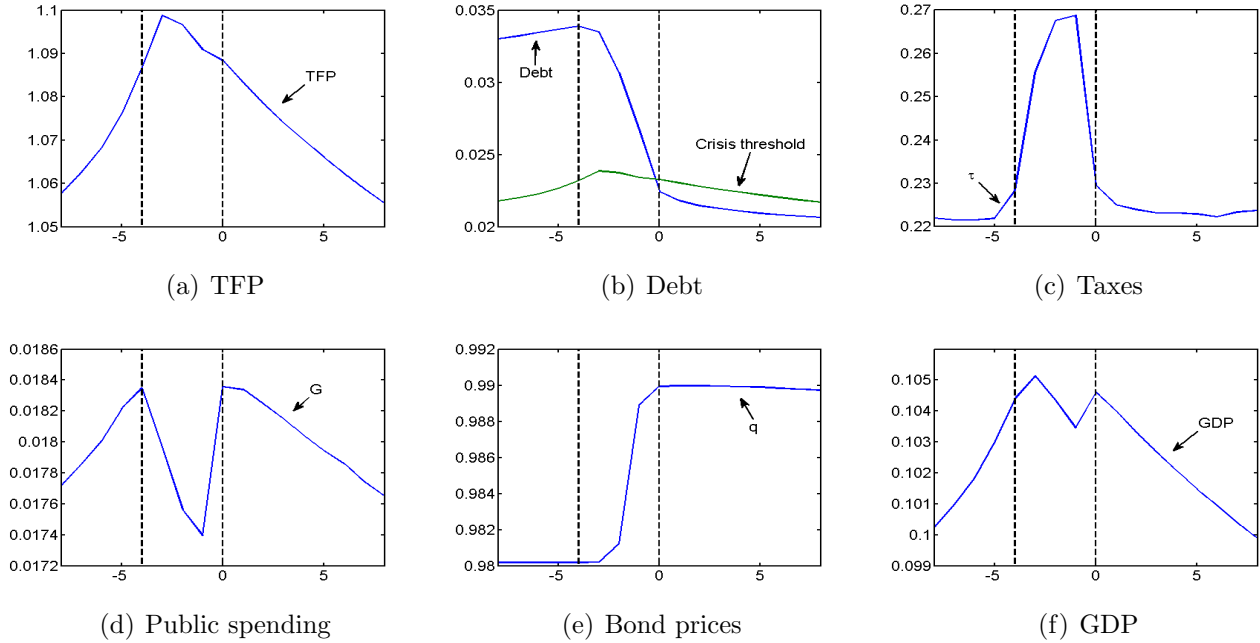


Figure 5: Fiscal policy when leaving the crisis zone ( $\beta = 0.96$ )

debt, which is driven below the crisis threshold within an average transition period of only four quarters.

Another important take-away is that consolidations to leave the crisis zone occur only during exceptional boom episodes, when productivity and output are significantly above trend. Indeed, the debt reversal at  $t = -4$  occurs when TFP is almost two standard deviations above average. This is remarkable insofar as movements into the crisis zone happen already during relatively mild recessions (cf. Figure 4, where the regime switch occurs when TFP is just about half a standard deviation below average). There is thus a fundamental asymmetry, which distinguishes transition events into and out of the crisis zone. This asymmetry can be explained with reference to condition (33), whose right-hand-side contains two factors implying a tendency towards debt accumulation rather than decumulation: the government's relative impatience and the discounting of the expected future marginal utility via the option value of default.

## 5.4 Empirical implications

We conclude this section by pointing out that the normative implications regarding optimal fiscal policy during transition episodes, that is, (i) the existence of large, fast-paced and countercyclical swings in government debt, and (ii) the asymmetry in the productivity trigger underlying these events, are supported by empirical evidence. Abbas, Belhocine, El-Ganainy, and Horton (2011) examine historical debt dynamics in advanced and emerging economies (1870-2007) and decompose 128 identified episodes of large debt increases and decreases into their respective budgetary determinants.<sup>18</sup> They document that strong growth is a consistent feature of most pronounced debt consolidations; (peacetime) debt buildups tend to be driven by weak growth or recessions, but in comparison to debt reductions the picture is somewhat less clear-cut. Another takeaway is that the documented *large* debt swings appear hard to rationalize as the cyclical response to underlying productivity shocks. Instead, our model, where – conditional on being in the crisis zone – rollover risk is not tied to fundamentals and thus constant, offers a plausible explanation: When fiscal policy is subject to a frontloading motive, the optimal policy exploits the fact that bond prices remain locally invariant to the level of liabilities and brings debt close to the default frontier. And conversely, successful fiscal consolidations must bring down the stock of debt sufficiently to escape the crisis zone.

## 6 Conclusions

This paper integrates rollover risk into a model of sovereign debt sustained by endogenous default costs. We use this framework to study how sovereign risk – both fundamental and self-fulfilling – shapes the government’s optimal debt policy and the cyclical behavior of taxes and public spending. Central to the fiscal policy problem is the trade-off between the government’s motives to frontload and to smooth consumption. Optimal taxes and spending are generally procyclical, but the incidence of rollover risk gives rise to infrequent episodes of severely countercyclical fiscal activity. These regime changes occur endogenously when the economy enters or

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<sup>18</sup>Even though Abbas, Belhocine, El-Ganainy, and Horton (2011) restrict their decomposition exercise to advanced economies, their data shows that large debt increases and decreases are common also in emerging economies and low-income countries. Notice also that debt reductions are engineered through default only in a small number (7) of the recorded cases (68).



leaves the crisis zone. Transitions into the crisis zone occur following relatively mild recessions that are sustained enough to make the government adopt expansionary measures in the form of tax cuts, an increase in spending and the issuance of public debt, although this increases future borrowing costs. By contrast, transitions out of the crisis zone happen only during exceptional boom periods, which are exploited by the government as an opportunity to decumulate debt in order to eliminate rollover risk and reduce future borrowing costs. In normal times, however, it is too costly for the government to escape the crisis zone, so that debt displays substantial persistence and is issued in a procyclical fashion.

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# A Appendix

## A.1 Optimal fiscal policy

*Proof of Proposition 1.* We prove the converse statement that  $\varepsilon \notin \Gamma(b, \xi)$  implies  $\varepsilon \notin \Gamma(\tilde{b}, \xi)$ . Let  $(c, L, g)$  denote the equilibrium values for consumption, labor supply and public expenditure under the optimal fiscal policy  $(d = 0, \tau, g, b')$  when debt is equal to  $b$ . Now consider the situation when debt is equal to  $\tilde{b} < b$  and suppose the government policy remains unchanged at  $(d = 0, \tau, g, b')$ . From (30), bond prices are then unchanged; and since the private-sector equilibrium is completely determined by  $(d = 0, \tau; \varepsilon)$ , the resulting allocation then has  $\tilde{c} = c$  and  $\tilde{L} = L$ . Hence, since  $\tilde{b} < b$ , the government budget constraint (21) implies that it is possible to increase public spending to  $\tilde{g} > g$ . Therefore, under the (possibly suboptimal) policy of keeping fiscal policy unchanged bar the residual adjustment of  $\tilde{g} > g$ , we have  $u(\tilde{c} - v(\tilde{L}), \tilde{g}) > u(c - v(L), g)$  and  $\tilde{b}' = b'$ . Hence, from (22),

$$\mathcal{V}^{nd}(\tilde{b}, \varepsilon) > \mathcal{V}^{nd}(b, \varepsilon).$$

Moreover, the default value  $\mathcal{V}^d(\varepsilon)$  is independent of the endogenous state  $b$ . By assumption,  $\varepsilon \notin \Gamma(b, \xi)$ , that is,

$$\mathcal{V}^{nd}(b, \varepsilon) \geq \mathcal{V}^d(\varepsilon).$$

It then follows from the last two inequalities that  $\varepsilon \notin \Gamma(\tilde{b}, \xi)$ . □

*Proof of Proposition 2.* The optimal tax policy must satisfy

$$u_c \frac{\partial c}{\partial \tau} - u_l \frac{\partial L}{\partial \tau} + u_g \frac{\partial g}{\partial \tau} = 0.$$

When the government has access to credit markets and  $b' > 0$ , its budget constraint (21) implies

$$g = \tau c + q(b', \varepsilon)b' - b,$$

whereby the bond price is determined by (30) and thus unaffected by variations in  $\tau$ . Moreover,

with GHH preferences as in (1), labor supply is independent of consumption and the optimality condition for consumption-leisure (3) implies

$$\frac{u_l}{u_c} = v'(L) = \frac{w}{1 + \tau}.$$

Finally, the household budget constraint (2) and the aggregation of income underlying (20) imply

$$c = \frac{1}{1 + \tau} [wL + \pi^f + \pi^m] = \frac{1}{1 + \tau} [y - P^*(r^*)m^*] = \frac{1}{1 + \tau} gdp.$$

Using the above three equations, the optimality condition for taxes becomes

$$0 = u_c \left[ \frac{\frac{\partial gdp}{\partial \tau}(1 + \tau) - gdp}{(1 + \tau)^2} \right] - u_c v'(L) \frac{\partial L}{\partial \tau} + u_g \left[ \frac{(gdp + \tau \frac{\partial gdp}{\partial \tau})(1 + \tau) - \tau gdp}{(1 + \tau)^2} \right],$$

or equivalently,

$$u_c \left\{ -\frac{\partial \frac{1}{1+\tau}}{\partial \tau} gdp - \frac{1}{1 + \tau} \frac{\partial gdp}{\partial \tau} + v'(L) \frac{\partial L}{\partial \tau} \right\} = u_g \left\{ -\frac{\partial \frac{1}{1+\tau}}{\partial \tau} gdp + \frac{\tau}{1 + \tau} \frac{\partial gdp}{\partial \tau} \right\}.$$

From the definition of GDP in (20),  $gdp = y - P^*(r^*)m^*$ , and since factors earn their marginal products, while the price  $p_j^*$  for imported inputs is exogenous, we have

$$\frac{\partial gdp}{\partial \tau} = \frac{\partial gdp}{\partial L} \frac{\partial L}{\partial \tau} + \frac{\partial gdp}{\partial m^*} \frac{\partial m^*}{\partial \tau} = w \frac{\partial L}{\partial \tau} + [P^*(r^*) - P^*(r^*)] \frac{\partial m^*}{\partial \tau} = w \frac{\partial L}{\partial \tau}.$$

Hence, the optimality condition for consumption-leisure (3) implies

$$-\frac{1}{1 + \tau} \frac{\partial gdp}{\partial \tau} + v'(L) \frac{\partial L}{\partial \tau} = \left[ -\frac{w}{1 + \tau} \frac{\partial gdp}{\partial \tau} + v'(L) \right] \frac{\partial L}{\partial \tau} = 0,$$

so that the the optimality condition for taxes becomes

$$u_c \left\{ -\frac{\partial \frac{1}{1+\tau}}{\partial \tau} gdp \right\} = u_g \left\{ -\frac{\partial \frac{1}{1+\tau}}{\partial \tau} gdp + \frac{\tau}{1 + \tau} \frac{\partial gdp}{\partial \tau} \right\}.$$

It is then immediate to verify that this condition implies  $u_c < u_g$ . □

*Proof of Proposition 3.* Observe first that the multiplier attached to the government budget constraint (21) is given by  $u_g > 0$ . Recall also (see proof of Proposition 2) that

$$c = \frac{1}{1 + \tau} gdp,$$

$$g = \tau c + q(b', \varepsilon) b' - b.$$

The optimality condition for  $b' > 0$  is then given by

$$u_g \left\{ \frac{\tau}{1 + \tau} \frac{\partial gdp}{\partial b'} + q + \frac{\partial q}{\partial b'} b' \right\} + \beta E \mathcal{V}_b(b', \varepsilon', \xi') = 0,$$

where  $\frac{\partial gdp}{\partial b'} = 0$  because, given the static nature of the private-sector equilibrium, the endogenous variable  $gdp$  is not affected by debt issuance. Since  $\mathcal{V}^d(\varepsilon)$ , the value function following default (no matter whether it occurred for fundamental or self-fulfilling reasons), does not depend on  $b$ , the envelope condition is

$$\mathcal{V}_b(b, \varepsilon) = \begin{cases} 0, & \text{if } \varepsilon \in \Gamma(b, \xi), \\ \mathcal{V}_b^{nd}(b, \varepsilon), & \text{if } \varepsilon \notin \Gamma(b, \xi), \end{cases}$$

where

$$\mathcal{V}_b^{nd}(b, \varepsilon) = u_c \left\{ \frac{1}{1 + \tau} \frac{\partial gdp}{\partial b} \right\} - u_c v'(L) \frac{\partial L}{\partial b} + u_g \left\{ \frac{\tau}{1 + \tau} \frac{\partial gdp}{\partial b} - 1 \right\}.$$

Since the level of maturing debt  $b$  affects the endogenous variables  $L$  and  $gdp$  not directly, but only through its impact on the government's fiscal control variables, we have  $\frac{\partial L}{\partial b} = \frac{\partial gdp}{\partial b} = 0$ , and the envelope condition further simplifies to  $\mathcal{V}_b^{nd}(b, \varepsilon) = -u_g$ . The generalized Euler equation for  $b'$  is then obtained as

$$u_g \left\{ q + \frac{\partial q}{\partial b'} b' \right\} = \beta E_{\xi'} E_{\varepsilon' \notin \Gamma(b', \xi')} u'_g.$$

From (30), bond prices are given by

$$q(b', \varepsilon) = \frac{1 - \rho(b', \varepsilon)}{1 + r^*} = E_{\xi'} E_{\varepsilon' \notin \Gamma(b', \xi')} \frac{1}{1 + r^*}.$$

The generalized Euler equation can therefore be rewritten as

$$u_g \{1 + \epsilon_q(b')\} = \beta(1 + r^*) E_{\xi'} E_{\xi' \notin \Gamma(b', \xi')} u'_g,$$

where  $\epsilon_q(b') = \frac{\partial q}{\partial b'} \frac{b'}{q}$  denotes the elasticity of bond prices with respect to debt issuance.  $\square$

## A.2 Business cycle statistics without rollover risk

Table 4 presents business cycle statistics for our benchmark model ( $\beta = 0.94$ ) with and without rollover risk. As expected, the model without rollover risk generates lower spreads, higher debt

Table 4: Business cycle statistics with and without rollover risk ( $\beta = 0.94$ )

Statistic	(I)	(II)	(III)
	Data	$p = 0.01$	$p = 0.00$
<i>Standard deviation relative to standard deviation of GDP</i>			
$C$	1.20	1.28	1.16
$G$	1.08	1.39	1.23
$\tau$	0.24	0.54	0.34
$R_s$	0.82	0.04	0.04
$TB$	0.65	0.47	0.29
<i>Correlation with GDP</i>			
$C$	0.93	0.95	0.98
$G$	0.38	0.92	0.96
$\tau$	-0.33	-0.46	-0.49
$R_s$	-0.64	-0.21	-0.21
$TB$	-0.68	-0.47	-0.48
$D$	-0.11	-0.13	-0.19
<i>Correlation with spread</i>			
$C$	-0.56	-0.22	-0.23
$G$	-0.16	-0.23	-0.23
$TB$	0.65	0.18	0.21
Mean debt-to-annual GDP	29%	7.6%	10.3%
Mean annual spread	4.27%	4.27%	0.14%

Note: The statistics reported are computed as averages over  $N = 2000$  simulations of length  $T = 500$ , with the first 100 observations truncated. The simulated series are logged and filtered. The empirical measure for taxes is based on data for 1980-2007 and taken from Cuadra, Sanchez, and Saprizza (2010).

positions and lower relative volatilities. But the various correlations, and particularly those



for the fiscal instruments, are basically unchanged. To understand this, observe that, when  $\beta = 0.94$ , the level of debt is relatively close to the default threshold. Fundamentals thus place the economy within the crisis zone for most of the time. Within the crisis zone, however, rollover risk is independent of the level of debt. Accordingly, bond prices are locally sensitive to variations in fundamental default risk only, and – despite being relatively small – this shapes the cyclical properties of the optimal policies for debt issuance, taxation and spending.

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