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Curtailment of Economic Activity and Labor Inequalities

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Abstract

The worrying combination of the labor market tightness and the wage inflation in the US since the pandemic raises a question on how the business closure orders affected the fragile segments of the labor force and contributed to mounting inflationary wage pressure. We develop a macroeconomic model with heterogeneous labor and a nested CES production function. We estimate the model using the newly collected data from the CPS and the BEA. The recent crisis leads to a contraction in total hours worked, makes wages more volatile, and sustains wage inflation. The model also generates differential effects of the business closure orders on productivity and the labor market in the US. The earning rates and hours responses to the crisis differ by age, skills, and origin of the worker.

Keywords: productivity shock, labor inequalities, heterogeneous labor, business closure orders.

JEL Codes: E20, E24, J01.

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1 Introduction

The Covid-19 outbreak had a tremendous impact on the U.S. labor market. The recent data from the U.S. Bureau of Labor Statistics confirm this thought. The total hours worked in the nonfarm business sector dropped sharply in 2020 after several states ordered citizens to stay home and imposed a shutdown of non-essential businesses. The U.S. second and third quarters' total hours (Y-o-Y) fell by 13.7 and 7%, respectively, relative to the second and third quarters of 2019, a year not affected by the Covid-19 crisis. This decline is perhaps the greatest in terms of its nature and the labor market implications of business closure orders need further analysis. The labor market inequalities in the U.S. appear as one of the most worrying trends, particularly among the most vulnerable workers (Yasenov, 2020). A similar conclusion is reached by Adams-Prassl et al. (2020), who show that the pandemic hit harder the fragile segments of the labor force. It thus seems that certain groups of workers such as young, low-skilled, and non-native workers are lagging far behind other segments in how they fully recover from the crisis.

There has been a well-documented decline in total productivity during the Covid-19 lockdown. The US economy experienced a huge decrease in the gross value added of nonfarm businesses (in the second quarter of 2020, it was 11.86% lower than it was in 2019) and it is then fundamental to analyze the repercussions that this drop in value added and production had on the careers of different types of workers. However, it remains unclear whether the responses of hours and wages to productivity shock are distributed more unequally across workers' types during the pandemic.

In this paper, we study the effects of the business closure orders and productivity shock on the US labor markets, in particular, the responses of labor and hourly earnings during the pandemic. We do so by developing a model with heterogeneous labor, nested CES production function, and three generations. These key elements allow the model to generate a differential effect of the productivity shock across workers that differs by age, skill, and origin. The model is estimated using the CPS and the BEA data. We focus

on the years 2019-2021 (pre- and post- Covid-19 crisis) so that we concentrate on the immediate labor impact and recovery after the shock across worker types in the different sectors. Our results suggest that the adoption of business closures orders by several states leads to the curtailment of economic activity and affected negatively the US labor market. The negative productivity shocks lead to a decline in total hours worked and excess wage inflation. While the shock hit sectors disproportionately, those sectors with non-essential activities were the most affected. Moreover, we find that the impact of this shock on workers is unequally distributed and the magnitude of the shock differs by age, origin, and skill of the worker.

This paper relates, first, to the literature on the effects of Covid-19 on the labor markets which shows that the disparity between workers in the reduced hours of work and employment is mostly due to the lower flexibility of jobs (Borjas and Cassidy, 2020; Yassenov, 2020) and to the higher spread of the virus among less “remotable” jobs (Basso et al., 2020).¹ Recent important contributions to this literature also include Cajner et al. (2020), Matias and Forsythe (2020), Cortes and Forsythe (2020), Adams-Prassl et al. (2020), and Leyva and Urrutia (2022). While the literature on the distributional effects of Covid-19 on labor markets mainly focuses on some segments of the labor force and could not address the macroeconomic implications of this shock, we instead propose an Overlapping Generations (OLG) model with heterogeneous labor to estimate and understand the impact of the productivity shock on labor markets.

Our study follows the literature adopting a structural approach to model heterogeneous labor. Our main reference is a recent paper by Busch et al. (2020), who employs an overlapping generations model to estimate the impact of refugees flow to Germany between 2015 and 2016 on different categories of workers. In particular, the authors assess the gross wage responses to the immigration inflow.² We instead develop a model with

¹ Fasani and Mazza (2020) use European data and find that extra-EU migrants are more likely to be exposed to a high risk of losing their job than natives, but only within industry and occupation, as many foreign workers are employed in what they identified as *essential occupations*.

² See also Imrohoğlu et al. (2017) who focuses on how immigration policy based on attracting high skilled

heterogeneous workers to assess the effects of the business closure orders on different types of workers, as we assume heterogeneity by age, origin, and skill. We also augment the model with a business closure shock to understand how the adoption of these orders in the US during the crisis contributed to the curtailment of productivity and affected substantially the dynamics in wages and hours worked. Other contemporaneous papers that explain the post-pandemic dynamics in the labor market are Faccini et al. (2022) who attribute the dynamics in the labor market to the high propensity of workers searching for a new job; Peri and Zaiour (2022) who emphasize the fall in the number of foreign workers as the main driver of US labor market dynamics; Domash and Summers (2022) who attempt to explain the labor market tightness to the increase in several vacant jobs in the US; and Tüzemen (2022) who documents a decline in the participation rate due to a large number of new retirees.³ Our paper differs from the aforementioned studies, as our model can generate differential effects of business closure orders on workers and suggest that the labor market is substantially tight.

Finally, we contribute to the literature that analyzes the labor inequality,⁴ and estimate the elasticity of substitution between the type of labor. This growing literature includes Berger et al. (2022) who focus on the differential effects of minimum wage. Other papers in this vein include Engbom and Moser (2021) and Hurst et al. (2022). We add to this literature by providing estimates of labor-labor elasticity which is crucial to understanding the distributional effects of the business closure orders on the labor market.⁵ Furthermore,

can mitigate Japan's fiscal imbalances problem. Storesletten (2000) employs an overlapping generations model to examine whether a selective immigration policy based on skills and age can have strong quantitative implications for U.S. public finance. He shows that an increased inflow of middle-aged and high and medium-skilled immigrants can sustain U.S. fiscal policy. Other papers adopting a structural approach to model immigration are Chassamboulli and Palivos (2014), Moreno-Galbis and Tritah (2016), and Ortega (2000), who focus on the effects of immigration on destination countries' labor markets.

³ See also Muehlemann and Leiser (2018) who show that labor market tightness is positively associated with hiring costs using Swiss data.

⁴ See for example Card et al. (2018), who interrogate whether differences in wages are driven by firm-specific productivity differentials.

⁵ To name a few of active literature on the estimation of elasticity of substitution, see, for instance, León-Ledesma et al. (2011) who use a normalized multi-level production function to estimate the elasticity of substitution; Borjas (2003) who use a simple supply-demand framework to estimate the labor supply elasticity of substitution. Alvarez-Cuadrado et al. (2018) who account for sectoral differences in the

the productivity shock mainly hit non-essential and less remearable industries, those industries are more likely to have a high share of low-skilled, young, and non-native workers. The model we present in this paper takes a stance by including these key features. We thus view our work as partly related to past papers which analyze the concentration of immigrants in specific sectors. Among these Fogel and Peri (2015), who show that migrants are concentrated in some low-skill sectors with high manual and low communication content, and their entry into the labor market leads low-skilled natives to shift towards non-manual (communication intensive) jobs that allow for native upgrading and an increase in natives' wages. In addition, Burstein et al. (2020) explore the labor impacts of immigration with worker heterogeneity in occupational productivity. They find that the effect varies across occupations, because of the concentration of high-skilled immigrant workers in computer-related jobs and low-skilled immigrants are concentrated in agriculture and manufacturing sectors.

Layout. The rest of the paper is as follows. Section 2 presents the trends in productivity and labor inequalities in the US. Section 3 laid out the theoretical model. Section 4 describes the data and the estimation of model parameters. Section 5 discusses the effects of productivity shocks on hours worked and wages. Section 6 concludes.

2 Labor Inequality and Business Closure: What Does the Data Tell us?

We start our analysis by documenting the recent trends in labor inequality. Next, we examine how the business closure orders make labor inequality worse, which we will later explain its implication on production and the labor market.

elasticity of substitution. Gechert et al. (2022) who revisit the existing approach in the estimation of the elasticity of substitution.

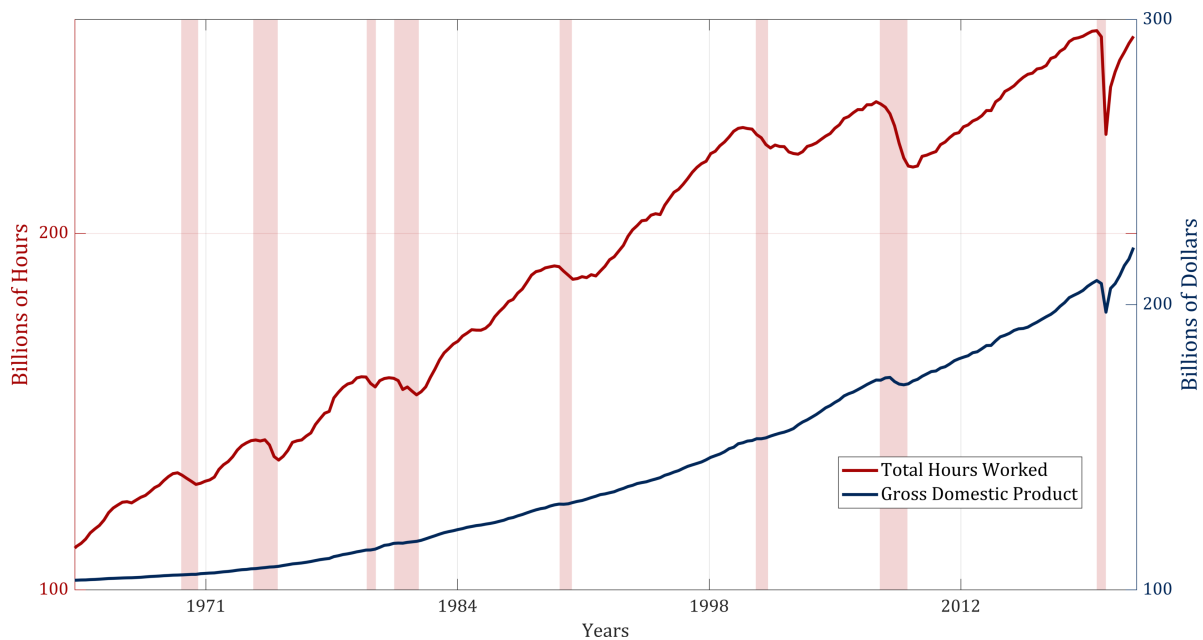
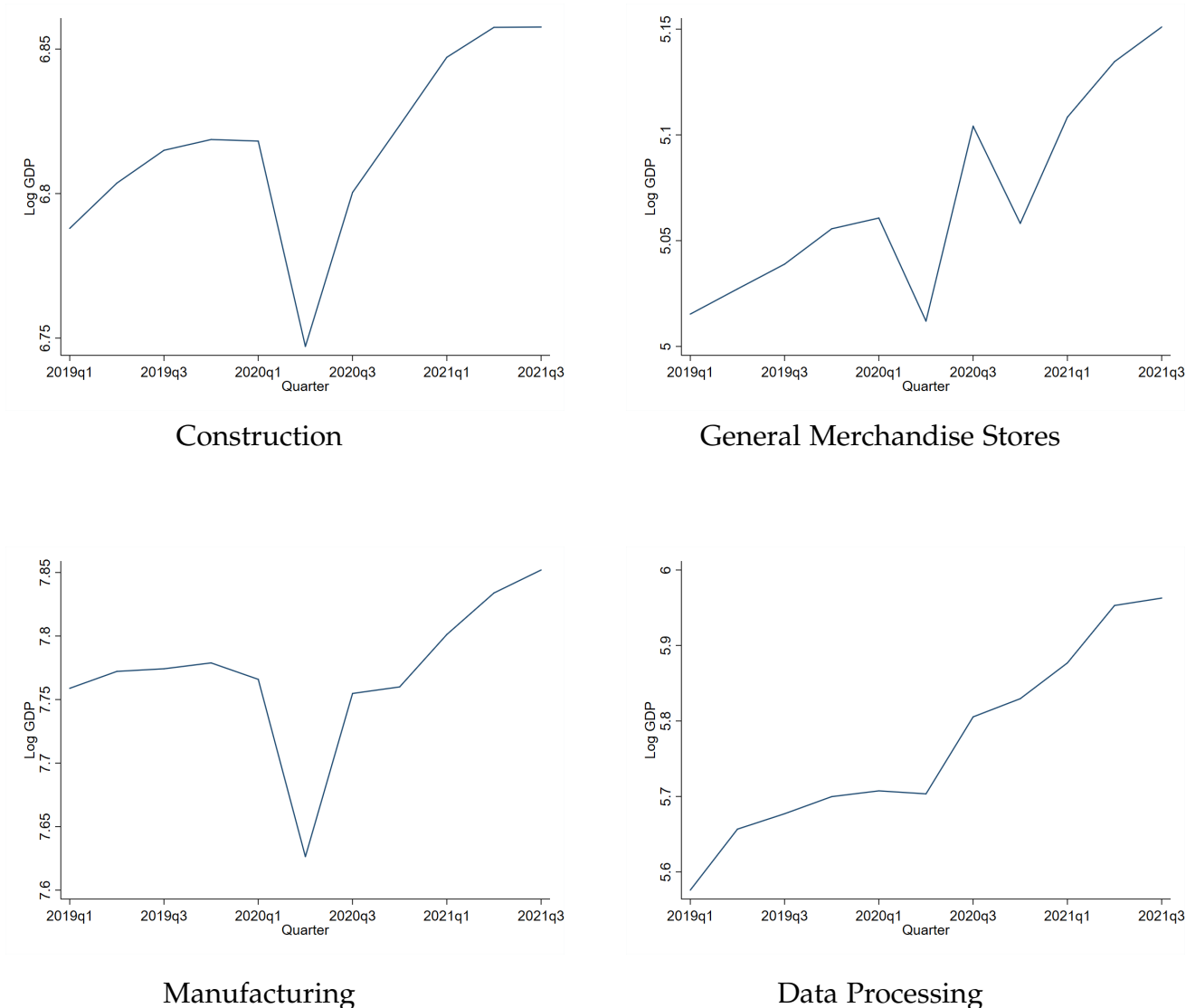


Figure 1: Hours Worked and Output Dynamics

Productivity and Hours Worked. As the U.S. economy emerged from the recent pandemic, it is worth examining the phases of this unprecedented crisis. Figure 1 plots the gross domestic product and total hours worked in the US over the period Q1-1964 to Q4-2021. Though the 2020 recession is the shortest, it has driven down the total hours. Indeed total hours worked fell dramatically in the second quarter of 2020 after most states across the US shut down the non-essential business to curb the spread of the virus. During this period, the gross domestic product reached the bottom in the second quarter. Many states have relaxed their restrictions on economic activities starting from the third quarter of the same year. Then, total hours worked went up, and following this trend, the gross domestic product rose rapidly giving a sign of economic recovery. This crisis brings attention to the impact of the reduction in productivity during the pandemic on the labor market. As Figure 2 shows, the impact of the crisis varied by sector, and this effect is likely due to the possibility of teleworking by sector and its *emergency* nature. For example, sectors such as Construction, Merchandise Stores, and Manufacturing, which require the physical presence of a high share of workers, were hardly hit by the crisis,

whereas it had a small (or even null) impact on the production of the Data Processing sector, which has a high share of workers who can work remotely.

Figure 2: Log-GDP by Sector: Some Examples



Distribution of the Average Hourly Wage Exploring the most recent data from the CPS during the period 2018-2021 reveals the potential difference in the hourly earning distribution of different types of workers. Figures 3- 6 show the average wage distribution of natives and foreign workers in four broad classes. Two important facts appear from these figures. Between 2018 and 2021, there is a much more unequal distribution

of wages between native and foreign workers when they are less skilled and middle-aged (Figure 3). Though this divide between these two types of workers (native vs foreigner) is less evident in the other classes. We doubt that this difference, observed in Figure 3, is because foreign workers newly arriving in the labor force in the US may face difficulties integrating the labor market. They may see their job experience not recognized or evaluated differently, and then continue to earn less than their counterparts.

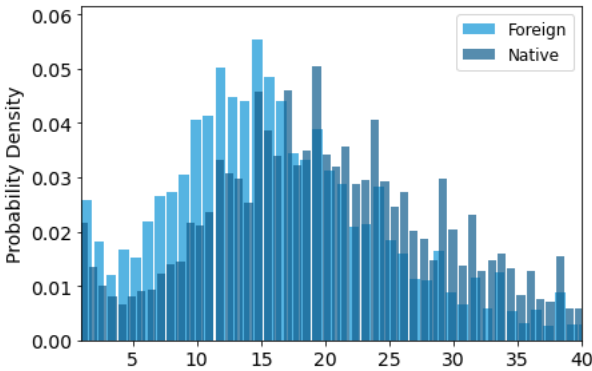


Figure 3: Middle-aged and Low Skill

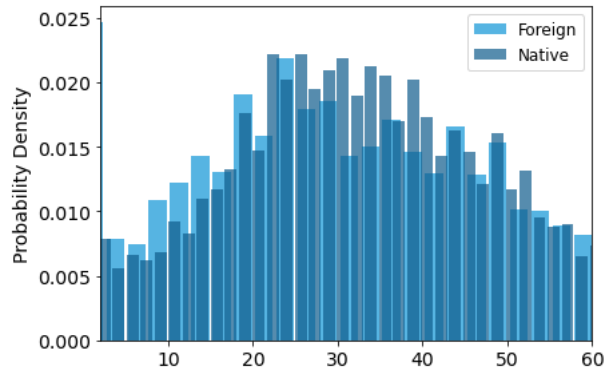


Figure 4: Middle-aged and High Skill

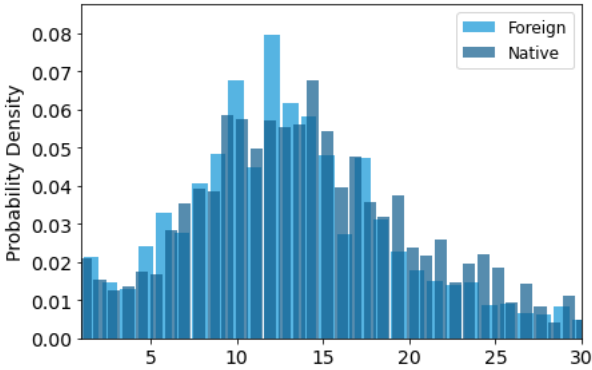


Figure 5: Young and Low Skill

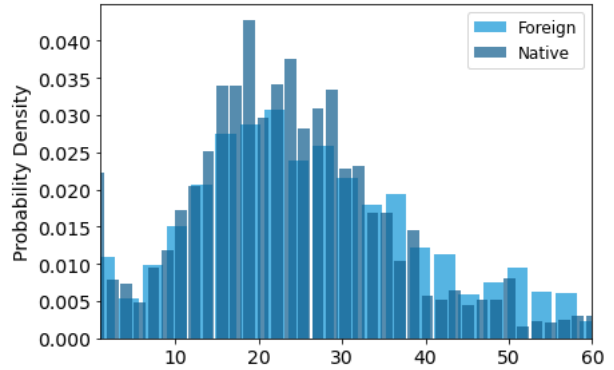


Figure 6: Young and High Skill

Figure 4 reports a small difference in the distribution of average wage rate between natives and foreigners when middle-aged and with high educational levels. It appears that there is a small shift of the native worker into the upper echelons of the hourly earnings distribution. Figure 5 illustrates that the average wage rate of workers with lower

educational levels lags further behind workers with high educational levels. However, among young workers with lower educational levels, there is no clear difference in the hourly wage rate between native and foreign workers (Figure 5).

Throughout the same period, Figure 6 indicates that the wage distribution of young workers with high skills is slightly skewed to the right. Reflecting the case that young workers are, on average, paid less than middle-aged workers with similar skills. Contrary to our expectations, we observe that the difference in earnings distribution between natives and foreigners when young and highly skilled is very small and the hourly earnings of foreign workers only become dominant at the top of the distribution (Figure 6).

Labor Inequalities: A Suggestive Evidence. If we think of labor inequalities, we might consider separately the average wage rate and hours worked by type of worker. Table 1 provides an example that illustrates substantial differences between wages and hours worked between comparable workers.

Productivity shocks may contribute to labor inequalities between workers of different

Table 1: Weekly Hourly Wage and Hours Worked 2018-2021

	<i>Hourly Wage</i>				<i>Hours</i>			
	low skill		high skill		low skill		high skill	
	immigrant	native	immigrant	native	immigrant	native	immigrant	native
<i>Young workers</i>								
2018	14.30	15.77	36.30	25.58	35.55	34.57	39.15	39.44
2019	14.27	15.89	36.32	25.65	35.55	34.54	39.15	39.41
2020	14.34	16.06	36.37	25.83	35.47	34.48	39.10	39.40
2021	14.35	16.10	37.18	25.88	35.47	34.56	39.16	39.41
<i>Middle aged workers</i>								
2018	17.65	21.97	37.84	38.37	38.82	40.03	40.07	41.07
2019	17.89	22.39	37.65	38.98	38.83	40.07	40.15	41.13
2020	17.82	23.70	38.04	39.13	38.61	39.45	39.98	40.70
2021	18.07	24.07	38.26	39.95	38.79	39.76	40.13	40.83

origins, skills, and ages. For instance, low-skilled workers performing non-remote-able jobs can be more vulnerable to government-mandated business closure. A temporarily closing of non-essential businesses can lead to worsening the pre-existing disparities

between workers with similar characteristics. Yet, in Table 1, we find that the average hours per week worked fell for all age and skill groups in 2020. A conceivable explanation for this variation may be the restrictions on economic activities during the pandemic outbreak. We also see a large difference between average hours worked per week by type of worker, these differences among workers are also intriguing in how they exacerbate labor disparities.

In Table 1, we report the average weekly wage rate from the CPS from 2018 to 2021. This table shows reliable evidence of an increase in hourly earnings inequality between native and immigrant workers within age and skill categories. Throughout the period, middle-aged foreign workers earn lower wages than native workers with similar skills and age. It is also true that a young foreign worker with low skills makes less than a comparable native worker. Perhaps surprisingly, the young immigrant with high skills earned more than what the native earned.

We provide an intuition of the patterns of wages and hours worked during the years 2018-2021 in Table 1. There are many plausible factors explaining the discrepancy between native and foreign workers. One explanation is the concentration of immigrants in low-paying occupations or sectors, it is also possible that there are some barriers to getting into some occupations that require some professional licensing. Not only the concentration of immigrants in specific sectors may be important in explaining low wages among these workers, but also those foreign workers may be less informed about the market wages and thus can accept lower wages.

We will use the relationship between wages and business closure orders to suggest a macroeconomic model that ideally accounts not only for the effect of productivity shock on labor but also for the effects of business closure on the wage and hours worked across different types of workers.

3 An OLG Model with Heterogeneous Labor

We start this section by presenting an overlapping generations model. In this model, we adopt a nested constant elasticity of substitutions (CES) production function, which allows us to consider the differential effect of a shock on productivity across workers. We allow for imperfect substitutability (i.e., the elasticity of substitution different from one) among workers of a given age (young vs. middle-aged) to understand how changes in productivity increase inequalities in the labor market, and later between workers of a given age (young vs. middle-aged), origin (native vs. foreign), and skills (high-skill vs. low-skill).

Households We start with a simple overlapping generations model and we consider three generations, each is alive at any point in time and maximizes the life time utility

$$U(c_{s,k,t}^j, l_{s,k,t}^j)$$

Let the instantaneous utility function $U(c)$ be a logarithmic function and $U(l)$ be a constant relative risk aversion $(l_t^j)^{1+\eta}/1+\eta$, where η is the curvature on the disutility of labor. We assume that the parameter η is the same for all types of workers. We have three generations at each time t where $j \in \{y, m, o\}$ identifies age groups: young, middle, and old age. We let s denote the skill of the agent with $s \in \{h, l\}$, and k represent the origin of the agent with $k \in \{f, n\}$. Note that each agent faces a specific budget constraint but the marginal utilities are the same across the three age groups. The budget constraint of each agent is therefore given by:

$$\begin{aligned} c_{s,k,t-1}^y + a_{s,k,t-1}^y &= \omega_{s,k,t-1}^y l_{s,k,t-1}^y \\ c_{s,k,t}^m + a_{s,k,t}^m &= \omega_{s,k,t}^m l_{s,k,t}^m + a_{s,k,t-1}^y (1 + r_{t-1}) \\ c_{s,k,t+1}^o &= a_{s,k,t}^m (1 + r_t) \end{aligned}$$

A household has units of time to spend on labor, let $l_{s,k}^y$ and $l_{s,k}^m$ represent the labor supply by young and middle age individuals, respectively. Moreover, old individuals (retirees) do not participate in the labor market. Let $\omega_{s,k}^j$ denote the average hourly earnings of a worker. $c_{s,k}^y, c_{s,k}^m, c_{s,k}^o$ denote the consumption of an agent belonging to the young, middle-aged and old generations. We further assume that both young and middle age agents save $(a_{s,k}^y, a_{s,k}^m)$, but only the middle age and old age agents receive the return at risk-free interest rate r . The young and middle age agents consume and work in each period, in the first and second period of life, while old agents earn no labor income in the third period of life but receive retirement income $a_{s,k,t}^m(1+r)$ and consume. Finally, in this economy, we assume that old agents are non-altruistic and consume all they have.⁶

Labor-Labor Substitution. Now, we shall present the labor force equation by assuming that the elasticity of substitution among workers for given age, origin, and skill are different and show that nesting the labor subgroups does change the results we have. In this economy, we distinguish labor by age (young and middle-aged), national origin (native and foreign), and sectors (high and low skilled sectors). Let l_t^y denote the total hours worked by young individuals and consider the separability of the labor factor. The hours worked by younger individuals are given by:

$$l_t^y = [(\phi_{h,f}(l_{h,f,t}^y)^{\frac{1}{\sigma}} + \phi_{h,n}(l_{h,n,t}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma-1}} + (\phi_{l,f}(l_{l,f,t}^y)^{\frac{1}{\sigma}} + \phi_{l,n}(l_{l,n,t}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma-1}}]^{\sigma-1},$$

this equation emphasizes that hours worked by younger individuals young is broken down into various category: origin (native or foreign) and skill (high or low) groups. We assume that labor inputs l_t^y is a vector of four types of labor: hours worked by foreign worker in high skilled sector $l_{h,f,t}^y$; hours worked by native worker in high skilled sector $l_{h,n,t}^y$; hours worked by foreign worker in low skilled sector $l_{l,f,t}^y$; and hours worked by

⁶ Our model adds the age category of the workers, which is an important feature in this model. Furthermore, the use of the three-period overlapping generations model with heterogeneous labor in which households decisions are age-dependent is convenient for our analysis.

native worker in low skilled sector $l_{l,n,t}^y$. Let $\varrho(v)$ denote the elasticity of substitution between native and foreign workers of young age and working in the high (low) skill sector. The parameter σ denotes the elasticity of substitution between high and low-skilled workers when young.

Similarly, the hours worked by middle-aged individuals are given by:

$$l_t^m = [(\theta_{h,f}(l_{h,f,t}^m)^{\frac{1}{\varepsilon}} + \theta_{h,n}(l_{h,n,t}^m)^{\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\zeta}} + (\theta_{l,f}(l_{l,f,t}^m)^{\frac{1}{\zeta}} + \theta_{l,n}(l_{l,n,t}^m)^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}}]^{\xi},$$

the term l_t^m represents the total hours worked by young (middle-aged) individuals and encompasses four labor groups. The hours worked by middle-aged individuals include hours worked by natives and foreigners and by high and low skilled. The labor inputs l_t^m is then a vector of four types of labor: hours worked by foreign worker in high skilled sector $l_{h,f,t}^m$; hours worked by native worker in high skilled sector $l_{h,n,t}^m$; hours worked by foreign worker in low skilled sector $l_{l,f,t}^m$; and hours worked by native worker in low skilled sector $l_{l,n,t}^m$. We also distinguish between the elasticity of substitution between middle-aged natives and foreign workers, with ε (ζ) that represents the elasticity of substitution between middle-aged workers with high (low) skill. We also assume that the elasticity of substitution between middle-aged high and low-skill workers is given by ξ .

Labor is disaggregated according to several criteria. That is, the relative supply elasticities between native and foreign workers are not identical given the age and the sector. A notable study on the substitution between workers is Ottaviano and Peri (2008), which adopted a nested constant elasticity of substitution production function. The key assumption in their nested CES framework is that for labor inputs, workers of different education levels are split into specific education subgroups, and those groups are then nested into groups with different experience levels. Within the same education and experience group, they identify US-born and foreign-born workers. Our framework instead considers age categories (young vs middle-aged workers) and assumes a split

between high-skilled native workers, high-skilled foreign workers, low-skilled native workers, and low-skilled foreign workers. Moreover, we do not restrict the elasticity of substitution to be the same between high and low-skilled workers of different ages (young and middle-aged). We also do not restrict the elasticity of substitution to be the same between foreign and native workers with different ages (young and middle-aged) and skills (high skill and low skill). Nonetheless, the framework provides new estimates of labor-labor elasticities, adding features such as age, skill, and origin of workers.

Technology Firms choose labor to maximize profits subject to the production function. The production function that relates outputs to inputs takes the constant elasticity of substitution (CES) form:

$$y_t = z_t (\alpha (l_t^y)^{\frac{s-1}{s}} + (1 - \alpha) (l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}} \quad (3.1)$$

where α is a distribution parameter that determines how important the two labor factors are in aggregate production, z_t is the productivity shock, and s denotes the elasticity of substitution between hours worked by young (l_t^y) and middle-aged workers (l_t^m). For each age group, we distinguish between natives and immigrants, and high and low skilled workers. In this case, we consider sectors that employ high numbers of high-skilled workers and sectors with a high share of low-skilled workers. When $s > 1$, the two types of workers are gross substitutes, and when $s < 1$, the two types of workers are complements. Following Gourio (2013), we assume

$$\ln(z_t) = \rho^z \ln(z_{t-1}) + \delta d_t + \epsilon_t^z; \quad (3.2)$$

where ρ^z is the productivity smoothing parameter δ is the productivity response to the business closure orders. Following (Koren and Pető, 2020) we construct a measure that reflects the share of workers affected by the business closure restrictions. We can interpret

this measure as the occurrence of the business closure orders and how it affects economic activity. For example, prior and after to the pandemic crisis, this share will be equal to zero, but when the Covid shock hits, the share will have a value between 0 and 1 and will vary across industries, given that some industries may have high or low affected shares.

Asset Market The total amount of asset in the economy evolves as follow $a_t = a_{t-1} + \tau_t$, where τ_t is the new allocated saving in this economy. The saving flow equation is given by $a_{t-1}(1 + r_{t-1}) = a_t$, $a_t \geq 0$, that simplifies to $r_{t-1}a_{t-1} = \tau_t$. This condition states that the return on saving equals the newly allocated saving. The market clearing condition is given by

$$a_t = y_t - c_{s,k,t}^y - c_{s,k,t}^m - c_{s,k,t}^o$$

where $a_t = a_{s,k,t}^y + a_{s,k,t}^m$, this equation reveals that total savings in this economy is equal to the difference between total output and the consumption of the three generations at time t .

The Definition of Equilibrium. We define an equilibrium as a collection of quantities, and prices such that, (i) Households choose $\{c_t^y, c_t^m, c_t^o, l_t^y, l_t^m, a_t\}$ in order to maximize their utility subject to the budget constraints; (ii) Producers choose how much of each input to employ $\{l_{h,f,t}^y, l_{h,f,t}^m, l_{l,f,t}^y, l_{l,f,t}^m, l_{h,n,t}^y, l_{h,n,t}^m, l_{l,n,t}^y, l_{l,n,t}^m\}$ to minimize their production cost. The first order conditions yields the market prices at the equilibrium $\{\omega_{h,f,t}^y, \omega_{h,f,t}^m, \omega_{l,f,t}^y, \omega_{l,f,t}^m, \omega_{h,n,t}^y, \omega_{h,n,t}^m, \omega_{l,n,t}^y, \omega_{l,n,t}^m\}$. (iii) Equilibrium requires that $a_t = y_t - c_t$, holds and by definition we have saving accumulation condition $a_t = a_{t-1} + \tau_t$, and the market for assets clears $a_t = a_{t-1}(1 + r_{t-1})$.⁷

⁷ Detailed computations are contained in the technical appendix.

4 Data and Estimation

In this section, we set the value of the main parameters of the model such that it matches the key facts of the U.S. labor market. We measure the remaining parameters directly from U.S. data.

4.1 Data, Summary Statistics and Construction of Variables.

Data. Our main data source of workers' data is the Current Population Survey (CPS) conducted by the Census Bureau. This data seeks to collect information on each individual of households over 15 years of age. This dataset has several advantages and collects information about the U.S. labor force characteristics every month, such as the average hours worked per week, education level, age (ages 14-24, ages 25-64), origin, industry, and county.

Additionally, we use quarterly data on Gross Domestic Product (Value Added) by industry at the county level as released by the Bureau of Economic Analysis (BEA) of the U.S. Department of Commerce, and focus on the period 2019-2021. The total factor productivity is calculated using the value added and labor input at the industry level.

In the US, several states have enforced business closure orders starting from the second quarter of 2020 to curb the spread of the virus, these restrictions have been largely lifted in the third quarter of 2020. This is why we use a sample that covers 2019-2021, as it covers the period before and after the public health restrictions on businesses.

Summary Statistics. The summary statistics in Table 2 shows the weekly average hours worked which is equal to 37 with a standard deviation of 11 hours. There is a substantial variation in hours worked and wages among individuals surveyed. Our initial dataset contains 4,274,781 individuals. We drop observations with missing or zero usual hours worked and missing wages and, then, winsorize wages at 1 and 99 percentiles. This leaves us with 1,910,679 (monthly) observations. 16 percent of individuals are immigrants,

Table 2: Summary statistics

Variable	Mean	Std. Dev.	Min.	Max.	N
Immigrant	0.158	0.365	0	1	1,910,679
Young (15-30)	0.232	0.422	0	1	1,910,679
Middle-aged (30-64)	0.692	0.462	0	1	1,910,679
Old (> 64)	0.076	0.265	0	1	1,910,679
High skilled (Degree or higher)	0.395	0.489	0	1	1,910,679
Low skilled (Less than degree)	0.605	0.489	0	1	1,910,679
Hours worked last week	38.653	13.034	1	198	1,910,679
Wage and salary income (yearly data)	53322.105	52007.357	0	320,000	222,240

23% are young, and slightly less than 40% are high-skilled (i.e., at least with a bachelor's degree). The yearly wage exhibits substantial volatility, with the mean equal to \$53,322 and the standard deviation being \$52,007.

Data for the estimation of the elasticities Using the CPS data, we computed weighted averages by skill level (we consider those who earned a degree as high-skilled and the others as low-skilled), age category (young: 15-30, middle-aged: 30-64, old: >64), industry, origin (foreigner or native), and county quarterly between 2019 and 2021. This collapse leaves us with a sample of 449,556 observations. To proceed with the estimation of elasticities, we reshape the data from long to wide and use the year-month-industry code-state-county as the cross-section identifiers. We will end up with data that is linked using these identifiers. There are also empty cells, for instance, it may happen that a specific industry does not exist in a specific county. To overcome this issue, we replace missing values with the mean of each column. Our sample thus reshaped now contains 98,709 observations.

Data for estimation of the productivity shock. To estimate the productivity shock, we start by computing the total hours worked. We need to collapse the CPS data by quarter and industry. Starting from the same initial dataset of 1,910,679 of monthly observations at the individual level, we collapse the CPS dataset by quarter (from Q1 2019 to Q3 2021) and industry. For what concerns the Covid-19 shock, we follow Koren and Pető (2020),

which used the Occupational Information Network dataset. This dataset includes the share of workers affected by Covid-19 restrictions in each industry at 3-digit NAICS. Some industries may have been more severely affected than others affected. One can see that the affected share is a measure of the percentage of workers hit by the restrictions, which in this case is an appropriate proxy of business closure orders.

Using the Bureau of Economic Analysis BEA (data), we can observe the total output at the quarter and sector level (with 671 observations). A sector is identified by the 3-digit NAICS code⁸. For the collapsed CPS data at the quarter level from Q1 2019 to Q3 2021 (with 3052 observations), we can observe the hours worked in each quarter and within each sector or sub-sector. We assign the specific industry (3-digit NAICS) code for each sub-sector to standardize it to the format of the BEA dataset. We drop data that contain 0 in the quarter column, this leaves us with 2782 observations. We then, aggregate the data at the sector level (3-digit NAICS code), to make the CPS data comparable to the BEA data, such that observation is an industry-quarter. After this, we end up with 693 observations. Additionally, we assign the affected share in each quarter and within each sector to the appropriate quarter-industry level.

We finally merge data from the BEA with the CPS data to obtain quarterly information on output, hours, and affected share at the industry level (3-digit NAICS code) between Q1-2019 to Q3-2021. As a result, our sample contains 671 observations that we will use for the estimation of the productivity shock parameters.

Construction of Variables. As the CPS includes questions on wages only in the ASEC module (the March module) and not monthly, we construct a measure of the hourly wage rate (measured in \$) as follows:

$$\text{Hourly Wage Rate} = \frac{\text{Wage Income Annual}}{\text{Weeks} \times \text{Hours Worked per Week}}$$

⁸ For a complete description of the industry code see Appendix C.

where *Hours Worked per Week* is computed from the monthly CPS.

To model the productivity shock due to the Covid-19 pandemic, we followed (Koren and Petř, 2020) which used the Occupational Information Network (ONET) dataset to measure the share of workers affected by the restrictions by industry. To compute this share, the authors build three indices (Teamwork, Customer, and Presence) that account for face-to-face interactions along these dimensions in more than 1,000 occupations included in the ONET dataset. Then, they calculate the share of affected workers based on the share of workers in each occupation by industry (NAICS).

Note that for our exercise the aggregate productivity shock z_t will depend on our measure of business closure order variable d_t , which is defined as follows:

$$d_{i,t} = \text{Affected Share}_{i,t}$$

the variable $d_{i,t}$ is proxied by the affected share in industry i at time t from the ONET dataset. Additionally, we approximate the value of total factor productivity z_t using the production function equation as follows:

$$\ln z_{i,t} = \ln y_{i,t} - \ln l_{i,t}$$

$$\ln \text{Total Factor productivity}_{i,t} = \ln \text{Value Added}_{i,t} - \ln \text{Hours Worked}_{i,t}$$

here again, we use the value added at the industry level from the BEA data and the total hours worked from the CPS data which is aggregated at the industry level.

4.2 Estimation

Preset parameters. We first set those parameters that are commonly used in the literature. We set the value of the discount factor $\beta = 0.97$ to target an interest rate of 3 percent. We set the value of the curvature on the disutility of labor $\eta = 1.5$ based on a standard

value in the literature, see for example (Chetty et al., 2011).

Estimated parameters for labor-labor Substitution. Our sample contains weighted averages of hours worked and wage rate by skill level, age category, industry, origin, county, and quarter-year. We reshaped the data in a way that we can estimate the elasticity of substitution between different types of labor. We first calculate the sum of hours supplied by young and middle-aged workers

$$l^y = l_{h,n}^y + l_{l,n}^y + l_{h,f}^y + l_{l,f}^y \quad l^m = l_{h,n}^m + l_{l,n}^m + l_{h,f}^m + l_{l,f}^m$$

The youth labor share parameter is given by $\alpha = l^y / (l^y + l^m)$, we use this equation to pin down α , implying a value of 0.48 (see Panel B in Table 4).

In the second step, we estimate the distribution parameters measuring the specific labor intensity of given labor to total labor supplied by younger workers with

$$\theta_{h,n} = l_{h,n,t}^y / l_t^y, \quad \theta_{h,f} = l_{h,f,t}^y / l_t^y, \quad \theta_{l,n} = l_{l,n,t}^y / l_t^y, \quad \theta_{l,f} = l_{l,f,t}^y / l_t^y.$$

The intensity of a type of labor can be simply measured by taking the ratio of a given type of labor to total labor supplied by middle-aged workers

$$\phi_{h,n} = l_{h,n,t}^m / l_t^m, \quad \phi_{h,f} = l_{h,f,t}^m / l_t^m, \quad \phi_{l,n} = l_{l,n,t}^m / l_t^m, \quad \phi_{l,f} = l_{l,f,t}^m / l_t^m.$$

Note that Panel C in Table 4 presents the values assigned to each parameter.

If we assume constant labor intensity parameters $\{\theta, \phi\}$ over time, one can easily estimate the elasticity parameters using the producer Euler equation. The estimation of the elasticity of substitution $\{\varrho, \nu, \varepsilon, \zeta\}$ between the type of labor can be easily identified using the Euler equations from the producer optimization. Our estimates of elasticity use the CPS data between 2018 and 2021. More formally, we take the logarithm of the Euler

equations:

$$\ln(\omega_{h,f,t}^y) - \ln(\omega_{h,n,t}^y) = (1/\varrho - 1) \left[\ln(l_{h,f,t}^y) - \ln(l_{h,n,t}^y) \right],$$

we introduce county α_i and year α_t fixed effects to absorb time trends and county characteristics that may affect the wage differentials. We can find the elasticity of substitution using the following specification $\ln \omega_{i,h,f,t}^y - \ln \omega_{i,h,n,t}^y = 0.14092(\ln l_{i,h,f,t}^y - \ln l_{i,h,n,t}^y) + \alpha_t + \alpha_i + \epsilon_{i,t}$. The estimation of the elasticity of substitution between foreign and native workers with equal skill and age shows that young-high skilled foreign and native workers are perfect substitutes (when the elasticity is positive, $1/\varrho - 1 = 0.14092$), this is different from the estimates of Ottaviano and Peri (2008) (see Table 3).

In a simple form, we can also estimate the elasticity of substitution between young-low skilled foreign and native workers using

$$\ln(\omega_{l,f,t}^y) - \ln(\omega_{l,n,t}^y) = (1/v - 1) \left[\ln(l_{l,f,t}^y) - \ln(l_{l,n,t}^y) \right],$$

Our empirical specification includes the time fixed effects as well as the county fixed effects. We also find a positive elasticity ($1/v - 1 = 0.11723$) between young-low skilled foreign and native workers, this reflects some degree of perfect substitution between foreign and native workers.

The elasticity between middle-aged foreign and native workers with high skills is captured by

$$\ln(\omega_{h,f,t}^m) - \ln(\omega_{h,n,t}^m) = (1/\epsilon - 1) \left[\ln(l_{h,f,t}^m) - \ln(l_{h,n,t}^m) \right],$$

On the other hand, our estimate favors a perfect substitution between middle-aged foreign and native workers with high skills. A positive elasticity $1/\epsilon - 1 = 0.05898$ implies that native workers when middle-aged and with high skill experience, suggest

little substitution between the two categories.

We also use the Euler equation to estimate the elasticity of substitution between middle-aged foreign and native workers with low skill

$$\ln(\omega_{l,f,t}^m) - \ln(\omega_{l,n,t}^m) = (1/\zeta - 1) \left[\ln(l_{l,f,t}^m) - \ln(l_{l,n,t}^m) \right].$$

We also find evidence of perfect substitution between middle-aged foreign and native workers with low skill, with positive elasticity of substitution $1/\zeta - 1 = 0.11216$. We report all the estimates of the elasticity of substitutions in Table 3

Before moving to the estimation of the elasticity between young and middle age workers, it is important to explain how our estimates relate to the emerging literature trying to estimate the labor-supply elasticity. In particular, the question of imperfect substitution between native and immigrant workers has been raised early on in Ottaviano and Peri (2008) and D'Amuri et al. (2009), which find imperfect substitutability between natives and immigrants. Though our estimates differ substantially, we think this is because we have a different specification of the estimated Euler equations.

An additional parameter that turns out to be important for our empirical analysis is the elasticity of substitution between middle-aged and young workers s . Note that the producer optimization implies an Euler equation which is given by

$$\frac{\omega_t^m}{\omega_t^y} = \frac{1 - \alpha}{\alpha} \left(\frac{l_t^m}{l_t^y} \right)^{-\frac{1}{s}}.$$

From the first order condition for producer's cost minimization, one can write it in logarithmic form

$$\ln(\omega_t^m) - \ln(\omega_t^y) = - (1/s) \left[\ln(l_t^m) - \ln(l_t^y) \right],$$

Table 3: Elasticity of substitution parameters

$1/\varrho - 1$	$1/\nu - 1$	$1/\varepsilon - 1$	$1/\xi - 1$	$1/s$	$1/\sigma - 1$	$1/\xi - 1$
0.14092*** (0.00558)	0.11723*** (0.00604)	0.05898*** (0.00593)	0.11216*** (0.00680)	0.07658*** (0.00623)	0.03924*** (0.00512)	0.07146*** (0.00638)

Note: The estimates in this table is obtained using a simple OLS with county and year fixed effects, the number of observation is equal to 98709. Homoscedastic standard error in parentheses $*p < .1$, $**p < .05$, $***p < .01$

Table 3 presents the estimates of the elasticity of substitution $\frac{1}{s}$, we include both county and year fixed effects in the regression. Here we note that the estimated $\frac{1}{s}$ is positive and statistically significant.

When we turn to the elasticity of substitution between high and low-skilled workers, our model yields two key Euler equations to estimate $(1/\sigma - 1)$ and $(1/\xi - 1)$ using the expressions

$$\ln(\omega_{h,t}^m) - \ln(\omega_{l,t}^m) = (1/\xi - 1) [\ln(l_{h,t}^m) - \ln(l_{l,t}^m)],$$

we find that the elasticity of substitution between high and low-skill workers is $1/\xi - 1 = 0.07$ among middle-aged workers (see Table 3), this suggests that high and low-skill middle-age workers are substitutes. We also run the regression for the Euler equation that captures the elasticity of substitution between high-skill and low-skill young workers

$$\ln(\omega_{h,t}^y) - \ln(\omega_{l,t}^y) = (1/\sigma - 1) [\ln(l_{h,t}^y) - \ln(l_{l,t}^y)],$$

Table 3 also presents the estimates of $1/\sigma - 1$. Our specification includes county and year fixed effects to remove time trends and specific county factors that can explain wage differentials. Ignoring the origin of the worker, once again the elasticity of substitution between high skill and low skill workers of young workers $1/\sigma - 1$ is positive and equals 0.03. These results imply that the two groups are perfect substitutes. These estimates are substantially different from the estimates of McAdam et al. (2011), who find high elasticity of substitution between skilled and unskilled labor.

Table 4: Summary of Parameters

Parameter	Description	Value	Source
<i>Panel A: Household</i>			
β	Discount factor	0.95	Standard
η	The curvature on the disutility of labor	1.5	Chetty et al. (2011)
<i>Panel B: Producer</i>			
α	Youth labor share parameter	0.48	CPS
s	Elasticity of substitution between labor factor l^y and l^m	-13.06	CPS
ρ^z	Autocorrelation parameter of technology shock	0.9832	BEA
σ^z	Standard deviation of technology shock	0.0835	BEA
δ	The responses of technology shock to the activity specific affected share	-0.0017	BEA/ ONET
<i>Panel C: Labor - Distribution parameters</i>			
$\phi_{h,f}$	high-skilled young foreign labor share parameter	0.26	CPS
$\theta_{h,f}$	high-skilled middle-aged foreign labor share parameter	0.25	CPS
$\phi_{h,n}$	high-skilled young native labor share parameter	0.27	CPS
$\theta_{h,n}$	high-skilled middle-aged native labor share parameter	0.26	CPS
$\phi_{l,f}$	low-skilled young foreign labor share parameter	0.24	CPS
$\theta_{l,f}$	low-skilled middle-aged foreign labor share parameter	0.24	CPS
$\phi_{l,n}$	low-skilled young native labor share parameter	0.23	CPS
$\theta_{l,n}$	low-skilled middle-aged native labor share parameter	0.25	CPS
<i>Panel D: Labor - Elasticity of substitution parameters (high vs low-skilled)</i>			
σ	Elasticity of substitution between labor factor l_h^y and l_l^y	0.96	CPS
ξ	Elasticity of substitution between labor factor l_h^m and l_l^m	0.93	CPS
<i>Panel E: Labor - Elasticity of substitution parameters (natives vs foreigners)</i>			
ϱ	Elasticity of substitution between labor factor $l_{h,f}^y$ and $l_{h,n}^y$	0.88	CPS
ν	Elasticity of substitution between labor factor $l_{l,f}^y$ and $l_{l,n}^y$	0.89	CPS
ε	Elasticity of substitution between labor factor $l_{h,f}^m$ and $l_{h,n}^m$	0.94	CPS
ζ	Elasticity of substitution between labor factor $l_{l,f}^m$ and $l_{l,n}^m$	0.89	CPS

Empirical specification of the productivity shock. Turning to the productivity shock, we now estimate the total factor productivity using the production function

$$y_{i,t} = F(z_{i,t}, l_{i,t})$$

where $y_{i,t}$ the amount of gross domestic product in industry i at time t depends on productivity shock $z_{i,t}$ in industry i at time t and labor supply $l_{i,t}$ in industry i at time t . Taking the logs we can write $\ln(z_{i,t}) = \ln(y_{i,t}) - \ln(l_{i,t})$ which can be estimated using data on gross domestic product and labor supply at industry level.

Due to the limitation in the data, we abstract from dis-aggregating labor by type for the

estimation of the total factor productivity z . We use quarterly data on gross domestic product and labor supply (hours worked) at the industry level to compute z . Then, we estimate a regression of the following form

$$\ln(z_{i,t}) = \rho^z \ln(z_{i,t-1}) + \delta d_{i,t} + \epsilon_{i,t}^z;$$

where ρ^z captures the persistence of productivity shocks and $\epsilon_{i,t}^z$ is an i.i.d. normal random variable with mean zero and variance σ^z . We want to capture the idea that business closures can affect the total factor productivity and thus affect aggregate supply. To do so, we add a variable $d_{i,t}$ to this specification, which reflect the affected share in industry i at time t . The term d measures the share of economic activities that have been disrupted by COVID-19. Lastly, we feed the estimates for $\rho^z = 0.98$ and $\sigma^z = 0.08$ estimated from the BEA and CPS data into our model.

5 Results

We divide our analysis of the impact of the productivity shock on the labor markets into three parts. First, we study the responses of total hours worked to a decline in productivity and business closure orders. Second, we investigate the implications of a negative productivity shock on the wage rate. Finally, we analyze how a business closure order affects the volatility of wages.

5.1 Labor Response to Productivity and Business Closure Shocks

In Figure 7, we document that aggregate hours worked declined considerably during the pandemic for all types of workers. The primary economic explanation of the responses of hours to a drop in productivity shock is that strict restrictions on businesses have suspended or entirely ceased business activity and which causes a significant decrease

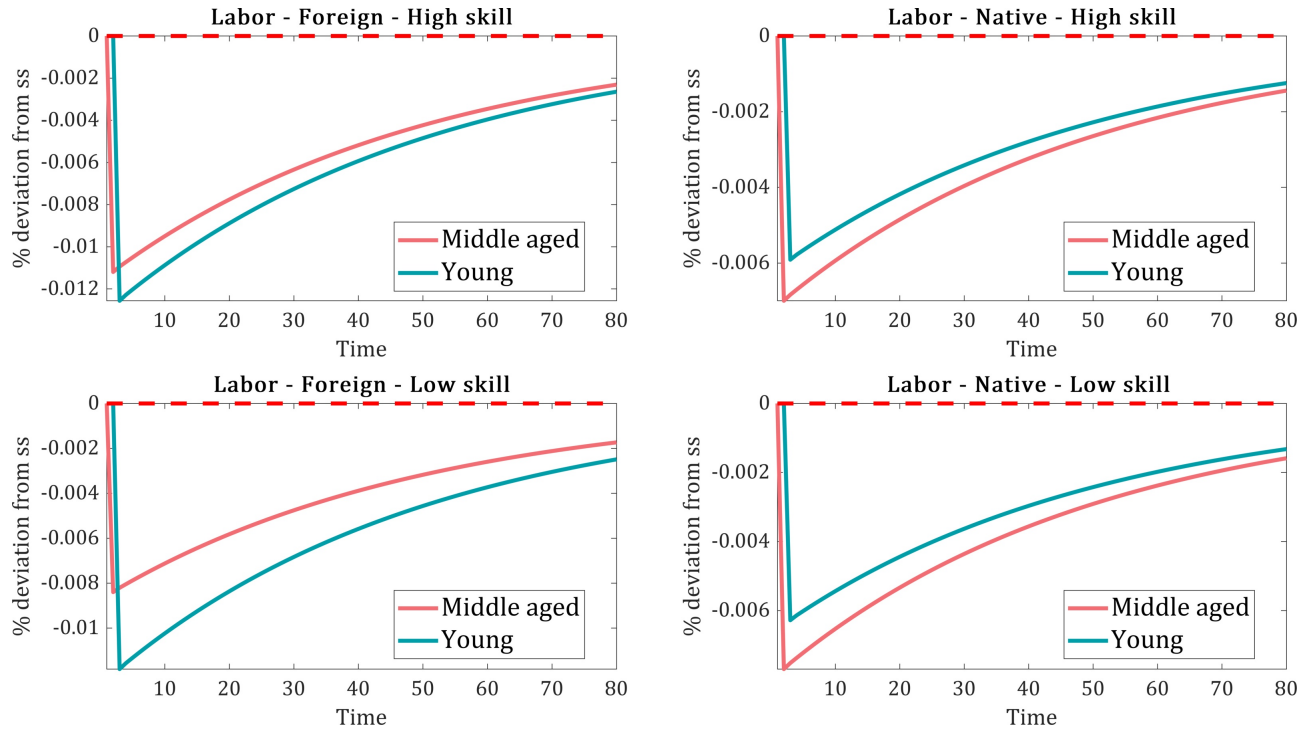


Figure 7: Hours responses to a negative productivity shock

in labor market activity. As panel A of Figure 7 illustrates, a business closure order that affects sectors disproportionately generates a fall in total hours worked of -0.014 percent for young foreign workers with high skills. On the other hand, middle-aged workers experienced a decline in the number of hours worked by -0.012 percent. The same economic forces led to a significant decline in hours worked by less-educated foreign workers (plotted in figure bottom-left), a decrease by -0.09 percent when middle-aged and -0.012 percent when young. These disparities between this labor group show that young workers suffered more heavily from the business closure than middle-aged workers, and high-skill workers experienced the largest reduction in hours worked. Certainly, the conclusion from these two figures is that age and education, to a certain extent, made a difference between foreign workers' responses to a negative productivity shock.

We now turn to the impact of productivity shock on native workers. Figure 7 panel B depicts the responses of hours worked. One consequence of productivity disturbance has been a considerable decline in hours worked of highly-educated middle-aged workers

by -0.007 percent. At the same time, young workers experienced a fall in hours of -0.006 percent. Furthermore, the decline in productivity affects low-skilled workers substantially (see Figure 7 panel C). A negative productivity shock depresses the total hours worked by middle-aged workers by -0.008 percent and -0.006 percent for young workers (see Figure 7 panel D). The response of hours worked by middle-aged workers to a fall in productivity is larger than the responses of hours worked by young workers.

This disturbance most obviously affects hours worked and merely adds to the existing disparities between workers. An interesting result is that the fall in productivity during the pandemic crisis lead to heterogeneous responses of hours worked. Higher exposure to productivity shock is observed among less-educated foreign young workers which experienced the largest decline in hours worked. We find that among native workers, middle-aged workers with low skills are the most disadvantaged by having the largest fall in total hours worked.

5.2 Hourly Earnings and Productivity Shocks

Consider the response of highly skilled native workers to the productivity shock, in panel B of Figure 8. A one percent decrease in productivity increases the wage rate of young workers by 2.9 percent and raises the hourly wage rate of the middle-aged by more than 4 percent. A similar effect is observed for the low-skilled native workers (See panel D of Figure 8). A one percent increases in productivity inflate the wage rate of young workers by 3 percent and middle-aged workers by 4.1 percent. Our model predicts that the effect on young workers is small compared to middle-aged workers. At the margin, one explanation for this result is that middle-aged workers are much more exposed to productivity shock, suggesting that they experience a shrink in their hourly wage. Young and middle-aged workers should be distinguished, and the fact that the hourly wage rate of middle-aged workers tends to be higher means that firms may have an incentive to reduce their labor costs in response to the chronic shock to productivity. This issue is

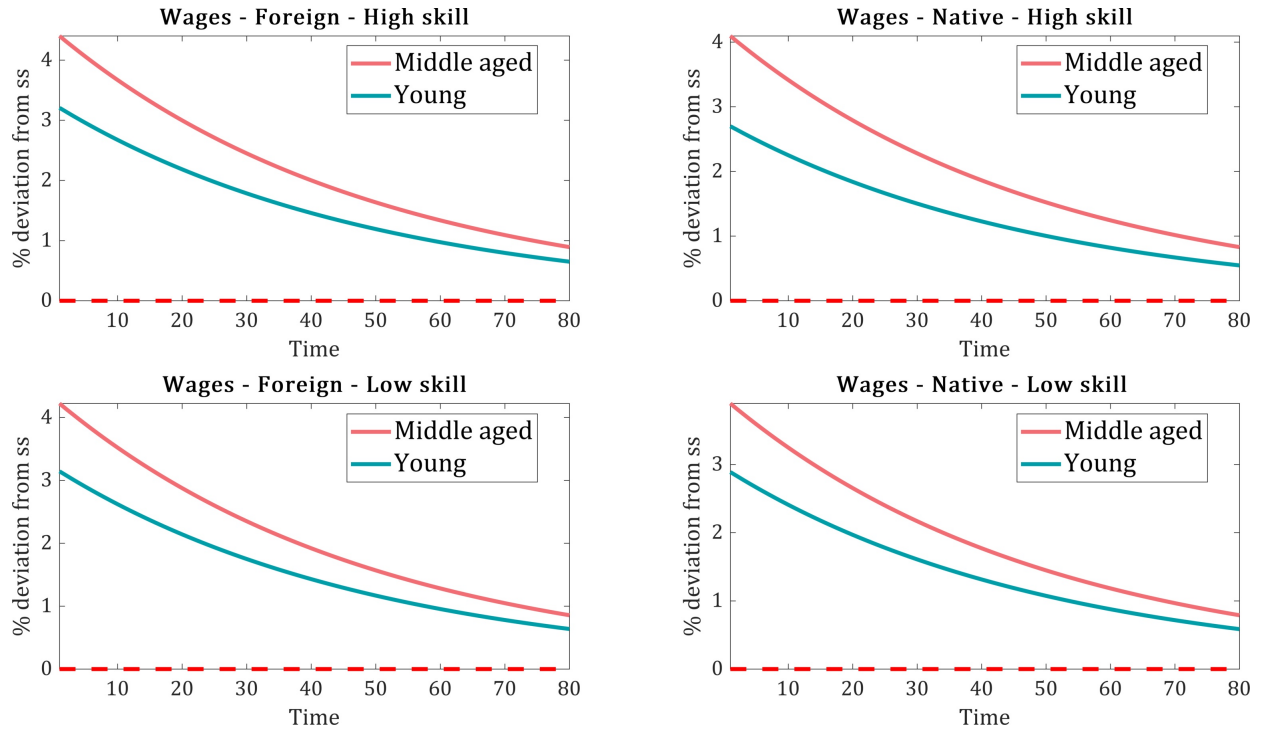


Figure 8: Wages responses to a negative productivity shock

especially apparent among workers with higher hourly earnings.

Returning to the case of foreign workers, we show that middle-aged workers experience rapid wage growth in response to a fall in productivity a result not very different from the case of native workers. The increase in the hourly wage rate is higher for middle-aged workers with an increase of 4.7 percent, and 3.4 percent for young workers (figure 8 panel A). Similarly, low-skilled workers enjoyed a rise in wage rate in response to the negative productivity shock, with an increase of 4.5 percent for middle aged-workers and an increase of 3.2 percent for young workers (figure 8 panel C).

Again age is making the difference in terms of wage growth. Middle-aged workers appear to be the most affected by the decline in productivity, whereas the wage growth of young workers generated by the pandemic is below the level of middle-aged workers. There are other possible explanations for these results. Intuitively, young workers may have suffered low and stagnant wages in the past which makes the rise in wages for this category less pronounced.

Table 5: Wage Volatility

	Benchmark	High δ
<i>Panel A: Standard deviation of average wages</i>		
High-skilled young foreign worker ω_{hf}^y	0.157	0.424
High-skilled young native worker ω_{hn}^y	0.151	0.406
Low-skilled young foreign worker ω_{lf}^y	0.154	0.415
Low-skilled young native worker ω_{ln}^y	0.154	0.416
High-skilled middle-aged foreign worker ω_{hf}^m	0.221	0.595
High-skilled middle-aged native worker ω_{hn}^m	0.217	0.584
Low-skilled middle-aged foreign worker ω_{lf}^m	0.207	0.557
Low-skilled middle-aged native worker ω_{ln}^m	0.207	0.558

^a Note: We report the standard deviation of the average hourly wage rate from the model simulation.

^b High δ reflect the case of high responsiveness of productivity to covid shock.

5.3 Wages Volatility in Unstable Times

Our model assumes that aggregate productivity shock conveys the economic impact of the pandemic, with important implications for workers' hourly earnings. The business closure shocks can lead to higher volatility in labor markets.

Table 5 reports the standard deviation of the average wage rate across labor categories generated by the model under the assumption that productivity shock is highly responsive to the business order closure shock by setting $\delta = 0.2$. The unprecedented supply shock across US industries has curtailed economic activity and caused a reduction in total hours worked. It is evident from the table that the pandemic increases the volatility of wage rates, firms face a combination of two economic phenomena, falling hours worked and rising wage rates. Somewhat surprising is that business closure orders contributed significantly to the excess wage inflation in the US.

Yet the problem of labor market tightness and skill shortage in the US is becoming increasingly worrying. As Domash and Summers (2022) suggested, there is evidence that the US labor market is extremely tight and the inflationary pressure from the labor market will persist. Mounting inflationary pressure on firms will push these firms to adjust their price and margin and eventually will pass through the wage cost to price

inflation. This model shows the mechanical effects of the pandemic as we assume that the responsiveness of productivity to this shock is high, and the underlying hourly earnings of all types of workers tend to become more volatile.⁹

6 Conclusion

In this paper, we study the effects of business closures during the pandemic crisis on the US labor market. The inclusion of restrictions on US business to the specification of the aggregate productivity shock generates a reduction in labor market activity and translates into a severe episode of economic downturn. The intuition for this result is that the business closure orders deliver a fall in total hours worked associated with a skill shortage. Recent work by Peri and Zaiour (2022) points out a presumably fall in the number of foreign workers in 2021. A similar conclusion is emphasized in Tüzemen (2022) who documents a decline in the labor force and the participation rate in the US at the onset of the pandemic. This combination of labor shortage and restrictions on businesses tends to push wages up.

Our paper emphasizes the estimation of labor-labor elasticity which is deemed crucial to map the wage differentials and differences in hours worked across the type of workers. Our model makes it clear that a negative shock to aggregate productivity leads to a contraction in total hours worked and sustained wage inflation. Though the magnitude of the shock responses differs by age, skill, and origin of the worker. This suggests that the productivity shock is unequally distributed over the type of workers during the pandemic.

One of the key contributions of this paper is to develop a model that generates labor market fluctuations consistent with what we observe in the real world. An interesting extension of our framework would be the inclusion of capital into the nested CES

⁹ There is a recent empirical study that documents large volatility of wages during heightened times as this improves workers' bargaining power (Den Haan et al., 2021).

production function, we believe including this feature in our model is important. One reason is that US firms in general have experienced a substantial change in the allocation of capital when several states imposed a shutdown of non-essential businesses. We believe our paper provides many directions for future investigation.

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TECHNICAL APPENDIX

Curtailement of Economic Activity and Labor Inequalities

Erminia Florio and Aicha Kharazi

Appendix A Model Computations

A.1 Nested production function

Technology We introduce a standard CES production function

$$y_t = z_t(\alpha(l_t^y)^{\frac{s-1}{s}} + (1-\alpha)(l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}} \quad (\text{A.1})$$

where z_t is the productivity shock, α is the share parameter of labor input. For simplicity, the producer use labor factor only as in input. l_t^y and l_t^m denote the hours worked by young and middle age workers, respectively. The parameter s represent the elasticity of substitution.

The hours worked by young is given by

$$l_t^y = [((\phi_{h,f}l_{h,f,t}^y)^{\frac{1}{\sigma}} + (\phi_{h,n}l_{h,n,t}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma-1}} + ((\phi_{l,f}l_{l,f,t}^y)^{\frac{1}{\sigma}} + (\phi_{l,n}l_{l,n,t}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma-1}}]^{\sigma-1}$$

where l_t^y denotes the total hours worked by young individuals. This equation emphasizes that hours worked by younger individuals young is broken down into various origin (native or foreign) and skill (high or low) groups. We assume that labor inputs l_t^y is a vector of four types of labor: hours worked by foreign worker in high skilled sector $l_{h,f,t}^y$; hours worked by native worker in high skilled sector $l_{h,n,t}^y$; hours worked by foreign worker in low skilled sector $l_{l,f,t}^y$; and hours worked by native worker in low skilled sector $l_{l,n,t}^y$.

Similarly, the hours worked by middle-age individuals is given by:

$$l_t^m = [((\theta_{h,f}l_{h,f,t}^m)^{\frac{1}{\xi}} + (\theta_{h,n}l_{h,n,t}^m)^{\frac{1}{\xi}})^{\frac{\xi}{\xi-1}} + ((\theta_{l,f}l_{l,f,t}^m)^{\frac{1}{\xi}} + (\theta_{l,n}l_{l,n,t}^m)^{\frac{1}{\xi}})^{\frac{\xi}{\xi-1}}]^{\xi-1}$$

the term l_t^m represents the total hours worked by young (middle aged) individuals. The hours worked by middle-aged individuals include hours worked by natives and foreigners and by high and low skilled. The labor inputs l_t^m is then a vector of four types of labor: hours worked by foreign worker in high skilled sector $l_{h,f,t}^m$; hours worked by native worker in high skilled sector $l_{h,n,t}^m$; hours worked by foreign worker in low skilled sector $l_{l,f,t}^m$; and hours worked by native worker in low skilled sector $l_{l,n,t}^m$.

Producer Problem The problem of the intermediate good producer is to minimize the cost

$$\omega_{h,f,t}^y l_{h,f,t}^y + \omega_{h,n,t}^y l_{h,n,t}^y + \omega_{l,f,t}^y l_{l,f,t}^y + \omega_{l,n,t}^y l_{l,n,t}^y + \omega_{h,f,t}^m l_{h,f,t}^m + \omega_{h,n,t}^m l_{h,n,t}^m + \omega_{l,f,t}^m l_{l,f,t}^m + \omega_{l,n,t}^m l_{l,n,t}^m$$

subject to the production function that relates outputs to inputs. The first order condition with respect to $l_{h,f,t}^y$ and $l_{h,n,t}^y$ are given by:

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial l_{h,f,t}^y} : \quad & \omega_{h,f,t}^y - \mu_t \alpha z_t (\phi_{h,f} l_{h,f,t}^y)^{\frac{1}{\sigma}-1} ((\phi_{h,f} l_{h,f,t}^y)^{\frac{1}{\sigma}} + (\phi_{h,n} l_{h,n,t}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma}-1} \\ & (((\phi_{h,f} l_{h,f,t}^y)^{\frac{1}{\sigma}} + (\phi_{h,n} l_{h,n,t}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma}} + ((\phi_{l,n} l_{l,n,t}^y)^{\frac{1}{\sigma}} + (\phi_{l,f} l_{l,f,t}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma}})^{\sigma-1} \\ & (l_t^y)^{\frac{s-1}{s}-1} (\alpha (l_t^y)^{\frac{s-1}{s}} + (1-\alpha) (l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial l_{h,n,t}^y} : \quad & \omega_{h,n,t}^y - \mu_t \alpha z_t (\phi_{h,n} l_{h,n,t}^y)^{\frac{1}{\sigma}-1} ((\phi_{h,n} l_{h,n,t}^y)^{\frac{1}{\sigma}} + (\phi_{h,f} l_{h,f,t}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma}-1} \\ & (((\phi_{h,n} l_{h,n,t}^y)^{\frac{1}{\sigma}} + (\phi_{h,f} l_{h,f,t}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma}} + ((\phi_{l,n} l_{l,n,t}^y)^{\frac{1}{\sigma}} + (\phi_{l,f} l_{l,f,t}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma}})^{\sigma-1} \\ & (l_t^y)^{\frac{s-1}{s}-1} (\alpha (l_t^y)^{\frac{s-1}{s}} + (1-\alpha) (l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0 \end{aligned}$$

The Euler equation is

$$\frac{\omega_{h,f,t}^y}{\omega_{h,n,t}^y} = \left(\frac{\phi_{h,f} l_{h,f,t}^y}{\phi_{h,n} l_{h,n,t}^y} \right)^{\frac{1}{\sigma}-1}$$

The first order condition with respect to $l_{l,f,t}^y$ and $l_{l,n,t}^m$ are given by:

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial l_{l,f,t}^y} : \quad & \omega_{l,f,t}^y - \mu_t \alpha z_t (\phi_{l,f} l_{l,f,t}^y)^{\frac{1}{\sigma}-1} ((\phi_{l,f} l_{l,f,t}^y)^{\frac{1}{\sigma}} + (\phi_{l,n} l_{l,n,t}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma}-1} \\ & (((\phi_{l,f} l_{l,f,t}^y)^{\frac{1}{\sigma}} + (\phi_{l,n} l_{l,n,t}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma}} + ((\phi_{h,n} l_{h,n,t}^y)^{\frac{1}{\sigma}} + (\phi_{h,f} l_{h,f,t}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma}})^{\sigma-1} \\ & (l_t^y)^{\frac{s-1}{s}-1} (\alpha (l_t^y)^{\frac{s-1}{s}} + (1-\alpha) (l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial l_{l,n,t}^m} : \quad & \omega_{l,n,t}^m - \mu_t \alpha z_t (\phi_{l,n} l_{l,n,t}^m)^{\frac{1}{\sigma}-1} ((\phi_{l,n} l_{l,n,t}^m)^{\frac{1}{\sigma}} + (\phi_{l,f} l_{l,f,t}^m)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma}-1} \\ & (((\phi_{l,n} l_{l,n,t}^m)^{\frac{1}{\sigma}} + (\phi_{l,f} l_{l,f,t}^m)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma}} + ((\phi_{h,n} l_{h,n,t}^m)^{\frac{1}{\sigma}} + (\phi_{h,f} l_{h,f,t}^m)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma}})^{\sigma-1} \\ & (l_t^m)^{\frac{s-1}{s}-1} (\alpha (l_t^y)^{\frac{s-1}{s}} + (1-\alpha) (l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0 \end{aligned}$$

The Euler equation is given by

$$\frac{\omega_{l,f,t}^y}{\omega_{l,n,t}^y} = \left(\frac{\phi_{l,f} l_{l,f,t}^y}{\phi_{l,n} l_{l,n,t}^y} \right)^{\frac{1}{v}-1}$$

The first order condition with respect to $l_{h,f,t}^m$ and $l_{h,n,t}^m$ are given by:

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial l_{h,f,t}^m} : \quad & \omega_{h,f,t}^m - \mu_t(1-\alpha)z_t(\theta_{hf}l_{h,f,t}^m)^{\frac{1}{\epsilon}-1}((\theta_{hf}l_{h,f,t}^m)^{\frac{1}{\epsilon}} + (\theta_{hn}l_{hnt}^m)^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\zeta}-1} \\ & (((\theta_{hf}l_{h,f,t}^m)^{\frac{1}{\epsilon}} + (\theta_{hn}l_{hnt}^m)^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\zeta}} + ((\theta_{ln}l_{lnt}^m)^{\frac{1}{\zeta}} + (\theta_{lf}l_{lft}^m)^{\frac{1}{\zeta}})^{\frac{\zeta}{\epsilon}})^{\zeta-1} \\ & (l_t^m)^{\frac{s-1}{s}-1}(\alpha(l_t^y)^{\frac{s-1}{s}} + (1-\alpha)(l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial l_{h,n,t}^m} : \quad & \omega_{h,n,t}^m - \mu_t(1-\alpha)z_t(\theta_{hn}l_{h,n,t}^m)^{\frac{1}{\epsilon}-1}((\theta_{hn}l_{h,n,t}^m)^{\frac{1}{\epsilon}} + (\theta_{hf}l_{hft}^m)^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\zeta}-1} \\ & (((\theta_{hn}l_{h,n,t}^m)^{\frac{1}{\epsilon}} + (\theta_{hf}l_{hft}^m)^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\zeta}} + ((\theta_{ln}l_{lnt}^m)^{\frac{1}{\zeta}} + (\theta_{lf}l_{lft}^m)^{\frac{1}{\zeta}})^{\frac{\zeta}{\epsilon}})^{\zeta-1} \\ & (l_t^m)^{\frac{s-1}{s}-1}(\alpha(l_t^y)^{\frac{s-1}{s}} + (1-\alpha)(l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0 \end{aligned}$$

The Euler equation is given by

$$\frac{\omega_{h,f,t}^m}{\omega_{h,n,t}^m} = \left(\frac{\theta_{hf}l_{h,f,t}^m}{\theta_{hn}l_{h,n,t}^m} \right)^{\frac{1}{\epsilon}-1}$$

The first order condition with respect to $l_{l,f,t}^m$ and $l_{l,n,t}^m$ are given by:

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial l_{l,f,t}^m} : \quad & \omega_{l,f,t}^m - \mu_t(1-\alpha)z_t(\theta_{lf}l_{l,f,t}^m)^{\frac{1}{\zeta}-1}((\theta_{lf}l_{l,f,t}^m)^{\frac{1}{\zeta}} + (\theta_{ln}l_{lnt}^m)^{\frac{1}{\zeta}})^{\frac{\zeta}{\epsilon}-1} \\ & (((\theta_{lf}l_{l,f,t}^m)^{\frac{1}{\zeta}} + (\theta_{ln}l_{lnt}^m)^{\frac{1}{\zeta}})^{\frac{\zeta}{\epsilon}} + ((\theta_{hn}l_{hnt}^m)^{\frac{1}{\epsilon}} + (\theta_{hf}l_{hft}^m)^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\zeta}})^{\zeta-1} \\ & (l_t^m)^{\frac{s-1}{s}-1}(\alpha(l_t^y)^{\frac{s-1}{s}} + (1-\alpha)(l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial l_{l,n,t}^m} : \quad & \omega_{l,n,t}^m - \mu_t(1-\alpha)z_t(\theta_{ln}l_{l,n,t}^m)^{\frac{1}{\zeta}-1}((\theta_{ln}l_{l,n,t}^m)^{\frac{1}{\zeta}} + (\theta_{lf}l_{lft}^m)^{\frac{1}{\zeta}})^{\frac{\zeta}{\epsilon}-1} \\ & (((\theta_{ln}l_{l,n,t}^m)^{\frac{1}{\zeta}} + (\theta_{lf}l_{lft}^m)^{\frac{1}{\zeta}})^{\frac{\zeta}{\epsilon}} + ((\theta_{hn}l_{hnt}^m)^{\frac{1}{\epsilon}} + (\theta_{hf}l_{hft}^m)^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\zeta}})^{\zeta-1} \\ & (l_t^m)^{\frac{s-1}{s}-1}(\alpha(l_t^y)^{\frac{s-1}{s}} + (1-\alpha)(l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0 \end{aligned}$$

The Euler equation

$$\frac{\omega_{l,f,t}^m}{\omega_{l,n,t}^m} = \left(\frac{\theta_{l,f} l_{l,f,t}^m}{\theta_{l,n} l_{l,n,t}^m} \right)^{\frac{1}{\zeta}-1}$$

A.2 Households

There are three generations, each is alive at any point in time. The lifetime utility is given by

$$\begin{aligned} \text{maximize } U(c_{s,k,t-1}^y, c_{s,k,t}^m, c_{s,k,t+1}^o, l_{s,k,t-1}^y, l_{s,k,t}^m) = & \left\{ \ln(c_{s,k,t-1}^y) - \frac{(l_{s,k,t-1}^y)^{1+\eta}}{1+\eta} \right\} \\ & + \beta \left\{ \ln(c_{s,k,t}^m) - \frac{(l_{s,k,t}^m)^{1+\eta}}{1+\eta} \right\} + \beta^2 \left\{ \ln(c_{s,k,t+1}^o) \right\} \end{aligned}$$

where $0 \leq \beta \leq 1$ is the discount factor and η is the curvature on the disutility of labor. All agents maximize the utility subject to the budget constraint and capital accumulation.

$$\begin{aligned} c_{s,k,t-1}^y + a_{s,k,t-1}^y &= \omega_{s,k,t-1}^y l_{s,k,t-1}^y \\ c_{s,k,t}^m + a_{s,k,t}^m &= \omega_{s,k,t}^m l_{s,k,t}^m + a_{s,k,t-1}^y (1 + r_{t-1}) \\ c_{s,k,t+1}^o &= a_{s,k,t}^m (1 + r_t) \end{aligned}$$

We let $c_{s,k}^j$ denote the per capita consumption across age groups j at time t where $j \in \{y, m, o\}$ identifies age groups: young, middle-age, old age. l^y and $l_{s,k}^m$ denote labor supply by young and middle age individuals at wage rate $\omega_{s,k}^j$. The young and middle age agents consume and work in each period, in the first and second period of life, while old agent earn no income in the third period of life but receive retirement income and consume. Both young and middle age agent save, but only the middle age and old age agents receive asset earnings at risk free interest rate r .

Using the substitution method we can then write the Lagrangian as follows

$$\begin{aligned} \mathcal{L}_t = & \left[\left\{ \ln(c_{s,k,t-1}^y) - \frac{(l_{s,k,t-1}^y)^{1+\eta}}{1+\eta} \right\} + \beta \left\{ \ln(c_{s,k,t}^m) - \frac{(l_{s,k,t}^m)^{1+\eta}}{1+\eta} \right\} + \beta^2 \left\{ \ln(c_{s,k,t+1}^o) \right\} \right. \\ & \left. + \lambda_t \left(\omega_{s,k,t}^m l_{s,k,t}^m + (\omega_{s,k,t-1}^y l_{s,k,t-1}^y - c_{s,k,t-1}^y)(1 + r_{t-1}) - c_{s,k,t}^m - \frac{c_{s,k,t+1}^o}{1 + r_t} \right) \right] \end{aligned}$$

The household optimality conditions with respect to consumption: $c_{s,k,t}^m$, $c_{s,k,t-1}^y$, $c_{s,k,t+1}^o$, and labor: $l_{s,k,t}^m$ and $l_{s,k,t-1}^y$ are derived as follow

$$\frac{\partial \mathcal{L}_t}{\partial c_{s,k,t-1}^y} : \quad c_{s,k,t-1}^y = \frac{1}{\lambda_t (1 + r_{t-1})} \quad (\text{A.2})$$

$$\frac{\partial \mathcal{L}_t}{\partial c_{s,k,t}^m} : \quad \frac{\beta}{c_{s,k,t}^m} = \lambda_t \quad (\text{A.3})$$

$$\frac{\partial \mathcal{L}_t}{\partial l_{s,k,t-1}^y} : \quad (l_{s,k,t-1}^y)^\eta = \lambda_t \omega_{s,k,t-1}^y (1 + r_{t-1}) \quad (\text{A.4})$$

$$\frac{\partial \mathcal{L}_t}{\partial l_{s,k,t}^m} : \quad \beta (l_{s,k,t}^m)^\eta = \lambda_t \omega_{s,k,t}^m \quad (\text{A.5})$$

$$\frac{\partial \mathcal{L}_t}{\partial c_{s,k,t+1}^o} : \quad \frac{\beta^2}{c_{s,k,t+1}^o} = \frac{\lambda_t}{(1 + r_t)} \quad (\text{A.6})$$

from A.3 and A.5 we obtain

$$\frac{1}{c_{s,k,t}^m} = \frac{(l_{s,k,t}^m)^\eta}{\omega_{s,k,t}^m}$$

using A.2 and A.4 we obtain

$$\frac{1}{c_{s,k,t-1}^y} = \frac{(l_{s,k,t-1}^y)^\eta}{\omega_{s,k,t-1}^y}$$

A.3 Alternative Specification of the Production Function

Suppose now that producers have access to the following production function

$$y_t = z_t (\alpha (l_t^y)^{\frac{s-1}{s}} + (1 - \alpha) (l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}} \quad (\text{A.7})$$

We assume that firm combine heterogeneous labor to produce good y . The hours worked by younger individuals is given by:

$$l_t^y = \phi_{h,f} l_{h,f,t}^y + \phi_{h,n} l_{h,n,t}^y + \phi_{l,f} l_{l,f,t}^y + \phi_{l,n} l_{l,n,t}^y$$

This equation emphasizes that hours worked by younger individuals young is broken down into various origin (native or foreign) and skill (high or low) groups. Similarly, the hours worked by middle-age individuals is given by:

$$l_t^m = \theta_{h,f} l_{h,f,t}^m + \theta_{h,n} l_{h,n,t}^m + \theta_{l,f} l_{l,f,t}^m + \theta_{l,n} l_{l,n,t}^m$$

the hours worked by middle-aged individuals include hours worked by natives and foreigners and by high and low skilled.

Producer Problem The problem of the intermediate good producer is to minimize the cost

$$\omega_{h,f,t}^y l_{h,f,t}^y + \omega_{h,n,t}^y l_{h,n,t}^y + \omega_{l,f,t}^y l_{l,f,t}^y + \omega_{l,n,t}^y l_{l,n,t}^y + \omega_{h,f,t}^m l_{h,f,t}^m + \omega_{h,n,t}^m l_{h,n,t}^m + \omega_{l,f,t}^m l_{l,f,t}^m + \omega_{l,n,t}^m l_{l,n,t}^m$$

subject to the production function that relates outputs to inputs. The first order condition with respect to $l_{h,f,t}^y$ and $l_{h,f,t}^m$ are given by:

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial l_{h,f,t}^y} : \quad & \omega_{h,f,t}^y - \mu_t \alpha z_t \phi_{h,f} (l_t^y)^{\frac{s-1}{s}-1} ((l_t^y)^{\frac{s-1}{s}} + (l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0, \\ \frac{\partial \mathcal{L}_t}{\partial l_{h,f,t}^m} : \quad & \omega_{h,f,t}^m - \mu_t (1 - \alpha) z_t \theta_{h,f} (l_t^m)^{\frac{s-1}{s}-1} ((l_t^y)^{\frac{s-1}{s}} + (l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0.\end{aligned}$$

We derive the Euler equation

$$\frac{\omega_{h,f,t}^y}{\omega_{h,f,t}^m} = \frac{\alpha}{(1 - \alpha)} \frac{\phi_{h,f}}{\theta_{h,f}} \left(\frac{l_t^y}{l_t^m} \right)^{-\frac{1}{s}},$$

The first order condition wrt $l_{h,n,t}^y$ and $l_{h,n,t}^m$ are:

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial l_{h,n,t}^y} : \quad & \omega_{h,n,t}^y - \mu_t \alpha z_t \phi_{h,n} (l_t^y)^{\frac{s-1}{s}-1} ((l_t^y)^{\frac{s-1}{s}} + (l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0, \\ \frac{\partial \mathcal{L}_t}{\partial l_{h,n,t}^m} : \quad & \omega_{h,n,t}^m - \mu_t (1 - \alpha) z_t \theta_{h,n} (l_t^m)^{\frac{s-1}{s}-1} ((l_t^y)^{\frac{s-1}{s}} + (l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0.\end{aligned}$$

Combining these two equations gives the following Euler equation

$$\frac{\omega_{h,n,t}^y}{\omega_{h,n,t}^m} = \frac{\alpha}{1 - \alpha} \frac{\phi_{h,n}}{\theta_{h,n}} \left(\frac{l_t^y}{l_t^m} \right)^{-\frac{1}{s}},$$

first order condition with respect to $l_{l,f,t}^y$ and $l_{l,f,t}^m$

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial l_{l,f,t}^y} : \quad & \omega_{l,f,t}^y - \mu_t \alpha z_t \phi_{l,f} (l_t^y)^{\frac{s-1}{s}-1} ((l_t^y)^{\frac{s-1}{s}} + (l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0, \\ \frac{\partial \mathcal{L}_t}{\partial l_{l,f,t}^m} : \quad & \omega_{l,f,t}^m - \mu_t (1 - \alpha) z_t \theta_{l,f} (l_t^m)^{\frac{s-1}{s}-1} ((l_t^y)^{\frac{s-1}{s}} + (l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0,\end{aligned}$$

The Euler equation

$$\frac{\omega_{l,f,t}^y}{\omega_{l,f,t}^m} = \frac{\alpha}{1 - \alpha} \frac{\phi_{l,f}}{\theta_{l,f}} \left(\frac{l_t^y}{l_t^m} \right)^{-\frac{1}{s}},$$

first order condition with respect to $l_{l,n,t}^y$ and $l_{l,n,t}^m$

$$\frac{\partial \mathcal{L}_t}{\partial l_{l,n,t}^y} : \quad \omega_{l,n,t}^y - \mu_t \alpha z_t \phi_{l,n} (l_t^y)^{\frac{s-1}{s}-1} ((l_t^y)^{\frac{s-1}{s}} + (l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0,$$

$$\frac{\partial \mathcal{L}_t}{\partial l_{l,n,t}^m} : \quad \omega_{l,n,t}^m - \mu_t(1 - \alpha)z_t\theta_{l,n}(l_t^m)^{\frac{s-1}{s}-1}((l_t^y)^{\frac{s-1}{s}} + (l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0,$$

The Euler equation

$$\frac{\omega_{l,n,t}^y}{\omega_{l,n,t}^m} = \frac{\alpha}{1 - \alpha} \frac{\phi_{l,n}}{\theta_{l,n}} \left(\frac{l_t^y}{l_t^m} \right)^{-\frac{1}{s}},$$

The producer marginal cost is

$$mc = \mu_t = \frac{\omega_t(l_t^y + l_t^m)}{z_t(\alpha(l_t^y)^{\frac{s-1}{s}} + (1 - \alpha)(l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}}},$$

Appendix B Steady State

As a first step we assume that $a_t^m = a_t^y$, given that the middle-aged budget constraint is $c_t^m + a_t^m = \omega_t l_t^m + a_{t-1}^y(1 + r_{t-1})$, and the total saving in this economy equals to $a_t = a_t^m + a_t^y$. The first order condition with respect to a_y is

$$\lambda_t = \lambda_{t+1}\beta(1 + r_t)$$

We can easily compute the steady state value of interest rate:

$$\begin{aligned} \lambda_{ss} &= \lambda_{ss}\beta(1 + r_{ss}) \\ r_{ss} &= \frac{1}{\beta} - 1 \end{aligned}$$

We set an initial value for the household's marginal utility λ_{ss} and we calculate the consumption value of middle-age agent

$$\begin{aligned} \lambda_t &= \frac{\beta}{c_t^m} \\ c_{ss}^m &= \frac{\beta}{\lambda_{ss}} \end{aligned}$$

the consumption of young agents

$$\begin{aligned} c_{t-1}^y &= \frac{1}{\lambda_t(1 + r_{t-1})} \\ c_{ss}^y &= \frac{1}{\lambda_{ss}(1 + r_{ss})} \end{aligned}$$

the consumption of the older

$$\frac{\beta^2}{c_{t+1}^o} = \frac{\lambda_t}{(1+r_t)}$$

$$c_{ss}^o = \frac{\beta^2(1+r_{ss})}{\lambda_{ss}}$$

We set an initial value for ω and we compute the value of labor l^m and l^y using the optimality conditions

$$\beta (l_t^m)^\eta = \lambda_t \omega_t \quad l_t^{y\eta} = \lambda_t \omega_{t-1} (1+r_{t-1})$$

$$l_{ss}^m = \left(\frac{(\lambda_{ss} \omega_{ss})}{\beta} \right)^{\frac{1}{\eta}} \quad l_{ss}^y = (\lambda_{ss} \omega_{ss} (1+r_{ss}))^{\frac{1}{\eta}}$$

using the labor intensity parameters we can derive the value of labor by types of workers

$$\begin{aligned} l_{t,h,f}^y &= \phi_{h,f} l_t^y & l_{ss,h,f}^y &= \phi_{h,f} l_{ss}^y \\ l_{t,h,n}^y &= \phi_{h,n} l_t^y & l_{ss,h,n}^y &= \phi_{h,n} l_{ss}^y \\ l_{t,l,f}^y &= \phi_{l,f} l_t^y & l_{ss,l,f}^y &= \phi_{l,f} l_{ss}^y \\ l_{t,l,n}^y &= \phi_{l,n} l_t^y & l_{ss,l,n}^y &= \phi_{l,n} l_{ss}^y \\ l_{t,h,f}^m &= \theta_{h,f} l_t^m & l_{ss,h,f}^m &= \theta_{h,f} l_{ss}^m \\ l_{t,h,n}^m &= \theta_{h,n} l_t^m & l_{ss,h,n}^m &= \theta_{h,n} l_{ss}^m \\ l_{t,l,f}^m &= \theta_{l,f} l_t^m & l_{ss,l,f}^m &= \theta_{l,f} l_{ss}^m \\ l_{t,l,n}^m &= \theta_{l,n} l_t^m & l_{ss,l,n}^m &= \theta_{l,n} l_{ss}^m \end{aligned}$$

Then, we set a initial value fro the producer marginal cost μ we use first order conditions with respect to to labor to compute the hourly wage rate

$$\begin{aligned} \omega_{h,f,ss}^y - \mu_{ss} \alpha z_{ss} (\phi_{h,f} l_{h,f,ss}^y)^{\frac{1}{\sigma}-1} ((\phi_{h,f} l_{h,f,ss}^y)^{\frac{1}{\sigma}} + (\phi_{h,n} l_{h,n,ss}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma}-1} \\ ((\phi_{h,f} l_{h,f,ss}^y)^{\frac{1}{\sigma}} + (\phi_{h,n} l_{h,n,ss}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma}} + ((\phi_{l,n} l_{l,n,ss}^y)^{\frac{1}{\sigma}} + (\phi_{l,f} l_{l,f,ss}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma}})^{\sigma-1} \\ (l_{ss}^y)^{\frac{s-1}{s}-1} (\alpha (l_{ss}^y)^{\frac{s-1}{s}} + (1-\alpha) (l_{ss}^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0, \\ \omega_{h,n,ss}^y - \mu_{ss} \alpha z_{ss} (\phi_{h,n} l_{h,n,ss}^y)^{\frac{1}{\sigma}-1} ((\phi_{h,n} l_{h,n,ss}^y)^{\frac{1}{\sigma}} + (\phi_{h,f} l_{h,f,ss}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma}-1} \\ ((\phi_{h,n} l_{h,n,ss}^y)^{\frac{1}{\sigma}} + (\phi_{h,f} l_{h,f,ss}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma}} + ((\phi_{l,n} l_{l,n,ss}^y)^{\frac{1}{\sigma}} + (\phi_{l,f} l_{l,f,ss}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma}})^{\sigma-1} \\ (l_{ss}^y)^{\frac{s-1}{s}-1} (\alpha (l_{ss}^y)^{\frac{s-1}{s}} + (1-\alpha) (l_{ss}^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0, \\ \omega_{l,f,ss}^y - \mu_{ss} \alpha z_{ss} (\phi_{l,f} l_{l,f,ss}^y)^{\frac{1}{\sigma}-1} ((\phi_{l,f} l_{l,f,ss}^y)^{\frac{1}{\sigma}} + (\phi_{l,n} l_{l,n,ss}^y)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma}-1} \end{aligned}$$

$$\begin{aligned}
& (((\phi_{lf}l_{l,f,ss}^y)^{\frac{1}{v}} + (\phi_{ln}l_{ln}^y)^{\frac{1}{v}})^{\frac{v}{\sigma}} + ((\phi_{hn}l_{hnt}^y)^{\frac{1}{\epsilon}} + (\phi_{hf}l_{hft}^y)^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\sigma}})^{\sigma-1} \\
& (l_{ss}^y)^{\frac{s-1}{s}-1}(\alpha(l_{ss}^y)^{\frac{s-1}{s}} + (1-\alpha)(l_{ss}^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0, \\
\omega_{l,n,ss}^y & - \mu_{ss}\alpha z_{ss}(\phi_{ln}l_{l,n,ss}^y)^{\frac{1}{v}-1}((\phi_{ln}l_{l,n,ss}^y)^{\frac{1}{v}} + (\phi_{lf}l_{lft}^y)^{\frac{1}{v}})^{\frac{v}{\sigma}-1} \\
& (((\phi_{ln}l_{l,n,ss}^y)^{\frac{1}{v}} + (\phi_{lf}l_{lft}^y)^{\frac{1}{v}})^{\frac{v}{\sigma}} + ((\phi_{hn}l_{hnt}^y)^{\frac{1}{\epsilon}} + (\phi_{hf}l_{hft}^y)^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\sigma}})^{\sigma-1} \\
& (l_{ss}^y)^{\sigma}^{\frac{s-1}{s}-1}(\alpha(l_{ss}^y)^{\frac{s-1}{s}} + (1-\alpha)(l_{ss}^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0, \\
\omega_{h,f,ss}^m & - \mu_{ss}(1-\alpha)z_{ss}(\theta_{hf}l_{h,f,ss}^m)^{\frac{1}{\epsilon}-1}((\theta_{hf}l_{h,f,ss}^m)^{\frac{1}{\epsilon}} + (\theta_{hn}l_{hnt}^m)^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\zeta}-1} \\
& (((\theta_{hf}l_{h,f,ss}^m)^{\frac{1}{\epsilon}} + (\theta_{hn}l_{hnt}^m)^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\zeta}} + ((\theta_{ln}l_{lnt}^m)^{\frac{1}{\zeta}} + (\theta_{lf}l_{lft}^m)^{\frac{1}{\zeta}})^{\frac{\zeta}{\epsilon}})^{\zeta-1} \\
& (l_{ss}^m)^{\frac{s-1}{s}-1}(\alpha(l_{ss}^y)^{\frac{s-1}{s}} + (1-\alpha)(l_{ss}^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0, \\
\omega_{h,n,ss}^m & - \mu_{ss}(1-\alpha)z_{ss}(\theta_{hn}l_{h,n,ss}^m)^{\frac{1}{\epsilon}-1}((\theta_{hn}l_{h,n,ss}^m)^{\frac{1}{\epsilon}} + (\theta_{hf}l_{hft}^m)^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\zeta}-1} \\
& (((\theta_{hn}l_{h,n,ss}^m)^{\frac{1}{\epsilon}} + (\theta_{hf}l_{hft}^m)^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\zeta}} + ((\theta_{ln}l_{lnt}^m)^{\frac{1}{\zeta}} + (\theta_{lf}l_{lft}^m)^{\frac{1}{\zeta}})^{\frac{\zeta}{\epsilon}})^{\zeta-1} \\
& (l_{ss}^m)^{\frac{s-1}{s}-1}(\alpha(l_{ss}^y)^{\frac{s-1}{s}} + (1-\alpha)(l_{ss}^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0, \\
\omega_{l,f,ss}^m & - \mu_{ss}(1-\alpha)z_{ss}(\theta_{lf}l_{l,f,ss}^m)^{\frac{1}{\zeta}-1}((\theta_{lf}l_{l,f,ss}^m)^{\frac{1}{\zeta}} + (\theta_{ln}l_{lnt}^m)^{\frac{1}{\zeta}})^{\frac{\zeta}{\epsilon}-1} \\
& (((\theta_{lf}l_{l,f,ss}^m)^{\frac{1}{\zeta}} + (\theta_{ln}l_{lnt}^m)^{\frac{1}{\zeta}})^{\frac{\zeta}{\epsilon}} + ((\theta_{hn}l_{hnt}^m)^{\frac{1}{\epsilon}} + (\theta_{hf}l_{hft}^m)^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\zeta}})^{\zeta-1} \\
& (l_{ss}^m)^{\frac{s-1}{s}-1}(\alpha(l_{ss}^y)^{\frac{s-1}{s}} + (1-\alpha)(l_{ss}^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0, \\
\omega_{l,n,ss}^m & - \mu_{ss}(1-\alpha)z_{ss}(\theta_{ln}l_{l,n,ss}^m)^{\frac{1}{\zeta}-1}((\theta_{ln}l_{l,n,ss}^m)^{\frac{1}{\zeta}} + (\theta_{lf}l_{lft}^m)^{\frac{1}{\zeta}})^{\frac{\zeta}{\epsilon}-1} \\
& (((\theta_{ln}l_{l,n,ss}^m)^{\frac{1}{\zeta}} + (\theta_{lf}l_{lft}^m)^{\frac{1}{\zeta}})^{\frac{\zeta}{\epsilon}} + ((\theta_{hn}l_{hnt}^m)^{\frac{1}{\epsilon}} + (\theta_{hf}l_{hft}^m)^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\zeta}})^{\zeta-1} \\
& (l_{ss}^m)^{\frac{s-1}{s}-1}(\alpha(l_{ss}^y)^{\frac{s-1}{s}} + (1-\alpha)(l_{ss}^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0
\end{aligned}$$

Then we compute the total cost

$$\begin{aligned}
\text{TC}_t &= l_{t,h,f}^m \omega_{t,h,f}^m + l_{t,h,n}^m \omega_{t,h,n}^m + l_{t,l,f}^m \omega_{t,l,f}^m + l_{t,l,n}^m \omega_{t,l,n}^m \\
&+ l_{t,h,f}^y \omega_{t,h,f}^y + l_{t,h,n}^y \omega_{t,h,n}^y + l_{t,l,f}^y \omega_{t,l,f}^y + l_{t,l,n}^y \omega_{t,l,n}^y \\
\text{TC}_{ss} &= l_{ss,h,f}^m \omega_{ss,h,f}^m + l_{ss,h,n}^m \omega_{ss,h,n}^m + l_{ss,l,f}^m \omega_{ss,l,f}^m + l_{ss,l,n}^m \omega_{ss,l,n}^m \\
&+ l_{ss,h,f}^y \omega_{ss,h,f}^y + l_{ss,h,n}^y \omega_{ss,h,n}^y + l_{ss,l,f}^y \omega_{ss,l,f}^y + l_{ss,l,n}^y \omega_{ss,l,n}^y
\end{aligned}$$

we compute the value of y

$$\begin{aligned}
y_t &= z_t(\alpha(l_t^y)^{\frac{s-1}{s}} + (1-\alpha)(l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}} \\
y_{ss} &= z_{ss}(\alpha(l_{ss}^y)^{\frac{s-1}{s}} + (1-\alpha)(l_{ss}^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}}
\end{aligned}$$

and marginal cost

$$\text{marginal cost}_t = \frac{\text{TC}_t}{y_t}$$
$$\text{marginal cost}_{ss} = \frac{\text{TC}_{ss}}{y_{ss}}$$

and using market clearing condition we can compute the total saving

$$a_t = y_t - c_t^m - c_t^y - c_t^o$$
$$a_{ss} = y_{ss} - c_{ss}^m - c_{ss}^y - c_{ss}^o$$

Appendix C Sample

We describe the 3 digits NIACS code for each sector of our sample.

Table 6: Industry code

Industry description	Code	Industry description	Code
Accommodation	721	Wood products	321
Administrative and support services	561	Air transportation	481
Ambulatory health care services	621	Broadcasting and telecommunications	515-517
Amusements, gambling, and recreation industries	713	Data processing, internet publishing, and other information services	518-519
Apparel and leather and allied products	315-316	Educational services	611
Chemical products	325	Farms	111-112
Computer and electronic products	334	Food and beverage stores	445
Electrical equipment, appliances, and components	335	Forestry, fishing, and related activities	113-114-115
Fabricated metal products	332	General merchandise stores	451-452-453-454
Federal Reserve banks, credit intermediation, and related activities	521-522	Management of companies and enterprises	551
Food and beverage and tobacco products	311-312	Mining, except oil and gas	212
Food services and drinking places	722	Motion picture and sound recording industries	512
Furniture and related products	337	Motor vehicle and parts dealers	441
Hospitals	622	Oil and gas extraction	211
Insurance carriers and related activities	524	Other retail	442-443-444-446-447-448
Machinery	333	Other transportation and support activities	487-488-491-492
Miscellaneous manufacturing	339	Pipeline transportation	486
Nonmetallic mineral products	327	Professional, scientific, and technical services	541
Nursing and residential care facilities	623	Publishing industries, except internet (includes software)	511
Paper products	322	Rail transportation	482
Performing arts, spectator sports, museums, and related activities	711-712	Support activities for mining	213
Petroleum and coal products	324	Transit and ground passenger transportation	485
Plastics and rubber products	326	Truck transportation	484
Primary metals	331	Warehousing and storage	493
Printing and related support activities	323	Water transportation	483
Real estate	531	Construction	23
Rental and leasing services and lessors of intangible assets	532-533	Utilities	221
Social assistance	624	Wholesale trade	423-424-425
Textile mills and textile product mills	313-314	Motor vehicles, bodies and trailers, and parts and Other transportation equipment	336
Waste management and remediation services	562	Funds, trusts, and other financial vehicles and Securities, commodity contracts, and investments	523-525
		Other services, except government	811-812-813

^a Note: We assign "Not specified utilities" to the 221 NAICS code, "Not specified food industries" to 312 NAICS code. We assign "Knitting mills, and apparel knitting mills" to 315 NAICS code, and 315M to 315. We assign "Not specified metal industries" to 332 NAICS code, and 333MS to 333. We assign "Not specified wholesale" trade to 424. We assign 44511 to 445, 45121 to 452, 454110 to 454. We assign "Not specified retail trade" to 454. We assign "Banking and related activities" to 522, "Securities, commodities, funds, trusts, and other financial investments" to 523, and 532M2 to 532. We assign "Commercial, industrial, and other intangible assets rental and leasing" to 533, we also assign 55 to 551.

^b In the Bureau of Economic Analysis data, the "Construction" industry is aggregated by sector, whereas we have affected share observations by three sub sectors of the "Construction" industry. To overcome this limitation, we assign the value of affected share of the sub-sector "Construction of buildings" to the overall sector "construction". Data sources: The Bureau of Economic Analysis.

Appendix D Reshaping the data for estimation of the elasticities

We reshape the data from long dimension (Table 7) to wide dimension (Table 8).

ind. code	month	year	county	state	l	ω	origin	skill	age category
					$l_{h,f}^y$	$\omega_{h,f}^y$	1	1	1
\vdots	\vdots	\vdots	\vdots	\vdots	$l_{h,n}^y$	$\omega_{h,n}^y$	0	1	1
					$l_{l,f}^m$	$\omega_{l,f}^m$	1	0	0

Table 7: The shape of data

ind. code	year	month	state	county	$l_{h,f}^y$	$l_{h,n}^y$	$\omega_{h,f}^y$	$\omega_{h,n}^y$	$l_{l,f}^y$	$l_{l,n}^y$	$\omega_{l,f}^y$	$\omega_{l,n}^y$
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots

Table 8: The shape of data

Appendix E Regression results: the elasticity of substitution

Table 9: Estimation of the elasticity of substitution $1/\zeta - 1$

	Dependent variable: $\ln(\omega_{l,f,t}^y) - \ln(\omega_{l,n,t}^y)$			
	(1)	(2)	(3)	(4)
Constant	0.00465** (0.00181)	0.00469** (0.00182)	0.00464** (0.00181)	0.00470*** (0.00182)
$\ln(I_{l,f,t}^m) - \ln(I_{l,n,t}^m)$	0.11263*** (0.00680)	0.11628*** (0.00683)	0.11216*** (0.00680)	0.11672*** (0.00683)
Fixed effects	County, Year		County	Year
F-statistic	274.1	289.5	271.7	291.8
No. Observations	98709	98709	98709	98709

Note: The estimates in this table is obtained using a simple OLS with county and year fixed effects, homoscedastic standard error in parentheses * $p < .1$, ** $p < .05$, *** $p < .01$

Table 10: Estimation of the elasticity of substitution $1/\zeta - 1$

	Dependent variable: $\ln(\omega_{h,t}^m) - \ln(\omega_{l,t}^m)$			
	(1)	(2)	(3)	(4)
Constant	0.64794*** (0.00117)	0.64793*** (0.00118)	0.64797*** (0.00117)	0.64790*** (0.00117)
$\ln(I_{h,t}^m) - \ln(I_{l,t}^m)$	0.07146*** (0.00638)	0.07175*** (0.00639)	0.07064*** (0.00638)	0.07258*** (0.00639)
Fixed effects	County, Year		County	Year
F-statistic	125.5	126.1	122.5	129.2
No. Observations	98709	98709	98709	98709

Note: The estimates in this table is obtained using a simple OLS with county and year fixed effects, homoscedastic standard error in parentheses * $p < .1$, ** $p < .05$, *** $p < .01$

Table 11: Estimation of the elasticity of substitution $1/\varrho - 1$

	Dependent variable: $\ln(\omega_{h,f,t}^y) - \ln(\omega_{h,n,t}^y)$			
	(1)	(2)	(3)	(4)
Constant	-0.09227*** (0.00126)	-0.09226*** (0.00126)	-0.09227*** (0.00126)	-0.09226*** (0.00126)
$\ln(l_{h,f,t}^y) - \ln(l_{h,n,t}^y)$	0.14092*** (0.00558)	0.14080*** (0.00558)	0.14093*** (0.00558)	0.14078*** (0.00558)
Fixed effects	County, Year		County	Year
F-statistic	637.8	635.9	638.0	635.8
No. Observations	98709	98709	98709	98709

Note: The estimates in this table is obtained using a simple OLS with county and year fixed effects, homoscedastic standard error in parentheses $*p < .1$, $**p < .05$, $***p < .01$

Table 12: Estimation of the elasticity of substitution $1/\varepsilon - 1$

	Dependent variable: $\ln(\omega_{h,f,t}^m) - \ln(\omega_{h,n,t}^m)$			
	(1)	(2)	(3)	(4)
Constant	-0.16092*** (0.00241)	-0.16091*** (0.00242)	-0.16091*** (0.00241)	-0.16092*** (0.00242)
$\ln(l_{h,f,t}^m) - \ln(l_{h,n,t}^m)$	0.05898*** (0.00593)	0.05850*** (0.00596)	0.05774*** (0.00594)	0.05972*** (0.00595)
Fixed effects	County, Year		County	Year
F-statistic	98.84	96.49	94.62	100.7
No. Observations	98709	98709	98709	98709

Note: The estimates in this table is obtained using a simple OLS with county and year fixed effects, homoscedastic standard error in parentheses $*p < .1$, $**p < .05$, $***p < .01$

Table 13: Estimation of the elasticity of substitution $1/\nu - 1$

	Dependent variable: $\ln(\omega_{l,f,t}^y) - \ln(\omega_{l,n,t}^y)$			
	(1)	(2)	(3)	(4)
Constant	0.35256*** (0.00075)	0.35256*** (0.00075)	0.35256*** (0.00075)	0.35256*** (0.00075)
$\ln(l_{l,f,t}^y) - \ln(l_{l,n,t}^y)$	0.11723*** (0.00604)	0.11781*** (0.00604)	0.11732*** (0.00604)	0.11771*** (0.00604)
Fixed effects	County, Year		County	Year
F-statistic	377.0	380.6	377.6	380.0
No. Observations	98709	98709	98709	98709

Note: The estimates in this table is obtained using a simple OLS with county and year fixed effects, homoscedastic standard error in parentheses $*p < .1$, $**p < .05$, $***p < .01$

Table 14: Estimation of the elasticity of substitution $1/s$

	Dependent variable: $\ln(\omega_t^m) - \ln(\omega_t^y)$			
	(1)	(2)	(3)	(4)
Constant	-0.22813*** (0.00076)	-0.22825*** (0.00077)	-0.22825*** (0.00076)	-0.22813*** (0.00077)
$\ln(l_t^m) - \ln(l_t^y)$	0.07658*** (0.00623)	0.07489*** (0.00627)	0.07487*** (0.00624)	0.07658*** (0.00626)
Fixed effects	County, Year		County	Year
F-statistic	151.0	142.9	144.1	149.6
No. Observations	98709	98709	98709	98709

Note: The estimates in this table is obtained using a simple OLS with county and year fixed effects, homoscedastic standard error in parentheses $*p < .1, **p < .05, ***p < .01$

Table 15: Estimation of the elasticity of substitution $1/\sigma - 1$

	Dependent variable: $\ln(\omega_{h,t}^y) - \ln(\omega_{l,t}^y)$			
	(1)	(2)	(3)	(4)
Constant	0.72139*** (0.00078)	0.72155*** (0.00079)	0.72142*** (0.00078)	0.72153*** (0.00079)
$\ln(l_{h,t}^y) - \ln(l_{l,t}^y)$	0.03924*** (0.00512)	0.03787*** (0.00512)	0.03904*** (0.00512)	0.03805*** (0.00512)
Fixed effects	County, Year		County	Year
F-statistic	58.73	54.67	58.13	55.21
No. Observations	98709	98709	98709	98709

Note: The estimates in this table is obtained using a simple OLS with county and year fixed effects, homoscedastic standard error in parentheses $*p < .1, **p < .05, ***p < .01$

Appendix F Regression results: productivity shock parameters

Table 16: Estimation of the productivity shock parameters

Dependent variable: $\ln(\text{tfp}_{i,t})$	Coefficients
Lagged total factor productivity $\ln(\text{tfp}_{i,t-1})$	0.98319*** (0.00251)
The affected share $d_{i,t}$	-0.00173*** (0.00060)
No. Observations:	610

Note: homoscedastic standard error in parentheses $*p < .1, **p < .05, ***p < .01$