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Quantile debt fan charts

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QUANTILE DEBT FAN CHARTS

Suzette Daggi, Paul Mariano, and Arjan Paulo Salvanera

NO. 664

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June 2022

**ADB ECONOMICS
WORKING PAPER SERIES**

ADB Economics Working Paper Series

Quantile Debt Fan Charts

Suzette Dagle, Paul Mariano, and Arjan Paulo Salvanera

No. 664 | June 2022

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ABSTRACT

The paper applies quantile regression technique, specifically, quantile vector autoregression to stochastic debt sustainability analysis (DSA) and the construction of public debt fan charts. Stochastic approach to DSA typically uses standard ordinary least squares vector autoregression (OLS VAR) and “fan charts” to depict the upside and downside risks surrounding public debt projections due to uncertain macroeconomic conditions. These VAR models rely on constant coefficients and random variables that are independent and identically distributed. However, empirical evidence suggests that macroeconomic variables are characterized by nonlinearities and asymmetries that linear regression models, such as OLS VAR, may not capture. Many attempt to show how such nonlinearities can be accounted for by using quantile regression methods. Quantile regression allows for varying parameters for each quantile and facilitates the analysis of asymmetric dynamics. It is also a natural environment for stress testing exercises by estimating the reaction of the endogenous variable to tail shocks or future quantile realizations.

Keywords: debt, quantile regression, fan charts

JEL codes: H63, H68, C31

I. INTRODUCTION

Fan chart analysis is one of the most common stochastic approaches to public debt sustainability analysis (DSA). It has proven to have the broadest applicability and data parsimony compared to other stochastic approaches (IMF and World Bank 2012, IMF 2013). Fan charts use historical information about the variations and correlation patterns among debt ratio determinants. They simulate a large number of random shocks, each representing a possible debt path and together fanning out as a bundle or fan. The pattern and shape of such fan charts convey information about the expected evolution of debt under uncertainty. Debt fan charts are featured regularly in the International Monetary Fund (IMF) and World Bank's standard DSA framework for both low-income and market-access countries (IMF 2021a).

Various methodologies have been proposed in the literature to construct debt fan charts in stochastic DSA. One approach is to devise a small number of standardized scenarios, where shocks are expressed as a proportion of the historical standard deviations of the variables (IMF 2003). Another approach, as proposed by Hostland and Karam (2005), sets up a stochastic simulation model involving reduced form equations for aggregate demand and supply and inflation process, with the parameters calibrated based on historical averages of a panel of comparable economies. But using unrestricted ordinary least squares (OLS) vector autoregression (VAR) has become the most common approach to constructing stochastic DSA fan charts. The correlation pattern calculated from the mean, variance, and covariance of the variables linked to the debt, such as real gross domestic product (GDP) growth, inflation, exchange rate, and nominal interest rates, is used to generate many random simulations. Here, the frequency distribution of the debt ratio is computed for each year in the forecast horizon. These are then mapped like a "fan" around the central projection, with the effects of the shocks multiplying over time and expanding the range of possible debt ratios (Garcia and Rigobon 2004; Celasun, Debrun, and Ostry 2006; Celasun and Keim 2010).

While debt fan chart analyses in existing literature rely mostly on standard OLS VAR, this paper proposes the use of quantile vector autoregression (VARQ) for the construction of debt fan charts. These are made possible by recent advancements within the econometric literature (White, Kim, and Manganelli 2015; Chavleishvili and Manganelli 2019; Montes-Rojas 2019). Quantile regression was introduced in 1978 by Koenker and Bassett as a semiparametric technique that allows covariates to affect different parts of the distribution, such that estimated coefficients may vary across quantiles. It models the relationship between the dependent variable and specific quantiles of independent variables using their conditional median. The dependent variable is assumed to be independently distributed and homoscedastic. Extreme sensitivity to outliers made the OLS regression a poor estimation technique in many non-Gaussian, long-tailed scenarios (Koenker and Bassett 1978). On the other hand, quantile regressions generate favorable results, producing robust estimation over a broad class of non-Gaussian error distribution. It is also important to note that quantile regression does not partition the data, and it uses all observations to estimate the parameters for every quantile.

Quantile regression methodology has recently found its way into the growth-at-risk framework, linking macrofinancial conditions to the distribution of future growth forecasts (Prasad et al. 2021). This is motivated by a wealth of empirical evidence that supports the view that macroeconomic variables are characterized by nonlinearities and asymmetries that linear regression models—such as OLS VAR—may not capture. Many papers show how such nonlinearities can be accounted for by using quantile regression methods instead. Unlike VAR-based methods, quantile regression does not rely on normality assumptions and the mean or average expected behavior of

variables. Here, asymmetric distribution is permissible, and coefficients are computed based on the median of each quantile of variables.

This paper uses quantile VAR to derive country-level forecasts of nonfiscal determinants of debt (GDP growth, inflation, nominal interest rate, and exchange rate) and panel quantile fiscal reaction function to derive the fiscal determinant or the primary balance. Quantile forecasts are generated for numerous combinations of the 10th, 20th, 50th, 80th, and 90th quantiles of the four nonfiscal variables and the primary balance for each country. Following a standard public debt dynamics equation, the forecasts are combined to project a wide range of debt-to-GDP ratios for the years 2021 to 2025. The resulting debt ratios are depicted in the fan charts for each country. All in all, nine developing member countries (DMCs) of the Asian Development Bank (ADB) are included in this paper which are selected based on the availability of quarterly data that are sufficient for the analysis. Several DMCs do not meet this requirement as either they do not have quarterly data or their quarterly series start much later. The nine DMCs included in this paper are India, Indonesia, Kazakhstan, Malaysia, the People's Republic of China (PRC), the Philippines, the Republic of Korea, Sri Lanka, and Thailand, with data spanning from the first quarter (Q1) of 1990 to Q1 2021.

The quantile fan charts in this paper reflect a broad range of equally likely pathways of debt given the prevailing historical trend and relationships of the macroeconomic variables in an economy. Quantile regression is a natural environment for stress testing as it estimates coefficients that are specific for quantiles, such that a combination of variables at their tail quantiles is, in effect, a shock scenario. This paper finds that this feature is extremely convenient in the context of stochastic DSA and fan charts—where a shock scenario is essentially one of the numerous permutations of quantile regression estimates, when one or all of the variables are in their tail quantiles, e.g., the 10th and the 90th quantiles. The use of quantile regression approach also expediently produces an array of debt paths showcased in the fan charts from combinations of variables at various quantiles without relying on correlation patterns to generate simulations. This paper also demonstrates that applying the quantile regression approach to stochastic DSA produces results that overcome the presence of outliers in macroeconomic determinants of debt. By using quantile VAR as an alternative, the estimated debt-to-GDP ratios presented in fan charts are more robust to outliers, unlike in the OLS VAR approach.

The results show that the quantile fan charts have median forecasts that are lower than the forecasts generated by the deterministic approach. Assumptions play a key role in the deterministic forecasts, such as possible new loans in the short term and deterioration of macroeconomic variables. It can be also observed from the results that the median projections have a downward trend due to the strong response of the primary balance to increasing public debt-to-GDP ratio at the median relative to other quantiles, as exhibited through an estimated fiscal reaction function. Using the latest primary balance forecasts from IMF World Economic Outlook produced median pathways in all DMCs that are less downward sloping and closer to the deterministic forecasts. Nevertheless, the shape of the fan cones remained almost the same. Across DMCs, this paper finds large variations in the fan cone widths or the range of debt pathways at the last year of the projection. Some DMCs, such as Sri Lanka, have large volatility in the determinants of debt, thus widening the range of debt projections, unlike others, such as Indonesia, with fan cone widths that are narrower and more stable.

II. BUILDING QUANTILE FAN CHARTS

Fan charts are constructed using forecast trajectories at various scenarios over a 5-year forecast horizon. Debt forecasts are produced using the standard debt dynamics equation, which relates the public debt-to-GDP ratio (d_t) to changes in real GDP growth rate (g_t), implicit real interest rates on domestic debt and foreign debt (r_t^d and r_t^f), shares of domestic currency debt and foreign currency debt (α_{t-1}^d and α_{t-1}^f), depreciation of the real exchange rate ($\Delta\epsilon_t$, where ϵ_t is expressed as local currency units per United States [US] dollar), foreign currency debt (d_{t-1}^f), the primary balance (% of GDP) (pb_t), and other factors creating debt in vector Z_t such as privatization receipts, contingent liabilities, and debt relief. The debt dynamics equation is expressed as:

$$d_t = \frac{1}{(1+g_t)} [d_{t-1} + (r_t^d \alpha_{t-1}^d + r_t^f \alpha_{t-1}^f) d_{t-1} + \Delta\epsilon_t (1 + r_t^f) d_{t-1}^f] - pb_t + Z_t \quad (1)$$

This equation requires forecasts for the nonfiscal and fiscal determinants of debt. To forecast the nonfiscal determinants of debt, this paper uses the VARQ approach. On the other hand, a fiscal reaction function estimated using the panel quantile approach is used to forecast the fiscal determinant or the primary balance.

As a first step, this paper estimates a VARQ model that applies the reduced form vector directional quantile (VDQ) model to an autoregressive context. The technical details are in Appendix (Quantile Regression Methodology). Following Montes-Rojas' (2019) approach, the relationship of the macroeconomic determinants of debt, which are real GDP growth rate, nominal interest rate, inflation rate, and real exchange rate, is thus expressed by the model:

$$Q(\tau|x_{t-1}) = B(\tau)x_{t-1} + A(\tau) \quad (2)$$

where Q and x are vectors that correspond to the multivariate quantiles of the macroeconomic variables, $B(\tau) = (B_1(\tau), \dots, B_n(\tau))^T$ is a matrix of coefficients at quantile τ , and $A(\tau)$ is a vector of constant coefficients at quantile τ . This equation describes how the endogenous variables are conditional on their past and how they simultaneously respond with other variables.

Note that this VARQ model is built for stationary processes, but unit root can be present in some quantiles while stationarity in others (Koenker and Xiao 2004). Unit root processes can be identified by the dynamic behavior of the variables.

In this paper, the quarterly VARQ forecasts are generated for numerous combinations of the 10th, 20th, 50th, 80th, and 90th percentiles of the four nonfiscal variables, or equivalent to 625 equations for each country. Five quantiles sufficiently represent the distribution, including outliers, but a different number or combination of quantiles could be applied to this analysis. To simplify the estimations, this paper fixes the quantile of a variable throughout the forecast horizon instead of projecting a variable with varying quantiles for each quarter. This approach minimizes the number of iterations but still allows for a full range of possible debt paths from nonfiscal determinants of debt, especially from the tails.

The second building block of the debt dynamics equation is the primary balance. A negative primary balance or deficit adds to the debt, and conversely, a positive balance or surplus reduces debt. In this paper, forecasts for the primary balance for each DMC are derived using a fiscal reaction function. It captures the role of fiscal behavior in shaping the risk profile of projected public debt ratios.

A fiscal reaction function describes how a government's primary surplus responds to changes in public debt. The theoretical foundations of fiscal reaction functions were laid out by Bohn (1998), where the relationship between primary surplus (s) and debt (d), expressed as proportions of GDP, can be written as a linear equation:

$$s_t = \rho d_{t-1} + \beta \mu_t + \varepsilon_t \quad \varepsilon \cdot (0, \sigma^2), \quad (3)$$

where μ_t pertains to the temporary factors affecting primary balance and ε_t is the error term.

The specification of the fiscal reaction function in this paper is given by:

$$pb_t = \alpha_0 + \rho d_{t-1} + ygap_t + \beta X_t + \varepsilon_t, \quad t = 1, \dots, T \quad (4)$$

where pb_t is the primary balance to GDP ratio in year t , d_{t-1} is the public debt-to-GDP ratio at $t - 1$, $ygap_t$ is the output gap (estimated as Christiano and Fitzgerald [2003] filtered GDP growth), and X_t is a vector of control variables. From this equation, the coefficient of the lagged public debt ratio, ρ , measures the fiscal reaction as the response of the primary balance to changes in the debt ratio. This coefficient must lie somewhere between 0 and 1 to achieve fiscal sustainability, as demonstrated by Bohn (1998), and a coefficient closer to one implies a stronger fiscal policy response to debt.

In the literature, fiscal reaction functions are often estimated using panel regression methods (Ogbeifun and Shobande 2020, Checherita-Westphal and Ždárek 2017, Cevic and Nanda 2020, Ferrarini and Ramayandi 2012, Everaert and Jansen 2018), except for Berti et al.'s (2016) paper, which estimates country-specific fiscal reaction functions. Although country-specific fiscal reaction functions offer some advantages over panel techniques, i.e., better capturing country specificities, they require very long time period data. For instance, Berti et al. (2016) estimate fiscal reaction functions of 13 European Union countries with data spanning from 1950 to 2013. Indeed, fiscal reaction functions estimated for a panel of countries only require shorter time period data, but they employ strict assumptions, such as the presence of country-invariant characteristics to fiscal behavior that are only partly mitigated by adding country fixed effects.¹

Given the limited data availability in each DMC, this paper employs a panel approach in estimating equation (3) using quantile regression with fixed effects. Similar to other studies, the challenge in estimating the fiscal reaction function is the lack of a long time series data in most countries. Instead, this paper uses annual data for a panel of countries. However, applying quantile methods with panel data is not straightforward because standard demeaning or differencing is not feasible in nonlinear models such as in quantile regression. This paper adopts the approach by Canay (2011) to combine panel estimation and quantile regression by applying a simple transformation of the data to remove fixed effects and produce estimators that are consistent and asymptotically normal when both n and T approach infinity. In this approach, fixed effects are considered location shifts and thus have constant coefficients across quantiles, τ .²

¹ Weichenrieder and Zimmer (2015) show that panel data results can be sensitive to the inclusion or exclusion of countries.

² While this paper acknowledges that fixed effects can have varying coefficients across quantiles and that the assumption of the approach adopted limits the kind of unobserved heterogeneity the model can handle, the results of this paper are expected to be in line with existing approaches to estimating fiscal reaction function, given appropriate controls for country heterogeneity.

The panel quantile fiscal reaction function is estimated at the 10th, 20th, 50th, 80th, and 90th quantiles, and the primary surplus is forecasted annually for these quantiles and for each country. These quantiles are selected to match the quantiles of the nonfiscal determinants forecasts derived using the quantile VAR approach. Quarterly forecasts of the nonfiscal determinants are annualized and combined with the yearly primary balance forecasts to recursively compute for the corresponding debt path using the debt dynamics equation. For each country, 3,125 forecasts of debt-ratio are generated over a 5-year horizon (2021–2025).

The quarterly data on GDP growth, inflation, nominal interest rate, and exchange rate (local currency units per US dollar) are from national sources as well as from Haver Analytics, and CEIC. Annual public debt (% of GDP) data are from the IMF World Economic Outlook Database. The selection of economies in this paper is based on the list of DMCs, considering the availability of a long time series data. The nine countries included in this paper are India, Indonesia, Kazakhstan, Malaysia, the PRC, the Philippines, the Republic of Korea, Sri Lanka, and Thailand, with data spanning from Q1 1990 to Q1 2021. For the panel fiscal reaction function estimation, annual data from 41 DMCs are used, covering the years 2014 to 2020. The detailed list of sources for each variable and DMCs is in Appendix Table A1. Movements of the nonfiscal determinants of debt across years and quarters are shown in Appendix Figure A1.

III. RESULTS AND DISCUSSION

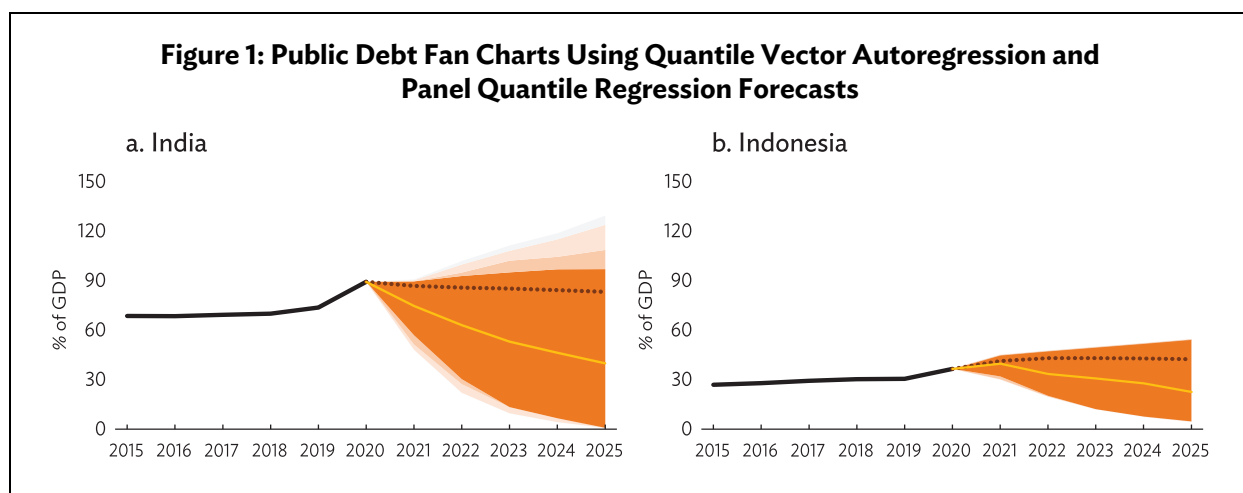
The quantile VAR estimates of the nonfiscal determinants of debt for each country reveal how they relate to each other at a particular quantile. In this paper, we observe the dynamics of the variables using two dimensional plots, with one of the variables set at the median, i.e., $\tau=50$ (see Appendix Figures A2.1 to A2.4.). These plots reveal the heterogeneity in responses of one variable at the median to changes in the other variables. For instance, in most DMCs it can be observed that when growth is at its median quantile, the coefficients of interest rate increase from one quantile to another, while the coefficients for inflation decrease. This translates to a stronger response of growth to higher interest rate, and stronger response to lower inflation. In addition, at the median interest rate, the coefficients for inflation and change in real exchange rate increase across quantiles. At median interest rate, the response is stronger at higher inflation and higher change in real exchange rate or depreciation. However, in other DMCs, there is no common pattern that can be observed with the movement of variables. In the presence of asymmetry and nonlinearity in the relationships of the variables, the choice of the quantile regression approach for the nonfiscal determinants of debt in this paper is justified.

Turning to the results of the estimation of the fiscal reaction function using panel quantile regression, the coefficients of the variable d_{t-1} , which measure the degree of fiscal reaction, have a median of 0.09% and a mean of 0.11%. These estimates align with the coefficient estimated by Checherita-Westphal and Žďárek (2015) for the world but higher than Ferrarini and Ramayandi (2012) estimates for Asia. The fiscal reaction coefficient estimates show that at the median level of debt or at the 50th quantile, the primary surplus increases by 0.29% for every 1% increase in debt, but at the lower quantiles or lower debt, the reaction is almost zero. At high levels of debt or at 90th quantile, primary surplus increases by 0.13% for every 1% rise in debt, and at the 80th quantile, it only increases by 0.04%. These findings suggest that the fiscal response to an increment in the public debt ratio is conditional on the size of the debt, and the fiscal response is stronger at the median than at higher

quantiles. Economies with very low or very high debt do not seem to adjust their primary balance significantly when debt increases, in contrast with those with median debt level. These results also provide evidence that the relationship between debt and fiscal response is asymmetric and nonlinear. Applying the quantile regression approach to the fiscal reaction function offers a more comprehensive approach in looking at a broader range of fiscal reactions to changes in debt and long-run outlook in the economy. A summary of the coefficients for the variables d_{t-1} as well as $ygap_t$ derived from the fiscal reaction function for the panel of economies is presented in Appendix Figure A3.

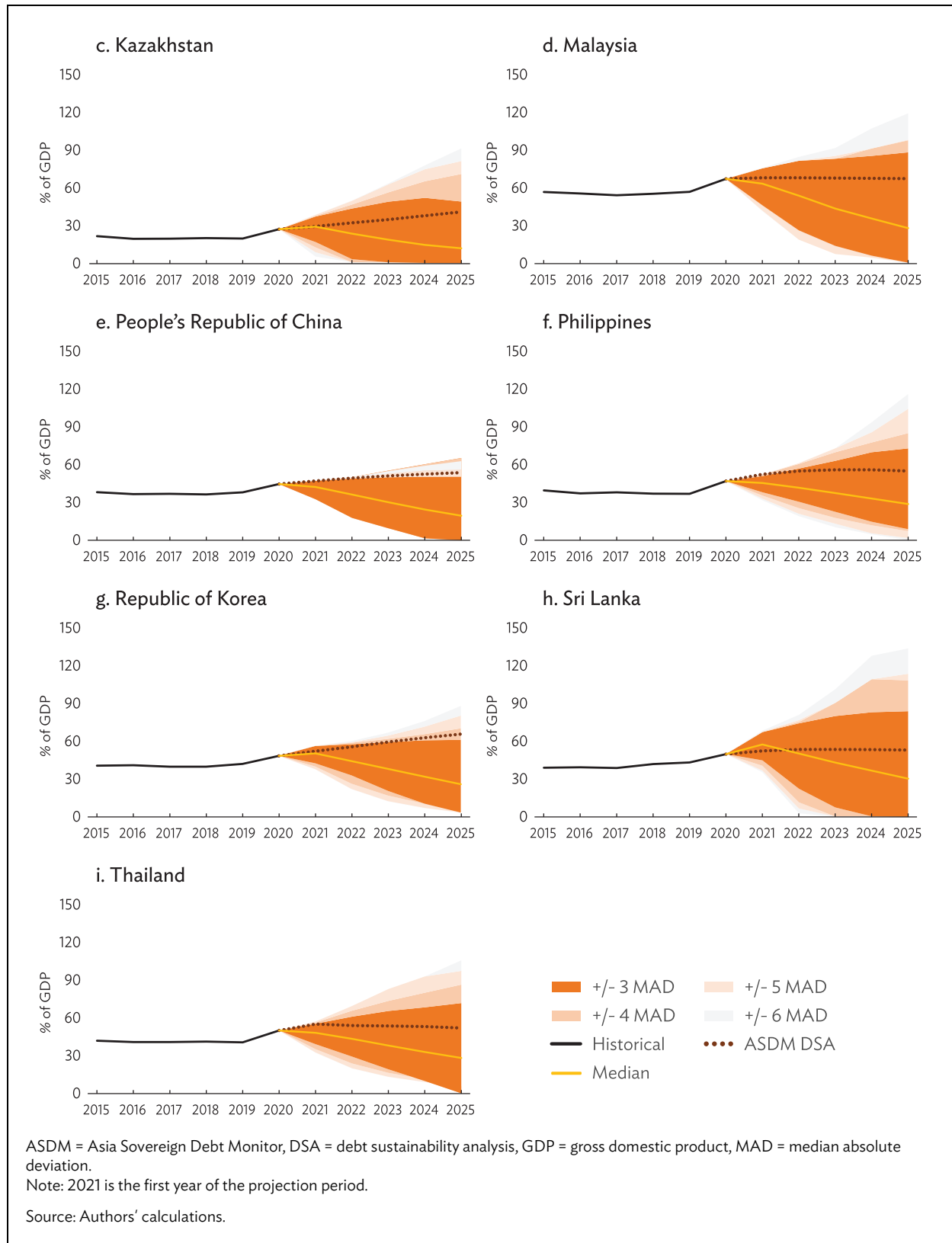
For each of the nine DMCs, the forecasts for nonfiscal and fiscal determinants of debt enter into the public debt dynamics equation (described in Section II) to come up with debt-to-GDP ratio forecasts for years 2021 to 2025. These debt paths are drawn into fan charts. The distributions of the projected debt ratios are presented in charts in Appendix Figure A4. Note that for most DMCs, extreme values of the estimated debt ratios are apparent, affecting the shape of the fan charts and widening the range tremendously. Given this, the extreme values, including their corresponding debt pathways, are filtered up to six absolute deviations from the median, and negative debt ratios were also automatically dropped.

The fan charts in Figure 1 show the DMCs' possible trajectories of debt and the degree of uncertainty in these projections. The fan cones represent the debt pathways from the 10th percentile to the 90th percentile. DMCs' fan charts widely differ in their fan cone widths or the range of debt pathways at the last year of the projection. The fan cone width captures the volatility of the determinants of debt and the possibility of adverse debt realizations in the future. Greater volatility in the determinants of debt will result in a wider fan cone width than one with more stable macroeconomic conditions. For example, Sri Lanka's fan chart has the largest fan cone width, spanning from the debt-to-GDP ratio of 1% to over 250% by 2025. On the other hand, Indonesia has the narrowest fan cone among DMCs, with the highest value at around 50% of GDP. It is important to emphasize however that the debt paths envisaged in these fan charts have equal likelihood, such that a path corresponding to a low debt by 2025 is just as likely to materialize to a debt path ending with high debt during the last year of projection.



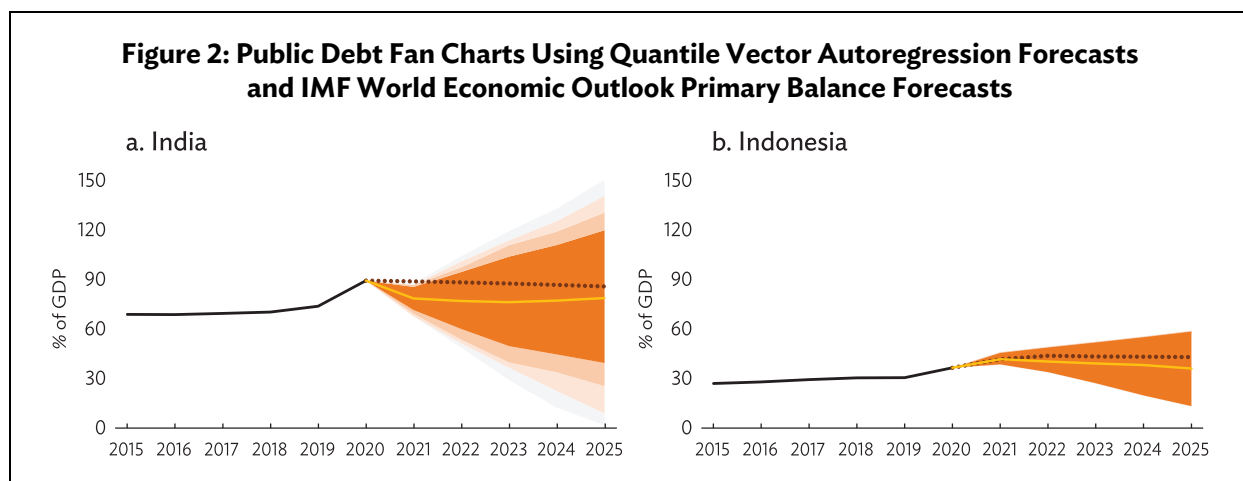
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Figure 1: continued



Overall, this paper finds that the median estimates are on a downward trend, especially from years 2022 to 2025. The trajectory of the median reflects the largely benign historical macroeconomic conditions underlying the estimation, with few crisis or upper tail episodes, which leads to projections that depict stabilization of debt at the median in the medium term. The median projections of the quantile fan charts in Figure 1 are also lower than the baseline forecasts derived using deterministic approach from Asia Sovereign Debt Monitor (ASDM) DSA produced by ADB.³ The ASDM baseline forecasts are found to be generally higher than the median debt forecasts, which can be explained by assumptions of new loans in the short term, projected higher interests on debt, and the macroeconomic assumptions underlying the forecasts. Quantile debt forecasts at the median paint a more sanguine picture of public debt in selected DMCs in the medium term than what is envisioned in the deterministic forecasts, which can be argued to be unrealistic or too optimistic, given current data. But the uncertainty in the forecast and the possibility of worse debt outcomes than the median path is precisely what the debt paths located at the upper part of the fan cone capture. The deterministic forecasts coinciding with debt paths depicting worsening debt ratios through to 2025 tell us that the coronavirus disease (COVID-19) pandemic as a scenario brought to life a combination of tail quantile events that can see debt ratios for the countries rising considerably. All in all, these fan charts depict possible upside, but more importantly further downside risks due to uncertain macroeconomic conditions.

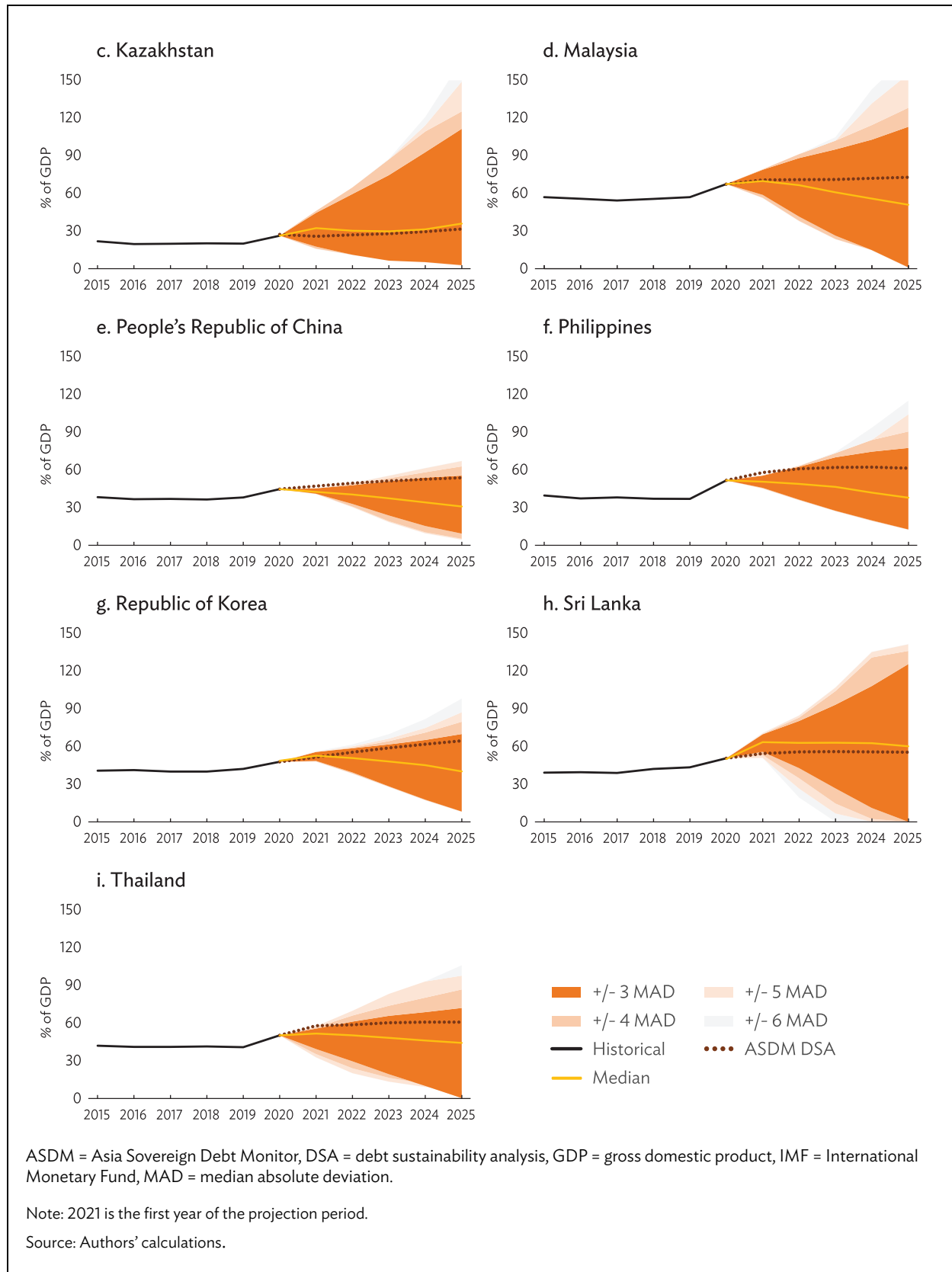
Furthermore, based on the results, this paper finds that the fiscal behavior captured by the fiscal reaction function estimates strongly influences the debt pathways in Figure 1, especially at the median. The higher coefficient of the variable lag of debt ratio at the 50th quantile translates into a larger correction of the primary balance in response to increasing debt, compared to other quantiles, and contributes to the downward sloping projected median debt ratios. While the quantile fiscal reaction function estimates allow for these asymmetries in fiscal response to changes in debt to be ascribed in the possible debt pathways (as shown in the fan charts), these findings also lead us to reconsider the reliability of such estimates when median projections are observed to fall below the historical trend for most of the countries. To check for the robustness of the debt-ratio forecast trajectories with primary balance estimates from the panel quantile fiscal reaction function, this paper also generates debt fan charts using primary balance data from the latest IMF World Economic Outlook Database, October 2021 (Figure 2). The median forecasts of the



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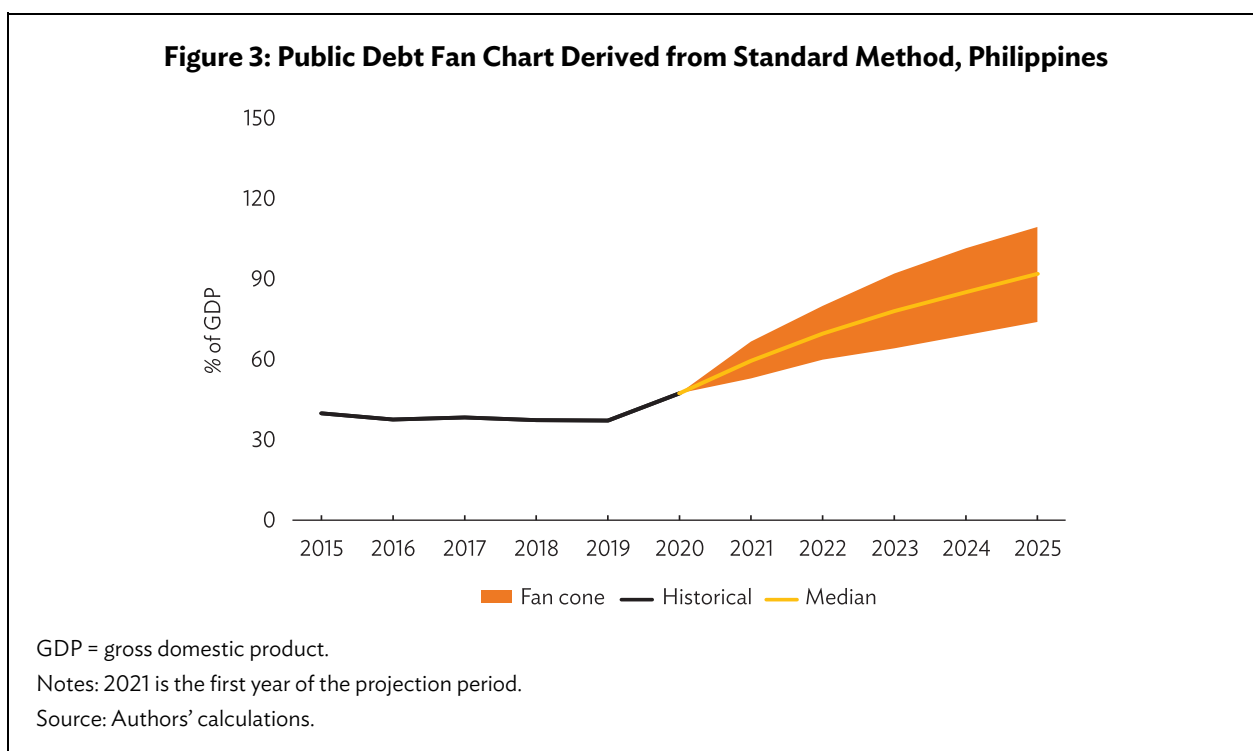
³ The Asia Sovereign Debt Monitor database by ADB estimates public debt ratios for DMCs in the medium term using forecasts from ADB's Asian Development Outlook, IMF's World Economic Outlook, World Bank's World Development Indicators, and national sources. It employs a deterministic approach using debt dynamics equation to compute for baseline debt ratio projections.

Figure 2: continued



fan charts in Figure 2 are closer in value to the deterministic forecasts, relative to Figure 1, and the fan cone shapes are generally similar to Figure 1.

The paper also compares a fan chart derived from the quantile approach to one computed from the standard stochastic method. Figure 3 shows a debt fan chart for the Philippines, which consists of various scenarios of debt ratios centered around the historical mean and shows dispersion that is consistent with the historical variance and covariance. Compared to Figure 1f, the fan chart in Figure 3 depicts a narrower range of possible debt ratios and significantly higher debt projections through 2025. As expected, the fan chart in Figure 3 shows that the range of the cone is symmetric since it uses the variance-covariance matrix of historical shocks. The standard stochastic method appears to depend on the assumptions used to produce the debt forecasts. Since the standard approach relies on mean estimation outliers such as the sudden deterioration in the macroeconomic and fiscal variables during the pandemic, as expected, drive projections upward and to paths that will not likely return to previous historical trend in the medium term based on the shape and width of the fan cone. The paper observes that in the standard approach outliers in the data can affect the projections significantly and alter the distribution of the paths, unlike in a quantile approach setting where estimations are taken at the median of each quantile.



A wider fan cone reflects a higher degree of uncertainty in the baseline forecasts, or how uncertain are the forecasts compared to the historical period (Razi and Loke 2017, Blix and Sellin 1998). The possibility of extreme shocks akin to historical crisis episodes impacting debt is represented as one of the debt pathways depicting a steep rise in the debt-to-GDP ratio. Likewise, a better-than-expected recovery resulting in significantly lower debt in the medium term is also shown, however unlikely it seems given current conditions. A wider fan cone presents a broader view of the upside and downside risks, while a narrower fan cone (such as observed in Figure 3) represents less volatility than a wider fan cone and a limited view on the range of uncertainties in the debt pathways.

In this exercise of generating debt fan charts, this paper observes three advantages of using the quantile approach. First, VARQ generates results that are more robust in the presence of outliers over the standard OLS VAR. As an example, the sudden deterioration in macroeconomic conditions at the onset of the pandemic significantly affect projections derived from a standard stochastic approach. The resulting fan chart tilts mostly to worse debt outcomes as seen in Figure 3. Outliers in the standard approach are affecting the mean estimates such that the results do not reflect the entire historical macroeconomic trend, rather just the few shocks in the system. This is in contrast with the quantile fan charts presented in Figures 1 and 2 that depict a broader range of debt pathways reflecting the benign historical macroeconomic conditions prevailing before the pandemic, as well as equally likely downside realizations.

Second, the quantile approach becomes a natural environment for stress testing by estimating specific coefficients for the variables at every quantile rather than constant coefficients based on the mean. In this approach, a scenario that combines the variables at their tail quantiles, e.g., 10th quantile and the 90th quantile, is in effect a shock scenario in a standard setting without coming up with simulations. For instance, a VARQ equation that consists of low growth at 10th quantile, and high interest rate, inflation, and exchange rate at 90th quantile depicts a scenario that results in worse debt outcomes than when growth, interest rate, inflation, and exchange rates are not in their tail quantiles. In this approach, it is possible to identify a combination of drivers of debt that leads to potential spike in debt ratios.

Lastly, the quantile approach applied to stochastic DSA expediently produces an array of debt outcomes without relying on the assumptions of debt forecasts and simulations generated from a variance-covariance matrix.

IV. CONCLUSION

The stochastic approach to DSA typically uses standard OLS VAR and “fan charts” to depict the upside and downside risks surrounding public debt projections due to uncertain macroeconomic conditions. These VAR models rely on constant coefficients and random variables that are independent and identically distributed. However, empirical evidence suggests that macroeconomic variables are characterized by nonlinearities and asymmetries that linear regression models, such as OLS VAR, may not capture. But in the literature, empirical evidence shows how such nonlinearities can be accounted for by using quantile regression methods. Quantile regression allows for varying parameters for each quantile and facilitates the analysis of asymmetric dynamics, i.e., quantiles can represent asymmetric responses to different types of shocks. The quantile regression approach is a natural environment for stress testing exercises, by estimating coefficients that are specific to tail shocks or future quantile realizations. This motivates the paper to apply quantile vector autoregression and panel quantile regression to stochastic DSA on the generation of the debt fan charts of nine DMCs.

Individual DMC-level forecasts of nonfiscal determinants of debt (real GDP growth, inflation, nominal interest rate, and exchange rate) are derived using VARQ. Primary balance estimates are derived from a panel quantile fiscal reaction function. These results are combined to project debt-to-GDP ratios, following a standard public debt dynamics equation, and debt ratios are depicted into fan charts for each DMC. The results show that the deterministic DSA forecasts are

higher than the median forecasts derived from the quantile regression, and are located at the upper limits of the fan charts. This can be explained by assumptions in the deterministic approach, such as possible new loans in the short term, projected higher interests on debt, and the macroeconomic assumptions underlying the forecasts. On the other hand, the quantile fan charts reflect all the possible pathways of debt given the prevailing historical trend and relationships of the macroeconomic variables in an economy. Compared to a fan chart produced using the standard OLS VAR, a quantile fan chart tends to be less sensitive to outliers due to sudden deterioration in the macroeconomic and fiscal variables.

This paper demonstrates that applying the quantile regression approach to stochastic DSA produces results that address some limitations of the existing stochastic DSA methods. The paper also introduces a panel quantile fiscal reaction function to estimate the primary balance. This is an initial attempt to implement quantile regression in stochastic DSA and could benefit from an improvement in the computation and execution in the future. Further computations improvements as future research endeavors could tackle forecasting the nonfiscal determinants with varying quantiles for each time period throughout the projection horizon and exploring other approaches in panel quantile regression to estimate the fiscal reaction function.

APPENDIX

Quantile Regression Methodology

(1) Quantile Vector Autoregression

This paper follows the approach proposed by Montes-Rojas (2019), which applies the reduced form vector directional quantile (VDQ) model to an autoregressive context, and thus derives the model for vector autoregressive quantile (VARQ). Montes-Rojas (2019) considers the model:

$$Q_{Y_t}(\tau|X_{t-1} = x_{t-1}) = B(\tau)x_{t-1} + A(\tau) \quad (1)$$

where Q is an $m \times 1$ vector that corresponds to the multivariate quantiles of the n random variables, $B(\tau) = (B_1(\tau), \dots, B_n(\tau))^T$ is an $n \times k$ matrix of coefficients with $B_j(\tau)$ for each $j \in \{1, \dots, n\}$, $k \times 1$ vector of coefficients of the j^{th} element in Y , and $A(\tau)$ is an $n \times 1$ vector of coefficients. In addition, let $B_h(\tau) = (B_{1h}(\tau), \dots, B_{nh}(\tau))$ with h -lag coefficients for all endogenous variables, for $h = 1, \dots, p$. Q is, therefore, a map of $X \times T^n \mapsto Y$.

In the context of time-series, the lag polynomials $B(\tau, L)$ can be defined where L is the lag operator, such that

$$B(\tau)X_{t-1} = B(\tau, L)Y_t = \sum_{k=1}^p B_k(\tau)L^k Y_T \quad (2)$$

and

$$Q_{Y_t}(\tau|x_{t-1}) = B(\tau, L)y_t + A(\tau),$$

where y_t represents the values of Y_T that are used in the equation.

To build the VARQ model, let us define $Q_{Y_t}(\tau|x_{t-1}) := \{q_1(\tau|x_{t-1}), \dots, q_n(\tau|x_{t-1})\}^T$ from the set of equations below:

$$\begin{aligned} \{q_1(\tau|x_{t-1})\} &:= c_1(\tau_1)^T q_{-1}(\tau|x_{t-1}) + b_1(\tau_1)^T x_{t-1} + a_1(\tau_1) \\ &: \\ \{q_n(\tau|x_{t-1})\} &:= c_n(\tau_n)^T q_{-n}(\tau|x_{t-1}) + b_n(\tau_n)^T x_{t-1} + a_n(\tau_n), \end{aligned} \quad (3)$$

where $\{c_j(\tau_j)\}_{j=1}^n$ and $\{b_j(\tau_j)\}_{j=1}^n$ are vectors with dimensions $(n-1) \times 1$ and $(k \times 1)$, respectively, and $\{a_j(\tau_j)\}_{j=1}^n$ are scalars. Note that $\{q_j(\tau|x_{t-1})\}_{j=1}^n$ pertains to individual time-series quantile regression models of each j component, that is Y_{jt} , on all others $-j$ components, that is Y_{-jt} , and the lags, that is X_t , where all are simultaneously evaluated at $Q(\tau|x_{t-1})$.

The coefficient matrices $C(\tau) := \{c_1(\tau_1), \dots, c_n(\tau_n)\}^T$ from equation (3) is an $(n \times n)$ matrix and the $n \times 1$ dimensional vector matrix $\{C_j(\tau_j)\}_{j=1}^n$ contains all the elements of the $n-1$ vector $\{c_j(\tau_j)\}_{j=1}^n$ augmented with a 0 in the corresponding j^{th} component. $b(\tau) = \{b_1(\tau_1), \dots, b_n(\tau_n)\}^T$ is an $n \times k$ matrix and $a(\tau) = \{a_1(\tau_1), \dots, a_n(\tau_n)\}^T$ is an $n \times 1$ vector. Finally, the VARQ model is defined as:

$$Q_{Y_t}(\tau|x_{t-1}) = \{I_n - C(\tau)\}^{-1}\{b(\tau)x_{t-1} + a(\tau)\} = B(\tau)x_{t-1} + A(\tau) \quad (4)$$

where I_n is an n -dimensional identity matrix, $B(\tau) := \{I_n - C(\tau)\}^{-1}b(\tau)$ and $A(\tau) := \{I_n - C(\tau)\}^{-1}a(\tau)$.

The expression above describes how multivariate random variables are conditional on their past and models their simultaneous response (Montes-Rojas 2019). In each of the j equations, the τ_j quantile model captures the conditional performance of the j^{th} endogenous variable on the values of the other variables and available past information. Hence, τ embodies the contribution of each endogenous variable in the system, after considering the effects of all other variables. This VARQ model is built for stationary processes, but unit root can be present in some quantiles while stationary in others (Koenker and Xiao 2004). Following Montes-Rojas (2019), unit root processes will be observed by looking at the dynamic behavior of the variables.

Drawing from Chavleishvili and Manganeli (2019), quantile forecasting can be imagined as branches of a tree, where each period forecast with more variables and quantiles results in a richer branch structure. For example, consider three variables, two forecast periods ahead, and three quantiles (5th, 50th, and 95th). At the starting node, the variable $\tilde{Y}_{1,t+1}$ has three branches corresponding to the three quantiles and then at the end of each branch, there are three more branches for the variable $\tilde{Y}_{2,t+1}$, or the one-step ahead forecast for the three quantiles conditional on the forecast for each quantile of the first variable. This process is repeated until period $t+2$ but can be applied to any number of quantiles, variables, and longer forecast horizon. The forecasting method for one-period ahead is implicitly defined in the VARQ model for all Y_{t+1} , given information from time t (Montes-Rojas 2019). It can be expressed as:

$$Q_{Y_{t+1}}(\tau|x_t) = Q_{Y_{t+1}}(\tau|\{y_t, y_{t-1}, \dots, y_{t-p}\}) = B(\tau, L)y_{t+1} + A(\tau), \quad (5)$$

where $Q_1(\tau|x_t) = Q_{Y_{t+1}}(\tau|x_t)$ is the one-period ahead forecast given all the information available at time t . Two-periods ahead forecast at $t+2$ for quantiles τ_2 depends on the response at $t+1$ and the quantile τ_1 . This is given by:

$$\begin{aligned} Q_2\{\tau_2, \tau_1|x_t\} &:= Q[\tau_2|\{Q_1(\tau_1|x_t), y_t, y_{t-1}, \dots, y_{t-p+1}\}] \\ &= B(\tau_2)\{Q_1(\tau_1|x_t)^T, y_t^T, \dots, y_{t-p+1}^T\}^T + A(\tau_2) \end{aligned} \quad (6)$$

It can be generalized for h -periods ahead forecast and written as:

$$Q_h\{\tau_h, \dots, \tau_1|x_t\} := B[\tau_h, L]Q_k\{(\tau_k, \dots, \tau_1)|x_t\} + A(\tau_h), \quad (7)$$

where $Q_k\{(\tau_k, \dots, \tau_1)|x_t\} = y_{t-k}$ if $L^k(t+h) \leq t$ and (τ_k, \dots, τ_1) , $k = 1, \dots, h-1$ refers to the k -periods quantile path. Further, the expression is generalized to:

$$Q_h\{\tau_h, \dots, \tau_1|x_t\} := \left\{ \prod_{k=1}^h B(\tau_k)x_t + \sum_{k=1}^{h-1} \left\{ \prod_{j=k+1}^h B(\tau_j) \right\} A(\tau_k) \right\} + A(\tau_h), \quad (8)$$

allowing for forecasting various quantile paths.

(2) Panel Quantile Regression

The theoretical foundations of fiscal reaction functions were laid out by Bohn (1998) and have been cemented in fiscal sustainability analysis. Fiscal sustainability is achieved when the government's

budget can be financed over time, without resulting in increases in public debt and money supply.⁴ This definition of fiscal sustainability implies that the difference between government revenues (R_t) and expenditures (G_t) in any period is reflected in the changes in outstanding public debt stock ($D_{t+1} - D_t$), expressed as a budget surplus:

$$R_t - G_t = -(D_{t+1} - D_t), \text{ for } t = 1, 2, 3, \dots, N \quad (9)$$

Further, the primary budget surplus is defined as the budget surplus minus interest payments. This is given by:

$$D_t = iD_t - S_t \text{ for } t = 1, 2, 3, \dots, N \quad (10)$$

which links public debt to the interest rate, i , and primary surplus S_t . Solving equation (10) forward, public debt in time t is given by:

$$D_t = \sum_{j=0}^{\infty} i_{t,t+j}^{-1} S_{t+j} + \lim_{T \rightarrow \infty} i_{t,t+T}^{-1} D_{t,T+1}, \quad (11)$$

where $D_{t,T+1}$ is the terminal debt stock and $i_{t,t+j}$ is the discount factor from time t and $t+j$. Dynamic fiscal sustainability requires that the terminal debt stock approaches zero as T approaches infinity and rules out “Ponzi-Madoff” schemes, in which debt is rolled over indefinitely. Under this condition, if the current debt stock is equal to zero, then the present value of all government expenditures (excluding interest payments) should match the present value of all government revenues, or

$$\sum_{j=0}^{\infty} i_{t,t+j}^{-1} G_{t+j} = \sum_{j=0}^{\infty} i_{t,t+j}^{-1} R_{t+j}. \quad (12)$$

The assessment of fiscal sustainability proposed by Bohn (1998) looks at how the government’s primary surplus responds to changes in public debt and other variables, or the adjustment of the primary surplus as public debt increases. Bohn (1998) adopts an uncertainty framework, replacing equation (11) with:

$$D_t = E \left(\frac{\sum_{j=0}^{\infty} \beta^j \mu'(c_{t+j})}{\mu'(c_t) S_{t+1}} \right) + \lim_{T \rightarrow \infty} E_T \beta^{T+1} u'(c_{t+T+1}) / u'(c_t) D_{t+1}, \quad (13)$$

where E is the mathematical expectations operator, $u'(c(t+j))/u'(c(t))$ is the marginal rate of substitution between consumption c in two time periods. This is analogous to equation (11) but incorporates uncertainty into the model and replaces the discount factor by the time-varying marginal rate of substitution in consumption. Under equation (13), the sustainability condition entails that the terminal debt stock discounted by the marginal rate of substitution in consumption approaches zero as T approaches infinity, and that equation (14) below holds.

$$D_t = E \left(\sum_{j=0}^{\infty} \beta^j u'(c_{t+j}) / u'(c_t) S_{t+1} \right) \quad (14)$$

The relationship between primary surplus (s) and debt (d), expressed as proportions of GDP can be written as a linear equation:

$$s_t = \rho d_{t-1} + \beta \mu_t + \varepsilon_t \quad \varepsilon \cdot (0, \sigma^2), \quad (15)$$

where μ_t pertains to the temporary factors affecting primary balance and ε_t is the error term.

⁴ Central banks theoretically can finance budget deficits of governments, but this is less common in Asia since most central banks’ charters do not allow for this within their inflation targeting framework. The role of money supply in fiscal policy is not considered in this paper.

Table A1: Data Sources

Country	Period Coverage	Variable	Measure	Data Source
1. India	Q2 1997–Q1 2021	Public debt	General Government gross debt (% of GDP)	IMF World Economic Outlook database
	Q2 1997–Q1 2021	GDP growth	Real GDP growth	Haver Analytics
	Q1 1993–Q1 2021	Nominal interest rates	Government Securities Yield: 1 Year	CEIC
	Q2 1997–Q1 2021	Inflation	Computed using percent change in the GDP deflator	Haver Analytics
	Q2 1997–Q1 2021	Change in real exchange rate	Computed using end-of-period exchange rate, LCU per dollar	IMF International Financial Statistics
	1997–2020	Primary balance	General government primary net lending/ borrowing, as percent of GDP	IMF World Economic Outlook database
2. Indonesia	Q1 1990–Q1 2021	Public debt	General Government gross debt (% of GDP)	IMF World Economic Outlook database
	Q1 1990–Q1 2021	GDP growth	Real GDP growth	Haver Analytics
	Q1 1990–Q1 2021	Nominal interest rates	State Bank lending rate for LCU- denominated loans	CEIC
	Q1 1990–Q1 2021	Inflation	Computed using percent change in the GDP deflator	Haver Analytics
	Q1 1990 – Q1 2021	Change in real exchange rate	Computed using end-of-period exchange rate, LCU per dollar	IMF International Financial Statistics
	1993–2020	Primary balance	General government primary net lending/ borrowing, as percent of GDP	IMF World Economic Outlook database
3. Kazakhstan	Q1 2000–Q4 2020	Public debt	General Government gross debt (% of GDP)	IMF World Economic Outlook database
	Q1 2000–Q4 2020	GDP growth	Real GDP growth	Haver Analytics
	Q1 1998–Q1 2021	Nominal interest rates	Avg Effective Yield: Govt Securities: MS: MEOKAM/Mid-term Treasury bills	CEIC
	Q1 2000–Q4 2020	Inflation	Computed using percent change in the GDP deflator	Haver Analytics
	Q1 2000–Q4 2020	Change in real exchange rate	Computed using end-of-period exchange rate, LCU per dollar	IMF International Financial Statistics
	2000–2020	Primary balance	General government primary net lending/ borrowing, as percent of GDP	IMF World Economic Outlook database
4. Malaysia	Q1 1992–Q1 2021	Public debt	General Government gross debt (% of GDP)	IMF World Economic Outlook database
	Q1 1992–Q1 2021	GDP growth	Real GDP growth	Haver Analytics
	Q1 1992–Q1 2021	Nominal interest rates	Government Securities Yield: 1 Year	CEIC

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Table A1: continued

Country	Period Coverage	Variable	Measure	Data Source
	Q1 1992–Q1 2021	Inflation	Computed using percent change in the GDP deflator	Haver Analytics
	Q1 1992–Q1 2021	Change in real exchange rate	Computed using end-of-period exchange rate, LCU per dollar	IMF International Financial Statistics
	1993–2020	Primary balance	General government primary net lending/ borrowing, as percent of GDP	IMF World Economic Outlook database
5. People's Republic of China	Q1 1990–Q1 2021	Public debt	General Government gross debt (% of GDP)	IMF World Economic Outlook database
	Q1 1993–Q1 2021	GDP growth	Real GDP growth	Haver Analytics
	Q1 1990–Q1 2021	Nominal interest rates	Prime Lending Rate (Average, % per annum)	Haver Analytics
	Q1 1993–Q1 2021	Inflation	Computed using percent change in the GDP deflator	Haver Analytics
	Q1 1993–Q1 2021	Change in real exchange rate	Computed using end-of-period exchange rate, LCU per dollar	IMF International Financial Statistics
	1993–2020	Primary balance	General government primary net lending/ borrowing, as percent of GDP	IMF World Economic Outlook database
6. Philippines	Q1 1990–Q1 2021	Public debt	General Government gross debt (% of GDP)	IMF World Economic Outlook database
	Q1 1990–Q1 2021	GDP growth	Real GDP growth	Haver Analytics
	Q1 1990–Q1 2021	Nominal interest rates	Treasury Bill rates (91-day)	IMF International Financial Statistics, Bureau of Treasury
	Q1 1990–Q1 2021	Inflation	Computed using percent change in the GDP deflator	Haver Analytics
	Q1 1990–Q1 2021	Change in real exchange rate	Computed using end-of-period exchange rate, LCU per dollar	IMF International Financial Statistics
	1993–2020	Primary balance	General government primary net lending/ borrowing, as percent of GDP	IMF World Economic Outlook database
7. Republic of Korea	Q1 1990–Q1 2021	Public debt	General Government gross debt (% of GDP)	IMF World Economic Outlook database
	Q1 1990–Q1 2021	GDP growth	Real GDP growth	Haver Analytics
	Q1 1990–Q1 2021	Nominal interest rates	Government bonds, percent per annum	IMF International Financial Statistics, Bureau of Treasury
	Q1 1990–Q1 2021	Inflation	Computed using percent change in the GDP deflator	Haver Analytics
	Q1 1990–Q1 2021	Change in real exchange rate	Computed using end-of-period exchange rate, LCU per dollar	IMF International Financial Statistics
	1993–2020	Primary balance	General government primary net lending/ borrowing, as percent of GDP	IMF World Economic Outlook database

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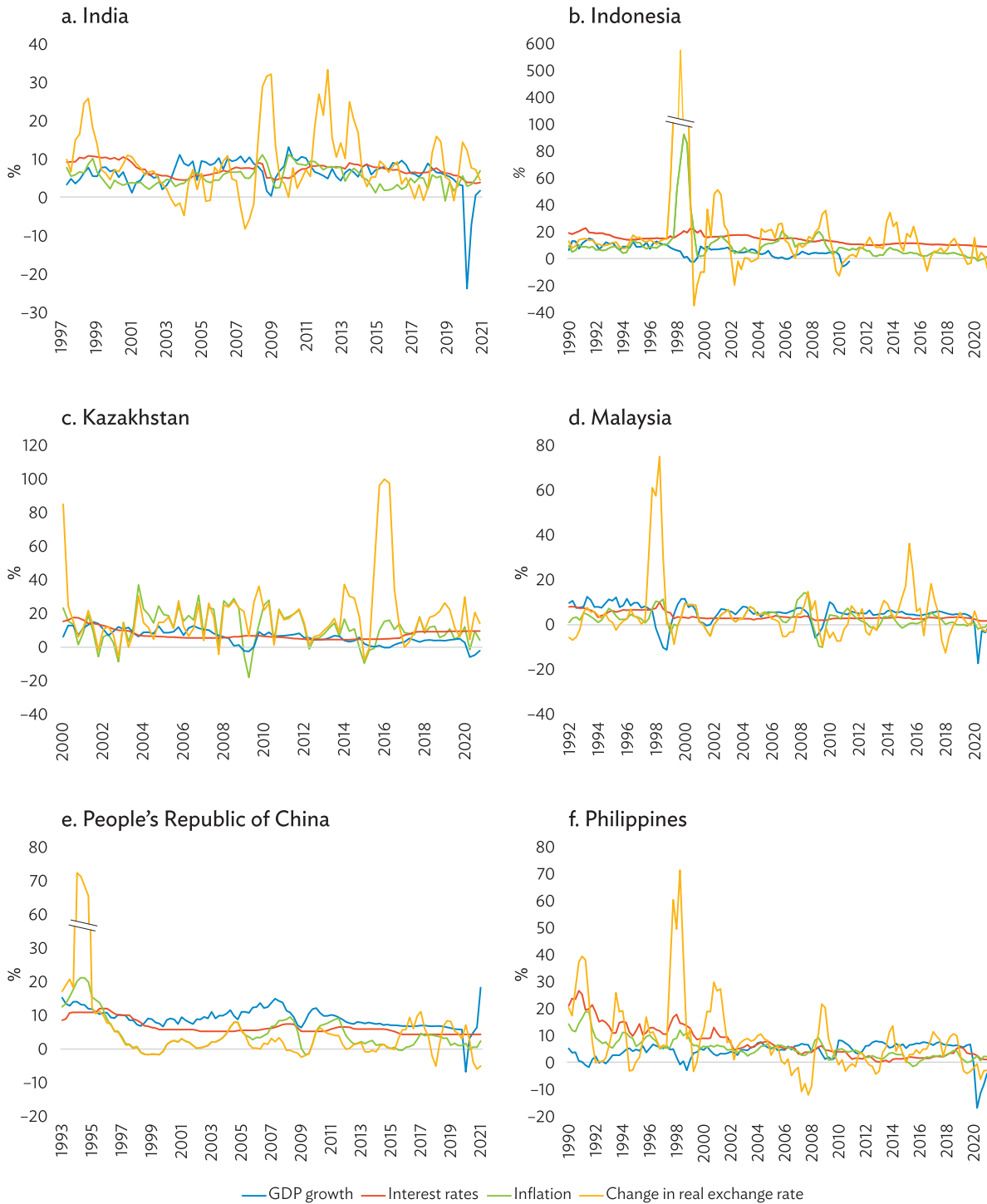
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Country	Period Coverage	Variable	Measure	Data Source
8. Sri Lanka	Q1 1997–Q4 2020	Public debt	General Government gross debt (% of GDP)	IMF World Economic Outlook database
	Q1 1997–Q4 2020	GDP growth	Real GDP growth	Haver Analytics
	Q1 1993–Q4 2020	Nominal interest rates	Treasury Bills Rate: Auction Market: 12 Months	CEIC
	Q1 1997–Q4 2020	Inflation	Computed using percent change in the GDP deflator	Haver Analytics
	Q1 1997–Q4 2020	Change in real exchange rate	Computed using end-of-period exchange rate, LCU per dollar	IMF International Financial Statistics
	1997–2020	Primary balance	General government primary net lending/ borrowing, as percent of GDP	IMF World Economic Outlook database
9. Thailand	Q2 1996–Q1 2021	Public debt	General Government gross debt (% of GDP)	IMF World Economic Outlook database
	Q2 1994–Q1 2021	GDP growth	Real GDP growth	Haver Analytics
	Q2 1996–Q1 2021	Nominal interest rates	Government bonds, percent per annum from Q2 1996 – Q2 1999 Government bond yield (%) Q3 1999–Q1 2021	IMF International Financial Statistics, Bureau of Treasury CEIC
	Q2 1994–Q1 2021	Inflation	Computed using percent change in the GDP deflator	Haver Analytics
	Q2 1994–Q1 2021	Change in real exchange rate	Computed using end-of-period exchange rate, LCU per dollar	IMF International Financial Statistics
	1993–2020	Primary balance	General government primary net lending/ borrowing, as percent of GDP	IMF World Economic Outlook database

GDP = gross domestic product, IMF = International Monetary Fund, LCU = local currency unit, Q = quarter.

Source: Authors.

Figure A1: Nonfiscal Determinants of Debt



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Figure A1: continued

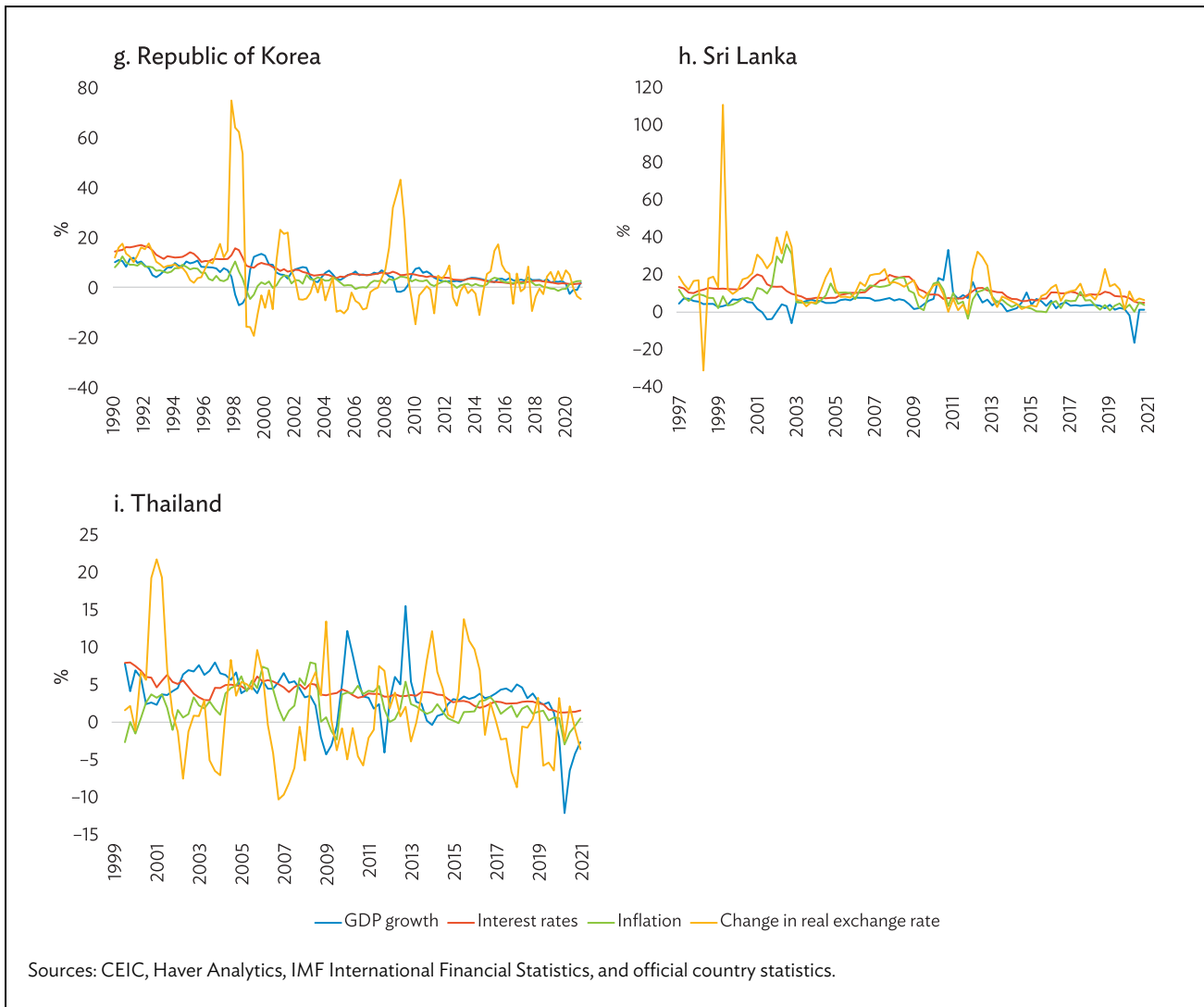
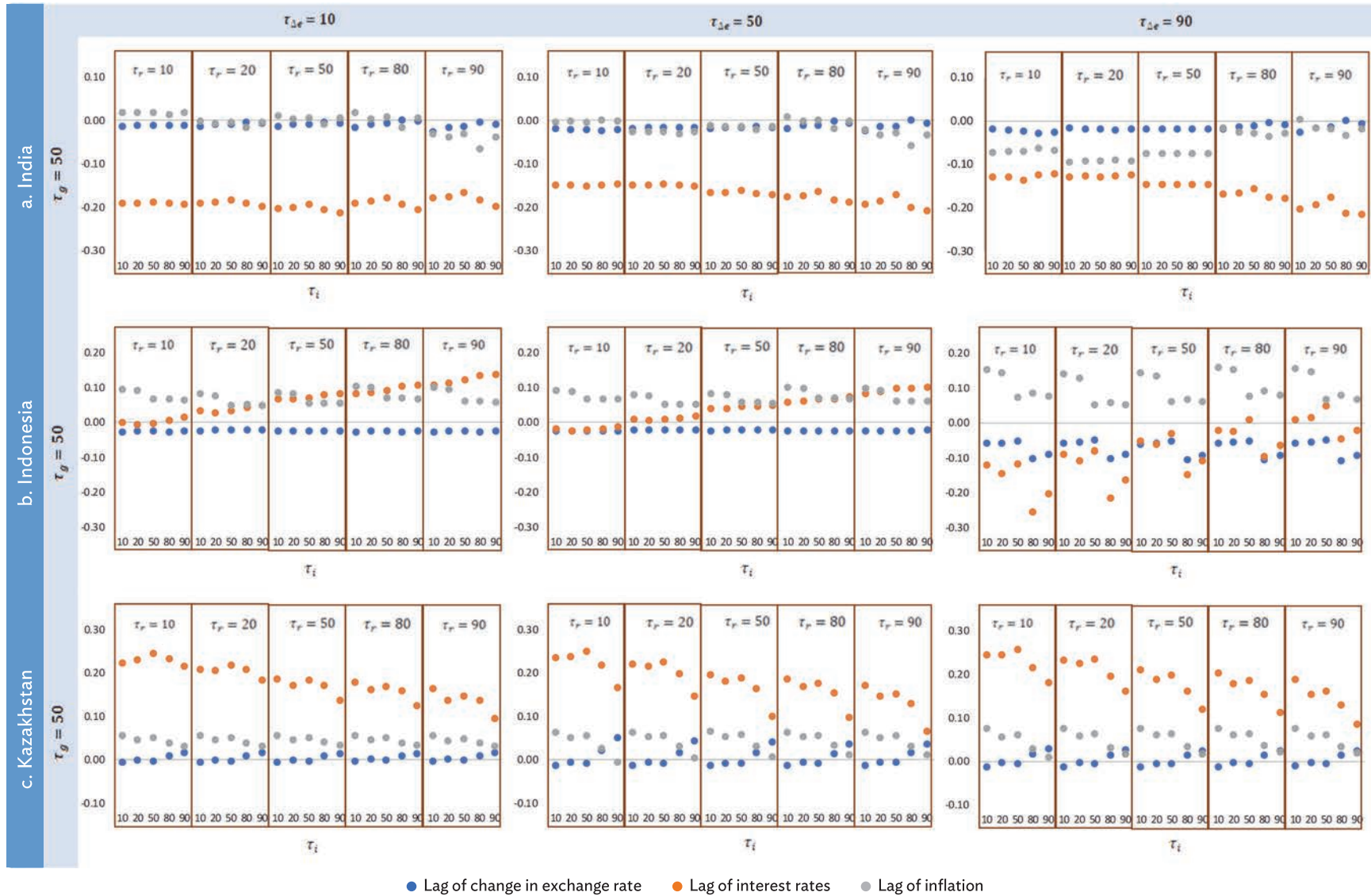
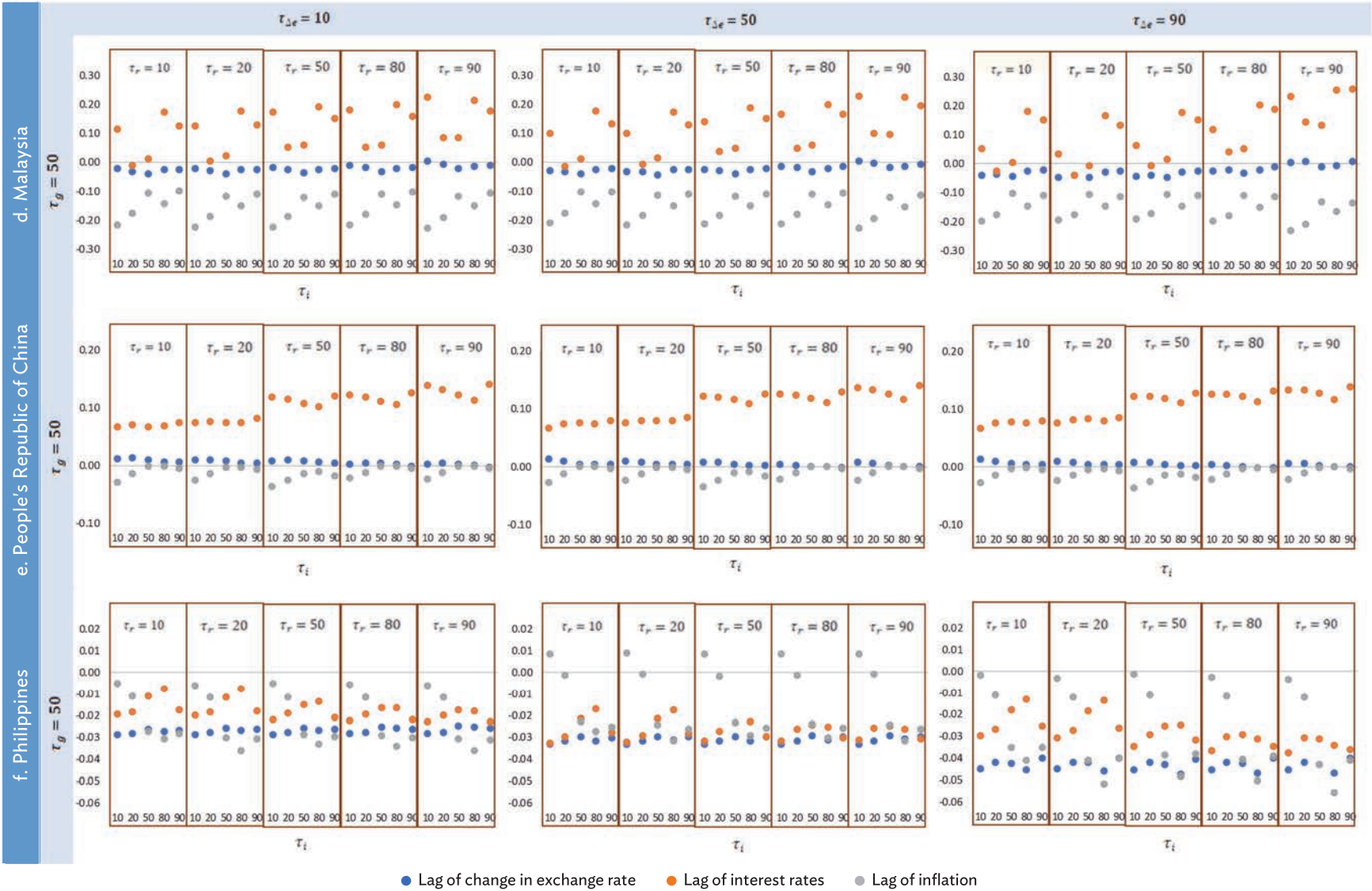


Figure A2.1: Quantile Vector Autoregression Estimated Coefficients from the Growth Equation ($\tau_g = 50$)



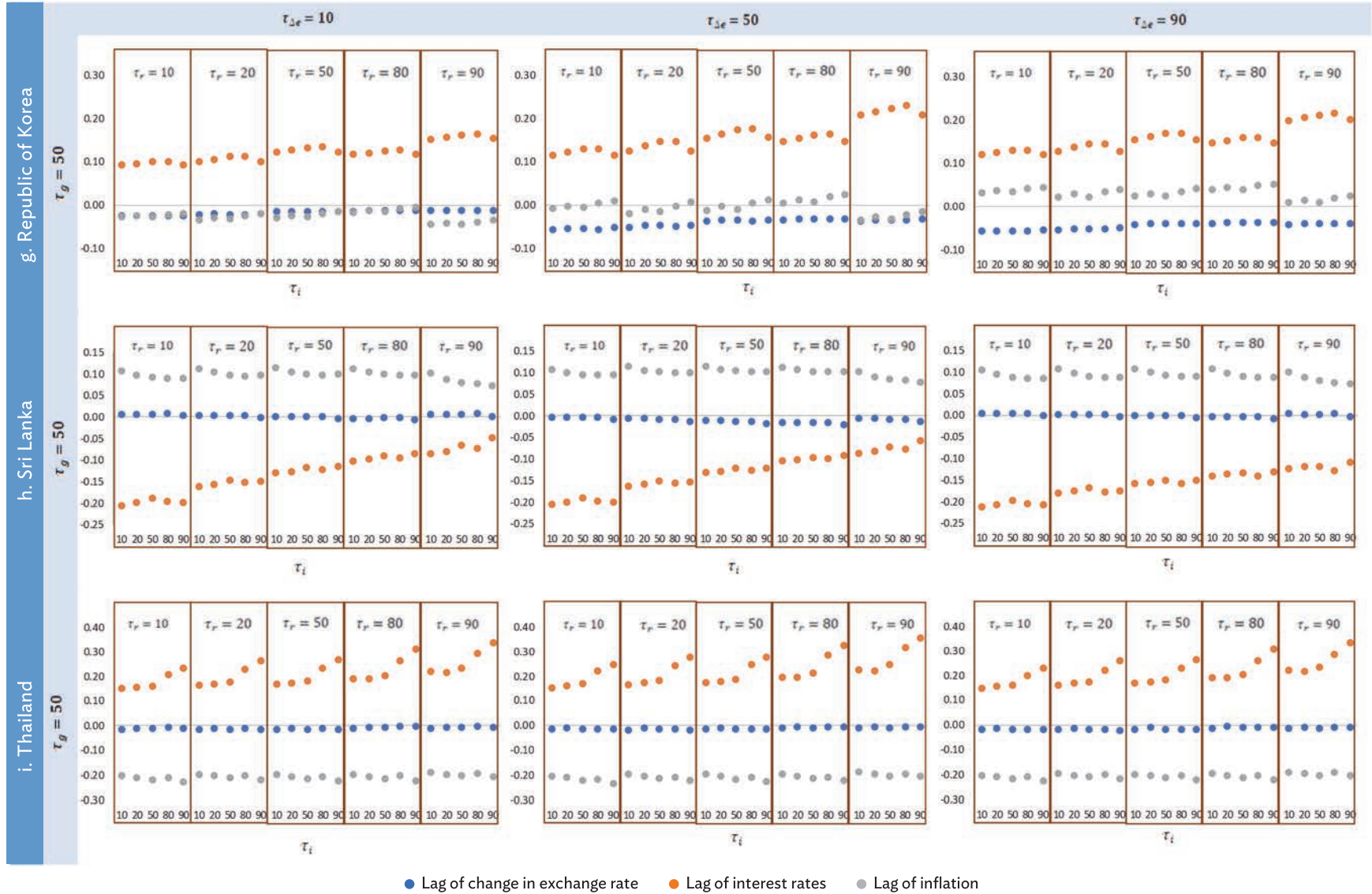
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Figure A2.1: continued



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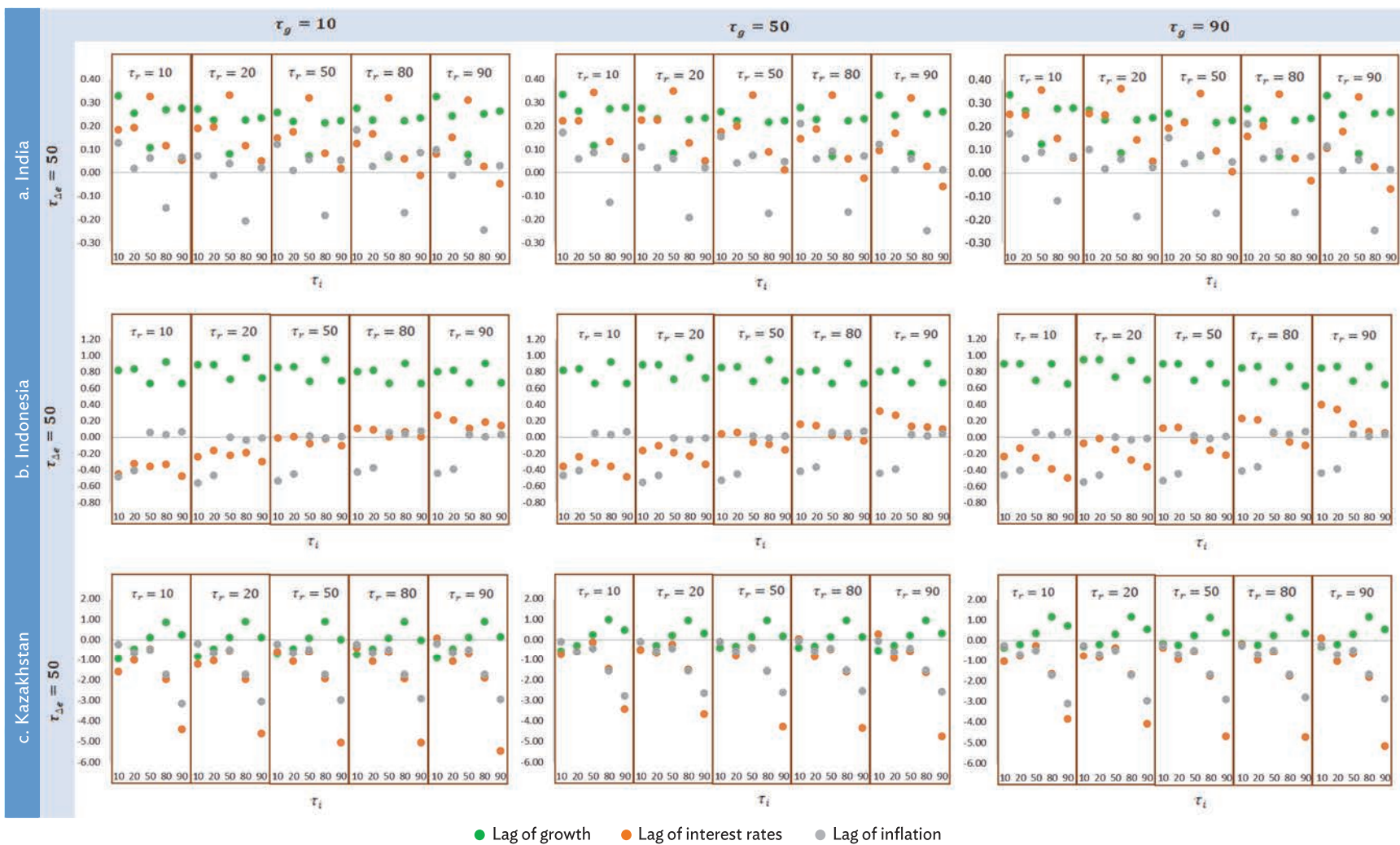


● Lag of change in exchange rate ● Lag of interest rates ● Lag of inflation

Note: τ_g is the quantile of the growth variable, $\tau_{\Delta e}$ is the quantile of the change in exchange rate, τ_r is the quantile of interest rates, and τ_i is the quantile of inflation.

Source: Authors' calculations.

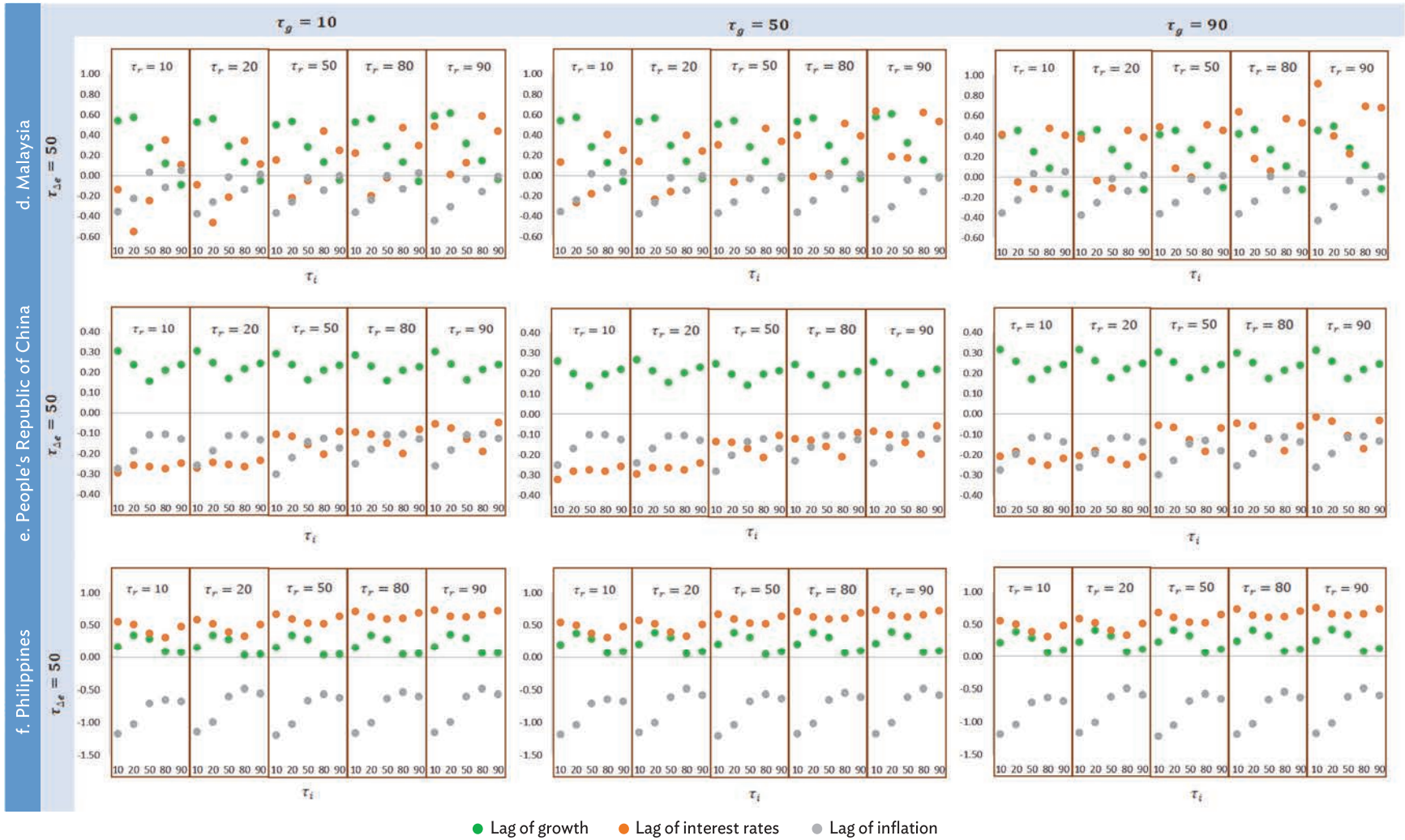
Figure A2.2 Quantile Vector Autoregression Estimated Coefficients from the Change in Exchange Rate Equation ($\tau_{\Delta e} = 50$)



● Lag of growth ● Lag of interest rates ● Lag of inflation

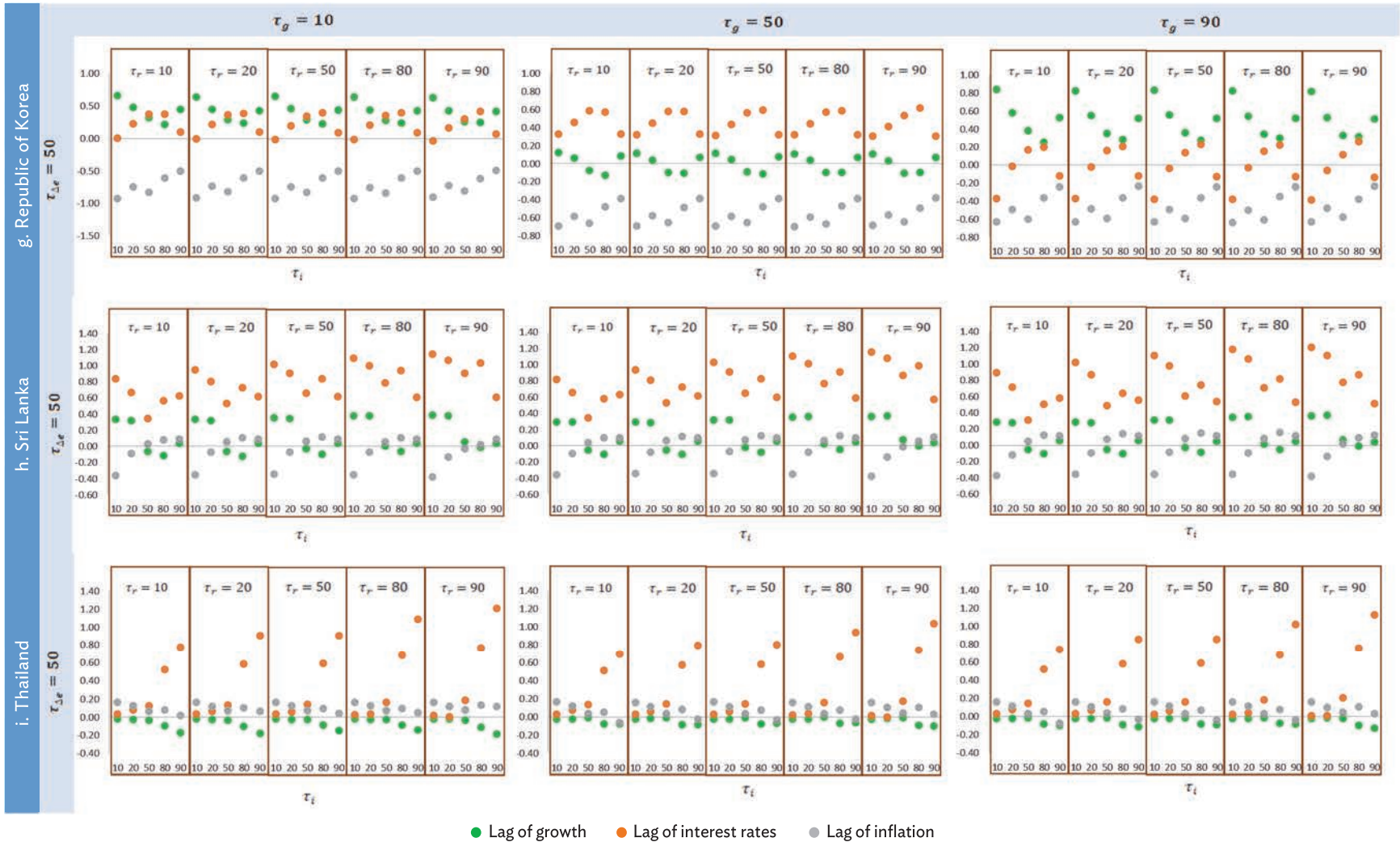
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Figure A2.2: continued



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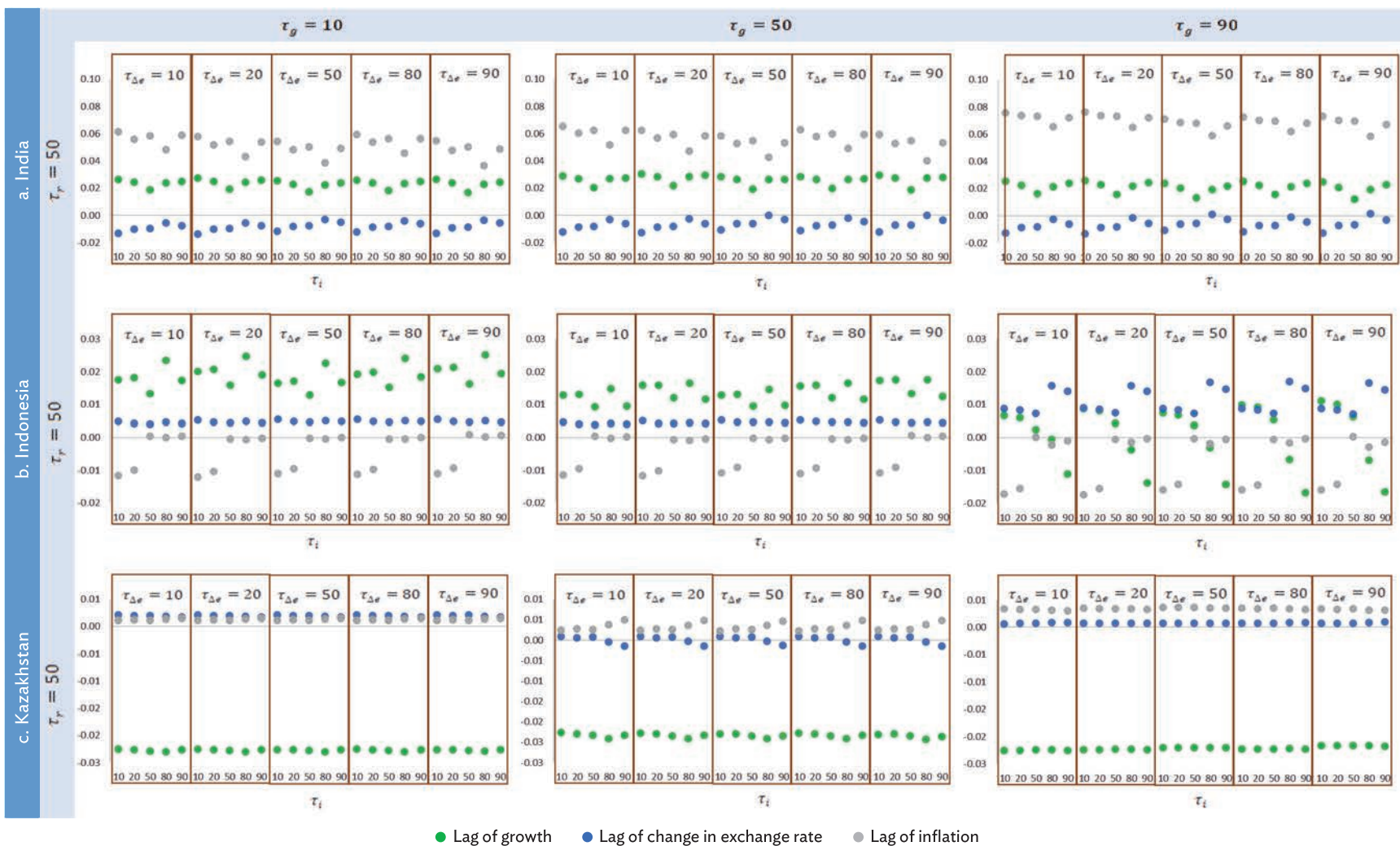
Figure A2.2: continued



Note: τ_g is the quantile of the growth variable, $\tau_{\Delta e}$ is the quantile of the change in exchange rate, τ_r is the quantile of interest rates, and τ_i is the quantile of inflation.

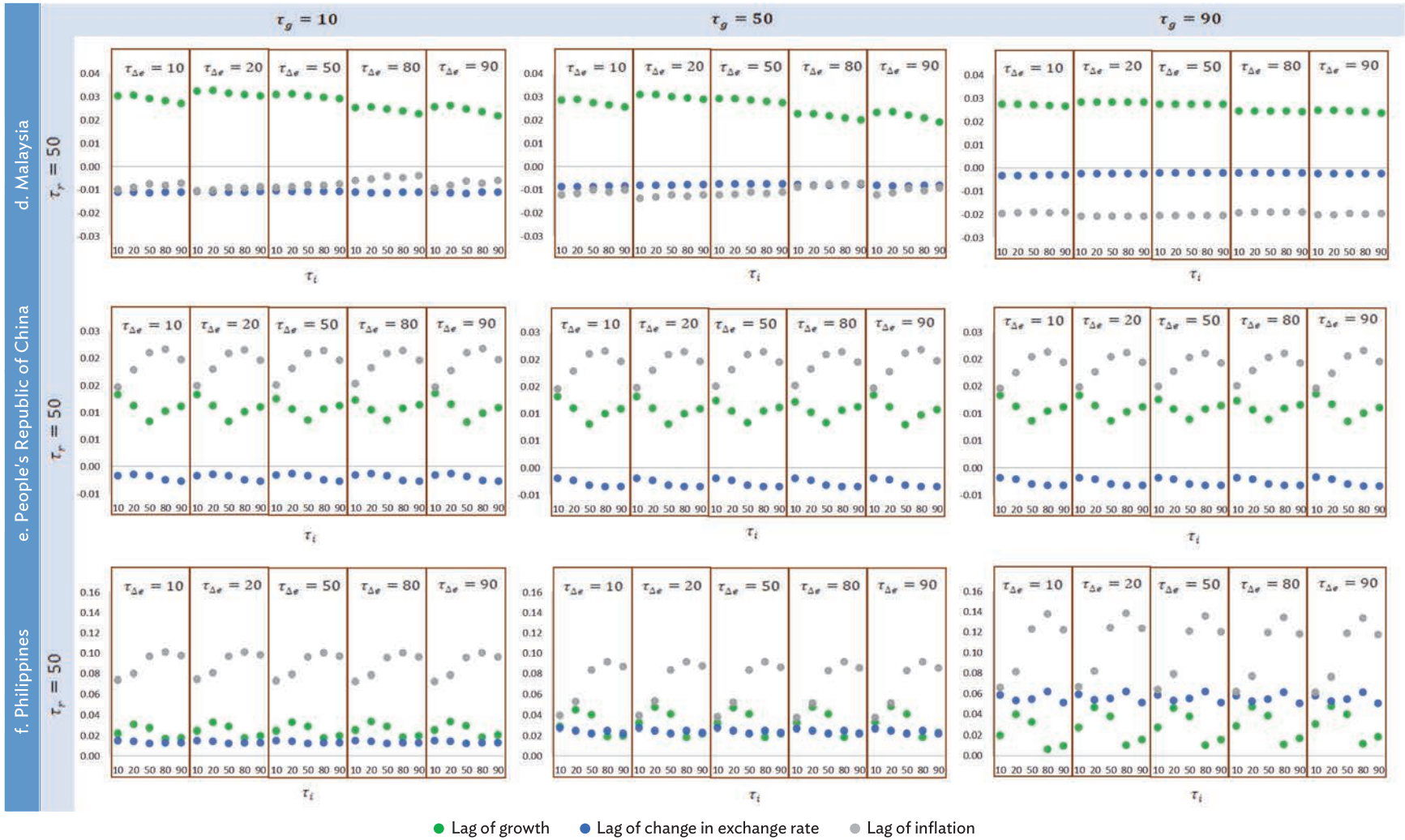
Source: Authors' calculations.

Figure A2.3: Quantile Vector Autoregression Estimated Coefficients from the Interest Rate Equation ($\tau_r = 50$)



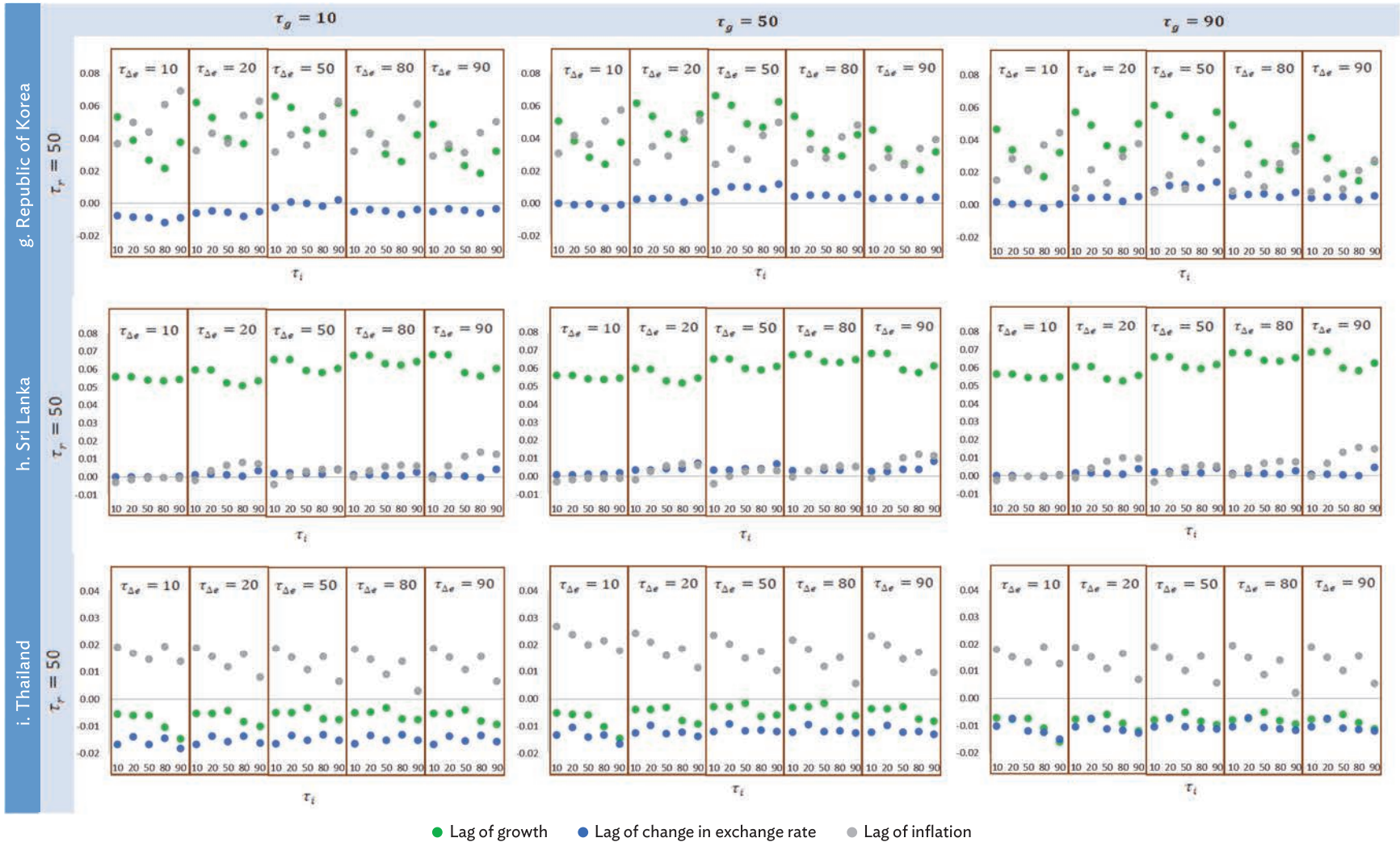
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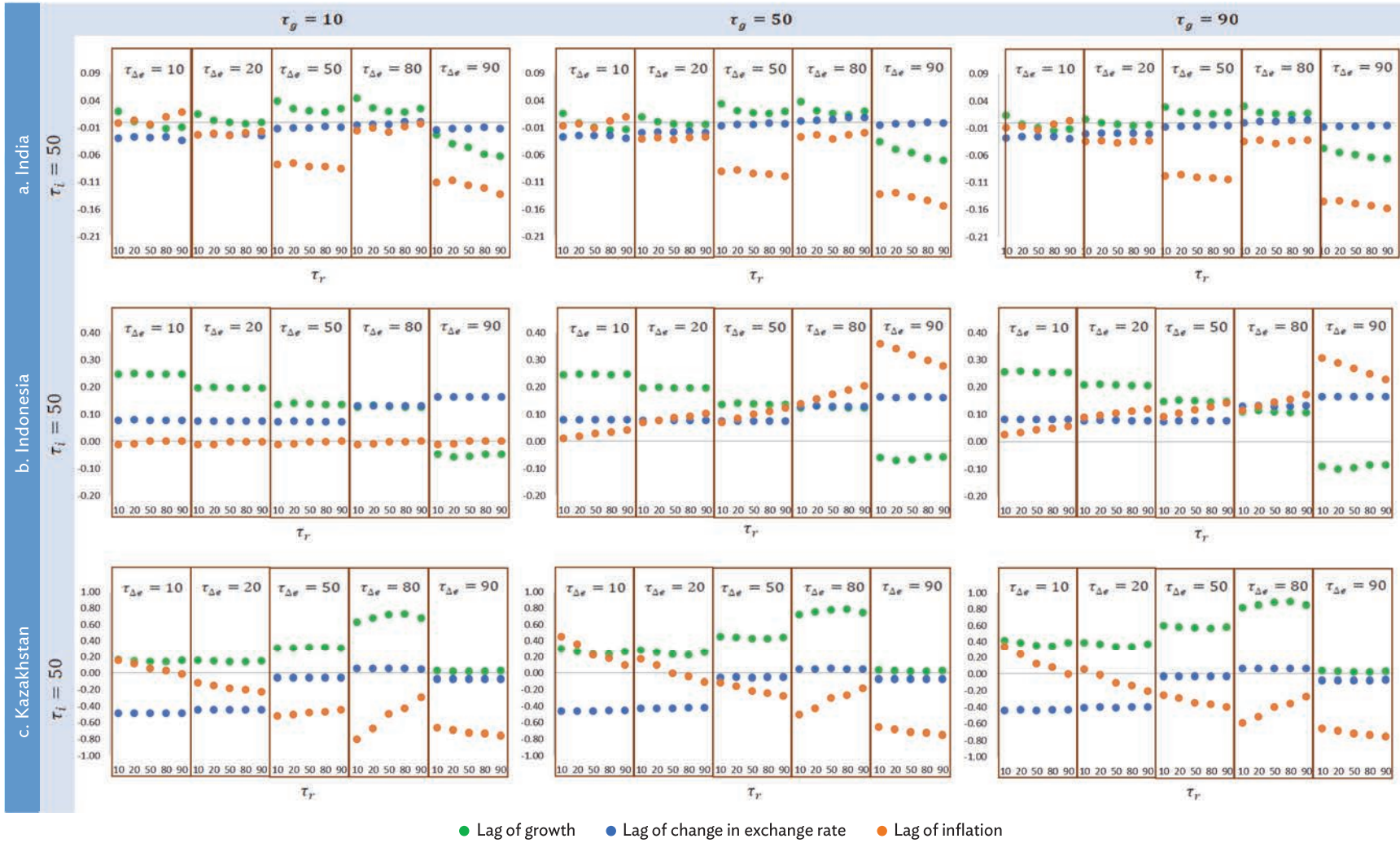
Figure A2.3: continued



Note: τ_g is the quantile of the growth variable, $\tau_{\Delta e}$ is the quantile of the change in exchange rate, τ_r is the quantile of interest rates, and τ_i is the quantile of inflation.

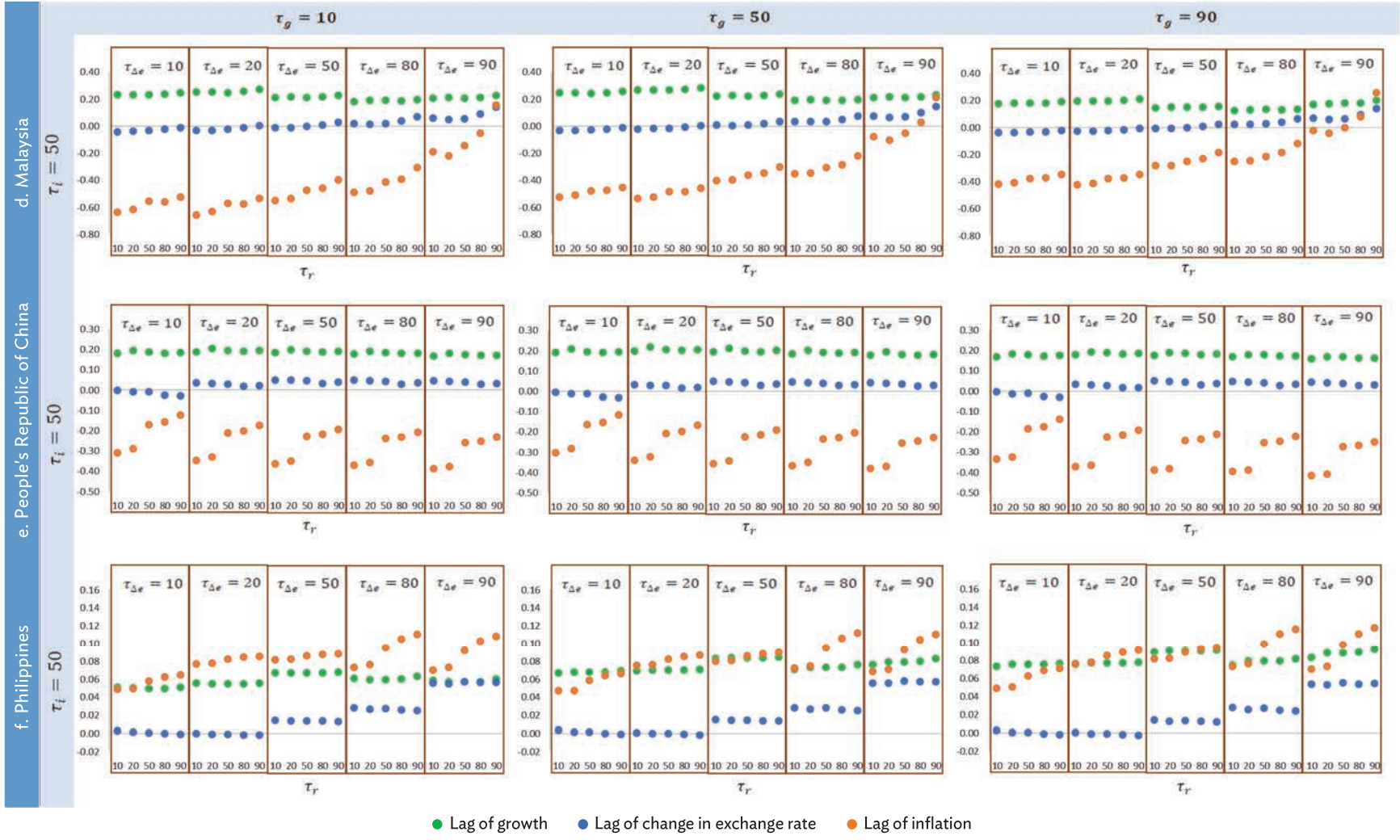
Source: Authors' calculations.

Figure A2.4: Quantile Vector Autoregression Estimated Coefficients from the Inflation Equation ($\tau_i = 50$)



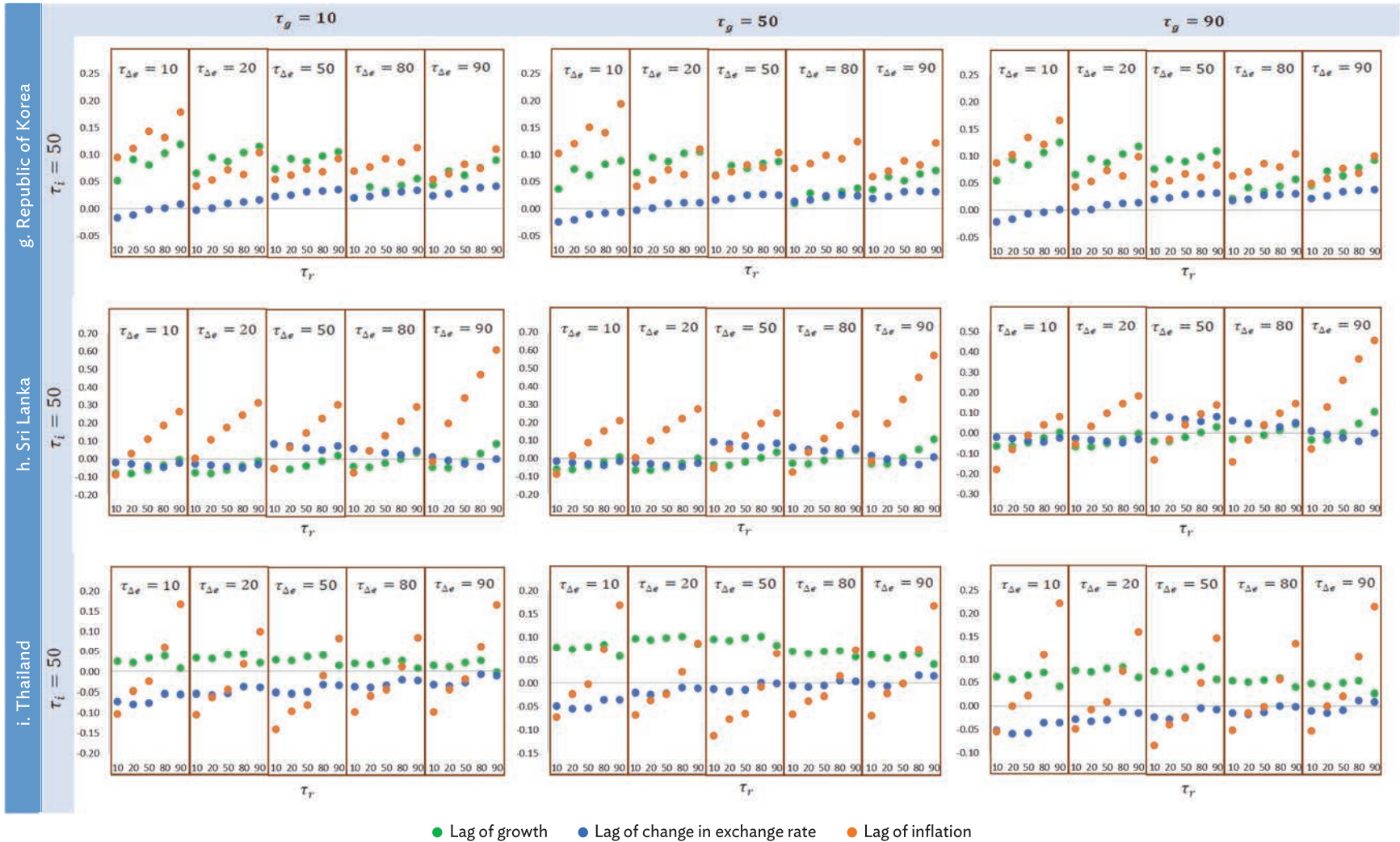
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Figure A2.4: continued



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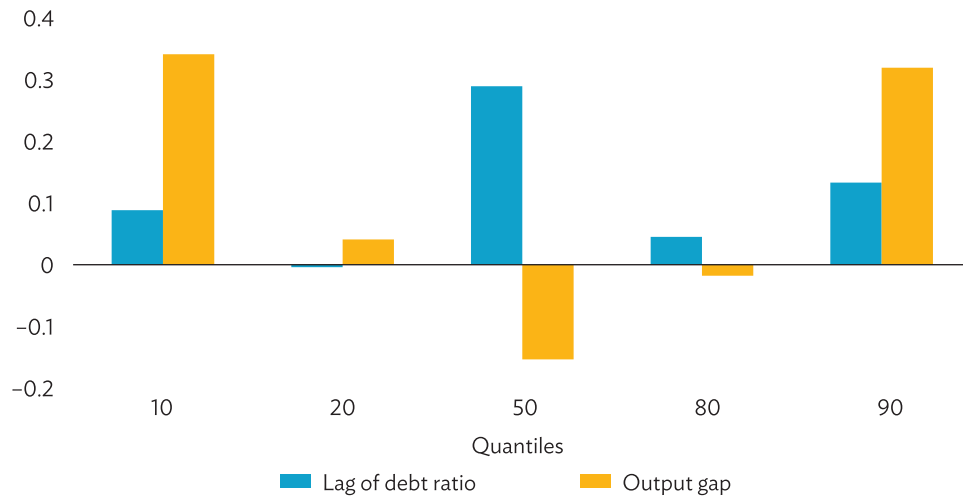
Figure A2.4: continued



Note: τ_g is the quantile of the growth variable, $\tau_{\Delta e}$ is the quantile of the change in exchange rate, τ_r is the quantile of interest rates, and τ_i is the quantile of inflation.

Source: Authors' calculations.

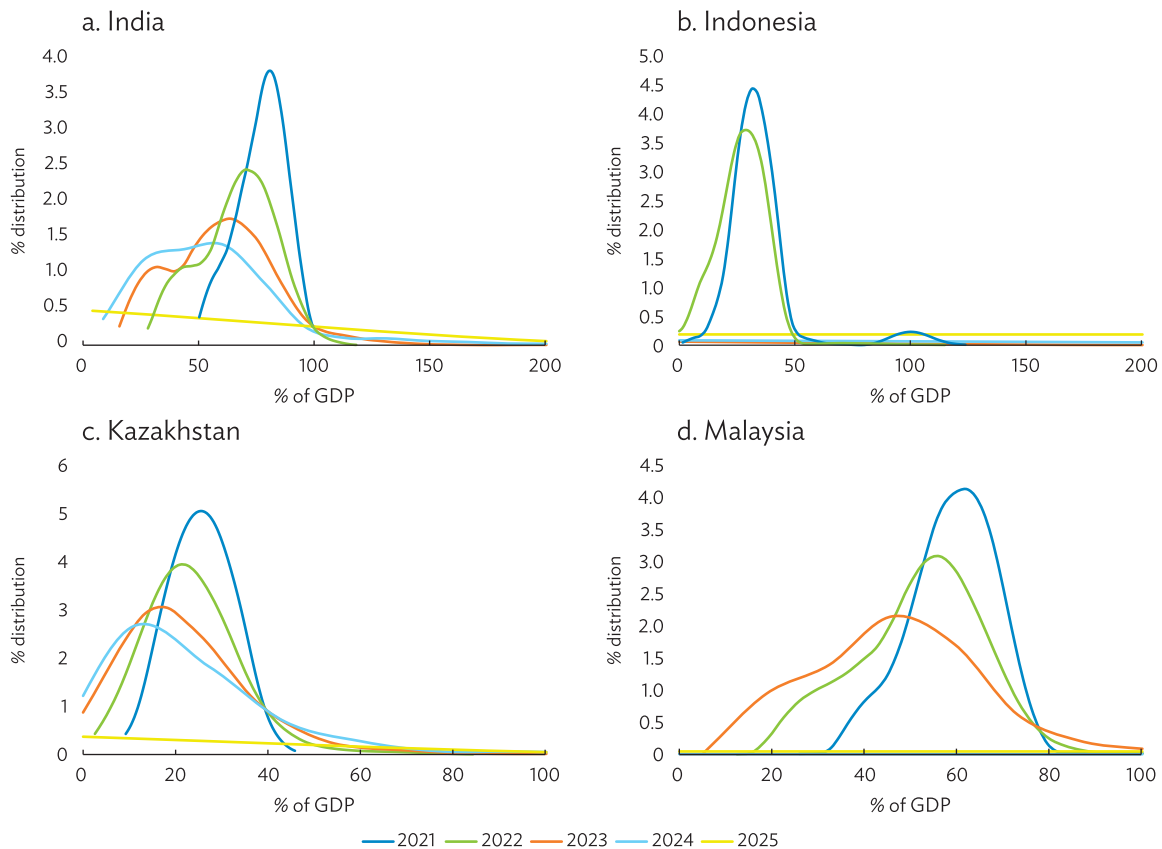
Figure A3: Fiscal Reaction Function Coefficients by Quantile



Notes: The coefficients are in percent (%) and interpreted as the % change in the primary surplus for every 1% change in lag of debt ratio and output gap. These are estimated using quantile regression with a panel of 44 ADB developing member countries, with annual data spanning from 2014 to 2020.

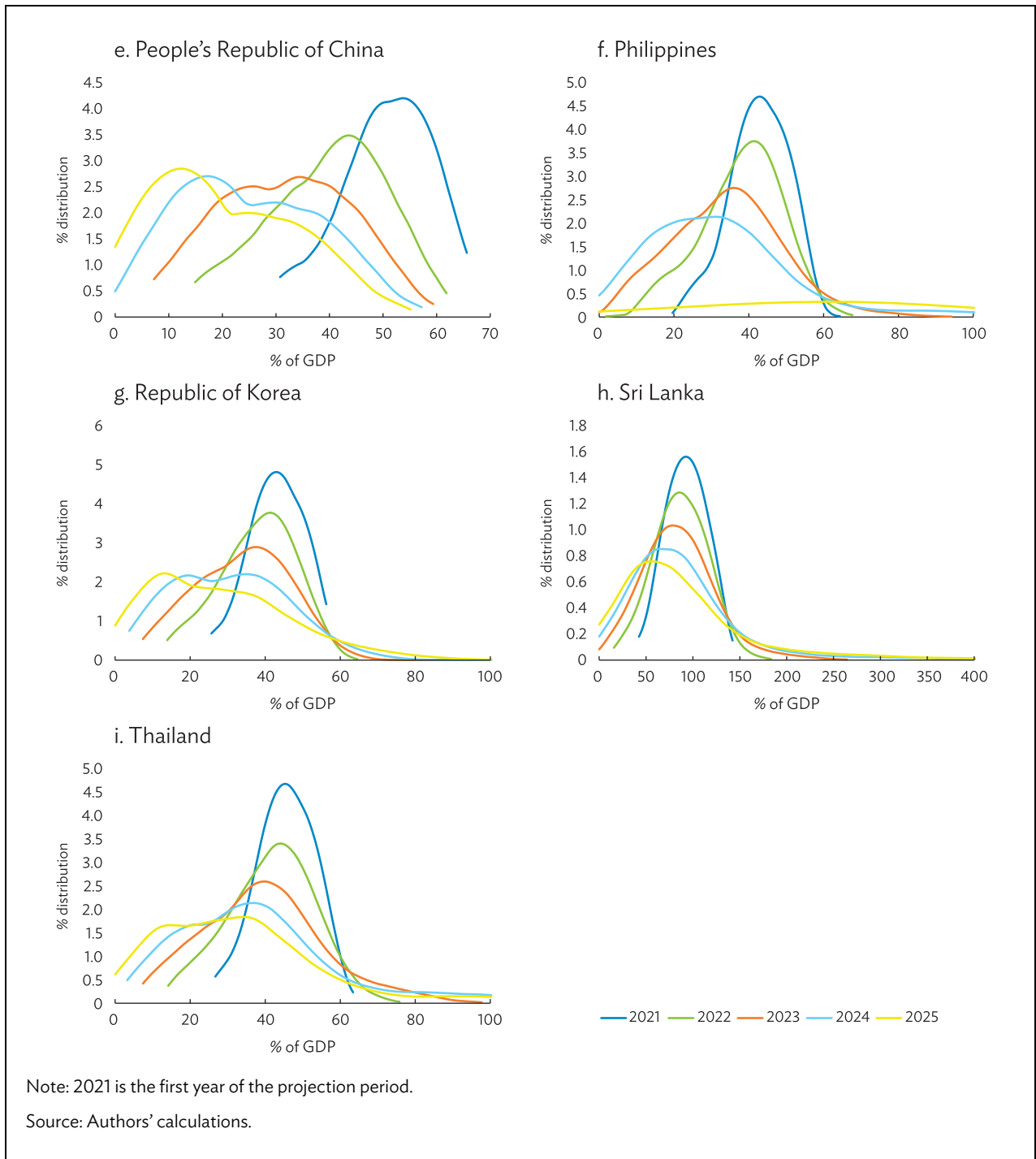
Source: Authors' calculations.

Figure A4: Distribution of Debt Ratio Quantile Vector Autoregression Estimates



continued on next page

Figure A4: continued



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Quantile Debt Fan Charts

This paper presents debt fan charts constructed using the quantile regression approach for nine developing member countries of the Asian Development Bank. Macroeconomic and fiscal determinants of debt are forecasted using quantile regression and the resulting projections are shown in the fan charts for India, Indonesia, Kazakhstan, Malaysia, the People’s Republic of China, the Philippines, the Republic of Korea, Sri Lanka, and Thailand. Furthermore, the fan charts present the uncertainty in the path of debt, especially in the aftermath of the coronavirus disease (COVID-19) pandemic.

About the Asian Development Bank

ADB is committed to achieving a prosperous, inclusive, resilient, and sustainable Asia and the Pacific, while sustaining its efforts to eradicate extreme poverty. Established in 1966, it is owned by 68 members—49 from the region. Its main instruments for helping its developing member countries are policy dialogue, loans, equity investments, guarantees, grants, and technical assistance.

