

Caporale, Guglielmo Maria; Cunado, Juncal; Gil-Alana, Luis A.

**Working Paper**

## Modelling long-run trends and cycles in financial time series data

CESifo Working Paper, No. 2330

**Provided in Cooperation with:**

Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

*Suggested Citation:* Caporale, Guglielmo Maria; Cunado, Juncal; Gil-Alana, Luis A. (2008) : Modelling long-run trends and cycles in financial time series data, CESifo Working Paper, No. 2330, Center for Economic Studies and ifo Institute (CESifo), Munich

This Version is available at:

<https://hdl.handle.net/10419/26375>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# Modelling Long-Run Trends and Cycles in Financial Time Series Data

GUGLIELMO MARIA CAPORALE  
JUNCAL CUÑADO  
LUIS A. GIL-ALANA

CESIFO WORKING PAPER NO. 2330  
CATEGORY 10: EMPIRICAL AND THEORETICAL METHODS  
JUNE 2008

*An electronic version of the paper may be downloaded*

- *from the SSRN website:* [www.SSRN.com](http://www.SSRN.com)
- *from the RePEc website:* [www.RePEc.org](http://www.RePEc.org)
- *from the CESifo website:* [www.CESifo-group.org/wp](http://www.CESifo-group.org/wp)

# Modelling Long-Run Trends and Cycles in Financial Time Series Data

## Abstract

This paper proposes a very general time series framework to capture the long-run behaviour of financial series. The suggested model includes linear and non-linear time trends, and stationary and nonstationary processes based on integer and/or fractional degrees of differentiation. Moreover, the spectrum is allowed to contain more than a single pole or singularity, occurring at zero and non-zero (cyclical) frequencies. This model is used to analyse four annual time series with a long span, namely dividends, earnings, interest rates and long-term government bond yields. The results indicate that the four series exhibit fractional integration with one or two poles in the spectrum. A forecasting comparison shows that a model with a non-linear trend along with fractional integration outperforms alternative models over long horizons.

JEL Code: C22, G1.

Keywords: fractional integration, financial time series data, trends, cycles.

*Guglielmo Maria Caporale*  
*Centre for Empirical Finance*  
*Brunel University*  
*Uxbridge, Middlesex UB8 3PH*  
*United Kingdom*  
*Guglielmo-Maria.Caporale@brunel.ac.uk*

*Juncal Cuñado*  
*University of Navarra*  
*Faculty of Economics*  
*31080 Pamplona*  
*Spain*  
*jcunado@unav.es*

*Luis A. Gil-Alana*  
*University of Navarra*  
*Faculty of Economics*  
*31080 Pamplona*  
*Spain*  
*alana@unav.es*

May 2008

The second and third named authors gratefully acknowledge financial support from the Ministerio de Ciencia y Tecnología (SEJ2005-07657, Spain).

## 1. Introduction

The statistical modelling of financial time series data such as asset prices plays an important role in portfolio management. Despite the extensive theoretical and empirical literature of the last thirty years, there is still no consensus on what might be the most adequate model specification for many financial series. For instance, whether asset returns of asset prices are predictable or not is still controversial. While the efficiency market hypothesis suggests that they should follow a random walk (see Fama, 1970; Summers, 1986), other authors have found evidence of mean reversion in their behaviour (see, e.g., Poterba and Summers, 1988 and Fama and French, 1988). The standard econometric approach to settle this issue empirically relies on establishing the (integer) order of integration of the series by carrying out nonstationary unit root tests. More recently, however, the possibility of fractional orders of integration has also been taken into account, with a slow rate of decay. Long memory specifications for realised volatility have been shown frequently to forecast very accurately (see, e.g., Li, 2002 or Martens and Zein, 2004), though in some cases the sum of short-memory (ARMA) specifications appears to forecast as accurately as a long-memory (ARFIMA) model (see, e.g., Pong et al, 2004). Using a fractional model, Caporale and Gil-Alana (2002) find that there is no permanent component in US stock market returns, since the series examined is close to being  $I(0)$ . Caporale and Gil-Alana (2007) decompose the stochastic process followed by US stock prices into a long-run component described by the fractional differencing parameter ( $d$ ) and a short-run (ARMA) structure. Empirical support for non-linear asset pricing models (such as the one by Dittmar, 2002) has also been found (see, inter alia, Hossein and Sonnie, 2006).

The present paper takes into account these various strands of the literature on modelling asset prices and proposes a very general time series framework to capture the

long-run behaviour of financial data. The suggested model includes linear and non-linear time trends, and stationary and nonstationary processes based on integer and/or fractional degrees of differentiation. Moreover, the spectrum is allowed to contain more than a single pole or singularity, occurring at the zero frequency but also at non-zero (cyclical) frequencies. This model is used to analyse four annual time series with a long span, namely dividends, earnings, interest rates and long-term government bond yields, obtainable from Robert Shiller's homepage (<http://www.econ.yale.edu/~shiller/>). We are able to show that the selected specifications (with linear and non-linear trends, fractional integration and cyclical fractional integration) have better forecasting properties than alternative models used in the literature to analyse these data.

The structure of the paper is as follows. Section 2 describes the model and the statistical approach employed in the paper. Section 3 presents the empirical analysis, considering first the case with linear trends and then the non-linear one. Section 4 assesses the forecasting performance of the selected models, whilst Section 5 offers some concluding comments.

## 2. The model

Let us assume that  $\{y_t, t = 1, 2, \dots, T\}$  is the time series we observe. We consider the following model:

$$y_t = f(t) + x_t, \quad (1)$$

$$(1 - L)^{d_1} (1 - 2\cos w_r L + L^2)^{d_2} x_t = u_t, \quad (2)$$

$$\phi_p(L) u_t = \theta_q(L) \varepsilon_t, \quad (3)$$

where  $f$  is a function of time that may be of a linear/non-linear nature;  $L$  is the lag operator (i.e.,  $L^s x_t = x_{t-s}$ );  $d_1$  is the order of integration corresponding to the long-run or zero frequency;  $w_r = 2\pi/r$ , with  $r$  representing the number of periods per cycle;  $d_2$  is the

order of integration with respect to the non-zero (cyclical) frequency, and  $u_t$  is assumed to be an  $I(0)$  process, that may follow a general stationary ARMA( $p, q$ ) process, where  $p$  and  $q$  indicate the orders of the autoregressive and moving average components respectively. Note that  $d_1$  and  $d_2$  are allowed be any real values and thus we do not restrict ourselves to integer degrees of differentiation.

The set-up described in (1) – (3) is fairly general, including the standard ARMA model (with or without trends), if  $d_1 = d_2 = 0$ ; the ARIMA case, if  $d_1$  is integer and  $d_2 = 0$ ; the ARFIMA specification, if  $d_1$  is fractional and  $d_2 = 0$ , along with other more complex representations.

We now focus on equation (2), and first assume that  $d_2 = 0$ . Then, the spectral density function of  $x_t$  is given by:

$$f(\lambda; \tau) = \frac{\sigma^2}{2\pi} \left| \frac{\theta(e^{i\lambda})}{\phi(e^{i\lambda})} \right|^2 |1 - e^{i\lambda}|^{-2d_1}, \quad (4)$$

and it contains a pole or singularity at the long-run or zero frequency. Further, note that the polynomial  $(1 - L)^{d_1}$  can be expressed in terms of its Binomial expansion, such that, for all real  $d_1$ ,

$$(1 - L)^{d_1} = \sum_{j=0}^{\infty} \binom{d_1}{j} (-1)^j L^j = 1 - d_1 L + \frac{d_1(d_1-1)}{2} L^2 - \dots, \quad (5)$$

implying that the higher is the value of  $d_1$ , the higher is the degree of association between observations distant in time. Thus, the parameter  $d_1$  plays a crucial role in determining the degree of persistence of the series. Although the time series literature for very long only considered the cases of integer values of  $d_1$  (stationarity if  $d_1 = 0$ , and nonstationarity with  $d_1 = 1$ ), more recently fractional values of  $d_1$  have been widely

employed when modelling macroeconomic and financial data.<sup>1</sup> Suppose now that  $d_1 = 0$  in (2). Then, the process  $x_t$  has a spectral density function given by:

$$f(\lambda; \tau) = \frac{\sigma^2}{2\pi} \left| \frac{\theta(e^{i\lambda})}{\phi(e^{i\lambda})} \right|^2 |2(\cos(\lambda) - \cos(w_r))|^{-2d_2} \quad (6)$$

and is characterised by having a pole at a non-zero frequency. Moreover, the polynomial  $(1 - 2\cos w_r L + L^2)^{d_2}$  can be expressed as a Gegenbauer polynomial, such that, defining  $\mu = \cos w_r$ , for all  $d_2 \neq 0$ ,

$$(1 - 2\mu L + L^2)^{-d_2} = \sum_{j=0}^{\infty} C_{j,d_2}(\mu) L^j, \quad (7)$$

where  $C_{j,d_2}(\mu)$  are orthogonal Gegenbauer polynomial coefficients recursively defined as:

$$C_{0,d_2}(\mu) = 1,$$

$$C_{1,d_2}(\mu) = 2\mu d_2,$$

$$C_{j,d_2}(\mu) = 2\mu \left( \frac{d_2 - 1}{j} + 1 \right) C_{j-1,d_2}(\mu) - \left( 2 \frac{d_2 - 1}{j} + 1 \right) C_{j-2,d_2}(\mu), \quad j = 2, 3, \dots$$

(see, inter alia, Magnus et al., 1966, or Rainville, 1960, for further details on Gegenbauer polynomials). Gray et al. (1989, 1994) showed that this process is stationary if  $d_2 < 0.5$  for  $|\mu = \cos w_r| < 1$  and if  $d_2 < 0.25$  for  $|\mu| = 1$ . If  $d_2 = 1$ , the process is said to contain a unit root cycle (Ahtola and Tiao, 1987; Bierens, 2001); other applications using fractional values of  $d_2$  can be found in Gil-Alana (2001), Ahn, Knopova and Leonenko (2004), Soares and Souza (2006), etc.<sup>2</sup>

---

<sup>1</sup> Empirical applications using fractional values of  $d_1$  include Diebold and Rudebusch (1989), Sowell (1992), Gil-Alana and Robinson (1997), etc.

<sup>2</sup> Models with multiple cyclical structures ( $k$ -factor Gegenbauer processes) with multiple poles in the spectrum have been examined, among others, by Ferrara and Guegan (2001), Sadek and Khotanzad (2004) and Gil-Alana (2007a).

In the empirical analysis carried out in the following section we use a method developed by Robinson (1994) that enables us to test a model such as (1) - (3). It is a testing procedure based on the Lagrange Multiplier (LM) principle that uses the Whittle function in the frequency domain. It can be used to test the null hypothesis:

$$H_o : d \equiv (d_1, d_2)^T = (d_{1o}, d_{2o})^T \equiv d_o, \quad (8)$$

in (1) – (3) where  $d_{1o}$  and  $d_{2o}$  may be any real values, thus encompassing stationary and nonstationary hypotheses. The specific form of the test statistic (denoted by  $\hat{R}$ ) is presented in the appendix. Under very general regularity conditions, Robinson (1994) showed that for this particular version of his tests,

$$\hat{R} \rightarrow_d \chi_2^2, \quad as \quad T \rightarrow \infty. \quad (9)$$

Thus, unlike in other procedures, we are in a classical large-sample testing situation. A test of (8) will reject  $H_o$  against the alternative  $H_a: d \neq d_o$  if  $\hat{R} > \chi_{2,\alpha}^2$ , where  $\text{Prob}(\chi_2^2 > \chi_{2,\alpha}^2) = \alpha$ . Furthermore the test is efficient in the Pitman sense against local departures from the null, that is, if the test is implemented against local departures of the form:  $H_a: d = d_o + \delta T^{-1/2}$ , for  $\delta \neq 0$ , the limit distribution is a  $\chi_2^2(v)$ , with a non-centrality parameter  $v$  that is optimal under Gaussianity of  $u_t$ .

There exist other procedures for estimating and testing the fractionally differenced parameters, some of them also based on the likelihood function. As in other standard large-sample testing situations, Wald and LR test statistics against fractional alternatives will have the same null and local limit theory as the LM tests of Robinson (1994). Ooms (1997) proposed tests based on seasonal fractional models: they are Wald tests, and thus require efficient estimates of the fractional differencing parameters. He used a modified periodogram regression estimation procedure due to Hassler (1994). Also, Hosoya (1997) established the limit theory for long-memory processes with the



singularities not restricted at the zero frequency, and proposed a set of quasi log-likelihood statistics to be applied to raw time series.<sup>3</sup> Unlike these previous methods, the tests of Robinson (1994) do not require estimation of the long-memory parameters since the differenced series have short memory under the null. Similarly, with respect to the zero frequency, Sowell (1992) employed a Wald testing procedure, though again this approach requires an efficient estimate of  $d_1$ , and while such estimates can be obtained, the LM procedure of Robinson (1994) seems computationally more attractive.<sup>4</sup>

### 3. Empirical Analysis

The data analysed in this paper have been obtained from Robert Shiller's homepage (<http://www.econ.yale.edu/~shiller/>). They are described in chapter 26 of Shiller's (1989) book on "Market Volatility", where further details can be found, and are constantly updated and revised. Specifically, they are the following series: dividends (an index), earnings (also an index), one-year interest rate (this series is the result of converting the January and July rates into an annual yield), long-term government bond yield (this is the yield on the 10-year Treasury bonds after 1953). The sample period goes from 1871 to 2006 for the first two series, 2004 for the third one, and 2007 for the fourth one. In all cases, we leave out the last ten observations to use them for the forecasting comparison carried out in Section 4.

**[INSERT FIGURE 1 ABOUT HERE]**

---

<sup>3</sup> Models of this form (with a pole at the non-zero frequency) were also considered, among others, by Giraitis, Hidalgo and Robinson (2001), Hidalgo and Soulier (2004) and Hidalgo (2005). These authors assume that the pole in the spectrum is unknown and suggest various parametric and semiparametric methods to estimate the fractional parameter ( $d_2$ ), along with the frequency of the pole in the spectrum (see also Arteche and Robinson, 2000, and Arteche, 2002).

<sup>4</sup> See also Tanaka (1999) for a time domain representation of Robinson's (1994) tests.

Figure 1 contains plots of the four series. As can be seen, both dividends and earnings appear to be quite stable for about a century, and then increase sharply in the last few decades of the sample. Interest rates and government bond yields fluctuate a lot more throughout the sample, but also seem to increase towards the end of the sample, before a significant fall.

In the following two subsections, we examine first a model with linear trends, and then one with non-linear structures. In both cases we allow for long-range dependence at the zero and non-zero (cyclical) frequencies.

### 3.1 The case of linear trends

First we consider the case of linear trends, and assume that the model contains two cyclical structures, one for the long-term behaviour of the series and the other for the cyclical component. We allow both components to display long-memory behaviour, and test the null hypothesis in (8),

$$H_o: d \equiv (d_1, d_2)^T = (d_{1o}, d_{2o})^T \equiv d_o,$$

in the following model,

$$y_t = \alpha + \beta t + x_t, \quad (10)$$

$$(1 - L)^{d_1} (1 - 2\cos w_r L + L^2)^{d_2} x_t = u_t, \quad (11)$$

under the assumption that the disturbance term  $u_t$  is white noise, AR(1) and AR(2) respectively. Higher AR orders were also employed and the results do not substantially alter the conclusions based on these two first orders. In all cases, we test  $H_o$  for  $(d_{1o}, d_{2o})$ -values from -1 to 3 with 0.01 increments, and  $r = 2, 3, \dots, T/2$ ,<sup>5</sup> choosing as

---

<sup>5</sup> Note that in case of  $r = 1$ , the polynomial  $(1 - 2\cos w_r L + L^2)^{d_2}$  becomes  $(1 - L)^{2d_2}$ , implying the existence of a pole at the long run or zero frequency.

estimates of  $d_1$  and  $d_2$  the values of  $d_{10}$  and  $d_{20}$  that produce the lowest statistics. These values should be an approximation to the maximum likelihood estimates, noting that Robinson's (1994) method is based on the Whittle function, which is an approximation to the likelihood function.<sup>6</sup> Given that some of the coefficients in (10) were not significant, we also carried out the computations in a model with only an intercept (i.e.  $\beta = 0$  a priori) and with no regressors at all ( $\alpha = \beta = 0$  a priori). The results for the four series are displayed in Tables 1A – 4A.

Also, noting that in some cases the order of integration for the cyclical part ( $d_2$ ) was not statistically significantly different from zero, we also perform the analysis with a single fractional differencing parameter, i.e., employing

$$(1 - L)^{d_1} x_t = u_t, \quad (12)$$

rather than (11). The results for this case are displayed in Tables 1B – 4B. We describe first of all the results for the trend-cyclical case.

### **[INSERT TABLES 1A – 4A ABOUT HERE]**

The first remark to make is that the parameter  $r$  (indicating the number of time periods per cycle) is constrained between 2 and 15 in all cases, being around 8 in the majority of cases. This is consistent with the empirical findings in the business cycle literature (Canova, 1998; Burnside, 1998; King and Rebelo, 1999; etc.) according to which cycles have a periodicity between five and ten years. It is also noteworthy that the order of integration at the long-run or zero frequency (i.e.,  $d_1$ ) is substantially higher than its corresponding value at the cyclical frequency ( $d_2$ ), especially for earnings and

---

<sup>6</sup> Several Monte Carlo experiments based on this approach were conducted by Caporale and Gil-Alana (2006), and Gil-Alana (2007). It is shown in these papers that this method correctly determines the orders of integration at the two frequencies for samples of similar size to those employed in this article.

interest rates. For these two series the unit root null cannot be rejected at the long-run frequency ( $d_1 = 1$ ), while  $d_2$  is found to be strictly below 1 in all cases. In Tables 1A – 4A we report in bold the cases where the null hypothesis of white noise errors cannot be rejected at conventional statistical levels. There are two such cases for dividends, a single one for earnings, six for interest rates and five for government bond yields. Among these selected models we choose the best specification on the basis of LR tests and other likelihood criteria. The selected model for each series is as follows. For dividends,

$$y_t = -6.891 + x_t; \quad (1-L)^{1.48} (1-2\cos w_{10}L+L^2)^{0.52} x_t = \varepsilon_t. \quad (1A)$$

For earnings,

$$y_t = 10.348 + x_t; \quad (1-L)^{1.19} (1-2\cos w_7L+L^2)^{0.39} x_t = u_t; \\ u_t = -0.284u_{t-1} - 0.503u_{t-2} + \varepsilon_t. \quad (2A)$$

For interest rates,

$$(1-L)^{0.76} (1-2\cos w_8L+L^2)^{0.18} x_t = u_t; \quad u_t = -0.217u_{t-1} - 0.211u_{t-2} + \varepsilon_t. \quad (3A)$$

Finally, for the government bond yields,

$$y_t = -23.090 + x_t; \quad (1-L)^{0.96} (1-2\cos w_8L+L^2)^{0.28} x_t = u_t. \\ u_t = -0.182u_{t-1} + \varepsilon_t. \quad (4A)$$

Considering the confidence bands for the orders of integration of these selected models we see that for dividends and earnings (Tables 1A and 2A),  $d_1$  is strictly above 1 while  $d_2$  is constrained between 0 and 1. On the other hand, for interest rates and bond yields (Tables 3A and 4A) we cannot reject the null hypotheses of  $d_1 = 1$  and  $d_2 = 0$ .

**[INSERT TABLES 1B – 4B ABOUT HERE]**

Next we examine the case of a single pole at the long-run or zero frequency (Tables 1B – 4B). Here we notice that for dividends the order of integration is much higher than 1, being even above 2 in three cases. For earnings and government bond yields, some values are below 1 while others are above 1. Finally, for interest rates, the estimated order of integration is below 1 in all cases and in six out of nine cases the unit root null is rejected in favour of smaller orders of integration. Using this specification, the selected model for dividends is the following:

$$(1-L)^{1.57} x_t = u_t; \quad u_t = 0.677u_{t-1} - 0.211u_{t-2} + \varepsilon_t, \quad (1B)$$

In case of earnings, the chosen specification is:

$$y_t = 0.575 - 0.434t + x_t; \quad (1-L)^{1.38} x_t = u_t; \quad u_t = 0.023u_{t-1} - 0.445u_{t-2} + \varepsilon_t. \quad (2B)$$

For interest rates,

$$(1-L)^{0.64} x_t = u_t; \quad u_t = 0.244u_{t-1} - 0.083u_{t-2} + \varepsilon_t, \quad (3B)$$

and finally, for government bond yields

$$(1-L)^{1.00} x_t = u_t; \quad u_t = -0.159u_{t-1} + \varepsilon_t. \quad (4B)$$

According to these models the four series are nonstationary, and the unit root hypothesis is rejected in favour of higher orders of integration in case of the dividend series.

### 3.2 The case of non-linear trends

Next we allow for possible non-linearities, and assume that the four series exhibit a single break.<sup>7</sup> Figure 1 suggests that there might be a break around 1973, the time of the first oil price crisis. We experimented with a change in the level, in the slope and in both of them, and came to the conclusion that a level change was the most plausible one for government bond yields, while for the remaining three series we allowed for a

---

<sup>7</sup> Multiple breaks could also be considered. However, we believe that the series examined in this paper can be adequately described including a single structural break. Note that allowing for multiple breaks would result in short subsamples and inaccurate estimates of the coefficients.

change in both level and slope after the break. Specifically, we consider a model of the form:

$$y_t = \alpha_1 I(t < 1973) + \alpha_2 I(t \geq 1973) + x_t, \quad (13)$$

for government bond yields, and

$$y_t = \alpha_1 I(t < 1973) + \alpha_2 I(t \geq 1973) + \beta t I(t \geq 1973) + x_t, \quad (14)$$

for the remaining three series, allowing two fractional structures as in (11) (in Tables 1C – 4C) and with a single fractional differencing polynomial at the zero frequency as in (12) (in Tables 1D – 4D):<sup>8</sup>

**[INSERT TABLES 1C – 4D ABOUT HERE]**

Starting again with the case of two poles in the spectrum (i.e., using equation 11), the selected models for dividends (with  $T^* = 1973$ ) in Tables 1C – 4C were:

$$\begin{aligned} y_t &= -6.006 I(t < T^*) - 20.204 I(t \geq T^*) + 0.452 t I(t \geq T^*) + x_t; \\ (1-L)^{1.56} (1 - 2 \cos w_{11} L + L^2)^{0.53} x_t &= u_t; \quad u_t = 0.308 u_{t-1} + \varepsilon_t. \end{aligned} \quad (1C)$$

For earnings,

$$\begin{aligned} y_t &= -121.003 I(t < T^*) - 104.15 I(t \geq T^*) - 1.039 t I(t \geq T^*) + x_t; \\ (1-L)^{1.32} (1 - 2 \cos w_7 L + L^2)^{0.52} x_t &= u_t; \quad u_t = -0.117 u_{t-1} - 0.260 u_{t-2} + \varepsilon_t. \end{aligned} \quad (2C)$$

For interest rates,

$$\begin{aligned} y_t &= -75.817 I(t < T^*) - 63.114 I(t \geq T^*) - 0.636 t I(t \geq T^*) + x_t; \\ (1-L)^{0.38} (1 - 2 \cos w_{15} L + L^2)^{1.23} x_t &= u_t; \quad u_t = -0.237 u_{t-1} + \varepsilon_t. \end{aligned} \quad (3C)$$

and finally, for government bond yields,

---

<sup>8</sup> We also considered other break dates for the four series, and the coefficients in (13) and (14) were insignificant in the majority of the cases.

$$\begin{aligned}
y_t &= -25.644I(t < T^*) - 28.013I(t \geq T^*) + x_t; \\
(1-L)^{0.95}(1-2\cos w_3L + L^2)^{0.14}x_t &= u_t; \quad u_t = -0.088u_{t-1} + \varepsilon_t.
\end{aligned} \tag{4C}$$

It can be seen that the number of periods per cycle varies substantially depending on the series. Specifically, it is 11 for dividends, 7 for earnings, 15 for interest rates, and 3 for government bond yields. The order of integration at the long-run or zero frequency is higher than the cyclical one for dividends, earnings and government bond yields, while the opposite holds for interest rates. For the first two series,  $d_1$  is significantly higher than 1, while  $d_2$  is in the interval (0, 1) for the three latter series. Surprisingly, for the interest rate  $d_1$  is strictly smaller than 1 while  $d_2$  is significantly above 1.

#### [INSERT TABLES 1D – 4D ABOUT HERE]

When we assume that there is a single pole occurring at the long-run or zero frequency, (Tables 1D – 4D), the deterministic terms are found to be mostly insignificant in the case of dividends and earnings, while they are all significant in the case of interest rates and government bond yields. The order of integration appears to be highly sensitive to the chosen specification for the disturbance term, especially for dividends and earnings. For instance, for dividends,  $d_1$  is above 2 in case of a white noise  $u_t$ ; it is 1.24 (and the unit root null cannot be rejected) if  $u_t$  is AR(1), and it is strictly smaller than 1 with an AR(2)  $u_t$ . The selected models in this case are the following: for dividends,

$$\begin{aligned}
y_t &= -1.744I(t < T^*) - 2.878I(t \geq T^*) + 0.485tI(t \geq T^*) + x_t; \\
(1-L)^{0.63}x_t &= u_t; \quad u_t = 1.372u_{t-1} - 0.745u_{t-2} + \varepsilon_t.
\end{aligned} \tag{1D}$$

For earnings,

$$\begin{aligned}
y_t &= 0.231I(t < T^*) + 0.269I(t \geq T^*) + 1.641tI(t \geq T^*) + x_t; \\
(1-L)^{1.17}x_t &= u_t; \quad u_t = 0.162u_{t-1} - 0.401u_{t-2} + \varepsilon_t.
\end{aligned} \tag{2D}$$

For interest rates,

$$\begin{aligned}
y_t &= 2.751I(t < T^*) + 7.105I(t \geq T^*) - 0.258tI(t \geq T^*) + x_t; \\
(1-L)^{0.45}x_t &= u_t; \quad u_t = 0.333u_{t-1} + 0.007u_{t-2} + \varepsilon_t.
\end{aligned} \tag{3D}$$

and for government bond yields,

$$\begin{aligned}
y_t &= 5.279I(t < T^*) + 5.761I(t \geq T^*) + x_t; \\
(1-L)^{0.99}x_t &= u_t; \quad u_t = -0.183u_{t-1} - 0.007u_{t-2} + \varepsilon_t.
\end{aligned} \tag{4D}$$

According to these specifications, dividends, government bond yields and earnings are nonstationary variables, while interest rates is the only one with a fractional differencing parameter in the stationary region ( $d < 0.5$ ). However, the confidence intervals indicate that dividends is the only series for which the unit root null ( $d = 1$ ) is rejected.

#### 4. Forecasting performance

This section examines the forecasting performance of the models previously selected. For each of the series we consider the four model specifications given by equations (1A) – (4D). First, we compute the  $k$  ( $=1, 2, \dots, 10$ )-ahead prediction errors of each model, obtained by expanding the fractional polynomials in (5) and (7). Tables 5 – 8 report the Root Mean Squared Errors (RMSE) of each specification for each series. It can be seen that for dividends (Table 5) model (1B) appears to be the best based on the 1-period ahead prediction. However, for longer horizons, model (1D) (i.e., a single fractional polynomial at the long-run frequency along with a non-linear trend) seems to perform best in all cases. A similar conclusion is reached for earnings (Table 6) and government bond yields (Table 8). Thus, based on the 1-period ahead predictions, the



model with a single fractional differencing polynomial (equations (2B) and (4B)) performs best, while the one with a non-linear trend is preferred in the remaining cases (equations (2D) and (4D)). Finally, for interest rates (Table 7) models (3A) and (3D) are the most adequate ones: based on the 1, 2, 5, 6 and 7-period ahead predictions, the model with a non-linear trend and a single fractional polynomial seems to be the most adequate. However, when the forecasting horizon is 3, 4, 8, 9 and 10 periods ahead, the specification with two fractional polynomials is the preferred one.

**[INSERT TABLES 5 – 8 ABOUT HERE]**

Overall, for dividends, earnings and government bond yields, the model with a single fractional polynomial at the zero frequency predicts better 1-period ahead; however, for longer horizons, a model with a non-linear trend (and also a fractional process at the zero frequency) outperforms the rival models. For interest rates, the results are slightly more ambiguous: the model with two polynomials (at the zero and the cyclical frequency) seems to be the most adequate one in some cases, but a non-linear model with a single polynomial at the zero frequency appears to be preferable in other cases.

The results presented so far as based on the RMSE. However, this criterion along with other methods such as the Mean Absolute Prediction Error (MAPE), Mean Squared Error (MSE), Mean Absolute Deviation (MAD), etc., is a purely descriptive device.<sup>9</sup> Several statistical tests for comparing different forecasting models are now available. One of them, widely employed in the time series literature, is the asymptotic

---

<sup>9</sup> The accuracy of different forecasting methods is a topic of continuing interest and research (see, e.g., Makridakis et al., 1998 and Makridakis and Hibon, 2000, for a review of the forecasting accuracy of competing forecasting models).

test for a zero expected loss differential due to Diebold and Mariano (1995).<sup>10</sup> Harvey, Leybourne and Newbold (1997) note that the Diebold-Mariano test statistic could be seriously over-sized as the prediction horizon increases, and therefore provide a modified Diebold-Mariano test statistic given by:

$$M - DM = DM \sqrt{\frac{n + 1 - 2h + h(h-1)/n}{n}},$$

where DM is the original Diebold-Mariano statistic, h is the prediction horizon and n is the time span for the predictions. Harvey et al. (1997) and Clark and McCracken (2001) show that this modified test statistic performs better than the DM test statistic (though still poorly in finite samples), and also that the power of the test is improved when p-values are computed with a Student t-distribution.

Using the M-DM test statistic, we further evaluate the relative forecast performance of the different models by making pairwise comparisons. We consider 2, 4, 6 and 8-period ahead forecasts on a 10-period horizon. The results are displayed in Tables 9 – 12, and are consistent with the previous ones.

### **[INSERT TABLES 9 – 12 ABOUT HERE]**

In particular, models (1D), (2D), (3D) and (4D) are preferred in most cases, especially based on the 2- and 4-period ahead prediction horizons. Only for interest rates does model (3A) outperform (3D) in some cases.

---

<sup>10</sup> An alternative approach is the bootstrap-based test of Ashley (1998), though his method is computationally more intensive.

## 5. Conclusions

In this paper we have introduced a new time series approach to modelling long-run trends and cycles in financial time series data. The proposed model is general enough to include linear and non-linear trends along with fractional integration at zero and non-zero (cyclical) frequencies. It is based on the testing procedure developed by Robinson (1994) for stationary and nonstationary hypotheses. We have used our framework to investigate the behaviour of four financial time series already examined in many earlier studies. Specifically, we have used the annual dataset including dividends, earnings, interest rates and government bond yields, which was constructed (and is constantly updated) by Robert Shiller.

The results can be summarised as follows. It appears that the four series of interest can be characterised in terms of long-memory processes with two poles in the spectrum, one corresponding to the long-run or zero frequency, and the other one to the cyclical component. The latter exhibits a periodicity ranging between 3 and 15 years depending on the series and the model considered. In general, the order of integration is higher at the zero frequency, implying that the degree of persistence is higher in this component. When non-linear trends are incorporated, the models outperform the linear ones in terms of their forecasting accuracy, especially over longer horizons.

This paper can be extended in several directions. First, multiple cyclical structures of the form advocated by Ferrara and Guegan (2001) and others can be considered. In fact, the interaction between cyclical (fractional) processes may produce autocorrelations decaying in a very complicated way that has not been much investigated yet. Other more complex non-linear structures (like the Threshold AutoRegressive, TAR, Momentum Threshold AutoRegressive, M-TAR or Smooth Transition Autoregressive, STAR-form (see, e.g. Enders and Granger, 1998; Enders and

Siklos, 2001; Skalin and Teräsvirta, 2002) can also be included in the regression model (1). Finally, the date(s) of the structural break(s) can be endogenously determined in the context of the general model described by equations (1) – (3). Future research will address these issues.

## Appendix

The test statistic proposed by Robinson (1994) for testing  $H_0$  (8) in (1) - (3) is given by:

$$\hat{R} = \frac{T}{\hat{\sigma}^4} \hat{a}' \hat{A}^{-1} \hat{a},$$

where  $T$  is the sample size, and

$$\hat{a} = \frac{-2\pi}{T} \sum_j^* \psi(\lambda_j) g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j); \quad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j),$$

$$\hat{A} = \frac{2}{T} \left( \sum_j^* \psi(\lambda_j) \psi(\lambda_j)' - \sum_j^* \psi(\lambda_j) \hat{\varepsilon}(\lambda_j)' \left( \sum_j^* \hat{\varepsilon}(\lambda_j) \hat{\varepsilon}(\lambda_j)' \right)^{-1} \sum_j^* \hat{\varepsilon}(\lambda_j) \psi(\lambda_j)' \right)$$

$$\psi(\lambda_j)' = [\psi_1(\lambda_j), \psi_2(\lambda_j)]; \quad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log g(\lambda_j; \hat{\tau}); \quad \psi_1(\lambda_j) = \log \left| 2 \sin \frac{\lambda_j}{2} \right|;$$

$$\psi_2(\lambda_j) = \log \left| 2 (\cos \lambda_j - \cos w_r) \right|, \text{ with } \lambda_j = 2\pi j/T, \text{ and the summation in } * \text{ is over all}$$

frequencies which are bounded in the spectrum.  $I(\lambda_j)$  is the periodogram of

$$\hat{u}_t = (1-L)^{d_{1o}} (1-2\cos w_r L + L^2)^{d_{2o}} y_t - \hat{\beta}' \bar{z}_t, \text{ with}$$

$$\hat{\beta} = \left( \sum_{t=1}^T \bar{z}_t \bar{z}_t' \right)^{-1} \sum_{t=1}^T \bar{z}_t (1-L)^{d_{1o}} (1-2\cos w_r L + L^2)^{d_{2o}} y_t;$$

$$\bar{z}_t = (1-L)^{d_{1o}} (1-2\cos w_r L + L^2)^{d_{2o}} z_t, \text{ evaluated at } \lambda_j = 2\pi j/T \text{ and } \hat{\tau} =$$

$\arg \min_{\tau \in T^*} \sigma^2(\tau)$ , with  $T^*$  as a suitable subset of the  $R^q$  Euclidean space. Finally, the

function  $g$  above is a known function coming from the spectral density of  $u_t$ :

$$f(\lambda; \tau) = \frac{\sigma^2}{2\pi} g(\lambda; \tau), \quad -\pi < \lambda \leq \pi.$$

Note that these tests are purely parametric and, therefore, they require specific modelling assumptions about the short-memory specification of  $u_t$ . Thus, if  $u_t$  is white

noise,  $g \equiv 1$ , and if  $u_t$  is an AR process of the form  $\phi(L)u_t = \varepsilon_t$ ,  $g = |\phi(e^{i\lambda})|^{-2}$ , with  $\sigma^2 = V(\varepsilon_t)$ , so that the AR coefficients are a function of  $\tau$ .

## References

- Ahtola, J. and Tiao, G.C., 1987, Distributions of least squares estimators of autoregressive parameters for a process with complex roots on the unit circle, *Journal of Time Series Analysis* 8, 1-14.
- Anh, V.V., V.P. Knopova and N.N. Leonenko, 2004, Continuous-time stochastic processes with cyclical long range dependence, *Australian and New Zealand Journal of Statistics* 46, 275-296.
- Arteche, J., 2002, Semiparametric robust tests on seasonal or cyclical long memory time series, *Journal of Time Series Analysis* 23, 251-268.
- Arteche, J. and P.M. Robinson, 2000, Semiparametric inference in seasonal and cyclical long memory processes, *Journal of Time Series Analysis* 21, 1-27.
- Ashley, R., 1998, A new technique for postsample model selection and validation, *Journal of Economics Dynamics and Control* 22, 647-665.
- Bierens, H.J., 2001, Complex unit roots and business cycles: Are they real? *Econometric Theory* 17, 962-983.
- Burnside, A.C., 1998, Detrending and business cycle facts. A comment, *Journal of Monetary Economics* 41, 513-532.
- Canova, F., 1998, Detrending and business cycle facts. A user's guide, *Journal of Monetary Economics* 41, 533-540.
- Caporale, G.M. and L.A. Gil-Alana, 2002, Fractional integration and mean reversion in stock prices, *Quarterly Review of Economics and Finance* 42, 599-609.
- Caporale, G.M. and L.A. Gil-Alana, 2006, Long memory at the long run and cyclical frequencies. Modelling real wages in England: 1260-1994, *Empirical Economics* 31(1), 83-92.

- Caporale, G.M. and L.A. Gil-Alana, 2007, Long run and cyclical dynamics in the US stock market, CESifo Working Paper no. 2046.
- Clark, T.E. and M.W. McCracken, 2001, Tests of forecast accuracy and encompassing for nested models, *Journal of Econometrics* 105, 85-110.
- Diebold, F.X. and R.S. Mariano, 1995, Comparing predictive accuracy, *Journal of Business, Economics and Statistics* 13, 253-263.
- Diebold, F.X. and G.D. Rudebusch, 1989, Long memory and persistence in the aggregate output. *Journal of Monetary Economics* 24, 189-209.
- Dittmar, R.F., 2002, Nonlinear Pricing Kernels, Kurtosis Preference, and Evidence from the Cross Section of Equity Returns, *Journal of Finance, American Finance Association*, vol. 57(1), 369-403.
- Enders, W. and Granger, C.W.J., 1998. Unit root tests and asymmetric adjustment with an example using the term structure of interest rates. *Journal of the American Statistical Association* 16(3), 304-311.
- Enders, W. and Siklos, P., 2001, Cointegration and threshold adjustment. *Journal of Business and Economic Statistics* 19(2), 166-176.
- Fama, E.F., 1970, Efficient capital markets: a review of theory and empirical work, *Journal of Finance* 25, 383-417.
- Fama, E.F. and K.R. French, 1988, Permanent and transitory components of stock prices, *Journal of Political Economy* 96, 246-273.
- Ferrara, L. and D. Guegan, 2001, Forecasting with k-factor Gegenbauer processes. Theory and Applications. *Journal of Forecasting* 20, 581-601.
- Gil-Alana, L.A., 2001, Testing stochastic cycles in macroeconomic time series. *Journal of Time Series Analysis* 22, 411-430.

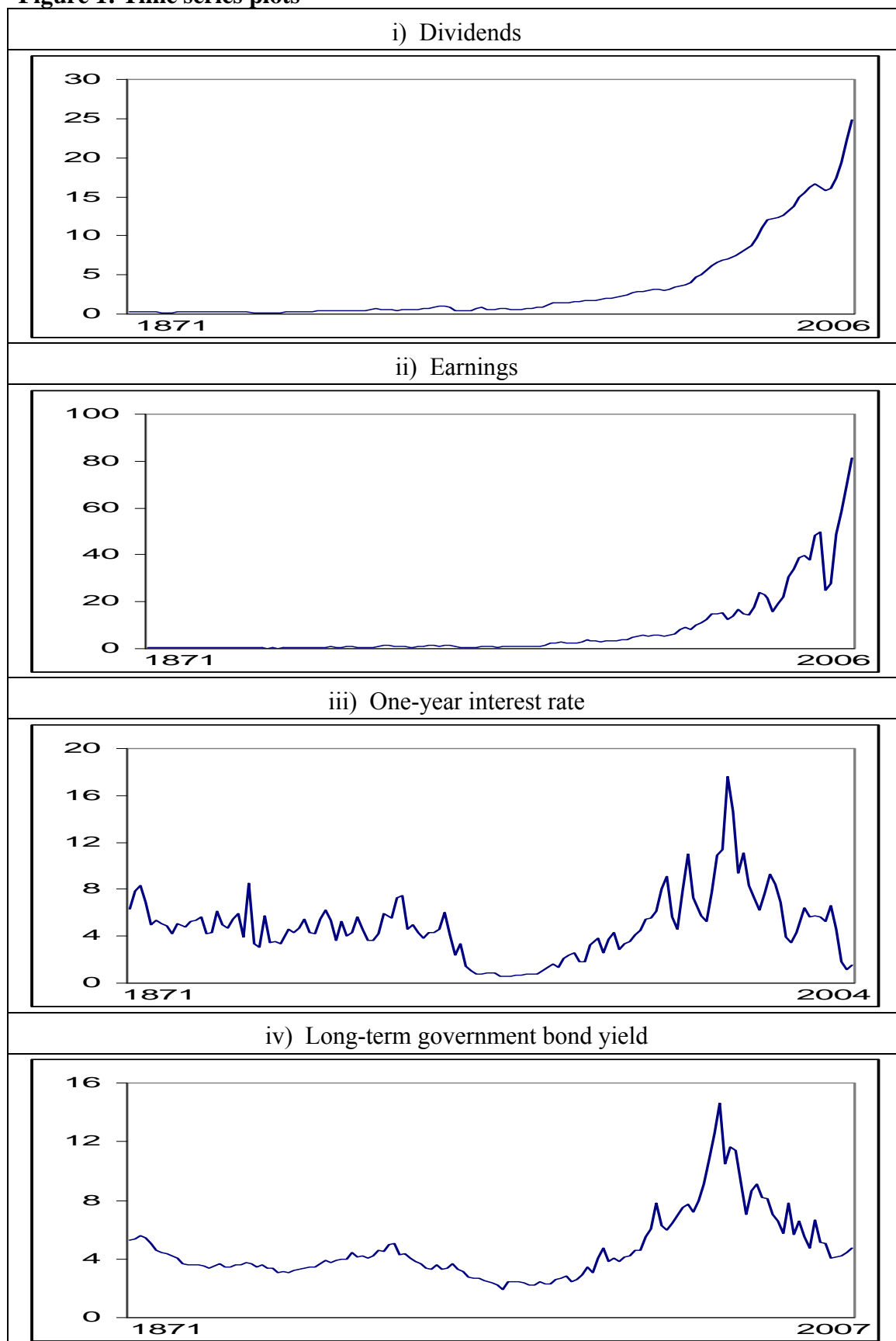


- Gil-Alana, L.A., 2007a, Testing the existence of multiple cycles in financial and economic time series. *Annals of Economics and Finance* 1, 1-20.
- Gil-Alana, L.A., 2007b, Long run and cyclical strong dependence in macroeconomic time series. *Nelson and Plosser revisited*, *Empirica* 34(2), 139-154.
- Gil-Alana, L.A. and P.M. Robinson, 1997, Testing of unit roots and other nonstationary hypotheses in macroeconomic time series. *Journal of Econometrics* 80, 241-268.
- Giraitis, L., J. Hidalgo and P.M. Robinson, 2001, Gaussian estimation of parametric spectral density with unknown pole, *Annals of Statistics* 29, 987-1023.
- Gray, H.L., Yhang, N. and Woodward, W.A., 1989, On generalized fractional processes, *Journal of Time Series Analysis* 10, 233-257.
- Gray, H.L., Yhang, N. and Woodward, W.A., 1994, On generalized fractional processes. A correction, *Journal of Time Series Analysis* 15, 561-562.
- Harvey, D.I., S.J. Leybourne and P. Newbold, 1997, Testing the equality of prediction mean squared errors, *International Journal of Forecasting* 13, 281-291.
- Hassler, U., 1994, Regression of spectral estimators with fractionally integrated time series, *Journal of Time Series Analysis* 14, 360-379.
- Hidalgo, J., 2005, Semiparametric estimation for stationary processes whose spectra have an unknown pole, *Annals of Statistics* 35, 1843-1889.
- Hidalgo, J. and P. Soulier, 2004, Estimation of the location and exponent of the spectral singularity of a long memory process, *Journal of Time Series Analysis* 25, 55-81.
- Hosoya, Y., 1997, A limit theorem for long run dependence and statistical inference on related models, *Annals of Statistics* 25, 105-137.
- Hossein, A. and K. Sonnie, 2006, Evaluating a nonlinear asset pricing model on international data, W.P. 2006:5, Department of Economics, School of Economics and Management, Lund University, Lund, Sweden.

- King, R.G. and S.T. Rebelo, 1999, Resuscitating real business cycles, in J.B. Taylor and M. Woodford eds., *Handbook in Macroeconomics* 1, 928-1001.
- Li, K., 2002, Long-memory versus option-implied volatility predictions, *Journal of Derivatives*, 9 (Fall), 9-25.
- Magnus, W., Oberhettinger, F. and R.P. Soni, 1966, *Formulas and theorems for the special functions of mathematical physics*. Springer, Berlin.
- Makridakis, S. and M. Hibon, 2000, The M-3 competition: results, conclusions and implications, *International Journal of Forecasting*, 16, 451-476.
- Makridakis, S., S. Wheelwright and R. Hyndman, 1998, *Forecasting methods and applications*, 3<sup>rd</sup> Edition, John Wiley & Sons.
- Martens, M. and J. Zein, 2004, Predicting financial volatility: high-frequency time-series forecasts vis-à-vis implied volatility, *Journal of Futures Markets*, 24, 1005-1028.
- Ooms, M., 1997, Flexible seasonal long memory and economic time series, *Econometrics Institute Report* 134, University of Rotterdam, Econometrics.
- Pong, S., M.B. Shackleton, S. J. Taylor and X. Xu, 2004, Forecasting currency volatility: a comparison of implied volatilities and AR(FI)MA models, *Journal of Banking and Finance*, 28, 2541-2563.
- Poterba, J.M. and L.H. Summers, 1988, Mean reversion in stock prices: evidence and implications, *Journal of Financial Economics* 22, 27-59.
- Rainville, E.D., 1960, *Special functions*, MacMillan, New York.
- Robinson, P.M., 1994, Efficient tests of nonstationary hypotheses, *Journal of the American Statistical Association* 89, 1420-1437.
- Sadek, N. and A. Khotanzad, 2004, K-factor Gegenbauer ARMA process for network traffic simulation. *Computers and Communications* 2, 963-968.
- Shiller, R., 1989, *Market Volatility*, Cambridge, Mass.: MIT Press.

- Skalin, J. and Teräsvirta, T., 2002. Modelling asymmetries and moving equilibria in unemployment rates. *Macroeconomic Dynamics*, 6, 202-241.
- Soares, L.J. and L.R. Souza, 2006, Forecasting electricity demand using generalized long memory, *International Journal of Forecasting* 22, 17-28.
- Sowell, F., 1992, Maximum likelihood estimation of stationary univariate fractionally integrated time series models, *Journal of Econometrics* 53, 165-188.
- Summers, L.H., 1986, Does the stock market rationally reflect fundamental values?, *Journal of Finance* 41, 591-601.
- Tanaka, K., 1999, The nonstationary fractional unit root, *Econometric Theory* 15, 549-582.

**Figure 1: Time series plots**



**Table 1A: Coefficient estimates with two fractional structures. Series: Dividends**

	$\alpha$	$\beta$	$d_1$	$r$	$d_2$	$\rho_1$	$\rho_2$
AR(2) with time trend	-462.89 (-61.42)	-0.075 (-0.80)	0.02 [-0.09, 0.14]	8	0.60 [0.47, 1.54]	0.804	-0.420
AR(2) with an intercept	-2.257 (-7.59)	---	1.16 [0.76, 1.54]	15	0.99 [0.33, 1.37]	-0.422	-0.212
<b>AR(2) with no regressors</b>	---	---	<b>1.51</b> <b>[0.84, 1.97]</b>	<b>9</b>	<b>0.53</b> <b>[0.03, 0.87]</b>	<b>-0.207</b>	<b>-0.099</b>
AR(1) with time trend	-2.023 (-2.23)	-24.501 (-98.28)	0.96 [0.91, 1.04]	13	0.98 [0.93, 1.42]	0.412	---
AR(1) with an intercept	42.283 (11.73)	---	1.47 [1.26, 1.87]	3	0.80 [0.64, 0.96]	-0.091	---
AR(1) with no regressors	---	---	1.36 [1.19, 1.53]	5	0.06 [0.00, 0.14]	0.691	---
White noise with trend	-431.63 (-33.46)	-1.710 (-10.66)	0.13 [0.06, 0.17]	15	1.01 [0.97, 1.07]	---	---
<b>White noise + intercept*</b>	<b>-6.891</b> <b>(-9.32)</b>	---	<b>1.48</b> <b>[1.31, 1.71]</b>	<b>10</b>	<b>0.52</b> <b>[0.30, 0.69]</b>	---	---
White noise with no reg.	---	---	1.17 [1.06, 1.35]	12	0.58 [0.47, 0.71]	---	---

In bold, the models for which the null hypothesis of white noise errors cannot be rejected. “\*” indicates the best model specification using LR tests and other likelihood criteria.

**Table 2A: Coefficient estimates with two fractional structures. Series: Earnings**

	$\alpha$	$\beta$	$d_1$	$r$	$d_2$	$\rho_1$	$\rho_2$
AR(2) with time trend	-54.618 (-3.45)	- 61.003	0.93 [0.86, 1.19]	8	0.73 [0.50, 0.91]	0.174	0.053
<b>AR(2) with an intercept*</b>	<b>10.348</b> <b>(0.89)</b>	---	<b>1.19</b> <b>[1.04, 1.81]</b>	<b>7</b>	<b>0.39</b> <b>[0.09, 0.51]</b>	<b>-0.284</b>	<b>-0.503</b>
AR(2) with no regressors	---	---	1.06 [0.60, 1.77]	8	0.10 [-0.37, 0.69]	0.130	-0.371
AR(1) with time trend	-10.996 (-0.98)	- 87.225	1.09 [0.89, 1.18]	11	0.74 [0.30, 0.94]	-0.165	---
AR(1) with an intercept	733.08 (11.79)	---	1.20 [0.98, 1.47]	2	0.92 [0.68, 1.07]	-0.069	---
AR(1) with no regressors	---	---	0.79 [0.28, 1.23]	3	0.16 [-0.02, 0.31]	0.560	---
White noise with trend	-51.997 (-3.50)	- 63.653	0.95 [0.89, 1.15]	10	0.72 [0.56, 0.97]	---	---
White noise with intercept	-131.86 (-7.26)	---	1.08 [1.01, 1.16]	8	0.42 [0.22, 0.66]	---	---
White noise with no reg.	---	---	0.93 [0.76, 1.21]	6	0.24 [0.01, 0.54]	---	---

In bold, the models for which the null hypothesis of white noise errors cannot be rejected. “\*” indicates the best model specification using LR tests and other likelihood criteria.

**Table 3A: Coefficient estimates with two fractional structures. Series: Interest rates**

	$\alpha$	$\beta$	$d_1$	$r$	$d_2$	$\rho_1$	$\rho_2$
AR(2) with time trend	-70.345 (-8.33)	-3.626 (-35.50)	0.68 [0.60, 1.54]	12	0.41 [0.07, 0.62]	0.115	0.137
AR(2) with an intercept	-144.46 (-8.19)	---	1.18 [0.80, 1.64]	4	0.11 [-0.30, 0.29]	0.091	-0.247
<b>AR(2) with no regressors*</b>	---	---	<b>0.76</b> [0.17, 1.11]	<b>8</b>	<b>0.18</b> [-0.15, 0.92]	<b>-0.217</b>	<b>-0.211</b>
AR(1) with time trend	62.566 (4.27)	14.998 (39.80)	1.17 [0.75, 1.32]	3	0.11 [-0.27, 0.34]	0.075	---
<b>AR(1) with an intercept</b>	<b>-120.01</b> (-7.96)	---	<b>1.08</b> [0.78, 1.47]	<b>2</b>	<b>0.09</b> [-0.05, 0.72]	<b>-0.166</b>	---
<b>AR(1) with no regressors</b>	---	---	<b>0.53</b> [0.12, 0.88]	<b>8</b>	<b>0.05</b> [-0.13, 0.38]	<b>0.204</b>	---
<b>White noise with trend</b>	<b>-48.096</b> (-5.45)	<b>-2.212</b> (-26.53)	<b>0.74</b> [0.71, 0.78]	<b>15</b>	<b>0.19</b> [-0.04, 0.46]	---	---
<b>White noise + intercept</b>	<b>-83.753</b> (-6.50)	---	<b>1.44</b> [1.32, 1.57]	<b>3</b>	<b>0.05</b> [-0.05, 0.29]	---	---
<b>White noise with no reg.</b>	---	---	<b>0.59</b> [0.45, 0.84]	<b>15</b>	<b>0.05</b> [-0.05, 0.20]	---	---

In bold, the models for which the null hypothesis of white noise errors cannot be rejected. “\*” indicates the best model specification using LR tests and other likelihood criteria.

**Table 4A: Coefficient estimates with two fractional structures. Series: Government bond yields**

	$\alpha$	$\beta$	$d_1$	$r$	$d_2$	$\rho_1$	$\rho_2$
<b>AR(2) with time trend</b>	<b>26.951</b> (2.29)	<b>3.889</b> (17.66)	<b>1.20</b> [0.86, 1.94]	<b>8</b>	<b>0.09</b> [-0.29, 0.44]	<b>-0.334</b>	<b>-0.047</b>
AR(2) with an intercept	-2.098 (-1.43)	---	0.95 [0.67, 1.09]	11	1.00 [0.40, 1.24]	0.998	-0.453
AR(2) with no regressors	---	---	0.10 [-0.14, 1.11]	4	0.76 [0.16, 1.53]	0.622	0.337
AR(1) with time trend	-84.161 (-10.97)	-4.990 (-54.51)	0.71 [0.61, 1.02]	14	0.42 [0.17, 0.58]	0.489	---
<b>AR(1) with an intercept*</b>	<b>-23.090</b> (-4.75)	---	<b>0.96</b> [0.88, 1.02]	<b>8</b>	<b>0.28</b> [-0.04, 0.87]	<b>-0.182</b>	---
<b>AR(1) with no regressors</b>	---	---	<b>0.75</b> [0.39, 1.16]	<b>3</b>	<b>0.01</b> [-0.17, 0.36]	<b>0.097</b>	---
White noise with trend	-36.744 (-3.53)	-2.563 (-4.123)	0.89 [0.85, 0.94]	15	0.08 [-0.04, 0.39]	---	---
<b>White noise + intercept</b>	<b>-39.023</b> (-6.84)	---	<b>1.30</b> [1.05, 1.46]	<b>3</b>	<b>0.09</b> [-0.04, 0.62]	---	---
<b>White noise with no reg.</b>	---	---	<b>0.82</b> [0.70, 1.01]	<b>2</b>	<b>0.01</b> [-0.06, 0.12]	---	---

In bold, the models for which the null hypothesis of white noise errors cannot be rejected. “\*” indicates the best model specification using LR tests and other likelihood criteria.

**Table 1B: Coefficient estimates with one fractional structure. Series: Dividends**

	$\alpha$	$\beta$	$d_1$	$\rho_1$	$\rho_2$
AR(2) with time trend	0.298 (1.448)	-0.113 (-0.522)	1.68 [1.07, 2.05]	0.577	-0.167
AR(2) with an intercept	0.241 (1.405)	---	1.73 [1.11, 2.05]	0.530	-0.150
<b>AR(2) with no regressors*</b>	---	---	<b>1.57</b> <b>[1.01, 2.00]</b>	<b>0.677</b>	<b>-0.211</b>
AR(1) with time trend	0.282 (1.268)	-0.091 (-1.049)	1.34 [1.21, 1.56]	0.699	---
AR(1) with an intercept	0.234 (1.068)	---	1.33 [1.20, 1.52]	0.707	---
AR(1) with no regressors	---	---	1.48 [1.32, 1.62]	0.619	---
White noise with trend	0.218 (0.960)	0.042 (0.127)	2.12 [1.93, 2.33]	---	---
White noise + intercept	0.239 (1.590)	---	2.12 [1.93, 2.35]	---	---
White noise with no reg.	---	---	2.11 [1.93, 2.35]	---	---

In bold, the models for which the null hypothesis of white noise errors cannot be rejected. “\*” indicates the best model specification using LR tests and other likelihood criteria.

**Table 2B: Coefficient estimates with one fractional structure. Series: Earnings**

	$\alpha$	$\beta$	$d_1$	$\rho_1$	$\rho_2$
<b>AR(2) with time trend*</b>	<b>0.575</b> <b>(0.185)</b>	<b>-0.434</b> <b>(-0.306)</b>	<b>1.38</b> <b>[0.75, 1.82]</b>	<b>0.023</b>	<b>-0.445</b>
AR(2) with an intercept	0.349 (0.115)	---	1.36 [0.83, 1.81]	0.038	-0.438
AR(2) with no regressors	---	---	1.37 [0.93, 1.81]	0.031	-0.441
AR(1) with time trend	-3.048 (-0.911)	0.091 (0.981)	0.73 [0.55, 0.98]	0.485	---
AR(1) with an intercept	-2.701 (-0.841)	---	0.70 [0.46, 0.98]	0.519	---
AR(1) with no regressors	---	---	0.81 [0.65, 0.96]	0.413	---
White noise with trend	-0.069 (-0.019)	0.709 (1.293)	1.13 [0.99, 1.34]	---	---
White noise + intercept	0.365 (0.103)	---	1.13 [0.99, 1.34]	---	---
White noise with no reg.	---	---	1.13 [0.99, 1.34]	---	---

In bold, the models for which the null hypothesis of white noise errors cannot be rejected. “\*” indicates the best model specification using LR tests and other likelihood criteria.

**Table 3B: Coefficient estimates with one fractional structure. Series: Interest rates**

	$\alpha$	$\beta$	$d_1$	$\rho_1$	$\rho_2$
AR(2) with time trend	6.332 (4.431)	-0.055 (-1.271)	0.75 [0.53, 1.16]	0.074	-0.149
<b>AR(2) with an intercept</b>	<b>4.923</b> (4.071)	---	<b>0.62</b> [0.37, 1.16]	<b>0.206</b>	<b>-0.098</b>
<b>AR(2) with no regressors*</b>	---	---	<b>0.64</b> [0.19, 1.11]	<b>0.244</b>	<b>-0.083</b>
AR(1) with time trend	<b>5.728</b> (4.544)	<b>-0.053</b> (-2.261)	<b>0.60</b> [0.40, 0.80]	<b>0.207</b>	---
<b>AR(1) with an intercept</b>	<b>3.751</b> (3.654)	---	<b>0.53</b> [0.39, 0.78]	<b>0.289</b>	---
AR(1) with no regressors	---	---	0.53 [0.40, 0.62]	0.340	---
White noise with trend	6.457 (4.487)	-0.019 (-0.471)	0.73 [0.61, 0.91]	---	---
<b>White noise + intercept</b>	<b>6.301</b> (4.514)	---	<b>0.73</b> [0.61, 0.89]	---	---
<b>White noise with no reg.</b>	---	---	<b>0.74</b> [0.64, 0.89]	---	---

In bold, the models for which the null hypothesis of white noise errors cannot be rejected. “\*” indicates the best model specification using LR tests and other likelihood criteria.

**Table 4B: Coefficient estimates with one fractional structure. Series: Government bond yields**

	$\alpha$	$\beta$	$d_1$	$\rho_1$	$\rho_2$
AR(2) with time trend	5.333 (7.139)	-0.038 (-0.526)	1.03 [0.80, 1.40]	-0.222	-0.038
<b>AR(2) with an intercept</b>	<b>5.300</b> (7.119)	---	<b>1.03</b> [0.82, 1.40]	<b>-0.222</b>	<b>-0.038</b>
<b>AR(2) with no regressors</b>	---	---	<b>1.20</b> [1.10, 1.81]	<b>-0.372</b>	<b>-0.134</b>
AR(1) with time trend	5.324 (7.119)	-0.038 (-0.608)	1.00 [0.86, 1.17]	-0.188	---
<b>AR(1) with an intercept</b>	<b>5.279</b> (7.085)	---	<b>0.99</b> [0.85, 1.17]	<b>-0.180</b>	---
<b>AR(1) with no regressors*</b>	---	---	<b>1.00</b> [0.82, 1.30]	<b>-0.159</b>	---
White noise with trend	5.278 (7.091)	0.0001 (0.0034)	0.88 [0.78, 1.01]	---	---
<b>White noise + intercept</b>	<b>5.278</b> (7.139)	---	<b>0.88</b> [0.80, 0.98]	---	---
<b>White noise with no reg.</b>	---	---	<b>0.88</b> [0.81, 0.98]	---	---

In bold, the models for which the null hypothesis of white noise errors cannot be rejected. “\*” indicates the best model specification using LR tests and other likelihood criteria.



**Table 1C: Estimates in the nonlinear case with two fractional structures. Series: Dividends**

	$\alpha_1$	$\alpha_2$	$\beta_2$	$d_1$	r	$d_2$	$\rho_1$	$\rho_2$
AR(2)	-0.682 (-2.855)	-0.999 (-0.006)	0.0076 (0.0016)	2.15 [1.99, 2.24]	7	1.01 [0.94, 1.09]	-1.059	-0.560
<b>AR(1)*</b>	<b>-6.066</b> <b>(-7.180)</b>	<b>-20.204</b> <b>(-0.036)</b>	<b>0.452</b> <b>(-0.028)</b>	<b>1.56</b> <b>[1.41, 1.72]</b>	<b>11</b>	<b>0.53</b> <b>[0.42, 0.71]</b>	<b>-0.308</b>	<b>---</b>
White Noise	-494.658 (-60.53)	-498.19 (-325)	-1.580 (-1.890)	0.08 [0.00, 0.11]	13	1.07 [1.01, 1.18]	---	---

In bold, the models for which the null hypothesis of white noise errors cannot be rejected. “\*” indicates the best model specification using LR tests and other likelihood criteria.

**Table 2C: Estimates in the nonlinear case with two fractional structures. Series: Earnings**

	$\alpha_1$	$\alpha_2$	$\beta_2$	$d_1$	r	$d_2$	$\rho_1$	$\rho_2$
<b>AR(2)*</b>	<b>-121.003</b> <b>(-2.891)</b>	<b>-104.15</b> <b>(-10.30)</b>	<b>-1.039</b> <b>(-3.34)</b>	<b>1.32</b> <b>[1.17, 1.55]</b>	<b>7</b>	<b>0.52</b> <b>[0.34, 0.63]</b>	<b>-0.117</b>	<b>-0.260</b>
AR(1)	-446.595 (-13.54)	396.206 (11.07)	-20.120 (-0.980)	0.17 [0.11, 0.30]	15	1.33 [1.21, 1.56]	-0.354	---
White Noise	-1090.33 (-62.53)	-1146.4 (-26.58)	-1.276 (-0.611)	0.09 [0.00, 0.13]	7	1.29 [1.17, 1.40]	---	---

In bold, the models for which the null hypothesis of white noise errors cannot be rejected. “\*” indicates the best model specification using LR tests and other likelihood criteria.

**Table 3C: Estimates in the nonlinear case with two fractional structures. Series: Interest rates**

	$\alpha_1$	$\alpha_2$	$\beta_2$	$d_1$	r	$d_2$	$\rho_1$	$\rho_2$
AR(2)	68.826 (3.409)	6.006 (0.0013)	1.728 (0.0123)	1.18 [1.04, 1.39]	5	0.63 [0.41, 0.88]	-0.199	-0.284
<b>AR(1)*</b>	<b>-75.817</b> <b>(-9.678)</b>	<b>-63.114</b> <b>(-7.089)</b>	<b>-0.636</b> <b>(-4.351)</b>	<b>0.38</b> <b>[0.30, 0.52]</b>	<b>15</b>	<b>1.23</b> <b>[1.13, 1.39]</b>	<b>-0.237</b>	<b>---</b>
White Noise	-41.014 (-5.193)	-37.947 (-4.720)	-0.851 (-5.028)	0.82 [0.77, 0.89]	3	0.01 [-0.09, 0.16]	---	---

In bold, the models for which the null hypothesis of white noise errors cannot be rejected. “\*” indicates the best model specification using LR tests and other likelihood criteria.

**Table 4C: Estimates in the nonlinear case with two fractional structures. Series: Government bond yields**

	$\alpha_1$	$\alpha_2$	$d_1$	r	$d_2$	$\rho_1$	$\rho_2$
AR(2)	-3.301 (-1.917)	-3.536 (-1.915)	0.95 [0.69, 1.06]	13	0.94 [0.75, 1.13]	-0.955	-0.424
<b>AR(1)*</b>	<b>-25.644</b> <b>(-4.446)</b>	<b>-28.013</b> <b>(-4.806)</b>	<b>0.95</b> <b>[0.89, 1.07]</b>	<b>3</b>	<b>0.14</b> <b>[0.03, 0.54]</b>	<b>-0.088</b>	<b>---</b>
<b>White Noise</b>	<b>-34.125</b> <b>(-6.275)</b>	<b>-36.569</b> <b>(-6.608)</b>	<b>0.94</b> <b>[0.86, 1.10]</b>	<b>8</b>	<b>0.27</b> <b>[0.08, 0.59]</b>	<b>---</b>	<b>---</b>

In bold, the models for which the null hypothesis of white noise errors cannot be rejected. “\*” indicates the best model specification using LR tests and other likelihood criteria.

**Table 1D: Coefficient estimates with one fractional structure. Series: Dividends**

	$\alpha_1$	$\alpha_2$	$\beta_2$	$d_1$	$\rho_1$	$\rho_2$
AR(2)*	<b>-1.744</b> (-8.287)	<b>-2.878</b> (-9.185)	<b>0.485</b> (31.232)	<b>0.63</b> [0.55, 0.70]	<b>1.372</b>	<b>-0.745</b>
AR(1)	0.225 (0.985)	-0.005 (-0.016)	0.459 (5.334)	1.24 [0.86, 1.49]	0.712	---
White noise	0.239 (1.592)	0.270 (1.001)	0.088 (0.263)	2.12 [1.93, 2.35]	---	---

In bold, the models for which the null hypothesis of white noise errors cannot be rejected. “\*” indicates the best model specification using LR tests and other likelihood criteria.

**Table 2D: Coefficient estimates with one fractional structure. Series: Earnings**

	$\alpha_1$	$\alpha_2$	$\beta_2$	$d_1$	$\rho_1$	$\rho_2$
AR(2)*	<b>0.231</b> (0.073)	<b>0.269</b> (0.059)	<b>1.641</b> (1.733)	<b>1.17</b> [0.51, 1.79]	<b>0.162</b>	<b>-0.401</b>
AR(1)	-8.748 (-4.465)	-13.960 (-4.363)	1.572 (11.162)	0.47 [0.30, 0.67]	0.497	---
White noise	-0.080 (-0.023)	-0.440 (-0.088)	1.634 (2.441)	1.03 [0.81, 1.31]	---	---

In bold, the models for which the null hypothesis of white noise errors cannot be rejected. “\*” indicates the best model specification using LR tests and other likelihood criteria.

**Table 3D: Coefficient estimates with one fractional structure. Series: Interest Rate**

	$\alpha_1$	$\alpha_2$	$\beta_2$	$d_1$	$\rho_1$	$\rho_2$
AR(2)*	<b>2.751</b> (3.330)	<b>7.105</b> (5.134)	<b>-0.258</b> (-4.019)	<b>0.45</b> [0.30, 1.12]	<b>0.333</b>	<b>0.007</b>
AR(1)	<b>2.462</b> (3.152)	<b>6.890</b> (5.159)	<b>-0.257</b> (-4.142)	<b>0.43</b> [0.16, 0.74]	<b>0.360</b>	---
White noise	<b>5.668</b> (4.328)	<b>9.466</b> (4.930)	<b>-0.259</b> (-2.365)	<b>0.69</b> [0.58, 0.85]	---	---

In bold, the models for which the null hypothesis of white noise errors cannot be rejected. “\*” indicates the best model specification using LR tests and other likelihood criteria.

**Table 4D: Coefficient estimates with one fractional structure. Series: Government bond yields**

	$\alpha_1$	$\alpha_2$	$d_1$	$\rho_1$	$\rho_2$
AR(2)*	<b>5.279</b> (7.100)	<b>5.761</b> (5.479)	<b>0.99</b> [0.80, 1.38]	<b>-0.183</b>	<b>-0.007</b>
AR(1)	<b>5.272</b> (6.985)	<b>5.760</b> (5.398)	<b>0.98</b> [0.82, 1.17]	<b>-0.173</b>	---
White noise	<b>5.126</b> (6.975)	<b>5.713</b> (5.505)	<b>0.87</b> [0.78, 0.98]	---	---

In bold, the models for which the null hypothesis of white noise errors cannot be rejected. “\*” indicates the best model specification using LR tests and other likelihood criteria.

**Table 5: RMSE of the k-ahead prediction errors. Series: Dividends**

k / Model	(1A)	(1B)	(1C)	(1D)
1	0.006234	<b>0.000591</b>	0.008026	0.018123
2	0.031326	0.025251	0.036914	<b>0.016958</b>
3	0.030545	0.024227	0.033172	<b>0.013850</b>
4	0.031718	0.025917	0.034087	<b>0.012656</b>
5	0.034013	0.028231	0.036075	<b>0.012689</b>
6	0.036629	0.030663	0.038195	<b>0.013575</b>
7	0.039581	0.033446	0.040673	<b>0.015228</b>
8	0.042596	0.036330	0.043256	<b>0.017238</b>
9	0.045808	0.039465	0.046073	<b>0.019673</b>
10	0.049288	0.042921	0.049189	<b>0.022551</b>

In bold the lowest value among models for each k-ahead prediction error.

**Table 6: RMSE of the k-ahead prediction errors. Series: Earnings**

k / Model	(2A)	(2B)	(2C)	(2D)
1	0.009811	<b>0.002347</b>	0.128726	0.041816
2	0.043473	0.047612	0.179285	<b>0.033633</b>
3	0.045904	0.054964	0.183457	<b>0.030183</b>
4	0.063079	0.073397	0.198744	<b>0.039817</b>
5	0.067516	0.080890	0.205456	<b>0.043198</b>
6	0.073898	0.087810	0.213325	<b>0.047346</b>
7	0.084659	0.097785	0.224043	<b>0.055428</b>
8	0.096819	0.109013	0.235610	<b>0.065194</b>
9	0.110367	0.122034	0.248383	<b>0.077060</b>
10	0.126495	0.138105	0.263549	<b>0.092292</b>

In bold the lowest value among models for each k-ahead prediction error.

**Table 7: RMSE of the k-ahead prediction errors. Series: Interest Rates**

k / Model	(3A)	(3B)	(3C)	(3D)
1	0.001353	0.001622	0.077673	<b>0.000636</b>
2	0.031361	0.032086	0.107293	<b>0.031284</b>
3	<b>0.057927</b>	0.058682	0.131547	0.058063
4	<b>0.085821</b>	0.086357	0.158054	0.085826
5	0.110345	0.110731	0.182517	<b>0.110294</b>
6	0.130518	0.130777	0.203123	<b>0.130446</b>
7	0.150488	0.150638	0.223397	<b>0.150430</b>
8	<b>0.167591</b>	0.167674	0.240997	0.167592
9	<b>0.180127</b>	0.180186	0.254282	0.180229
10	<b>0.188005</b>	0.188063	0.263042	0.188227

In bold the lowest value among models for each k-ahead prediction error.

**Table 8: RMSE of the k-ahead prediction errors. Series: Government bond yields**

k / Model	(4A)	(4B)	(4C)	(4D)
1	0.021940	<b>0.000892</b>	0.026910	0.006647
2	0.043353	0.022852	0.046478	<b>0.019229</b>
3	0.060376	0.040011	0.063574	<b>0.035646</b>
4	0.079280	0.059526	0.083255	<b>0.054984</b>
5	0.096742	0.077144	0.101070	<b>0.072477</b>
6	0.112871	0.093214	0.117255	<b>0.088455</b>
7	0.126838	0.106984	0.131291	<b>0.102144</b>
8	0.139513	0.119445	0.143957	<b>0.114540</b>
9	0.151303	0.131048	0.155693	<b>0.126092</b>
10	0.162691	0.142301	0.167085	<b>0.137308</b>

In bold the lowest value among models for each k-ahead prediction error.

**Table 9: Modified DM statistic: 2-step ahead forecasts**

Dividends	1A	1B	1C
1B	<b>65.654 (1B)</b>	XXXXXX	XXXXXX
1C	<b>-3.154 (1A)</b>	<b>-14.420 (1B)</b>	XXXXXX
1D	<b>4.574 (1D)</b>	<b>3.120 (1D)</b>	<b>5.217 (1D)</b>
Earnings	2A	2B	2C
2B	<b>-3.998 (2A)</b>	XXXXXX	XXXXXX
2C	<b>-63.060 (2A)</b>	<b>-174.975 (2B)</b>	XXXXXX
2D	<b>2.813 (2D)</b>	<b>3.130 (2D)</b>	<b>17.222 (2D)</b>
Interest rates	3A	3B	3C
3B	<b>-3.501 (3A)</b>	XXXXXX	XXXXXX
3C	<b>-139.916 (3A)</b>	<b>-137.877 (3B)</b>	XXXXXX
3D	0.565	<b>2.905 (3D)</b>	<b>129.224 (3D)</b>
Gov bond yield	4A	4B	4C
4B	<b>125.925 (4B)</b>	XXXXXX	XXXXXX
4C	<b>-20.277 (4A)</b>	<b>-93.416 (4B)</b>	XXXXXX
4D	<b>22.406 (4D)</b>	<b>3.069 (4D)</b>	<b>28.210 (4D)</b>

Critical value: 1.833 (95% level, with 9 degrees of freedom). In parentheses, the preferred model in a pairwise comparison.

**Table 10: Modified DM statistic: 4-step ahead forecasts**

Dividends	1A	1B	1C
1B	<b>50.144 (1B)</b>	XXXXXX	XXXXXX
1C	<b>-2.420 (1A)</b>	<b>-11.013 (1B)</b>	XXXXXX
1D	<b>3.493 (1D)</b>	<b>2.383 (1D)</b>	<b>3.985(1D)</b>
Earnings	2A	2B	2C
2B	<b>-3.054 (2A)</b>	XXXXXX	XXXXXX
2C	<b>-48.163 (2A)</b>	<b>-133.64 (2B)</b>	XXXXXX
2D	<b>2.148 (2D)</b>	<b>2.390 (2D)</b>	<b>13.153 (2D)</b>
Interest rates	3A	3B	3C
3B	<b>-2.674 (3A)</b>	XXXXXX	XXXXXX
3C	<b>-106.863 (3A)</b>	-105.305	XXXXXX
3D	0.431	<b>2.219 (3D)</b>	<b>98.696 (3D)</b>
Gov bond yield	4A	4B	4C
4B	<b>96.177(4B)</b>	XXXXXX	XXXXXX
4C	<b>-15.487(4A)</b>	<b>-71.347 (4B)</b>	XXXXXX
4D	<b>17.113 (4D)</b>	<b>2.344 (4D)</b>	<b>21.545 (4D)</b>

Critical value: 1.833 (95% level, with 9 degrees of freedom). In parentheses, the preferred model in a pairwise comparison.

**Table 11: Modified DM statistic: 6-step ahead forecasts**

Dividends	1A	1B	1C
1B	<b>34.602 (1B)</b>	XXXXXX	XXXXXX
1C	-1.662	<b>-7.600 (1B)</b>	XXXXXX
1D	<b>2.410 (1D)</b>	1.644	<b>2.750 (1D)</b>
Earnings	2A	2B	2C
2B	<b>-2.107 (2A)</b>	XXXXXX	XXXXXX
2C	<b>-33.235 (2A)</b>	<b>-92.220 (2B)</b>	XXXXXX
2D	1.482	1.649	<b>9.076 (2D)</b>
Interest rates	3A	3B	3C
3B	<b>-1.845 (3A)</b>	XXXXXX	XXXXXX
3C	<b>-73.742 (3A)</b>	<b>-72.667 (3B)</b>	XXXXXX
3D	0.298	1.531	<b>68.107 (3D)</b>
Gov bond yield	4A	4B	4C
4B	<b>66.368 (4B)</b>	XXXXXX	XXXXXX
4C	<b>-10.687 (4A)</b>	<b>-49.234 (4B)</b>	XXXXXX
4D	<b>11.809 (4D)</b>	1.617	<b>14.868 (4D)</b>

Critical value: 1.833 (95% level, with 9 degrees of freedom). In parenthesis, the preferred model in a pairwise comparison.

**Table 12: Modified DM statistic: 8-step ahead forecasts**

Dividends	1A	1B	1C
1B	<b>18.952 (1B)</b>	XXXXXX	XXXXXX
1C	-0.910	<b>-4.162 (1B)</b>	XXXXXX
1D	1.320	0.900	1.506
Earnings	2A	2B	2C
2B	-1.154	XXXXXX	XXXXXX
2C	<b>-18.204 (2A)</b>	<b>-50.511 (2B)</b>	XXXXXX
2D	0.812	0.903	<b>4.971 (2D)</b>
Interest rates	3A	3B	3C
3B	-1.010	XXXXXX	XXXXXX
3C	<b>-40.390 (3A)</b>	<b>-39.801 (3B)</b>	XXXXXX
3D	0.163	0.838	<b>37.303 (3D)</b>
Gov bond yield	4A	4B	4C
4B	<b>36.351 (4B)</b>	XXXXXX	XXXXXX
4C	<b>-5.853 (4A)</b>	<b>-26.966 (4B)</b>	XXXXXX
4D	<b>6.468 (4D)</b>	0.885	<b>8.143 (4D)</b>

Critical value: 1.833 (95% level, with 9 degrees of freedom). In parenthesis, the preferred model in a pairwise comparison.

# CESifo Working Paper Series

for full list see [www.cesifo-group.org/wp](http://www.cesifo-group.org/wp)

(address: Poschingerstr. 5, 81679 Munich, Germany, [office@cesifo.de](mailto:office@cesifo.de))

---

- 2269 Thomas Aronsson and Erkki Koskela, Outsourcing and Optimal Nonlinear Taxation: A Note, April 2008
- 2270 Gary E. Bolton, Claudia Loebbecke and Axel Ockenfels, How Social Reputation Networks Interact with Competition in Anonymous Online Trading: An Experimental Study, April 2008
- 2271 Nikolaus Wolf, Scylla and Charybdis. Explaining Europe's Exit from Gold, January 1928 – December 1936, April 2008
- 2272 Michael Funke and Marc Gronwald, The Undisclosed Renminbi Basket: Are the Markets Telling us something about where the Renminbi – US Dollar Exchange Rate is Going?, April 2008
- 2273 Thor Olav Thoresen and Annette Alstadsæter, Shifts in Organizational Form under a Dual Income Tax System, April 2008
- 2274 Helge Berger and Volker Nitsch, Too many Cooks? Committees in Monetary Policy, April 2008
- 2275 Yin-Wong Cheung and Eiji Fujii, Deviations from the Law of One Price in Japan, April 2008
- 2276 Michael S. Michael, Sajal Lahiri and Panos Hatzipanayotou, Integrated Reforms of Indirect Taxes in the Presence of Pollution, April 2008
- 2277 Bas Jacobs, Is Prescott Right? Welfare State Policies and the Incentives to Work, Learn and Retire, April 2008
- 2278 Burkhard Heer and Alfred Maußner, Value Function Iteration as a Solution Method for the Ramsey Model, April 2008
- 2279 Jarko Fidrmuc and Christa Hainz, Integrating with their Feet: Cross-Border Lending at the German-Austrian Border, April 2008
- 2280 Kristof Dascher and Alexander Haupt, The Political Economy of Regional Integration Projects at Borders where Rich and Poor Meet: The Role of Cross-Border Shopping and Community Sorting, April 2008
- 2281 Katrin Assenmacher-Wesche and M. Hashem Pesaran, A VECX\* Model of the Swiss Economy, April 2008
- 2282 Christophe Rault, Robert Sova and Ana Maria Sova, Modeling International Trade Flows between CEEC and OECD Countries, April 2008

- 2283 Timo Boppart, Josef Falkinger, Volker Grossmann, Ulrich Woitek and Gabriela Wüthrich, Qualifying Religion: The Role of Plural Identities for Educational Production, April 2008
- 2284 Armin Falk, David Huffman and W. Bentley MacLeod, Institutions and Contract Enforcement, April 2008
- 2285 Axel Dreher and Stefan Voigt, Does Membership in International Organizations Increase Governments' Credibility? Testing the Effects of Delegating Powers, April 2008
- 2286 Xavier Freixas and Bruno M. Parigi, Lender of Last Resort and Bank Closure Policy, April 2008
- 2287 Regina Dionisius, Samuel Muehlemann, Harald Pfeifer, Günter Walden, Felix Wenzelmann and Stefan C. Wolter, Cost and Benefit of Apprenticeship Training – A Comparison of Germany and Switzerland, April 2008
- 2288 Francesco Daveri and Cecilia Jona-Lasinio, Off-Shoring and Productivity Growth in the Italian Manufacturing Industries, April 2008
- 2289 Mikael Priks, Do Surveillance Cameras Affect Unruly Behavior? A Close Look at Grandstands, April 2008
- 2290 Marianna Belloc and Daniela Federici, A Two-Country NATREX Model for the Euro/Dollar, April 2008
- 2291 Nicolas Treich, The Value of a Statistical Life under Ambiguity Aversion, April 2008
- 2292 J. Atsu Amegashie, Socially-Tolerable Discrimination, April 2008
- 2293 M. Hashem Pesaran and Andreas Pick, Forecasting Random Walks Under Drift Instability, April 2008
- 2294 Steven Brakman, Gus Garita, Harry Garretsen and Charles van Marrewijk, Unlocking the Value of Cross-Border Mergers and Acquisitions, May 2008
- 2295 Eric O'N. Fisher and Kathryn G. Marshall, The Structure of the American Economy, May 2008
- 2296 Claudia M. Buch and Martin Schlotter, Regional Origins of Employment Volatility: Evidence from German States, May 2008
- 2297 Helmuth Cremer, Philippe De Donder, Dario Maldonado and Pierre Pestieau, Taxing Sin Goods and Subsidizing Health Care, May 2008
- 2298 Reinhilde Veugelers and Frederick van der Ploeg, Reforming European Universities: Scope for an Evidence-Based Process, May 2008



- 2299 Jon H. Fiva and Lars J. Kirkeb  en, Does the Housing Market React to New Information on School Quality?, May 2008
- 2300 Tina Klautke and Alfons J. Weichenrieder, Interest Income Tax Evasion, the EU Savings Directive, and Capital Market Effects, May 2008
- 2301 Harald Badinger and Peter Egger, GM Estimation of Higher Order Spatial Autoregressive Processes in Panel Data Error Component Models, May 2008
- 2302 Jan K. Brueckner, Slot-Based Approaches to Airport Congestion Management, May 2008
- 2303 S  ren Blomquist, Vidar Christiansen and Luca Micheletto, Public Provision of Private Goods and Nondistortionary Marginal Tax Rates, May 2008
- 2304 Dan Anderberg and Alessandro Balestrino, The Political Economy of Post-Compulsory Education Policy with Endogenous Credit Constraints, May 2008
- 2305 Tomer Blumkin, Yoram Margalioth and Efraim Sadka, The Role of Stigma in the Design of Welfare Programs, May 2008
- 2306 Vesa Kannianen and Paolo M. Panteghini, Tax Neutrality: Illusion or Reality? The Case of Entrepreneurship, May 2008
- 2307 Thomas Dohmen, Armin Falk, David Huffman and Uwe Sunde, The Intergenerational Transmission of Risk and Trust Attitudes, May 2008
- 2308 Guglielmo Maria Caporale and Mario Cerrato, Using Chebyshev Polynomials to Approximate Partial Differential Equations, May 2008
- 2309 Peter Egger and Doina Maria Radulescu, Labour Taxation and Foreign Direct Investment, May 2008
- 2310 Laurent Linnemer, Dissipative Advertising Signals Quality even without Repeat Purchases, May 2008
- 2311 Jordi Jofre-Monseny and Albert Sol  -Oll  , Which Communities should be afraid of Mobility? The Effects of Agglomeration Economies on the Sensitivity of Firm Location to Local Taxes, May 2008
- 2312 Andreas Haufler and Ferdinand Mittermaier, Unionisation Triggers Tax Incentives to Attract Foreign Direct Investment, May 2008
- 2313 Ronel Elul and Piero Gottardi, Bankruptcy: Is it enough to Forgive or must we also Forget?, May 2008
- 2314 Andreas Irmen and Johanna Kuehnel, Productive Government Expenditure and Economic Growth, May 2008

- 2315 Beate Henschel, Carsten Pohl and Marcel Thum, Demographic Change and Regional Labour Markets: The Case of Eastern Germany, May 2008
- 2316 Gabriel Felbermayr, Wido Geis and Wilhelm Kohler, Restrictive Immigration Policy in Germany: Pains and Gains Foregone?, May 2008
- 2317 Michael Hofmann, Gerhard Kempkes and Helmut Seitz, Demographic Change and Public Sector Budgets in a Federal System, May 2008
- 2318 Paul De Grauwe, Macroeconomic Modeling when Agents are Imperfectly Informed, June 2008
- 2319 Johann K. Brunner and Susanne Pech, Optimum Taxation of Inheritances, June 2008
- 2320 Thomas Eichner and Marco Runkel, Corporate Income Taxation of Multinationals in a General Equilibrium Model, June 2008
- 2321 Rainald Borek and Matthias Wrede, Subsidies for Intracity and Intercity Commuting, June 2008
- 2322 Patricia Apps and Ray Rees, Testing the Pareto Efficiency of Household Resource Allocations, June 2008
- 2323 Amihai Glazer, Vesa Kanninen and Panu Poutvaara, Firms' Ethics, Consumer Boycotts, and Signalling, June 2008
- 2324 Claudia M. Buch, Jörg Döpke and Kerstin Stahn, Great Moderation at the Firm Level? Unconditional vs. Conditional Output Volatility, June 2008
- 2325 Helmuth Cremer, Philippe De Donder, Dario Maldonado and Pierre Pestieau, Forced Saving, Redistribution and Nonlinear Social Security Schemes, June 2008
- 2326 M. Hashem Pesaran and Paolo Zaffaroni, Optimal Asset Allocation with Factor Models for Large Portfolios, June 2008
- 2327 Harald Badinger and Peter Egger, Horizontal versus Vertical Interdependence in Multinational Activity, June 2008
- 2328 Jan K. Brueckner and Harris Selod, A Theory of Urban Squatting and Land-Tenure Formalization in Developing Countries, June 2008
- 2329 Paolo M. Panteghini, Corporate Debt, Hybrid Securities and the Effective Tax Rate, June 2008
- 2330 Guglielmo Maria Caporale, Juncal Cuñado and Luis A. Gil-Alana, Modelling Long-Run Trends and Cycles in Financial Time Series Data, June 2008