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Responsive state-dependent or habitual state-independent congestion pricing under dynamic congestion

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Responsive state-dependent or habitual state-independent
congestion pricing under dynamic congestion

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Abstract

In the face of capacity disruptions (due, for example, to traffic incidents or poor weather), information provision and congestion pricing are alternative alleviating policies. A state-dependent toll equals the state-dependent marginal external cost (MEC), which is higher if traffic condition is in a bad state. This raises efficiency and thus welfare, but it may also be even more unpopular with the populace than state-independent tolling. We study this using dynamic bottleneck congestion with an uncertain capacity that can have two states: high or low. We consider two congestion pricing regimes: responsive state-dependent congestion pricing and habitual state-independent pricing, and three information provision regimes: no information, perfect information and imperfect information. We find that, without information provision, the habitual toll equals the expected MEC. With information provision, this is a weighted average of the MEC over all states; with weights depending on the capacity distribution, the price sensitivity of demand, the values of schedule delay and the quality of the information. Responsive pricing leads to higher welfare and a lower expected price than habitual pricing, but in our numerical model the differences tend to be small. When only one policy is implemented, information provision and congestion pricing both raise welfare. Information provision is preferable when uncertainty is high, as information is more valuable at this time.

Keywords: Uncertainty; Bottleneck congestion; Information provision; Responsive pricing; State-dependent pricing

JEL codes: R41; R48; D62; D80

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1. Introduction

Traffic congestion is a major problem around the world, especially at peak hours. The amount of congestion is uncertain due to such factors as accidents, rain, snow and demand fluctuations. Modern technology can reduce this uncertainty by providing information on traffic conditions. Congestion pricing can also improve matters by pricing congestion externalities. Tolls for most roads are usually constant or state-independent. This form of pricing can be called habitual pricing. If traffic conditions vary unpredictably, an alternative is to adjust prices according to known traffic conditions. Following Arnott et al. (1991) this will be called responsive pricing, which is state-dependent.

In face of traffic congestion, state-dependent ‘responsive tolls’ can improve the situation further: if traffic is bad, marginal external costs (MECs) are high and a responsive toll equals this higher externality (e.g., Daniel, 1995). This makes the system more efficient and raises welfare. However, such state-dependent tolls can be even more unpopular with the public than state-independent ‘habitual tolls’ and hence even harder to sell politically. For example, it means that if it is raining and traffic is heavy, you will also have to pay more in tolls if you want to drive.

Hence, the questions this paper considers are as follows: What is the welfare loss from using habitual state-independent tolls? Are state-dependent tolls worth the hassle? How well off are consumers in the two settings? One might expect that the efficiency gain from responsive state-dependent tolling means that the expected cost and travel price will be lower. Therefore, there exists a trade-off between the efficiency gains of responsive state-dependent pricing and the simplicity and predictability of habitual state-independent pricing. We also study what happens if we do not charge a toll and only supply information. What are the effects on welfare and travelers? What role does information provision play under uncertainty and dynamic congestion? Finally, we consider imperfect information—as defined in Arnott et al. (1991)—whereby information reduces uncertainty but does not eliminate it.

To do this, we use the Vickrey (1969) bottleneck model with the capacity being uncertain and having a binominal distribution with two possible states: high and low. The bottleneck model has been widely recognized as an important tool for modeling dynamic congestion.¹ Researchers have proposed various tolling regimes to reduce the congestion externality in dynamic congestion, such as a fully time-varying toll (e.g., Arnott et al., 1993; Yang and Huang, 1997; Van den Berg and Verhoef, 2011a, b); a step toll (Arnott et al., 1990; Laih, 1994, 2004; Lindsey et al., 2012; Van den Berg, 2012), and a time-independent flat toll (Xiao et al., 2012; Silva et al., 2014; Fu et al., 2018).

When traffic conditions are uncertain, departure time adaption appears to be one of

¹ For comprehensive reviews of the bottleneck model, please see Small (2015) and Li et al. (2020).

the most important behavioral changes in attempting to arrive at work on time and to reduce the probability of arriving late (e.g., Siu and Lo, 2013; Li et al., 2016). Various stochastic bottleneck models have been proposed in order to investigate travelers' travel behavior and regulators' optimal tolling under uncertainty (e.g., Arnott et al., 1991, 1996, 1999; Li et al., 2007; Fosgerau, 2008; Guo et al., 2018; Zhu et al., 2019). Studies have found that with a stochastic bottleneck system, uncertainty will lead to significant changes in departure time patterns and tolling designs. For instance, it will cause travelers to depart earlier and spread departure flows over a longer time period (e.g., Li et al., 2007; Xiao et al., 2015); it will also lead time-varying tolling to spread traffic more evenly over time, such that the queue length can be significantly reduced (e.g., Daniel, 1995; Lindsey, 1999; Yao et al., 2010; Xiao et al., 2015; Jiang et al., 2021). Nevertheless, to the best of our knowledge, little is known about the role of responsive state-dependent pricing and habitual state-independent pricing under dynamic congestion. In particular, it is widely demonstrated in the mode- and route-choice literature that uncertainty has considerable impacts on habitual decision making (e.g., Pinjari and Bhat, 2006; Liu et al., 2007; Small, 2015). Furthermore, innovative technologies now allow tolls to be varied dynamically and methods for developing and analyzing responsive pricing strategies must be applied.

With respect to the effects of information provision, there have been numerous efforts during the last decade to study various aspects of information provision in isolation, including the evaluation of its impacts on travelers and on the transport system; see Chorus et al. (2006) and de Palma et al. (2012) for reviews. A majority of the studies concluded that information is likely to be socially beneficial, although some identified conditions under which adverse responses occur (e.g., Arnott et al., 1996, 1999; Lo and Szeto, 2004; Lindsey et al., 2014; Rapoport et al., 2014; Liu and Liu, 2018; Zhu et al., 2019; Han et al., 2021; Yu et al., 2021). Recently, Engelson and Fosgerau (2020) established relationships between the accuracy of the travel time information and its value for an individual traveler. They found that a signal always increases the expected utility compared to the situation without any signal. Ye et al. (2021) investigated how the advanced traveler information affects the stability of the day-to-day flow evolution of a transportation system, and found that changing publishing information from historical information to predicted information will change the system stability. However, these studies mainly focus on information provision in isolation, such that they only consider that the regulator aims at disseminating travel information to the public but do not explore the congestion pricing issue when information is provided to travelers.

There is a large body of literature on uncertainty and static congestion that also suggests the need to consider information provision and road pricing (e.g., Emmerink et al., 1996; Verhoef et al., 1996; de Palma and Lindsey, 1998; Yang, 1999; Gardner et

al., 2011) as measures to reduce congestion and uncertainty. The work of Verhoef et al. (1996) is most closely related to our paper: using static congestion and uncertain cost functions for two parallel roads. This study is, however, different from Verhoef et al. (1996) in the following major aspects. Obviously, we use a different and dynamic congestion technology, but there are also differences in the results: the second-best habitual toll rule is markedly different and numerically we see that information provision tends to lower the expected travel price and habitual pricing tends to raise it. When combined with information provision, habitual state-independent pricing leads to a lower expected toll and a higher travel price than responsive state-dependent pricing. Second, as perfect and accurate information is difficult to achieve in reality, we have studied imperfect information, which reduces uncertainty but does not remove it. Verhoef et al. (1996) only consider regimes of no information and perfect information.

In addition to very pertinent policy questions, this paper also has strong methodological contributions to make to the literature on uncertainty and the bottleneck model. First, a joint departure time choice and tolling design model is presented. It enriches congestion pricing studies by including price-sensitive demand and proposing responsive state-dependent pricing and habitual state-independent pricing to deal with uncertainty. Thus, we move beyond prior studies that rely on state-independent time-varying tolling with fixed demand. Second, we derive what the ‘second-best’ habitual toll would be. Without information provision, a habitual toll will equal the expected MEC; with perfect information, it equals the weighted average of the state i -dependent MEC $_i$ ’s with weights being a function of the price sensitivity of demand, the values of schedule delay and the capacity distribution. Although travelers are assumed to be risk-neutral, the equilibrium travel demand still tends to fall with the degree of uncertainty, as the expected travel price is raised by the habitual toll. With imperfect information, the habitual toll equals the weighted average of the expected MEC under each signal, with the weights depending on the probability distribution of the capacity, and the expected MEC under a signal depends on the conditional probability distribution of the capacity. Finally, this paper enriches existing information studies by exploring the various interactions between information provision and congestion pricing. Using analytical and numerical methods, we analyze the welfare and price effects under different combinations of road pricing and information provision regulations.

The remainder of this paper is organized as follows. The next section describes the basic components of the model. Section 3 proposes a joint departure time choice and tolling design model and derives the optimal toll under different regimes. Section 4 presents a numerical model and evaluates the welfare and price effects under zero information and perfect information. Section 5 considers the quality of information and investigates imperfect information. Section 6 concludes the paper.

For ease of reference, Table 1 below summarizes the notations used throughout the paper. The notation will also be introduced in the text.

Table 1. Notation list.

Parameters	Definition
t^*	Preferred arrival time
s	Capacity of the bottleneck
π	Probability of low capacity
s_H	High capacity state
s_L	Low capacity state
σ	Bottleneck capacity degradation rate
t	Arrival time
$T_i(t)$	Queuing time at arrival time t under capacity state i , $i = H, L$
α	Value of time
β	Value of schedule delay early
γ	Value of schedule delay late
δ	Compound preference parameter, with $\delta = \beta\gamma/(\beta + \gamma)$
$c_i(t)$	Travel cost at arrival time t
$E[c(t)]$	Expected travel cost with respect to arrival time t
$E[c]$	Expected travel cost in the equilibrium without information
N	Total number of travelers
D	Inverse demand function
d_0	Intercept of the inverse demand function
d_1	The inverse of the slope of the inverse demand function
$E[W]$	Expected welfare
τ	Optimal habitual tolling
$E[MEC]$	Expected marginal external cost
N_i	Travel demand under capacity state i
c_i	Equilibrium travel cost under state i
W_i	Welfare under state i
p_i	Equilibrium travel price under state i
$E[p]$	Expected travel price in the equilibrium
MEC_i	Marginal external cost under capacity state i
λ	The weight of MEC_L under habitual pricing with information
ω	Relative efficiency
Q	Quality of information
η	Signal of the capacity
$\pi(\eta)$	Conditional probability of low capacity

2. Model description

Suppose that every morning, travelers travel from home to the workplace along a single road that has a bottleneck. Everyone wishes to arrive at their workplace at an identical preferred arrival time, t^* . Without loss of generality, the free-flow travel time is normalized to zero. Thus, a traveler arrives at the bottleneck immediately after leaving home and arrives at work immediately after leaving the bottleneck. When the arrival rate of the bottleneck exceeds the bottleneck's capacity, a queue develops. Those who arrive early or late encounter a schedule delay cost.

Unlike the deterministic bottleneck model, in this paper, we consider a bivariate distribution of capacity reflecting the effects of uncertainty on users' travel cost. The capacity of the bottleneck, s , has two states: high capacity (H) and low capacity (L), varying from day to day because of bad weather, road accidents, etc. The probability of low capacity is denoted by π , which may range from 0 to 1 and describes the frequency of bottleneck capacity reductions. The capacity of the bottleneck thus follows:

$$s = \begin{cases} s_H, & \text{with probability } 1-\pi \\ s_L = \sigma s_H < s_H, & \text{with probability } \pi \end{cases} \quad (1)$$

where s_H and s_L are bottleneck capacities in good and bad traffic conditions, respectively, and σ ($0 \leq \sigma \leq 1$) is the bottleneck capacity degradation rate, indicating the severity of capacity reductions. The above bivariate distribution of bottleneck capacity was also adopted by Arnott et al. (1991) and Yu et al. (2021).

We consider homogeneous travelers. Travelers are assumed to be risk-neutral and to have rational expectations with respect to travel costs (e.g., Emmerink et al., 1998; Arnott et al., 1999; Zhang and Verhoef, 2006). Their perception of the distribution of capacity realizations are assumed to be accurate. All travelers are deemed to be rational self-optimizers and to seek to minimize their travel price, defined as the sum of the travel cost and any tolls charged by the regulator.

For now, we consider two extreme information regimes: no information and perfect information. Later, we will also consider imperfect information, whereby the degree of information can range continuously from none to perfect. All travelers either have no information or they are fully informed. It is assumed that information is provided at zero cost. With perfect information, travelers are fully informed of the travel conditions before traveling. They choose their departure times to minimize the actual travel price under each capacity realization. The travel behavior and tolling thus depend on the actual capacity realization. Without information provision, travelers only know the probability distribution of the capacity and have no access to the actual capacity realization before departing. As a result, they minimize the expected travel price, based on the expected travel cost and tolls.

We now formulate the travel cost under different information regimes. With perfect information, the travel cost reduces to the deterministic case under the corresponding capacity realized on that day. Let $c_i(t)$ denote the travel cost for travelers arriving at t under capacity state i , with $i = H, L$. The travel cost consists of two parts. The first part is the queuing time cost associated with waiting at the bottleneck, which equals the product of the value of time, denoted by α , and queuing time under capacity state i , denoted by $T_i(t)$. The second part is the schedule delay cost. For travelers arriving early, the cost is the product of how early they arrive, measured by $t - t^*$, and the schedule delay value of arriving early, denoted by β . The schedule delay cost is defined similarly for people arriving late, where γ denotes the schedule delay value of arriving late. In sum, the travel cost for arriving at t , $c_i(t)$ is:

$$c_i(t) = \alpha \cdot T_i(t) + \begin{cases} \beta(t^* - t), & \text{if } t \leq t^* \\ \gamma(t - t^*), & \text{if } t > t^* \end{cases} \quad i = H, L, \quad (2)$$

where the first term measures the queuing time cost and the second measures the schedule delay cost.

Without information provision, the travel cost functions are replaced with a single function $E[c(t)]$, representing the expectation over all capacity realizations:

$$E[c(t)] = \alpha \cdot E[T(t)] + \begin{cases} E[\beta(t^* - t)], & \text{if } t < t^* \\ E[\gamma(t - t^*)], & \text{if } t \geq t^* \end{cases} \quad (3)$$

where $E[\cdot]$ is the expectation operator.

Empirical research shows that transport demand varies with the travel price. Hence, we consider price-sensitive demand, such that, if the travel price falls, more travelers will travel and this increases congestion. To keep the presentation of results manageable and tractable, we have kept the model simple by assuming that demand is linear over the relevant ranges. The analysis can be easily extended to other elastic demand functions.

This section compares two pricing regimes: habitual state-independent pricing and responsive state-dependent pricing. As we are focusing on whether and how tolls should vary over actual traffic conditions, both regimes investigate a time-independent toll. Time-varying tolling is not considered in this paper but will be explored in a subsequent study.

The regulator aims at improving social welfare through setting tolls. In the habitual pricing case, the regulator sets a flat toll that does not vary over the realization of capacity. In the responsive pricing case, the regulator varies the flat toll in response to capacity realizations. The social welfare measure is total benefits, represented by the

area below the demand curve, minus total cost.²

The two information regimes and two congestion pricing regimes result in five regulation scenarios: information provision only, habitual pricing only, responsive pricing only, the combination of information provision and habitual pricing, and the combination of information provision and responsive pricing. The travelers' and the regulator's decision objectives under these regulation regimes are summarized in Table 2. The next section provides more precise definitions of traveler and regulator behavior. We also compare the results with an unregulated case, in which neither information provision nor congestion pricing exists.

Note that when only implementing responsive pricing and not providing information, travelers receive no information on the actual capacity realization, neither is there any value in varying the toll accordingly as travelers cannot adapt their travel choice to the toll on that day. Thus, the travel demand will be constant from day to day, regardless of any toll variation attempting to account for capacity uncertainty; this scenario is a special case of habitual pricing only and can be solved as such. In the following, we ignore this case.

Table 2. Decision objectives in different regulation regimes.

Regulation regime	Travelers	Regulator
Habitual pricing only	Minimize expected travel price	Maximize expected welfare
Responsive pricing only	Minimize expected travel price	Maximize expected welfare
Information provision only	Minimize actual travel price	Maximize actual welfare
Information provision & Habitual pricing	Minimize actual travel price	Maximize expected welfare
Information provision & Responsive pricing	Minimize actual travel price	Maximize actual welfare

3. Theoretical results

In this section, we study the travel equilibrium and optimal tolling under various regulation regimes. The properties of the model solutions are also analyzed and

² With perfect information, the welfare under capacity state i , W_i , is consumer surplus minus the total travel cost, where the consumer surplus is measured by the area below the demand curve. Without information, the expected welfare, $E[W]$, is the expected consumer surplus minus the expected total travel cost. For the exact equations per case, see below:

$$W_i = \int_0^{N_i} D(n)dn - N_i \cdot C_i(N_i), \quad E[W] = \int_0^N D(n)dn - E[c] \cdot N.$$

compared. In particular, when proposing a regulation policy for society, a major concern is how travelers' travel price will be affected because this has a strong impact on social and political feasibility. A policy is likely to meet resistance from travelers if they are made worse off. It is also important to conduct a full assessment and compare the welfare effects of different regulations before implementation. In the following analysis, we further investigate the effects of different regulation regimes on travelers' travel price and welfare.

3.1 Habitual pricing only and thus no information provision

We first investigate the case of habitual pricing only. For this case, information is not available, and we seek a uniform toll that does not vary over the actual capacity realization. Travelers minimize the expected travel price by choosing departure times and whether to travel based on the expected travel cost and the expected toll. The regulator maximizes expected welfare by setting a uniform toll.

According to Arnott et al. (1991) and Xiao et al. (2015), there are three departure time intervals to consider: travelers always arrive early; depending on the distribution of capacity, travelers may arrive early or late; and travelers always arrive late. Specifically, when $\pi = 0$, capacity is high with certainty: the last traveler arrives late but never encounters a queue. As π increases, the last traveler faces a growing probability of a longer trip and a greater degree of lateness. To keep equilibrium, departure times shift earlier, including that of the last traveler, which partially offsets that traveler's increase in costs. This continues until the last traveler is departing at t^* . Further increases in π do not shift the last departure any earlier. The last traveler thus bears the full brunt of the increase in expected schedule delay cost, and the expected travel cost rises more rapidly with π . As π continues to increase, however, departures begin ever earlier, the increase in cost suffered by the last traveler when low capacity occurs becomes smaller, and the expected travel cost flattens out.

Following the conventions, we use the following compound preference parameter: $\delta = \beta\gamma/(\beta + \gamma)$. To facilitate the presentation, we introduce the following definitions: $\pi_1 = \beta\sigma/((\alpha + \gamma)(1 - \sigma))$ and $\pi_2 = \gamma/(\alpha + \gamma)$. The expected travel cost in the equilibrium, $E[c]$, is (Arnott et al., 1991):

$$E[c] = \begin{cases} \delta \left(1 - \frac{(1 - \pi)(\alpha + \gamma) - \alpha}{\gamma} (1 - \sigma) \right) \frac{N}{s_L}, & \text{if } 0 \leq \pi < \pi_1 \\ \frac{\beta(\alpha + \gamma)\pi}{\beta + (\alpha + \gamma)\pi} \frac{N}{s_L}, & \text{if } \pi_1 \leq \pi < \pi_2 \\ \frac{\delta N}{s_L}, & \text{if } \pi_2 \leq \pi \leq 1 \end{cases}, \quad (4)$$

where N is the total number of travelers. The expected travel price is the sum of the expected travel cost and the habitual toll.

The inverse demand function, $D(N)$, which measures the willingness to pay in terms of the travel price, is assumed to be linear:³

$$D(N) = d_0 - d_1 \cdot N, \quad (5)$$

where d_0 is travelers' maximum willingness to pay, and d_1 the slope. In user equilibrium, the inverse demand equals the expected travel price.

The expected welfare, $E[W]$, is the consumer benefits minus the total expected travel cost. The associated welfare-maximization problem is:

$$\max_{N, \tau} E[W] = \int_0^N D(n)dn - E[c] \cdot N, \quad (6)$$

$$s.t. \quad D(N) = E[c] + \tau, \quad (7)$$

where τ is the toll under habitual pricing that is independent of the state. Eq. (7) is the user equilibrium condition: travelers' willingness to pay equals the travel price, which is the sum of the travel cost and the toll.

Proposition 1a. *With habitual pricing only, the optimal toll:*

- (i) *equals the expected marginal external cost of travelers;*
- (ii) *is a linear increasing function of π if $\pi \leq \pi_1$, is an increasing and concave function of π if $\pi_1 < \pi < \pi_2$, and is independent of π if the probability of low capacity exceeds π_2 ;*
- (iii) *is between $\delta N/s_H$ and $\delta N/s_L$. Specifically, when $\pi = 0$, the optimal toll is $\delta N/s_H$; when $\pi = 1$, the optimal toll is $\delta N/s_L$.*

Proof. Substituting Eq. (4) into Eqs. (6)-(7) and solving the first-order conditions yields the following optimal toll:

$$\tau = \begin{cases} \delta \left(1 - \frac{(1-\pi)(\alpha+\gamma)-\alpha}{\gamma} (1-\sigma) \right) \frac{N}{s_L}, & \text{if } 0 \leq \pi < \pi_1 \\ \frac{\beta(\alpha+\gamma)\pi}{\beta + (\alpha+\gamma)\pi} \frac{N}{s_L}, & \text{if } \pi_1 \leq \pi < \pi_2 \\ \frac{\delta N}{s_L}, & \text{if } \pi_2 \leq \pi \leq 1 \end{cases}, \quad (8)$$

implying that the optimal toll equals the expected travel cost of travelers.

According to Eq. (4), travelers' expected marginal external cost, $E[MEC]$, is:

³ The use of linear functions is sufficient to serve the goal of the current paper: enhancing our insight into the welfare economic effects of information provision and congestion pricing on a dynamic congestion model.

$$E[MEC] = E\left[\frac{dc}{dN} \cdot N\right] = \frac{dE[c]}{dN} \cdot N = E[c]. \quad (9)$$

Hence, the optimal toll in Eq. (8) equals travelers' expected marginal external cost.

Taking the derivate of Eq. (8) with respect to π yields:

$$\frac{\partial \tau}{\partial \pi} = \begin{cases} \frac{\delta \cdot (\alpha + \gamma)}{\gamma} (1 - \sigma) \frac{N}{s_L} & \text{if } 0 \leq \pi < \pi_1 \\ \frac{\beta^2 (\alpha + \gamma)}{(\beta + (\alpha + \gamma)\pi)^2} \frac{N}{s_L} & \text{if } \pi_1 \leq \pi < \pi_2 \\ 0 & \text{if } \pi_2 \leq \pi \leq 1 \end{cases} \quad (10)$$

Combining Eq. (8) and Eq. (10), one can obtain the properties in Proposition 1a(ii) and 1a(iii). This completes the proof of Proposition 1a. ■

Proposition 1b. *With habitual pricing only, the number of users N :*

(i) *is non-increasing with π , i.e., $\frac{\partial N}{\partial \pi} \leq 0$;*

(ii) $\frac{d_0 s_L}{2\beta\sigma + d_1 s_L} \leq N \leq \frac{d_0 s_H}{\delta + d_1 s_H}$ when $0 \leq \pi \leq \pi_1$; $\frac{d_0 s_L}{\delta + d_1 s_L} \leq N \leq \frac{d_0 s_L}{2\beta\sigma + d_1 s_L}$ when $\pi_1 \leq \pi \leq \pi_2$; $N = \frac{d_0 s_L}{2\delta + d_1 s_L}$ when $\pi_2 \leq \pi \leq 1$.

Proof. Substituting Eq. (4) and Eq. (5) into Eq. (7) yields the number of travelers:

$$N = \begin{cases} \frac{d_0 \gamma s_L}{2\delta\pi(1 - \sigma)(\alpha + \gamma) + \gamma(2\delta\sigma + d_1 s_L)}, & \text{if } 0 \leq \pi \leq \pi_1 \\ \frac{d_0 (\beta + (\alpha + \gamma)\pi) s_L}{(2\beta + d_1 s_L)(\alpha + \gamma)\pi + \beta d_1 s_L}, & \text{if } \pi_1 \leq \pi \leq \pi_2 \\ \frac{d_0 s_L}{2\delta + d_1 s_L}, & \text{if } \pi_2 \leq \pi \leq 1 \end{cases} \quad (11)$$

To investigate how uncertainty affects travel demand, we take the derivative of Eq. (11) with respect to π and find $\partial N / \partial \pi \leq 0$ always holds. This completes the proof of Proposition 1b (i). $\partial N / \partial \pi \leq 0$ indicates that the travel demand decreases with the probability of low capacity. We can then further obtain the lower and upper bounds of the travel demand, which provide valuable information for the regulator in controlling traffic congestion, as presented in Proposition 1b (ii). ■

Combining Eqs. (4)-(6) and Eq. (11), we can find the expression of the expected

welfare:

$$E[W] = \begin{cases} \frac{d_0^2 \gamma s_L}{4\delta\gamma(\pi(1-\sigma) + \sigma) + 4\alpha\delta\pi(1-\sigma) + 2d_1\gamma s_L}, & \text{if } 0 \leq \pi < \pi_1 \\ \frac{d_0^2 (\beta + (\alpha + \gamma)\pi) s_L}{2(2\beta\gamma\pi + \beta d_1 s_L + d_1 \gamma \pi s_L + \alpha\pi(2\beta + d_1 s_L))}, & \text{if } \pi_1 \leq \pi < \pi_2 \\ \frac{d_0^2 s_L}{4\delta + 2d_1 s_L}, & \text{if } \pi_2 \leq \pi \leq 1 \end{cases} \quad (12)$$

Taking the derivative of Eq. (12) with respect to π , we can find $\partial E[W]/\partial \pi \leq 0$ always holds, meaning that the expected welfare is also non-increasing with π . Specifically, when $\gamma/(\alpha + \gamma) \leq \pi \leq 1$, $E[W]$ is independent of the probability of low capacity.

To further explore the role of habitual pricing under uncertainty, we compare the results with the no congestion pricing case. When congestion is not priced, the travel equilibrium can be directly obtained by equalizing the inverse demand and the travel cost, where the travel cost is again given in Eq. (4). We mainly focus on the effects of habitual pricing on welfare, travel demand and travelers' travel price. The findings are summarized in the following proposition.

Proposition 1c. *Compared to an unregulated regime, the introduction of habitual pricing will: (i) lower the number of travelers; (ii) raise the travel price for travelers; and (iii) improve welfare.*

Proof of Proposition 1c: See Appendix A.

3.2 Information provision only and thus no tolling

In equilibrium, no one can reduce the travel cost by altering the departure time. Hence, travel cost is constant over the rush hour, i.e., $dc_i(t)/dt = 0$. Considering price-sensitive demand, when capacity drops, some users may decide to use an alternative mode, to stay home or to go to a different destination. Therefore, in equilibrium, the travel demand may consist of two states:

$$N = \begin{cases} N_H, & \text{when } s = s_H \\ N_L, & \text{when } s = s_L \end{cases}, \quad (13)$$

where N_H and N_L are the number of travelers in good and bad conditions, respectively.

The equilibrium travel cost in each state i , c_i , equals the travel cost when the capacity is deterministic at s_i :

$$c_i = \frac{\delta N_i}{s_i}, \text{ with } \delta = \frac{\beta\gamma}{\beta + \gamma}, \quad (14)$$

implying that equilibrium is independent of the capacity in the other state.

Solving the associated user equilibrium $D(N_i) = c_i + \tau_i$ yields the following number of travelers:

$$N_H = \frac{d_0 s_H}{\delta + d_1 s_H}, \quad N_L = \frac{d_0 s_L}{\delta + d_1 s_L}. \quad (15)$$

The welfare under each capacity realization is $W_i = \int_0^{N_i} D(n)dn - c_i \cdot N_i$.

Combining Eq. (5) and Eq. (14)-(15), we can derive the expression of the actual welfare with information provision:

$$W_H = \frac{d_0^2 d_1 s_H^2}{2(\delta + d_1 s_H)^2}, \quad W_L = \frac{d_0^2 d_1 s_L^2}{2(\delta + d_1 s_L)^2}. \quad (16)$$

In order to analyze the effects of information provision, we again compare it to the unregulated regime in Proposition 2.

Proposition 2. *Introducing information will:*

- (i) *raise the number of users when the state of the bottleneck capacity is high, and lower the number of users when the state of the bottleneck capacity is low;*
- (ii) *lower the travel cost when the state of the bottleneck capacity is high, and raise the travel cost when the state of the bottleneck is low;*
- (iii) *improve the welfare when the state of the bottleneck capacity is high, and decrease the welfare when the state of the bottleneck capacity is low.*

Proof of Proposition 2. See Appendix B.

3.3 Information provision and habitual pricing

With perfect information and habitual pricing, travelers decide their travel behavior based on the actual price rather than the price expected. Hence, the travel demand again consists of two states, N_H and N_L . In setting the habitual toll, which is independent of the actual capacity, the regulator can do no better than to maximize expected welfare. The regulator finds the optimal toll, τ , by maximizing the expected welfare subject to user equilibrium condition based on the actual capacity realization. In this situation, the expected welfare, $E[W]$, is the probability-weighted average of the actual welfare over all capacity states. That is, we maximize:

$$\max_{N_H, N_L, \tau} E[W] = \left((1-\pi) \cdot \int_0^{N_H} D(n)dn + \pi \cdot \int_0^{N_L} D(n)dn \right) - ((1-\pi) \cdot c_H \cdot N_H + \pi \cdot c_L \cdot N_L), \quad (17)$$

$$\begin{aligned} s.t. \quad & D(N_H) - c_H - \tau = 0 \\ & D(N_L) - c_L - \tau = 0 \end{aligned} \quad (18)$$

where c_H and c_L are given in Eq. (14), and for ease of presentation the inverse demand function is assumed to be $D(N_i) = d_0 - d_1 N_i$. The first term of Eq. (17) is the expected consumer benefits, and the last term is the expected total travel cost; that is, the average cost over all states.

Proposition 3. *Under habitual pricing with perfect information,*

(i) *the optimal toll is the weighted average of the marginal external costs over all states, with weight λ depending on the capacity distribution, the demand sensitivity parameter d_1 and the schedule delay parameter δ :*

$$\tau = \lambda \cdot MEC_L + (1 - \lambda) \cdot MEC_H, \quad (19)$$

with $MEC_i = \delta N_i / s_i$, and

$$\lambda = \frac{\pi \cdot \left(d_1 + \frac{\delta}{s_H} \right)}{(1 - \pi) \left(d_1 + \frac{\delta}{\sigma s_H} \right) + \pi \left(d_1 + \frac{\delta}{s_H} \right)}; \quad (20)$$

(ii) *the weight λ satisfies: $\frac{\partial \lambda}{\partial \pi} > 0$, $\frac{\partial \lambda}{\partial \sigma} \leq 0$, and $\frac{\partial \lambda}{\partial d_1} \geq 0$.*

Proof of Proposition 3. See Appendix C.

Proposition 3 reveals that the weights are, in the first place, positively related to the probability of bad traffic conditions. Therefore, as π increases, the weight of MEC_L increases and the weight of MEC_H decreases. Specifically, when $\pi = 0$, the weight of MEC_L is zero and thus the optimal toll equals MEC_H . In contrast, when $\pi = 1$, the optimal toll equals MEC_L . In addition, the weights are positively related to the slope of the inverse demand function: the flatter the demand curves, the smaller the weight of MEC_L .

Proposition 4a. *Compared to information provision only, the combination of habitual pricing and information provision: (i) lowers the number of users; (ii) raises the travel price.*

Proof of Proposition 4a. With the combination of information provision and habitual pricing, the number of travelers under each state is:

$$N_H = \frac{d_0 s_H}{(2\delta + d_1 s_H) + \frac{d_1 \delta \lambda(s_H - s_L)}{(\delta + d_1 s_L)}}, N_L = \frac{d_0 s_L}{(2\delta + d_1 s_H)(\delta + d_1 s_L) + \frac{d_1 \delta \lambda(s_H - s_L)}{(\delta + d_1 s_H)}}. \quad (21)$$

The travel demand under information provision only is given in Eq. (15). Specifically, for the high-capacity case, because $(2\delta + d_1 s_H) \geq (\delta + d_1 s_H)$ and $\frac{d_1 \delta \lambda(s_H - s_L)}{(\delta + d_1 s_L)} \geq 0$,

we can obtain that $\frac{d_0 s_H}{(2\delta + d_1 s_H) + \frac{d_1 \delta \lambda(s_H - s_L)}{(\delta + d_1 s_L)}} \leq \frac{d_0 s_H}{\delta + d_1 s_H}$ always holds. Similarly,

for the low-capacity case, because $(2\delta + d_1 s_H) \geq (\delta + d_1 s_H)$ and $\frac{d_1 \delta \lambda(s_H - s_L)}{(\delta + d_1 s_L)} \geq 0$,

we find $\frac{d_0 s_L}{(2\delta + d_1 s_H)(\delta + d_1 s_L) + \frac{d_1 \delta \lambda(s_H - s_L)}{(\delta + d_1 s_H)}} \leq \frac{d_0 s_L}{\delta + d_1 s_L}$ holds.

Therefore, compared to information only, the combination of information and habitual pricing lowers the travel demand. As a result of the user equilibrium conditions, the travel price is raised. ■

Proposition 4b. *Compared to habitual pricing only, the combination of habitual pricing and information provision: (i) raises the number of travelers when the capacity state is high, and lowers the number of travelers when the capacity state is low; (ii) lowers the travel price when the capacity state is high, and lowers the travel price when the capacity state is low.*

To save space, no proof similar to Proposition 4a was made for Proposition 4b as the proofs are highly similar. Details are available upon request from the authors.

3.4 Information provision and responsive pricing

Responsive pricing means that the regulator can adjust tolls based on the actual capacity realization, enabling the regulator to use pricing as a response mechanism for managing traffic. The optimal tolls follow from maximizing social welfare under actual capacity subject to the user equilibrium constraint based on perfect information. That is, we maximize:

$$\begin{aligned} \max_{N_H, N_L, \tau_H, \tau_L} \quad & W_i = \int_0^{N_i} D(n) dn - c_i \cdot N_i \\ \text{s.t.} \quad & D(N_H) - c_H - \tau_H = 0, \\ & D(N_L) - c_L - \tau_L = 0 \end{aligned} \quad (22)$$

where W_i is the welfare under state i , and τ_i is the responsive toll under state i .

Solving the first-order condition of problem (20) gives the following tolls:

$$\tau_H = \frac{\delta N_H}{s_H}, \quad \tau_L = \frac{\delta N_L}{s_L}, \quad (23)$$

implying that under perfect information provision, the toll equals the MEC under the actual capacity realization.

Proposition 5. *Compared to information only, the combination of information provision and responsive pricing tends to: (i) lower the travel demand; and (ii) raise the travel price.*

Proof of Proposition 5. Solving the first-order condition of problem (22) yields:

$$N_H = \frac{d_0 s_H}{2\delta + d_1 s_H}, \quad N_L = \frac{d_0 s_L}{2\delta + d_1 s_L}. \quad (24)$$

$$\tau_i = \frac{\delta N_i}{s_i}, \quad W_i = \frac{d_0^2 s_i}{4\delta + 2d_1 s_i}. \quad (25)$$

The travel price under each state, i , is thus:

$$p_i = \frac{2\delta N_i}{s_i} = \frac{2\delta d_0}{2\delta + d_1 s_i}. \quad (26)$$

Comparing Eq. (24) and Eq. (21), because $2\delta + d_1 s_i \geq \delta + d_1 s_i$ always holds, we can find the introduction of responsive pricing lowers the travel demand. From Eq. (14) and Eq. (15), under information only, the travel price is $\delta d_0 / (\delta + d_1 s_i)$. As a result of $2\delta d_0 / (2\delta + d_1 s_i) \geq \delta d_0 / (\delta + d_1 s_i)$, we can derive that the introduction of responsive pricing raises the travel price. ■

Proposition 6. *When combined with information provision, compared to habitual pricing, responsive pricing tends to: (i) raise the travel demand when the capacity is high and lower the travel demand when the capacity is low; and (ii) lower the travel price when the capacity is high and raise the travel price when the capacity is low.*

Proof of Proposition 6. (i). Comparing Eq. (24) and Eq. (21), when the capacity is

high, $\frac{d_0 s_H}{(2\delta + d_1 s_H) + \frac{d_1 \delta \lambda (s_H - s_L)}{(\delta + d_1 s_L)}} \leq \frac{d_0 s_H}{(2\delta + d_1 s_H)}$ always holds. This implies that the

travel demand under responsive pricing exceeds the travel demand under habitual pricing. In contrast, when the capacity is low, according to Eq. (21), because $\lambda \leq 1$, we have:

$$\begin{aligned} \frac{(2\delta + d_1 s_H)(\delta + d_1 s_L)}{(\delta + d_1 s_H)} + \frac{d_1 \delta \lambda (s_H - s_L)}{(\delta + d_1 s_H)} &\leq \frac{(2\delta + d_1 s_H)(\delta + d_1 s_L)}{(\delta + d_1 s_H)} + \frac{d_1 \delta (s_H - s_L)}{(\delta + d_1 s_H)} \\ &= \frac{(2\delta + d_1 s_L)(\delta + d_1 s_H)}{(\delta + d_1 s_H)} = 2\delta + d_1 s_L \end{aligned} \quad (27)$$

Hence,

$$\frac{\frac{d_0 s_L}{(2\delta + d_1 s_H)(\delta + d_1 s_L)} + \frac{d_1 \delta \lambda (s_H - s_L)}{(\delta + d_1 s_H)}}{\frac{d_0 s_L}{2\delta + d_1 s_L}} \geq \frac{d_0 s_L}{2\delta + d_1 s_L}. \quad (28)$$

This implies that, when the capacity state is low, the travel demand under responsive pricing is lower than the travel demand under habitual pricing.

(ii) In equilibrium, inverse demand equals the travel price. Hence, a higher travel demand corresponds to a lower travel price, as presented in Proposition 6(ii). ■

In sum, for controlling dynamic congestion under uncertainty, information provision and congestion pricing generate different travel equilibria for travelers. As a result, information provision and congestion pricing also have different implications for welfare that are interesting for both policy makers and regulators. It is difficult to analyze the relationship between welfare effects under different regimes, due to the complicated expressions of the welfare functions, especially for comparisons with and without information provision. We use simulation to examine them in the next section. We also examine the expected travel prices and optimal tolling.

4. Numerical analysis

In the previous sections, the closed-form solutions and solution properties were obtained. In this section, we present numerical results for the proposed models. The results are also compared with an unregulated regime in which there are no congestion pricing and no information provision.

For the parameters, we set the value of time, the value of schedule delay early, and the value of schedule delay late as $\alpha = 10$, $\beta = 6.09$ and $\gamma = 23.77$, respectively. This ensures that the ratios between the cost parameters are as in Small (1982); following Van den Berg and Verhoef (2011b), the value of time is updated so that it is close to the current official Dutch value in order to work with current values⁴. In the base case, we set $\sigma = 0.8$ and $\pi = 0.2$. For comparability, the expected capacity is 3,600 cars per hour. This means that when traffic condition is good, the capacity of the bottleneck is $s_H = 3750$; when it is bad, the capacity is $s_L = 3000$. The inverse demand function is constructed to ensure that under an unregulated regime with high

⁴ The currency used in this paper is Euro.

capacity, the price elasticity of demand is -0.35 and the number of users is 9,000. Therefore, $d_0 = 44.88$ and $d_1 = 0.0037$ are chosen.

As we consider various combinations of information provision and congestion pricing regimes, the welfare implications of different regulations will be studied by relative efficiency, ω , which is a performance indicator and measures the relative welfare improvement of a certain policy. The relative efficiency ω is:

$$\begin{aligned}\omega &= \frac{\text{Welfare Gain (policy considered)}}{\text{Welfare Gain (Responsive pricing \& information provision)}} \\ &= \frac{SW(\text{policy considered}) - SW(\text{no information \& no pricing})}{SW(\text{information \& responsive pricing}) - SW(\text{no information \& no pricing})},\end{aligned}\quad (29)$$

where ‘policy considered’ can refer to information only, habitual pricing only, information and habitual pricing. The relative efficiency indicates the welfare gain from a non-optimal policy as a proportion of the highest achievable welfare gain. By definition, ω cannot exceed 1.

Below, we first present the numerical results under the base case and study the effects of uncertainty by varying the probability distribution of capacity. The optimal tolling under different regimes is presented in Section 4.4. Section 4.5 investigates the effects of price elasticity.

4.1 Base case equilibrium

The numerical results are given in detail in Table 3. For each regime, we compute the actual travel demand under each state N_i , the expected travel demand $E[N]$, the actual travel cost under each state c_i , the expected travel cost $E[c]$, the optimal responsive toll τ_i , the optimal habitual toll τ , the travel price under each state p_i , the expected travel price $E[p]$, the welfare under each state W_i , the expected welfare $E[W]$, and the relative efficiency ω . To reveal the relationship between the marginal external cost and the optimal toll, we also present the marginal external cost MEC_i , and the expected marginal external cost, $E[MEC_i]$.

Compared to the unregulated regime, the introduction of information provision lowers the expected travel price for travelers, and the introduction of habitual pricing raises it. This is because information provision allows travelers to make decisions on whether to travel and departure time choices based on the actual capacity realization, thereby eliminating uncertainty from the traffic conditions. In contrast, habitual pricing reduces congestion by charging an additional congestion fee. As a result, although habitual pricing leads to a lower expected travel cost, the expected travel price is still higher than that under information provision.

Table 3. Outcomes for the base calibration.

	No information		Perfect information		
	No pricing	Habitual pricing	No pricing	Habitual pricing	Responsive pricing
N_L	-	-	8,776	7,065	6,875
N_H	-	-	9,290	7,478	7,527
$E[N]$	9,138	7,330	9,187	7,395	7,397
c_L	-	-	12.41	9.99	9.72
c_H	-	-	10.51	8.46	8.52
$E[c]$	11.07	8.88	10.89	8.77	8.76
p_L	-	-	12.41	18.74	19.44
p_H	-	-	10.51	17.21	17.03
$E[p]$	11.07	17.76	10.89	17.52	17.51
τ_L	0	-	0	-	9.72
τ_H	0	-	0	-	8.52
$E[\tau]$	0	8.88	0	8.75	8.76
MEC_L	-	-	12.41	9.99	9.72
MEC_H	-	-	10.51	8.46	8.52
$E[MEC]$	11.07	8.88		8.77	8.76
W_L	-	-	142,482	-	154,276
W_H	-	-	159,650	-	168,911
$E[W]$	154,469	164,477	156,217	165,954	165,984
ω	0	0.869	0.152	0.997	1

Under habitual pricing, the introduction of information lowers the toll from 8.88 to 8.75. Indeed, without information provision, fewer users will choose to travel when the actual state is high, and more users will choose to travel when the actual state is low. The resulting welfare losses thus make it worthwhile to set a higher toll. This indicates that information provision reduces the loss caused by uncertainty.

For the comparison of habitual pricing and responsive pricing, we find that when combined with information, the habitual toll is 8.75, which is higher than the responsive toll under a high-capacity state (8.52) and lower than the responsive toll of a low-capacity state (9.72). According to our theoretical results, this is because the responsive toll equals the MEC under each state, whereas a habitual toll is a weighted average of the marginal external costs. Due to the higher toll, the travel price under responsive pricing is slightly lower than that under habitual pricing.

It can be observed that compared to an unregulated regime, all regulations are welfare-improving. In particular, responsive pricing performs best since the toll can be adjusted responsively according to the actual capacity realization, which helps lower the welfare loss from uncertainty and congestion. When implementing regulations in isolation, habitual pricing performs better than information provision.

4.2 Uncertainty and welfare effects

What happens to the welfare effects of different regimes when the degree of uncertainty changes? It is important to carry out a full assessment and compare the effects before policies are implemented. In the following subsections, we first present the numerical results of the welfare effects by varying the ratio of low capacity to high capacity s_L/s_H from 0 to 1, and then by varying the probability of low capacity π from 0 to 1.

4.2.1 Varying s_L/s_H

We compare the results under $\pi = 0.2$ and $\pi = 0.8$. As we keep the average capacity at 3600, a larger s_L/s_H raises the capacity under bad traffic conditions and lowers the capacity under good traffic conditions.

Fig. 1 shows that information provision only may perform better or worse than habitual pricing only in improving welfare. This is because information is more valuable when uncertainty is high and congestion pricing is more efficient when more travelers are on the road. Hence, a lower s_L/s_H brings more uncertainty and makes information provision more efficient. When s_L/s_H exceeds a certain value, habitual pricing performs better.

As s_L/s_H increases, the relative efficiency of information provision decreases, and that of habitual pricing increases. Specifically, when $s_L/s_H = 0$, information provision is at its most efficient, and the relative efficiency is 1. When $s_L/s_H = 1$, information provision is completely useless and the relative efficiency is 0. For habitual pricing only, an increase in s_L/s_H lowers the expected travel cost and raises the expected travel demand, thereby improving the relative efficiency. Specifically, when $s_L/s_H = 0$, the expected travel cost becomes infinite and the expected travel demand is zero. Habitual pricing thus becomes useless. When $s_L/s_H = 1$, habitual pricing has the same performance as responsive pricing.

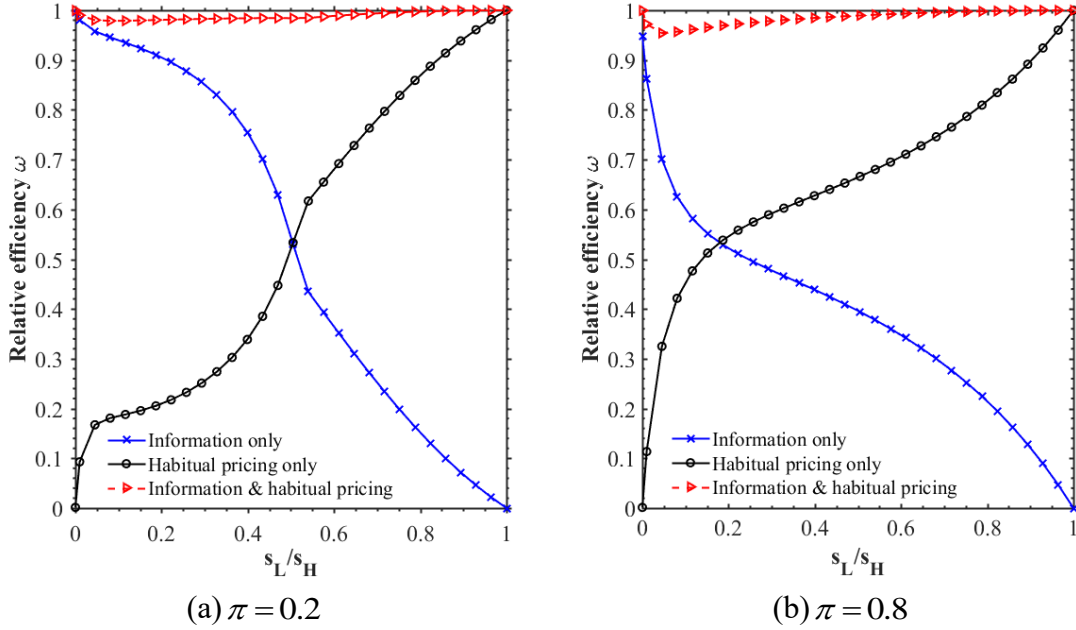


Fig. 1. s_L/s_H and welfare effects.

Note: Relative efficiency measures the relative welfare improvement of a certain policy compared to the combination of information provision and responsive pricing.

For the combination of information provision and habitual pricing, as s_L/s_H increases, the relative efficiency first decreases and then increases. This is caused by the interaction between information provision and habitual pricing: the former tends to lower efficiency and the latter tends to raise efficiency. The results indicate that when s_L/s_H is low, the effects of information dominate and when s_L/s_H exceeds a certain value, the effects of habitual pricing dominate.

Comparing Fig. 1(a) and Fig. 1(b), we find that the relative efficiency changes at a faster rate under $\pi = 0.8$ than under $\pi = 0.2$. This implies that uncertainty accelerates changes in relative efficiency. In addition, when combined with information

provision, habitual pricing and responsive pricing show very similar performance in terms of the welfare gain when s_L/s_H exceeds 0.6. However, when s_L/s_H is small, responsive pricing performs better than habitual pricing. Hence, responsive pricing is preferred when the capacity drop is large.

Fig. 2 plots the percentage change in welfare difference between habitual and responsive pricing. Fig. 2 confirms that responsive pricing is slightly more efficient in improving welfare than habitual pricing. As s_L/s_H increases, the welfare difference increases initially and then decreases. Specifically, when $s_L/s_H = 0$ and $s_L/s_H = 1$, habitual pricing and responsive pricing have the same welfare.

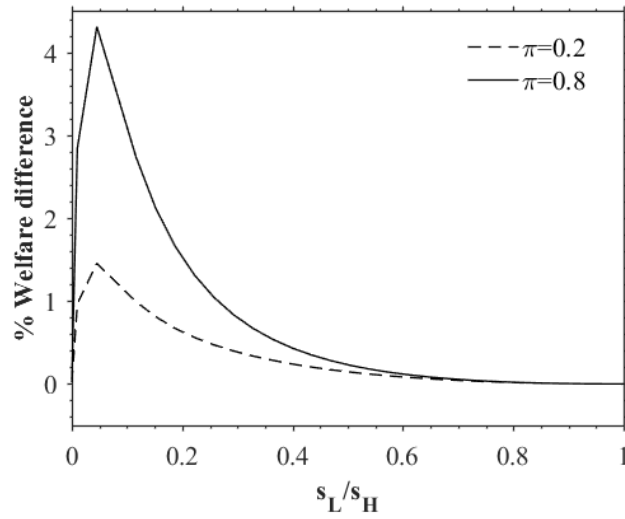


Fig. 2. s_L/s_H and % welfare difference between habitual and responsive pricing.

Note: % welfare difference is measured by $100 \cdot (SW_{res} - SW_{hab}) / SW_{hab}$.

4.2.2 Varying π

We compare the results under $\sigma=0.2$ and $\sigma=0.8$. Fig. 3 shows that information provision and habitual pricing again show opposite patterns. Specifically, as π increases, the relative efficiency of information provision increases initially and then decreases, whereas the relative efficiency of habitual pricing decreases first and then increases. This indicates that information provision and habitual pricing are highly complementary instruments. When $\pi=0$ or $\pi=1$, the relative efficiency of habitual pricing is 1, implying that habitual pricing is as efficient as responsive pricing if the capacity applies with certainty.

Comparing Fig. 3(a) and Fig. 3(b), we find that when the distribution of capacity change is small, the combination of information provision and habitual pricing yields an expected welfare that is almost as high as responsive pricing. Nevertheless, when the distribution of capacity change is large, habitual pricing becomes less efficient owing to the increasingly important shortcomings of fee adaptation. This indicates that

the advantage of responsive pricing is more obvious under a higher degree of uncertainty.

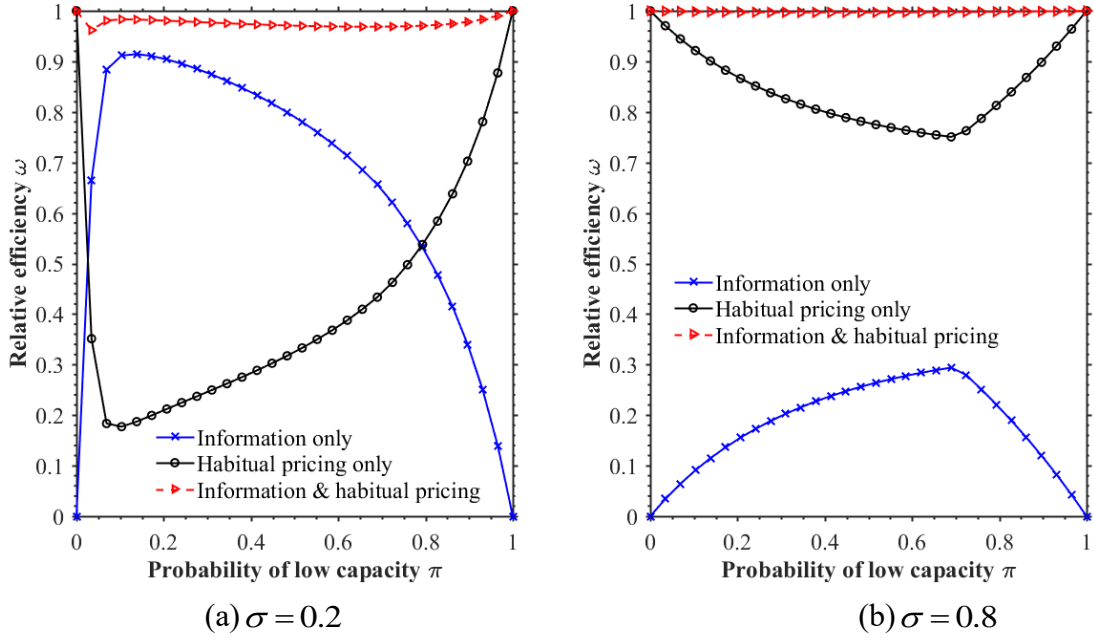


Fig. 3. π and welfare effects.

Note: Relative efficiency measures the relative welfare improvement of a certain policy compared to the combination of information provision and responsive pricing.

Fig. 4 plots the percentage change in welfare difference between habitual and responsive pricing. Fig. 4 shows that as π increases, the welfare difference first increases and then decreases. Specifically, when capacity is deterministic, i.e., $\pi = 0$ and $\pi = 1$, responsive pricing and habitual pricing result in the same welfare.

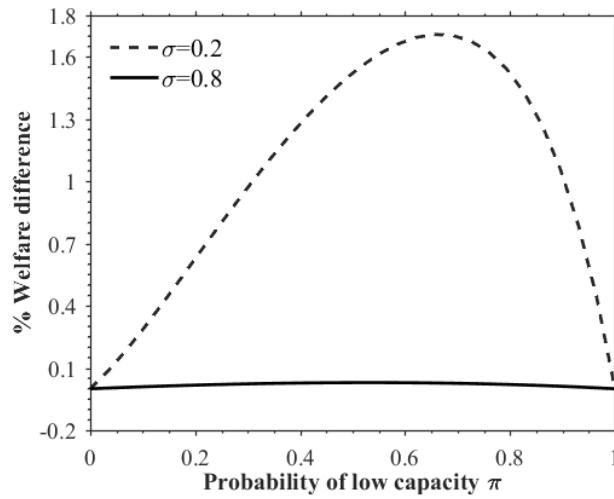


Fig. 4. s_L/s_H and % welfare difference between habitual and responsive pricing.

Note: % welfare difference is measured by $100 \cdot (SW_{res} - SW_{hab}) / SW_{hab}$.

4.3 Uncertainty and price effects

In addition to social welfare concerns, policy makers and regulators are also interested in price effects. Section 4.3.1 investigates the impact of habitual pricing and responsive pricing on travel price under perfect information.

4.3.1 Information provision vs habitual pricing

Fig. 5 shows that information provision tends to lower the expected travel price for travelers, because travelers can change their travel decision according to the actual capacity realization. In contrast, habitual pricing tends to raise the expected travel price for travelers, since they need to pay a congestion toll. This implies that habitual pricing is likely to meet resistance from travelers unless the allocation of toll revenues convinces them otherwise.

Second, with information only, the change in the expected travel price decreases with s_L/s_H , because information is more valuable when the capacity fluctuation is large. Nevertheless, under habitual pricing only, the change in the expected travel price does not vary monotonically, as it first increases and then slightly decreases. When s_L/s_H reaches a certain value, information provision does not make sense. Habitual pricing and the combination of information and habitual pricing have the same expected price.

Third, with information only, the change in the expected price does not vary monotonically with π , as it first decreases and then increases. This is because when π exceeds $\gamma/(\alpha + \gamma)$, the expected travel cost does not vary with π , which lowers the role of information. Specifically, when $\pi = 0$ and $\pi = 1$, information provision is completely useless, and has the same expected travel price as the unregulated case. In contrast, the change in the expected price change of habitual pricing only increases monotonically with π , and increases more slowly when π exceeds $\gamma/(\alpha + \gamma)$. The reason is that a higher probability of low capacity raises the travel cost and the toll by exacerbating the traffic congestion.

Finally, with the combination of information provision and habitual pricing, the expected travel price may be raised or lowered, depending on how they interact. Specifically, when s_L/s_H is low, the effects of information provision dominate, so that the expected travel price is lowered; when s_L/s_H is high, the effects of habitual pricing dominate, so that the expected travel price is raised.

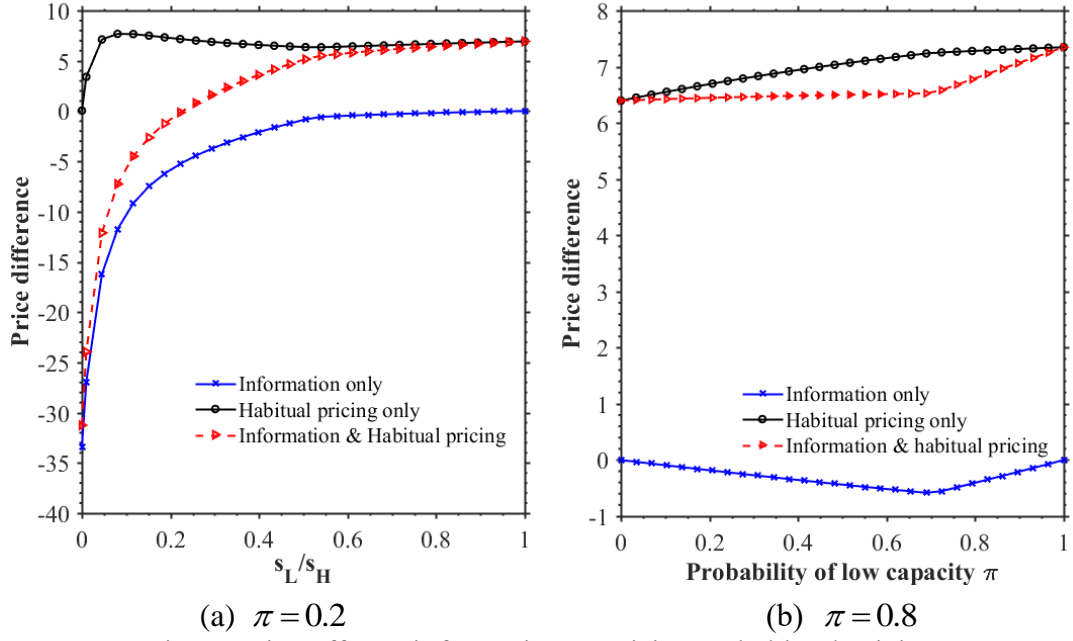


Fig. 5. Price effects: information provision vs habitual pricing.

Note: Price difference is measured by the difference between the travel price of the considered policy and an unregulated regime. A positive value means the introduced policy raises the expected travel price, and a negative value means the introduced policy lowers the expected travel price.

4.3.2 Habitual pricing vs responsive pricing

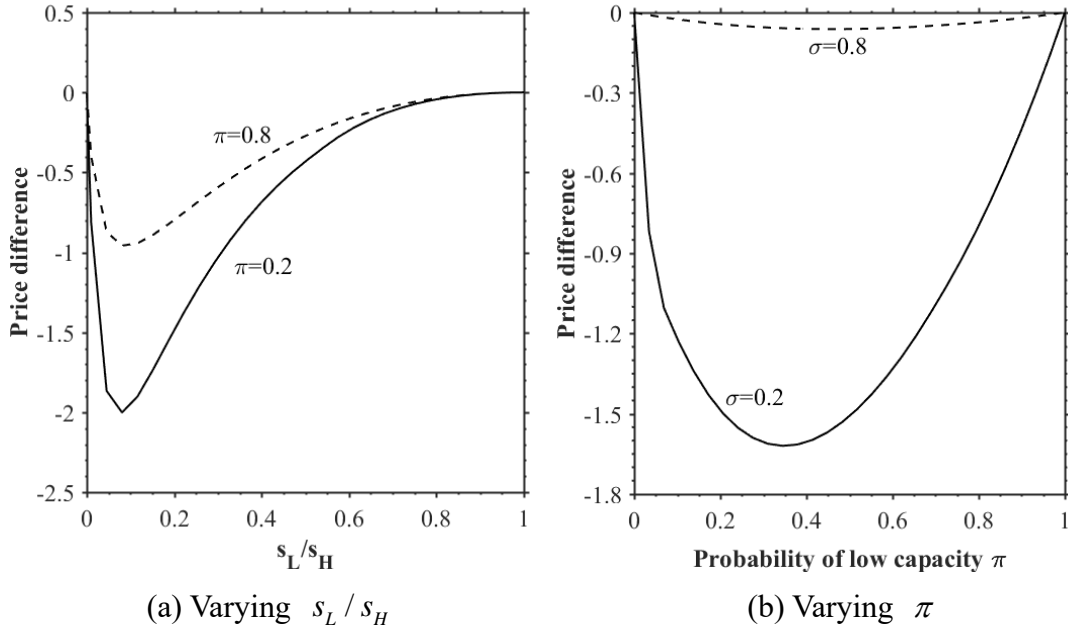


Fig. 6. Price effects: habitual pricing vs responsive pricing.

Note: In Fig. 6, price difference is measured by the difference in the expected travel price between responsive pricing and habitual pricing. A negative value means responsive pricing leads to a lower expected price than habitual pricing.

Fig. 6 shows that when combined with information provision, responsive pricing leads to a lower expected price than habitual pricing. This means that responsive pricing raises the travel price less than habitual pricing. This is intuitive because responsive pricing lowers the travel price more, as it allows regulators to adjust the toll according to the actual capacity realization, thereby lowering the welfare losses from congestion and uncertainty. In addition, responsive pricing is more conducive to lowering travel prices than habitual pricing.

4.4 Uncertainty and optimal tolling

This subsection examines the effects of uncertainty on optimal tolling. Subsection 4.4.1 investigates the effects of s_L / s_H on optimal tolling by varying s_L / s_H from 0 to 1, and subsection 4.4.2 investigates the effects of π on optimal tolling by varying π from 0 to 1.

4.4.1 Varying s_L / s_H

The toll under habitual pricing only is the highest. Without information provision, more travelers will travel when there is low capacity. The resulting welfare losses are then considerable and make it apparently worthwhile to set the habitual toll relatively high. Consequently, travelers not only benefit from information because of its informational value; a secondary benefit is that tolls will be lower.

In a comparison between responsive pricing and habitual pricing, Fig. 7 shows that when combined with information provision, the expected responsive toll, surprisingly, exceeds the habitual toll. This is because the habitual toll is a weighted average of the MEC, with the weight of MEC_L decreasing with s_L / s_H , and the weight of MEC_H increasing with s_L / s_H (see Proposition 3). Therefore, this toll is biased towards the congestion under a high-capacity state and is lower than the expected responsive toll.

As s_L / s_H increases, the toll differences between different regimes become smaller. Indeed, a larger s_L / s_H lowers the value of information, thereby reducing the toll difference between habitual pricing only and the combination of information provision and habitual pricing. For responsive and habitual pricing, because MEC_L becomes closer to MEC_H , the difference between the expected responsive toll and habitual toll also becomes smaller.

Comparing Fig. 7a and Fig. 7b, we find that the toll changes more quickly under $\pi = 0.8$ than $\pi = 0.2$, verifying that uncertainty plays an important role in deciding the optimal toll.

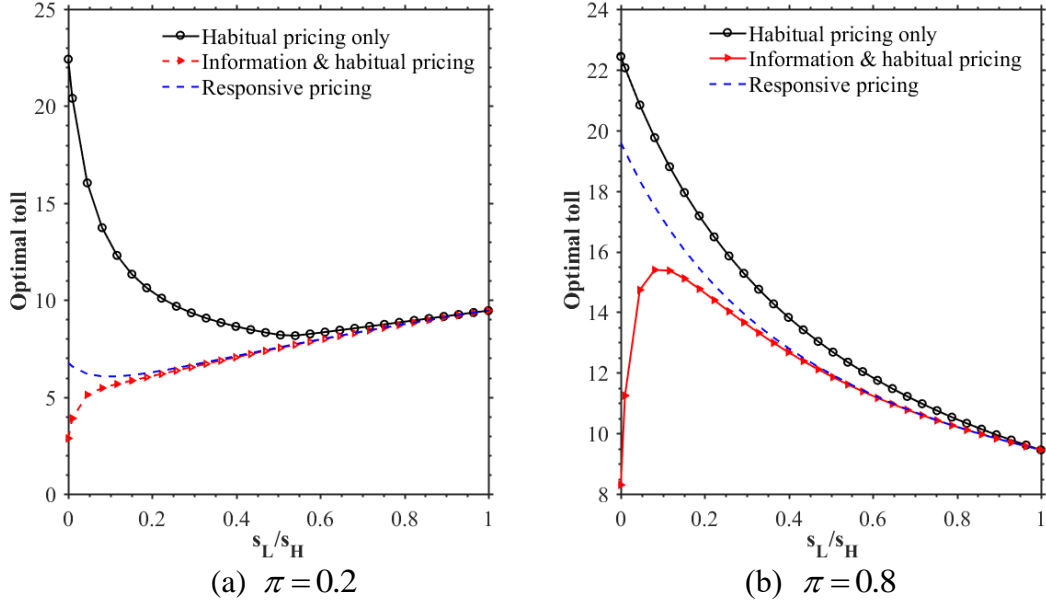


Fig. 7. s_L/s_H and optimal toll.

4.4.2 Varying π

Fig. 8 shows that under all regimes, expected toll increases with the probability of low capacity, π . This is because a higher π raises the welfare losses from congestion, which makes it beneficial to set an increasing toll. Similar to the results in Section 4.4.1, the toll under habitual pricing only is the highest, followed by the combination of information provision and responsive pricing, and the toll under the combination of information provision and responsive pricing is the lowest.

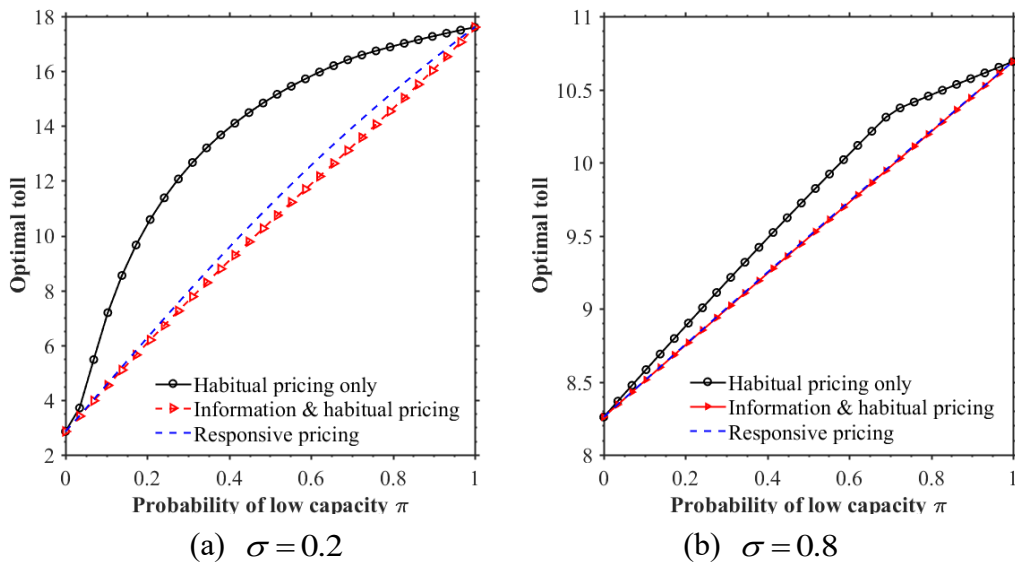


Fig. 8. π and optimal toll.

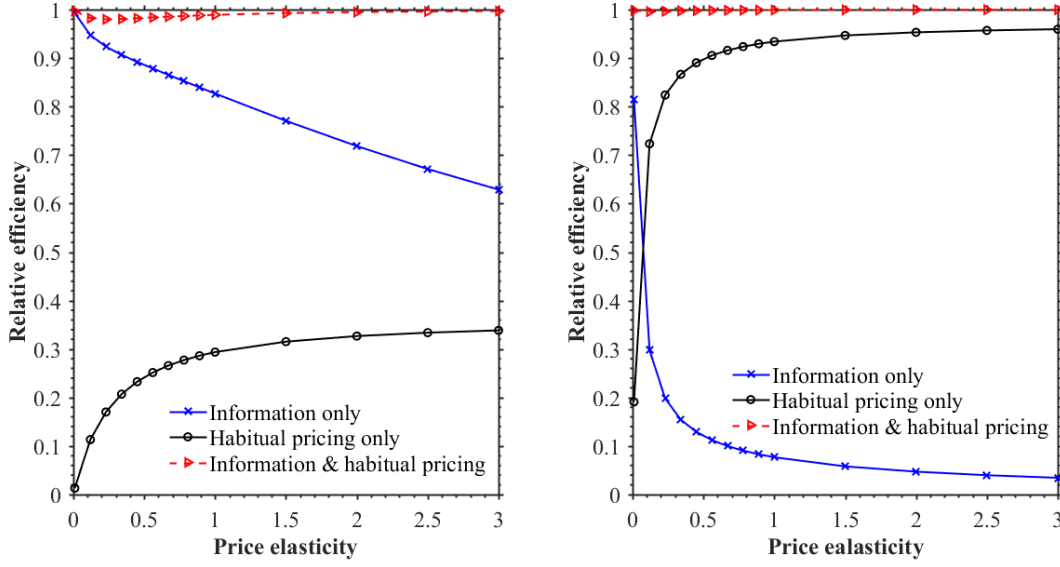
In the comparison of responsive pricing and habitual pricing, a responsive toll increases linearly with π , and a habitual toll increases non-linearly. This is because a habitual toll is a weighted average of the MEC, with the weight of MEC_L increasing with π , and the weight of MEC_H decreasing with π (see Proposition 3). Therefore, as the probability of low capacity increases, the weight of MEC_L increases, which leads to an increasing toll. The toll difference between these two regimes first increases and then decreases. Specifically, when the capacity is deterministic (i.e., $\pi = 0$ or $\pi = 1$), habitual pricing and responsive pricing have the same tolling.

With respect to the comparison between habitual pricing only and the combination of information provision and habitual pricing, the toll difference also first increases and then decreases. This is because without information, the habitual toll equals the expected travel price, which is an increasing function of π if $\pi < \pi_H$, and is independent of π if the probability of low capacity exceeds π_H (see Proposition 1a).

Comparing Fig. 8(a) and Fig. 8(b), we observe that the toll varies more quickly under $\sigma = 0.2$ than under $\sigma = 0.8$. Therefore, it is of great importance to take uncertainty into account when deciding a toll.

4.5 Varying price elasticity

We vary the price elasticity from 0 to 3. It can be observed from Fig. 9 that as travelers become more sensitive, the relative efficiency of information provision only decreases, and the relative efficiency of habitual pricing only increases. In contrast, for the combination of information provision and habitual pricing, relative efficiency first decreases and then increases. This is because when travelers' price elasticity is small, the effects of information provision dominate. As price elasticity increases, travelers become more sensitive to the travel price and the effects of habitual pricing dominate. When demand is perfectly elastic, habitual pricing and responsive pricing have the same performance under perfect information, implying that marginal external cost equals expected external cost.



(a) $\sigma = 0.2$

(b) $\sigma = 0.8$

Fig. 9. Price elasticity and relative efficiency.

5. Effects of imperfect information

5.1 Analytics

Although traffic information can be collected and disseminated, it may be difficult to receive high-quality information since traffic conditions may change in the morning peak period, and travelers may miss communication dispatches. Consequently, in practice, the information provided is usually imperfect (Arnott et al., 1991). In this section, we examine how imperfect information affects travelers.

We characterize imperfect information about s by a signal, η . A signal $\eta = \eta_L$ indicates that s is likely to be low, and a signal $\eta = \eta_H$ that it is likely high. It is assumed that travelers are informed of the signal of the capacity. The *ex-post* probability of low capacity, conditional on η_L and η_H , is specified in Table 4, where $\pi(\eta)$ is the probability of low capacity conditional on signal η , $1 - \pi(\eta)$ is the probability of high capacity conditional on signal η , and $Q \in [0, 1]$ is an index of signal quality. If $Q = 0$, the signal conveys no information: the conditional probability distribution of capacity equals the unconditional distribution. At the other extreme, $Q = 1$ means the signal is perfectly accurate and that the information is accurate.

Table 4. Conditional probability distribution of capacity

Signal	Probability of low capacity	Probability of high capacity
	$\pi(\eta)$	$1 - \pi(\eta)$
None	π	$1 - \pi$
Indicates high capacity	$\pi(1 - Q)$	$1 - \pi(1 - Q)$
Indicates low capacity	$1 - (1 - \pi)(1 - Q)$	$(1 - \pi)(1 - Q)$

For all values of Q_i , the signal is assumed to be unbiased in the sense that forecasts are made with the same frequency that they occur. From Table 4, the expected *ex-post* probability of low capacity is:

$$(1 - \pi) \cdot \pi(1 - Q) + \pi \cdot (1 - (1 - \pi)(1 - Q)) = \pi, \quad (30)$$

and the expected *ex-post* probability of high capacity is:

$$(1 - \pi) \cdot (1 - \pi(1 - Q)) + \pi \cdot ((1 - \pi)(1 - Q)) = 1 - \pi. \quad (31)$$

After receiving η_L and η_H , travelers face the same problem as under zero information, but with the conditional probability distributions of capacity instead of unconditional ones. According to Eq. (4), the expected travel cost under signal η_i , $Ec(\pi(\eta_i))$, is:

$$Ec(\pi(\eta_i)) = \begin{cases} \delta \left(1 - \frac{(1 - \pi(\eta_i))(\alpha + \gamma) - \alpha}{\gamma} (1 - \sigma) \right) \frac{N}{s_L}, & \text{if } 0 \leq \pi(\eta_i) < \pi_1 \\ \frac{\beta(\alpha + \gamma)\pi(\eta_i)}{\beta + (\alpha + \gamma)\pi(\eta_i)} \frac{N}{s_L}, & \text{if } \pi_1 \leq \pi(\eta_i) < \pi_2 \\ \frac{\delta N}{s_L}, & \text{if } \pi_2 \leq \pi(\eta_i) \leq 1 \end{cases}. \quad (32)$$

Given the unbiased nature of the signals, the probability that signal η_i is received is π_i . With expected information provision, the expected travel cost, $E[c]$, is thus:

$$E[c] = \pi \cdot Ec(\pi(\eta_L)) + (1 - \pi) \cdot Ec(\pi(\eta_H)). \quad (33)$$

The expected welfare is again the consumer benefits minus the total expected travel cost. The habitual toll now equals the weighted average of the expected MEC under each signal, with the weights depending on the probability distribution of the capacity. The expected MEC under a given signal depends on the conditional probability distribution of the capacity.

5.2 Numerical model of the imperfect information setting

We now return to our numerical model. We compare the results under imperfect

information only with the combination of imperfect information and habitual pricing, by varying the quality of information, Q , from 0 to 1.

Fig. 10 shows that as the quality of information increases, the expected travel price decreases, especially when the capacity degradation is larger. The kinks are caused by the shift in the expected travel cost function. According to Eq. (32), the expected travel cost is a linear increasing function of $\pi(\eta_i)$ if $0 \leq \pi(\eta_i) < \pi_1$, is an increasing and concave function of $\pi(\eta_i)$ if $\pi_1 \leq \pi(\eta_i) < \pi_2$, and is independent of $\pi(\eta_i)$ if the probability of low capacity exceeds $\pi_2 \leq \pi(\eta_i) \leq 1$.

Fig. 11 shows that relative efficiency increases with the quality of the information. This demonstrates that although imperfect information may not be accurate, information provision still performs better than no information in reducing the expected travel price and improving welfare. We also see that adding a habitual toll to imperfect information raises welfare but also the expected price. These effects get stronger when the capacity degradation is larger.

It can be observed from Fig. 12 that a higher quality of information tends to lower the optimal habitual toll. This is intuitive because more welfare loss caused by uncertainty is reduced by a higher quality of information provision.

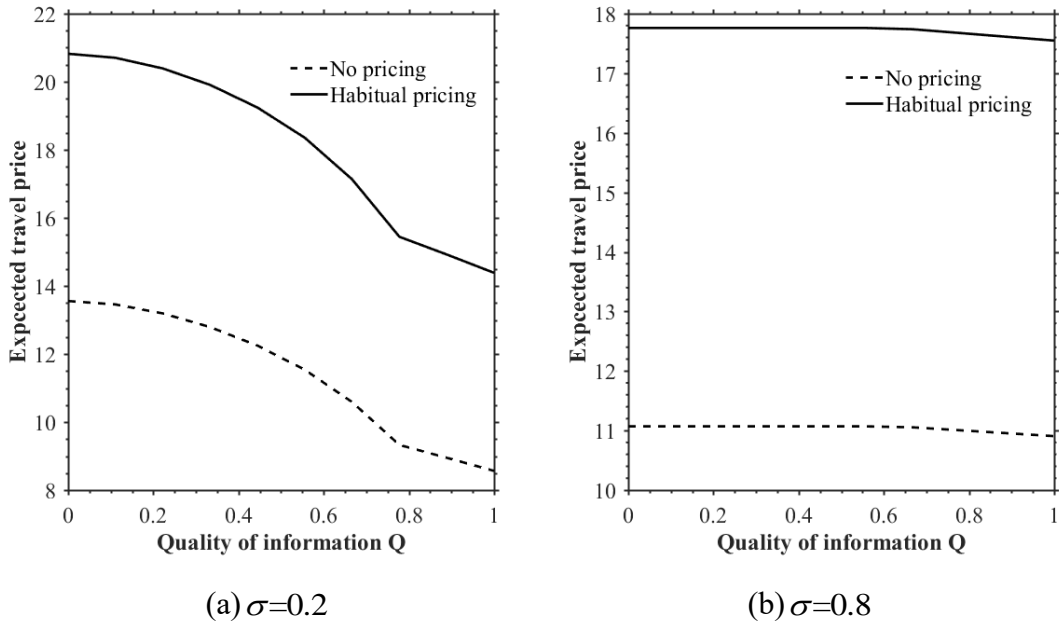
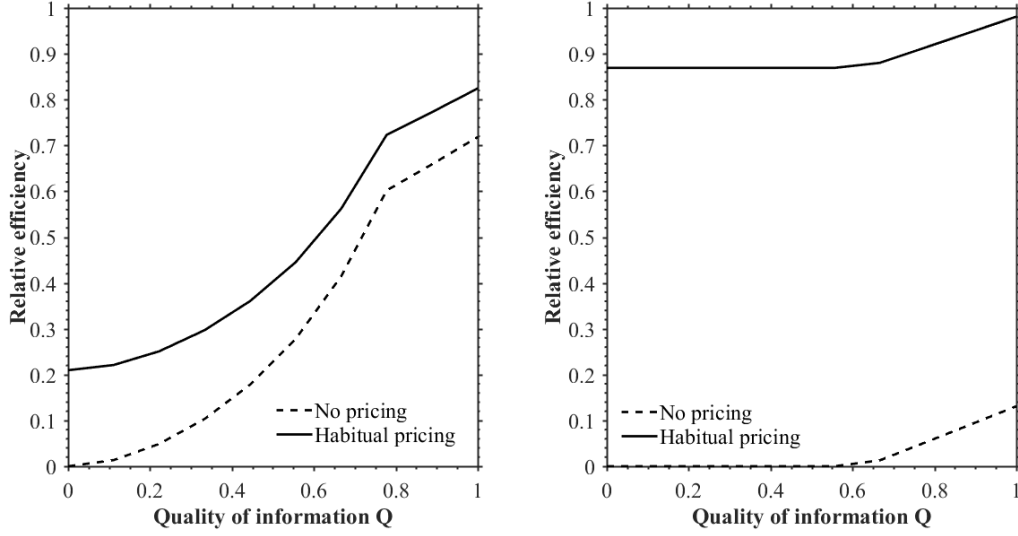


Fig. 10. Information quality and price effects.



(a) $\sigma = 0.2$

(b) $\sigma = 0.8$

Fig. 11. Information quality and relative efficiency.

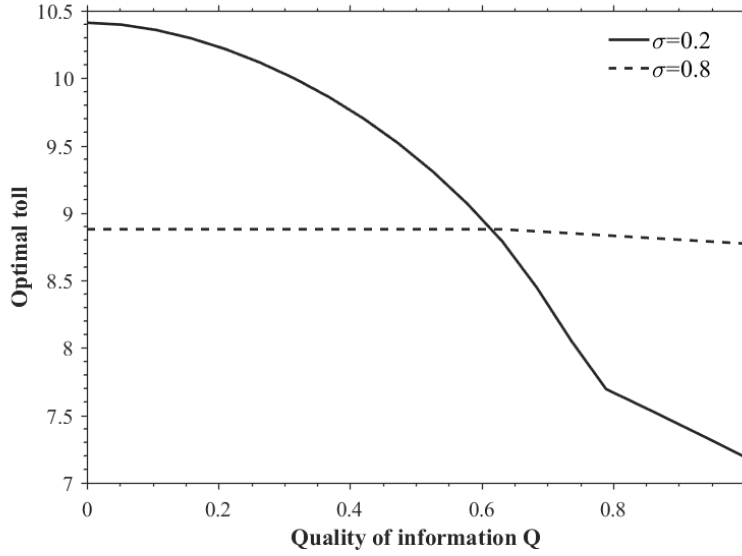


Fig. 12. Information quality and optimal toll.

To conclude our analysis of imperfect information, we note that imperfect information always raises welfare and lowers the expected price. A habitual toll would be the weighted average of the expected MEC under each signal and adding it to imperfect information further raises welfare but also the expected price. Toll are lowered by a higher quality of information.

6. Conclusion

With travelers and regulators becoming increasingly concerned about the

uncertainty and congestion in transport systems, we have developed a joint departure time choice and tolling design model for solving travelers' travel equilibrium and regulators' regulatory decisions under uncertainty and dynamic congestion. The combination of information provision and congestion pricing is proposed to reduce uncertainty and congestion. We considered three information regimes (no information, perfect information and imperfect information) and two congestion pricing regimes: responsive state-dependent pricing and habitual state-independent pricing. We derived theoretical solutions and discussed the welfare and price effects under different regulation regimes, providing guidance to public agencies in developing toll policies and information provision decisions.

Our study shows that the introduction of information provision lowers the habitual toll. In the absence of information provision, the habitual toll equals users' expected MEC. With information provision, the habitual toll is a weighted average of the MECs over all states, with the weights depending on the probability distribution of capacity, travelers' price sensitivity of demand and the value of schedule delay parameter. With imperfect information, the weights also depend on the quality of the signal.

When only one policy is implemented, information provision and congestion pricing are both welfare-improving. Information provision is preferable when uncertainty is high, as information is more valuable at this time. Otherwise, congestion pricing performs better. With respect to the travel price, information provision tends to lower the expected travel price and congestion pricing tends to raise it. Hence, congestion pricing is more likely to meet resistance from travelers.

When pricing and information provision are implemented together, responsive pricing performs better than habitual pricing. In our numerical model, however, differences tend to be very small unless the difference between the low and high capacity is very large. Although both policies will raise the expected price for travelers, habitual pricing raises it more. Hence, if the regulator aims at improving welfare, combining information provision and responsive pricing may be the best choice. However, habitual tolls tend to perform very similarly in our numerical model, so responsive tolls may not be worth the extra unpopularity they might entail.

Various interesting and important extensions can be dealt with in the future.

- First, we have assumed that all travelers either have no information, perfect information or imperfect information, and that the information is provided at no cost. It would be interesting to endogenize the choice of being informed and to allow for partial or sometimes incorrect information.⁵
- Second, it would be interesting to study the impacts of information provision

⁵ See, e.g., Ben-Akiva and de Palma (1991), Arnott et al. (1996, 1999), Zhang and Verhoef (2006), Enrique Fernández et al. (2009), Khan and Amin (2018) and Xie and Liu (2021).

and tolling when travelers are heterogeneous, for instance, in terms of varying the values of time, as in Xiao et al. (2014).

- Third, extending the analysis to more general networks, for example, larger networks with many parallel competing routes and networks that allow for route diversion.⁶
- Fourth, we assume that users have no risk aversion and attach the same weights to losses or gains, which is not in line with prospect theory. What changes when we allow for such factors, where people may even have different risk aversions and asymmetries for various considerations such as tolls, travel times and schedule delays?⁷
- Fifth, we assume away toll collection cost, but such costs can be substantial. Such costs may very well be higher for the more complex state-dependent responsive tolls, if only because they need to be announced each day and this makes administration more complex.
- Sixth, we only considered capacity uncertainty in which the realized level of capacity is the same for the entire day and only has two states. What changes if the capacity can fluctuate over the day due to, say, traffic incidents or rain showers? What changes if demand is also uncertain? What about more realistic capacity distributions?
- Seventh, we only considered flat tolls that are constant for the day. How would the situation change if tolls were fully time variant or varied in steps, as in Singapore and Stockholm and with many US toll roads and bridges? Lindsey (1994, 1999), Xiao et al. (2015) and Jiang et al. (2021) show that uncertainty affects the design of such systems.⁸ Therefore, there is a need to incorporate time-varying tolling when analyzing information provision.

Appendices

Appendix A. Proof of Proposition 1c.

Proof. (i) In an unregulated regime, solving $D(N) = d_0 - d_1 \cdot N = E[c]$ yields:

⁶ See, e.g., Emmerink et al. (1995), Verhoef et al. (1996), Emmerink et al. (1997), Yang (1999), and Xiao et al. (2017).

⁷ See, e.g., de Borger and Fosgerau (2008), Börjesson et al. (2012), Kouwenhoven et al. (2014) and Beaud et al. (2016) for empirical studies on this, and de Palma and Picard (2006), Fosgerau and Karlström. (2010) and Liu and Liu (2018) for theoretical analyses. One could also add restrictions such as a travel cost budget, as in Liu et al. (2020).

⁸ None of these papers looked at information provisions. Lindsey examined fully time-variant optimal pricing and the other authors looked at second-best time-variant pricing—where the toll is forced to be linear over time and keeps the generalized price unchanged—and at step tolling.

$$N = \begin{cases} \frac{d_0 \gamma s_L}{\delta \pi (\alpha + \gamma)(1 - \sigma) + \gamma(d_1 s_L + \delta \sigma)}, & \text{if } 0 \leq \pi \leq \pi_1 \\ \frac{(\beta + (\alpha + \gamma)\pi) \cdot d_0 s_L}{\beta d_1 s_L + (\beta + d_1 s_L)(\alpha + \gamma)\pi}, & \text{if } \pi_1 \leq \pi \leq \pi_2 \\ \frac{d_0 s_L}{\delta + d_1 s_L}, & \text{if } \pi_2 \leq \pi \leq 1 \end{cases} \quad (A1)$$

Comparing Eq. (A1) with Eq. (11), we can directly find the travel demand under habitual pricing is lower than that under the unregulated regime.

(ii) In equilibrium, the marginal willingness to pay equals the travel price. According to the relationship between the travel demands under different regimes, we find that habitual pricing tends to raise the travel price.

(iii) According to Eq. (12), when $\pi = 0$, the expected welfare is $E[W] = \frac{d_0^2 \gamma s_L}{4\delta \gamma \sigma + 2d_1 \gamma s_L}$, and when $\pi = 1$, the expected welfare is $E[W] = \frac{d_0^2 s_L}{4\delta + 2d_1 s_L}$. Since $\partial E[W]/\partial \pi \leq 0$ always holds, we can obtain that under habitual pricing, the expected welfare is between $\frac{d_0^2 s_L}{4\delta + 2d_1 s_L}$ and $\frac{d_0^2 \gamma s_L}{4\delta \gamma \sigma + 2d_1 \gamma s_L}$.

In the unregulated regime, the expected welfare satisfies:

$$E[W] = \begin{cases} \frac{d_0^2 d_1 \gamma^2 s_L^2}{2(\delta \gamma (\pi(1 - \sigma) + \sigma) + \alpha \delta \pi(1 - \sigma) + d_1 \gamma s_L)^2}, & \text{if } 0 \leq \pi < \pi_1 \\ \frac{d_0^2 d_1 (\beta + (\alpha + \gamma)\pi)^2 s_L^2}{2(\beta d_1 s_L + (\alpha + \gamma)\pi(\beta + d_1 s_L))^2}, & \text{if } \pi_1 \leq \pi < \pi_2 \\ \frac{d_0^2 d_1 s_L^2}{2(\delta + d_1 s_L)^2}, & \text{if } \pi_2 \leq \pi \leq 1 \end{cases} \quad (A2)$$

Taking the derivative of Eq. (A2) with respect to π , we find that $\partial E[W]/\partial \pi \leq 0$ still holds. Specifically, when $\pi = 0$, $E[W] = \frac{d_0^2 d_1 s_H^2}{2(\delta + d_1 s_H)^2}$; when $\pi = 1$,

$$E[W] = \frac{d_0^2 d_1 s_L^2}{2(\delta + d_1 s_L)^2}.$$

As the expected welfare is non-decreasing under both regimes, we only need to compare the relationship at the end points. More specifically, at $\pi = 0$, we have:

$$\frac{d_0^2 \gamma s_L}{4\delta \gamma \sigma + 2d_1 \gamma s_L} = \frac{d_0^2 d_1 s_L s_H}{2(2\delta s_L d_1 + d_1^2 s_L s_H)} = \frac{d_0^2 d_1 s_H^2}{2(2\delta d_1 s_H + d_1^2 s_H^2)} \geq \frac{d_0^2 d_1 s_H^2}{2(\delta + d_1 s_H)^2}; \quad \text{and at}$$

$$\pi = 1, \text{ we have } \frac{d_0^2 s_L}{4\delta + 2d_1 s_L} = \frac{d_0^2 s_L}{2(2\delta + d_1 s_L)} \geq \frac{d_0^2 d_1 s_L^2}{2(\delta + d_1 s_L)^2}.$$

Therefore, introducing habitual pricing improves welfare. ■

Appendix B. Proof of Proposition 2.

Proof. (i) According to Eq. (A1), we find that under the unregulated regime, the number of users, N , is non-increasing with π , i.e., $\partial N / \partial \pi \leq 0$. When $0 \leq \pi \leq \pi_1$,

$$\frac{d_0 s_L}{\beta \sigma + d_1 s_L} \leq N \leq \frac{d_0 s_H}{\delta + d_1 s_H}; \text{ when } \pi_1 \leq \pi \leq \pi_2, \quad \frac{d_0 s_L}{\delta + d_1 s_L} \leq N \leq \frac{d_0 s_L}{\beta \sigma + d_1 s_L}; \text{ when } \pi_2 \leq \pi \leq 1, \quad N = \frac{d_0 s_L}{\delta + d_1 s_L}.$$

Therefore, the travel demand is between $\frac{d_0 s_L}{\delta + d_1 s_L}$ and $\frac{d_0 s_H}{\delta + d_1 s_H}$. Combining Eq. (13), we find that information tends to raise the number of

users when the state of the bottleneck capacity is high, and lower the number of users when the state of the bottleneck is low, as presented in Proposition 1a.

(ii) The property on the travel price can be obtained directly from (i).

(iii) This is because in the unregulated regime, the expected welfare is between

$$\frac{d_0^2 d_1 s_H^2}{2(\delta + d_1 s_H)^2} \quad \text{and} \quad \frac{d_0^2 d_1 s_L^2}{2(\delta + d_1 s_L)^2}.$$

After introducing information, welfare is $\frac{d_0^2 d_1 s_H^2}{2(\delta + d_1 s_H)^2}$ when the actual capacity is high, and $\frac{d_0^2 d_1 s_L^2}{2(\delta + d_1 s_L)^2}$ when the actual

capacity is low. As a result, welfare is raised by information provision when the capacity is high, and lowered when the capacity is low. ■

Appendix C. Proof of Proposition 3.

Proof. The associated Lagrangian of the maximization problem under Eq. (17) and Eq. (18) is:

$$L = \left((1-\pi) \cdot \int_0^{N_H} D(n) dn + \pi \cdot \int_0^{N_L} D(n) dn \right) - \left((1-\pi) \cdot c_H \cdot N_H + \pi \cdot c_L \cdot N_L \right) + \lambda_H \cdot (D(N_H) - c_H - \tau) + \lambda_L \cdot (D(N_L) - c_L - \tau) \quad (C1)$$

Taking the derivative of Eq. (C1) with respect to N_H , N_L , τ , λ_H and λ_L yields:

$$\begin{aligned} \frac{\partial L}{\partial N_H} &= (1-\pi) \cdot D(N_H) - (1-\pi) \cdot c_H - (1-\pi) \cdot \frac{\partial c_H}{\partial N_H} \cdot N_H + \lambda_H \cdot \left(D' - \frac{\partial c_H}{\partial N_H} \right) = 0; \\ \frac{\partial L}{\partial N_L} &= \pi \cdot D(N_L) - \pi \cdot c_L - \pi \cdot \frac{\partial c_L}{\partial N_L} \cdot N_L + \lambda_L \cdot \left(D' - \frac{\partial c_L}{\partial N_L} \right); \\ \frac{\partial L}{\partial \lambda_H} &= D(N_H) - c_H - \tau = 0; \quad \frac{\partial L}{\partial \lambda_L} = D(N_L) - c_L - \tau = 0; \\ \frac{\partial L}{\partial \tau} &= -\lambda_H - \lambda_L = 0 \quad \lambda_H + \lambda_L = 0 \quad \lambda_L = -\lambda_H. \end{aligned} \quad (C2)$$

Solving the above problem yields:

$$\lambda_H = \frac{\pi \cdot \tau - \pi \cdot \frac{\partial c_L}{\partial N_L} \cdot N_L}{D' - \frac{\partial c_L}{\partial N_L}}, \lambda_L = -\lambda_H. \quad (C3)$$

$$\tau = \frac{\pi \cdot \left(D' - \frac{\partial c_H}{\partial N_H} \right) \cdot \frac{\partial c_L}{\partial N_L} \cdot N_L + (1-\pi) \cdot \left(D' - \frac{\partial c_L}{\partial N_L} \right) \cdot \frac{\partial c_H}{\partial N_H} \cdot N_H}{(1-\pi) \left(D' - \frac{\partial c_L}{\partial N_L} \right) + \pi \left(D' - \frac{\partial c_H}{\partial N_H} \right)}. \quad (C4)$$

$$\text{Let } \lambda = \frac{\pi \cdot \left(D' - \frac{\partial c_H}{\partial N_H} \right)}{(1-\pi) \left(D' - \frac{\partial c_L}{\partial N_L} \right) + \pi \left(D' - \frac{\partial c_H}{\partial N_H} \right)}. \text{ Substituting Eq. (14) into Eq. (C4),}$$

the toll can be rewritten as:

$$\tau = \lambda \cdot MEC_L + (1-\lambda) \cdot MEC_H. \quad (C5)$$

with

$$\lambda = \frac{\pi \cdot \left(d_1 + \frac{\delta}{s_H} \right)}{(1-\pi) \left(d_1 + \frac{\delta}{s_L} \right) + \pi \left(d_1 + \frac{\delta}{s_H} \right)} \leq 1. \quad (C6)$$

Taking the derivate of λ with respect to π and d_1 yields:

$$\frac{\partial \lambda}{\partial \pi} = \frac{1}{\pi^2} \cdot \frac{\left(d_1 + \frac{\delta}{s_H} \right) \left(d_1 + \frac{\delta}{s_L} \right)}{\left(\left(1 - \frac{1}{\pi} \right) \left(d_1 + \frac{\delta}{s_L} \right) - \left(d_1 + \frac{\delta}{s_H} \right) \right)^2} > 0. \quad (C7)$$

$$\begin{aligned} \frac{\partial \lambda}{\partial d_1} &= \frac{\pi \left(d_1 + \frac{(1-\pi)\delta}{s_L} + \frac{\pi\delta}{s_H} \right) - \pi \cdot \left(d_1 + \frac{\delta}{s_H} \right)}{\left(d_1 + \frac{(1-\pi)\delta}{s_L} + \frac{\pi\delta}{s_H} \right)^2} = \frac{\pi \left(\frac{(1-\pi)\delta}{s_L} + \frac{\pi\delta}{s_H} - \frac{\delta}{s_H} \right)}{\left(d_1 + \frac{(1-\pi)\delta}{s_L} + \frac{\pi\delta}{s_H} \right)^2} \\ &= \frac{\pi \cdot (1-\pi) \cdot (\delta/s_L - \delta/s_H)}{\left(d_1 + \frac{(1-\pi)\delta}{s_L} + \frac{\pi\delta}{s_H} \right)^2} \geq 0 \end{aligned} \quad (C8)$$

Proposition 3 has been proved. ■

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