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**Working Paper**

## Advantageous selection with moral hazard (with an application to life care annuities)

Document de travail, No. 2022-04

**Provided in Cooperation with:**

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*Suggested Citation:* De Donder, Philippe; Leroux, Marie-Louise; Salanié, François (2022) : Advantageous selection with moral hazard (with an application to life care annuities), Document de travail, No. 2022-04, Université du Québec à Montréal, École des sciences de la gestion (ESG UQAM), Département des sciences économiques, Montréal

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**Mai 2022**

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# Advantageous selection without moral hazard <sup>\*</sup>

(with an application to life care annuities)

Philippe De Donder<sup>†</sup>, Marie-Louise Leroux<sup>‡</sup>, François Salanié<sup>§</sup>

May 10, 2022

## Abstract

Advantageous (or propitious) selection occurs when an increase in the premium of an insurance contract induces high-cost agents to quit, thereby reducing the average cost among remaining buyers. Hemenway (1990) and many subsequent contributions motivate its advent by differences in risk-aversion among agents, implying different prevention efforts. We argue that it may also appear in the absence of moral hazard, when agents only differ in riskiness and not in (risk) preferences. We first show that profit-maximization implies that advantageous selection is more likely when markup rates and the elasticity of insurance demand are high. We then move to standard settings satisfying the single-crossing property and show that advantageous selection may occur when several contracts are offered, when agents also face a non-insurable background risk, or when agents face two mutually exclusive risks that are bundled together in a single insurance contract. We exemplify this last case with life care annuities, a product which bundles long-term care insurance and annuities, and we use Canadian survey data to provide an example of a contract facing advantageous selection.

**Keywords:** Propitious selection, positive or negative correlation property, contract bundling, long-term care insurance, annuity

**JEL Codes:** D82, I13

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<sup>\*</sup>This project was started while Marie-Louise Leroux was visiting TSE, which she thanks for its hospitality. Financial support from the ANR (Programme d'Investissement d'Avenir ANR-17-EURE-0010), from the Chair "Marché des risques et création de valeurs, fondation du risque/Scor", from CRSH-SSRHC and FRQSC is gratefully acknowledged. The authors also thank P.-Y. Yanni for his expert help with the data.

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# 1 Introduction

Since the classical contribution of Rothschild and Stiglitz (1976), most of the literature on asymmetric information in insurance markets has highlighted the difficulties associated with adverse selection. As is well-known, these difficulties include the under-provision of insurance, and even sometimes a market breakdown, without trade at all (Hendren, 2013). By contrast, the seminal paper by Hemenway (1990) argues that sometimes selection can be advantageous. In such a case, insurers should be more eager to increase insurance provision, thereby yielding very different market outcomes, as well as different testable predictions.

Hemenway (1990, 1992) and most subsequent papers (de Meza and Webb, 2001; Finkelstein and McGarry, 2006; Cohen and Einav, 2007; Davidoff and Welke, 2007; Fang et al. 2008; Wang et al. 2009; De Donder and Hindriks, 2009) envision advantageous selection as arising from differences in risk-aversion among agents: more risk-averse agents buy more coverage, and simultaneously exert more self-protection efforts, so that they may end up less risky overall.<sup>1</sup> Our paper instead argues that advantageous selection may occur under various circumstances, including regular settings where agents differ only in riskiness, exhibiting the same attitude towards risk, and where moral hazard is excluded. To do so, we first have to provide a precise definition for the type of selection.

Indeed, two definitions of selection can be drawn from the literature. Chiappori and Salanié (2000) focus on testing for the existence of private information, and to do so they propose a correlation test: under adverse selection, it is expected that riskier agents choose higher coverage, thus yielding a Positive Correlation Property (PCP hereafter) between coverage and riskiness. Conversely, a negative correlation (NCP) points toward the existence of advantageous selection. Notice that such a test is global, and requires data about distinct populations of agents, each buying a different contract. More recently, Einav and Finkelstein (2011) aim at estimating welfare changes, and to do so they mostly focus on the case where only one insurance contract

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<sup>1</sup>Hemenway (1990) makes a distinction between favorable and propitious selection, in that the latter is a special case of the former. We use the word advantageous in this paper, noting that the recent literature uses the terms advantageous, favorable and propitious interchangeably.

is offered, to agents who are privately informed of their own exogenous riskiness. A change in the contract premium then modifies the agents' decision to insure, and therefore changes the indemnities the insurer expects to pay. This article defines the type of selection by the sign of the slope of the insurer's average cost function: Advantageous selection occurs when "as price is lowered and more individuals opt into the market, the marginal individual opting in has higher expected cost than infra-marginal individuals" (p.124).<sup>2</sup> This local definition thus only requires data on a population of agents buying the same contract.

In this paper, we adopt this definition, because it is better tailored to the study of pricing by the insurer. We also follow this paper by considering that adverse selection is a serious impediment to the provision of insurance, while on the opposite advantageous selection tends to favor over-insurance. Notice that these two definitions are strongly related, at least in this one-contract setting with exogenous riskiness: in the case of advantageous selection, the local definition states that lowering the premium attracts relatively more costly agents from the pool of uninsured agents, so that the average cost among clients increases; while the negative correlation property says that insured agents display on average lower costs than uninsured agents. This intuition will however be shown to fail in richer settings with more than one contract.

We begin by noticing that in many theoretical approaches with two types of agents, perfect competition or binding incentive constraints imply that demand functions are infinitely elastic, even for small changes in the premium (see for example the analysis in de Meza and Webb (2001), among many others). In Section 3, we adopt instead a general framework where demand functions are continuous so that each insurer enjoys some degree of market power, and chooses the premium to maximize the profit from the contract this insurer offers. A simple result follows. A contract may face advantageous selection (using the marginal cost definition) only when the premium markup over average cost, and the elasticity of demand for this contract, are high enough. The first element points in the direction of market power. The second one means that

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<sup>2</sup>Making the link with previous empirical investigations, they also note that "making inferences about marginal individuals is difficult, however. As a result, the early empirical approaches developed strategies that attempt to get around this difficulty by, instead, focusing on comparing averages", and more precisely by "comparing the expected cost of those with insurance to the expected cost of those without (or comparing those with more insurance coverage to those with less coverage)" (p.127). We are thus back to the global correlation test.

a significant fraction of consumers (presumably the high-cost consumers) do not enjoy much rent, compared to other contracts, so that these consumers may leave *en masse* when the premium is raised.

To which contract these consumers switch is in fact a key question. Section 4 studies a classical model à la Rothschild-Stiglitz (1976), in which a single-crossing property ensures that riskier agents end up with higher coverage. This means that in any case, the definition based on a correlation between riskiness and coverage results in the impossibility of advantageous selection (i.e., the PCP holds). We then observe that such a conclusion is not warranted when one applies the marginal-cost definition to a given contract. Indeed, an increase in the premium makes some agents quit, either to subscribe to a contract with less coverage (then these agents must be relatively low-risk agents, according to the single-crossing property), or to a contract with more coverage (for the relatively high-risks agents). The balance between these two opposite effects is thus ambiguous, and so is the nature of selection. Hence, even in the most structured case where coverage is positively correlated with riskiness, a contract may well face advantageous selection. The global measure of selection (PCP) thus disagrees with the local test proposed by Einav et al. (2011), due to the existence of both contracts with lower coverage (including the absence of insurance), *and* of contracts with higher coverage.

We then explore more realistic settings with two exclusive risks in Section 5. Once more, there is no moral hazard, and agents differ only through their riskiness, which is now a two-dimensional variable. To avoid the effects described above, we only allow for one insurance contract. We first assume that only one risk is insurable, while the other one is a non-insurable background risk. As a consequence, when buying insurance for the first risk, a consumer should take into account the fact that he pays the premium even in the state when the insurable risk did not occur, while the background risk did. This reduces his willingness-to-pay for an insurance contract covering only the first risk, and more so for an agent whose probability to face the background risk is higher. Therefore, participation in an insurance contract depends on private exposure to the background risk. We show by means of an example that agents with a high probability to incur the insurable loss may choose not to insure, because they also display a

high probability of the background risk. This proves that advantageous selection may occur even when a single contract is offered, provided the correlation of the insurable risk with the background risk is high enough.

In the same model, but allowing both risks to be insured, we also show how a contract bundling together coverage against the two risks may face advantageous selection, even though all contracts that would cover only one of these risks would face adverse selection. This leads to the observation that issuing bundled insurance contracts may help insurers avoid the difficulties associated with adverse selection.

Section 6 applies this idea to the bundling of longevity and Long-Term Care (LTC hereafter) risks. Longevity risk is usually insured through annuity contracts, for which empirical studies tend to support the existence of adverse selection.<sup>3</sup> LTC corresponds to the risk of becoming dependent at old age and of needing “day-to-day help with activities such as washing and dressing, or help with household activities such as cleaning and cooking” (OECD, 2011). While this risk realizes with high probability and implies significant expenses, most people choose not to insure against it. OECD (2011) has estimated that only 2% of LTC expenditures are financed by private LTC insurance (LTCI hereafter) in OECD countries, while the figure is 7% in the US. This is what is often referred to as the LTCI puzzle. Adverse selection is often mentioned as one reason why this market is so little developed.<sup>4</sup> Notice that in both cases, moral hazard can be assumed away, so that only riskiness matters.

We claim that proposing an insurance contract that would insure against both the longevity and the LTC risks could alleviate simultaneously both adverse selection problems. Such a contract was first proposed by Pauly (1990) who saw it as a way to foster the purchase of LTCI in a context where individuals rationally decide not to buy LTCI. Such a contract has then be referred to as a ‘life care annuity’ contract in Murtaugh et al. (2001) and Brown and Warshawsky (2013), who see it as a way to alleviate underwriting problems (quite common on

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<sup>3</sup>Some classical references are Finkelstein and Poterba (2002, 2004), and Rothschild (2009).

<sup>4</sup>On the LTC insurance puzzle and the importance of adverse selection, see among others, Pauly (1990), Sloan and Norton (1997), Brown and Finkelstein (2007, 2009), Pestieau and Ponthiere (2012), Lockwood (2018) and Boyer et al. (2019, 2020).

the LTCI market) as well as adverse selection problems.<sup>5</sup> Intuitively, agents with poor health face high prices for LTC insurance, while they are less costly on the annuity side because of their reduced life expectancy. The bundling of insurance would thus increase the participation of poor health/low life expectancy individuals and reduce the extent of adverse selection. Our argument is different since we argue that bundling could even generate advantageous selection. In fact, using Canadian survey data we are able to recover the two-dimensional riskiness probabilities for a dataset of 2,000 agents, and under reasonable assumptions on preferences we exhibit an example of a bundled contract displaying advantageous selection.

## 2 Framework and definitions

In this section, we describe a general insurance economy with hidden types. As discussed above, we depart from most of the existing literature by providing rationales for advantageous selection which do not hinge on moral hazard.

Consider a fixed population of observationally identical consumers. Each consumer faces a risk, which is represented by a stochastic, individual-specific state of the world, whose realization is public. An insurance contract is a premium  $P \geq 0$  to be paid upfront, and an indemnity function  $I(\cdot)$ , which specifies the amount to be paid as a function of the realization of the state.

Each consumer is endowed with a private information  $\alpha$ , that may bear both on his preferences and riskiness, and whose distribution in the population is known.<sup>6</sup> An individual with type  $\alpha$  who buys the contract  $(P, I(\cdot))$  obtains a payoff  $V(P, I(\cdot), \alpha)$ , and for the moment we only assume that this payoff is decreasing with the premium  $P$ . The insurer's revenue is  $P$ , minus the cost associated to the payment of indemnities that we represent as a function  $c(I(\cdot), \alpha)$ . Notice that a consequence of excluding moral hazard is that this cost does not depend on  $P$ .<sup>7</sup>

As an illustration, in the Rothschild-Stiglitz (1976) model, agents face a binomial risk on their wealth and differ through their riskiness  $\alpha$  distributed on  $[0, 1]$ . Then, an insurance contract

<sup>5</sup>Ameriks et al. (2011) provides survey evidence of a significant potential demand for such a product.

<sup>6</sup>From Section 4 on, we assume that agents differ only in riskiness.

<sup>7</sup>Moral hazard would re-introduce  $P$  as an argument of  $c$ , either because the agent's choice of effort may be impacted by the value of the premium through wealth effects, or – more perversely – because the agent may be indifferent between several effort levels and may change his selection as a function of the premium.

is simply a pair  $(P, I)$ , and payoffs and costs are:

$$\begin{aligned} V(P, I, \alpha) &= \alpha u(w - L + I - P) + (1 - \alpha)u(w - P) \\ c(I, \alpha) &= \alpha I, \end{aligned}$$

where  $w$  is the initial wealth level,  $L$  is the loss, and  $u(\cdot)$  is an increasing and concave function. By contrast, our general framework encompasses in addition non-expected utility, multiple states, or general cost functions.

The market situation we consider throughout the paper is as follows. The focus is on a particular contract  $(P, I(\cdot))$ , while other available contracts  $(P_j, I_j(\cdot))_{j \in J}$  are fixed. A consumer with type  $\alpha$  will select the contract under study if and only if the following participation constraint holds:

$$PC(\alpha) : \quad V(P, I(\cdot), \alpha) \geq \max_{j \in J} V(P_j, I_j(\cdot), \alpha).$$

This inequality defines a set of subscribers, together with the subscribers' distribution of types. If this set is nonempty, and ignoring ties, we can define the insurer's average cost for this contract as a function

$$AC(P, I(\cdot), (P_j, I_j(\cdot))_{j \in J}) = E[c(I(\cdot), \alpha) | PC(\alpha) \text{ holds}],$$

where the expectation is taken over the set of types. Because the emphasis will be put on changes in the premium  $P$ , we shall often omit arguments and use the lighter notation  $AC(P)$ . As in Einav et al. (2010a) and Einav and Finkelstein (2011), we adopt the following definition:

**Definition 1** *A contract  $(P, I(\cdot))$  is said to face adverse selection when  $AC(P)$  is increasing with  $P$ , while it faces advantageous selection when  $AC(P)$  is decreasing with  $P$ .*

These definitions formalize the idea that a change in premium affects the composition of the pool of subscribers, in a manner that may harm or benefit the insurer. In fact, in the absence of moral hazard this composition effect is the unique channel through which a change in premium has some impact on the expected cost of the contract.<sup>8</sup>

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<sup>8</sup>Interestingly, the survey in Einav et al. (2010b) allows for moral hazard, but uses a different definition:

As recalled in the Introduction, Hemenway (1990, 1992) and many subsequent works single out a rationale for advantageous selection which relies on agents having different attitudes towards risk, and choosing both their coverage and their prevention effort. The driving force is that more risk-averse agents choose both a higher coverage and higher prevention efforts, and it may be that this second effect is strong enough to reduce the riskiness of insured agents below that of uninsured agents. Accordingly, the nature of selection is an empirical question, and the test these works use is based on the correlation between riskiness and coverage. This positive correlation property (PCP, see Chiappori and Salanié (2000)) states that riskier agents buy more coverage, and it is often a consequence of well-structured theoretical models à la Rothschild-Stiglitz (1976). Chiappori et al. (2006) use a revealed preference argument to show that a similar property also obtains in very general models, provided profits are non-increasing with coverage. As underlined in de Meza and Webb (2001), a negative correlation property (NCP) may also appear in a competitive equilibrium when administrative costs are significant.

From an empirical viewpoint, our definition focuses on a single contract, but requires to estimate how demand changes as a function of the premium; while the correlation test requires to collect data for at least two contracts (or to compare the population of insured to the population of uninsured agents). As mentioned in the Introduction, Einav and Finkelstein (2011) argue that both definitions are equivalent in the specific setting they study, where  $AC(P)$  is assumed monotonic in  $P$ .

In the next section, we study the case of a profit-maximizing insurer facing a continuous insurance demand and we relate advantageous selection to the equilibrium markup and elasticity for the contract proposed.

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There is adverse selection if  $AC(P)$  is greater than the average cost of insuring the *whole* population with the contract  $(P, I(\cdot))$ , assuming that each agent would select the optimal effort under that contract. This definition has the drawback of requiring a precise knowledge of the whole population, while the definition we use allows to only consider the effects of an increase in the premium of a contract on the population of subscribers to this contract.

### 3 Advantageous selection and profit maximization

Many theoretical analyses of insurance markets assume a discrete set of types, together with perfect competition in prices. This implies that equilibrium incentive constraints bind for a nonzero fraction of consumers, leading to discontinuous demand functions.<sup>9</sup> Maybe more realistically, we assume in this Section that the insurer offering the contract under study enjoys some market power, so that he faces a continuous demand function  $D(P)$ , equal to the measure of agents for whom the participation constraint  $PC(\alpha)$  holds. Assuming differentiability, we define the demand elasticity,  $\varepsilon$ , as usual:

$$\varepsilon(P) = -\frac{PD'(P)}{D(P)} > 0,$$

with  $D'(P) < 0$ . Then, the insurer's profits are

$$\Pi(P) = (P - AC(P))D(P).$$

Assuming that the insurer sets the premium to maximize profits from this contract, the first-order condition is

$$\Pi'(P) = (1 - AC'(P))D(P) + (P - AC(P))D'(P) = 0.$$

Advantageous selection occurs when  $AC'(P) < 0$ . Replacing in the above condition, this yields:

$$\varepsilon(P) \frac{P - AC(P)}{P} > 1.$$

**Result 1** *When the insurer enjoys market power on a given insurance contract, advantageous selection requires that the product of the markup rate above average cost, and of the elasticity of demand for this contract, be above one.*

Advantageous selection is thus more likely to appear when competition is weak and when high-cost consumers derive few benefits from the contract, so that they are ready to quit *en masse* as soon as the premium increases. It is interesting to note that such a result can only arise when

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<sup>9</sup>This property indeed holds for equilibria in Rothschild and Stiglitz (1976) or de Meza and Webb (2001).

profits are positive; if they are zero, then the first-order condition for profit maximization implies  $AC'(P) = 1$ , and one must conclude to the presence of adverse selection. Finally, the first-order condition itself need not hold if the insurer optimizes profits jointly over the multiple contracts it offers, and it is meaningless if demand is discontinuous at  $P$ ; in the latter case, a precise study of incentive constraints is needed.

In any case, whatever the market structure, a study of selection requires to know who the consumers are, and to which other contract they switch. This is what we intend to do in the next sections. We make no specific assumption regarding competition in the insurance market and rather look at the conditions under which advantageous selection (as given by Definition 1) occurs for a given set of insurance contracts. The next section studies the important class of settings where the single-crossing condition holds, implying that the PCP also holds, and shows that advantageous selection may nevertheless occur when several contracts are offered.

## 4 Advantageous selection when the single crossing property holds

We now focus on a particular class of settings that have been widely used in the literature in order to study the effects of adverse selection. In the Rothschild and Stiglitz (1976) model, agents differ only in riskiness, and those with a higher probability of loss are more costly to the insurer. For the same reason, they are also more eager to buy more coverage. This single-crossing property creates a rich structure. For a given set of insurance contracts, the subscribers of each contract form an interval of risk types, and these intervals are ordered by coverage so that the PCP necessarily holds.

We first show that this structure also arises in equilibrium for a much more general class of models. Formally, consider the case where  $\alpha$  is unidimensional, with a differentiable distribution  $F$  on the interval  $[0, 1]$ . Suppose that consumers face a monetary risk  $L$  continuously distributed over some interval, and that higher types face a higher risk, in the sense of the Monotone Likelihood Ratio Property (MLRP hereafter).<sup>10</sup> Each agent is an expected utility maximizer,

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<sup>10</sup>This property expresses that the ratio of conditional densities  $g(L|\alpha')/g(L|\alpha)$  is increasing in  $L$  when  $\alpha' > \alpha$ ,

and gets the payoff

$$V(P, I(\cdot), \alpha) \equiv E[u(w - P + I(L) - L) | \alpha]$$

when he subscribes to the insurance contract  $(P, I(\cdot))$ . Note that all agents share the same utility function, and thus the same risk attitude.

As in Chiappori et al. (2006), we assume that available contracts specify indemnity functions  $I(\cdot)$  that are increasing with the loss, and in addition that the amount which is not covered increases with the loss, i.e.,  $L - I(L)$  is increasing with  $L$  (for example, because one does not want the agent to hide the true extent of their losses). Finally, we assume that available contracts can be ranked by coverage, in the following sense: a contract  $(P_2, I_2(\cdot))$  covers more than a contract  $(P_1, I_1(\cdot))$  if and only if the difference  $I_2(L) - I_1(L)$  is increasing in  $L$ . Many classes of real-life contracts satisfy these requirements, for example contracts with deductibles ( $I(L) = \max\{L - D, 0\}$ ), or co-insurance contracts ( $I(L) = \delta L$ ,  $\delta < 1$ ).

Our last assumption is that the individual cost function  $c(I(\cdot), \alpha)$  increases with the expected payment  $E[I(L) | \alpha]$ . Then MLRP yields two properties. First, higher types are more costly to insure. Second, if a type prefers a contract that covers more to a contract that covers less, then so do all higher types.<sup>11</sup> This single-crossing property implies that the population of types is segmented in intervals, with higher (and thus costlier) types choosing contracts with higher coverage. This generalized model thus yields the same market structure as the Rothschild-Stiglitz model, and this implies that the PCP holds, for every pair of contracts.

The set of subscribers to a contract  $(P, I(\cdot))$  thus forms an interval of types  $[\underline{\alpha}(P), \bar{\alpha}(P)]$ . Notice that the lower bound,  $\underline{\alpha}(P)$  is increasing with  $P$ , while the upper bound,  $\bar{\alpha}(P)$  is decreasing with  $P$ . Indeed, when  $P$  increases, low-risk agents prefer switching to a contract with less coverage, and high-risk agents to one with more coverage.

Consider first the particular case where only one contract is offered on the market. This

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thus making higher losses more likely for higher types.

<sup>11</sup>A proof of this result is provided in Appendix.

contract attracts all types above the threshold  $\underline{\alpha}(P)$ , so that  $\bar{\alpha}(P) = 1$ , and we obtain

$$AC(P) = \frac{\int_{\underline{\alpha}(P)}^1 c(I(\cdot), \alpha) dF(\alpha)}{1 - F(\underline{\alpha}(P))}.$$

In this case, adverse selection must prevail: the marginal consumers who leave when the premium is increased are those consumers with the lowest types  $\underline{\alpha}(P)$ , and these types display the lowest cost among all subscribers. Indeed, one easily shows that the derivative  $AC'(P)$  has the sign of

$$f(\underline{\alpha}(P))\underline{\alpha}'(P)[AC(P) - c(I(\cdot), \underline{\alpha}(P))],$$

which is nonnegative.

However, in the more general case where several contracts are offered, there might exist a contract offering a higher coverage, and in that case the upper bound  $\bar{\alpha}(P)$  is not trivial anymore. Then we have

$$AC(P) = \frac{\int_{\underline{\alpha}(P)}^{\bar{\alpha}(P)} c(I(\cdot), \alpha) dF(\alpha)}{F(\bar{\alpha}(P)) - F(\underline{\alpha}(P))},$$

and we get that the derivative  $AC'(P)$  has the same sign as

$$f(\underline{\alpha}(P))\underline{\alpha}'(P)[AC(P) - c(I(\cdot), \underline{\alpha}(P))] + f(\bar{\alpha}(P))\bar{\alpha}'(P)[c(I(\cdot), \bar{\alpha}(P)) - AC(P)]. \quad (1)$$

An increase in  $P$  now has two effects. Marginal subscribers with the lowest types decide to migrate to a contract with a lower coverage (the first term above), while marginal subscribers with the highest type instead migrate to a contract with higher coverage (the second term). The sum of these effects is ambiguous, as  $\bar{\alpha}'(P) \leq 0 \leq \underline{\alpha}'(P)$ .<sup>12</sup> Once more, the nature of selection appears to be more an empirical question than a theoretical one.

To summarize, we have shown that *adverse and advantageous selection are mirror images of each other*. They appear simultaneously, for each contract, as soon as the set of contracts is sufficiently rich. The assumptions we made in this section are often said to formalize the idea of adverse selection – but in fact they generate advantageous selection as well. At the extremes, the

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<sup>12</sup>Even when postulating uniformly distributed types and costs  $c(I(\cdot), \alpha)$  linear in  $\alpha$ , expression (1) only tells us to check the sign of  $\underline{\alpha}'(P) + \bar{\alpha}'(P)$ . Despite these strong assumptions, this sign still depends on detailed properties of neighboring contracts and remains ambiguous.

contract with the lowest coverage (maybe the null contract) must exhibit advantageous selection, while the contract with the highest coverage must exhibit adverse selection; in between, the nature of selection depends not only on costs, preferences, and the distribution of types, but also on the presence and characteristics of other contracts.

**Result 2** *Under single-crossing, advantageous selection may arise when there exists an alternative insurance contract offering higher coverage.*

## 5 Advantageous selection, background risk and bundling

In this section, we explore two other possible sources of advantageous selection, namely the existence of a background risk, and the bundling of two risks into a unique insurance contract. Once more, this exploration will be conducted in a standard framework, with agents privately informed of their riskiness. Given the previous result, we also assume that there are no other contracts available on the market, so that a result of advantageous selection cannot be attributed to the existence of alternative contracts providing higher coverage.

### 5.1 The setting

For each agent, there are three mutually exclusive individual states of the world, labelled 0, 1 and 2. An agent ending up in state  $i$  incurs a monetary loss equal to  $L_i$ , with  $L_0 = 0$  so that state 0 is the state without loss. An agent's type  $\alpha = (\alpha_1, \alpha_2)$  specifies the probabilities of ending up in state  $i = \{1, 2\}$ , with  $\alpha_1 + \alpha_2 < 1$ . Types are distributed in the population of agents according to a c.d.f.  $F$ . An insurer proposes to these agents a single contract  $(P, I_1, I_2)$ . If an agent of type  $(\alpha_1, \alpha_2)$  buys it, then the insurer's cost is simply the expectation of indemnities paid:

$$c(I_1, I_2, \alpha) = \alpha_1 I_1 + \alpha_2 I_2,$$

and the expected utility of an agent with initial wealth  $w$  is<sup>13</sup>

$$V(P, I_1, I_2, \alpha) = \alpha_1 u(w - L_1 + I_1 - P) + \alpha_2 u(w - L_2 + I_2 - P) + (1 - \alpha_1 - \alpha_2) u(w - P).$$

where  $u$  is strictly increasing and strictly concave in income, and is the same for all agents.

## 5.2 Background risk

One intriguing question is whether advantageous selection can occur for contracts that cover a single risk, say the risk of being in state 1, while individuals face also a second uninsurable risk (equivalently, a background risk), say risk 2. Thus, consider the contract  $(P, I_1, 0)$ . A type  $\alpha$  buys this contract if and only if  $V(P, I_1, 0, \alpha) \geq V(0, 0, 0, \alpha)$ , or equivalently

$$\begin{aligned} \alpha_1 u(w - P - L_1 + I_1) + \alpha_2 u(w - L_2 - P) + (1 - \alpha_1 - \alpha_2) u(w - P) \\ \geq \alpha_1 u(w - L_1) + \alpha_2 u(w - L_2) + (1 - \alpha_1 - \alpha_2) u(w). \end{aligned}$$

This participation constraint can be rewritten as

$$\begin{aligned} \alpha_1 u(w - P - L_1 + I_1) + (1 - \alpha_1) u(w - P) &\geq \alpha_1 u(w - L_1) + (1 - \alpha_1) u(w) \\ &+ \alpha_2 [u(w - L_2) - u(w - L_2 - P) - (u(w) - u(w - P))]. \end{aligned}$$

We recognize on the first line the usual trade-off associated to the risk  $\alpha_1$  of a single loss  $L_1$ , but on the second-line an additional term appears. The bracketed expression turns out to be positive under risk-aversion, and is increasing in  $P$ . This means that a higher  $\alpha_2$  makes the agent more reluctant to participate, and more sensitive to an increase in the premium. Intuitively, because risks are exclusive, the only impact of the background risk is to increase the utility cost of paying the contract premium. This simple observation means that even in the case of a contract that covers only one risk, the participation frontier is not trivial.

We now show that this contract may face advantageous selection, even though it is the only contract on the market. Consider the simple case of full coverage:  $I_1 = L_1$ . Then the

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<sup>13</sup>In this section, we abstract from the possibility of state-dependent preferences as our rationale for the emergence of advantageous selection is orthogonal to this issue. In Section 6 we consider state-dependent preferences because they seem indispensable when modeling the risk of death.

participation constraint can be rewritten as

$$b\alpha_1 - a \geq \alpha_2,$$

where

$$a = \frac{u(w) - u(w - P)}{u(w - L_2) - u(w - L_2 - P) - (u(w) - u(w - P))} > 0$$

$$b = \frac{u(w) - u(w - L_1)}{u(w - L_2) - u(w - L_2 - P) - (u(w) - u(w - P))} > 0.$$

where  $a$  may increase or decrease with  $P$  (because of wealth effects) while  $b$  decreases with  $P$ . This participation frontier is represented in the space of types in Figure 1 by an ascending solid line. Agents with types  $(\alpha_1, \alpha_2)$  above the frontier prefer not to buy the contract while those with types below do. The descending solid line is the line  $\alpha_2 + \alpha_1 = 1$ , that constrains feasible types. The bottom triangle is thus the set of subscribers to the contract under study.

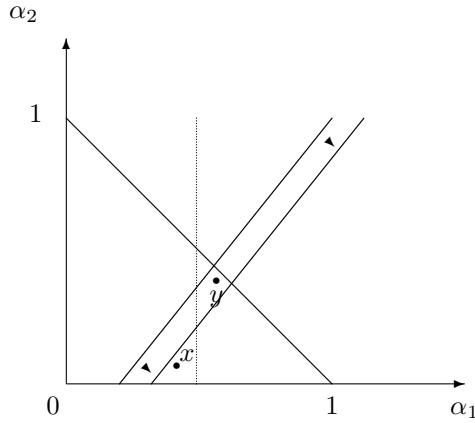


Figure 1: Advantageous selection with a background risk

Note: Types in the bottom triangle buy the contract; the dotted line is an iso-cost curve.

To illustrate our argument, consider the special case where types take only two values  $x$  and  $y$ , as shown in the figure. The iso-cost curve corresponding to the average cost is the vertical dotted line, with an equation given by

$$AC(P) = E[\alpha_1 I_1 | b\alpha_1 - a \geq \alpha_2].$$

Being an average cost, it must thus lie in between  $x$  and  $y$ , as shown on Figure 1. An increase in  $P$  then translates the ascending solid line in the direction of the arrows, into a new ascending line (it is parallel to the initial line under constant absolute risk-aversion). One sees on Figure 1 that this move indeed induce type  $y$  agents to quit buying the contract because their high probability of a background risk reduces their willingness-to-pay for insurance, while type  $x$  agents choose to stay. In this case, an increase in  $P$  leads to a reduction in  $AC(P)$ , and advantageous selection occurs.<sup>14</sup> We have shown the following result:

**Result 3** *The presence of a background risk makes advantageous selection possible, even in the simple case of an insurance contract designed to cover only one risk.*

Note that the introduction of the background risk  $L_2$  is the only difference with the Rothschild-Stiglitz model. To check for the role of correlations in the above result, one has to carefully distinguish two notions. We assumed that the two risks were exclusive, and thus a strong form of *negative correlation between losses*. Alternatively, one could assume that the two losses are independent, thus creating four different states of nature instead of three. It can be shown that under constant absolute risk-aversion, the new participation constraint is independent from the value of  $\alpha_2$ , and therefore that we are back to a standard model where adverse selection must hold; taking wealth effects into account considerably complicates the picture, and we will not follow this avenue further. Going back to our case where risks are exclusive, one sees from the above reasoning that a form of *positive correlation between riskinesses* has to hold for advantageous selection to prevail: when the premium is increased, agents with high values for  $\alpha_1$  are the first to leave because they also display high values for  $\alpha_2$ , and thus are more sensitive to this increase in the premium.

### 5.3 Contract bundling

Let us now study whether advantageous selection may occur when both risks are insured through a single contract. We proceed step-by-step.

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<sup>14</sup>The NCP holds for sure in the final situation, as riskier types are not covered anymore. The correlation property thus agrees with our definition in this very specific case.

**Participation and expected cost under bundling** A bundling contract  $(P, I_1, I_2)$  provides insurance against both risks. We therefore assume  $I_1, I_2 > P > 0$ . An agent of type  $(\alpha_1, \alpha_2)$  buys the contract if and only if  $V(P, I_1, I_2; \alpha) \geq V(0, 0, 0; \alpha)$ , or equivalently

$$PC(\alpha) : \quad \alpha_1 A_1 + \alpha_2 A_2 \geq H,$$

where

$$H = u(w) - u(w - P) > 0$$

can be interpreted as the utility cost of the premium in the absence of loss, while

$$A_i = u(w - L_i + I_i - P) - u(w - L_i) + H > 0 \quad i = 1, 2 \quad (2)$$

adds to  $H$  the utility gain from being insured when state  $i$  realizes. Note that  $A_1$ ,  $A_2$  and  $H$  are independent of the agent's type. Figure 2 represents this set of types, whose frontier is the solid line  $\alpha_1 A_1 + \alpha_2 A_2 = H$ . Agents whose type lies above the frontier buy the insurance contract  $(P, I_1, I_2)$ , while types below the frontier do not. Because agents dislike increases in the premium, such an increase must shift the frontier upwards, meaning that both  $H/A_1$  and  $H/A_2$  must increase with  $P$ ; this is readily verified using the above formulas.

Moving to the insurer's side, an iso-cost curve is  $\alpha_1 I_1 + \alpha_2 I_2 = c$  for some constant cost  $c$ . One of these iso-cost curves is represented by the dotted line in Figure 2. Note that there is no a priori reason why iso-cost and participation lines should have the same slope, a point on which we shall come back soon. Finally, the expected cost of the contract is

$$AC(P) = E[\alpha_1 I_1 + \alpha_2 I_2 | PC(\alpha) \text{ holds}]. \quad (3)$$

**Necessary conditions for the existence of advantageous selection** Advantageous selection necessitates that an increase in the premium lead some high-cost types to unsubscribe. These marginal subscribers' types must thus belong to the participation frontier, with individual costs  $c(I_1, I_2, \alpha)$  above the average cost  $AC(P)$ . At this point, essentially two cases may occur.

It may be that all types on the participation frontier have the same cost. In that situation, the slope of the participation frontier equals the slope of an iso-cost curve, i.e.,  $I_1/I_2 = A_1/A_2$ .

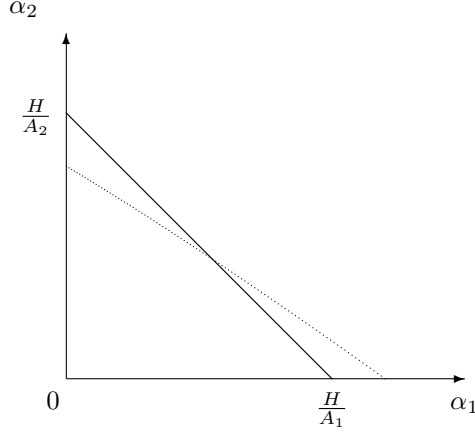


Figure 2: Advantageous selection under contract bundling

Note: Types above the solid line buy the bundling contract; the dotted line is an iso-cost curve.

In this case, all types above the frontier display a higher cost than those at the frontier, and the iso-cost corresponding to the average cost  $AC(P)$  must also lie above the participation frontier. Then advantageous selection cannot occur: an increase in  $P$  moves the participation frontier upward and all consumers who stop buying the contract are the lowest cost ones.

Alternatively and generically, types on the participation frontier may exhibit different costs. To fix ideas, suppose that the participation frontier is steeper than iso-cost curves, i.e.,  $I_1/I_2 < A_1/A_2$ , which is the case illustrated on Figure 2 (the opposite inequality would define a symmetrical case and would be dealt with similarly). Then, individual costs decrease when one goes down the frontier, from the northwest to the southeast. A necessary condition to obtain advantageous selection is that the highest cost on the frontier (which equals  $\alpha_2 I_2$  at  $(\alpha_1, \alpha_2) = (0, H/A_2)$ ) is larger than  $AC(P)$ . Since  $AC(P)$  is an average cost, the corresponding iso-cost line must lie above the one representing the lowest cost in the population of subscribers, which is also the lowest cost on the frontier (which equals  $\alpha_1 I_1$  at  $(\alpha_1, \alpha_2) = (H/A_1, 0)$ ). Therefore, a necessary condition for advantageous selection to occur is

$$\frac{I_1}{A_1} < \frac{AC(P)}{H} < \frac{I_2}{A_2}. \quad (4)$$

Moreover, if the premium increases, some agents decide not to buy the contract anymore, and

the participation constraint moves upward. For advantageous selection to arise, an additional necessary condition is that the measure of agents exiting the market to the left of the intersection of the iso-cost and the frontier (that is, with costs above the average) be sufficiently large compared to the measure of agents exiting the market to the right of the intersection (with costs below the average).

**Existence of advantageous selection under full insurance** We now provide a simple example to show that advantageous selection may indeed occur. To do so, we assume full insurance for both risks ( $I_1 = L_1$  and  $I_2 = L_2$ ), so that we now have

$$A_i = u(w) - u(w - L_i), \quad i = 1, 2 \quad H = u(w) - u(w - P).$$

In particular, the slope of the participation frontier  $A_1/A_2$  becomes independent of  $P$ . This is also the case for the slope of the iso-cost curve which now equals  $L_1/L_2$ . It is interesting to note that, due to risk-aversion, the ratio

$$\frac{A_i}{L_i} = \frac{u(w) - u(w - L_i)}{L_i} \tag{5}$$

is increasing in  $L_i$ . Therefore, when  $L_1 = L_2$  the two slopes are equal, and advantageous selection cannot occur, as already observed. In fact, in such a situation, we are back to a Rothschild-Stiglitz model with one contract covering against a single loss that realizes with probability  $\alpha_1 + \alpha_2$ .

Assume now  $L_1 > L_2$ . Since  $A_i/L_i$  increases with  $L_i$ , the slope of the participation frontier is larger (in absolute value) than the slope of the iso-costs, as in Figure 2 and therefore the highest-cost types on the participation frontier are those with a type close to  $(0, H/A_2)$ . We obtain the paradoxical result that, although the indemnity is larger for the first loss than for the second one (since  $I_1 = L_1 > I_2 = L_2$ ), the minimum cost for the insurer on the participation frontier is attained for a type with a high probability of the largest loss (i.e.  $\alpha_1 = H/A_1$ ). The intuition for this result comes from the behavior of agents. Since  $L_1 > L_2$ , insurance against state 1 is more valuable to agents than insurance against state 2. This translates into a smaller participation threshold (i.e., minimum risk  $\alpha_i$  to buy the insurance contract) for contract 1 (i.e.

$H/A_1 < H/A_2$ ). Due to risk-aversion, this effect on the loss probability for the insurer more than compensates for the higher loss  $L_1$ .

We now propose a distribution of types under which adverse selection holds for every unbundled contract of the form  $(P, I_1, 0)$  or  $(P, 0, I_2)$ , while advantageous selection occurs for at least one bundled contract. Consider the case where a proportion  $1/2$  of agents face only risk 1, with a loss probability  $\alpha_1$  uniformly distributed on  $[0, 1]$ ; and the other half face only risk 2, with  $\alpha_2$  also uniformly distributed. Then only agents in the first group may be attracted by a contract  $(P, I_1, 0)$ , and this simple structure à la Rothschild-Stiglitz immediately implies that adverse selection must prevail for such a contract; and similarly for contracts of the form  $(P, 0, I_2)$ .

There remains to show that advantageous selection may happen for some full-insurance contract  $(P, L_1, L_2)$ . We need participation from both groups, so let us set  $P$  so that  $P < L_2 < L_1$ , or equivalently  $H < A_2 < A_1$ . Then the average cost is

$$AC(P) = \frac{\int_{\frac{H}{A_1}}^1 \alpha_1 L_1 d\alpha_1 + \int_{\frac{H}{A_2}}^1 \alpha_2 L_2 d\alpha_2}{\int_{\frac{H}{A_1}}^1 d\alpha_1 + \int_{\frac{H}{A_2}}^1 d\alpha_2}.$$

**Result 4** *One can find a Constant Relative Risk Aversion utility function and parameters  $(w, L_1, L_2)$  such that  $AC(P)$  is decreasing on a non-empty interval of values of  $P$ , for the full insurance contract  $(P, I_1 = L_1, I_2 = L_2)$ .*

A constructive proof with a relative risk aversion equal to 2 is provided in the Appendix.

## 6 The life care annuity market

We now switch to the study of a practical case where bundling different risks into a single contract may create advantageous selection. These two risks are the longevity risk, and the LTC risk. In what follows, we distinguish three exclusive states of nature: state 0 corresponds to a short life; state 1 is associated to a long life with bad health, during which costly LTC is needed; finally, state 2 is associated to a long and healthy life, with intermediate needs for higher revenues.

Data come from a survey conducted in 2016 on 2,000 respondents aged 50 to 70, from Ontario and Québec, representative of the older population in these two provinces.<sup>15</sup> The survey includes many socio-demographic and health information about each participant. A health microsimulation model (COMPAS) provides estimates of personalized lifetime exposure to death and disability (Boisclair et al., 2016) from socio-economic and health individual characteristics. This microsimulation model estimated the individuals' probability to be alive at 85 years old as well as the probability of ever entering a nursing home. This enabled us to recover the type  $(\alpha_1, \alpha_2)$  of each individual since the probability to be alive at 85 corresponds to  $\alpha_1 + \alpha_2$  while the probability of entering a nursing equals  $\alpha_1$ . The probability of dying before age 85 is then given by  $1 - \alpha_1 - \alpha_2$ . Figure 3 represents this cloud of 2,000 points in the type space. We also present descriptive statistics in Table 1.

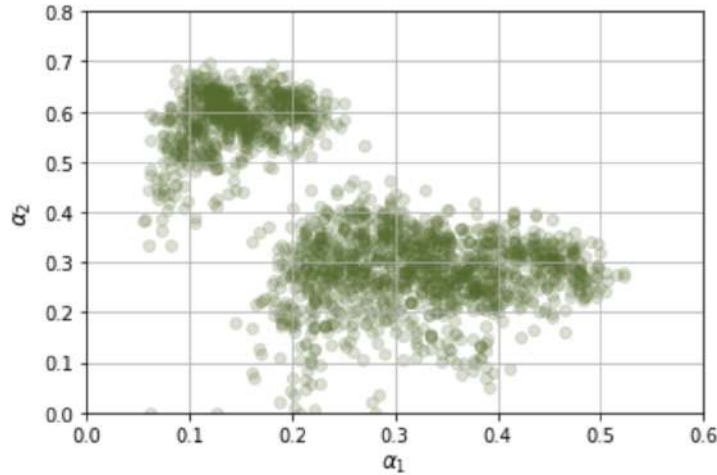


Figure 3: Probability distribution of  $(\alpha_1, \alpha_2)$ .

Interestingly, the figure allows to distinguish two groups. The lucky group in the Northwest displays a high probability  $\alpha_2$  of good health; while the other (unlucky) group displays a high risk of needing LTC (a high  $\alpha_1$ ). Note that the correlation coefficient between  $\alpha_1$  and  $\alpha_2$  is

<sup>15</sup>Data are available at <https://dataverse.scholarsportal.info/dataset.xhtml?persistentId=doi:10.5683/SP2/PP5U7Y>. To correct for the under- or overrepresentation of some socio-demographic groups in the sample, we reweighed the data using the Labor Force Survey of 2014. More details about this survey are given in Boyer et al. (2019, 2020).

	$\alpha_1$	$\alpha_2$	$1 - \alpha_1 - \alpha_2$
mean	0.262	0.386	0.352
std	0.112	0.16	0.119
min	0.055	0.000	0.125
max	0.523	0.698	0.938

Table 1: Distribution of types for the 2,000 individuals.

equal to  $-0.67$ . This suggests that those agents who need LTC do not have a high demand for annuities which are served only in good health, and vice-versa. For each risk insured separately, we thus expect adverse selection to prevail, as it is a one-dimensional risk without moral hazard, and the correlation of riskinesses is negative (see our comments at the end of subsection [5.2](#)).

Recall that the three possible states of the world correspond to early death (state 0), dependency (state 1), and good health (state 2). Then the loss  $L_2$  in state 2 can be construed as the income needed when alive (and, hence, justifying the buying of annuities), while  $L_1 - L_2 > 0$  can be seen as the cost of LTC services. A life care annuity contract  $(P, I_1, I_2)$  consists in providing an annuity  $I_2$  together with LTC insurance coverage ( $I_1 - I_2 > 0$ ). For an agent with income  $w$  and type  $\alpha = (\alpha_1, \alpha_2)$ , the expected utility from buying this contract is

$$V(P, I_1, I_2, \alpha) = \alpha_1 u(w - L_1 + I_1 - P) + \alpha_2 u(w - L_2 + I_2 - P) + (1 - \alpha_1 - \alpha_2) v(w - P),$$

where we assume the same utility function  $u(x)$  when alive (whenever dependent or not), but a different utility function  $v(x)$  when dead.<sup>[16](#)</sup>

Using this model, we now show that it is possible to design life care annuity contracts with advantageous selection by bundling together the risk of a long life under good health, and the risk of becoming dependent. In this numerical exercise, we assume the following functional forms and parameter values:

$$u(x) = \frac{x^{1-\varepsilon}}{1-\varepsilon}, \quad v(x) = \beta u(x)$$

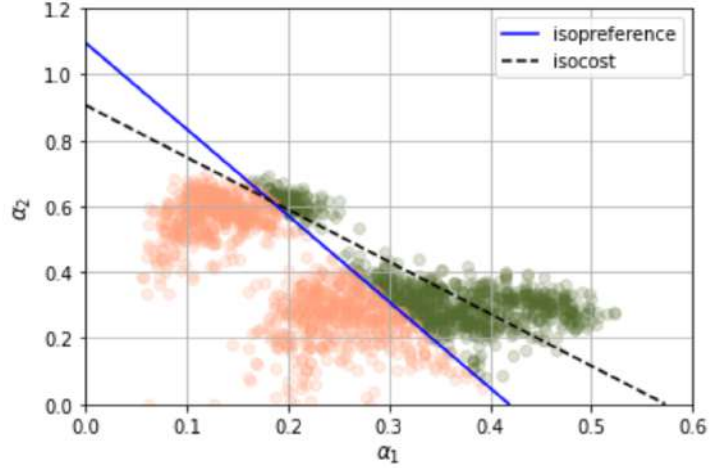
where we set  $\beta = 0.8$  and  $\varepsilon = 0.8$ .<sup>[17](#)</sup> The utility of leaving a bequest is thus smaller than

<sup>16</sup>This function  $v(x)$  can be seen as a "joy of giving" utility—i.e., the individual derives utility from leaving some bequests in case of death. The modeling is similar, for instance, to Glomm and Ravikumar (1992), Kopczuk and Lupton (2007), Piketty and Saez (2013) and Fleurbaey et al. (2022).

<sup>17</sup>Assuming a coefficient of relative risk aversion (i.e.  $\varepsilon$ ) smaller than one is supported by empirical evidence

the utility of consuming the same amount when alive. In our sample, the average income is 107,000 CAD with median 72,000 CAD, so we have set  $w = 100,000$  CAD,  $L_1 = 87,000$  CAD,  $L_2 = 55,000$  CAD.

Figure 4 is the simulated equivalent of Figure 2 in Section 5.3 for a premium  $P = 57,850$  CAD. In the figure, the blue line represents the participation frontier and the dotted line represents the iso-cost line corresponding to the average cost  $AC(P)$ . In this example, the necessary condition for advantageous selection (4) is satisfied since the participation frontier has a steeper slope than the iso-average-cost line, and since both intersect. Orange dots account for agents choosing not to buy the contract, while green dots represent individuals buying the contract.



Note: the intersection arises at (0.183, 0.618).

Figure 4: Advantageous selection

For this contract, we find that a local increase in  $P$  lowers the average cost of the contract, generating advantageous selection. When  $P$  increases, some individuals located on or immediately above the participation frontier stop buying the contract. Note that the isocost curves are flatter than the participation constraint in the  $(\alpha_1, \alpha_2)$  space. As  $P$  increases, two groups of agents stop buying the contract. The first group (located to the left of the intersection between

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(see for instance, Karagyzova and Siegelman, 2012; Holt and Laury, 2002; Chetty, 2006). Assuming different values for  $\beta$  and  $\varepsilon$  does not qualitatively change our results.

the two lines) has a large probability of remaining healthy but a probability to need formal LTC low enough to induce them to stop buying the bundled contract. The second group (located to the right of the intersection) exhibits a large LTC risk, but a probability of a long healthy life small enough that they quit buying the more expensive contract. The former group (made of agents with a larger-than-average cost among contract buyers) is large enough compared to the latter (with lower-than-average cost) that the average cost of the contract decreases when its price is increased. Advantageous selection may then occur when bundling the longevity and LTC risks within life care annuities. Note that this argument necessitates a negative correlation between the longevity risk and the LTC risk, so that, as we just described the agents who are more (resp. less) costly on the annuity side of the contract are also less (resp. more) costly on the LTC side of the contract.

## 7 Conclusion

In this paper, we propose several novel rationales for advantageous selection. If the demand for a specific insurance contract is continuous in its price, we show that advantageous selection requires that the equilibrium markup and the price elasticity of demand be large enough. We then study a large class of environments where the single-crossing property implies the positive correlation property, and show how adverse and advantageous selections are mirror images of each other. More precisely, when several contracts are offered, the increase in the price of a single contract induces both low-cost and high-cost agents to stop buying this contract, so that the type of selection at equilibrium is essentially an empirical question. Finally, we study an environment with two mutually exclusive risks, and show that advantageous selection may arise either when one risk is a background, uninsurable one, or when the two risks are bundled into the same contract. The necessary condition for advantageous selection to arise has a simple geometrical interpretation, stating that the iso-preference curve (delineating the set of buyers from non buyers) has to intersect the iso-average-cost line in the probabilities space. We then provide two numerical examples showing that advantageous selection can indeed occur, the first

based on the simplest setting (uniform probability distributions of perfectly negatively correlated risks) and the second obtained from a Canadian survey covering the survival and long-term care risks.

These results show that advantageous selection may appear in various settings without moral hazard, when agents only differ in their riskiness, and not in their attitude toward risk. This is due to a composition effect: the set of clients to a contract changes as the premium changes, in ways that are specific to each situation. This effect is also impacted by the presence of market power, and by the existence of contracts offering higher coverage. Therefore, the nature of selection is likely to remain an empirical question, in particular when it comes to bundling different risks. Our study of this complicated geometry opens up the possibility of building new contracts for which issues associated with adverse selection are alleviated or even annihilated, and that would fare better on markets plagued by asymmetric information.

## References

- [1] Ameriks J., Caplin C., Laufer S., and S. Van Nieuwerburgh, 2011, The Joy of Giving or Assisted Living? Using Strategic Surveys to Separate Public Care Aversion from Bequest Motives, *Journal of Finance*, 66(2), 519-561.
- [2] Boisclair, D., Cote-Sergent, A., Laliberte-Auger, F., Marchand, S., and P.-C. Michaud, 2016, A health microsimulation model for Quebec and Canada, Technical report, HEC Montreal - CEDIA.
- [3] Boyer, M. M., De Donder, P., Leroux, M.-L., Fluet, C., and P.-C. Michaud, 2019, A Canadian parlor room-type approach to the long-term care insurance puzzle. *Canadian Public Policy*, 42(2), 262-282.
- [4] Boyer, M.M., P. De Donder, C. Fluet, M-L. Leroux, P-C. Michaud, 2020, Long-term Care Insurance : Information Frictions and Selection, *American Economic Journal: Economic Policy*, 12 (3), 134-69.

- [5] Brown, J. R. and Finkelstein, A., 2007, Why is the market for long-term care insurance so small? *Journal of Public Economics*, 91(10), 1967–1991.
- [6] Brown, J. R. and Finkelstein, A., 2009, The private market for long-term care insurance in the united states: A review of the evidence, *Journal of Risk and Insurance*, 76(1), 5-29.
- [7] Brown J. and M. Warshawsky, 2013, The Life Care Annuity: A New Empirical Examination of an Insurance Innovation That Addresses Problems in the Markets for Life Annuities and Long-Term Care Insurance, *The Journal of Risk and Insurance*, 80(3), 677-703.
- [8] Chetty, R., 2006, A new method of estimating risk aversion, *American Economic Review*, 96(5), 1821-1834.
- [9] Chiappori P-A., B. Jullien, B. Salanié, and F. Salanié, 2006, Asymmetric information in insurance: some testable implications, *Rand Journal of Economics*, 37(4), 783-798.
- [10] Chiappori P-A. and B. Salanié, 2000, Testing for Asymmetric Information in Insurance Markets, *The Journal of Political Economy*, 108(1), 56-78.
- [11] Cohen, A. and L. Einav, 2007, Estimating Risk Preferences from Deductible Choice, *American Economic Review*, 97 (3), 745-788.
- [12] Davidoff T. and G. Welke, 2007, Selection and Moral Hazard in the Reverse Mortgage Market, <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.320.143&rep=rep1&type=pdf>
- [13] de Meza D. and D. C. Webb, 2001, Selection in Insurance Markets, *The RAND Journal of Economics*, 32(2), pp. 249-262.
- [14] De Donder P. and J. Hindriks, 2009, Adverse selection, moral hazard and propitious selection, *Journal of Risk and Uncertainty*, 38, 73–86.
- [15] Einav, L. and A. Finkelstein, 2011, Selection in Insurance Markets: Theory and Empirics in Pictures, *Journal of Economic Perspectives*, 25(1), 115–138.

- [16] Einav, L., A. Finkelstein, and M.R. Cullen, 2010a, Estimating Welfare in Insurance Markets Using Variation in Prices, *Quarterly Journal of Economics*, 125(3), 877-921.
- [17] Einav, L., A. Finkelstein, and J. Levin, 2010b, Beyond Testing: Empirical Models of Insurance Markets, *Annual Review of Economics*, 2, 311–36.
- [18] Fang H., M. Keane and D. Silverman, 2008, Sources of Advantageous Selection: Evidence from the Medigap Insurance Market, *Journal of Political Economy*, 116(2), 303-350.
- [19] Finkelstein, A. and K. McGarry, 2006, Multiple Dimensions of Private Information: Evidence from the Long-Term Care Insurance Market, *American Economic Review*, 96, 938–958.
- [20] Finkelstein, A. and J. Poterba, 2002, Selection Effects in the United Kingdom Individual Annuities Market, *Economic Journal*, 112, 28-50.
- [21] Finkelstein, A. and J. Poterba, 2004, Adverse Selection in Insurance Markets: Policyholder Evidence from the U.K. Annuity Market, *Journal of Political Economy*, 112(1), 183-208.
- [22] Fleurbaey, M., M-L. Leroux, P. Pestieau, G. Ponthiere and S. Zuber, 2019, Premature Deaths, Involuntary Bequests and Fairness, *Scandinavian Journal of Economics*, <https://doi.org/10.1111/sjoe.12478>
- [23] Glomm, G. and B. Ravikumar, 1992, Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality, *Journal of Political Economy*, 100(4), 818-834.
- [24] Hendren N., 2013, Private information and insurance rejections, *Econometrica*, 81(5), 1713-1762.
- [25] Hemenway, H., 1990, Propitious Selection, *The Quarterly Journal of Economics*, 105(4), 1063-1069.
- [26] Hemenway, H., 1992, Propitious Selection in Insurance, *Journal of Risk and Uncertainty*, 5(3), 247-251.

- [27] Holt, CA and SK Laury, 2002, Risk aversion and incentive effects, *American Economic Review*, 92(5), 1644-1655.
- [28] Karagyozyova, T. and P. Siegelman, 2012, Can propitious selection stabilize insurance markets?, *Journal of Insurance Issues*, 35 (2), 121-158.
- [29] Kopczuk, W. and J.P. Lupton, 2007, To Leave or Not to Leave: The Distribution of Bequest Motives, *Review of Economic Studies*, 74(1), 207-235.
- [30] Lockwood, L., 2018, Incidental bequests: Bequest motives and the choice to self-insure late-life risks, *American Economic Review*, 108(9), 2513–2550.
- [31] Murtaugh, C., B. Spillman, and M. Warshawsky, 2001, In Sickness and in Health: An Annuity Approach to Financing Long-Term Care and Retirement Income, *Journal of Risk and Insurance*, 68(2), 225-254.
- [32] OECD (2011). Help Wanted? Providing and Paying for Long-Term Care, available at [http://www.oecd.org/document/23/0,3746,en\\_2649\\_37407\\_47659479\\_1\\_1\\_1\\_37407,00.html](http://www.oecd.org/document/23/0,3746,en_2649_37407_47659479_1_1_1_37407,00.html)
- [33] Pauly, M., 1990, The rational non-purchase of long-term care insurance, *Journal of Political Economy*, 95, 153–168.
- [34] Pestieau, P. and G. Ponthière, 2012, Long-term care insurance puzzle. In *Financing long-term care in Europe*, pages 41–52.
- [35] Piketty, T. and E. Saez, 2013, A Theory of Optimal Inheritance Taxation, *Econometrica*, 81 (5), 1851-1886.
- [36] Rothschild, M., and J. Stiglitz. 1976, Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information, *The Quarterly Journal of Economics* 90 (4), 629-649.

- [37] Rothschild, C., 2009, Adverse selection in annuity markets: Evidence from the British Life Annuity Act of 1808, *Journal of Public Economics* 93, 776-784.
- [38] Sloan, F. A. and Norton, E. C., 1997, Adverse Selection, Bequests, Crowding Out, and Private Demand for Insurance: Evidence from the Long-term Care Insurance Market, *Journal of Risk and Uncertainty*, 15(3), 201–219.
- [39] Wang K., Huang R. and Tzeng L., 2009, Empirical Evidence for Advantageous Selection in the Commercial Fire Insurance Market, *The Geneva Risk and Insurance Review*, 34, 1-19.

## Appendix

### A study of the single-crossing property in Section 4

Let  $\alpha < \alpha'$ , and consider two contracts  $(P_1, I_1(\cdot))$  and  $(P_2, I_2(\cdot))$ . Define  $w_i(L) = w - P_i - L + I_i(L)$ . If the second contract covers more, then there exists  $L_0$  such that  $u(w_2(L)) - u(w_1(L))$  is at most zero when  $L < L_0$ , and is at least zero when  $L > L_0$ . Then the difference in payoffs for type  $\alpha'$

$$\int_L [u(w_2(L)) - u(w_1(L))] dG(L|\alpha')$$

can be split in two integrals

$$\int_{L < L_0} [u(w_2(L)) - u(w_1(L))] \frac{g(L|\alpha')}{g(L|\alpha)} dG(L|\alpha) + \int_{L > L_0} [u(w_2(L)) - u(w_1(L))] \frac{g(L|\alpha')}{g(L|\alpha)} dG(L|\alpha).$$

Recall that MLRP states that the ratio of densities is increasing with  $L$ . Therefore, because  $u(w_2) - u(w_1)$  changes sign at  $L_0$ , this expression is at least  $g(L_0|\alpha')/g(L_0|\alpha)$ , times

$$\int_{L < L_0} [u(w_2(L)) - u(w_1(L))] dG(L|\alpha) + \int_{L > L_0} [u(w_2(L)) - u(w_1(L))] dG(L|\alpha),$$

which is the difference in payoffs for type  $\alpha$ . This proves that if  $\alpha$  prefers the second contract to the first, then so does  $\alpha'$ , as announced.

**Proof of Result 4:** We have

$$AC(P) = \frac{1}{2} \frac{L_1(1 - (\frac{H}{A_1})^2) + L_2(1 - (\frac{H}{A_2})^2)}{1 - \frac{H}{A_1} + 1 - \frac{H}{A_2}} = \frac{1}{4} (L_1 + L_2) \frac{1 - bH^2}{1 - aH},$$

where

$$a = \frac{1}{2} \left( \frac{1}{A_1} + \frac{1}{A_2} \right) \in \left[ \frac{1}{A_1}, \frac{1}{A_2} \right] \quad b = \frac{1}{L_1 + L_2} \left( \frac{L_1}{A_1^2} + \frac{L_2}{A_2^2} \right).$$

Since  $H = u(w) - u(w - P)$  is an increasing function of  $P$ , and the  $A_i = u(w) - u(w - L_i)$ ,  $i = 1, 2$ , do not depend on it, we directly differentiate with respect to  $H$  to get that  $AC'(P)$  has the same sign as

$$-2bH(1 - aH) + (1 - bH^2)a = -2bH + abH^2 + a. \quad (6)$$

This expression is decreasing in  $H$  for  $H < 1/a$ , which holds since  $H < A_2 < A_1$ . Therefore, if the derivative of the average cost is negative at  $H = A_2$ , then it is negative on a nonempty interval  $[H_0, A_2]$ , where  $H_0$  is the smallest solution to equation (6). There remains to provide a numerical example. Take for example a CRRA function with relative risk-aversion equal to 2, and the following parameters:

$$u(x) = -1/x, w = 10, L_1 = 9, L_2 = 7.$$

Then we obtain

$$A_1 = 0.9, A_2 = 0.233, a = 2.698, b = 8.73$$

and the derivative  $(-2bH + abH^2 + a)$  is negative for  $H \in [0.22, 0.2333]$ , or equivalently for  $P \in [6.871, 7]$ .