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# Mergers of Complements: On the Absence of Consumer Benefits\*

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## Abstract

*According to Cournot's (1838) standard argument, mergers of complements benefit consumers – because merged entities internalise negative externalities and decrease prices. I model a vertically related industry where upstream suppliers may or may not be constrained by competition. A merger of sufficiently constrained suppliers does not decrease prices. I show that observing pre-merger margins can reveal whether a merger decreases prices. This enables me to develop a simple, practicable merger test. And to identify when the standard prediction of merger benefits is inconsistent with observable facts. Finally, instead of yielding benefits, profitable mergers of complements can cause unambiguous consumer harm.*

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# 1 Introduction

Absent anticompetitive effects, mergers of complements are typically thought to benefit consumers (Etro, 2019) – through the Cournot effect. According to this effect, mergers of complements eliminate negative externalities and thus decrease consumer prices.<sup>1</sup> Antitrust authorities acknowledge this merger benefit. In particular, the US DOJ & FTC in their 2020 Antitrust guidelines, as well as the EC in its latest merger guidelines.<sup>2</sup>

Meanwhile, mounting evidence of insufficient Antitrust enforcement has fuelled widespread concerns that merger policy is too lax.<sup>3,4</sup> This calls for a re-examination of what are deemed to be merger benefits. In this paper I show how to identify whether a merger of complements does – or does not – yield the beneficial Cournot effect. This can help Antitrust authorities identify mergers which do not yield this benefit, and help detect mergers which are, instead, harmful to consumers.

Cournot (1838) first demonstrated the effect named after him. His argument can be synthesised as follows. Suppose that two monopolists each supply one input to downstream consumers, that each monopolist sets the unit price for the input it supplies, and that the two inputs are perfect complements. The two suppliers each set their unit price without considering that a higher price diminishes the profit of the other firm through a lower (common) demand. If the two monopolists merge, this negative externality is internalised. Post-merger prices fall<sup>5</sup> and demand increases. This benefits all – the merging parties and downstream consumers.

In contrast, when there is perfect competition in each input market, suppliers earn zero profit pre-merger.<sup>6</sup> Post-merger the merged entity does not decrease price (doing so would be unprofitable). Hence the Cournot effect does not materialise.<sup>7</sup>

For all competitive landscapes in input markets between the two polar extremes, of monopoly and perfect competition, there is no general result on whether the Cournot

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<sup>1</sup>Some think of this as a horizontal internalisation of double mark-ups. The problem of negative externalities is also referred to as “tragedy of the anticommons” (Buchanan and Yoon, 2000; Heller, 1998).

<sup>2</sup>Quotes from the guidelines are provided in section 4.

<sup>3</sup>See e.g. Shapiro (2018, 2019). Such evidence includes ex post merger studies, and findings that market power and margins have been rising. For reviews of ex post studies, see Kwoka (2014), Ashenfelter, Hosken, and Weinberg (2014). Specific studies include Ashenfelter and Hosken (2010), Ashenfelter, Hosken, and Weinberg (2013), Gaynor and Town (2012), Gaynor (2018). On increasing margins, see De Loecker, Eeckhout, and Unger (2020), Barkai (2020). Hall (2018) finds more modest increases.

<sup>4</sup>The rise of minority shareholdings, in the form of common and cross ownership, is linked to rising margins. See Azar, Raina, and Schmalz (2019); Schmalz (2018); Azar, Schmalz, and Tecu (2018). On Antitrust scrutiny – or the lack thereof – of such effective concentration see e.g. Moiseyev (2016) for the US, and Miller, Raven, and Went (2012) for the EU. Some argue more desirable causes explain rising margins: efficiency gains of “superstar firms” (Autor et al., 2020). In the public realm, overly lenient courts are identified as a cause (Finley, 2019; Teachout, 2021). See also Melamed (2020).

<sup>5</sup>To internalise the Cournot externality after a merger can also raise investment levels, see Etro (2019).

<sup>6</sup>For constant returns to scale.

<sup>7</sup>On this see Kühn, Stillman, and Caffarra (2005) and Dari-Mattiacci and Parisi (2006) among others.

effect materialises. This gap is problematic for Antitrust authorities and applied theorists alike. During a merger review an Antitrust authority needs to weigh positive merger effects with negative effects from any theories of harm it may have. However it may lack the data to ascertain the net impact.<sup>8</sup> More fundamentally, it does not have a test or tool to determine when exactly it can outright dismiss positive Cournot effects. The problem for an applied theorist is knowing how to recognise whether a model correctly predicts that a merger benefits consumers via Cournot effects, or whether that prediction is inconsistent with observable facts.

I re-examine the Cournot effect in a vertically-related industry. In my model, a downstream firm needs different inputs to manufacture a consumer good. In stage 1, each upstream producer sets the unit price (the bid) at which it offers to supply an input to the downstream firm.<sup>9</sup> In stage 2, based on input prices the downstream firm selects its suppliers and sets the price for its consumer product. This model differs from Cournot's original model (Cournot, 1838, p.112-7) in two crucial ways. First, while Cournot exclusively modelled suppliers, I consider both upstream and downstream firms as profit-maximising entities. This enables me to obtain results on upstream versus downstream margins. Second, I allow for any number of producers of each input. This captures the polar cases of monopoly and perfect competition, but also intermediate cases of competition. With this model I develop solutions to the problems laid out in the previous two paragraphs.

Theorem 1 closes the gap in the literature: it can be used to determine whether a proposed merger among suppliers of complements would benefit consumers – or whether it would not, for all competitive landscapes, from the polar extreme of monopoly to the polar extreme of perfect competition. One can also use Theorem 1 to identify whether particular models of mergers of complements, and their predictions, are consistent with observable facts of an industry under analysis.

The test, a corollary of Theorem 1, constitutes a practicable new merger tool.<sup>10</sup> Antitrust authorities can use it to determine whether proposed mergers would benefit consumers. If a particular merger would not yield that positive effect, an Antitrust authority can dismiss claims that the merger would benefit consumers via the Cournot effect. Absent other claimed benefits, and if the Antitrust authority has a theory of harm, the authority would then have unambiguous grounds to act against the proposed merger.

The intuition for Theorem 1 and the test goes as follows. When a downstream firm passes through to consumers less than 100% of an input price increase (i.e. when it “absorbs” some of the increase<sup>11</sup>) the elasticity of demand upstream is lower than it is

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<sup>8</sup>The magnitude of the Cournot effect may depend on parameters which, in practice, may be unknown or hard to estimate reliably – e.g. the elasticity of demand (see Etro (2019) and Karlinger et al. (2020)).

<sup>9</sup>When suppliers set two-part tariffs, with prices at marginal costs there are no negative externalities pre-merger such that, trivially, there is no Cournot effect. On this, see Karlinger et al. (2020).

<sup>10</sup>The test is useful precisely because of its simplicity – and generality. It is “transparent”, “simple and speedy”: it meets criteria for a useful merger test listed by Farrell and Shapiro (2010).

<sup>11</sup>See Amir, Maret, and Troege (2004), Adachi and Ebina (2014).

downstream. Consequently, by the intuition of the Lerner index,<sup>12</sup> the equilibrium unit dollar margin of any upstream supplier must exceed that of the downstream firm.<sup>13</sup> If instead the unit dollar margin of a supplier is smaller than that of the downstream firm, then the supplier must be constrained – by some outside option the downstream firm has. Such an alternative can be e.g. to source an input from an alternative supplier, to produce the input in-house,<sup>14</sup> or not to purchase and use the input at all. A constrained supplier maximises profit by maximising price: it sets its unit price to the level beyond which the downstream firm would resort to an outside option. If instead the supplier were unconstrained it would not maximise price, it would trade-off an increase in the unit price it charges with a decrease in consumer demand.

The test comes down to a comparison of pre-merger margins.<sup>15</sup> It states: if the merging suppliers earn a combined unit dollar margin which is smaller than the unit dollar margin of the downstream firm, then post-merger prices would not decrease. Hence consumers would reap no benefit from the merger. Effectively the test verifies and reveals whether the merging suppliers are collectively sufficiently constrained pre-merger to rule out a price decrease post-merger. Margins can contain sufficient information on outside options (constraints) to predict merger effects.

As a practical illustration: data shows that Apple earns more dollars of profit per iPhone sold than all of its suppliers combined. (The price of an iPhone, e.g. the iPhone 11, is around \$1100, while its total unit cost to Apple is estimated at \$490 – less than half.)<sup>16</sup> Hence, to apply the test yields the following prediction: if any two of Apple’s suppliers were to merge – or even if all of its suppliers were to merge – the merged entity would not find it profitable to lower input prices.

Theorem 1 and the test can identify the absence of consumer benefits, but not the presence of harm. In section 3 I augment the model to allow for profitable strategies harmful to consumers. I present two separate theories of harm. In the first, I add a (single) compatibility parameter to the model. In the second, I apply a central idea from Whinston (1990) to my model. In each theory of harm, the merged entity weakens the downstream firm’s outside options (i.e. relaxes price constraints), raises price, and thus causes unambiguous consumer harm.

The remainder of the paper is structured as follows. After a literature review, in section 2 I describe the baseline model and develop the main results. Section 3 relates to merger harm. In section 4 I discuss implications for Antitrust review, and show how to apply the results in practice. In section 5 I show results hold with downstream competition. Section 6 contains extensions. Sections 7 and 8 relate to the robustness of results. In section 9 I conclude. Appendix A contains proofs, Appendix B contains the underlying workings

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<sup>12</sup>See Lerner (1934) and Elzinga and Mills (2011).

<sup>13</sup>Adachi and Ebina (2014) obtain this result for the special case of single-product successive monopolies.

<sup>14</sup>On Apple gradually incorporating the production of its inputs in-house, see Bradshaw (2018).

<sup>15</sup>Data Antitrust authorities can request from the relevant parties.

<sup>16</sup>See Yang, Wegner, and Cowsky (2019) and Rosalsky (2021).

for sections 5, 7, and 8.

## Literature review

Since Cournot (1838) many others have modelled mergers of suppliers of complements as resulting in the Cournot effect, e.g. Choi (2008), Quint (2014), and Etro (2019). The effect appears in the latter two papers because upstream firms are assumed to be monopolist suppliers of essential inputs. More generally the effect appears in all three aforementioned papers because, when setting its price, a supplier is assumed to trade off its unit price with aggregate consumer demand. The three papers model upstream suppliers as strategic actors, while the downstream level is not explicitly modelled. The present paper takes a different approach and explicitly models both upstream and downstream firms as profit-maximising entities.

Some papers have already shown specific instances when the Cournot effect does not materialise. Masson, Dalkir, and Eisenstadt (2014) criticise what they perceive as a strong presumption of competition authorities that mergers of complements can benefit consumers and show Cournot effects “do not exist” for a certain specific consumer preferences. However Masson, Dalkir, and Eisenstadt (2014) analyse downstream mergers.<sup>17</sup> My results cannot be applied to mergers of firms which directly sell to customers:<sup>18</sup> such settings do not have a downstream margin to which the upstream margins can be compared to.<sup>19</sup> Antitrust reviews have highlighted specific contractual features as the reason why Cournot effects may not play out. Karlinger et al. (2020) mention the example of contracts that fix prices (such that there is no scope for post-merger price changes).

The key idea I develop is that margins – thanks to their endogenous nature – convey information on whether firms are constrained, and that margins can therefore be used to assess whether Cournot effects would materialise. This idea is new. While there exists a debate and a literature on whether margins should be used to inform Antitrust review, the debate is largely focused on horizontal mergers. For example, Inderst and Valletti (2011) consider pre-merger margins of the merging parties, and argue these are insufficient to predict post-merger incentives. Bresnahan and Reiss (1985) explicitly compare upstream to downstream margins, but not in a merger setting.

The toolbox of Antitrust authorities already includes tests to estimate harm from proposed horizontal mergers (Farrell and Shapiro, 2010; Moresi and Salop, 2009) and from

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<sup>17</sup>Similarly, Alvisi, Carbonara, and Parisi (2011) study the tragedy of the anticommons in a setting where producers of complements sell directly to consumers.

<sup>18</sup>However the intuition of the test still applies: the merged entity may not want to offer consumers a lower price for the two inputs than pre-merger.

<sup>19</sup>A broad literature explicitly analyses complementary goods sold directly to consumers, with a focus on compatibility. Matutes and Regibeau (1988, 1992) explain why competing firms may want to offer compatible products, Farrell, Monroe, and Saloner, 1998 and Denicolo (2000) consider why competing firms would benefit from a lack of interoperability.

proposed vertical mergers (Moresi and Salop, 2013). The present paper adds a test for mergers of complements. I compare this new test to existing tests in section 4.

The theory of harm laid out in section 3 is akin to “raising a rival’s cost” (RRC) via tying. See Whinston (1990) for an early and seminal exposition of harmful tying.<sup>20</sup> Carlton and Waldman (2002) as well as Choi (2004) consider how a monopolist in one market can leverage its monopoly power to monopolise a second market. In contrast, in the present paper the tying good is not provided by a monopolist, but instead by a firm which merely has a competitive edge over its rivals. Even in such a setting, post-merger bundling can harm consumers. In fact, in my setting it is precisely because the merging parties lack monopoly power that they can profit from anticompetitive bundling. Here, unambiguous consumer harm results from the interplay of two factors: the absence of Cournot effects and bundling.

## 2 No merger benefit

In this section I address the following question: yes or no, would a proposed merger among suppliers of complementary inputs benefit consumers, as a result of the Cournot effect (‘internalisation of double mark-ups’)?

### A. Baseline model

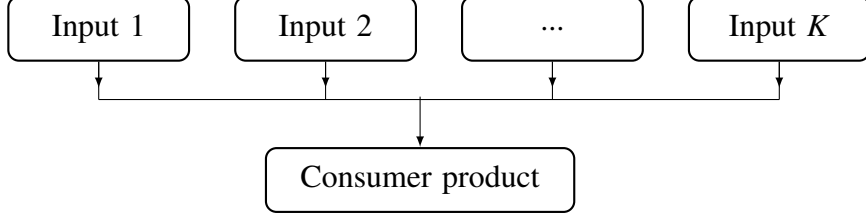
I model a vertically related industry. The downstream market is monopolised. The downstream monopolist needs  $K \geq 2$  different inputs to manufacture a consumer product. It requires one unit of each input per unit of output. (It has a Leontief production function.)<sup>21</sup> Let  $K$  also denote the set of inputs. Each input is produced by two firms upstream. The more efficient of the two has constant unit marginal cost of production  $c_i \geq 0$ ,  $i \in K$ , the other has constant unit marginal cost  $c_i^{[2]} > c_i$ .

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<sup>20</sup>He argued against the Chicago school one-monopoly-profit theorem (on this, see Posner (1976), Bork (1978)). For recent work on harm from mergers of complements, see Akgün et al. (2020).

<sup>21</sup>I treat the production function as exogenous. In principle the downstream firm may choose one of several production functions, but to endogenise this choice adds little to the paper.

Figure 1: Inputs to the downstream firm’s consumer product



There are two stages. In the first stage all  $2K$  upstream firms simultaneously submit an offer to the downstream firm. Each upstream firm submits a unit price at which it offers to supply any quantity of its input. The downstream firm then selects its suppliers, it single-sources each input from the firm which offered the lowest price,  $v_i$ , for input  $i \in K$ .<sup>22</sup> Consequently its unit cost is the sum of unit input prices  $\sum v_i$ . Effectively, the two producers of each input compete à la Bertrand. (In equilibrium, the supplier of an input is the most efficient producer of that input.)

In the second stage the downstream firm sets uniform consumer price  $P$ . The product market clears and determines the quantity  $Q(P)$  demanded by consumers. The downstream firm has profit  $\Pi_D = (P - \sum v_i)Q$ . Supplier  $i \in K$  has profit  $\Pi_i = (v_i - c_i)Q$ .

I assume demand has the following characteristics.

**Assumption A.** *Consumer demand  $Q(P)$  is (i) strictly downward-sloping, (ii) weakly log concave, and (iii) it induces non-decreasing pass-through:  $Q'(P) < 0$ ,  $(\ln Q(P))'' \leq 0$ ,  $\frac{\partial^2 P^*}{\partial v_i^2} \geq 0$ , where  $P^* = \operatorname{argmax} \Pi_D$ .*

Example demand functions which satisfy this assumption are provided in Table 1. As I show in robustness section 7, Theorem 1 does not rest on log concavity. In contrast, the test (Corollary 2) does rely on log concavity. On the pervasiveness of the log concavity assumption across Microeconomics, see Bagnoli and Bergstrom (2005).<sup>23</sup> Intuitively, non-decreasing pass-through ensures that a supplier’s profit does not keep rising in the price it sets.<sup>24</sup> Finally, while the baseline model features complete information – this is not necessary (as discussed in the robustness section 8).

<sup>22</sup>I assume that bidding is costless; that no firm bids below its cost; and that, at equal bids, the most efficient producer is selected.

<sup>23</sup>Log concavity allows for highly convex functions, but not for overly convex functions. Amir (2005) writes the limit case takes the form  $Q(P) = -\ln(P)$  “which is convex and log linear”.

<sup>24</sup>Non-decreasing pass-through is sufficient for any supplier’s optimal price to be unique, but it is not necessary for Theorem 1 and the test.



## B. Equilibrium

The game is solved by backwards induction. In the second stage the downstream firm maximises its profit  $\Pi_D = (P - \sum_{i=1}^K v_i)Q(P)$ . The first order condition determines the optimal price  $P^*$  and yields the downstream firm's unit dollar margin  $m_D^*$ .<sup>25</sup>

$$P^* - \sum_{i=1}^K v_i = -\frac{Q(P^*)}{Q'(P^*)} \quad (1)$$

Applying the implicit function theorem to the downstream firm's first order condition yields the pass-through rate  $\rho$

$$\rho := \frac{\partial P^*}{\partial v_i} = \frac{1}{2 - \frac{Q''(P^*)Q(P^*)}{Q'(P^*)^2}}$$

Weak log concavity of demand is equivalent to  $Q'(P)^2 \geq Q''(P)Q(P)$ , hence  $\rho \in (0, 1]$ .<sup>26</sup>

In the first stage, the most efficient producer of input  $i$  sets its profit-maximising offer price  $v_i^* \leq c_i^{[2]}$ .<sup>27</sup>

$$v_i^* = \min\{v_i^{*u}, c_i^{[2]}\} \quad (2)$$

where  $v_i^{*u}$  denotes the optimal unconstrained (or monopoly) price: the price the supplier would set if it were a monopolist producer of input  $i$ . Thus, the most efficient producer of input  $i$  is either constrained in the price it offers, or it is unconstrained.

**Definition.** Supplier  $i$  is *unconstrained* if  $v_i^* = v_i^{*u}$ .

**Definition.** Supplier  $i$  is *constrained* if  $v_i^* \neq v_i^{*u}$ .

The optimal unconstrained price maximises the profit

$$\Pi_i = (v_i - c_i)Q(P^*(\sum_{j=1}^K v_j))$$

The first order condition is

$$Q + (v_i^{*u} - c_i) \frac{\partial Q}{\partial P} \frac{\partial P^*}{\partial v_i} = 0$$

Rearranging, the producer's optimal unconstrained unit dollar margin  $m_i^{*u}$  is:<sup>28</sup>

$$v_i^{*u} - c_i = -\frac{Q(P^*(v_i^{*u} + \sum_{-i} v_j))}{Q'(P^*(v_i^{*u} + \sum_{-i} v_j))} \times \frac{1}{\rho(P^*(v_i^{*u} + \sum_{-i} v_j))} \quad (3)$$

<sup>25</sup>The second order condition yields  $\frac{\partial^2 \Pi_D}{\partial P^2} \Big|_{P=P^*} = \frac{2Q'(P)^2 - Q''(P)Q(P)}{Q'(P)}$ . Log concavity of demand is equivalent to  $Q'(P)^2 \geq Q''(P)Q(P)$ . Hence  $\frac{\partial^2 \Pi_D}{\partial P^2} \Big|_{P=P^*} < 0$ .

<sup>26</sup>See Amir, Maret, and Troege (2004) and Weyl and Fabinger (2013).

<sup>27</sup>Supplier  $i$ 's profit is strictly increasing up to its single peak.

<sup>28</sup>The second order condition is satisfied, see the proof of Proposition 1.

### C. Identifying constrained suppliers

Comparing (1) with (3), it immediately follows that

$$m_i^{*u} \times \rho = m_D^* \quad (4)$$

In words: the equilibrium margin of the downstream firm is related to the margin of any unconstrained supplier. The pass-through rate is (weakly) less than 1. Therefore, by (4) the unit dollar margin of an unconstrained supplier exceeds that of the downstream firm.

**Observation.** *Any unconstrained supplier has a (weakly) bigger unit dollar margin than the downstream firm.*

The intuition for this observation is straightforward. For any log concave demand function the pass-through rate is less than 100%: the downstream firm absorbs some of an input price increase Adachi and Ebina, 2014; Amir, Maret, and Troege, 2004. Consequently the elasticity of demand upstream is lower than it is downstream. Therefore – by the intuition of the Lerner index – the margin upstream must be bigger than downstream.<sup>29</sup>

If there are several unconstrained suppliers, each earns a higher (unit dollar) margin than the downstream firm. All unconstrained suppliers earn the same (unit dollar) margin in equilibrium.

A supplier is constrained if (and only if) its margin, times the pass-through rate, is smaller than the downstream firm’s margin.

**Proposition 1.** *Supplier  $i$  is constrained if and only if  $m_i^* \times \rho < m_D^*$ .*

Figure 2 illustrates Proposition 1. It depicts three intermediate results. First, at supplier  $i$ ’s optimal unconstrained price the difference  $\Delta(v_i) = m_i(v_i)\rho(v_i) - m_D^*(v_i)$  equals zero (see (4)). Second, the difference increases (strictly) monotonically in  $v_i$ . This is shown in the proof of Proposition 1, but can be seen easily. The margin  $m_i$  strictly increases in price  $v_i$ , the pass-through rate  $\rho$  is non-decreasing (by Assumption A), and the margin  $m_D^*$  is weakly decreasing (due to less than full pass-through) in  $v_i$ . Third, any demand function that satisfies Assumption A induces a single-peaked profit function for a supplier. (As confirmed by the second-order condition, shown in the proof of Proposition 1.)

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<sup>29</sup>While the Lerner index relates percentage margins to the elasticity of demand, I relate upstream to downstream unit dollar margins.

Figure 2: Illustration of the equivalence in Proposition 1

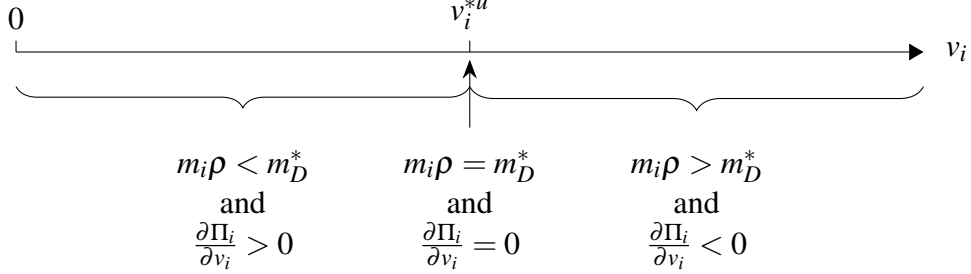


Figure 2 illustrates that only at  $v_i = v_i^{*u}$  does the difference  $m_i \rho - m_D^*$  equal zero. For any  $v_i > v_i^{*u}$  it is profitable to decrease the unit price, and  $m_i \rho > m_D^*$ . For any  $v_i < v_i^{*u}$  the supplier would like to obtain a higher unit price, and  $m_i \rho < m_D^*$ . Therefore if in equilibrium  $m_i^* \rho < m_D^*$ , the supplier would like to increase price but cannot – because it is constrained.

If a supplier's unit dollar margin is smaller than that of the downstream firm, then it is constrained.<sup>30</sup>

**Corollary 1.** *If  $m_i^* < m_D^*$ , then supplier  $i$  is constrained.*

Hence empirical facts can reveal whether a supplier is constrained. If data shows the margin of supplier  $i$  is smaller than that of the downstream firm, then that supplier cannot be setting its unconstrained price. The supplier must be constrained. In this way, if the exact pass-through rate is unknown but is known not to exceed 100%, constrained suppliers can be identified.

**Remark.** *Suppose each input is supplied by a monopolist. Then each equilibrium input price is a function of all other input prices. An exogenous change in one input price affects all other equilibrium input prices. However a constrained supplier does not necessarily change the price it sets in response to exogenous changes in other input prices.*

*This can be interpreted as dramatically reducing the information suppliers need to set their optimal price: a supplier who correctly believes it will be constrained in equilibrium need not know anything about the demand function and any other input. To “know its market” by knowing  $c_i^{[2]}$  suffices to set its optimal price.*

## D. Merger effects: equivalence & test

Consider a merger among suppliers (the most efficient producers) of different inputs  $i \in M \subseteq K$ : a merger of complements. As before the merger, in the second stage the

<sup>30</sup>The sufficient condition  $v_i^* < m_D^*$  may be used if the supplier's cost is not known.

downstream firm sets the optimal consumer price, as a function of input prices. Post-merger, the downstream firm has the same best response function  $P^*(\sum v_j)$  as pre-merger. Thus if the merger leaves input prices unaltered, the consumer price remains unaltered too. The level of the optimal consumer price changes post-merger only if the merger leads to a change in input prices. For a given sum of input prices, which might be different after the merger, post-merger the downstream firm's margin is given by (1):

$$m_D^* = -\frac{Q(P^*)}{Q'(P^*)}$$

In the first stage, post-merger the merged entity is either constrained in the total unit price it offers for inputs  $i \in M$  (at  $\sum_{i \in M} c_i^{[2]}$ ), or it is unconstrained. Let  $v_M$  denote the total unit price the merged entity offers for the  $M$  inputs.<sup>31</sup> Then the merged entity has profit

$$\Pi_M = (v_M - \sum_{i \in M} c_i) Q(P^*(\sum_{j=1}^K v_j))$$

Similar to (3), the first order condition yields the merged entity's optimal unconstrained unit dollar margin  $m_M^{*u}$ :

$$v_M^{*u} - \sum_{i \in M} c_i = -\frac{Q(P^*)}{Q'(P^*)} \frac{1}{\rho} \quad (5)$$

Again because the pass-through rate is (weakly) lower than 1, the merged entity has an optimal unconstrained price at which its dollar margin (weakly) exceeds that of the downstream firm. Finally, the merged entity sets the optimal total unit price

$$v_M^* = \min \left\{ v_M^{*u}, \sum_{i \in M} c_i^{[2]} \right\} \quad (6)$$

Due to the Cournot effect,  $v_M^{*u} < \sum_{i \in M} v_i^{*u}$ . In words: the merged entity has a lower optimal unconstrained price than the sum of the merging parties' pre-merger unconstrained prices. This is Cournot's (1838) finding: a merger of monopolists would lead to lower prices.

In Theorem 1 I answer the central question addressed in this paper: does a given merger of complements benefit consumers – via the Cournot effect (a price decrease) – or does it not? In that theorem I also answer a follow-up question of direct practical relevance. Suppose an Antitrust authority reviews a proposed merger for which it has data on margins and the pass-through rate. Does such data reveal whether the proposed merger would result in the Cournot effect?

**Theorem 1** (No merger benefit). *The merger yields no price decrease if and only if pre-merger  $\rho \times \sum_{i \in M} m_i^* \leq m_D^*$ .*

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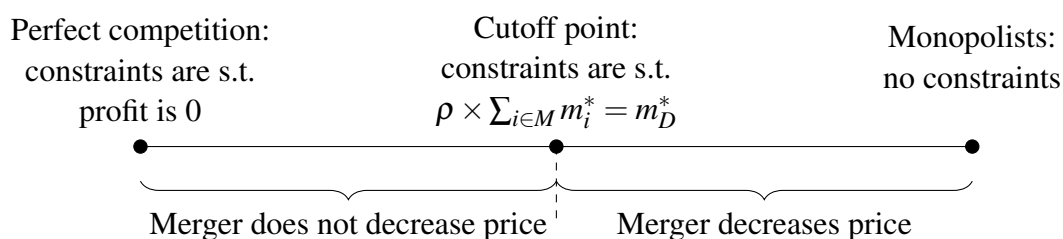
<sup>31</sup>The merged entity's profit depends only on the total unit price it receives.

Theorem 1 states: a merger among suppliers of complements does not benefit consumers if and only if the product of (a) the combined unit dollar margin earned by the merging suppliers and (b) the pass-through rate, is smaller than (c) the unit dollar margin of the downstream firm.

Intuitively, the merging parties need to be collectively sufficiently constrained pre-merger for the merged entity not to want to decrease price post-merger. Theorem 1 identifies the cutoff point where a merger of complements yields no consumer price decrease and consequently does not benefit consumers. When the condition  $\rho \times \sum_{i \in M} m_i^* \leq m_D^*$  holds, it is immediately clear that no single merging party is unconstrained. (For any unconstrained supplier alone already has  $\rho \times m_i^* = m_D^*$ .) Therefore all merging suppliers must be constrained. However, for the merger to yield no price decrease it does not suffice that each supplier be constrained. The merging parties must be collectively sufficiently constrained. Indeed, if each merging party is constrained but only slightly so, then the merged entity would find it profitable to decrease price. The Cournot effect would materialise and consumers would benefit from the merger.

Cournot (1838) found a merger between monopolist suppliers lowers price while at the other extreme a merger between suppliers who face perfect competition does not change the consumer price. Between these two extremes, the literature has hitherto not provided a result on whether consumers benefit from the Cournot effect. Theorem 1 fills this gap in the literature. It shows: past a certain cutoff point, where the merging parties are sufficiently constrained, no price decrease occurs at all. In the figure below, I depict that point graphically. To the right of the cutoff a merger of suppliers  $i \in M$  lowers price, to the left a merger does not.

Figure 3: Illustration of Theorem 1



In practice, it is hard to estimate the precise level of the pass-through rate. Imagine you do not know the pass-through rate, but know it does not exceed 100%. For such a situation, I now develop a new merger test. The test comes in the form of a sufficient condition. It states: if before the merger (a) the unit dollar margin of the downstream firm is larger than (b) the combined unit dollar margin earned by the merging suppliers, then

the merger of complements does not result in a lower price.<sup>32</sup> For a merger among two firms, suppliers of inputs  $A, B \in K$ :

**Corollary 2 (Test).** *If pre-merger  $m_A^* + m_B^* \leq m_D^*$ , then the merger yields no price decrease.*

If the merging parties together earn a combined unit dollar margin smaller than the margin of the downstream firm, then each merging party is constrained pre-merger, and their combined competitive edge over their competitors is small. Sufficiently small such that post-merger the merged entity would not want to decrease price.

The simplicity, practicability and transparency of the test renders it powerful for use by Antitrust authorities. It requires little data, merely three observable data points. Knowing only pre-merger margin data can suffice to identify whether the merging parties face sufficiently strong constraints pre-merger to exclude a price decrease post-merger.

The following result is important for section 3, which considers merger harm. The merging parties  $i \in M$  are **collectively constrained** if  $\rho \times \sum_{i \in M} m_i^* < m_D^*$ . When they are collectively constrained, not only does the merger and the ‘internalisation of double markups’ yield no price decrease but – to the contrary – the merged entity would want to increase price, if only it could.

**Lemma 1.** *The merged entity is constrained if and only if pre-merger  $\rho \times \sum_{i \in M} m_i^* < m_D^*$ .*

**Remark.** *Given quantity sold is the same upstream as downstream, one can compare dollar profits instead of unit dollar margins in Theorem 1, the test (Corollary 2), and in Lemma 1.*

### 3 Merger harm

In the previous section, a merger either reduces prices – to the benefit of consumers – or leaves prices unchanged. In this section, I discuss how the merger can harm consumers. Lemma 1 establishes whether the merging parties are collectively constrained. When the merging parties are collectively constrained pre-merger, the merged entity would find it profitable to obtain a higher price – but is limited by constraints. Intuitively, harm arises whenever the merger weakens those constraints.

I present two different theories of harm. Each lays out how constraints can be weakened post-merger. For both theories of harm, I assume the merged entity can credibly commit to pure bundling (to sell either both its components to the downstream firm or none at all). This assumption is both crucial and typical in theories for harm from mergers of complements.<sup>33</sup> To simplify the exposition (and without loss of generality), let the

<sup>32</sup>A different, more restrictive, sufficient condition is  $v_A^* + v_B^* < m_D^*$ .

<sup>33</sup>See Whinston (1990). See Kühn, Stillman, and Caffarra (2005) for factors which influence the credibility of pure bundling and “technical bundling”.

consumer product consist of two inputs, A and B. There are two producers of each input: firms A1 and A2, and firms B1 and B2.

For the first theory of harm, I add one compatibility parameter to the model. Let A1 and B1 be the more efficient producers of their respective components. As in the baseline model, the input produced by A1 is perfectly compatible with that of B1 and B2. Similarly, the input produced by B1 is perfectly compatible with that of A1 and A2. However, suppose the downstream firm incurs an extra unit cost  $x \geq 0$  to overcome compatibility issues if it sources from the worst producers (the high-cost producers) of each input, A2 and B2. If inputs from A2 and B2 are perfectly compatible,  $x = 0$ , if they are outright incompatible  $x$  is prohibitively large. To assume different levels of compatibility among various complementary products is common-place in the literature on compatibility, see e.g. Matutes and Regibeau (1988).

Pre-merger outcomes are the same as in section 2. Firm A1 knows that, if it offers a price in excess of  $c_A^{[2]}$ , the downstream firm would source from A2. (Similarly, B1 cannot obtain more than  $c_B^{[2]}$ .) Consequently, pre-merger the downstream firm pays a total unit price of  $v_A^* + v_B^* = c_A^{[2]} + c_B^{[2]}$  for the two inputs.

Let A1 merge with B1 and credibly commit to pure bundling. Post-merger, if the downstream firm does not source inputs from the merged entity, it sources from A2 and B2 and incurs the extra cost  $x$ . Consequently, the merged entity knows it can obtain a unit price up to  $c_A^{[2]} + c_B^{[2]} + x$  for its bundle. Hence, the merged entity can extract a higher unit price post-merger. This harms consumers, whenever the merged entity has an incentive to raise price.

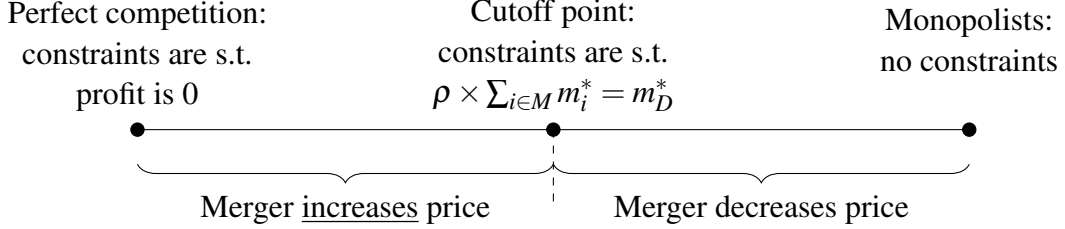
Intuitively, pre-merger neither merging party can generate a profit from the extra cost  $x$  because neither merging party is – alone – necessary to the downstream firm to avoid that cost. But the merging parties A1 and B1 are – jointly – necessary. If the merged entity does not supply, the downstream firm incurs  $x$ . Intuitively, with pure bundling the downstream firm's outside option – the constraint on A1 and B1 – is weakened post-merger.

**Lemma 2** (Ability). *If  $x > 0$  and pure bundling is credible, then a merger among suppliers of inputs A, B enables the merged entity to raise price.*

**Theorem 2** (Merger Harm). *If  $x > 0$  and pure bundling is credible, then a merger among suppliers of inputs A, B harms consumers if and only if pre-merger  $\rho(m_A^* + m_B^*) < m_D^*$ .*

These results are illustrated in the figure below. Figure 4 replicates Figure 3, with one difference. When  $x > 0$  and pure bundling is credible, a merger of collectively constrained suppliers no longer leaves prices unchanged. Instead consumers are unambiguously harmed by the merger.

Figure 4: Illustration of Theorem 2



For the second theory of harm, I apply a central idea from Whinston (1990) to the model of section 2. In his seminal work on tying, Whinston (1990) finds that if a merged entity can credibly commit to pure bundling, a merger of complements can induce a competitor to exit the market – thus changing the market structure. This way a merger of complements can reduce competition, lead to higher prices, and harm consumers.

Whinston (1990) does not explicitly analyse supply chains, i.e. vertically related firms. I now make a few changes to the baseline model of section 2 to apply, in spirit, his theory of harm to the vertical structure considered in the present paper. Let there be several downstream firms, who all compete with one another. Each downstream firm manufactures a differentiated version of the same consumer product. The two producers of A have respective constant unit costs  $c_{A1}$  and  $c_{A2}$ , where  $c_{A2} - c_{A1} > 1$ . Each producer of B has a unit cost specific to each different downstream firm  $j$ . Constant unit costs of the two producers to supply downstream firm  $j$ ,  $c_{B1j}$  and  $c_{B2j}$ , are randomly drawn from the same c.d.f. on  $[0, 1]$ . Each producer of B incurs fixed (sunk) costs.

The timing is as follows.<sup>34</sup> First, firms B1 and B2 decide whether to enter and thereby incur fixed entry costs. A firm which enters is able to produce B and thus bid to supply the downstream firm. Second, upstream producers (of A and B) submit offers to the downstream firm. Third, the downstream firm selects its suppliers, one of each input, and sets the consumer price. Post-merger, an initial stage (stage 0) is added where the merged entity decides whether to engage in pure bundling.

Pre-merger, the more efficient producer of A supplies all downstream firms. It obtains unit price  $v_A = c_{A2}$ . B1 supplies those downstream firms for which it is more efficient than B2. From any such downstream firm  $j$ , B1 obtains unit price  $v_{Bj} = c_{B2j} \geq c_{B1j}$ . Conversely, B2 supplies those downstream firms for which  $c_{B2j} < c_{B1j}$ . Again, these pre-merger outcomes are essentially the same as in section 2.

Let A1 merge with one of the producers of B, say with B1, and credibly commit to pure bundling. Post-merger, the non-integrated producer B2 knows it will no longer supply a downstream firm. At best, its cost-advantage over B1 is of magnitude 1. But the

<sup>34</sup>The timing is identical to that of Whinston, 1990, except here there is an additional stage as I model both the upstream and downstream levels.



cost-advantage of A1 over A2 is greater than 1. Therefore, the merged entity can always undercut the total unit price for the two inputs offered by B2 and A2. Consequently, post-merger B2 earns no revenue, cannot cover its fixed costs, and exits the market. Thus the market structure for input B changes. This change in market structure allows the merged entity to extract a higher price for inputs A and B than what downstream firms paid pre-merger.<sup>35</sup>

The two theories of harm do not encompass all possibilities for merger-related harm. For each proposed merger of complements there may be a specific theory of harm. While theories of harm may be merger-specific, I provide a general test of incentives: Lemma 1 identifies whether a merged entity would want to increase prices. Whenever such incentive exists, an Antitrust authority may have reason to worry about and investigate possible post-merger strategies detrimental to consumers.

## 4 Antitrust discussion

### A. No trade-off

When a competition authority reviews a proposed merger of complements, it might develop a theory of harm regarding that merger. However Antitrust guidelines in the US and the EU lay out how such mergers can benefit consumers. Therefore, in order to act against the merger, an Antitrust authority needs to show that any harm exceeds the benefit.

“The merged firm may also have an incentive to offer lower prices [...] If the Agencies conclude that both countervailing price effects are likely to be present post-merger, the Agencies will conduct a balancing of the effects to determine the net effect on the prices customers will likely pay.” U.S. Department of Justice & The Federal Trade Commission (2020), p.9.

Similarly, the European Commission merger guidelines state

“The integration of complementary activities or products within a single firm may [...] provide an increased incentive to seek to decrease prices.” European Commission (2014), p.212.

The present paper provides a practicable empirical test. Antitrust authorities (and ultimately the courts) can use this test to dismiss claimed positive effects. If positive effects of a proposed merger can be dismissed, then the merger might not entail a trade-off. In the absence of benefits, and if the Antitrust authority has a theory of harm, the authority has unambiguous grounds to act against the proposed merger.

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<sup>35</sup>After the market exit, the merged entity would act as a monopolist producer of B. However, if there is a competitive fringe in B, of producers with unit cost  $c_B \geq 1$ , the price the merged entity charges for its bundle may increase but nevertheless still be constrained post-merger.

## B. Discussion of the test

The Cournot effect holds true with regards to monopoly prices: if monopolist suppliers merge, the merged entity wants to decrease price. However, if the facts of the case show that the sum of the pre-merger dollar margins of the merging parties is smaller than the downstream firm's margin, the merging parties are severely constrained in setting their prices pre-merger, to such an extent that the merged entity has no incentive to lower price.

In this way, facts on margins can inform us to expect no merger benefit for consumers (and for third party suppliers, and the downstream firm). Margins are highly informative and revealing of the competitive situation because the profit functions upstream and downstream are identical, except for the pass-through and thus the margins.

The two theorems and the test hold even if the market share of the merging parties is 100%. And even if they 'dominate' their market. What matters are not market shares, not percentage margins, but data on dollar margins.<sup>36</sup>

Finally, note that horizontal and vertical mergers have standard effects in the present setting and model. A merger of the two most efficient suppliers of a particular input leads to a loss of competition, the offered price of the input rises (absent synergies). A vertical merger between the downstream firm and the most efficient firm reduces the consumer price through the elimination of double marginalisation on that input.

## C. Example: how to apply the test in practice

One of the attractive features of the test is that it relies solely on verifiable pre-merger data (margin figures), rather than demand elasticities (which may be difficult to estimate reliably, Karlinger et al., 2020) or any counterfactual or hypothetical post-merger outcomes.

To illustrate how the test can be applied in practice, let's consider Apple. Apple's unit cost of an iPhone (taking the iPhone 11 as an example) is estimated at \$490, including inputs produced in-house and assembly.<sup>37</sup> But Apple "charges more than double that amount to consumers"<sup>38</sup> (about \$1100). In other words, Apple's unit dollar margin on an iPhone 11 is greater than the unit price paid to all suppliers combined. Consequently its unit dollar margin surpasses that of all its suppliers combined. A direct application of the test concludes: *even if all of Apple's numerous suppliers merged, pre-merger margins reveal suppliers are collectively sufficiently constrained such that post-merger they would have no incentive to decrease price.* The Cournot effect would not materialise. Consumers would not benefit from such a merger of complements.<sup>39</sup>

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<sup>36</sup>If pass-through data is available, Theorem 1 can be used.

<sup>37</sup>Yang, Wegner, and Cowsky (2019).

<sup>38</sup>See e.g. Rosalsky (2021).

<sup>39</sup>Similarly for the iPhone X. Apple's gross unit dollar margin (sales price minus input costs) for the iPhone X has been estimated at around \$620 (McGuire, 2018). The total unit price it pays to all of its suppliers is estimated at \$370 (IHS Markit, 2017). Such that the combined margin of all of iPhone suppliers

## D. Applying Theorem 1 in practice

I want to emphasise that Theorem 1 can also be used by Antitrust authorities to inform merger review even in the absence of pass-through data. To see this, consider a proposed merger among suppliers of inputs A and B. Stating Theorem 1 differently: a merger yields no benefit to consumers via the Cournot effect if

$$\rho < \frac{m_D^*}{m_A^* + m_B^*}$$

By calculating the right-hand side fraction, Theorem 1 can be used to “back out” the lowest necessary pass-through rate for the merger to benefit consumers via the Cournot effect.

Using three data points on margins an Antitrust authority can calculate  $\frac{m_D^*}{m_A^* + m_B^*}$ . While an Antitrust authority may not know the level of the pass-through rate, it might have a good enough understanding to determine what level is unrealistic – unreasonably high. For example, suppose two suppliers propose to merge, one has a (pre-merger) unit dollar margin of \$10, the other of \$60. The downstream firm has a unit margin of \$490. To insert these numbers into the fraction yields a minimum pass-through rate of 700%. An Antitrust authority may deem this level implausible and conclude that the merger would not benefit consumers via Cournot effects.

## E. Antitrust review: fact-check claimed benefits

The EC, FTC, DOJ and many other authorities have come to rely on insights from Industrial Organization to help decide on Antitrust cases. But both proponents and opponents to a particular case (e.g. a merger case) typically submit economic analyses (often from academic economists) with opposing conclusions and Antitrust implications. In a comment on the field of Industrial Organization Paul Krugman writes “there came a [... time] when a smart grad student could produce a model to justify anything” Krugman (2014).

Models are by definition a simplification of reality. The pertinent and critical question is whether the manner in which a particular merger model simplifies reality leads to predictions which distort or are even opposite to the true merger effects. When different models are submitted to an Antitrust authority by parties respectively favourable and opposed to a proposed merger, the test provides a practicable, simple, and transparent facts-based condition to dismiss arguments regarding claimed benefits from Cournot effects. Claimed benefits can be dismissed as inconsistent with available data.

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is less than \$370. If Apple has in-house unit costs of less than \$250 its margin is greater than \$370, and thus the test would conclude a merger of even all of Apple’s suppliers would not yield a price decrease.

## **F. The Antitrust toolbox: a comparison to existing merger tests**

A number of merger tests have been developed for use by Antitrust authorities. For horizontal mergers Farrell and Shapiro (2010) have introduced the concept of upwards-pricing pressure (UPP) to replace the older HHI.<sup>40</sup> The UPP uses price-cost margins of the merged entity and diversion ratios<sup>41</sup> to infer the extent to which the merged entity would want to raise prices post-merger (due to weakened competition) or decrease prices (due to efficiency savings). Moresi and Salop (2009) cited the evidentiary burden for efficiencies and other factors and argued for the FTC to adopt the GUPPI (gross UPP index) which focuses on the incentive to increase prices.<sup>42</sup>

More recently, for vertical mergers the vGUPPI has been developed by Moresi and Salop (2013) to gauge the incentive of the merged (vertically integrated) entity to raise the input price for a third party firm which is simultaneously a downstream customer and rival. I.e. to estimate the incentive to engage in partial input foreclosure by losing upstream profits but gaining downstream profits. Again the interplay of margins and diversion ratios plays a key role in the calculation.<sup>43</sup>

Compared to the aforementioned tests, the present test requires fewer data points: it merely requires three data points on unit dollar margins. (Diversion ratios and market shares play no role here.) Furthermore, the type of conclusions reached differs: in the horizontal and vertical GUPPIs margins are used to estimate the extent of a post-merger price increase by the merging parties. The test developed in the present paper provides a binary yes or no answer to the question: do sufficient constraints exist to rule out merger-specific gains to consumers?

## **G. Merger harm**

When the merged entity does not find it profitable to decrease price, it would like to increase price. But it cannot because of constraints. Merger harm arises when the merger leads those constraints to be weakened – thus allowing price to rise post-merger. This is illustrated in section 3.

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<sup>40</sup>The HHI for an upstream market does not necessarily offer much insight for the present context. If the HHI shows there are several producers, this does not exclude that the supplier can charge the monopoly price. And if the HHI shows there is no other producer, then a monopolist supplier may nevertheless be constrained, e.g. by the ability of the downstream firm to produce an input in-house.

<sup>41</sup>Which may not be available, and in practice are typically estimated via market shares – see Moresi and Zenger (2018).

<sup>42</sup>The GUPPI estimates the incentive of the merged entity to raise prices, it does not estimate the response of other firms and the equilibrium price change. Moresi, Salop, and Woodbury (2010) briefly lays out the calculation of the index. An ongoing discussion in Europe revolves around whether data on margin levels should be used in Antitrust merger reviews. See Padilla (2018) and Valletti and Zenger (2018). For a brief policy discussion, see also Caffarra et al. (2018).

<sup>43</sup>See also Moresi and Salop (2020) on this.

## 5 Downstream competition

Suppose there are several downstream firms, who compete with one another. Let each produce a differentiated version of the same consumer product. Let the quantity sold by one downstream firm increase in the price of another. And suppose Assumption A still holds, such that each downstream firm faces a demand which is log concave in its price (this excludes some extreme forms of competition, such as perfect competition). In stage 1, upstream producers set their prices. Producers can price discriminate across downstream firms. In stage 2, all downstream firms simultaneously set their price.

Further below in this section, I provide intuitions on how downstream competition affects the theorems and the test. Formal workings are deferred to the appendix. For conciseness of the formal workings, I follow Moresi and Salop (2013)<sup>44</sup> in assuming “passive beliefs”. According to the passive beliefs assumption, a downstream firm does not know the input prices of its competitors and forms point beliefs regarding those prices – beliefs which are correct in equilibrium.<sup>45</sup> (The passive beliefs assumption is not necessary, see Kadner-Graziano (2022).)

There is a distinction between two cases. Either a supplier supplies only one downstream firm. Or it supplies several competing downstream firms. I refer to the latter case as overlapping supply chains. If the merging parties supply only one (and the same) downstream firm pre-merger, Theorem 1, the test, and Theorem 2 apply unaltered. Intuitively, nothing changes as the profit function of a supplier does not change (as it still supplies only one downstream firm), and the pass-through rate still lies below 100% (because the downstream firm faces a log concave demand).

If instead one or more merging parties supply several downstream firms, there is a minor tweak to the two theorems. Because with overlapping supply chains, a new effect appears. When a supplier decides which price to set to a downstream firm, it knows that a higher input price leads that downstream firm to set a higher consumer price, which in turn leads to more sales for other downstream firms. The supplier earns profits on diverted sales, as it supplies other downstream firms. Intuitively, because of this new effect, the ‘elasticity of total quantity sold’ upstream relative to downstream is even lower than in section 2. Therefore, the ratio of (i) the unit dollar margin a supplier earns on sales to a given downstream firm, to (ii) the unit dollar margin of that downstream firm, is even higher than in section 2.

Due to this new effect, when supply chains overlap Theorems 1 and 2 provide sufficient conditions rather than equivalences. The test already provides a sufficient condition, thus it remains unchanged with downstream competition.

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<sup>44</sup>See also Rey and Tirole, 2007 on this.

<sup>45</sup>The key technical and simplifying consequence of such beliefs is that, when the supplier sets its optimal input price  $v_i$ , a change in  $v_i$  is assumed to change only  $P_i$ , and not the consumer prices of other downstream firms (as they set their prices based on their point beliefs regarding  $v_i$ .)

## 6 Extensions

Theorem 1, the test, and Theorem 2 are obviously robust to a series of extensions, which I sketch briefly.

First, suppose the downstream firm has some constant unit marginal cost of assembly. This alters none of the results, because the results relate downstream to upstream dollar margins (net of any unit costs).

Second, adding more producers of an input does not change results: the constraint on a supplier of any input is determined by the second-most efficient producer, out of any number of potential producers.

Third, suppose the downstream firm is able to produce a given input  $i \in K$  in-house, at some constant unit marginal cost of production. If it is more efficient than the most efficient upstream producer of that input, it will produce the input in-house. Otherwise, the upstream supplier of that input simply faces an extra constraint (in addition to  $c_i^{[2]}$ ).

Fourth, suppose some input  $i \in K$  is non-essential: the downstream firm can do without it and still produce a valuable consumer product. Then, beyond some limit price for that input, the downstream firm chooses to do without it.<sup>46</sup> An input being non-essential simply constitutes a further explanation as to what may constrain a supplier. The theorems and the test do not change.

Fifth, suppose an upstream producer supplies several inputs to the downstream firm – all inputs  $i \in J \subset K$ . Then its optimal total unit price for those inputs is

$$v^* = \min \left\{ v^{*u}, \sum_{i \in J} c_i^{[2]} \right\}$$

This strongly resembles (6), the optimal price of the merged firm. Alike a merged entity, a multi-product supplier sets prices for the different inputs so as to set its optimal total unit price. Theorem 1, the test, and Theorem 2 apply unaltered. One merely needs to ensure that the unit margins used in the theorems and the test reflect the total unit margin earned by the merging parties for the entirety of inputs they supply.

Sixth, suppose a merger among suppliers of inputs  $A, B \in K$  creates synergies such that the merged entity has a constant unit marginal cost  $c_M < c_A + c_B$ . If suppliers are monopolists, such synergies lead to a lower price post-merger. Synergies would then benefit all: suppliers of complementary inputs, the downstream firm, and consumers. In contrast, when the merging parties are collectively constrained synergies may or may not be passed on. Consumers benefit from synergies if and only if  $v_M^*(c_M) < \sum_{i \in M} c_i^{[2]}$ . Synergies are not passed on when the new, lower, optimal unconstrained price exceeds the level at which the merged entity is constrained. With synergies the theorems and test change somewhat. Sufficient conditions for benefits not to arise are  $\rho \sum_{i \in M} v_i^* < m_D^*$  or, in the spirit of the test,  $\sum_{i \in M} v_i^* < m_D^*$ . These conditions ensure that, even if synergies

<sup>46</sup>The limit price is unique, because the downstream firm's profit strictly decreases in  $v_i$ .

are so extreme as to eliminate the merging parties' costs completely, the merging parties are collectively sufficiently constrained pre-merger for there to be no price decrease post-merger.

## 7 Discussion of Assumption A and robustness

### A. Discussion of Assumption A

I assume demand is strictly decreasing and weakly log concave in price. These are general and standard assumptions. I also assume that demand induces non-decreasing pass-through ( $\rho' := \frac{\partial^2 P^*}{\partial v_i^2} \geq 0$ ). In the rightmost column of the table below, I show that this latter assumption is satisfied for all the main log concave distributions.<sup>47</sup>

Table 1: Distributions of consumers' willingness-to-pay, and pass-through

Log concave distribution	$Q(P)$	$\rho$	$\rho'$
Uniform	$1 - P$	$\frac{1}{2}$	0
Gaussian	no closed form	$\in [\frac{P^2+1}{P^2+2}, 1]$	$\geq 0$
Laplace	$\frac{1}{2}e^{-P}$	1	0
Exponential	$e^{-\lambda P}$	1	0
Logistic	$\frac{1}{1+e^{-P}}$	$\frac{1}{1+e^{-P}}$	$> 0$
Extreme value	$1 - e^{-e^{-P}}$	$\frac{1}{1 - \frac{1}{e^{-P}} - \frac{1}{e^{-e^{-P}}} + \frac{1}{e^{-P}e^{-e^{-P}}}}$	$> 0$
Weibull, $c \geq 1$	$e^{P^c}$	$\frac{cP^c}{cP^c + c - 1}$	$> 0$
Rayleigh	$e^{P^2}$	$\frac{2P^2}{2P^2 + 1}$	$> 0$
Power, $\beta > 1$	$1 - P^\beta$	$\frac{1}{1 + \frac{1}{\beta} + (1 - \frac{1}{\beta})\frac{1}{P^\beta}}$	$> 0$
Inverse of power reliability, $\beta > 0$	$(1 - P)^\beta$	$\frac{\beta}{\beta + 1}$	0

Source: own workings on  $\rho$  and  $\rho'$ .

Appendix C contains further permissible demand functions and technical discussions.

<sup>47</sup>Let each consumer have unit demand with utility  $x - P$ , where  $x$  represents an individual consumer's willingness-to-pay for the consumer good. A consumer buys when  $x - P \geq 0$ , where  $F(x)$  is the c.d.f of consumers' willingness-to-pay. Demand, then, takes the form  $Q(P) = 1 - F(P)$ .

That the supports of some of these distributions contain negative willingness-to-pay is no issue: left (and right) side truncations of the c.d.f. preserve log concavity of the reliability (Bagnoli and Bergstrom, 2005, p.8-9).

## B. Pass-through above 100%

I now do away with the assumption of log concave demand. I replace Assumption A with the more general assumption below.

**Assumption B.** *Consumer demand  $Q(P)$  induces single-peaked profit functions for the downstream firm and its suppliers. It induces non-decreasing pass-through.*

Non-decreasing pass-through was already assumed in Assumption A. An example of a log convex demand function that satisfies Assumption B is provided Appendix C.

Researchers or Antitrust practitioners might have a reliable estimate of the pass-through rate for a specific setting under analysis. The question I address now is: if the pass-through rate lies above one (above 100%), can Theorems 1 and 2 as well as the test still be used? The test cannot: it specifically applies to log concave demands (where the pass-through rate is less than one.) In contrast, under Assumption B the two theorems continue to hold. Thus even for pass-through rates higher than 100% Theorems 1 and 2 can still be used to inform on the effects of a proposed merger.

Workings are provided in the appendix.

## 8 Robustness: price formation & information structure

In this section I consider different information structures and different ways input prices are formed in stage 1. I discuss three possibilities: second price auctions, first price auctions, and bargaining. For each of the three possibilities, the focus in this section is three-fold: to assess whether the test and theorems continue to hold; to assess whether a merger of complements is profitable; and, if results change, to provide intuitions. Workings on first price auctions and bargaining are provided in Appendix B.

### A. Second price auctions – with endogenous quantity

Suppose that competing producers of the same input do not know each other's cost.<sup>48</sup> Let input prices be determined via second-price auctions (SPAs). Then outcomes are identical to those of sections 2 and 3 – without complete information and with SPAs nothing changes.

Pre-merger, each producer of input  $i \in K$  submits a bid equal to its constant unit marginal cost. The downstream firm and its chosen supplier agree on the unit price given by (2):  $v_i^* = \min\{c_i^{[2]}, v_i^{*u}\}$ . (If the chosen supplier finds it profitable to offer a lower price

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<sup>48</sup>I retain the assumption that the downstream firm does not know the costs of its upstream suppliers, and retain the assumption that upstream monopolist suppliers – to the extent there are any – know the costs of suppliers of other inputs.



than the SPA outcome, then it is mutually beneficial to agree to a lower price.) Post-merger, producers still bid their cost. The sole merger effect is the Cournot effect. The unconstrained price of the merged entity is lower than the sum of unconstrained prices of its pre-merger entities:  $v_M^{*u} < \sum_{i \in M} v_i^{*u}$ . The theorems and the test continue to hold.

**Remark.** *The SPA can be interpreted as repeated rounds of negotiations, where the downstream firm negotiates with upstream producers, and where producers can repeatedly bid down the offer price.*

## B. First price auctions – with endogenous quantity

Suppose that competing producers of the same input do not know each other’s cost. Let input prices be determined via first price auctions (FPAs). (The downstream firm selects that supplier of a given input which offers the lowest unit price.) Whether the theorems and test continue to hold depends, in part, on the cost distributions of competing input suppliers.

Trivially, to know the costs of competitors is the degenerate case of incomplete information. Intuitively, the findings of sections 2 and 3 continue to hold when competing suppliers have cost distributions which are non-overlapping and where the most efficient firm prices such that it wins with certainty. An example of this is provided in the appendix. Therefore with incomplete information and FPAs, the theorems and test can continue to hold.

However, there are settings in which the theorems and test do not hold. For example when suppliers have the same cost distributions. To understand why, it helps to appreciate a result from Hansen (1988). He shows that with endogenous demand the revenue equivalence theorem between FPAs and SPAs<sup>49</sup> no longer holds. Because, intuitively, in a FPA with endogenous demand the “cost” of bidding slightly “too low” (relative to the optimal bid with fixed quantity) is reduced, because a lower bid increases demand. Therefore parties will bid more aggressively (submit lower offer prices), to the benefit of the downstream firm.<sup>50</sup>

Pre-merger, FPA bids of competing producers are determined by three incentives. First, a producer has an incentive to bid low in order win more frequently. Second, it has an incentive to bid low in order to sell more units if it does win. Finally, it has an incentive to bid high in order to obtain a high price when it wins.

Post-merger, the merged entity has an added incentive to bid low: to sell more units of the newly acquired input when winning. Intuitively, this leads to lower bids after the merger. Bids decrease even if suppliers were bidding a price far below their monopoly price pre-merger. In such a setting, the theorems and test do not hold.

<sup>49</sup>Due to Vickrey (1961), Myerson (1981), Riley and Samuelson (1981) as well as Harris and Raviv (1981).

<sup>50</sup>Hansen (1988), p.50.

I provide pre- and post-merger workings on FPAs – with endogenous demand – in the appendix, along with more detailed intuitions.

### C. Nash-in-Nash bargaining – with endogenous quantity

Suppose the downstream firm knows the costs of upstream producers. And let the downstream firm bargain with producers over the unit input price. To solve the game, I use the Nash-in-Nash bargaining solution.<sup>51</sup> This bargaining solution is well-suited to deal with monopolist suppliers of essential inputs (Schmidt, 2014; Spulber, 2017). So to begin with, consider the case where each input is produced by a monopolist.

In this setting, a merger of complements always leads to lower input prices. This results in a lower consumer price, and benefits consumers. The main merger effect is, effectively, a loss of bargaining surplus upstream.

With exogenous demand, a merger of complements is never profitable for the merging parties. Intuitively: in the bargaining solution each essential upstream supplier earns one  $K^{\text{th}}$  of the total upstream profit. Twice this share,  $\frac{2}{K}$ , exceeds the share  $\frac{1}{K-1}$  the merged entity would receive. Consequently a merger diminishes profit upstream. (This result bears some semblance to the unprofitability of Cournot mergers, where for  $K \geq 3$  a merger between two firms is strictly unprofitable.)<sup>52</sup>

With endogenous demand, the bargaining solution takes into account that lower input prices increase quantity and thus “the size of the pie”. This added effect exerts downward pressure on input prices. Post-merger, the input price decreases further. The resulting quantity increase benefits the merged entity. However, but for the most extreme cases (e.g. with very few inputs, and with extremely convex demand), this quantity effect is not sufficient to render the merger profitable for the merged entity.

Consequently, with bargaining (and absent anticompetitive effects), mergers of complements always benefit consumers, because prices fall. The findings of section 2 and 3 do not hold. But because such mergers are mostly unprofitable, such mergers might not occur. Such that the question of whether the test and theorems apply to such a proposed merger may, in practice, not arise.

Finally, when there are multiple producers of the same input, algebraic workings becomes significantly more cumbersome. But the equilibrium price still reflects that a lower price increases quantity. The merger always decreases price. Therefore the test and the

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<sup>51</sup>Harsanyi (1963) extended Nash bilateral bargaining solution (Nash, 1950) to the  $(n + 1)$ -player case, where a buyer faces  $n$  sellers of perfect complements. The Shapley value (see Shapley, 1952) with perfect complements yields the same solution as Nash-in-Nash bargaining (as with perfect complements all essential suppliers are required for a coalition to have a non-zero value), except that the Shapley value effectively imposes equal bargaining powers. Collard-Wexler, Gowrisankaran, and Lee (2019) provide non-cooperative micro-foundations for the Nash-in-Nash solution.

<sup>52</sup>On this, see Salant, Switzer, and Reynolds (1983) and Belleflamme and Peitz (2015), p.374-6. See also Motta (2004), p.243-50.

theorems do not hold. But, as discussed, the price decrease is not a result of the Cournot effect. Rather it results from a loss of bargaining surplus. Only in extreme cases would such a merger be profitable.

## 9 Conclusion

Standard intuition on Cournot effects; guidelines from Antitrust authorities; as well as many models in the literature suggest mergers of complements can benefit not only the merging parties, but also other suppliers, downstream firms, and consumers. Such that, when a merger of complements is reviewed by competition authorities, any potential negative merger effect ought to be weighed against at least the positive effect from internalising Cournot externalities. This complicates the work of competition authorities. The trade-off may be difficult to ascertain. It may be ambiguous.

In this paper I have developed tools to identify whether a merger of complements will benefit consumers, via the Cournot effect – or whether the merger will not. The main finding, Theorem 1, closes the gap in the literature. It deals with all competitive landscapes between the polar extremes of monopoly and perfect competition. Antitrust authorities would typically have a good understanding of the three relevant unit dollar margins (of the two merging parties and of the downstream firm). Hence Antitrust authorities can use the tools developed herein to inform merger review.

When a merger-related price decrease can be dismissed – as being inconsistent with facts of the case – any possible negative effects need not be weighed against positive Cournot effects. In the section on merger harm I show how, absent benefits, the merger can unambiguously harm consumers. The authority may then have clear grounds to act against a proposed merger.

The findings are general. They extend, among other, to downstream competition and to non-essential inputs. Furthermore, the main result is robust to pass-through rates in excess of 100%. The findings hold for different informational assumptions when second-price auctions are used. With first-price auctions, whether the results hold is specific to the exact setting. With bargaining the findings do not hold, but with bargaining the merger is (but for extreme cases) not profitable. Such that a bargaining setting might not be relevant to the present analysis.

If the tools developed herein indicate that Cournot effects do materialise, this implies that – absent anticompetitive behaviour – the merger would benefit consumers. However accounting for anticompetitive behaviour may still result in the merger being harmful overall. Thus the tools cannot provide a free pass to a merger.

Further helpful work would encompass empirical studies which compare margins of manufacturers (downstream firms) and of their suppliers (upstream firms), for different industries. Further helpful work would also include empirical studies on the extent to which Bertrand competition among suppliers is a good way to characterise competitive

interactions, or whether instead FPAs, SPAs, bargaining, or other price formation processes better characterise how prices are agreed upon with given manufacturers, or in given industries.

In short, this paper shows observable data can reveal useful information to fact-check existing theory, and to predict merger effects of consequence to consumers.

## APPENDIX

### A. Proofs

*Proof of Proposition 1.* I use three intermediate results for the proof. I lay these out now.

First, supplier  $i$ 's profit function is single-peaked. This is shown here: the second-order condition is

$$\begin{aligned} \left. \frac{\partial^2 \Pi_i^u}{\partial v_i^2} \right|_{v_i=v_i^{*u}} &= 2 \frac{\partial Q}{\partial P} \frac{\partial P}{\partial C} + (v_i^{*u} - c_i) \left[ \frac{\partial^2 Q}{\partial P^2} \left( \frac{\partial P}{\partial C} \right)^2 + \frac{\partial Q}{\partial P} \frac{\partial P^2}{\partial C^2} \right] \\ &= [2Q'(P)^2 - Q(P)Q''(P)] \frac{\rho}{Q'(P)} - Q(P) \frac{\rho'}{\rho} \\ &< 0 \end{aligned}$$

as  $Q'(P)^2 > Q(P)Q''(P)$  (demand is log concave) and as  $\rho' \geq 0$  (by Assumption A).

Second, at supplier  $i$ 's optimal unconstrained price  $v_i^{*u}$ , equation (4) shows that  $m_i^{*u} \times \rho - m_D^* = 0$ .

Third, the difference  $m_i \times \rho - m_D^* = 0$  strictly increases in  $v_i$ . This is shown here:

$$m_i \times \rho - m_D^* = (v_i - c_i) \times \rho - [P^* (\sum v_j) - \sum v_j]$$

Differentiating this difference with respect to  $v_i$  yields

$$\begin{aligned} \rho + (v_i - c_i) \frac{\partial \rho}{\partial v_i} - \frac{\partial P^*}{\partial v_i} + 1 \\ = 1 + (v_i - c_i) \frac{\partial \rho}{\partial v_i} \\ > 0 \end{aligned}$$

as the pass-through rate is non-decreasing (by Assumption A).

It follows that  $v_i^{*u}$  is the unique price  $v_i$  at which the difference equals zero.

I now use these three intermediate results. If  $m_i^* \times \rho < m_D^*$ , then by the first two intermediate results above it must be that  $v_i^* < v_i^{*u}$ . If  $v_i^* < v_i^{*u}$  then, because profit is strictly increasing below price  $v_i^{*u}$ , it must be that  $v_i^* = c_i^{[2]}$ . (On this see (2).) Hence the supplier is constrained.

Proving the other direction: if supplier  $i$  is constrained, then  $v_i^* \neq v_i^{*u}$ . The cost  $c_i^{[2]}$  cannot lie above supplier's unconstrained price, for it did, the supplier could set its unconstrained price. Therefore  $c_i^{[2]}$  must lie below supplier  $i$ 's unconstrained price:  $c_i^{[2]} < v_i^{*u}$ . From the first two intermediate results above we know that at any such input price  $v_i^* = c_i^{[2]}$ , the inequality  $m_i^* \times \rho < m_D^*$  holds.  $\square$

*Proof of Corollary 1.* Demand is log concave, hence the pass-through rate  $\rho \in (0, 1]$ . Consequently Corollary 1 follows directly from Proposition 1.  $\square$

*Proof of Theorem 1.* (The proof is mostly identical to the proof of Proposition 1.) I begin by listing the three intermediate results used for the proof.

First, the profit of the merged entity – as of any supplier – is single-peaked in  $v_M$ . See the proof of Proposition 1.

Second, at the merged entity's optimal unconstrained price  $v_M^{*u}$  equation (5) shows that  $m_M^{*u} \times \rho = m_D^*$ .

Third, the difference  $m_M \times \rho - m_D^*$  increases (strictly) monotonically in the price  $v_M$ . This is shown here:

$$m_M \times \rho - m_D^* = (v_M - \sum_{i \in M} c_i) \times \rho - [P^*(\sum v_j) - \sum v_j]$$

Differentiating the difference with respect to  $v_M$  yields

$$\begin{aligned} & \rho + m_M \frac{\partial \rho}{\partial v} - \frac{\partial P^*}{\partial v} + 1 \\ &= \rho + m_M \frac{\partial \rho}{\partial v} - \rho + 1 \\ &= 1 + m_M \frac{\partial \rho}{\partial v} \\ &> 0 \end{aligned}$$

as the pass-through rate is non-decreasing (by Assumption A).

It follows that, for any given sum of prices of the non-merging suppliers  $\sum_{j \in K, j \notin M} v_j$ , the price  $v_M^{*u}$  is the unique price where the difference equals zero.

Proving one direction: suppose that pre-merger  $\rho \times \sum_{i \in M} m_i^* \leq m_D^*$ . Imagine the merged entity were to fix its price at  $v_M = \sum_{i \in M} v_i^*$ . At this price, in equilibrium  $\rho \times m_M \leq m_D^*$ . (This immediately shows that the price  $v_M = \sum_{i \in M} v_i^*$  is not the optimal unconstrained price.) Therefore, given the direction of the inequality, the optimal unconstrained price  $v_M^{*u}$  must lie above  $\sum_{i \in M} v_i^*$ . Given single-peakedness, at any price below the optimum the merged entity finds it profitable to obtain a higher price. Consequently, if  $\rho \times \sum_{i \in M} m_i^* \leq m_D^*$  it would find it strictly unprofitable to decrease price. The merged entity does not decrease price post-merger.

Proving the other direction: Suppose the merged entity does not decrease price post-merger. Then it must be that profit is weakly increasing at the point  $v_M = \sum_{i \in M} v_i^*$ . Profit is weakly increasing only where  $\rho \times m_M \leq m_D^*$ . Where  $m_M = \sum_{i \in M} (v_i^* - c_i) = \sum_{i \in M} m_i^*$ . Therefore if the merged entity does not decrease price post-merger then it must be that  $\rho \times \sum_{i \in M} m_i^* \leq m_D^*$ .  $\square$

*Proof of Corollary 2.* Demand is log concave, hence the pass-through rate  $\rho \in (0, 1]$ . Consequently Corollary 1 follows directly from Theorem 1.  $\square$

*Proof of Lemma 1.* The proof parallels the proof of Theorem 1. In light of the proof of Theorem 1, the proof of Lemma 1 is obvious.  $\square$

*Proof of Lemma 2.* This follows from the text.  $\square$

*Proof of Theorem 2.* By Lemma 2, if  $x > 0$  and pure bundling is credible then the merged entity has the ability to raise price. By Lemma 1, the merged entity wants to obtain a higher price if and only if pre-merger  $\rho \times \sum_{i \in M} m_i^* < m_D^*$ . The merged entity uses its ability to raise price whenever it has the incentive to do so.  $\square$

## B. Deferred workings

### B.1 Workings for section 5

I briefly lay out the setting of section 5. There are  $N > 1$  downstream firms. (Let  $N$  also denote the set of those firms.) All downstream firms produce the same consumer product, but each makes a differentiated version thereof. Downstream, there is differentiated price competition. Firms play a supermodular game with  $\text{sign}\{\frac{\partial \Pi_{Di}}{\partial P_j}\} = \text{sign}\{\frac{\partial Q_i}{\partial P_j}\} > 0$  for any  $i \neq j, i, j \in N$ . (This demand structure includes e.g. Shubik Levitan linear demand systems.)<sup>53</sup> Assumption A is maintained. And I assume downstream firms have “passive beliefs”.

In stage 2, downstream firms simultaneously set the price of their respective consumer product. Let  $\mathbf{P}$  denote the vector of downstream consumer prices. Downstream firm  $n \in N$  faces demand  $Q_n(\mathbf{P})$ . It has profit

$$\Pi_{Dn} = (P_n - \sum_{i=1}^K v_{in}) Q_n(\mathbf{P})$$

Its optimal price  $P_n^*$  is such that its unit dollar margin  $m_{Dn}^*$  equals  $-\frac{Q_n}{Q_n'}$  (see (1)).

In stage 1 upstream firms set their offer prices. For conciseness only, I present the following two cases. First, the case where suppliers supply multiple downstream firms, with  $2K$  firms upstream, two producers of each input  $i \in K$ , where the two producers of each input compete in their offer to each of the  $N$  downstream firms. Second, the case

<sup>53</sup>Consider a representative consumer with a Levitan Shubik quasilinear quadratic utility model (QQUM). Such utility functions induce linear demand and profit for downstream firm  $n \in N$ :

$$\Pi_{Dn} = (P_n - C_n)(\alpha_n - \beta_n P_n + \sum_{j \neq n}^N \beta_j P_j)$$

I explicitly model the upstream level, such that  $C_n = \sum_i v_{in}$ . Note: as can be gathered from Choné and Linnemer (2020), the Bertrand equilibrium price is linear in unit cost, such that pass-through is constant (and non-decreasing).

where each supplier supplies only one downstream firm, with  $2KN$  firms upstream,  $2N$  producers of each input, where two producers of each input compete in their offer to one (and only one) downstream firm. In other words, in the former case each upstream supplier supplies every downstream firm, in the latter case no upstream firm supplies competing downstream firms.

If the merging parties supply only one (and the same) downstream firm pre-merger, nothing changes compared to sections 2 and 3. Then Theorem 1, the test, and Theorem 2 continue to hold.

If supply chains overlap, supplier  $i \in K$  has profit (allowing for price discrimination across downstream firms)

$$\Pi_i = \sum_{n=1}^N (v_{in} - c_i) Q_n(\mathbf{P})$$

The optimal unconstrained price to downstream firm  $n$ , unit price  $v_{in}^{*u}$ , is given by the first order condition

$$\begin{aligned} \frac{\partial \Pi_i}{\partial v_{in}} &= Q_n + (v_{in} - c_i) \frac{\partial Q_n}{\partial v_{in}} + \sum_{j \neq n}^N (v_{ij} - c_i) \frac{\partial Q_j}{\partial v_{in}} \\ &= Q_n + (v_{in} - c_i) \frac{\partial Q_n}{\partial P_n} \frac{\partial P_n}{\partial v_{in}} + \sum_{j \neq n}^N (v_{ij} - c_i) \frac{\partial Q_j}{\partial P_n} \frac{\partial P_n}{\partial v_{in}} \\ &= Q_n + m_{in} Q_n' \rho_n + \rho_n \sum_{j \neq n}^N m_{ij} \frac{\partial Q_j}{\partial P_n} \end{aligned} \quad (7)$$

From (1) and (7) it directly follows that

$$\begin{aligned} m_{Dn}^* &= \rho_n \times m_{in}^{*u} + \frac{1}{Q_n} \rho_n \sum_{j \neq n}^N m_{ij} \frac{\partial Q_j}{\partial P_n} \\ &\Rightarrow m_{Dn}^* < \rho_n \times m_{in}^{*u} \\ &\Rightarrow m_{Dn}^* < m_{in}^{*u} \end{aligned}$$

because  $\frac{\partial Q_n}{\partial P_n} < 0$ ,  $\rho_n \in (0, 1]$ , and  $\sum_{j \neq n}^N \frac{\partial Q_j}{\partial P_n}$  is positive.

As in section 2, the upstream supplier's margin surpasses the downstream firm's margin because less than full pass-through by the downstream firm generates lower elasticity of demand upstream. However, a second effect now appears. A higher price by one downstream firm leads to higher quantities sold at other downstream firms. Consequently, and intuitively, a supplier faces a 'total demand elasticity' that is lower still. Therefore, when an upstream firm supplies several competing downstream firms, Theorem 1 provides a sufficient though no longer necessary condition. The same is true of Theorem 2. The test already provides a sufficient condition: it remains unaltered. <sup>54</sup>

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<sup>54</sup>If Apple and Samsung are the downstream firms of interest, Stackelberg price competition might better



## B.2 Workings for section 7

Here I deal with the robustness of results to pass-through rates in excess of 100%. I demonstrate that Assumption B suffices for Theorems 1 and 2 to hold. The three intermediate results used to prove the two theorems remain unaltered under Assumption B. To see this, consider each intermediate result in turn. First, at the merged entity's optimal unconstrained unit input price the result in (4) still holds. Such that

$$m_M^{*u} \times \rho = m_D^*$$

Second, the difference  $m_M \times \rho - m_D^*$  increases (strictly) monotonically in  $v_M$ . To see this: differentiating the difference with respect to  $v_M$  yields

$$\rho + m_M \frac{\partial \rho}{\partial v} - \frac{\partial P^*}{\partial v} + 1 = \rho + m_M \frac{\partial \rho}{\partial v} - \rho + 1 = 1 + m_M \frac{\partial \rho}{\partial v} > 0$$

as pass-through is non-decreasing (by Assumption B). It follows that  $v_M^{*u}$  is the unique price at which the difference equals zero. Third, the profit of a supplier is single-peaked (Assumption B). These three intermediary results are illustrated in Figure 2.

Now suppose pre-merger data shows that  $\rho(m_A^* + m_B^*) \leq m_D^*$ . Imagine the merged entity were to fix its total unit price for the two inputs at the level of pre-merger prices:  $v_A^* + v_B^*$ . Then the condition  $\rho m_M \leq m_D^*$  would of course be satisfied. This would thus indicate that at this unit price the merged entity would find it profitable to obtain a higher price. It would not find it profitable to decrease price (Theorem 1). If the merged entity has the ability to increase price, then whenever  $\rho(m_A^* + m_B^*) < m_D^*$  it would do so post-merger (Theorem 2). Hence under Assumption B the two theorems remain unchanged.

Finally, as an example of a log-convex demand function – i.e. one which induces pass-through in excess of one – consider the Pareto distribution. When consumer valuations are distributed according to this distribution, consumer demand is  $Q(P) = P^{-\beta}$  (for  $\beta > 1$ ).<sup>55</sup> The associated pass-through rate is

$$\rho = \frac{1}{2 - \frac{Q(P)Q''(P)}{Q'(P)^2}} = \frac{\beta}{\beta - 1}$$

This strictly exceeds 1 and is non-decreasing (it is constant). The Pareto demand func-

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fit their competitive interaction. When firms set prices sequentially, a Shubik Levitan linear demand system still yields  $\rho'_n = 0$  and a pass-through rate  $\in (0, 1)$ , such that results of sections 2 and 3 go through. On price leadership games, see e.g. Ono (1982) and Braid (1986).

<sup>55</sup>See the next footnote.

tion also induces a single-peaked profit functions downstream and upstream.<sup>56,57</sup> It thus satisfies Assumption B.

## B.3 Workings for section 8

### B.3.1 First price auctions – with endogenous quantity

Here I consider incomplete information and FPAs. The following provides workings for a setting where the results no longer hold, as well as workings for a setting where the results continue to hold.

I now construct a situation to show how and why my main results do not necessarily hold with incomplete information and FPAs. I use the setting in Hansen (1988) as the pre-merger situation. Then I add to it by modelling a merger, and discuss results.

Hansen (1988)'s setup has two producers of an input. Let that input be  $r \in K$ . The two producers have respective constant unit marginal cost  $c_i$  and  $c_j$ . Each firm's cost is i.i.d. on c.d.f  $F$  with continuous density  $f$  on  $[\underline{c}, \bar{c}]$ . The two firms have common knowledge about their shared cost distribution. Unit costs  $c_i$  and  $c_j$  are private information. Let firm  $j$  use bidding function  $b_j(c_j)$  with  $b'_j > 0$ , then firm  $i$  has expected profit from offering a unit price, i.e. from bid  $b_i$

$$\begin{aligned}\mathbb{E}(\Pi_i) &= \Pr(\text{win}|b_i)(b_i - c_i)Q(b_i) \\ &= [1 - F(b_j^{-1}(b_i))](b_i - c_i)Q(b_i)\end{aligned}$$

where  $Q(b_i)$  is shorthand notation for  $Q(P(b_i + \sum_{j \neq i}^K v_j))$ . Assuming both firms use the same bidding function  $b$ , and letting firm  $i$  choose its reported  $c$  yields

$$\mathbb{E}(\Pi_i) = [1 - F(c)](b(c) - c_i)Q(b(c))$$

Differentiating the expected profit with respect to  $c$  and rearranging yields the differential equation  $\frac{\partial b}{\partial c}$ . An equilibrium exists when the two producers choose ‘‘honest revelation’’,  $c = c_i$ . Pre-merger,

$$\left[\frac{\partial b}{\partial c}\right]_{pre} = \frac{f(c_i)[b(c_i) - c_i]Q(b(c_i))}{(1 - F(c_i))[Q(b(c_i)) + (b(c_i) - c_i)\rho Q'(P)]}$$

<sup>56</sup>Either one of the following two adjustments are necessary to avoid the problem of infinite demand as the price  $P$  approaches zero. A rightward shift of the Pareto function, of the form  $Q(P) = (P + \alpha)^{-\beta}$  for  $\alpha > 0$ , ensures demand does not approach infinity as the consumer price approaches zero, and ensures that profit is not maximised at a price close to zero. Alternatively, no change to the demand function is required if the supplier has a large enough unit cost – large enough such that it would not set a price close to zero. With either of those two adjustments, the supplier's profit function is indeed single-peaked in the unit price  $v$  it sets.

Demand  $Q(P) = (P + \alpha)^{-\beta}$  leads the downstream firm to set the optimal consumer price  $P^* = \frac{\beta}{\beta-1}v_M^*$ , while the merged entity has the optimal unconstrained unit price  $v_M^{*u} = \frac{\alpha}{\beta-1}$ .

<sup>57</sup>That profits are single-peaked is shown in the appendix, on the workings on the Pareto distribution.

The above is Hansen (1988)'s result with endogenous quantity.<sup>58</sup>

Let's now add to Hansen's result by analysing post-merger outcomes. Imagine the following situation. The downstream firm uses at least 3 different inputs,  $r, s, t \in K$ . The supplier of input  $s$  and the supplier of input  $t$  each have complete information about their respective cost advantage over the second-most efficient producer of  $s$  and  $t$ . Let the competitive advantages be equal:  $c_s^{[2]} - c_s = c_t^{[2]} - c_t > 0$ . Let both be constrained: the suppliers of  $s$  and  $t$  respectively bid and obtain optimal price  $v_s = c_s^{[2]}$  and  $v_t = c_t^{[2]}$  (with certainty). To retain post-merger symmetry, I consider two simultaneous mergers. One of the two symmetric producers of input  $r$  merges with the supplier of  $s$ , the other merges with the supplier of  $t$ .

Each merged entity bids to supply input  $r$ . Each also sells another input ( $s$  or  $t$  respectively) to the downstream firm. The expected profit of a merged entity  $i$  given its bid  $b_i$  for input  $r$  and bid  $v_s$  for the other input  $s$  is<sup>59</sup>

$$\begin{aligned}\mathbb{E}(\Pi_i) &= \Pr(\text{win}|b_i)(b_i - c_i + v_s - c_s)Q(b_i) \\ &\quad + [1 - \Pr(\text{win}|b_i)](v_s - c_s)\mathbb{E}(Q(b^{[1]})|b^{[1]} < b_i) \\ &= \Pr(\text{win}|b_i)(b_i - c_i)Q(b_i) + (v_s - c_s)\mathbb{E}(Q(b^{[1]})) \\ &= [1 - F(c)](b(c) - c_i)Q(b(c)) + (v_s - c_s)\mathbb{E}(Q(b^{[1]}))\end{aligned}$$

where again the last line assumes both firms use the same bidding function  $b$ , and lets firm  $i$  choose  $c$ . Note that  $\frac{\partial \mathbb{E}(Q(b^{[1]}))}{\partial b} < 0$ .

Suppose that pre-merger  $\rho \times (m_r^* + m_s^*) < m_D^*$ . Then by Theorem 1 the merged entity does not lower its price for input  $s$ . Because by Theorem 1

$$\left. \frac{\partial \mathbb{E}(\Pi_i)}{\partial v_s} \right|_{v_s < c_s^{[2]}} > 0$$

However the merged entity does change its bid for input  $r$ . Differentiating the expected profit with respect to  $c$  and rearranging to obtain the differential equation yields, at  $c = c_i$

$$\left[ \frac{\partial b}{\partial c} \right]_{post} = \frac{f(c_i)[b(c_i) - c_i]Q(b(c_i))}{(1 - F(c_i))[Q(b(c_i)) + (b(c_i) - c_i)\rho Q'(P)] + (v_s - c_s)\frac{\partial \mathbb{E}(Q(b^{[1]}))}{\partial b}}$$

This is identical to the pre-merger result, except for an additional negative term in the denominator. Hence

$$\left[ \frac{\partial b}{\partial c} \right]_{post} > \left[ \frac{\partial b}{\partial c} \right]_{pre}$$

<sup>58</sup>  $\rho Q'(P)$  in the present paper corresponds to  $\frac{\partial q}{\partial b}$  in Hansen (1988), p.48-9.

<sup>59</sup> Implicitly I assume that the merged entities do not offer (pure) bundles. But note that through pure bundling the merged entities risk foregoing the safe profit on input  $r$  and  $s$  respectively.

and (given  $b(\bar{c}) = \bar{c}$ )<sup>60</sup>

$$b(c_i)_{post} < b(c_i)_{pre}$$

The post-merger offer price  $b$  for input  $r$  is lower than the pre-merger bid for any given cost realisation  $c_i < \bar{c}$ .

This artificially constructed example nicely illustrates how the intuitions from section 2 and 3 are preserved or altered.

- Theorem 1 identifies whether each merged entity lowers the input price for, respectively, input  $s$  and  $t$ .
- For input  $r$ , the optimal offer price (bid) decreases post-merger, because of an additional effect. The effect pushing in the direction of a high bid is unchanged: (i) to earn a high margin on input  $r$  when winning. The two pre-merger effects pushing for a low bid – (ii) to win more frequently, and (iii) to sell a higher quantity of input  $r$  when winning – are now joined by a further effect: (iv) to increase the expected quantity sold of input  $s$  and  $t$  respectively when winning. Thus Hansen’s result combines with the Cournot effect to yield a post-merger price decrease. The price decrease occurs because of incomplete information coupled with the Cournot effect.
- In equilibrium, the probability to win and supply input  $r$  remains a half. But post-merger bids are more aggressive (lower), therefore the expected consumer price decreases. Thus the merger benefits consumers.

I mentioned that the test and the theorems can continue to hold. Now I illustrate this point, using an example in Maskin and Riley (2000) on asymmetric FPAs – to which I add endogenous demand. Let there be two suppliers of input  $i \in K$ , with

$$c_i \sim U[0, 1] \text{ and } c_i^{[2]} \sim U[2, 3]$$

The profit of the most efficient supplier from bidding unit price  $b_i$  is

$$\mathbb{E}(\Pi_i) = \Pr(\text{win}|b_i)(b_i - c_i^{[1]})Q(b_i)$$

(Recall  $Q(b_i)$  is shorthand for  $Q(P(b_i + \sum_{j \neq i}^K v_j)) = Q(b_i)$ .) We have  $\Pr(\text{win}|b_i \leq 2) = 1$ ,  $\Pi_i(2) = (2 - c_i)Q(2)$ , and  $\mathbb{E}(c_i) = 0.5$ . Maskin and Riley (2000) show it is an equilibrium for the most efficient firm to bid a unit price of 2, while the less efficient supplier submits a bid equal to its revealed cost.<sup>61</sup> Letting demand be endogenous renders the most efficient

<sup>60</sup>The steps in the argument to obtain this bid comparison based on the comparison of the bids’ derivatives pre- and post-merger are given in Hansen (1988), p.49.

<sup>61</sup>Following Maskin and Riley (2000): given a bid of 2 by the more efficient firm, the second-most efficient producer of input  $i$  may play the strategy of bidding its realised unit cost. Given this, the most efficient supplier finds it profitable to bid 2 than anything higher. (Thus there is a Nash equilibrium in bidding strategies.) To see this: the probability of winning when bidding  $v_i \in [2, 3]$  is  $3 - v_i$ . Thus, ignoring the endogenous demand for now, we have  $\Pi_i = (3 - v_i)(v_i - c_i)$ ,  $v_i^* = \frac{3+c_i}{2} \leq 2$ , which is strictly dominated by  $v_i^* = 2$ .

supplier (weakly) more aggressive: if its behaviour changed at all, the most efficient firm would submit a lower bid. But by Proposition 1, if at a price of 2 the supplier's margin is below that of the downstream firm there is no benefit to lower price in exchange for a higher quantity.

If the most efficient supplier of input  $i$  merges with the supplier of some other input  $j \in K$  where supplier  $j$  also bids in a manner so as to win with certainty, then the test and the theorems apply unaltered. Intuitively, if suppliers bid (weakly) below the point where they may lose, then incomplete information plays no role. Suppliers trade off an offer price with a deterministic quantity.

### B.3.2 Nash-in-Nash bargaining – with endogenous quantity

Here I provide algebraic workings for the merger effects discussed in section 8 with bargaining. Let suppliers be monopolists. Consider the Nash-in-Nash bargaining solution with endogenous demand.

In stage 1 the downstream firm bargains with all upstream producers simultaneously. Let  $\beta \in (0, 1)$  denote the downstream firm's bargaining power.<sup>62</sup> When the downstream firm bargains with the monopolist producer of input  $i$ , the Nash-in-Nash solution yields unit price

$$\begin{aligned} v_i^* &= \operatorname{argmax}_{v_i} \Pi_i^{1-\beta} \Pi_D^\beta \\ &= \operatorname{argmax}_{v_i} [v_i - c_i]^{1-\beta} \left[ P^* - \sum_{j=1}^K v_j \right]^\beta Q(P^*) \end{aligned} \quad (8)$$

(Recall:  $P^*$  is a function of input prices, including  $v_i$ .) The above shows: the Nash-in-Nash bargaining solution with endogenous quantity accounts for the increase in the size of the “pie” as a result of lower input prices. The first order condition yields

$$m_i^* = (1 - \beta) \left[ \frac{\beta(1 - \rho)}{m_D^*} - \rho \frac{Q'(P^*)}{Q(P^*)} \right]^{-1}$$

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<sup>62</sup>The level of the bargaining power is taken to be exogenous here. For empirical studies investigating determinants of the level of the bargaining power, see for example Haucap et al. (2013). Crawford et al. (2018) focus on merger effects and conduct an empirical study where, among others, the level of the bargaining parameters are also empirically estimated. Rubinstein (1982) offers a well-known theoretical explanation of the level of bargaining powers. (Some issues with this well-known model are discussed in Ponsati and Sakovics (1998).) Whatever the value of the parameter, experimental evidence suggests it is neither 1 nor 0 and instead strictly in  $(0, 1)$  (see e.g. Güth, Schmittberger, and Schwarze (1982) and Güth (1995)).

Plugging in (1) yields<sup>63</sup>

$$m_i^* = -\frac{Q(P^*)}{Q'(P^*)} \frac{1-\beta}{\rho + \beta(1-\rho)}$$

Let suppliers of inputs  $A, B \in K$  integrate and form the merged entity  $M$ . After the integration, the merged entity obtains unit price

$$v_M^* = \operatorname{argmax}_{v_M} \left[ v_M - c_A - c_B \right]^{1-\beta} \left[ P^* - \sum_{j \neq A, B}^K v_j - v_M \right]^\beta Q(P^*)$$

The first order condition is identical to (8), the sole difference is that post-merger there are  $K-1$  upstream players rather than  $K$ .

To illustrate the intuitions, I use the log concave demand function  $Q = (1-P)^\gamma$ . This demand function can be highly concave or highly convex. Using (8) to calculate equilibrium profits the merger is strictly profitable when<sup>64</sup>

$$\begin{aligned} \Pi_i(K-1) &> 2\Pi_i(K) \\ \Leftrightarrow \frac{(1-\beta)(1-c-\sum_i^K c_i)}{(K-1)(1-\beta)+\gamma+\beta} \left[ \frac{\gamma(\gamma+\beta)}{\gamma+1} \frac{1-c-\sum_i^K c_i}{(K-1)(1-\beta)+\gamma+\beta} \right]^\gamma &> 2 \frac{(1-\beta)(1-c-\sum_i^K c_i)}{K(1-\beta)+\gamma+\beta} \left[ \frac{\gamma(\gamma+\beta)}{\gamma+1} \frac{1-c-\sum_i^K c_i}{K(1-\beta)+\gamma+\beta} \right]^\gamma \\ \Leftrightarrow \left[ \frac{K(1-\beta)+\gamma+\beta}{(K-1)(1-\beta)+\gamma+\beta} \right]^{\gamma+1} &> 2 \end{aligned}$$

For small  $K$ , low  $\beta$ , and highly convex demand the merger can be profitable. Equal bargaining powers ( $\beta = 0.5$ ) suffices to render the merger unprofitable, as does large  $K$ . This is shown in the table below.

<sup>63</sup>Section 2 showed an unconstrained supplier earns a larger margin than the downstream firm. This is not the case here. Comparing the downstream margin to the unconstrained (monopoly) upstream margin yields  $m_D^* > m_i^*$  if

$$\begin{aligned} 1 &> \frac{1-\beta}{\rho + \beta(1-\rho)} \\ \Leftrightarrow \rho &> \frac{1-2\beta}{1-\beta} \end{aligned}$$

For the Laplace and Exponential distributions of consumers' willingness-to-pay, the pass-through rate  $\rho = 1$  (see Table 1). Hence for those distributions the downstream firm's margin exceeds any upstream margin, whatever  $\beta$ .

The comparison of pre-merger upstream and downstream margins is less insightful with bargaining. Not only does the pass-through rate play a role, but so does the level of the bargaining power – which may be unknown in practice.

<sup>64</sup>When  $P$  and thus  $Q$  are treated as fixed a merger of two suppliers is never profitable:

$$\begin{aligned} \Pi_i(K-1) &< 2\Pi_i(K) \\ \Leftrightarrow \frac{1}{K-1}(1-\beta)(P-c-\sum_i^K c_i) &< \frac{2}{K}(1-\beta)(P-c-\sum_i^K c_i) \\ \Leftrightarrow 0 &< (K-2)(1-\beta) + \beta \end{aligned}$$

The merger solely leads to lower input prices.

Table 2: Merger profitability with bargaining,  $Q = (1 - P)^\gamma$

$K$	10	10	10	3	3
$\beta$	0.5	0.5	0.01	0.5	0.1
$\gamma$	1	10	10	100	4
$\Pi_i(K - 1) - 2\Pi_i(K)$	$-0.8 < 0$	$-0.6 < 0$	$-0.2 < 0$	$-0.4 < 0$	$0.03 > 0$

## C. Demand functions

### C.1 Distributions and pass-through rates shown in Table 1

The pass-through rate is

$$\rho = \frac{1}{2 - \frac{Q(P)Q''(P)}{Q'(P)^2}}$$

An individual consumer's willingness-to-pay for the consumer good is given by  $x$ ,  $F(x)$  and  $f(x)$  denote, respectively, the cdf and pdf of consumers' willingness-to-pay. Then

$$Q(P) = 1 - F(P) \quad Q'(P) = -f(P) \quad Q''(P) = -f'(P)$$

To determine the sign of  $\rho'$  it helps to note that

$$\text{sign}\left\{\frac{\partial^2 P^*}{\partial C^2}\right\} = \text{sign}\left\{\frac{\partial \rho}{\partial C}\right\} = \text{sign}\left\{\frac{\partial \rho}{\partial P} \frac{\partial P}{\partial C}\right\} = \text{sign}\left\{\frac{\partial \rho}{\partial P}\right\}$$

For the log concavity of the distributions considered below, see Bagnoli and Bergstrom (2005). I now derive the pass-through rate  $\rho$  as well as  $\text{sign}\{\rho'\}$ .

The uniform distribution has support  $[0, 1]$  with  $F(x) = x$ . Hence

$$\begin{aligned} Q(P) &= 1 - P & Q'(P) &= -1 & Q''(P) &= 0 \\ \Rightarrow \rho &= \frac{1}{2} & \text{and } \rho' &= 0 \end{aligned}$$

The Gaussian distribution has support  $(-\infty, +\infty)$  with no closed form cdf. We can use the bounds on the Gaussian cdf for  $Q(P)$ , and the pdf  $\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$  for  $Q'(P)$ .

$$\begin{aligned} Q(P) &\in \left[ \frac{P}{P^2+1} \frac{1}{\sqrt{2\pi}} e^{-\frac{P^2}{2}}, \frac{1}{P} \frac{1}{\sqrt{2\pi}} e^{-\frac{P^2}{2}} \right] \\ Q'(P) &= -\frac{1}{\sqrt{2\pi}} e^{-\frac{P^2}{2}} \\ Q''(P) &= P \frac{1}{\sqrt{2\pi}} e^{-\frac{P^2}{2}} \\ Q'''(P) &= (1 - P^2) \frac{1}{\sqrt{2\pi}} e^{-\frac{P^2}{2}} \end{aligned}$$

We can then obtain  $\frac{Q(P)Q''(P)}{Q'(P)^2} \in [\frac{P^2}{P^2+1}, 1]$  such that

$$\rho \in [\frac{P^2+1}{P^2+2}, 1]$$

We can determine the sign of  $\rho'$  by determining the sign of the derivative of  $\frac{Q(P)Q''(P)}{Q'(P)^2}$ .

$$\begin{aligned} \text{sign}\{\rho'\} &= \text{sign}\left\{\frac{\partial[Q(P)Q''(P)Q'(P)^{-2}]}{\partial P}\right\} \\ &= \text{sign}\left\{\frac{Q'''(P)Q(P)}{Q'(P)^2} + \frac{Q''(P)Q'(P)}{Q'(P)^2} - 2\frac{Q''(P)^2Q(P)}{Q'(P)^3}\right\} \\ &= \text{sign}\left\{Q'''(P) + \frac{Q''(P)Q'(P)}{Q(P)} - 2\frac{Q''(P)^2}{Q'(P)}\right\} \end{aligned}$$

Using the lower bound<sup>65</sup> on  $Q(P)$  to verify whether the RHS  $\geq 0$  yields:

$$\begin{aligned} \text{RHS} &= (1-P^2)\frac{1}{\sqrt{2\pi}}e^{-\frac{P^2}{2}} - (P^2+1)\frac{1}{\sqrt{2\pi}}e^{-\frac{P^2}{2}} + 2P^2\frac{1}{\sqrt{2\pi}}e^{-\frac{P^2}{2}} \\ &= 0 \end{aligned}$$

Therefore

$$\rho' \geq 0$$

for the Gaussian distribution.

The Laplace distribution has support  $(-\infty, +\infty)$  with  $F(x) = 1 - \frac{1}{2}e^{-x}$  for  $x \geq 0$ .

$$\begin{aligned} Q(P) &= \frac{1}{2}e^{-P} \quad Q'(P) = -\frac{1}{2}e^{-P} \quad Q''(P) = \frac{1}{2}e^{-P} \\ \Rightarrow \rho &= 1 \quad \text{and} \quad \rho' = 0 \end{aligned}$$

The exponential distribution has support  $(0, \infty)$  with  $F(x) = 1 - e^{-\lambda x}$ .

$$\begin{aligned} Q(P) &= e^{-\lambda P} \quad Q'(P) = -\lambda e^{-\lambda P} \quad Q''(P) = \lambda^2 e^{-\lambda P} \\ \Rightarrow \rho &= 1 \quad \text{and} \quad \rho' = 0 \end{aligned}$$

The logistic distribution has support  $(-\infty, +\infty)$  with  $F(x) = \frac{1}{1+e^{-x}}$ .

$$\begin{aligned} Q(P) &= \frac{e^{-P}}{1+e^{-P}} \quad Q'(P) = -\frac{e^{-P}}{(1+e^{-P})^2} \quad Q''(P) = \frac{e^{-P}(e^{-P}-1)}{(1+e^{-P})^3} \\ \Rightarrow \rho &= \frac{1}{1+e^{-P}} \quad \text{and} \quad \rho' > 0 \end{aligned}$$

<sup>65</sup>I use the bounds  $\frac{1}{\sqrt{2\pi}} \frac{x}{x^2+1} \exp\{-\frac{x^2}{2}\} \leq 1 - F(x) \leq \frac{1}{\sqrt{2\pi}} \frac{1}{x} \exp\{-\frac{x^2}{2}\}$  for  $x \geq 0$ , from the textbook Pishro-Nik (2014).



The power function distribution has support  $(0, 1]$  with  $F(x) = x^\beta$ . With  $\beta > 1$

$$Q(P) = 1 - P^\beta \quad Q'(P) = -\beta P^{(\beta-1)} \quad Q''(P) = -\beta(\beta-1)P^{(\beta-2)}$$

$$\Rightarrow \rho = \frac{1}{1 + \frac{1}{\beta} + (1 - \frac{1}{\beta})\frac{1}{P^\beta}} \quad \text{and} \quad \rho' = 0$$

The inverse of the power function's reliability is log concave<sup>66</sup> with  $Q(P) = (1 - P)^\beta$ . With support  $[0, 1)$  and  $\beta > 0$

$$Q(P) = (1 - P)^\beta \quad Q'(P) = -\beta(1 - P)^{(\beta-1)} \quad Q''(P) = \beta(\beta-1)(1 - P)^{(\beta-2)}$$

$$\Rightarrow \rho = \frac{\beta}{\beta+1} \quad \text{and} \quad \rho' = 0$$

The extreme value distribution has support  $(-\infty, +\infty)$  with  $F(x) = e^{-e^{-x}}$ . With  $\beta > 1$

$$Q(P) = 1 - e^{-e^{-P}} \quad Q'(P) = e^{-P} e^{-e^{-P}} \quad Q''(P) = (e^{-P} - 1)e^{-P} e^{-e^{-P}}$$

$$\Rightarrow \rho = \left[ 1 - \frac{1}{e^{-P}} - \frac{1}{e^{-e^{-P}}} + \frac{1}{e^{-P} e^{-e^{-P}}} \right]^{-1} \quad \text{and} \quad \rho' > 0$$

where  $\rho$  was plotted graphically to determine that  $\rho' > 0$ .

The Weibull distribution has support  $[0, \infty)$  with  $F(x) = 1 - e^{-x^c}$  for  $x \geq 0$ . With  $c \geq 1$

$$Q(P) = e^{-P^c} \quad Q'(P) = -cP^{c-1} e^{-P^c} \quad Q''(P) = [cP^c - (c-1)]cP^{c-2} e^{-P^c}$$

$$\Rightarrow \rho = \frac{cP^c}{cP^c + c - 1} \quad \text{and} \quad \rho' > 0 \text{ if } c > 1, \text{ else } 0$$

With the form of the Weibull distribution given above, the Rayleigh distribution is a Weibull distribution with  $c = 2$ .

## C.2 Further permissible distributions

For the test and Theorem 1 to hold  $\rho' \geq 0$  is not necessary (it is not necessary for the demand function to induce non-increasing pass-through). The test holds for any strictly decreasing and weakly log concave demand function which induces a unique maximum upstream. (With  $\frac{\partial^2 \Pi_i^u}{\partial v_i^2} \Big|_{v_i=v_i^{*u}} < 0$ .) Theorem 1 requires, in addition, that the demand function induces increasing  $\rho \times m_i$  for  $v_i < v_i^{*u}$ .

First let's show  $\rho \times m_i^*$  is weakly increasing for any weakly log concave function:

$$\rho \cdot m_i^* = -\frac{Q'(P)}{Q(P)} \quad \text{from (3)}$$

$$\Rightarrow \frac{\partial(\rho \cdot m_i^*)}{\partial v_i} = -\frac{Q''(P)}{Q(P)} \rho + \frac{Q'(P)^2}{Q(P)^2} \rho$$

$$= [Q'(P)^2 - Q''(P)Q(P)] \frac{\rho}{Q(P)^2} \geq 0$$

<sup>66</sup>Three equivalent ways to confirm are the following. First,  $\ln Q(P) = \beta \ln(1 - P)$  with  $(\ln Q(P))'' = -\frac{\beta}{(1-P)^2} < 0$ . Second,  $Q(P)Q''(P) - Q'(P)^2 = -\beta(1-P)^{(\beta-2)} < 0$  for  $\beta > 0$ . Third,  $\rho \in (0, 1]$ .

There exist log concave demand functions with  $\rho' < 0$  which meet those weaker requirements. The limit case of log concave functions is one example:  $Q(P) = -\ln P$  (or any affine transformation thereof)<sup>67</sup> yields  $\rho = \frac{1}{2+\ln P}$ , with  $\rho' < 0$ , but still yields a unique upstream maximum.

Let's show the demand function  $Q(P) = -\ln(P)$  for  $0 \leq P \leq 1$  yields a unique equilibrium upstream. From the proof of Proposition 1 we have the second order condition

$$\left. \frac{\partial^2 \Pi_i^u}{\partial v_i^2} \right|_{v_i=v_i^{*u}} = [2Q'(P)^2 - Q(P)Q''(P)] \frac{\rho}{Q'(P)} - Q(P) \frac{\rho'}{\rho} < 0$$

Plugging in  $Q(P) = -\ln(P)$ ,  $Q'(P) = -\frac{1}{P}$ ,  $Q''(P) = \frac{1}{P^2}$ ,  $\rho = \frac{1}{2+\ln P}$ ,  $\rho' = -\frac{1}{P(2+\ln P)^2}$  and simplifying yields

$$\left. \frac{\partial^2 \Pi_i^u}{\partial v_i^2} \right|_{v_i=v_i^{*u}} = -\frac{1}{P} - \frac{P \ln P}{(2+\ln P)^2}$$

Whilst the second derivative can be positive for  $P \geq 0$  it is always negative for  $P \geq P^*(0) = e^{-1}$ .<sup>68</sup> Thus  $Q(P) = -\ln P$  is a log concave demand function with decreasing pass-through which nonetheless satisfies the requirements for Theorem 1 and the test to hold.

Furthermore, while Bagnoli and Bergstrom (2005) conclude the power function for  $\beta \in (0, 1)$  has a mixed reliability this is unproblematic for the present paper: the power function for  $\beta \in (0, 1)$  is log concave on the relevant price range  $P \geq P^*(C=0)$ .<sup>69</sup> While  $\rho' < 0$  for  $\beta \in (0, 1)$ , this demand function also induces a unique maximum upstream.

Let's show that the power demand function  $1 - P^\beta$  for  $\beta \in (0, 1)$  induces a unique upstream equilibrium. From the proof of Proposition 1 we have the second order condition

$$\left. \frac{\partial^2 \Pi_i^u}{\partial v_i^2} \right|_{v_i=v_i^{*u}} = [2Q'(P)^2 - Q(P)Q''(P)] \frac{\rho}{Q'(P)} - Q(P) \frac{\rho'}{\rho} < 0$$

Plugging in

$$\begin{aligned} Q(P) &= 1 - P^\beta & Q'(P) &= -\beta P^{\beta-1} & Q''(P) &= -\beta(\beta-1)P^{\beta-2} \\ \Rightarrow \rho &= \frac{1}{1+\frac{1}{\beta}+(1-\frac{1}{\beta})\frac{1}{P^\beta}} & \text{and } \rho' &= (\beta-1)P^{-\beta-1}\rho^3 \end{aligned}$$

<sup>67</sup>See Lemma 3 of Bagnoli and Bergstrom (2005) on affine transformations of log concave functions.

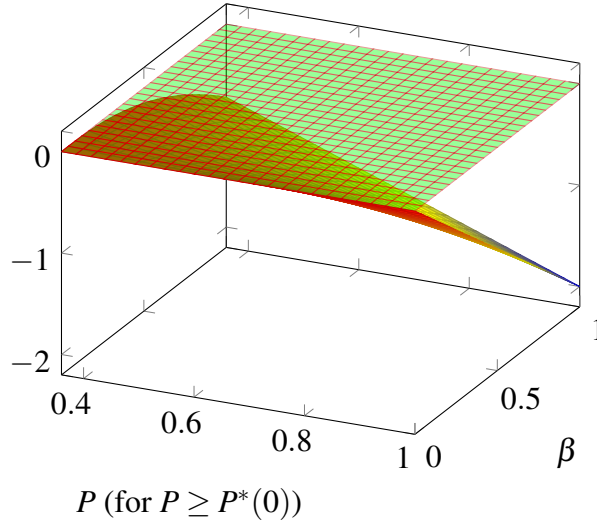
<sup>68</sup>This can be verified graphically by plotting  $-\frac{1}{P} - \frac{P \ln P}{(2+\ln P)^2}$  on  $e^{-1} \leq P \leq 1$ .

<sup>69</sup>The log concavity condition  $Q''(P)Q(P) \leq Q'(P)^2$  yields  $1 - \gamma < P^\gamma$ , which only holds for some  $P \in [0, 1]$  – hence the mixed reliability. The minimum optimal price a firm would set is  $P^*(C=0)$ . Maximising profit  $PQ(P)$  yields  $P^{*\gamma} = \frac{1}{1+\gamma}$ . This exceeds  $1 - \gamma$ , such that the optimal price is always in the log concave range of the demand curve, i.e. where the pass-through rate is inferior to 100%.

and simplifying yields

$$\text{sign}\left\{\frac{\partial^2 \Pi_i^u}{\partial v_i^2}\bigg|_{v_i=v_i^{*u}}\right\} = -\text{sign}\left\{(1+\beta)P^\beta - (1-\beta) - \frac{\beta(1-\beta)(1-P^\beta)}{(1+\beta)P^\beta - (1-\beta)}\right\}$$

where the right hand side is negative for all  $P \geq P^*(0) = \left(\frac{1}{1+\beta}\right)^{\frac{1}{\beta}}$  – this has been verified with Mathematica, the right hand side is also depicted as the surface below the green  $z = 0$  plane in the graph below.



Therefore the power function for  $\beta \in (0, 1)$  is another log concave demand function with decreasing pass-through which nonetheless satisfies the requirements for Theorem 1 and the test to hold.

### C.3 Deferred workings on the Pareto distribution

Here I show that the Pareto distribution induces single-peaked demands for the downstream firm and its suppliers. (While I use  $Q(P) = P^{-\beta}$  to show this, do note footnote 57.)

Let's evaluate the second-order condition of the downstream firm.

$$\begin{aligned}\Pi_D &= (P - \sum_i v_i)P^{-\beta} \\ \frac{\partial \Pi_D}{\partial P} &= P^{-\beta} - \beta(P - \sum_i v_i)P^{-\beta-1} \\ \frac{\partial^2 \Pi_D}{\partial P^2} &= -2\beta P^{-\beta-1} + \beta(\beta + 1)(P - \sum_i v_i)P^{-\beta-2} \\ \text{sign}\left\{\frac{\partial^2 \Pi_D}{\partial P^2}\right\}\bigg|_{P=P^*} &= \text{sign}\left\{-2\beta P^* + \beta(\beta + 1)(P - \sum_i v_i)\right\}\end{aligned}$$

Using

$$P^* = \beta(P^* - \sum v_i)$$

to evaluate the sign of the second derivative yields:

$$\begin{aligned} \text{sign}\left\{\frac{\partial^2 \Pi_D}{\partial P^2}\right\}\Big|_{P=P^*} &= \text{sign}\{-2\beta P^* + (\beta + 1)P^*\} \\ &= \text{sign}\{1 - \beta\} \\ &< 0 \quad \text{as } \beta > 1 \end{aligned}$$

Now let's evaluate the second-order condition of a supplier.

$$\begin{aligned} \Pi_i &= (v_i - c_i)P^{-\beta} \\ \frac{\partial \Pi_i}{\partial v_i} &= P^{-\beta} - \beta(v_i - c_i)P^{-\beta-1} \frac{\partial P^*}{\partial v_i} \\ \frac{\partial^2 \Pi_i}{\partial v_i^2} &= -2\beta P^{-\beta-1} \rho + \beta(\beta + 1)(v_i - c_i)P^{-\beta-2} \rho^2 \\ \text{sign}\left\{\frac{\partial^2 \Pi_i}{\partial v_i^2}\right\}\Big|_{v_i=v_i^{*u}} &= \text{sign}\left\{-2P^* + (\beta + 1)(v_i^{*u} - c_i)\rho\right\} \end{aligned}$$

Using the pass-through rate  $\rho = \frac{\beta}{\beta-1}$  and the optimal unconstrained margin

$$v_i^{*u} - c_i = \frac{\beta - 1}{\beta^2} P^*$$

to evaluate the sign of the second derivative yields:

$$\begin{aligned} \text{sign}\left\{\frac{\partial^2 \Pi_i}{\partial v_i^2}\right\}\Big|_{v_i=v_i^{*u}} &= \text{sign}\left\{-2P^* + (\beta + 1)\frac{\beta - 1}{\beta^2} P^* \frac{\beta}{\beta - 1}\right\} \\ &= \text{sign}\left\{-2 + \frac{\beta + 1}{\beta}\right\} \\ &= \text{sign}\{1 - \beta\} \\ &< 0 \quad \text{as } \beta > 1 \end{aligned}$$

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