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Clustered Local Average Treatment Effects: Fields of Study and Academic Student Progress

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DISCUSSION PAPER SERIES

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ABSTRACT

Clustered Local Average Treatment Effects: Fields of Study and Academic Student Progress*

Multiple unordered treatments with a binary instrument for each treatment are common in policy evaluation. This multiple treatment setting allows for different types of changes in treatment status that are non-compliant with the activated instrument. Therefore, instrumental variable (IV) methods have to rely on strong assumptions on the subjects' behavior to identify local average treatment effects (LATEs). This paper introduces a new IV strategy that identifies an interpretable weighted average of LATEs under relaxed assumptions, in the presence of clusters with similar treatments. The clustered LATEs allow for shifts across treatment clusters that are consistent with preference updating, but render IV estimation of individual LATEs biased. The clustered LATEs are estimated by standard IV methods, and we provide an algorithm that estimates the treatment clusters. We empirically analyze the effect of fields of study on academic student progress, and find violations of the LATE assumptions in line with preference updating, clusters with similar fields, treatment effect heterogeneity across students, and significant differences in student progress due to fields of study.

JEL Classification: C36, I21, I23

Keywords: multiple treatments, instrumental variables, treatment clusters, field of study

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1 Introduction

Many policy evaluations include multiple unordered treatments. For instance, randomized control trials with multiple treatment arms (Duflo, Glennerster, and Kremer, 2007), location decisions by firms or families (Chetty, Hendren, and Katz, 2016), and fields of study choices by students (Kirkeboen, Leuven, and Mogstad, 2016). Since treatments are often endogenous, instrumental variable (IV) approaches are used to estimate the local average treatment effects (LATEs). To illustrate the multiple unordered treatment setting, consider three treatments – A, B, and C–, an additional control group that does not receive any treatment, and one binary instrument for each treatment. There are five possible changes in treatment status induced by the instrument for treatment A: (1) from control group to treatment A, known as compliers; (2) between control group and treatment B or C; (3) between treatments B and C; (4) from treatment B or C to treatment A; (5) away from treatment A, known as defiers. In this paper, we develop an IV framework that identifies the weighted average of LATEs in the presence of treatment shifts (1-4).

In a binary treatment setting, Imbens and Angrist (1994) show that IV can identify a local average treatment effect (LATE) for the compliers (1). This result relies on a monotonicity assumption that ensures that the instrument does not induce individuals to shift away from the treatment group, and hence excludes (5). In addition to the compliers and defiers, the multiple treatment setting also includes shifts (2-4). Since it is likely that these type of shifts are present in many multiple unordered treatment applications, and we do not observe the type of shifts, these settings pose an additional identification problem.

The current literature on multiple unordered treatments estimates LATEs under the assumption that all shifts can be excluded except the compliers. Under this assumption, Behaghel, Crépon, and Gurgand (2013) show that LATEs can be estimated with standard IV methods in the presence of a natural control group. Kirkeboen, Leuven, and Mogstad (2016) extend this result to settings where there is no natural control group. Using individual information on the preferred and next-best treatment, they identify the LATE of one treatment relative to a particular next-best alternative. This approach has been used to study the impact of fields of study on wages (Kirkeboen, Leuven, and Mogstad, 2016; Heinesen and Hvid, 2019; Dahl, Rooth, and Stenberg, 2020), and the impact of institutions of study on marriage outcomes (Kirkeboen, Leuven, and Mogstad, 2021). Provided that there is treatment heterogeneity, these approaches are biased in the presence of shifts (2-4). These shifts arise if, for

instance, individuals update treatment preferences in response to treatment assignment, which is found in a large empirical literature on preference updating (see *e.g.*, [Kuziemko, Norton, Saez, and Stantcheva \(2015\)](#) and [Schildberg-Hörisch \(2018\)](#)).

This paper introduces an IV strategy that identifies multiple unordered treatment effects, while allowing for shifts across treatment clusters. We identify clustered local average treatment effects (CLATE): a weighted average of the LATEs of all pairs of treatments across the treatment cluster and control cluster, and the instruments corresponding to these clusters. CLATE allows for any shift from a treatment that is in the same cluster as the control group, towards a treatment that is in the same cluster as the active instrument. The weighted average of LATEs is therefore not restricted to LATEs of compliers, allows for preference updating, and does not require any homogeneity assumptions. We show that CLATE can be applied to empirical settings with a natural control group, or with individual information on the preferred and next-best treatment. CLATE is estimated by standard IV methods, and we provide an algorithm that estimates treatment clusters without requiring additional data.

The CLATE approach overcomes the bias in existing approaches for estimating LATE, if the multiple unordered treatments can be partitioned into disjoint treatment clusters, and there are only shifts across, and not within, these clusters. With many unordered treatments, some treatments may be more similar than others, and therefore the actual treatment shifts are likely to follow this pattern. This can be explained by preference updating among, and low switching costs across, similar treatments. For instance, in the analysis of the causal effects of fields of study, students assigned to Economics may be more likely to update their preference to Business than to Medicine. Moreover, moving from Economics to Medicine might be too costly ([Altonji, Arcidiacono, and Maurel, 2016](#)). In this case, the instrument of Economics may induce shifts to a cluster of fields of study, where the cluster includes fields similar to Economics, but does not shift students to fields outside this cluster. In general, individuals may obtain a treatment from a cluster after being initially assigned to a treatment in the cluster. Since the initially assigned treatment is in the cluster, but may not be considered after preference updating, a cluster is different from a consideration set ([Mehta, Rajiv, and Srinivasan, 2003](#); [Nierop, Bronnenberg, Paap, Wedel, and Franses, 2010](#)).

Clustering multiple unordered treatments relaxes the assumptions imposed on the instrument induced treatment shifts, while retaining an interpretable treatment effect. In contrast, [Angrist and Imbens \(1995\)](#) show that with a single multi-level treatment, standard IV identifies a weighted average of LATEs across multiple compliers: the monotone shifts

across all treatment levels, which is sometimes argued to lack interpretability (Andresen and Huber, 2021). Binarizing the treatment variable into a control group below a cutoff and a treatment group above, excludes all compliers who are shifting on one side of the cutoff, which may result in a biased treatment effect (Marshall, 2016; Andresen and Huber, 2021). With multiple unordered treatments, each instrument only has one complier: the shift from the control group to the treatment of the activated instrument. CLATE allows for additional shifts across treatment clusters, instead of excluding compliers. Moreover, since CLATE only includes the LATEs of shifts between the control cluster and treatment cluster, instead of all compliers across all treatments, it still identifies a local average treatment effect.

We show that CLATE is able to identify treatment effect heterogeneity. In the presence of non-compliant shifts (2-4), we derive which homogeneity assumptions are necessary for IV to identify LATE. Under these assumptions, CLATE equals a weighted average of identifiable LATEs, which allows for the examination of treatment effect heterogeneity. Moreover, we show that the type of shift determines the strength of the required homogeneity. This result is different from Kirkeboen, Leuven, and Mogstad (2016), who show that IV is unbiased when treatment effects are common across all individuals. This complete treatment effect homogeneity has been rejected in many economic settings (Angrist and Pischke, 2008).

We develop an algorithm that estimates the treatment clusters for settings in which the clusters are not observed. The LATE assumptions pose a testable necessary condition on the first stage IV estimates of the unclustered treatments (Behaghel, Crépon, and Gurgand, 2013), which the algorithm uses to identify violations of the LATE assumptions. This is consistent with Imbens and Rubin (2015), who argue that treatments have to be considered at the individual unclustered level, instead of an arbitrary clustering, as otherwise violations may stay undetected. Next, the algorithm clusters these treatments in such a way that the set of treatment clusters has the highest level of granularity that satisfies the CLATE assumptions. The algorithm ensures the practical applicability of CLATE, as treatments do not have to be clustered ad hoc in absence of information on the treatment clustering. Arbitrary clustering may bias the results, as is also shown in the literature on categorization of ordered treatment variables (Angrist and Imbens, 1995; Marshall, 2016; Andresen and Huber, 2021).

We use CLATE to study the causal effect of field of study on academic student progress. Sixty percent of the higher education students in OECD countries experiences study delay (OECD, 2019). This is costly to both governments and students: The cost of higher education, excluding R&D activities, is about 10.000 euros per student per year across OECD

countries (OECD, 2020), and nobody reaps the full benefits until the degree is completed. Decreasing study delay is therefore a first-order policy objective in a majority of the European countries (European Commission and Directorate-General for Education, Youth, Sport and Culture et al., 2015). Our causal estimates may detect exemplary and problematic fields in terms of student progress, and inform changes to curriculum design and pedagogy, the allocation of placement slots and resources, and study choice for prospective students.

We have access to an administrative data set that includes four complete cohorts in the Portuguese higher education system, from which we exploit a natural experiment. Prospective students apply to higher education via the submission of a ranking of courses based on their preferences. A course is a field of study at a particular institution (e.g., Law at the University of Lisbon). Each course ranks their applicants by a score that consists of high school and national exam grades, after which a deferred acceptance mechanism assigns all applicants to one course (DGES, 2019). This mechanism generates (many) application cut-off scores for each field, where an applicant scoring just above (below) the cutoff for her preferred field in the submitted ranking is assigned to her preferred (next-best) field. This allows us to estimate CLATEs for applicants with the same next-best field via a fuzzy regression discontinuity design while using the initial field assignments as instruments, the second-year fields as treatments, and the number of credit points collected in the second year through the European Credit Transfer System (ECTS) as outcome variable.

We find many violations of the LATE assumptions, and they are consistent with preference updating and switching costs. For instance, students assigned to IT in the first year, enroll in the similar field Engineering in the second year. Our clustering algorithm clusters IT and Engineering together. With this treatment cluster, CLATEs are identified in the presence of the additional shift towards Engineering while a student is assigned to IT. We find evidence for treatment effect heterogeneity, which results in biased LATE estimates, while it does not affect the validity of our CLATE estimates. The CLATE estimates suggest that, for instance, Law has a negative and Information and journalism a positive impact on academic student progress.

Our empirical study is related to a large body of research that studies the effect of varying interventions on academic student outcomes, such as group assignment policies (Booij, Leuven, and Oosterbeek, 2017), academic dismissal policies (Lindo, Sanders, and Oreopoulos, 2010), professor quality (Carrell and West, 2010), and restricting access to alcohol (Carrell, Hoekstra, and West, 2011) and cannabis (Marie and Zölitz, 2017). Similar to our analysis,

academic outcomes are often measured via the number of study credit points collected. Instead of comparing study credit points within fields of study as a measure for student skills, we compare study credit points across fields of study as a measure of study progress.

Studying the causal effect of fields of study, instead of interventions within fields of study, has proven to be a challenging task. Even in the presence of an instrument for each field, this context likely has many unwanted shifts (2-4) due to students updating their fields of study preferences and low switching costs within clusters of fields. Therefore, most previous research towards the relationship between fields of study and academic progress documents correlations (Lassibille, 2011), which may only reflect causal effects under strong conditional independence assumptions (Kirkeboen, Leuven, and Mogstad, 2016).

Our CLATE estimates identify the causal effects for the students who are induced to shift field of study by their instrument, and are at the margin of entry to particular fields. Since it is often infeasible to run a randomized experiment with full compliance (Imbens, 2010), especially in multiple unordered treatment settings, additional assumptions are necessary to identify average treatment effects. For instance, Heckman and Vytlačil (2007), Brinch, Mogstad, and Wiswall (2017), and Mogstad, Santos, and Torgovitsky (2018), among others, study marginal treatment effects (MTEs) under different sets of assumptions. Heckman, Urzua, and Vytlačil (2008) identify MTEs with multiple unordered treatments. Alternatively, bounds can be derived on the average treatment effect (Manski, 2003). Mogstad and Torgovitsky (2018) provide an overview on the methods that extend LATE estimates from IV methods to average treatment effects.

The paper proceeds as follows. In Section 2 we discuss the identification of LATEs in the presence of multiple unordered treatments, and the bias that results from violations of the LATE assumptions. In Section 3 we introduce the less strict CLATE assumptions, and derive the treatment effects identified by CLATE. Section 4 introduces our algorithm for estimating treatment clusters, and Section 5 extends CLATE to a fuzzy regression discontinuity design. Section 6 presents our empirical analysis of the effects of field of study on academic student progress. Section 7 concludes.

2 The multiple unordered treatment IV model

This section discusses the standard multiple unordered treatment IV model. First, we introduce the model and the assumptions under which the IV coefficients identify LATEs. Sec-

ond, we discuss the LATE identification result. Third, we discuss potential violations of the assumptions and derive the corresponding bias in the IV coefficients.

2.1 Setting and assumptions

Suppose we have J different treatment choices. Define the dummy variable d_j that equals one if treatment $j = 1, \dots, J$ is chosen and zero otherwise. The instruments z_j are dummy variables that equal one if the cost to obtain treatment j is decreased and zero otherwise. Define $d = \sum_{j=1}^J j \times d_j$ and $z = \sum_{j=1}^J j \times z_j$. The value of z can be interpreted as the initial treatment assignment, where it is costly to obtain treatment $d \neq z$.

We are interested in the effect on y of taking any treatment j compared to treatment k , without loss of generality. Consider the second stage model

$$y = \alpha_k + \sum_{j \neq k} \beta_{jk} d_j + \varepsilon_k. \quad (1)$$

The first stage equations are

$$d_j = \delta_{jk} + \sum_{l \neq k} \pi_{jlk} z_l + u_{jk}, \quad \text{for all } j \neq k. \quad (2)$$

We define the potential outcomes d^z and $y^{d,z}$, which represent the value of d for each z and the value of y for each combination of d and z , respectively. The dummy variable d_j^z equals one if treatment j is chosen for a given value of z . Define the J -dimensional vector $\mathbf{z}_{-k} = (1, z_1, \dots, z_{k-1}, z_{k+1}, \dots, z_J)'$ and the $(J-1)$ -dimensional vector $\mathbf{d}_{-k} = (d_1, \dots, d_{k-1}, d_{k+1}, \dots, d_J)'$. We make the four standard IV assumptions:

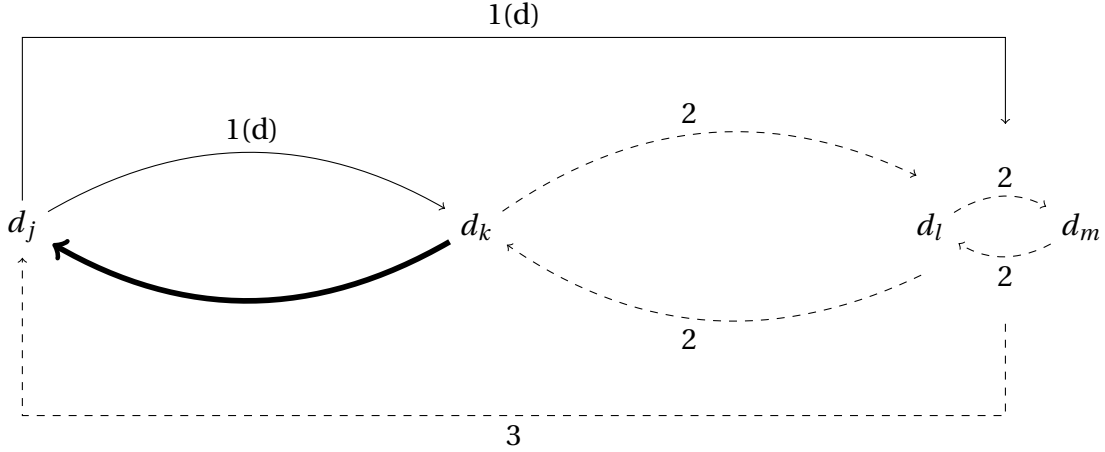
Assumption 1. *IV assumptions treatment j compared to treatment k*

- a. (Exclusion) $y^{d,z} = y^d$ for all d, z .
- b. (Independence) $y^d, d^z \perp z_l$ for all d, z, l .
- c. (Rank) $\mathbb{E}[\mathbf{d}_{-k} \mathbf{z}_{-k}']$ has full rank.
- d. (Monotonicity) $d_j^j \geq d_j^k$.

It follows from Assumption 1(a) that the potential outcomes can be linked to the observed outcomes via $y = \sum_{j=1}^J y^j \times d_j$ and $d_l = \sum_{j=1}^J d_l^j \times z_j$ for each l .

With multiple unordered treatments, Assumption 1 is not sufficient to identify LATEs as it does not rule out additional shifts next to the group of compliers: shifts *towards* and *away*

Figure 1: An overview of all the possible shifts when z switches from k to j



Notes: the arrows represent a shift from one treatment to another when z switches from k to j . For example, $d_j \rightarrow d_k$ is a shift from treatment j to k when z switches from k to j . The numbers represent the assumption that prevents the shift of the corresponding arrow.

from treatments $l \neq j$ are unrestricted. Figure 1 visualizes this problem, where each arrow represents a possible change in treatment d if z switches from k to j . In case a shift is ruled out by an assumption, the arrow is accompanied by the assumption that prevents the shift. For instance, $d_j \rightarrow d_k$ reflects a change from treatment j towards k due to a switch in z from k to j . This shift is prevented by assumption 1(d). The compliers are represented by the bold-faced arrow corresponding to the shift from treatment k towards j . However, we also find multiple dashed arrows that represent shifts that are not excluded by Assumption 1(d).

Recent literature introduces additional assumptions in multiple unordered treatments settings to prevent the remaining unwanted shifts (see [Behaghel, Crépon, and Gurgand \(2013\)](#), [Heinesen and Hvid \(2019\)](#), [Dahl, Rooth, and Stenberg \(2020\)](#), [Altmejd, Barrios-Fernández, Drlje, Goodman, Hurwitz, Kovac, Mulhern, Neilson, and Smith \(2021\)](#), and [Kirkeboen, Leuven, and Mogstad \(2021\)](#)). We follow the approach of [Kirkeboen, Leuven, and Mogstad \(2016\)](#).

Assumption 2. *Irrelevance assumption treatment j compared to treatment k*

(Irrelevance) If $d_j^j = d_j^k = 0$, then $d_l^j = d_l^k$ for all $l \neq j, k$.

This assumption prevents all shifts *across* treatments $l \neq j, k$ and $m \neq j, k$, and all shifts *across* treatments $l \neq j, k$ and k . Together with Assumption 1(d), this implies that switching z from k to j cannot induce an individual to shift *towards* treatment $l \neq j, k$. To see this,

write a shift towards treatment l as $d_l^j > d_l^k$, which implies that $d_j^j = 0$, and according to Assumption 1(d) that $d_j^j = d_k^j = 0$, which contradicts Assumption 2. Moreover, it also implies that an individual cannot shift *towards* treatment k : $d_k^j > d_k^k$ implies that $d_j^j = d_j^k = 0$ by Assumption 1(d), hence the shift towards k must come from treatment $l \neq j, k$ with $d_l^j < d_l^k$, which is again a contradiction of Assumption 2.

Figure 1 visualizes which shifts are prevented by Assumption 2. Note that there is still one additional group of shifts next to the compliers: from treatment $l \neq j, k$ towards treatment j . To also exclude this group, we focus on the subsample of individuals that revealed treatment k as their next-best treatment. More specifically, assume that each individual has a preference ranking over treatments. Define r_m as the treatment that received rank m . When revealed before z , the ranking is a predetermined characteristic, and the independence assumption 1(b) remains valid after conditioning on r_m .

Assumption 3. *Information on next-best treatment k*

(Next-best) If $r_2 = k$ and $d_j^j = 1$, then $d_l^k = 0$ for all $l \neq j, k$.

This assumption implies that, after conditioning the sample on individuals with k as next-best treatment, switching z from k to j cannot induce an individual to shift away from treatment l towards j . To see this, note that Assumption 3 prevents that $d_j^j > d_j^k$ with $d_l^j < d_l^k$. This assumption is less strict than the next-best assumption in Kirkeboen, Leuven, and Mogstad (2016). Their formulation of Assumption 3 prevents shifts already prevented by Assumption 2, and excludes the presence of always takers for treatment l .

Figure 1 visualizes that Assumptions 1(d), 2, and 3 together imply that $d_l^j = d_l^k$ for all $l \neq j, k$. As a consequence, the only shifts in the multiple unordered treatment model are the compliers. This allows us to identify the average treatment effect for the compliers.

2.2 Local average treatment effects

The following theorem formalizes what the IV coefficients β_{jk} identify.

Theorem 1. *Under Assumptions 1-3 and the model in (1) and (2), it holds for the individuals with $r_2 = k$ that*

$$\beta_{jk} = \mathbb{E}[y^j - y^k | d_j^j - d_j^k = 1, d_k^j - d_k^k = -1, r_2 = k], \quad \text{for all } j \neq k. \quad (3)$$

A formal proof can be found in Appendix A. Kirkeboen, Leuven, and Mogstad (2016) and Behaghel, Crépon, and Gurgand (2013) show that IV with three multiple unordered treatments can identify LATEs under appropriate assumptions. We prove the general case under a less strict next-best Assumption 3, while using the relationship between the IV coefficients and the reduced form and first stage coefficients.

We obtain the reduced form by substituting the first stage equations in (2) into the second stage equation in (1),

$$y = \alpha_k + \underbrace{\sum_{j \neq k} \beta_{jk} \delta_{jk}}_{\theta_k} + \underbrace{\sum_{l \neq k} \sum_{j \neq k} \beta_{jk} \pi_{jlk} z_l}_{\theta_{lk}} + \underbrace{\sum_{j \neq k} \beta_{jk} u_{jk}}_{\epsilon_k} + \epsilon_k, \quad (4)$$

where θ_{lk} is a reduced form coefficient. It follows that the IV parameters β_{jk} equal

$$\beta_{jk} = \frac{\theta_{jk}}{\pi_{jjk}} - \sum_{l \neq j, k} \frac{\pi_{ljk}}{\pi_{jjk}} \beta_{lk}, \quad \text{for all } j \neq k. \quad (5)$$

Equation (5) shows that the IV coefficient β_{jk} consists of two terms. The first term is the reduced form coefficient on z_j divided by the first stage coefficient of d_j on z_j . This term is akin to the expression for the IV coefficient in a binary treatment setting. However, in a multiple treatment setting, there is a second term that is an average of the IV coefficients β_{lk} for the remaining treatments l , weighted by the first stage coefficients of d_l on z_j .

For β_{jk} to identify a LATE, the second term in (5) has to be zero. This term is a function of the first stage coefficients in (2), for which we define the $(J-1) \times (J-1)$ matrix Π_k with the first stage coefficients $\pi_{ljk} = \mathbb{E}[d_l | z = j] - \mathbb{E}[d_l | z = k] = \mathbb{E}[d_l^j - d_l^k]$ as elements. The subscript l corresponds to the outcome variables d_l across the rows of Π_k , and the subscript j corresponds to the instruments z_j across the columns, with $l, j \neq k$. If $l = j$, the element is on the diagonal of Π_k , and we will refer to it as a diagonal first stage coefficient. Conversely, if $l \neq j$, we will refer to it as an off-diagonal first stage coefficient. The off-diagonal first stage coefficients are a function of $d_l^j - d_l^k$ with $l \neq j, k$, which we will refer to as off-diagonal shifts. The second term in (5) sums over all off-diagonal first stage coefficients in one column, which correspond to the coefficients of all d_l with $l \neq j, k$ on z_j . It follows that if there are no off-diagonal shifts, so that $\pi_{ljk} = 0$, the second term in (5) equals zero.

The off-diagonal first stage coefficients can be written as

$$\pi_{ljk} = \mathbb{E}[d_l^j - d_l^k] = \mathbb{P}[d_l^j - d_l^k = 1] - \mathbb{P}[d_l^j - d_l^k = -1], \quad \text{for all } l \neq j, k. \quad (6)$$

Figure 1 shows that Assumption 1(d) and 2 prevent that $d_l^j - d_l^k = 1$, and if we condition the sample upon $r_2 = k$, Assumption 2 and 3 further prevent $d_l^j - d_l^k = -1$. That is why Assumption 2 and 3 are pivotal to the proof of Theorem 1.

2.3 Individual behavior that generates off-diagonal treatment shifts

To prevent off-diagonal shifts, Assumptions 2 and 3 impose strong restrictions on individual behavior. We illustrate this in two different empirically relevant settings.

First, consider the setting with $z \in \{j, k\}$ for individuals with $r_1 = j$ and $r_2 = k$, so that instrument z can only switch from an individual's next-best treatment k to her preferred treatment j . This is common in empirical applications of multiple unordered treatment IV models (Kirkeboen, Leuven, and Mogstad, 2016; Heinesen and Hvid, 2019; Dahl, Rooth, and Stenberg, 2020), and is also our empirical setting in Section 6. As it is costly to obtain treatment $d \neq z$, the individuals with $z = k$ take their next-best treatment $d = k$ or make costs to obtain the preferred treatment $d = j$. For $z = j$, these individuals will obtain their preferred treatment $d = j$. This means that these individuals either have to be compliers with $d_j^j = d_k^k = 1$, or always takers with $d_j^j = d_j^k = 1$.

The assumptions rule out that individuals update their preference ranking due to exposure to treatment z . If there is a treatment $l \neq j, k$ with a higher *updated* preference rank than j or k , the individual may obtain treatment l with a lower *revealed* preference ranking than j or k . If the updated rank of l depends on the value of z , preference updating may result in off-diagonal shifts.

Figure 1 shows that Assumptions 2 and 3 prevent five off-diagonal shifts when z switches from k to j , which may all be possible under preference updating. First, an individual can update preferences for either $z = j$ or $z = k$. In case initial exposure to treatment $z = j$ induces an individual to prefer treatment l , but initial exposure to treatment $z = k$ results in treatment $d = k$, we may have $d_k \rightarrow d_l$. If $z = k$ induces an individual to prefer l but $z = j$ results in $d = j$, $d_l \rightarrow d_j$. Second, individuals can deviate from revealed preferences for both z , so that treatment m has a higher updated preference rank than k if $z = k$, and treatment l a higher updated preference rank than j when $z = j$. This explains $d_m \rightarrow d_l$, but also $d_l \rightarrow d_m$ if we reverse m and l , and $d_m \rightarrow d_k$ if $l = k$.

Preference updating in response to treatment assignment z has been demonstrated in several settings. First, preference parameters often used to describe individual behavior, such as risk aversion, are affected by economic crises, natural catastrophes, and violent

conflicts (Schildberg-Hörisch, 2018). Second, preferences for support of public policy are affected by information provision (Kuziemko, Norton, Saez, and Stantcheva, 2015; Lergetporer, Werner, and Woessmann, 2020). Third, changes in preferences have been demonstrated in the ranking of job type characteristics (Cotofan, Cassar, Dur, and Meier, 2021) and fields of study (Altonji, Arcidiacono, and Maurel, 2016). Heinesen and Hvid (2019) discuss the role of preference updating in violations of Assumptions 2 and 3 while analyzing the returns to fields of study in a multiple unordered treatment setting.

Preference updating is unlikely to generate violations of Assumption 1(d). As individuals have to make costs to obtain the revealed preferred treatment $d = j$ if $z = k$, it can be argued that their preference for the revealed preferred treatment j is strong. This, in turn, makes it unlikely these individuals move away from treatment j if $z = j$.

Second, consider the setting in which the preference ranking is unobserved. The instrument z may switch from treatment k to any j for all individuals, where treatment k is a single natural control group rather than one of many next-best treatments. This is the setting in randomized control trials (RCTs) with multiple treatment arms, where individuals assigned to the natural control group do not receive any treatment (see *e.g.*, Behaghel, Crépon, and Gurgand (2013); Duflo, Glennerster, and Kremer (2007)). In such RCTs, a preference ranking is not elicited, but individuals may nevertheless have one based upon the perceived outcome of each treatment and the costs to obtain treatment $d \neq z$.

If individuals are indifferent ex-ante, Assumptions 2 and 3 rule out preference updating in response to treatment assignment z . For instance, if assignment to the natural control group $z = k$ induces individuals to prefer treatment m , whereas $z = j$ induces a preference for l , we may have $d_m \rightarrow d_l$. If individuals are not indifferent ex-ante, Assumptions 2 and 3 impose even stronger restrictions: Even without preference updating, individuals with $r_1 = l$ may make costs to obtain their preferred treatment l if assigned to the natural control group $z = k$ and stay in treatment j if $z = j$, which would generate $d_l \rightarrow d_j$.

2.4 The bias from off-diagonal treatment shifts

In the presence of off-diagonal shifts, the Assumptions 2 and 3 are violated and the IV coefficients in (1) are biased. The following corollary provides an expression for the bias.

Corollary 1. *Under Assumptions 1-3 and the model in (1) and (2), with Assumption 2 or 3 only violated by $\mathbb{P}[d_m^j - d_m^k = 1, d_n^j - d_n^k = -1 | r_2 = k] > 0$ with $m \neq n$ and $n \neq j$, it holds for the*

individuals with $r_2 = k$ that

$$\beta_{jk} = \Delta_{jkjk} + (\Delta_{jkmn} - (\Delta_{mkmk} - \Delta_{nknk})) \frac{\mathbb{P}[d_m^j - d_m^k = 1, d_n^j - d_n^k = -1 | r_2 = k]}{\mathbb{P}[d_j^j - d_j^k = 1 | r_2 = k]}, \quad (7)$$

where $\Delta_{jkmn} = \mathbb{E}[y^m - y^n | d_m^j - d_m^k = 1, d_n^j - d_n^k = -1, r_2 = k]$.

The proof is deferred to Appendix B. Corollary 1 shows that in the presence of a violation of Assumption 2 or 3, the IV coefficient β_{jk} equals the LATE of the compliers Δ_{jkjk} plus a bias term that consists of a weighted combination of additional LATEs. Note that if $m \neq j$, the bias results from a violation of Assumption 2, and if $m = j$ of Assumption 3.

The bias term includes the difference between Δ_{jkmn} , which is the treatment effect for the off-diagonal shifts from treatment n to m as a result of a switch in the instrument from k to j , and $\Delta_{mkmk} - \Delta_{nknk}$, which includes the LATEs of two different compliers. The LATE Δ_{jkmn} is not identified, but Theorem 1 shows that $\beta_{mk} = \Delta_{mkmk}$ and $\beta_{nk} = \Delta_{nknk}$.

Depending on the off-diagonal shift, a different treatment effect homogeneity assumption sets the bias to zero. The dashed arrows in Figure 1 represent the off-diagonal shifts violating Assumptions 2 and 3. For $d_k \rightarrow d_l$ or $d_l \rightarrow d_k$, $\Delta_{jklk} = \Delta_{lklk}$ results in zero bias. Therefore, these shifts only require treatment effect homogeneity across different values of z , while treatment effects are allowed to be heterogeneous across d . For $d_l \rightarrow d_m$ or $d_m \rightarrow d_l$, we require $\Delta_{jkmk} = \Delta_{mkmk} - \Delta_{lklk}$. This only holds under the strict assumption that the LATEs are homogeneous across different values of both d and z . In general, treatment effect homogeneity has been rejected in many economic settings. For instance, Angrist and Pischke (2008) discuss examples where several credible instruments estimate different treatment effects for the same causal relation.

The bias is weighted by the probability of observing the off-diagonal shift $\mathbb{P}[d_m^j - d_m^k = 1, d_n^j - d_n^k = -1 | r_2 = k] > 0$ divided by the first stage coefficient $\pi_{jjk} = \mathbb{P}[d_j^j - d_j^k = 1 | r_2 = k]$. Hence, the bias increases in the probability of a violation of Assumption 2 or 3, and decreases with a stronger first stage for the compliers. The first stage plays a similar role in the bias that follows from violating the exclusion restriction in Assumption 1(a). This bias is studied in binary treatment IV models, that is (1)-(2) with $J = 2$, and known to be small if the first stage is strong (Bound, Jaeger, and Baker, 1995; Angrist, Imbens, and Rubin, 1996).

There are also differences between the bias of a violation of the exclusion restriction and the bias in Corollary 1, which potentially exacerbate the effect of the latter. Since an off-diagonal shift cannot be a complier at the same time, the presence of off-diagonal shifts,

in addition to always- and never takers, must weaken the first stage. In contrast, a violation of the exclusion restriction does not need to reduce the first stage. Moreover, there can be multiple off-diagonal shifts in a multiple treatment setting: Each of these shifts add an additional bias term and further reduce the first stage. Section 6 shows the presence of off-diagonal shifts in our empirical context, and Appendix C argues that they may also be present in other applications of the multiple unordered treatment model.

3 Treatment cluster IV model

This section introduces the multiple unordered treatment IV model in which we identify CLATEs: clustered local average treatment effects. First, we adapt the IV assumptions to the cluster level. Second, we derive the treatment effects identified by CLATE. Third, we discuss the settings in which CLATEs can be identified, but LATEs cannot. Finally, we show that CLATEs can identify treatment effect heterogeneity.

3.1 Setting and assumptions

Suppose we have $c = 1, \dots, C$ mutually exclusive clusters of treatments S_c . Define the set $S = \{S_c\}_{c=1}^C$ that includes all treatment clusters. Define $\tilde{d}_c = \sum_{n \in S_c} d_n$ that equals one if a treatment in treatment cluster S_c is chosen and zero otherwise. Similarly, $\tilde{z}_c = \sum_{n \in S_c} z_n$ equals one if the cost of one of the treatments in cluster S_c is decreased, and zero otherwise. We are interested in the effect on y of taking treatment from cluster S_a compared to taking treatment from cluster S_b . Consider the second stage model

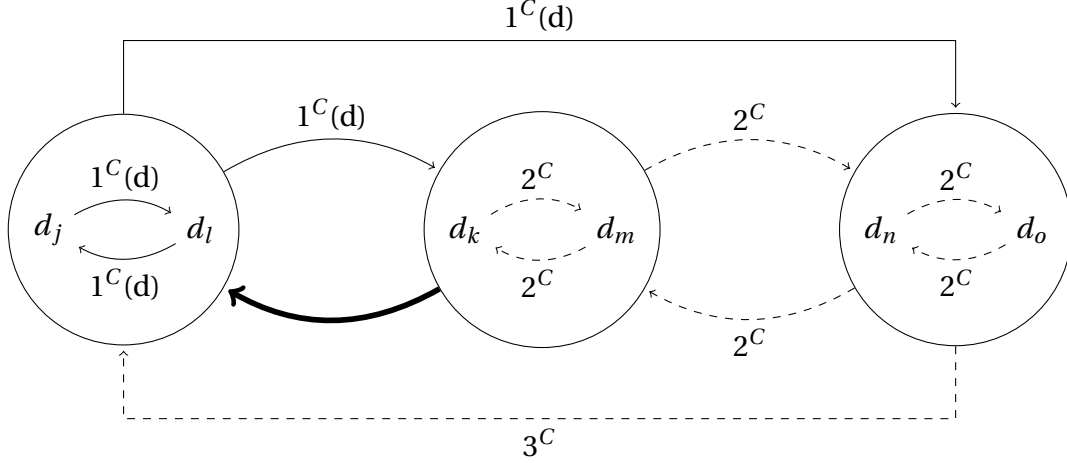
$$y = \tilde{\alpha}_b + \sum_{a \neq b} \tilde{\beta}_{ab} \tilde{d}_a + \tilde{\varepsilon}_b, \quad (8)$$

where the first stage equations are

$$\tilde{d}_a = \tilde{\delta}_{ab} + \sum_{c \neq b} \tilde{\pi}_{acb} \tilde{z}_c + \tilde{u}_{ab}, \quad \text{for all } a \neq b. \quad (9)$$

We adapt the IV assumptions 1(d), 2, and 3 in Section 2 to the cluster setting. Figure 2 shows all possible shifts in d when z switches from one $k \in S_b$ to one $j \in S_a$. The circles represent three different clusters. In this setting, the LATE assumptions only allow for one group of compliers: the treatment shifts from k towards j . However, with multiple treatments in the clusters, the CLATE assumptions allow shifts from all treatments in cluster S_b towards

Figure 2: An overview of all the possible shifts when z switches from $k \in S_b$ to $j \in S_a$



Notes: $j, l \in S_a$, $k, m \in S_b$, and $n, o \in S_c$. The arrows represent a shift from one treatment to another when z switches from $k \in S_b$ to $j \in S_a$. The numbers represent the assumption that prevents the shift of the corresponding arrow.

all treatments in cluster S_a . That is, CLATE allows for the presence of $|S_a| \times |S_b|$ shifts instead of only one. When each cluster contains one treatment, Figure 2 boils down to Figure 1, and the adapted IV assumptions boil down to their standard form.

Assumption 1^C. *Monotonicity treatment cluster S_a compared to S_b*

d. (Monotonicity) $d_l^j \geq d_l^k$ for all $j, l \in S_a$ and $k \in S_b$.

For treatment j within cluster S_a , Assumption 1^C(d) requires that $d_l^j \geq d_l^k$ for each treatment l within cluster S_a . Standard monotonicity 1(d) only requires this for treatment $l = j$. Hence, in isolation Assumption 1^C(d) may be viewed as a stronger assumption than 1(d). However, Assumption 1(d) can only identify LATEs in conjunction with Assumptions 2 and 3, which together require that $d_l^j = d_l^k$ for all $l \neq j, k$, which is a substantially stronger set of restrictions than imposed by Assumption 1^C(d) and the following two assumptions.

Assumption 2^C. *Irrelevance assumption treatment cluster S_a compared to S_b*

(Irrelevance) If $d_l^j = d_l^k = 0$ for all $l \in S_a$, then $d_n^j = d_n^k$ for all $n \notin S_a$, $j \in S_a$ and $k \in S_b$.

Following the same logic as with Assumption 2, Assumption 2^C implies together with Assumption 1^C(d) that switching z from k to j cannot induce an individual *towards* a treatment outside cluster S_a . In contrast to the stronger Assumption 2, this does not impose any

restriction on the shifts towards the treatments $l \neq j$ within cluster S_a . Figure 2 shows how Assumption 2^C prevents any shifts towards treatments outside cluster S_a , while Figure 1 shows how Assumption 2 prevents any shifts towards $l \neq j$.

Assumption 3^C. *Information on next-best treatment cluster S_b*

(Next-best) If $r_2 = k$ and there is an $l \in S_a$ for which $d_l^j = 1$, then $d_n^m = 0$ for all $j \in S_a$, $k, m \in S_b$ and $n \notin \{S_a, S_b\}$.

Following the same logic as with Assumption 3, Assumption 3^C implies that, after conditioning the sample on individuals with a next-best treatment in S_b , switching z from k to j cannot induce an individual to shift *away* from a treatment outside clusters S_a and S_b *towards* a treatment within cluster S_a . In contrast to the stronger Assumption 3, this does not impose a restriction on all treatments $l \neq j, k$, but only on the treatments that are not included in the clusters S_a and S_b . This is also illustrated by Figures 1 and 2, where the first shows that only treatment k is excluded from the next-best assumption, and the latter shows that all treatments in the cluster S_b are excluded.

3.2 Clustered local average treatment effects

Similar as in Section 2, we substitute the first stage equations in (9) into the second stage equations in (8), to obtain the reduced form,

$$y = \underbrace{\tilde{\alpha}_b + \sum_{a \neq b} \tilde{\beta}_{ab} \tilde{\delta}_{ab}}_{\tilde{\theta}_b} + \sum_{c \neq b} \underbrace{\sum_{a \neq b} \tilde{\beta}_{ab} \tilde{\pi}_{acb} \tilde{z}_c}_{\tilde{\theta}_{cb}} + \underbrace{\sum_{a \neq b} \tilde{\beta}_{ab} \tilde{u}_{ab} + \tilde{\epsilon}_b}_{\tilde{\epsilon}_b}, \quad (10)$$

from which follows that the IV parameters $\tilde{\beta}_{ab}$ equal

$$\tilde{\beta}_{ab} = \frac{\tilde{\theta}_{ab}}{\tilde{\pi}_{aab}} - \sum_{c \neq a, b} \frac{\tilde{\pi}_{cab}}{\tilde{\pi}_{aab}} \tilde{\beta}_{cb}, \quad \text{for all } a \neq b, \quad (11)$$

where the expression is similar as in (5).

Lemma 1. *Under Assumptions 1(a)-(c), 1^C(d)-3^C and the model in (8) and (9), it holds for the individuals with $r_2 \in S_b$ that*

$$\tilde{\pi}_{cab} = 0 \text{ for all } c \neq a, b \text{ and } a \neq b.$$

The proof is deferred to Appendix D. Lemma 1 shows that in the treatment cluster IV model, the multiple treatment bias disappears under less strict assumptions than for LATE. We discuss this in more detail in Section 3.3.

Theorem 2 shows that $\tilde{\beta}_{ab}$ is a weighted average of the LATEs of all pairs of treatments in which a treatment from cluster S_a is compared to a treatment from cluster S_b . We will refer to this as CLATE: clustered LATE.

Theorem 2. *Under Assumptions 1(a)-(c), $I^C(d)$ - \mathcal{Z}^C and the model in (8) and (9), it holds for the individuals with $r_2 \in S_b$ that*

$$\tilde{\beta}_{ab} = \sum_{j \in S_a} \sum_{k \in S_b} \sum_{l \in S_a} \sum_{m \in S_b} \lambda_{jklm} \mathbb{E}[y^l - y^m | d_l^j - d_l^k = 1, d_m^j - d_m^k = -1, r_2 \in S_b], \quad (12)$$

for all clusters $a \neq b$, with

$$\lambda_{jklm} = \frac{\mathbb{P} \left[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1 \right] \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b]}{\sum_{j \in S_a} \sum_{k \in S_b} \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b] \sum_{l \in S_a} \sum_{m \in S_b} \mathbb{P} \left[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1 \right]}, \quad (13)$$

where the probabilities implicitly condition on $r_2 \in S_b$.

The proof is deferred to Appendix E.

Since $0 \leq \lambda_{jklm} \leq 1$ and $\sum_{j \in S_a} \sum_{l \in S_a} \sum_{k \in S_b} \sum_{m \in S_b} \lambda_{jklm} = 1$, $\tilde{\beta}_{ab}$ is a weighted average of LATEs $\Delta_{jklm} = \mathbb{E}[y^l - y^m | d_l^j - d_l^k = 1, d_m^j - d_m^k = -1, r_2 \in S_b]$. The weighted average is taken over all possible pairs of treatments with one treatment from S_a and one from S_b , and for each of those treatment pairs all possible pairs of instruments with one from S_a and one from S_b .

The LATE Δ_{jklm} equals the causal effect of a change from an individual treatment $m \in S_b$ to treatment $l \in S_a$, for the individuals that shift from treatment m to l as a result of a switch in the instrument from $k \in S_b$ to $j \in S_a$. Although Theorem 2 shows that under the CLATE assumptions the weighted average of the LATEs $\tilde{\beta}_{ab}$ is identified, the individual LATEs Δ_{jklm} are not identified under these assumptions.

When the LATE assumptions are satisfied, it follows from Theorem 1 that one can identify $\beta_{jk} = \Delta_{jkjk}$. At the same time, note that under the strict LATE assumptions, one can not identify the treatment effects Δ_{jklm} with either $j \neq l$ or $k \neq m$. When we compare the CLATEs in Theorem 2 to the LATEs in Theorem 1, CLATE equals a (weighted) local average treatment effect for a more comprehensive sample. This sample does not only include the shifts from k to j as a result of a switch in the instrument from k to j , but all the shifts from S_b to S_a as a result of a switch in the instrument from S_b to S_a .

The weights λ_{jklm} show which LATEs Δ_{jklm} contribute the most to $\tilde{\beta}_{ab}$. The weight λ_{jklm} represents the normalized proportion of individuals that shift from treatment m to l as a result of a switch in the instrument from k to j . The weight is normalized by the cumulative proportion of all possible shifts from a treatment within S_b to a treatment within S_a as a result of a switch in the instrument from S_b to a S_a .

Since we observe for each individual only the treatment choice for one instrument value, the individuals for which we observe a treatment choice from S_a could have shifted from any treatment in S_b , and the other way around. Therefore, the normalized proportion λ_{jklm} of individuals that shift from treatment $m \in S_b$ to $l \in S_a$ cannot be point-identified when both S_a and S_b contain multiple treatments. Corollary 2 identifies bounds on these weights under the CLATE assumptions.

Corollary 2. *Under Assumptions 1(a)-(c), 1^C(d)-3^C and the model in (8) and (9), it holds for the individuals with $r_2 \in S_b$ that the weights λ_{jklm} in (13) with $j, l \in S_a$ and $k, m \in S_b$ are bounded as*

$$\frac{\max(0, -\pi_{mjk} - \sum_{n \in \{S_a \setminus l\}} \pi_{njk}) \omega_{jk}}{\sum_{j \in S_a} \sum_{k \in S_b} \omega_{jk} \sum_{l \in S_a} \pi_{ljk}} \leq \lambda_{jklm} \leq \frac{\min(\pi_{ljk}, -\pi_{mjk}) \omega_{jk}}{\sum_{j \in S_a} \sum_{k \in S_b} \omega_{jk} \sum_{l \in S_a} \pi_{ljk}}, \quad (14)$$

where $-\pi_{kjk} = \sum_{l \in S_a} \pi_{ljk} + \sum_{m \in \{S_b \setminus k\}} \pi_{mjk}$, π_{ljk} with $l \neq k$ is a first-stage coefficient in (2), and $\omega_{jk} = \mathbb{P}[z = j | z \in S_a, r_2 \in S_b] \mathbb{P}[z = k | z \in S_b, r_2 \in S_b]$.

The proof is deferred to Appendix F. Corollary 2 shows that the proportion of shifts from treatment $m \in S_b$ to $l \in S_a$ is upper bounded by both the total proportion of shifts away from m and the total proportion of shifts towards l . A lower bound is constructed by the difference between the total proportion of shifts away from m and the total proportion of shifts towards $\{S_a \setminus l\}$. In the special cases where either S_a or S_b contains only one treatment, the first stage coefficients respectively identify where the shifts come from or go to. Hence, the bounds in Corollary 2 boil down to point identification:

$$\lambda_{jkjm} = \frac{-\pi_{mjk} \mathbb{P}[z = k | z \in S_b, r_2 \in S_b]}{\sum_{k \in S_b} \mathbb{P}[z = k | z \in S_b, r_2 \in S_b] \pi_{jjk}} \text{ and } \lambda_{jklk} = \frac{\pi_{ljk} \mathbb{P}[z = j | z \in S_a, r_2 \in S_b]}{\sum_{j \in S_a} \mathbb{P}[z = j | z \in S_a, r_2 \in S_b] \sum_{l \in S_a} \pi_{ljk}}, \quad (15)$$

with respectively $S_a = \{j\}$ and $S_b = \{k\}$.

Under the LATE assumptions it holds that $\pi_{ljk} = 0$ for all $l \neq j, k$. In that case, it follows from Corollary 2 that $\lambda_{jkjk} = \pi_{jjk} \omega_{jk} / (\sum_{j \in S_a} \sum_{k \in S_b} \pi_{jjk} \omega_{jk})$ and $\lambda_{jklm} = 0$ if $l \neq j$ and

$m \neq k$. Then, CLATE boils down to $\tilde{\beta}_{ab} = (\sum_{j \in S_a} \sum_{k \in S_b} \Delta_{jkjk} \pi_{jjk} \omega_{jk}) / (\sum_{j \in S_a} \sum_{k \in S_b} \pi_{jjk} \omega_{jk})$, which is a weighted average of LATEs Δ_{jkjk} for the regular compliers. Moreover, if each cluster has only one individual treatment, as in the standard multiple unordered treatment IV model, $\tilde{\beta}_{ab} = \Delta_{jkjk}$, and the CLATE in Theorem 2 equals the LATE in Theorem 1.

3.3 When CLATEs are identified, but LATEs are not

The CLATE assumptions allow for off-diagonal treatment shifts that are ruled out by LATE. Similar as in Section 2.3, we illustrate this using two empirically relevant applications of the multiple unordered treatment model.

First, consider the individuals with preference ranking $r_1 = j$ and $r_2 = k$, $z \in \{j, k\}$, and a switch in z from k to j . As it is costly to obtain treatment $d \neq z$, the individuals with $z = k$ take their next-best treatment $d = k$ or make costs to obtain the preferred treatment $d = j$. For $z = j$, these individuals will obtain their preferred treatment $d = j$. The shift of the compliers $d_j^j = d_k^k = 1$ is the only one that is allowed by the LATE assumptions.

In addition to this diagonal shift, Figure 2 precisely demonstrates which off-diagonal shifts are allowed under the CLATE Assumptions. Initial exposure to treatment $z = j$ may result in treatment $l \in S_a$, and initial exposure to treatment $z = k$ may result in treatment $m \in S_b$. CLATE allows for the off-diagonal shift $d_m \rightarrow d_l$, for all $m \in S_b$ and $l \in S_a$. However, if $l \notin S_a$, the off-diagonal shift is prevented by Assumption 2^C, and for $l \in S_a$ and $m \notin S_b$, the off-diagonal shift is prevented by Assumption 3^C. Therefore, S_a and S_b have to be disjoint clusters of treatments.

The off-diagonal shifts from cluster S_b to cluster S_a , which are allowed under the CLATE assumptions, may be explained by preference updating within clusters. Initial exposure to treatment $z = j$ in cluster S_a may result in preferring and obtaining any other treatment $d = l$ in cluster S_a . The same holds for $z = k$ and $d = m$ in cluster S_b .

CLATE allows for preference updating within, and not across, disjoint clusters. Consistent with disjoint clusters in a setting with fields of study as multiple unordered treatments, [Wiswall and Zafar \(2015\)](#) find evidence that student preferences are similar for majors within large sets, such as Natural sciences and Mathematics, and Business and Economics. Disjoint sets are also expected in settings in which the switching costs are lower between treatments within, and not across, clusters. In this case, the costs to obtain treatment $d \neq z$ will depend on the initial treatment assignment z . An example of this is provided by [Altonji, Arcidiacono, and Maurel \(2016\)](#), who discuss that the costs to enroll in certain majors depend on

an individual's educational history.

In the second setting, the instrument z can switch from k to any j for all individuals, where $k \in S_b$ is a single natural control group. Preference updating, and low switching costs between treatments, within clusters may generate shifts from treatment $m \in S_b$ to $l \in S_a$ when instrument z switches from any $n \in S_b$ to any $j \in S_a$. Ultimately, the presence of disjoint treatment clusters is an empirical question, to which we find a positive answer in our analysis of the effect of fields of study in Section 6.

3.4 An illustration with five treatments

In order to show which restrictions in LATE are eased by CLATE, consider an example with $J = 5$ unordered treatments clustered into $S_1 = \{1,2\}$, $S_2 = \{3\}$, and $S_3 = \{4,5\}$. We are interested in the treatment effects for the individuals with $r_2 = 5$. For these individuals, S_1 and S_2 are the preferred treatment clusters, and S_3 is the next-best cluster.

The assumptions in LATE and CLATE impose different restrictions on the first stage matrix for this sample. The LATE assumptions prevent all off-diagonal shifts in Π_5 :

$$\Pi_5^{\text{LATE}} = \begin{bmatrix} \pi_{115} > 0 & \pi_{125} = 0 & \pi_{135} = 0 & \pi_{145} = 0 \\ \pi_{215} = 0 & \pi_{225} > 0 & \pi_{235} = 0 & \pi_{245} = 0 \\ \pi_{315} = 0 & \pi_{325} = 0 & \pi_{335} > 0 & \pi_{345} = 0 \\ \pi_{415} = 0 & \pi_{425} = 0 & \pi_{435} = 0 & \pi_{445} > 0 \end{bmatrix}, \quad (16)$$

where the restrictions follow from Assumptions 1-3. The CLATE assumptions, however, only prevent off-diagonal shifts towards or from a treatment d that is not in the same preferred or next-best treatment cluster as the values of the instrument z :

$$\Pi_5^{\text{CLATE}} = \begin{bmatrix} \pi_{115} > 0 & \pi_{125} \geq 0 & \pi_{135} = 0 & \pi_{145} \leq 0 \\ \pi_{215} \geq 0 & \pi_{225} > 0 & \pi_{235} = 0 & \pi_{245} \leq 0 \\ \pi_{315} = 0 & \pi_{325} = 0 & \pi_{335} > 0 & \pi_{345} \leq 0 \\ \pi_{415} \leq 0 & \pi_{425} \leq 0 & \pi_{435} \leq 0 & \pi_{445} \geq 0 \end{bmatrix}, \quad (17)$$

where the restrictions follow from Assumptions 1^C-3^C.

When the off-diagonal shift corresponds to a treatment d that is in the same preferred or next-best treatment cluster as one of the instrument values z , the CLATE assumptions impose an inequality instead of an equality restriction on the off-diagonal shifts. Assumption 1^C(d) ensures that there are only shifts *towards* treatments in the preferred cluster,

whereas Assumption 2^C and 3^C ensure that there are only shifts *away from* treatments in the next-best cluster.

These inequality restrictions in (17) allow for five off-diagonal shifts. First, $\pi_{215} \geq 0$ and $\pi_{415} \leq 0$ allow for preference updating within clusters S_1 and S_3 when z switches from 5 to 1. This behavior may result in the shifts $d_5 \rightarrow d_2$, $d_4 \rightarrow d_1$, and $d_4 \rightarrow d_2$. Similarly, $\pi_{125} \geq 0$ and $\pi_{425} \leq 0$ may result in $d_5 \rightarrow d_1$, $d_4 \rightarrow d_2$, and $d_4 \rightarrow d_1$ after switching z from 5 to 2. Finally, $\pi_{435} \leq 0$ explains $d_4 \rightarrow d_3$ when z switches from 5 to 3.

CLATE does not include switches in z from one treatment in the next-best cluster to another treatment in the next-best cluster. Therefore, the fourth column of Π_k in (17) is unrestricted: Since both treatment 4 and 5 are in the next-best cluster S_3 , CLATE will not consider switches in z from 5 to 4. It follows that the possible shifts $\pi_{l45} = \mathbb{E}[d_l^4 - d_l^5]$, with $l = 1, \dots, 4$, are unrestricted.

3.5 CLATE and heterogeneous treatment effects

Corollary 1 shows that β_{jk} in (1) does not equal Δ_{jkjk} if Assumption 2 or 3 is violated, and the LATEs are heterogeneous. Since Δ_{jkmn} is only identified if $m = j$ and $n = k$, we cannot identify from the IV coefficients whether the LATEs are indeed heterogeneous. However, the corollary below shows that CLATE can identify treatment effect heterogeneity.

Corollary 3. *Under Assumptions 1(a)-(c), 1^C(d)-3^C and the model in (8) and (9), it holds for the individuals with $r_2 \in S_b$ that under the following two conditions:*

1. *If $j \in S_a$, $\Delta_{jklm} = \Delta_{lklk} - \Delta_{mkmk}$ for $l \in S_a$ and $k, m \in S_b$ for all $a \neq b$,*
2. *If $j \in S_b$, $\Delta_{jklm} = \Delta_{lklk} - \Delta_{mkmk}$ for $k \in S_b$ and all $l, m \in S$,*

the IV coefficients equal

$$\tilde{\beta}_{ab} = \frac{\sum_{j \in S_a} \sum_{k \in S_b} \sum_{l \in S_a} \beta_{lk} \pi_{ljk} \omega_{jk}}{\sum_{j \in S_a} \sum_{k \in S_b} \sum_{l \in S_a} \pi_{ljk} \omega_{jk}} - \frac{\sum_{j \in S_a} \sum_{k \in S_b} \sum_{m \in S_b} \beta_{mk} \pi_{mjk} \omega_{jk}}{\sum_{j \in S_a} \sum_{k \in S_b} \sum_{m \in S_b} \pi_{mjk} \omega_{jk}}, \quad (18)$$

where $\omega_{jk} = \mathbb{P}[z = j | z \in S_a, r_2 \in S_b] \mathbb{P}[z = k | z \in S_b, r_2 \in S_b]$.

The proof is deferred to Appendix G. In addition to the CLATE assumptions, Corollary 3 imposes homogeneity assumptions under which the IV coefficient β_{jk} identifies Δ_{jkjk} , even if the LATE Assumptions 2 and 3 are violated. In this case, the weighted average of IV coefficients in (18) equals the corresponding CLATE. Hence, the homogeneity assumptions are violated if the equality in (18) does not hold.

Corollary 1 shows that the bias in the IV coefficients from an off-diagonal shift between treatments l and m is zero with $\Delta_{jklm} = \Delta_{lklk} - \Delta_{mkmk}$. The CLATE assumptions prevent off-diagonal shifts across treatments within preferred clusters S_a with $a \neq b$ as a result of a switch in z from $k \in S_b$ to $j \in S_a$. Therefore, we only require homogeneity assumptions on the effects between treatments across a preferred cluster and a next-best cluster (condition 1) and for all treatment changes as a result of a switch in z within S_b (condition 2). These shifts correspond to π_{215} and π_{125} in (17), and the final row and column in (17), respectively.

Kirkeboen, Leuven, and Mogstad (2016) show that LATE is unbiased under the stronger homogeneity assumption that $\mathbb{E}[y^l - y^m]$ is common across all individuals for all treatments l and m . Under this assumption $\Delta_{jklm} = \mathbb{E}[y^l - y^k] - \mathbb{E}[y^m - y^k]$, from which the conditions in Corollary 3 follow. In case the next-best treatment cluster has only one individual treatment $S_b = \{k\}$, the assumptions in Corollary 3 boil down to $\Delta_{jklk} = \Delta_{lklk}$, which imposes that the treatment effects are common across individuals for which the instrument switches from k . In this case, $\tilde{\beta}_{ab} = \sum_{l \in S_a} \beta_{lk} \sum_{j \in S_a} \lambda_{jklk}$.

4 Granularity estimation

In general, the treatment clusters $\{S_c\}_{c=1}^C$ that satisfy the CLATE assumptions are not observed. This section explains how these treatment clusters can be estimated from individual treatment data for d and z . We estimate the set of treatment clusters S with the highest level of granularity so that treatment effects are estimated at the finest level.

To obtain the highest level of granularity, we estimate the set of treatment clusters S separately for each sample of individuals with next-best treatment $r_2 = k$, with $k = 1, \dots, J$. Moreover, we start at the individual treatment level and only cluster treatments for which we find violations of Assumptions 2 and 3. As we show in Figure 1, each assumption excludes different off-diagonal shifts, but together with Assumption 1(d) they exclude all off-diagonal shifts. This results in the necessary condition that $\pi_{ljk} = 0$, which serves as the null-hypothesis for a two-sided t-test in (2). If the necessary condition is rejected, this is evidence of the presence of off-diagonal shifts, which implies that at least one of the Assumptions 1(d), 2 or 3 is violated.

Next, we cluster the treatments corresponding to the violations of the LATE assumptions in such a way that the resulting set of treatment clusters satisfies the CLATE assumptions. Since $\pi_{ljk} = \mathbb{P}[d_l^j - d_l^k = 1] - \mathbb{P}[d_l^j - d_l^k = -1]$, a $\hat{\pi}_{ljk} > 0$ implies the presence of off-diagonal

shifts towards treatment l , and $\hat{\pi}_{ljk} < 0$ off-diagonal shifts from treatment l . According to Figure 2, shifts towards l only satisfy the CLATE assumptions if l is in the same treatment cluster as treatment j . On the other hand, shifts from treatment l are only allowed if l is in the same treatment cluster as treatment k . Therefore, we merge the clusters of l and j when $\hat{\pi}_{ljk} > 0$, and of l and k when $\hat{\pi}_{ljk} < 0$.

We complete the clustering algorithm with a rule for the treatments for which the necessary condition is rejected and one of the following statements hold: (i) $\hat{\pi}_{jlk} > 0$ together with $\hat{\pi}_{ljk} < 0$, or (ii) $\hat{\pi}_{mjk} < 0$ together with $\hat{\pi}_{mlk} > 0$. To ensure disjoint clusters of treatments, we merge in both cases the cluster of treatment l with the next-best cluster of k . In (ii) we additionally merge treatment m with the next-best cluster of k . The CLATE assumptions do allow for shifts from the next-best cluster, and hence $\hat{\pi}_{ljk} < 0$ in (i) and $\hat{\pi}_{mjk} < 0$ in (ii) are accommodated. Since CLATE does not consider treatment shifts or instrument switches within the next-best cluster, the CLATE assumptions do not impose restrictions on the first stage coefficients in the column related to z_l : π_{jlk} in (i) or π_{mlk} in (ii). This clustering rule is automatically satisfied if clusters are first merged according to $\hat{\pi}_{ljk} < 0$, and the merge of a preferred treatment cluster and next-best treatment cluster results in a next-best treatment cluster.

Algorithm 1 summarizes the CLATE estimation algorithm. The algorithm tests the necessary condition for all l and j with $l \neq j$. In case the necessary condition is rejected, there is evidence for the presence of off-diagonal shifts, and hence the LATE assumptions are violated, and clusters are merged. On the other hand, if we cannot reject $\pi_{ljk} = 0$, there is no evidence of violation, and no further action is required. The final set of treatment clusters S satisfies the CLATE assumptions in Theorem 2 for the sample of individuals with $r_2 = k$. The cluster that includes treatment k is referred to as the next-best treatment cluster. Conditional on S , CLATE can be estimated using two-stage least squares.

Note that the CLATE assumptions do not necessarily hold for the resulting set of CLATEs. Failure to reject the necessary condition $\pi_{ljk} = 0$ does not necessarily mean that Assumptions 2 and 3 are satisfied, as the probability of the shifts $d_l^j - d_l^k = 1$ and $d_l^j - d_l^k = -1$ might cancel out in $\pi_{ljk} = \mathbb{P}[d_l^j - d_l^k = 1] - \mathbb{P}[d_l^j - d_l^k = -1]$. Therefore, this procedure is useful for testing which set of treatment clusters do not satisfy the CLATE assumptions, but cannot guarantee for which set the CLATE assumptions are satisfied.

Algorithm 1 estimates separate CLATEs $\tilde{\beta}_{ab}$ for J groups of individuals with $r_2 = k$, $k = 1, \dots, J$, to obtain the highest level of granularity. However, treatment effects may be

Algorithm 1 CLATE estimation for the sample of individuals with next-best treatment k

- 1: Set J treatment clusters: one for each treatment j
 - 2: Estimate Π_k in (2) with least squares
 - 3: **for all** $l \neq j$ **do**
 - 4: Two-sided t-test for $H_0 : \pi_{ljk} = 0$
 - 5: **if** $H_0 : \pi_{ljk} = 0$ is rejected and $\hat{\pi}_{ljk} < 0$ **then**
 - 6: merge the clusters of l and k
 - 7: **else if** $H_0 : \pi_{ljk} = 0$ is rejected and $\hat{\pi}_{ljk} > 0$ **then**
 - 8: merge the clusters of l and j
 - 9: **end if**
 - 10: **end for**
 - 11: Define S as the set of nonempty clusters
 - 12: Given S , estimate $\tilde{\beta}_{ab}$ for all $a \neq b$ with $k \in S_b$ in (8) and (9) with two-stage least squares
-

hard to identify when the sample corresponding to $r_2 = k$ is small. Appendix H shows that the CLATEs can potentially be more precisely estimated under a homogeneity assumption across the individuals with $r_2 \in S_b$.

5 The fuzzy regression discontinuity design

Most applications of the multiple unordered treatment IV model have used a fuzzy regression discontinuity design (RDD) (Kirkeboen, Leuven, and Mogstad, 2016; Heinesen and Hvid, 2019; Dahl, Rooth, and Stenberg, 2020; Kirkeboen, Leuven, and Mogstad, 2021). This section provides an expression for CLATE in the fuzzy RDD used in this literature. Appendix I derives an expression for CLATE with general covariates. We leave nonparametric IV models with covariates –as studied by Frölich (2007) in a binary treatment setting– for future research, but note that the typical sample sizes in multiple unordered treatment settings do not allow for nonparametric estimation.

In a fuzzy RDD, treatment assignment z is determined by the value of a continuous running variable lying on either side of a fixed cutoff. We analyze the setting where the running variable determines assignment to the multiple unordered treatments as follows:

$$z = \begin{cases} r_1 & s \geq 0, \\ r_2 & s < 0, \end{cases} \quad (19)$$

where s is defined as the running variable minus the cutoff. Hence, by conditioning the sample on individuals with $r_2 = k$ as next-best treatment, z either equals k or the preferred treatment $j \neq k$.

We extend the treatment cluster IV model in (8) and (9) by including the running variable and fixed effects for preferring treatment j and having treatment $k \in S_b$ as next-best treatment. This results in the second stage model

$$y = \sum_{l \neq k} \nu_{lk} x_l + \sum_{a \neq b} \tilde{\beta}_{ak} \tilde{d}_a + \gamma_k s + \tilde{\varepsilon}_k, \quad (20)$$

and the corresponding first stages are

$$\tilde{d}_a = \sum_{l \neq k} \eta_{alk} x_l + \sum_{c \neq b} \tilde{\pi}_{ack} \tilde{z}_c + \psi_{ak} s + \tilde{u}_{ak}, \quad \text{for all } a \neq b, \quad (21)$$

where the fixed effect x_j equals one if $r_1 = j$ and zero otherwise. Therefore, the instrument z can only switch from $r_2 = k$ to $r_1 = x$, with $x = \sum_{j \neq k} j x_j$, for the individuals with $r_2 = k$. Recall that in the case that each treatment cluster contains only one treatment, this model boils down to the (unclustered) multiple unordered treatment IV model, and hence the results in this section hold both for LATE and CLATE with a fuzzy RDD.

The fuzzy RDD replaces the independence assumption 1(b) by the local continuity assumption (Dong, 2018; Imbens and Lemieux, 2008; Hahn, Todd, and Van der Klaauw, 2001):

Assumption 1^D. *Continuous potential outcomes at the cutoff $s = 0$*

b. (Continuity) y^d and d^z are continuous in s at $s = 0$ for all d, z .

This assumption implies that individuals just above the cutoff are similar to individuals just below the cutoff. Theorem 3 shows that the following CLATEs are identified under the local continuity assumption.

Theorem 3. *Under Assumptions 1(a), 1^D(b), 1(c), 1^C(d)-3^C, and the model in (20) and (21), it holds for the individuals with $r_2 = k$ that*

$$\tilde{\beta}_{ak} = \sum_{j \in S_a} \sum_{l \in S_a} \sum_{m \in S_b} \lambda_{jklm} \mathbb{E} \left[y^l - y^m \mid d_l^j - d_l^k = 1, d_m^j - d_m^k = -1, x = j, s = 0, r_2 = k \right], \quad (22)$$

for all a for which $k \notin S_a$ and $k \in S_b$, with

$$\lambda_{jklm} = \frac{\mathbb{P} \left[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1 \mid x = j, s = 0 \right] \lim_{v \rightarrow +0} \mathbb{P} \left[x = j \mid z \in S_a, s = v \right]}{\sum_{j \in S_a} \lim_{v \rightarrow +0} \mathbb{P} \left[x = j \mid z \in S_a, s = v \right] \sum_{l \in S_a} \sum_{m \in S_b} \mathbb{P} \left[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1 \mid x = j, s = 0 \right]},$$

where the probabilities implicitly condition on $r_2 = k$.

The proof is deferred to Appendix J. The IV coefficients in the fuzzy RDD IV model identify a weighted average of LATEs, with weights $0 \leq \lambda_{jklm} \leq 1$ and $\sum_{j \in S_a} \sum_{l \in S_a} \sum_{m \in S_b} \lambda_{jklm} = 1$. Similar to Theorem 2, the weights represent a normalized proportion of shifts corresponding to each LATE: the individuals corresponding to the fixed effect $x = j$ that shift from treatment m to treatment l as a result of a switch in the instrument from k to j . However, the weighted average of LATEs does not include a summation over $k \in S_b$, as it conditions on the sample of individuals with $r_2 = k$. Appendix K derives CLATE with a fuzzy RDD applied to the total sample of individuals.

6 The effect of field of study on academic student progress

This section studies the effect of field of study on academic student progress. To do so, we exploit a natural experiment in the Portuguese higher education system to estimate CLATEs.

6.1 Empirical context and strategy

6.1.1 Higher education admission in Portugal

In the final year of high school, prospective students in Portugal apply to public higher education via a centralized admission process of the Direção-Geral do Ensino Superior (DGES; Directorate General for Higher Education). Only 17 percent of the students use a different enrollment system to enroll into private higher education ([Pordata, 2022](#)). The public application process is commonly referred to as Regime Geral de Acesso ([General Access Regime](#)). Applicants apply to a field and institution simultaneously, which we will refer to as a course (*e.g.*, Law at the University of Lisbon). On applying, applicants submit a ranking of up to six courses based on their preferences.

Each course has a quota (q) of applicants that can be admitted, which are set in agreement between the institutions and the Ministério da Ciência, Tecnologia e Ensino Superior (MCTES; Ministry of Science, Technology and Higher Education) and are stable across years. DGES ranks for each course the applicants by their application score. This score is a weighted average of an applicant's high school and national exam grades. Since each course weighs the high school grades and the different national exams differently, the application scores are both applicant- and course-specific (Article 33 until 36 of [DGES \(2019\)](#)).

The applicants are assigned to courses using an iterative process (Article 37 of [DGES](#)

(2019)), similar to the college-proposing Gale-Shapley algorithm (Gale and Shapley, 1962):

- Step 1: Each course makes offers to the first q applicants on its ranked list. Each applicant keeps the best offer according to her submitted ranking and rejects the rest.
- Step $s \geq 2$: Any course rejected in step $s - 1$ by $n \geq 1$ applicants proposes to the next n applicants on its list. Each applicant keeps the best offer according to her submitted ranking among all the proposals received in step s (including the one retained in step $s - 1$) and rejects the rest.

The algorithm terminates when the allocation is not updated, and matches each course to the set of q most preferred applicants who did not reject it.

One to two weeks before the start of the academic year, each applicant receives the offer from her assigned course. Within that week, applicants have to accept the offer, participate in a second cycle, or withdraw from the application process. The slots that have remained or were not accepted after the first cycle are then allocated to the applicants who participate in the second cycle according to the same algorithm. The process concludes with an identical third cycle. Second and third cycle applicants must submit a new ranking of up to six courses among the remaining courses only. Our analysis will use a student's first cycle from the first observed application.

Active students that want to change field of study have to make an official request at DGES. This process is commonly referred to as *Mudança de Curso* (Change of Course). DGES (2015) describes that approval of such a request depends on course availability and minimum legal requirements. This process also includes the potential accreditation of relevant subjects from a student's previous course.

6.1.2 Preferred and next-best field of study

The matching algorithm generates (ex-ante) unknown application score cutoffs for each course. Each cutoff represents the lowest application score that qualifies an applicant to receive an offer from that course. The preferred and next-best field of study can be constructed from these course cutoffs and course rankings in three steps. First, we only consider applicants who apply to at least two different fields. Second, we construct preference blocks, where a block gives all consecutive ranked courses within the same field the same rank. For example, if an applicant ranks a course in a field from different institutions as first and second in her course ranking, then this applicant's first block contains two courses.

Third, we construct a sample of applicants with two blocks of preferences by only considering an applicant's assigned block, the block above (if available), and the block below (if available). If the application score is below the cutoff for each course within the block below the assigned block, we remove this block. This ensures that the applicant would have been assigned to the field of the block below if she had scored below the course cutoff(s) of her assigned block. Subsequently, we drop applicants that have neither an available block above or below her assigned block. For applicants that have both an available block above and below her assigned block, we drop the block below.

In this sample, the highest ranked block is our measure for the preferred field r_1 , and the lowest ranked block is our measure for the next-best field r_2 . This sample includes 57 percent of the total applicants. The instrument z_j for enrollment in field j equals one if j is the preferred field and the application score for j exceeds the cutoff. In our fuzzy RDD, the running variable s is the distance between the application score and the cutoff. We measure this as the maximum re-centered application score amongst all courses within the highest ranked block. For applicants assigned to their preferred (next-best) field, this value is always positive (negative) and represents the margin of being assigned to their next-best (preferred) field.

Table 1 illustrates the construction of the preferred and next-best field. Panel A shows the course ranking of an applicant with field j in the first block and field k in the second block. The ex-post realized cutoff to be admitted to field j at institution A is 180. The applicant has an application score for field j of 180.1, and so she receives an offer from field j . This applicant also scores above the cutoff for field k at institution A in the second block. Hence, field j (k) is the applicant's preferred (next-best) field.

Panel B illustrates the fuzzy RDD design by comparing the top applicant to an applicant that has the same preferred and next-best field, but has a slightly lower application score for preferred field j at institution A, namely 179.9. The fuzzy RDD compares the students just above and below the cutoff, and uses a scoring above the cutoff of 180 as an instrument for field j enrollment (instead of k). As students are likely to be similar near the cutoff, this identifies the causal effect of field j assignment.

Panel B illustrates variation in fields of study while keeping the institution fixed. However, in our analysis, we also indicate courses with the same field of study from different institutions as the same treatment. For instance, field k at institution A or D are indicated as the same treatment k . Panel C illustrates this, where our analysis also compares the top ap-

Table 1: Illustration of preferred and next-best fields and the instruments

Ranking	Field	Institution	Score	Cutoff	Block	z	s
Panel A: Course ranking							
1st	j	B	< 190	190	1	j	0.1
2nd	j	A	180.1	180	1	j	0.1
3rd	k	A	≥ 170	170	2	j	0.1
4th	k	B	.	.	2	j	0.1
5th	l	B	.	.	3	j	0.1
6th	j	C	.	.	4	j	0.1
Panel B: Preferred and next-best fields							
Preferred (r_1)	j	A	180.1	180	1	j	0.1
Next-best (r_2)	k	A	≥ 170	170	2	j	0.1
Preferred (r_1)	j	A	179.9	180	.	k	-0.1
Next-best (r_2)	k	A	≥ 170	170	.	k	-0.1
Panel C: Preferred and next-best fields while institution changes							
Preferred (r_1)	j	A	180.1	180	1	j	0.1
Next-best (r_2)	k	A	≥ 170	170	2	j	0.1
Preferred (r_1)	j	C	169.9	170	.	k	-0.1
Next-best (r_2)	k	D	≥ 180	180	.	k	-0.1

Notes: panel A provides an illustrative example of an applicant's course ranking and the corresponding outcome of the admission mechanism. Panel B and C illustrate how this ranking and outcome can be used in a fuzzy RDD. See the text for details on the construction of the columns Block, z , and s .

plicant to an applicant that has preferred field j at a different institution than her next-best field k .

In case the preferred and next-best field are measured with error, switches in the instrument are not restricted between r_2 and r_1 . Section 2.3 shows that the additional switches also generate off-diagonal shifts, even without preference updating. There might be two reasons for the measurement error. First, our measure of the preferred and next-best field

may not refer to the top two fields in the course ranking. However, the descriptive statistics in the next section show that our measure of r_1 corresponds to the top field in the course ranking for 88% of our sample. By construction, if r_1 corresponds to the first ranked field, r_2 corresponds to the second ranked field.

Second, the college-proposing Gale-Shapley algorithm is not strategy proof from the applicants' perspective. In particular, it has been shown that an applicant might have incentives to misrepresent her preferences from the second ranked course onwards (Roth, 1982). Hence, if applicants truly understand the allocation mechanism, the preferences in the course ranking are not necessarily an applicants true preferences from the second ranked course onwards. This may especially generate error in our measure for the next-best field.

6.1.3 Data

We use data from two different sources. First, the higher education application data from the DGES contains for each applicant the six listed courses, their application scores, the assigned course, and the course cutoff for all courses, from 2008-09 unto 2019-20. We consider for each student the first cycle from the first observed application. We refer to students that applied within the same year as a cohort.

The second data set has the student outcome data from the Direção-Geral de Estatísticas da Educação e Ciência (DGEEC; Directorate for Education Statistics). For each enrolled student, this data set contains the enrolled course, whether the student is active in the first, second, or third year of that course, and the study points collected in that course in the previous year, from 2013-14 unto 2019-20. These study points are denoted by ECTS (European Credit Transfer System). This measure for student progress accommodates the transfer of students between European universities. One ECTS is equivalent to 28 hours of studying. A course of five (ten) ECTS is considered as a medium-sized (large) course. Sixty ECTS account for one year of study.

The two data sets are merged so that we observe for each student the course ranking (r_1, r_2) , the assigned course (z) , the application score minus the cutoff (s) , the first observed enrollment in the second year of a course (d) , and the ECTS collected in that second year (y) . A student's enrollment in the first year needs to be observed to combine the two data sets. Hence, the first cohort for which we can combine the data sets is 2013-14. We take 2016-17 as the final cohort, so that we can observe course enrollment in the second year up unto

Table 2: Descriptive statistics for estimation sample

Panel A: Means and standard deviations					
	Mean	Std. dev.			
Running variable (re-centered)	8.56	18.17			
$\mathbb{1}(\text{running variable} > 0)$	0.68				
No. of courses ranked	5.46	0.99			
$\mathbb{1}(\text{top two courses} = \{r_1, r_2\})$	0.88				
$\mathbb{1}(d \neq z)$ in year 2	0.08				
No. of ECTS in year 2	54.18	13.52			
Observations (cohorts)	50252 (4)				

Panel B: Common preferred fields per next-best					
Next-best field	Total obs.	Largest pref. field:		2nd largest pref. field:	
		Name	Obs.	Name	Obs.
Agriculture, forestry and fisheries	404	Veterinary	155	Life sciences	65
Architecture and construction	2132	Engineering	1009	Arts	780
Arts	1785	Arch. and constr.	602	Humanities	273
Business sciences	7498	Soc. and beh. sci.	3014	Comm. services	878
Commercial services	3036	Business sci.	993	Health	453
Engineering	5258	Health	1906	Life sciences	1002
Health	2913	Life sciences	1055	Engineering	726
Humanities	2903	Soc. and beh. sci.	885	Business sci.	550
IT	1807	Engineering	1369	Business sci.	294
Information and journalism	1358	Business sci.	338	Soc. and beh. sci.	290
Law	1411	Business sci.	477	Soc. and beh. sci.	464
Life sciences	4402	Health	2351	Engineering	1164
Manufacturing	1153	Engineering	682	Health	102
Mathematics and statistics	968	Engineering	390	Business sci.	175
Physics	2705	Engineering	1135	Life sciences	610
Social and behavioral sciences	7062	Business sci.	2507	Law	1174
Social services	1290	Soc. and beh. sci.	409	Teacher	273
Teacher	1417	Soc. and beh. sci.	512	Social services	212
Veterinary	750	Health	590	Life sciences	59

Notes: columns of Panel A display descriptive statistics of our estimation sample, based upon 50252 applicants from 4 cohorts. The number of ECTS and $\mathbb{1}(d \neq z)$ refer to the first observed enrollment in the second year of a course. Panel B provides an overview of the number of applicants per combination of next-best and preferred field. For instance, 477 of 1411 applicants with Law as next-best prefer Business sciences.

2018-19, allowing for a one year delay. This results in the final data set with four application cohorts from 2013-14 unto 2016-17.

Panel A of Table 2 shows descriptive statistics of the final estimation sample. The final estimation sample includes 36 percent of the total number of applicants in the four application cohorts. The mean of the running variable is positive, and 68% of the students are assigned to their preferred field. The applicants have on average 5.46 courses on their course ranking. In 88% of these rankings, the top two fields equal our measure for the preferred and next-best field. The enrolled field in year two is different from the initially assigned field for 8% of the students. Next to always- and never takers, this suggests the presence of off-diagonal treatment shifts.

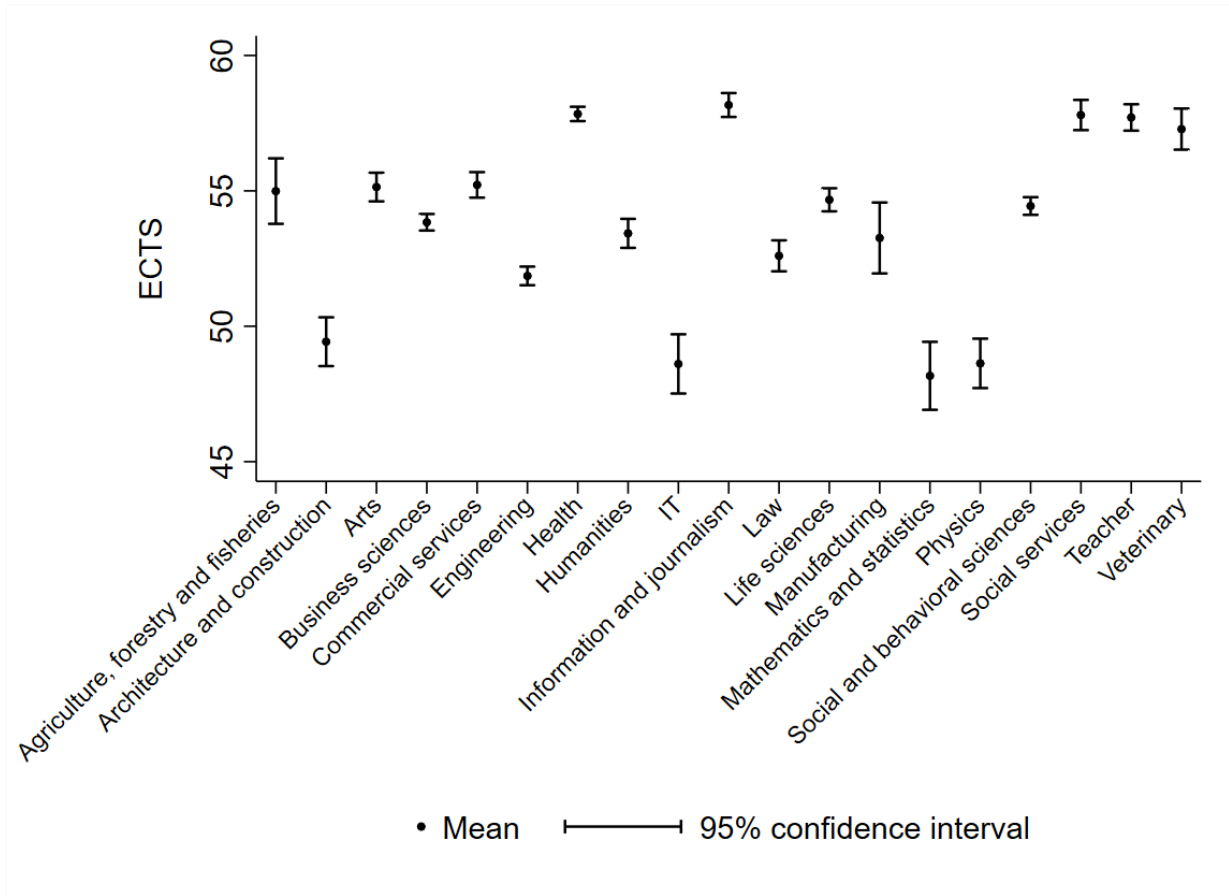
Panel B of Table 2 provides an overview of the most common preferred fields for each next-best. We have nineteen (unclustered) fields of study, which we write with a capital letter for clarity. We find next-best fields with a skewed distribution of students across preferred fields. For instance, 75% (15%) of the students with IT as next-best prefer Engineering (Business sciences). The remaining 10% prefer any of the other 16 fields. Panel B also shows that the number of observations differs across next-best fields. For example, there are 7498 students with Business sciences as next-best versus 404 students with Agriculture, forestry and fisheries.

Figure 3 shows the mean of the second year ECTS across all students enrolled in a field of study, together with the 95% confidence interval. There is substantial variation in student progress between fields, with less second year progress for fields like Architecture and construction and Physics, and more progress for Health and Information and journalism. The fields Law and Social and behavioral sciences are close to the mean ECTS of 54.18 across all fields of study (see panel A of Table 2). These mean comparisons are in line with previous research that documents correlations between fields of study and measures of academic progress. For instance, it is often found that Health students have higher on-time completion rates ([Lassibille, 2011](#)).

6.1.4 Model specification and estimation

The outcome variable y equals the number of ECTS collected in the first observed enrollment in the second year of a course. The endogenous treatment variable d_j equals one if the corresponding second-year course enrollment is in field j . We estimate the CLATEs in the fuzzy RDD model as specified in (20) and (21) per sample of applicants with $r_2 = k$. The

Figure 3: Mean ECTS per field of study

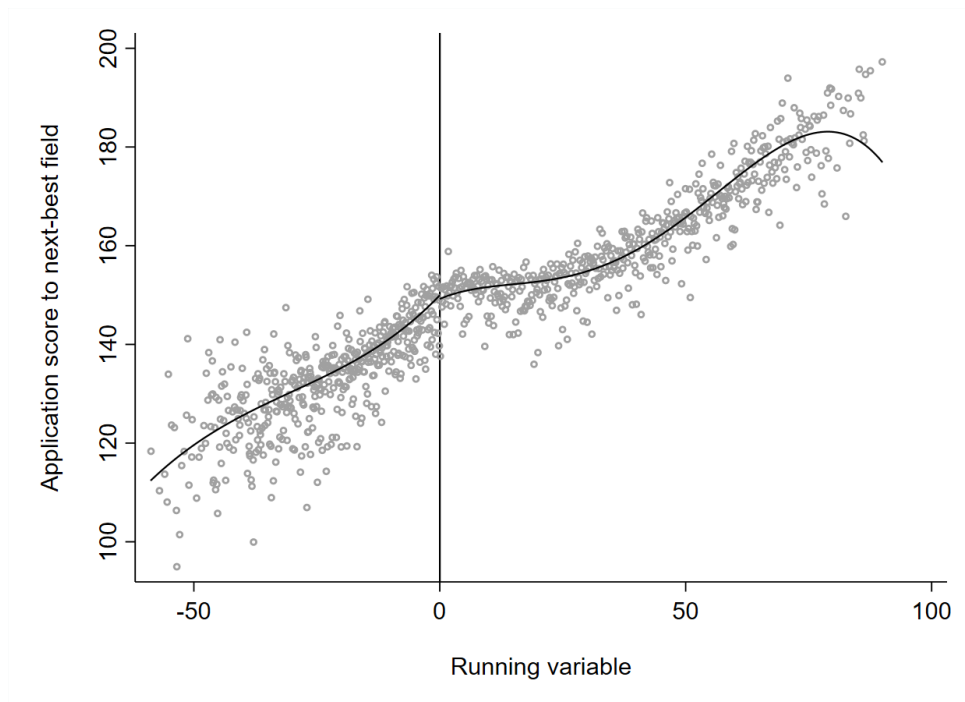


Notes: this figure shows the mean second year ECTS across all students enrolled in a field of study, together with the 95% confidence interval of the mean.

clusters are estimated with Algorithm 1, the coefficients with 2SLS, and we employ robust standard errors.

Our sample sizes do not allow for local nonparametric estimation. Therefore we follow [Imbens and Lemieux \(2008\)](#), and assess the robustness of our results against more flexible model specifications. First, we include an additional interaction term in the first stage between s and an indicator for scoring above the cutoff across all preferred fields ($\sum_{c \neq b} \tilde{z}_c$). We find that 87 percent of the first stage estimates corresponding to $\sum_{c \neq b} \tilde{z}_c \times s$ are statistically insignificant at the 1%-level. Second, we include s^2 as an additional variable in both the first and second stage. Similarly, 86 (95) percent of the first (second) stage estimates on s^2 are statistically insignificant at the 1%-level. Moreover, in both cases the estimates of interest are qualitatively similar.

Figure 4: Balancing test



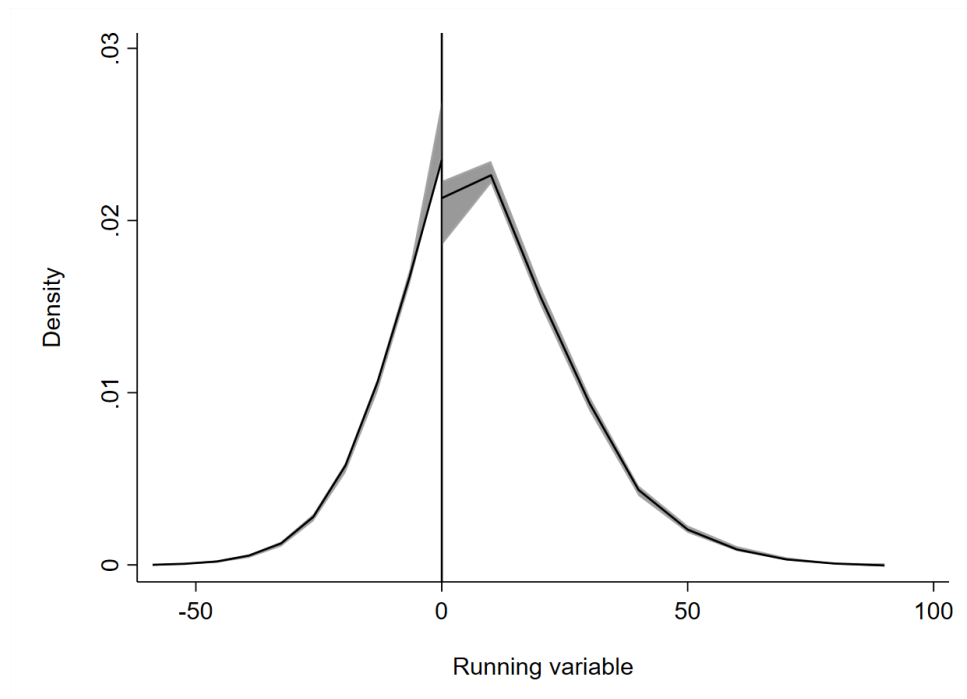
Notes: this figure shows a scatterplot of the application score to the next-best field against the running variable with a fourth-order polynomial estimated with a uniform kernel on the full estimation sample (the default of the Stata program rdplot).

6.1.5 Validity of the continuity assumption

This section assesses the validity of the local continuity assumption (Assumption 1^D). First, since we observe the application score to the next-best field for all students, we can examine whether it is continuous at the cutoff. Figure 4 plots the maximum score across all courses in the preference block of the next-best field against the running variable. Consistent with continuous potential outcomes, this score does not show a discontinuity around the cutoff. This result is confirmed with a pooled reduced form regression on the students from all next-best fields: We regress the application score to the next-best field on the running variable and a single treatment dummy indicating whether students scored above the cutoff, and the estimate on this treatment dummy is equal to -0.29 and has a p -value of 0.19.

Second, we examine the density of the students near the course cutoffs (McCrary, 2008). If applicants have precise control over their application score, we might observe bunching just above the cutoffs and the local continuity assumption may not be valid (Lee and Lemieux, 2010). In contrast, Figure 5 shows excess mass just below the cutoffs. Moreover,

Figure 5: Discontinuity in density test



Notes: this figure shows the density of the running variable estimated with a second-order polynomial and a third-order polynomial for the bias-correction estimate, an optimally-chosen bandwidth with the full estimation sample as plot range, a triangular kernel, and 95% confidence intervals with jackknifed standard errors (the default of the Stata program rddensity).

the bias corrected discontinuity test introduced in [Cattaneo, Jansson, and Ma \(2018\)](#) has a test statistic equal to 0.11 with a p -value of 0.91, implying that we cannot reject the null hypothesis of no discontinuity.

The continuity assumption is also consistent with the setting at hand. The number of applications is different each year, and hence the cutoffs change from year to year and are unknown at the time of application. The cutoffs only become publicly available when the applicants receive their offer. A regression of the course cutoffs across years on course dummies has an R-squared of 0.81, which confirms the movement in the cutoffs across years.

6.2 Results

6.2.1 LATE first stage

We use the nineteen unclustered fields of study to estimate the first stage coefficients in (21) for each sample corresponding to the different next-best fields. Figure 6 and 7 show the

estimated first stage matrices corresponding to the next-best fields Architecture and construction and Social and behavioral sciences, respectively. The rows represent the treatment variables and the columns the instruments. A black (grey) element indicates that the estimate is positive (negative) and significantly different from zero at the 1%-level.

Figure 6 shows that six of the fifteen off-diagonal first stage estimates for the Engineering treatment are negative and significantly different from zero. These estimates suggest that individuals with initial exposure to the next-best Architecture and construction update their preference towards Engineering. This results in the treatment shift from Engineering towards the preferred field in the corresponding column, when the instrument switches from the next-best Architecture and construction towards this preferred field.

Figure 7 shows four treatments with negative off-diagonal first stage estimates that are significantly different from zero, with next-best Social and behavioral sciences. The significant negative estimates on the treatment Humanities suggest that individuals with initial exposure to the next-best Social and behavioral sciences change their preference towards Humanities. The other significant negative off-diagonal estimates for the treatments Arts, Business sciences, and Law imply that individuals assigned to the next-best Social and behavioral sciences also change their preferences towards Arts, Business sciences, and Law.

Figure 7 also shows a significant positive off-diagonal estimate for the treatments Business sciences and Information and Journalism. The positive estimate of the instrument Law on treatment Information and journalism suggests that initial exposure to the preferred field Law may result in preferring Information and journalism. This explains a treatment shift from the next-best Social and behavioral sciences towards Information and journalism, when the instrument switches from the next-best towards the preferred field Law.

The first stage estimates corresponding to the other seventeen next-best fields are deferred to Appendix L. Eight of these first stage matrices have off-diagonal estimates significantly different from zero. This implies that for nine of the nineteen next-best fields, such as Health and Manufacturing, we find diagonal first stage matrices. The first column in Table 3 provides an overview of the ten fields with significant off-diagonal estimates.

6.2.2 Treatment clusters

Next, we apply Algorithm 1 to the first stage estimates of the unclustered fields of study to infer the treatment clusters. The fields corresponding to rows with negative off-diagonal estimates will be clustered with the next-best field. For next-best Architecture and construction

Figure 6: LATE first stage estimates with next-best Architecture and construction

Agriculture, forestry and fisheries	1.000	-0.000	-0.000	-0.063	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
Arts	-0.004	0.800	-0.005	-0.004	-0.006	-0.005	0.245	-0.004	-0.002	-0.005	-0.007	-0.004	-0.007	-0.007	-0.006		-0.004	
Business sciences	-0.002	-0.018	0.883	-0.002	-0.013	0.074	-0.003	-0.002	-0.001	-0.253	-0.004	-0.085	-0.004	-0.004	-0.065		-0.002	
Commercial services	-0.003	-0.003	0.024	0.885	-0.009	-0.003	-0.004	-0.003	-0.002	-0.004	0.078	-0.003	-0.005	0.023	-0.004		-0.003	
Engineering	-0.018	-0.027	-0.110	-0.156	0.716	-0.021	-0.024	-0.017	-0.010	-0.023	-0.032	-0.232	-0.032	-0.262	-0.025		-0.019	
Health	0.002	0.003	0.002	0.002	0.007	0.675	0.002	0.002	0.001	-0.248	0.003	0.002	0.003	0.003	0.002		0.002	
Humanities	0.001	0.002	0.001	0.001	0.002	0.001	0.751	0.001	0.001	0.001	0.002	0.001	0.002	-0.075	0.002		0.001	
IT	-0.001	-0.001	-0.001	-0.001	-0.007	-0.001	-0.001	0.999	-0.000	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001		-0.001	
Information and journalism	0.001	0.001	0.001	0.001	0.002	0.001	0.001	0.001	1.000	0.001	0.001	0.001	0.001	0.001	0.001		0.001	
Law	0.001	0.004	0.002	0.001	0.004	0.002	0.002	0.001	0.001	1.002	0.002	0.001	0.002	0.002	0.002		0.001	
Life sciences	-0.000	-0.000	-0.000	-0.000	-0.004	-0.000	-0.000	-0.000	-0.000	-0.000	0.791	-0.000	-0.001	-0.001	-0.000		-0.000	
Manufacturing	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.000	-0.001	-0.001	0.922	-0.001	-0.001	-0.001		-0.001	
Mathematics and statistics	-0.001	-0.002	-0.001	-0.001	-0.002	-0.001	-0.001	-0.001	-0.001	-0.001	-0.002	-0.001	0.998	-0.002	-0.001		-0.001	
Physics	-0.005	-0.016	-0.006	-0.005	-0.005	-0.006	-0.007	-0.005	-0.003	-0.006	-0.009	-0.005	-0.009	0.671	-0.007		-0.005	
Social and behavioral sciences	0.001	0.004	-0.058	0.001	0.007	0.001	0.001	0.001	0.001	0.001	0.002	0.001	0.002	0.057	1.001		0.001	
Social services	-0.001	-0.002	-0.001	-0.001	-0.002	-0.001	-0.001	-0.001	-0.001	-0.001	-0.002	-0.001	-0.002	-0.002	-0.001		-0.001	
Teacher	0.001	-0.005	0.002	0.001	0.002	0.002	0.002	0.001	0.001	0.002	0.003	0.001	0.003	0.002	0.002		1.002	
Veterinary	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.001	-0.124	0.001	0.001	0.029	0.001		0.001	

Notes: this figure shows the estimated first stage coefficients corresponding to the next-best field Architecture and construction. The rows represent the treatments and the columns the instruments. A black (grey) element indicates a positive (negative) estimate significantly different from zero at the 1%-level. There is no variation in the instruments for Social services and Veterinary conditional on the fixed effects.

in Figure 6, this implies that Engineering is clustered with next-best. For next-best Social and behavioral sciences in Figure 7, the fields Arts, Business sciences, Humanities, and Law are clustered with next-best.

For next-best Social and behavioral sciences in Figure 7, we find two positive off-diagonal estimates: treatment Information and journalism on instrument Law, and treatment Business sciences on instrument Commercial services. As Law is already clustered with next-best, and CLATE does not consider instrument switches within the next-best clus-

Figure 7: LATE first stage estimates with next-best Social and behavioral sciences

Agriculture, forestry and fisheries	-0.000	-0.000	-0.000	-0.018	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
Architecture and construction	0.833	-0.001	-0.001	-0.001	-0.016	-0.000	-0.001	-0.000	-0.000	-0.000	-0.000	-0.001	0.012	-0.000	-0.000	-0.000	-0.000	-0.000	
Arts	0.000	0.796	-0.003	0.000	0.006	0.000	0.004	0.000	-0.033	-0.007	0.007	0.018	0.012	0.000	0.000	0.000	0.000		
Business sciences	0.052	-0.030	0.926	0.035	0.000	0.009	0.016	0.008	-0.037	0.005	-0.020	0.029	-0.077	0.008	0.018	0.007	0.007		
Commercial services	0.003	0.002	-0.000	0.901	0.005	-0.006	-0.010	0.002	0.002	-0.007	0.009	-0.034	0.002	0.002	0.011	0.002	0.002		
Engineering	-0.002	-0.002	-0.003	-0.002	0.940	-0.011	0.000	-0.251	0.002	-0.002	0.005	-0.002	-0.002	-0.001	-0.002	-0.001	-0.001		
Health	0.043	0.001	-0.002	0.011	0.012	0.898	0.003	0.001	-0.007	0.003	-0.024	0.019	0.001	-0.015	0.001	0.001	0.001		
Humanities	-0.014	-0.032	-0.009	-0.013	-0.010	-0.008	0.872	-0.011	-0.021	-0.007	0.001	-0.016	-0.002	-0.012	-0.004	-0.004	-0.009		
IT	0.001	0.012	-0.000	0.001	0.001	0.001	0.001	0.751	-0.003	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001		
Information and journalism	-0.001	0.010	-0.005	-0.001	-0.001	-0.000	0.001	-0.000	0.874	0.010	-0.001	-0.001	-0.001	-0.000	-0.001	-0.001	-0.000		
Law	-0.005	-0.005	-0.012	-0.022	-0.004	-0.003	-0.003	-0.004	-0.019	0.831	-0.004	-0.005	-0.005	-0.004	-0.004	-0.004	-0.003		
Life sciences	-0.002	-0.002	-0.002	-0.002	-0.002	-0.011	-0.002	-0.002	-0.005	-0.002	0.876	-0.002	0.010	-0.002	0.007	-0.001	-0.001		
Manufacturing	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000		
Mathematics and statistics	-0.004	-0.004	-0.003	-0.004	-0.004	-0.003	-0.002	0.247	-0.003	-0.004	-0.003	0.758	-0.004	-0.003	-0.004	-0.003	-0.003		
Physics	-0.000	-0.000	0.000	-0.000	-0.013	-0.000	-0.000	-0.000	-0.000	0.001	0.013	0.018	0.889	-0.016	-0.000	-0.000	-0.000		
Social services	0.000	0.000	-0.002	0.000	0.000	0.002	0.001	0.000	0.003	0.000	0.000	0.001	0.000	0.947	-0.026	0.000	0.000		
Teacher	0.003	0.003	0.002	0.003	0.003	0.006	0.005	0.002	0.005	0.004	0.002	0.003	0.015	0.002	0.952	0.002	0.002		
Veterinary	-0.000	-0.000	-0.000	-0.000	0.008	0.004	-0.000	-0.000	-0.000	-0.000	0.006	-0.000	-0.000	-0.000	-0.000	-0.000	1.000		

Notes: this figure shows the estimated first stage coefficients corresponding to the next-best field Social and behavioral sciences. The rows represent the treatments and the columns the instruments. A black (grey) element indicates a positive (negative) estimate significantly different from zero at the 1%-level. There is no variation in the instruments for Agriculture, forestry, and fisheries and Manufacturing conditional on the fixed effects.

ter, the first positive off-diagonal does not result in an additional cluster. Since Business sciences is also clustered with next-best, and a merge between a preferred and next-best treatment cluster results in a next-best treatment cluster, we have one large next-best treatment cluster with Commercial services, Arts, Business sciences, Humanities, Law, and Social and behavioral sciences.

Across all next-best fields, we find one preferred treatment cluster. With Commercial

Table 3: Clusters for the next-best fields with significant off-diagonal estimates

Next-best field k	m clustered with k	l clustered with j
Agriculture, forestry and fisheries	Veterinary	–
Architecture and construction	Engineering	–
Commercial services	–	Business sciences and Humanities
Engineering	Business sciences	–
Humanities	Information and journalism	–
IT	Engineering	–
Life sciences	Veterinary	–
Mathematics and statistics	Engineering	–
Social and behavioral sciences	Arts, Business sci., Commercial services, Humanities, and Law	–
Teacher	Life sciences	–

Notes: the nine next-best fields without significant off-diagonal estimates are Arts, Business sciences, Health, Information and journalism, Law, Manufacturing, Physics, Social services, and Veterinary.

services as next-best, the treatment Business sciences and instrument Humanities have a significant positive off-diagonal estimate, which results in a preferred treatment cluster including Business sciences and Humanities.

We find ten treatment clusters in total, which are in general in line with preference updating. Table 3 documents the clusters for all ten next-best fields with significant off-diagonal estimates. According to preference updating, the treatment clusters contain similar fields. We find that, for instance, Engineering is in clusters with Architecture and construction, IT, and Mathematics and statistics. These clusters contain STEM (Science, Technology, Engineering and Mathematics) fields. On the other hand, Humanities clusters with fields of study from the Social sciences umbrella: Arts, Business sciences, Commercial services, Information and journalism, Law, and Social and behavioral sciences. The clusters are also consistent with the presence of lower switching costs between similar fields of study. Similar fields may share parts of the curriculum and subjects, which makes the accreditation of previous subjects easier. The first stage CLATE estimates, using the treatment clusters in Table 3, do not show evidence of remaining violations.

6.2.3 CLATE second stage

Figure 8 shows the CLATE estimates. The rows correspond to next-best samples and the columns to treatments or treatment clusters. Table 3 shows that we find treatment clusters for ten next-best samples. We find one preferred treatment cluster, Business sciences and Humanities, with Commercial services as next-best. The corresponding CLATE is in the element corresponding to Business sciences, the element of Humanities is left empty, and both elements are indicated by an asterisk.

Figure 8 contains 235 CLATE estimates. From these estimates, 111 (44) are larger than five (ten) in absolute value, and 24 (12) of these 111 (44) CLATE estimates are also significantly different from zero at the 1%-level. This implies that fields of study speed up or slow down student academic progress with the equivalence of a medium-sized or even large course per year. As many European universities, including those in Portugal, have academic dismissal policies that can dismiss students in case they do not obtain a minimum number of ECTS ([Sneyers and Witte, 2018](#)), this result implies that the choice of field of study has a substantial impact on academic student progress. Note that in the United States and Canada these academic dismissal policies are often based upon a student's GPA ([Lindo, Sanders, and Oreopoulos, 2010](#); [Ost, Pan, and Webber, 2018](#)).

The CLATE estimates further identify fields that have a positive or negative impact on student progress. For instance, the estimates in Figure 8 suggest that Law (Information and journalism) slows down (speeds up) student academic progress. Across all next-best fields, the majority of the CLATE estimates of treatment Law (Information and journalism) are negative (positive), whereas with Law (Information and journalism) as next-best the majority of the CLATE estimates are positive (negative). Our finding on Law is consistent with [Garrett and Bahia \(2020\)](#), who argue that, in Portugal, Law is considered as a conservative field with traditional pedagogical practices. The authors also discuss that Law professors recognize the need to modernize and invest in their pedagogical practices. For some fields the picture is more heterogeneous. For instance, consider the field Architecture and construction as next-best. Compared to the preferred field Social and behavioral sciences this field slows students down, whereas this field increases academic progress compared to Teaching.

In absence of causal estimates, policy makers and university administrators might resort to simple descriptive statistics to identify slow and exemplary fields. The CLATE estimates in Figure 8 suggest that the simple mean comparisons in ECTS can be misleading. For instance, the CLATE estimates identify Law as a slow field, but Law is close to the average

fields of study choice. Consider two students with the same next-best field, Architecture and construction, but with different preferred fields, Social and behavioral sciences versus Teaching. The positive CLATE estimate of 12.10 from Social and behavioral sciences versus next-best Architecture and construction might inform the first student to choose her preferred field. In contrast, the negative CLATE estimate of -5.72 from Teaching versus next-best Architecture and construction may inform the second student to choose next-best.

6.2.4 Heterogeneous treatment effects

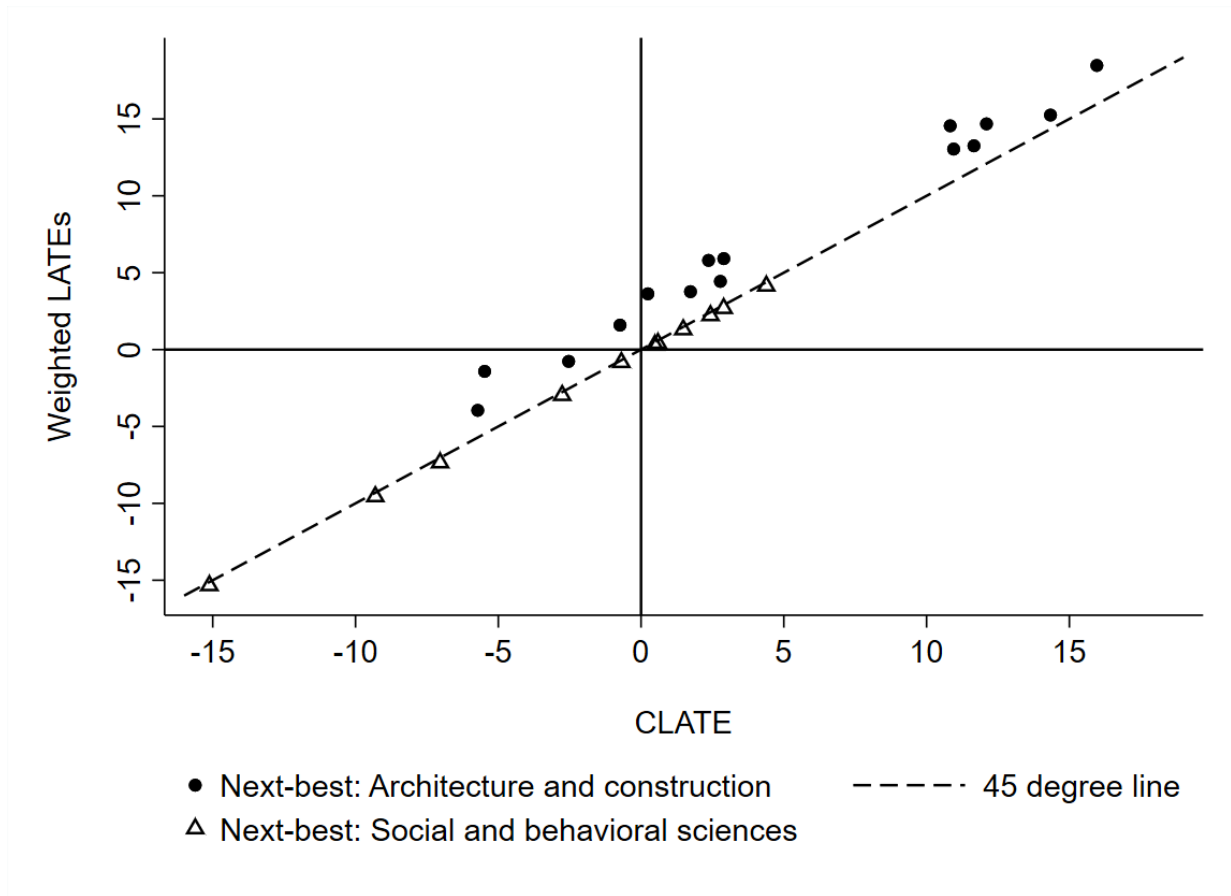
Within each next-best sample, we find at most one treatment cluster. Corollary 2 shows that in this case CLATE equals a weighted average of LATEs, in which the weights are point identified even in the presence of heterogeneous treatment effects.

We find non-negligible weights for the LATEs corresponding to the off-diagonal treatment shifts. With the next-best sample Architecture and construction (Social and behavioral sciences) the next-best cluster contains two (six) treatments, and the CLATEs consist of two (six) LATEs multiplied by the weights. For example, the CLATE of Physics in the next-best sample Architecture and construction, is a weighted average of the LATE between Physics and this next-best, and the LATE between Physics and Engineering, with weights 0.610 and 0.390 respectively. The CLATE of Information and journalism in the next-best sample Social and behavioral sciences, is a weighted average of the LATE between Information and journalism and this next-best, and the five LATEs between Information and journalism and Arts, Business sciences, Commercial services, Humanities, and Law. The weights equal 0.876, 0.038, 0.042, and smaller than 0.030 respectively.

Under homogeneous treatment effect assumptions, Corollary 3 shows that CLATE equals a weighted sum of identifiable LATEs. It follows that a difference between CLATE and the weighted sum of LATEs suggests the presence of treatment effect heterogeneity. The presence of off-diagonal first stage coefficients different from zero and treatment effect heterogeneity, implies that the LATE estimates are biased.

Figure 9 plots the estimated weighted sum of LATEs against the estimated CLATEs for the next-bests Architecture and construction and Social and behavioral sciences. For the latter next-best we cannot find evidence of treatment effect heterogeneity as the differences between the CLATEs and the weighted sum of LATEs are small. One reason for this result, next to potential treatment effect homogeneity, is that the weights corresponding to the off-diagonal shifts are small in magnitude. Hence, even in the presence of treatment effect het-

Figure 9: Weighted sum of LATEs versus CLATE



Notes: this figure shows the estimated CLATEs with next-best sample Architecture and construction (circles) and Social and behavioral sciences (triangles) on the x-axis and the corresponding weighted sum of LATEs as in Corollary 3 on the y-axis.

erogeneity one would expect small differences between CLATE and the weighted LATEs. For Architecture and construction as next-best we do find evidence for the presence of heterogeneous treatment effects: the CLATEs are substantially different from the weighted sum of LATEs across all treatment variables.

7 Conclusion

Since many modern policies consist of multiple treatments, it becomes increasingly unrealistic to restrict treatment evaluation to a binary treatment setting. At the same time, the increase in sophisticated policy design and data collection makes it possible to construct valid instruments in a wide variety of multiple treatment settings. However, existing IV ap-

proaches to multiple unordered treatment evaluation only identify treatment effects under strong assumptions on the behavior of the individuals.

This paper shows that we can identify clustered local average treatment effects (CLATE) under relaxed assumptions in the presence of clusters with similar treatments. Since these assumptions impose less restrictions on individual behavior, they are more likely to hold in empirical settings. Moreover, they can be motivated by the preference updating literature that finds that differences across preferences for similar treatments are usually small, and switching costs between dissimilar treatments are high. CLATE is estimated by standard IV, and we provide a simple algorithm for estimating the treatment clusters.

CLATE identifies a weighted average of unidentifiable LATEs of all pairs of treatments across the treatment cluster and control cluster, and the instruments corresponding to these clusters. The weights represent the proportion of shifts corresponding to each LATE, for which we derive the settings in which they are set and point identified. We show that CLATE is able to identify treatment effect heterogeneity by deriving the homogeneity assumptions under which CLATE equals a weighted average of identifiable LATEs.

The relevance of CLATE is further emphasized by our empirical analysis of the effect of field of study on academic student progress. We find many violations of the LATE assumptions, evidence for treatment clusters that are in line with preference updating, and a strong indication of the presence of treatment effect heterogeneity. Since CLATE retains a causal interpretation in these settings, we use the IV CLATE estimates to derive implications for policy makers, university administrators, and students.

The CLATE approach might also be well suited for treatment effect estimation in other multiple treatment settings. Multiple ordered treatments, such as years of education, are commonly modelled as a single multi-valued treatment (Angrist and Imbens, 1995). The literature on dynamic treatment assignment (Kasy and Sautmann, 2021), also has to deal with multiple type of changes in treatment status. Moreover, our paper provides an avenue for the structural modelling of treatment clusters.

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A Proof Theorem 1

Equation (5) shows,

$$\beta_{jk} = \frac{\theta_{jk}}{\pi_{jjk}} - \sum_{l \neq j, k} \frac{\pi_{ljk}}{\pi_{jjk}} \beta_{lk}, \quad \forall j \neq k. \quad (23)$$

We start with the first stage estimates. Realize that $\pi_{ljk} = \mathbb{E}[d_l | z = j] - \mathbb{E}[d_l | z = k]$, substitute for potential outcomes $d_j = \sum_{l=1}^J d_l^j \times z_l$, and then use assumption 1(b) to arrive at,

$$\pi_{ljk} = \mathbb{E}[d_l^j - d_l^k] = \mathbb{P}[d_l^j - d_l^k = 1] - \mathbb{P}[d_l^j - d_l^k = -1], \quad \forall l \neq k. \quad (24)$$

Monotonicity assumption 1(d) implies that $\mathbb{P}[d_l^j - d_l^k = -1] = 0$, so that π_{jjk} simplifies to

$$\pi_{jjk} = \mathbb{P}[d_j^j - d_j^k = 1]. \quad (25)$$

Assumption 2, together with Assumption 1(d), guarantees first that $\mathbb{P}[d_l^j - d_l^k = 1] = \mathbb{P}[d_l^j - d_l^k = 1, d_j^j = 0, d_j^k = 0] = \mathbb{P}[d_l^j - d_l^k = 1, d_j^j = 0, d_j^k = 0] = 0$, so that π_{ljk} simplifies to

$$\pi_{ljk} = -\mathbb{P}[d_l^j - d_l^k = -1], \quad \forall l \neq j, k. \quad (26)$$

Second, Assumption 2, together with Assumption 1(d), guarantees that when $d_l^j - d_l^k = -1$ it must be that $d_j^j - d_j^k = 1$. Hence,

$$\pi_{ljk} = -\mathbb{P}[d_l^j - d_l^k = -1, d_j^j - d_j^k = 1], \quad \forall l \neq j, k. \quad (27)$$

Then assumption 3 implies that $\mathbb{P}[d_l^j - d_l^k = -1, d_j^j = 1] = \mathbb{P}[d_l^j - d_l^k = -1, d_j^j - d_j^k = 1] = 0$ for individuals with $r_2 = k$, and it follows that for these individuals

$$\pi_{ljk} = 0, \quad \forall l \neq j, k. \quad (28)$$

In turn, the individuals with $d_j^j - d_j^k = 1$ must have $d_k^j - d_k^k = -1$. We rewrite π_{jjk} by being explicit that the shifts towards treatment j come from treatment k ,

$$\pi_{jjk} = \mathbb{P}[d_j^j - d_j^k = 1, d_k^j - d_k^k = -1 | r_2 = k], \quad (29)$$

and as $\pi_{ljk} = 0$ with $l \neq j, k$ it follows that conditional on $r_2 = k$,

$$\beta_{jk} = \frac{\theta_{jk}}{\pi_{jjk}}, \quad \forall j \neq k. \quad (30)$$

Next, we continue with the reduced form. Realize that $\theta_{jk} = \mathbb{E}[y | z = j] - \mathbb{E}[y | z = k]$, use assumption 1(a) to substitute for potential outcomes $y = \sum_{j=1}^J y^j \times d_j$, and then use assumption 1(b) to arrive at,

$$\theta_{jk} = \mathbb{E}[y^j (d_j^j - d_j^k)] + \mathbb{E}[y^k (d_k^j - d_k^k)] + \sum_{l \neq j, k} \mathbb{E}[y^l (d_l^j - d_l^k)], \quad (31)$$

where the expectations implicitly condition on individuals with $r_2 = k$. From the above we know that $d_l^j - d_l^k = 0$ with $l \neq j, k$, that $d_k^j - d_k^k = -1$ if $d_j^j - d_j^k = 1$, and that $d_j^j - d_j^k = 1$ if $d_k^j - d_k^k = -1$, and so θ_{jk} simplifies,

$$\begin{aligned}\theta_{jk} &= \mathbb{E}[y^j(d_j^j - d_j^k)] + \mathbb{E}[y^k(d_k^j - d_k^k)] \\ &= \mathbb{E}[y^j - y^k | d_j^j - d_j^k = 1, d_k^j - d_k^k = -1] \mathbb{P}[d_j^j - d_j^k = 1, d_k^j - d_k^k = -1].\end{aligned}\quad (32)$$

Finally, using (29) and (32) we have

$$\beta_{jk} = \frac{\theta_{jk}}{\pi_{jjk}} = \mathbb{E}[y^j - y^k | d_j^j - d_j^k = 1, d_k^j - d_k^k = -1, r_2 = k], \quad \forall j \neq k, \quad (33)$$

where assumption 1(c) ensures β_{jk} is finite.

B Proof Corollary 1

Note that in this proof all expectations and probabilities implicitly condition on $r_2 = k$. For the case with $\mathbb{P}[d_m^j - d_m^k = 1, d_n^j - d_n^k = -1] > 0$ and $m \neq j$, it follows from (5) that

$$\beta_{jk} = \frac{\theta_{jk}}{\pi_{jjk}} - \frac{\pi_{mjk}}{\pi_{jjk}} \beta_{mk} - \frac{\pi_{njc}}{\pi_{jjk}} \beta_{nk}, \quad (34)$$

where $\pi_{jjk} = \mathbb{P}[d_j^j - d_j^k = 1, d_k^j - d_k^k = -1]$ as derived in (29) in Appendix A, and $\pi_{mjk} = -\pi_{njc} = \mathbb{P}[d_m^j - d_m^k = 1, d_n^j - d_n^k = -1]$ as follows from Assumption 1(d), 2, and 3. From (31) in Appendix A follows that

$$\begin{aligned}\theta_{jk} &= \mathbb{E}[y^j - y^k | d_j^j - d_j^k = 1, d_k^j - d_k^k = -1] \mathbb{P}[d_j^j - d_j^k = 1, d_k^j - d_k^k = -1] + \\ &\quad \mathbb{E}[y^m - y^n | d_m^j - d_m^k = 1, d_n^j - d_n^k = -1] \mathbb{P}[d_m^j - d_m^k = 1, d_n^j - d_n^k = -1].\end{aligned}\quad (35)$$

Substituting (35) into (34) gives

$$\begin{aligned}\beta_{jk} &= \mathbb{E}[y^j - y^k | d_j^j - d_j^k = 1, d_k^j - d_k^k = -1] + \\ &\quad \frac{\mathbb{P}[d_m^j - d_m^k = 1, d_n^j - d_n^k = -1]}{\mathbb{P}[d_j^j - d_j^k = 1, d_k^j - d_k^k = -1]} (\mathbb{E}[y^m - y^n | d_m^j - d_m^k = 1, d_n^j - d_n^k = -1] + \beta_{nk} - \beta_{mk}).\end{aligned}\quad (36)$$

Since all assumptions for Theorem 1 hold, we have that $\beta_{nk} = \mathbb{E}[y^n - y^k | d_n^n - d_n^k = 1, d_k^n - d_k^k = -1]$ and $\beta_{mk} = \mathbb{E}[y^m - y^k | d_m^m - d_m^k = 1, d_k^m - d_k^k = -1]$. Hence

$$\beta_{jk} = \Delta_{jkjk} + (\Delta_{jkmn} - (\Delta_{mkmk} - \Delta_{nknk})) \frac{\mathbb{P}[d_m^j - d_m^k = 1, d_n^j - d_n^k = -1]}{\mathbb{P}[d_j^j - d_j^k = 1]}. \quad (37)$$

For the case with $\mathbb{P}[d_j^j - d_j^k = 1, d_n^j - d_n^k = -1] > 0$, it follows from (5) that

$$\beta_{jk} = \frac{\theta_{jk}}{\pi_{jjk}} - \frac{\pi_{njc}}{\pi_{jjk}} \beta_{nk}. \quad (38)$$

Similarly as in the first case, we have that $\pi_{njk} = -\mathbb{P}[d_j^j - d_j^k = 1, d_n^j - d_n^k = -1]$, $\theta_{jk} = \mathbb{E}[y^j - y^k | d_j^j - d_j^k = 1, d_k^j - d_k^k = -1] \mathbb{P}[d_j^j - d_j^k = 1, d_k^j - d_k^k = -1] + \mathbb{E}[y^j - y^n | d_j^j - d_j^k = 1, d_n^j - d_n^k = -1] \mathbb{P}[d_j^j - d_j^k = 1, d_n^j - d_n^k = -1]$, and $\beta_{nk} = \mathbb{E}[y^n - y^k | d_n^n - d_n^k = 1, d_k^n - d_k^k = -1]$. However, $\pi_{jjk} = \mathbb{P}[d_j^j - d_j^k = 1] = \mathbb{P}[d_j^j - d_j^k = 1, d_k^j - d_k^k = -1] + \mathbb{P}[d_j^j - d_j^k = 1, d_n^j - d_n^k = -1]$. Hence,

$$\begin{aligned} \beta_{jk} &= \mathbb{E}[y^j - y^k | d_j^j - d_j^k = 1, d_k^j - d_k^k = -1] \mathbb{P}[d_j^j - d_j^k = 1, d_k^j - d_k^k = -1] / \pi_{jjk} + \\ &\quad (\mathbb{E}[y^j - y^n | d_j^j - d_j^k = 1, d_n^j - d_n^k = -1] + \beta_{nk}) \mathbb{P}[d_j^j - d_j^k = 1, d_n^j - d_n^k = -1] / \pi_{jjk} \\ &= \Delta_{jkjk} - \Delta_{jkjk} \frac{\mathbb{P}[d_j^j - d_j^k = 1, d_n^j - d_n^k = -1]}{\mathbb{P}[d_j^j - d_j^k = 1]} + (\Delta_{jkjn} + \Delta_{nknk}) \frac{\mathbb{P}[d_j^j - d_j^k = 1, d_n^j - d_n^k = -1]}{\mathbb{P}[d_j^j - d_j^k = 1]} \\ &= \Delta_{jkjk} + (\Delta_{jkjn} - (\Delta_{jkjk} - \Delta_{nknk})) \frac{\mathbb{P}[d_j^j - d_j^k = 1, d_n^j - d_n^k = -1]}{\mathbb{P}[d_j^j - d_j^k = 1]}. \end{aligned} \tag{39}$$

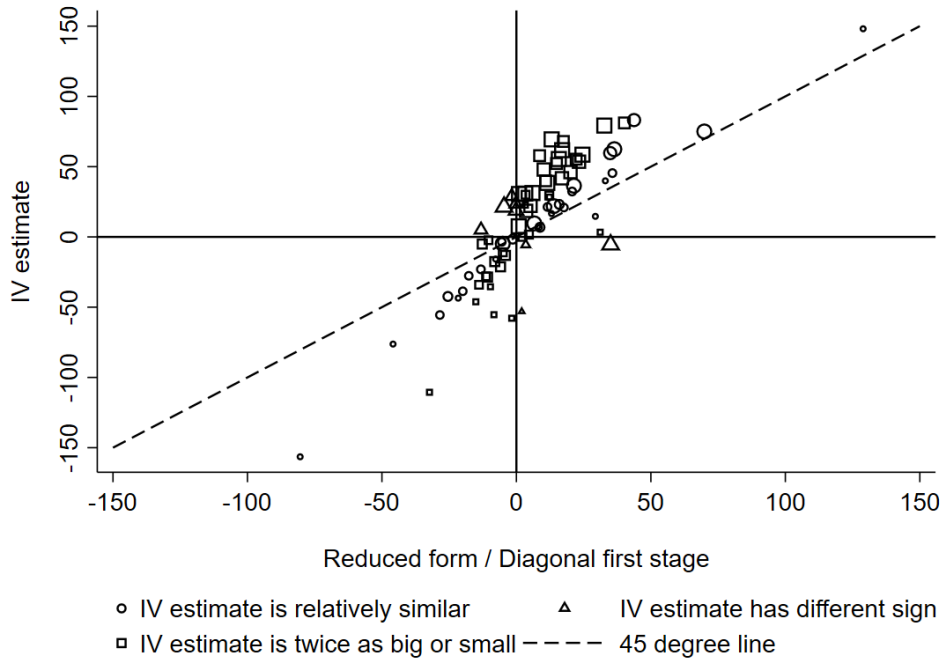
C Empirical relevance of off-diagonal treatment shifts

Kirkeboen, Leuven, and Mogstad (2016) combine the identification result generalized in Theorem 1 with credible instruments for each field of study to estimate the returns to fields of study in higher education in Norway. Figure 10(a) plots their IV estimates ($\hat{\beta}_{jk}$) against the numbers that result from dividing the corresponding reduced form estimate ($\hat{\theta}_{jk}$) by the diagonal first stage estimate ($\hat{\pi}_{jjk}$). It reveals deviation from the 45 degree line. For instance, 43 of the 81 IV estimates are more than twice as big or small compared to the number that divides the corresponding reduced form by the diagonal first stage.

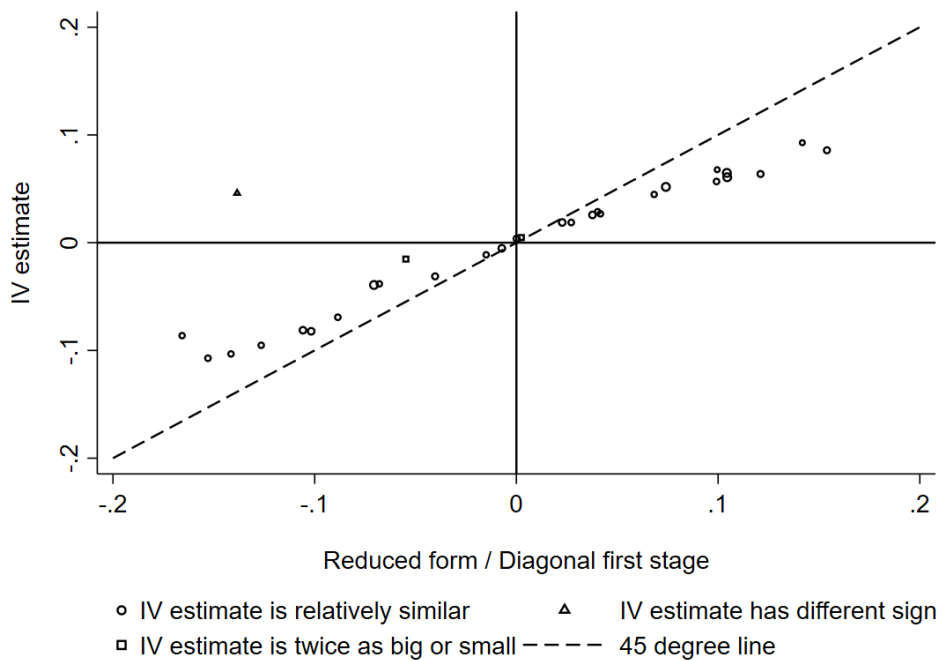
Figure 10(b) does the same for the estimates of Dahl, Rooth, and Stenberg (2020), who study the returns to fields of study in secondary school in Sweden with a similar credible IV approach as Kirkeboen, Leuven, and Mogstad (2016). The estimates are close to the 45 degree line. Dahl, Rooth, and Stenberg (2020) deal with five broad fields of study in secondary school, whereas Kirkeboen, Leuven, and Mogstad (2016) deal with many more granular fields in university.

Figure 10: Plotting the IV estimates $\hat{\beta}_{jk}$ against the numbers that result from dividing the reduced form by the diagonal first stage $\frac{\hat{\theta}_{jk}}{\hat{\pi}_{jjk}}$

(a) The estimates of Kirkeboen, Leuven, and Mogstad (2016)



(b) The estimates of Dahl, Rooth, and Stenberg (2020)



Notes: all estimates used in the top and bottom figure can be found in Kirkeboen, Leuven, and Mogstad (2016) and Dahl, Rooth, and Stenberg (2020), respectively. The figures plot the IV estimates ($\hat{\beta}_{jk}$) against the numbers that result from dividing the corresponding reduced form estimate ($\hat{\theta}_{jk}$) by the diagonal first stage estimate ($\hat{\pi}_{jjk}$). The size of the characters reflects the number of observations in the next-best sample.

D Proof Lemma 1

Write the off-diagonal first stage estimate $\tilde{\pi}_{cab}$ for all $c \neq a, b$ as

$$\begin{aligned}\tilde{\pi}_{cab} &= \mathbb{E}[\tilde{d}_c | \tilde{z} = a] - \mathbb{E}[\tilde{d}_c | \tilde{z} = b] = \sum_{n \in S_c} \mathbb{E}[d_n | \tilde{z} = a] - \sum_{n \in S_c} \mathbb{E}[d_n | \tilde{z} = b] \\ &= \sum_{j \in S_a} \sum_{n \in S_c} \mathbb{E}[d_n | z = j] \mathbb{P}[z = j | z \in S_a] - \sum_{k \in S_b} \sum_{n \in S_c} \mathbb{E}[d_n | z = k] \mathbb{P}[z = k | z \in S_b].\end{aligned}\quad (40)$$

Then we substitute for potential outcomes $d_n = \sum_{l=1}^J d_n^l \times z_l$, and use assumption 1(b) to write

$$\tilde{\pi}_{cab} = \sum_{j \in S_a} \sum_{n \in S_c} \mathbb{E}[d_n^j] \mathbb{P}[z = j | z \in S_a] - \sum_{k \in S_b} \sum_{n \in S_c} \mathbb{E}[d_n^k] \mathbb{P}[z = k | z \in S_b]. \quad (41)$$

Note that $\sum_{j \in S_a} \mathbb{P}[z = j | z \in S_a] = 1$ and that $\sum_{k \in S_b} \mathbb{P}[z = k | z \in S_b] = 1$. We can write

$$\begin{aligned}\tilde{\pi}_{cab} &= \sum_{j \in S_a} \sum_{k \in S_b} \sum_{n \in S_c} \mathbb{E}[d_n^j - d_n^k] \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b] \\ &= \sum_{j \in S_a} \sum_{k \in S_b} \sum_{n \in S_c} \mathbb{P}[d_n^j - d_n^k = 1] \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b] - \\ &\quad \sum_{j \in S_a} \sum_{k \in S_b} \sum_{n \in S_c} \mathbb{P}[d_n^j - d_n^k = -1] \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b].\end{aligned}\quad (42)$$

Assumption 1^C(d) guarantees first that $\mathbb{P}[d_n^j - d_n^k = 1] = \mathbb{P}[d_n^j - d_n^k = 1, d_l^j = 0] = \mathbb{P}[d_n^j - d_n^k = 1, d_l^j = 0, d_l^k = 0]$ with $l \in S_a$. Since this holds for all $l \in S_a$, we have according to Assumption 2^C that $\mathbb{P}[d_n^j - d_n^k = 1] = 0$ for all $n \notin S_a$, $j \in S_a$ and $k \in S_b$. This implies that $\tilde{\pi}_{cab}$ simplifies to

$$\tilde{\pi}_{cab} = - \sum_{j \in S_a} \sum_{k \in S_b} \sum_{n \in S_c} \mathbb{P}[d_n^j - d_n^k = -1] \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b]. \quad (43)$$

Second, assumption 2^C guarantees, together with Assumption 1^C(d), that when $d_n^j - d_n^k = -1$ it must be that $d_l^j - d_l^k = 1$ for one treatment $l \in S_a$. Hence, we are explicit in $\tilde{\pi}_{cab}$ where the shifts go to,

$$\tilde{\pi}_{cab} = - \sum_{j \in S_a} \sum_{k \in S_b} \sum_{n \in S_c} \sum_{l \in S_a} \mathbb{P}[d_n^j - d_n^k = -1, d_l^j - d_l^k = 1] \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b]. \quad (44)$$

Then assumption 3^C implies that for the individuals with $r_2 = k$, $\mathbb{P}[d_n^j - d_n^k = -1, d_l^j = 1] = \mathbb{P}[d_n^j - d_n^k = -1, d_l^j - d_l^k = 1] = 0$ for all $n \notin \{S_a, S_b\}$, $j \in S_a$ and $k \in S_b$. We leave implicit that we condition on the individuals with $r_2 = k$ and write,

$$\tilde{\pi}_{cab} = 0, \quad \forall c \neq a, b. \quad (45)$$

E Proof Theorem 2

It follows from (11) and Lemma 1 that for the individuals with $r_2 = k$,

$$\tilde{\beta}_{ab} = \frac{\tilde{\theta}_{ab}}{\tilde{\pi}_{aab}}, \quad \forall a \neq b. \quad (46)$$

We start with the first stage estimate $\tilde{\pi}_{aab} = \mathbb{E}[\tilde{d}_a | \tilde{z} = a] - \mathbb{E}[\tilde{d}_a | \tilde{z} = b]$, for which we derived an expression in the proof of Lemma 1,

$$\begin{aligned} \tilde{\pi}_{aab} = & \sum_{j \in S_a} \sum_{k \in S_b} \sum_{l \in S_a} \mathbb{P} \left[d_l^j - d_l^k = 1 \right] \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b] - \\ & \sum_{j \in S_a} \sum_{k \in S_b} \sum_{l \in S_a} \mathbb{P} \left[d_l^j - d_l^k = -1 \right] \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b]. \end{aligned} \quad (47)$$

The monotonicity assumption 1^C(d) implies that $\mathbb{P}[d_l^j - d_l^k = -1] = 0$ for all $j, l \in S_a$ and $k \in S_b$, so that $\tilde{\pi}_{aab}$ simplifies to

$$\tilde{\pi}_{aab} = \sum_{j \in S_a} \sum_{k \in S_b} \sum_{l \in S_a} \mathbb{P} \left[d_l^j - d_l^k = 1 \right] \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b]. \quad (48)$$

Since Assumption 3^C shows that the shifts $d_l^j - d_l^k = 1$ with $r_2 = k$ move away from S_a or S_b , and Assumption 1^C(d) shows that these shifts cannot come from S_a , they must have $d_m^j - d_m^k = -1$ with $m \in S_b$. We rewrite $\tilde{\pi}_{aab}$ by being explicit that the shifts towards $l \in S_a$ must come from $m \in S_b$,

$$\tilde{\pi}_{aab} = \sum_{j \in S_a} \sum_{k \in S_b} \sum_{l \in S_a} \sum_{m \in S_b} \mathbb{P} \left[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1 \right] \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b], \quad (49)$$

where the probabilities implicitly condition on $r_2 = k$. Next, we continue with the reduced form. Realize that $\tilde{\theta}_{ab} = \mathbb{E}[y | \tilde{z} = a] - \mathbb{E}[y | \tilde{z} = b]$, use assumption 1(a) to substitute for potential outcomes $y = \sum_{l=1}^J y^l \times d_l$, and write

$$\begin{aligned} \tilde{\theta}_{ab} = & \sum_{l=1}^J \mathbb{E}[y^l d_l | \tilde{z} = a] - \sum_{l=1}^J \mathbb{E}[y^l d_l | \tilde{z} = b] \\ = & \sum_{j \in S_a} \sum_{l=1}^J \mathbb{E} \left[y^l d_l^j | z = j \right] \mathbb{P}[z = j | z \in S_a] - \sum_{k \in S_b} \sum_{l=1}^J \mathbb{E} \left[y^l d_l^k | z = k \right] \mathbb{P}[z = k | z \in S_b], \end{aligned} \quad (50)$$

then use assumption 1(b) to arrive at,

$$\tilde{\theta}_{ab} = \sum_{j \in S_a} \sum_{k \in S_b} \sum_{l=1}^J \mathbb{E} \left[y^l (d_l^j - d_l^k) \right] \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b], \quad (51)$$

which we split per cluster as follows,

$$\begin{aligned} \tilde{\theta}_{ab} = & \sum_{j \in S_a} \sum_{k \in S_b} \sum_{l \in S_a} \mathbb{E} \left[y^l (d_l^j - d_l^k) \right] \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b] + \\ & \sum_{j \in S_a} \sum_{k \in S_b} \sum_{m \in S_b} \mathbb{E} \left[y^m (d_m^j - d_m^k) \right] \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b] + \\ & \sum_{j \in S_a} \sum_{k \in S_b} \sum_{n \notin S_a, S_b} \mathbb{E} \left[y^n (d_n^j - d_n^k) \right] \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b]. \end{aligned} \quad (52)$$

From Lemma 1 we know that $d_n^j - d_n^k = 0$ for all $n \notin \{S_a, S_b\}$ for the individuals with $r_2 = k$. Hence, the last term in (52) is equal to zero conditional on $r_2 = k$, and from hereon the probabilities in this proof implicitly condition on $r_2 = k$.

For the second term in (52), assumption 1^C(d) guarantees that $\mathbb{P}[d_m^j - d_m^k = 1] = \mathbb{P}[d_m^j - d_m^k = 1, d_l^j = 0, d_l^k = 0]$ for all $l \in S_a$. Then we have according to Assumption 2^C that $\mathbb{P}[d_m^j - d_m^k = 1] = 0$ for all $m \in S_b, j \in S_a$ and $k \in S_b$.

From $d_n^j - d_n^k = 0$ for $n \notin S_a, S_b$, it follows that $d_m^j - d_m^k = -1$ if $d_l^j - d_l^k = 1$, and that $d_l^j - d_l^k = 1$ if $d_m^j - d_m^k = -1$, for all $j, l \in S_a$ and $k, m \in S_b$, and so $\tilde{\theta}_{ab}$ can be written as

$$\begin{aligned} \tilde{\theta}_{ab} &= \sum_{j \in S_a} \sum_{k \in S_b} \sum_{l \in S_a} \sum_{m \in S_b} \mathbb{E}[y^l - y^m | d_l^j - d_l^k = 1, d_m^j - d_m^k = -1, r_2 = k] \times \\ &\quad \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1] \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b]. \end{aligned} \quad (53)$$

Finally, combining (49) and (53) we have

$$\tilde{\beta}_{ab} = \frac{\tilde{\theta}_{ab}}{\tilde{\pi}_{aab}} = \sum_{j \in S_a} \sum_{k \in S_b} \sum_{l \in S_a} \sum_{m \in S_b} \lambda_{jklm} \mathbb{E}[y^l - y^m | d_l^j - d_l^k = 1, d_m^j - d_m^k = -1, r_2 = k], \quad (54)$$

for all $a \neq b$, with

$$\lambda_{jklm} = \frac{\mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1] \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b]}{\sum_{j \in S_a} \sum_{k \in S_b} \sum_{l \in S_a} \sum_{m \in S_b} \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1] \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b]}, \quad (55)$$

where assumption 1(c) ensures that $\tilde{\beta}_{ab}$ is finite.

F Proof Corollary 2

Note that in this proof all probabilities implicitly condition on $r_2 = k$. From Theorem 2 we have that

$$\lambda_{jklm} = \frac{\mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1] \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b]}{\sum_{j \in S_a} \sum_{k \in S_b} \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b] \sum_{l \in S_a} \sum_{m \in S_b} \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1]}. \quad (56)$$

The denominator can be written as

$$\sum_{j \in S_a} \sum_{k \in S_b} \omega_{jk} \sum_{l \in S_a} \sum_{m \in S_b} \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1] = \sum_{j \in S_a} \sum_{k \in S_b} \omega_{jk} \sum_{l \in S_a} \pi_{ljk}, \quad (57)$$

where we define $\omega_{jk} = \mathbb{P}[z = j | z \in S_a] \mathbb{P}[z = k | z \in S_b]$.

For the probability in the numerator we have that

$$\mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1] \leq \min\left(\mathbb{P}[d_l^j - d_l^k = 1], \mathbb{P}[d_m^j - d_m^k = -1]\right), \quad (58)$$

$$\mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1] \geq \max\left(0, \mathbb{P}[d_m^j - d_m^k = -1] - \sum_{n \in \{S_a \setminus l\}} \mathbb{P}[d_n^j - d_n^k = 1]\right). \quad (59)$$

From Assumption 1(b) it follows that

$$\pi_{ljk} = \mathbb{E}[d_l^j - d_l^k] = \mathbb{P}[d_l^j - d_l^k = 1] - \mathbb{P}[d_l^j - d_l^k = -1], \quad \forall l \neq k. \quad (60)$$

From Assumption 1^C(d) it follows that $\mathbb{P}[d_l^j - d_l^k = -1] = 0$ for all $l \in S_a$, and from Assumption 1^C(d) and 2^C that $\mathbb{P}[d_l^j - d_l^k = 1] = 0$ for all $l \in S_b$. Therefore, we can write

$$\max\left(0, -\pi_{mjk} - \sum_{n \in \{S_a \setminus l\}} \pi_{njk}\right) \leq \mathbb{P}\left[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1\right] \leq \min(\pi_{ljk}, -\pi_{mjk}). \quad (61)$$

Note that π_{ljk} is a first stage coefficient in (2) for $l \neq k$. For π_{kjk} , which is not in (2), we use that $\sum_{l=1}^J \mathbb{E}[d_l^j - d_l^k] = \sum_{l \in S_a, S_b} \mathbb{E}[d_l^j - d_l^k] = 0$, and so we can write

$$\begin{aligned} -\pi_{kjk} &= \mathbb{P}[d_k^j - d_k^k = -1] = \sum_{l \in S_a} \mathbb{P}[d_l^j - d_l^k = 1] - \sum_{m \in \{S_b \setminus k\}} \mathbb{P}[d_m^j - d_m^k = -1] \\ &= \sum_{l \in S_a} \pi_{ljk} + \sum_{m \in \{S_b \setminus k\}} \pi_{mjk}. \end{aligned} \quad (62)$$

G Proof Corollary 3

From (31) follows that

$$\begin{aligned} \theta_{jk} &= \sum_{l=1}^J \mathbb{E}[y^l (d_l^j - d_l^k)] \\ &= \sum_{l=1}^J \mathbb{E}[y^l | d_l^j - d_l^k = 1] \mathbb{P}[d_l^j - d_l^k = 1] - \sum_{m=1}^J \mathbb{E}[y^m | d_m^j - d_m^k = -1] \mathbb{P}[d_m^j - d_m^k = -1] \\ &= \sum_{l=1}^J \sum_{m \neq l} \mathbb{E}[y^l | d_l^j - d_l^k = 1, d_m^j - d_m^k = -1] \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1] - \\ &\quad \sum_{m=1}^J \sum_{l \neq m} \mathbb{E}[y^m | d_l^j - d_l^k = 1, d_m^j - d_m^k = -1] \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1] \\ &= \sum_{l=1}^J \sum_{m \neq l} \Delta_{jklm} \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1]. \end{aligned} \quad (63)$$

First, for all $j, l \in S_a$ and $k, m \in S_b$ with $a \neq b$, the CLATE assumptions simplify (63) to

$$\theta_{jk} = \sum_{l \in S_a} \sum_{m \in S_b} \Delta_{jklm} \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1]. \quad (64)$$

Assume that $\Delta_{jklm} = \Delta_{lklk} - \Delta_{mkmk}$ to rewrite (64) to

$$\begin{aligned} \theta_{jk} &= \sum_{l \in S_a} \sum_{m \in S_b} (\Delta_{lklk} - \Delta_{mkmk}) \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1] \\ &= \sum_{l \in S_a} \Delta_{lklk} \sum_{m \in S_b} \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1] - \\ &\quad \sum_{m \in S_b} \Delta_{mkmk} \sum_{l \in S_a} \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1] \\ &= \sum_{l \in S_a} \Delta_{lklk} \mathbb{P}[d_l^j - d_l^k = 1] - \sum_{m \in S_b} \Delta_{mkmk} \mathbb{P}[d_m^j - d_m^k = -1] = \sum_{l \in \{S_a, S_b\}} \Delta_{lklk} \pi_{ljk}. \end{aligned} \quad (65)$$

Second, for $j, k \in S_b$, the CLATE assumptions do not impose any restrictions on (63). We assume that $\Delta_{jklm} = \Delta_{lklk} - \Delta_{mkmk}$ for all l and m , from which it follows that

$$\begin{aligned}
\theta_{jk} &= \sum_{l=1}^J \sum_{m \neq l} (\Delta_{lklk} - \Delta_{mkmk}) \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1] \\
&= \sum_{l=1}^J \Delta_{lklk} \sum_{m \neq l} \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1] - \\
&\quad \sum_{m=1}^J \Delta_{mkmk} \sum_{l \neq m} \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1] \\
&= \sum_{l=1}^J \Delta_{lklk} \mathbb{P}[d_l^j - d_l^k = 1] - \sum_{m=1}^J \Delta_{mkmk} \mathbb{P}[d_m^j - d_m^k = -1] = \sum_{l \neq k} \Delta_{lklk} \pi_{ljk}.
\end{aligned} \tag{66}$$

Therefore, it is sufficient to assume that $\Delta_{jklm} = \Delta_{lklk} - \Delta_{mkmk}$ to arrive at $\theta_{jk} = \sum_{l \neq k} \Delta_{lklk} \pi_{ljk}$ in both the first and second case.

From (5) follows that for all $j \neq k$ with $k \in S_b$,

$$\theta_{jk} = \sum_{l \neq k} \beta_{lk} \pi_{ljk}. \tag{67}$$

Combining (67) with the result that $\theta_{jk} = \sum_{l \neq k} \Delta_{lklk} \pi_{ljk}$ for all $j \neq k$ with $k \in S_b$ we have the homogeneous system of equations

$$\sum_{l \neq k} (\Delta_{lklk} - \beta_{lk}) \pi_{ljk} = 0, \quad \text{for all } j \neq k, \tag{68}$$

which has the unique solution $\beta_{lk} = \Delta_{lklk}$ for all $l \neq k$ if Π_k has full rank. Since $\mathbb{E}[\mathbf{d}_{-k} \mathbf{z}'_{-k}]$ has full rank according to Assumption 1(c) and $\mathbb{E}[\mathbf{z}_{-k} \mathbf{z}'_{-k}]$ has full rank, $\mathbb{E}[\mathbf{d}_{-k} \mathbf{z}'_{-k}] \mathbb{E}[\mathbf{z}_{-k} \mathbf{z}'_{-k}] = [\delta_k, \Pi_k]$ has full rank with $\delta_k = (\delta_{1k}, \dots, \delta_{k-1,k}, \delta_{k+1,k}, \dots, \delta_{J,k})'$. Therefore, Π_k has full rank with elements π_{ljk} for all $l \neq k$ and $j \neq k$.

Finally, from Theorem 2 follows that

$$\tilde{\beta}_{ab} = \sum_{j \in S_a} \sum_{k \in S_b} \sum_{l \in S_a} \sum_{m \in S_b} \lambda_{jklm} \Delta_{jklm}, \tag{69}$$

where we use that $\Delta_{jklm} = \Delta_{lklk} - \Delta_{mkmk} = \beta_{lk} - \beta_{mk}$ for $j, l \in S_a$ and $k, m \in S_b$ with $a \neq b$ to write

$$\tilde{\beta}_{ab} = \sum_{j \in S_a} \sum_{k \in S_b} \left(\sum_{l \in S_a} \beta_{lk} \sum_{m \in S_b} \lambda_{jklm} - \sum_{m \in S_b} \beta_{mk} \sum_{l \in S_a} \lambda_{jklm} \right) \tag{70}$$

$$= \frac{\sum_{j \in S_a} \sum_{k \in S_b} \sum_{l \in S_a} \beta_{lk} \pi_{ljk} \omega_{jk}}{\sum_{j \in S_a} \sum_{k \in S_b} \sum_{l \in S_a} \pi_{ljk} \omega_{jk}} - \frac{\sum_{j \in S_a} \sum_{k \in S_b} \sum_{m \in S_b} \beta_{mk} \pi_{mjk} \omega_{jk}}{\sum_{j \in S_a} \sum_{k \in S_b} \sum_{m \in S_b} \pi_{mjk} \omega_{jk}}, \tag{71}$$

where $\omega_{jk} = \mathbb{P}[z = j | z \in S_a, r_2 \in S_b] \mathbb{P}[z = k | z \in S_b, r_2 \in S_b]$.

H Treatment clusters for the total sample

Algorithm 1 estimates separate CLATES $\tilde{\beta}_{ab}$ for J groups of individuals with $r_2 = k$, $k = 1, \dots, J$, which we denote as $\tilde{\beta}_{ab}(k)$. Since the variance of the 2SLS estimates for CLATE decreases in the number of observations, the CLATES can potentially be more precisely estimated under a homogeneity assumption across the individuals with $r_2 \in S_b$.

Assumption 4^C. Homogeneity across the individuals with different $r_2 \in S_b$

(Homogeneous treatment effects) $\mathbb{E}[y^l - y^m | d_l^j - d_l^k = 1, d_m^j - d_m^k = -1, r_2 = k] = \mathbb{E}[y^l - y^m | d_l^j - d_l^k = 1, d_m^j - d_m^k = -1, r_2 \in S_b]$ for all $j, l \in S_a$ and $k, m \in S_b$.

(Homogeneous treatment shifts) $\mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1 | r_2 = k] = \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1 | r_2 \in S_b]$ for all $j, l \in S_a$ and $k, m \in S_b$.

(Homogeneous treatment assignments) $\mathbb{P}[z = l | z \in S_c, r_2 = k] = \mathbb{P}[z = l | z \in S_c, r_2 \in S_b]$ for all $l \in S_c, k \in S_b$, and S_c .

Under Assumption 4^C we have that $\tilde{\beta}_{ab}(k) = \tilde{\beta}_{ab}(m)$ for all $k, m \in S_b$, and hence $\tilde{\beta}_{ab}$ can be estimated on all individuals with $r_2 \in S_b$. Especially if the number of observed individuals with $r_2 = k$ is small, and the group of individuals with $r_2 \in S_b$ is large, this can result in substantial improvements in the variance of the CLATE estimates. In case Assumption 4^C does not hold, 2SLS is not an unbiased estimator of the CLATE as defined in Theorem 2, but an unbiased estimator of a weighted average of LATEs that depend on samples of individuals with different r_2 . However, the mean squared error of the CLATE estimates might still improve as a result of the decrease in variance.

Assumption 4^C allows us to estimate one set of treatment clusters across the whole sample. Instead of starting with a set of individual treatments for each sample of individuals with next-best treatment $r_2 = k$, as in line 1 of Algorithm 1, we apply the loop in line 2-12 iteratively to each sample with $r_2 = k$ for $k = 1, \dots, J$ to update the set of clusters. In this way, if a LATE assumption is violated in one subsample, the corresponding treatments are clustered for all subsamples. The resulting set of treatments satisfies the CLATE assumptions across all subsamples, and therefore identifies CLATE on the whole sample. Algorithm 2 outlines these steps for estimating CLATE on the total sample.

Algorithm 2 CLATE estimation for the total sample of individuals

- 1: Set J treatment clusters: one for each treatment j
 - 2: **for** all k **do**
 - 3: Estimate Π_k in (2) with least squares
 - 4: **for** all $l \neq j$ **do**
 - 5: Two-sided t-test for $H_0 : \pi_{ljk} = 0$
 - 6: **if** $H_0 : \pi_{ljk} = 0$ is rejected and $\hat{\pi}_{ljk} < 0$ **then**
 - 7: merge the clusters of l and k
 - 8: **else if** $H_0 : \pi_{ljk} = 0$ is rejected and $\hat{\pi}_{ljk} > 0$ **then**
 - 9: merge the clusters of l and j
 - 10: **end if**
 - 11: **end for**
 - 12: **end for**
 - 13: Define S as the set of nonempty clusters
 - 14: Given S , estimate $\tilde{\beta}_{ab}$ for all a, b with $a \neq b$ in (8) and (9) with two-stage least squares
-

I Clustered local average treatment effects with covariates

Including control variables in the multiple unordered treatment effect analysis is not straightforward. Goldsmith-Pinkham, Hull, and Kolesár (2021) show that including controls in a multiple unordered treatment regression model does not identify a weighted average of treatment effects. Instead, they show that the controls have to be included as interactions with the treatment variables, to prevent each treatment coefficient to be confounded with the effects of all other treatments. We extend this reduced form result to the multiple unordered treatment IV model, and provide an expression for CLATE with covariates.

Let x denote a vector of covariates including an intercept. We extend the treatment cluster IV model in (8) and (9) by including the covariates in the second stage model

$$y = \tilde{v}'x + \sum_{a \neq b} \tilde{\beta}_{ab} \tilde{d}_a + \tilde{\varepsilon}_b, \quad (72)$$

and as interactions with the full set of instruments in the first stage equations

$$\tilde{d}_a = \sum_c \tilde{\eta}'_{acb} \tilde{z}_c x + \sum_{c \neq b} \tilde{\pi}_{acb} \tilde{z}_c + \tilde{u}_{ab}, \quad \text{for all } a \neq b. \quad (73)$$

The following result generalizes Theorem 2 to CLATE with covariates.

Theorem 4. *Under Assumptions 1(a)-(c), 1^C(d)-3^C and the model in (72) and (73), it holds that for the individuals with $r_2 \in S_b$ that*

$$\tilde{\beta}_{ab} = \sum_{j \in S_a} \sum_{k \in S_b} \sum_{l \in S_a} \sum_{m \in S_b} \mathbb{E} \left[\lambda_{jklm}(x) \mathbb{E}[y^l - y^m | d_l^j - d_l^k = 1, d_m^j - d_m^k = -1, r_2 \in S_b, x] \right], \quad (74)$$

for all clusters $a \neq b$, with

$$\lambda_{jklm}(x) = \frac{\sigma^2(x) \omega_{jk}(x) \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1 | x]}{\mathbb{E} \left[\sigma^2(x) \sum_{j \in S_a} \sum_{k \in S_b} \omega_{jk}(x) \sum_{l \in S_a} \sum_{m \in S_b} \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1 | x] \right]}, \quad (75)$$

where $\sigma^2(x) = \left(1 - \sum_{j \in S_a} \mathbb{P}[z = j | x] \right) \sum_{j \in S_a} \mathbb{P}[z = j | x]$, $\omega_{jk}(x) = \mathbb{P}[z = j | z \in S_a, x] \mathbb{P}[z = k | z \in S_b, x]$, and the probabilities implicitly condition on $r_2 \in S_b$.

Proof: Substitute (73) into (72) to obtain the reduced form:

$$y = \sum_c \tilde{\mu}'_{cb} w_c + \sum_{c \neq b} \tilde{\theta}_{cb} \tilde{z}_c + \tilde{\mathcal{E}}_b, \quad (76)$$

with $\tilde{\mu}_{cb} = \tilde{v} + \sum_{a \neq b} \tilde{\beta}_{ab} \tilde{\eta}_{acb}$, $w_c = \tilde{z}_c x$, and $\tilde{\theta}_{cb} = \sum_{a \neq b} \tilde{\beta}_{ab} \tilde{\pi}_{acb}$. It follows that

$$\tilde{\beta}_{ab} = \frac{\tilde{\theta}_{ab}}{\tilde{\pi}_{aab}} - \sum_{c \neq a, b} \frac{\tilde{\pi}_{cab}}{\tilde{\pi}_{aab}} \tilde{\beta}_{cb}. \quad (77)$$

Below we show that $\tilde{\pi}_{cab} = 0$ if $c \neq a$ and derive an expression for $\tilde{\theta}_{ab} / \tilde{\pi}_{aab}$.

Applying the Frisch-Waugh-Lovell Theorem to (73) and (76) results in

$$\tilde{\pi}_{cab} = \frac{\mathbb{E}[(\tilde{z}_c - \mathbb{E}[\tilde{z}_c | \tilde{z}_{-c}, w]) \tilde{d}_a]}{\mathbb{E}[(\tilde{z}_c - \mathbb{E}[\tilde{z}_c | \tilde{z}_{-c}, w])^2]} \text{ and } \tilde{\theta}_{ab} = \frac{\mathbb{E}[(\tilde{z}_a - \mathbb{E}[\tilde{z}_a | \tilde{z}_{-a}, w]) y]}{\mathbb{E}[(\tilde{z}_a - \mathbb{E}[\tilde{z}_a | \tilde{z}_{-a}, w])^2]}, \quad (78)$$

where $\tilde{z}_{-c} = \{\tilde{z}_e\}_{e \neq b, c}$ and $w = \{w_c\}_{c=1}^C$. To simplify (78), define Z_{-c} as a vector that stacks $\{w_e\}_{e \neq c}$ and \tilde{z}_{-c} , and $Z = (w'_c, Z'_{-c})$. We can write

$$\begin{aligned} \mathbb{E}[\tilde{z}_c | \tilde{z}_{-c}, w] &= (w'_c \quad Z'_{-c}) \begin{pmatrix} \mathbb{E}[w_c w'_c] & \mathbb{E}[w_c Z'_{-c}] \\ \mathbb{E}[w_c Z'_{-c}]' & \mathbb{E}[Z_{-c} Z'_{-c}] \end{pmatrix}^{-1} \begin{pmatrix} \mathbb{E}[w_c \tilde{z}_c] \\ \mathbb{E}[Z_{-c} \tilde{z}_c] \end{pmatrix} \\ &= w'_c \mathbb{E}[w_c w'_c]^{-1} \mathbb{E}[w_c \tilde{z}_c] = \mathbb{E}[\tilde{z}_c | x], \end{aligned}$$

where the second line uses that the instruments are mutually exclusive, and hence w_c only contains nonzero elements if Z_{-c} is zero. It follows that $\mathbb{E}[w_c Z'_{-c}]$ and $Z_{-c} \tilde{z}_c$ only have zero elements. Hence, (78) simplifies to

$$\tilde{\pi}_{cab} = \frac{\mathbb{E}[(\tilde{z}_c - \mathbb{E}[\tilde{z}_c | x]) \tilde{d}_a]}{\mathbb{E}[(\tilde{z}_c - \mathbb{E}[\tilde{z}_c | x])^2]} \text{ and } \tilde{\theta}_{ab} = \frac{\mathbb{E}[(\tilde{z}_a - \mathbb{E}[\tilde{z}_a | x]) y]}{\mathbb{E}[(\tilde{z}_a - \mathbb{E}[\tilde{z}_a | x])^2]}. \quad (79)$$

We simplify the numerator of $\tilde{\pi}_{cab}$ in (79) using that $\tilde{d}_a = \sum_{n \in S_a} d_n = \sum_{l=1}^J \sum_{n \in S_a} d_n^l z_l$:

$$\begin{aligned} \tilde{d}_a &= \mathbb{E}[\tilde{d}_a | \tilde{z}, w] + \tilde{u}_{ab} = \sum_{l=1}^J \sum_{n \in S_a} \mathbb{E}[d_n^l z_l | \tilde{z}, w] + \tilde{u}_{ab} \\ &= \sum_{k \in S_b} \sum_{n \in S_a} \mathbb{E}[d_n^k | z = k, x] \mathbb{P}[z = k | z \in S_b, x] + \sum_{e \neq b} \tilde{z}_e \tilde{\pi}_{aeb}(x) + \tilde{u}_{ab}, \end{aligned} \quad (80)$$

where

$$\tilde{\pi}_{aeb}(x) = \sum_{j \in S_e} \sum_{k \in S_b} \left(\sum_{n \in S_a} \mathbb{E}[d_n^j | z = j, x] - \mathbb{E}[d_n^k | z = k, x] \right) \mathbb{P}[z = j | z \in S_e, x] \mathbb{P}[z = k | z \in S_b, x].$$

Since $\mathbb{E}[d_n^j - d_n^k | x] = \mathbb{E}[d_n^j | z = j, x] - \mathbb{E}[d_n^k | z = k, x]$, it follows from the proof of Lemma 1 in Appendix D that $\tilde{\pi}_{aeb}(x) = 0$ if $a \neq e$ and from the proof of Theorem 2 in Appendix E that

$$\tilde{\pi}_{aab}(x) = \sum_{j \in S_a} \sum_{k \in S_b} \omega_{jk}(x) \sum_{l \in S_a} \sum_{m \in S_b} \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1 | x], \quad (81)$$

with $\omega_{jk}(x) = \mathbb{P}[z = j | z \in S_a, x] \mathbb{P}[z = k | z \in S_b, x]$. Substituting (80) into the numerator of $\tilde{\pi}_{cab}$ in (79) shows that $\mathbb{E}[(\tilde{z}_c - \mathbb{E}[\tilde{z}_c | x]) \tilde{d}_a]$ equals

$$\sum_{k \in S_b} \sum_{n \in S_a} \mathbb{E}[(\tilde{z}_c - \mathbb{E}[\tilde{z}_c | x]) \mathbb{E}[d_n^k | z = k, x] \mathbb{P}[z = k | z \in S_b, x]] + \mathbb{E}[(\tilde{z}_c - \mathbb{E}[\tilde{z}_c | x]) \tilde{z}_a \tilde{\pi}_{aab}(x)],$$

since $\mathbb{E}[\tilde{u}_{ab} | z, x] = 0$ by definition. The law of total expectation shows that the first term with $c \neq b$ equals zero: $\mathbb{E}[(\tilde{z}_c - \mathbb{E}[\tilde{z}_c | x]) \mathbb{E}[d_n^k | z = k, x] \mathbb{P}[z = k | z \in S_b, x]] =$

$$\begin{aligned} &\mathbb{E}\left(\mathbb{E}\left[(\tilde{z}_c - \mathbb{E}[\tilde{z}_c | x]) \mathbb{E}[d_n^k | z = k, x] \mathbb{P}[z = k | z \in S_b, x] \mid x\right]\right) = \\ &\mathbb{E}\left(\mathbb{E}\left[(\tilde{z}_c - \mathbb{E}[\tilde{z}_c | x]) \mid x\right] \mathbb{E}[d_n^k | z = k, x] \mathbb{P}[z = k | z \in S_b, x]\right) = 0, \end{aligned} \quad (82)$$

since $\mathbb{E}[(\tilde{z}_c - \mathbb{E}[\tilde{z}_c|x])|x] = \mathbb{E}[\tilde{z}_c|x] - \mathbb{E}[\tilde{z}_c|x] = 0$. Similarly, the second term can be written as

$$\mathbb{E}[(\tilde{z}_c - \mathbb{E}[\tilde{z}_c|x])\tilde{z}_a\tilde{\pi}_{aab}(x)] = \mathbb{E}(\mathbb{E}[(\tilde{z}_c - \mathbb{E}[\tilde{z}_c|x])\tilde{z}_a|x]\tilde{\pi}_{aab}(x)), \quad (83)$$

which equals zero if $a \neq c$ because, conditional on x , there is only one c with \tilde{z}_c nonzero. For $a = c$ we have that

$$\mathbb{E}[(\tilde{z}_a - \mathbb{E}[\tilde{z}_a|x])\tilde{d}_a] = \mathbb{E}(\text{var}[\tilde{z}_a|x]\tilde{\pi}_{aab}(x)). \quad (84)$$

Now consider the numerator of $\tilde{\theta}_{ab}$ in (79). Similar as for \tilde{d}_a , use $y = \sum_{l=1}^J y^l d_l$ to write

$$y = \mathbb{E}[y|\tilde{z}, w] + \tilde{\mathcal{E}}_b = \sum_{k \in S_b} \sum_{l=1}^J \mathbb{E}[y^l d_l^k|x] \mathbb{P}[z = k|z \in S_b, x] + \sum_{c \neq b} \tilde{z}_c \tilde{\theta}_{cb}(x) + \tilde{\mathcal{E}}_b, \quad (85)$$

where $\tilde{\theta}_{cb}(x) = \mathbb{E}[y^l d_l|\tilde{z} = c, x] - \mathbb{E}[y^l d_l|\tilde{z} = b, x] =$

$$\sum_{j \in S_c} \sum_{k \in S_b} \left(\sum_{l=1}^J \mathbb{E}[y^l d_l^j|x] - \mathbb{E}[y^l d_l^k|x] \right) \mathbb{P}[z = j|z \in S_c, x] \mathbb{P}[z = k|z \in S_b, x]. \quad (86)$$

Since $\mathbb{E}[y^l(d_l^j - d_l^k)|x] = \mathbb{E}[y^l d_l^j|x] - \mathbb{E}[y^l d_l^k|x]$, it follows from the proof of Theorem 2 in Appendix E that

$$\tilde{\theta}_{ab}(x) = \sum_{j \in S_a} \sum_{k \in S_b} \omega_{jk}(x) \sum_{l \in S_a} \sum_{m \in S_b} \Delta_{jklm}(x) \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1|x], \quad (87)$$

with $\Delta_{jklm}(x) = \mathbb{E}[y^l - y^m | d_l^j - d_l^k = 1, d_m^j - d_m^k = -1, x]$.

Substituting (85) into the numerator of $\tilde{\theta}_{ab}$ in (79) gives $\mathbb{E}[(\tilde{z}_a - \mathbb{E}[\tilde{z}_a|x])y] =$

$$\sum_{k \in S_b} \sum_{l=1}^J \mathbb{E}[(\tilde{z}_a - \mathbb{E}[\tilde{z}_a|x])\mathbb{E}[y^l d_l^k|z = k, x] \mathbb{P}[z = k|z \in S_b, x]] + \sum_{c \neq b} \mathbb{E}[(\tilde{z}_a - \mathbb{E}[\tilde{z}_a|x])\tilde{z}_c \tilde{\theta}_{cb}(x)],$$

since $\mathbb{E}[\tilde{\mathcal{E}}_b|z, x] = 0$. In a similar way as for the numerator of $\tilde{\pi}_{cab}$, the law of total expectation shows that the first term equals zero and the second term $\mathbb{E}(\mathbb{E}[(\tilde{z}_a - \mathbb{E}[\tilde{z}_a|x])\tilde{z}_c|x]\tilde{\theta}_{cb}(x))$, where we also use that conditional on x , there is only one c with \tilde{z}_c nonzero.

Finally, we have that

$$\tilde{\beta}_{ab} = \frac{\tilde{\theta}_{ab}}{\tilde{\pi}_{aab}} = \frac{\mathbb{E}(\text{var}[\tilde{z}_a|x]\tilde{\theta}_{ab}(x))}{\mathbb{E}(\text{var}[\tilde{z}_a|x]\tilde{\pi}_{aab}(x))} = \sum_{j \in S_a} \sum_{k \in S_b} \sum_{l \in S_a} \sum_{m \in S_b} \mathbb{E}[\lambda_{jklm}(x)\Delta_{jklm}(x)], \quad (88)$$

where $\tilde{\theta}_{ab}(x)$ is defined in (87), $\tilde{\pi}_{aab}(x)$ in (81), and $\Delta_{jklm}(x)$ below (87). It follows that

$$\lambda_{jklm}(x) = \frac{\sigma^2(x)\omega_{jk}(x)\mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1|x]}{\mathbb{E}\left[\sigma^2(x)\sum_{j \in S_a} \sum_{k \in S_b} \omega_{jk}(x)\sum_{l \in S_a} \sum_{m \in S_b} \mathbb{P}[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1|x]\right]}, \quad (89)$$

where $\omega_{jk}(x) = \mathbb{P}[z = j|z \in S_a, x]\mathbb{P}[z = k|z \in S_b, x]$ and

$$\sigma^2(x) = \text{var}[\tilde{z}_a|x] = (1 - \mathbb{P}[\tilde{z} = a|x])\mathbb{P}[\tilde{z} = a|x] = \left(1 - \sum_{j \in S_a} \mathbb{P}[z = j|x]\right) \sum_{j \in S_a} \mathbb{P}[z = j|x]. \quad (90)$$

J Proof Theorem 3

It follows from the first stage regressions in (21) that for $c \neq b$

$$\begin{aligned}\mathbb{E}[\tilde{d}_a|\tilde{z} = c, x, s] &= \sum_{l \neq k} \mathbb{E}[\tilde{d}_a|\tilde{z} = c, x = l, s] \mathbb{P}[x = l|\tilde{z} = c, s] \\ &= \sum_{l \in S_c} \mathbb{E}[\tilde{d}_a|\tilde{z} = c, x = l, s] \mathbb{P}[x = l|\tilde{z} = c, s] \\ &= \sum_{l \in S_c} \eta_{alk} \mathbb{P}[x = l|\tilde{z} = c, s] + \tilde{\pi}_{ack} + \psi_{aks},\end{aligned}\tag{91}$$

where we first use that $\mathbb{P}[x = l|\tilde{z} = c, s] = 0$ if $x \notin S_c$ and second that $\sum_{l \in S_c} \mathbb{P}[x = l|\tilde{z} = c, s] = 1$.

We take the limit of s towards zero:

$$\begin{aligned}\lim_{\nu \rightarrow +0} \mathbb{E}[\tilde{d}_a|\tilde{z} = c, x, s] &= \lim_{\nu \rightarrow +0} \sum_{l \in S_c} \mathbb{E}[\tilde{d}_a|\tilde{z} = c, x = l, s = \nu] \lim_{\nu \rightarrow +0} \mathbb{P}[x = l|\tilde{z} = c, s = \nu] \\ &= \sum_{l \in S_c} \eta_{alk} \lim_{\nu \rightarrow +0} \mathbb{P}[x = l|\tilde{z} = c, s = \nu] + \tilde{\pi}_{ack}.\end{aligned}\tag{92}$$

For $\tilde{z} = b$, we can write

$$\sum_{l \in S_c} \mathbb{E}[\tilde{d}_a|\tilde{z} = b, x = l, s] \lim_{\nu \rightarrow +0} \mathbb{P}[x = l|\tilde{z} = c, s = \nu] = \sum_{l \in S_c} \eta_{alk} \lim_{\nu \rightarrow +0} \mathbb{P}[x = l|\tilde{z} = c, s = \nu] + \psi_{aks},$$

after which we take the limit of s towards zero:

$$\lim_{\nu \rightarrow +0} \sum_{l \in S_c} \mathbb{E}[\tilde{d}_a|\tilde{z} = b, x = l, s = \nu] \lim_{\nu \rightarrow +0} \mathbb{P}[x = l|\tilde{z} = c, s = \nu] = \sum_{l \in S_c} \eta_{alk} \lim_{\nu \rightarrow +0} \mathbb{P}[x = l|\tilde{z} = c, s = \nu],$$

from which it follows that $\tilde{\pi}_{ack}$ equals

$$\sum_{l \in S_c} \left(\lim_{\nu \rightarrow +0} \mathbb{E}[\tilde{d}_a|\tilde{z} = c, x = l, s = \nu] - \lim_{\nu \rightarrow +0} \mathbb{E}[\tilde{d}_a|\tilde{z} = b, x = l, s = \nu] \right) \lim_{\nu \rightarrow +0} \mathbb{P}[x = l|\tilde{z} = c, s = \nu].$$

Next, we write the expectations in terms of potential outcomes. For $c \neq b$ we have that

$$\begin{aligned}\mathbb{E}[\tilde{d}_a|\tilde{z} = c, x = l, s = \nu] &= \sum_{n \in S_a} \mathbb{E}[d_n|\tilde{z} = c, x = l, s = \nu] \\ &= \sum_{n \in S_a} \sum_{j \in S_c} \mathbb{E}[d_n|z = j, x = l, s = \nu] \mathbb{P}[z = j|z \in S_c, x = l, s = \nu] \\ &= \sum_{n \in S_a} \mathbb{E}\left[d_n^l|x = l, s = \nu\right],\end{aligned}\tag{93}$$

which follows from the fact that given $z \in S_c$, and therefore $z \neq k$, we have that $z = x = l$. For $c = b$ holds that

$$\begin{aligned}\mathbb{E}[\tilde{d}_a|\tilde{z} = b, x = l, s = \nu] &= \sum_{n \in S_a} \mathbb{E}[d_n|\tilde{z} = b, x = l, s = \nu] \\ &= \sum_{n \in S_a} \sum_{j \in S_b} \mathbb{E}[d_n|z = j, x = l, s = \nu] \mathbb{P}[z = j|z \in S_b, x = l, s = \nu] \\ &= \sum_{n \in S_a} \mathbb{E}\left[d_n^k|x = l, s = \nu\right],\end{aligned}\tag{94}$$

which follows from the fact that for $x = l \notin S_b$, z either equals l or k .

Substitute the expressions for the expectations into the expression for $\tilde{\pi}_{ack}$:

$$\begin{aligned}\tilde{\pi}_{ack} &= \sum_{l \in S_c} \sum_{n \in S_a} \left(\lim_{\nu \rightarrow +0} \mathbb{E} \left[d_n^l | x = l, s = \nu \right] - \lim_{\nu \rightarrow -0} \mathbb{E} \left[d_n^k | x = l, s = \nu \right] \right) \lim_{\nu \rightarrow +0} \mathbb{P} [x = l | \tilde{z} = c, s = \nu] \\ &= \sum_{l \in S_c} \sum_{n \in S_a} \mathbb{E} \left[d_n^l - d_n^k | x = l, s = 0 \right] \lim_{\nu \rightarrow +0} \mathbb{P} [x = l | \tilde{z} = c, s = \nu],\end{aligned}\quad (95)$$

which follows from Assumption 1^D. Now it follows from Lemma 1 and Theorem 2 that

$$\tilde{\beta}_{ak} = \sum_{j \in S_a} \sum_{l \in S_a} \sum_{m \in S_b} \lambda_{jklm} \mathbb{E} \left[y^l - y^m | d_l^j - d_l^k = 1, d_m^j - d_m^k = -1, x = j, s = 0, r_2 = k \right], \quad (96)$$

for all a for which $k \notin S_a$, with

$$\lambda_{jklm} = \frac{\mathbb{P} \left[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1 | x = j, s = 0 \right] \lim_{\nu \rightarrow +0} \mathbb{P} [x = j | z \in S_a, s = \nu]}{\sum_{j \in S_a} \sum_{l \in S_a} \sum_{m \in S_b} \mathbb{P} \left[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1 | x = j, s = 0 \right] \lim_{\nu \rightarrow +0} \mathbb{P} [x = j | z \in S_a, s = \nu]}, \quad (97)$$

where the probabilities implicitly condition on $r_2 = k$.

K Fuzzy RDD on the total sample

The RDD setting discussed in Section 5 defines separate CLATEs for each sample of individuals with $r_2 = k$ with $k \in S_b$. CLATE can also be defined for the total sample of individuals. The RDD model in Section 5 for this sample is

$$y = \sum_{k \in S_b} \sum_{l \neq k} \nu_{lk} x_{lk} + \sum_{a \neq b} \tilde{\beta}_{ab} \tilde{d}_a + \tilde{\gamma}_b s + \tilde{\varepsilon}_b, \quad (98)$$

$$\tilde{d}_a = \sum_{k \in S_b} \sum_{l \neq k} \eta_{alk} x_{lk} + \sum_{c \neq b} \tilde{\pi}_{acb} \tilde{z}_c + \tilde{\psi}_{ab} s + \tilde{u}_{ab}, \quad \text{for all } a \neq b, \quad (99)$$

where the fixed effect x_{jk} equals one if $(r_1, r_2) = (j, k)$ and zero otherwise. The theorem below provides an expression for CLATE on the total sample of individuals.

Theorem 5. *Under Assumptions 1(a), 1^D(b), 1(c), 1^C(d)-3^C, and the model in (98) and (99), it holds for the individuals with $r_2 \in S_b$ that*

$$\tilde{\beta}_{ab} = \sum_{j \in S_a} \sum_{k \in S_b} \sum_{l \in S_a} \sum_{m \in S_b} \lambda_{jklm} \mathbb{E} \left[y^l - y^m | d_l^j - d_l^k = 1, d_m^j - d_m^k = -1, x_{jk} = 1, s = 0 \right], \quad (100)$$

for all $a \neq b$, with

$$\lambda_{jklm} = \frac{\mathbb{P} \left[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1 | x_{jk} = 1, s = 0 \right] \lim_{\nu \rightarrow +0} \mathbb{P} [x_{jk} = 1 | z \in S_a, s = \nu]}{\sum_{j \in S_a} \sum_{k \in S_b} \lim_{\nu \rightarrow +0} \mathbb{P} [x_{jk} = 1 | z \in S_a, s = \nu] \sum_{l \in S_a} \sum_{m \in S_b} \mathbb{P} \left[d_l^j - d_l^k = 1, d_m^j - d_m^k = -1 | x_{jk} = 1, s = 0 \right]},$$

where $x_{jk} = 1$ indicates the individuals with $r_1 = j$ and $r_2 = k$.

Proof: It follows from the first stage regressions in (99) that for $c \neq b$

$$\begin{aligned}
\mathbb{E}[\tilde{d}_a | \tilde{z} = c, x, s] &= \sum_{k \in S_b} \sum_{l \neq k} \mathbb{E}[\tilde{d}_a | \tilde{z} = c, x_{lk} = 1, s] \mathbb{P}[x_{lk} = 1 | \tilde{z} = c, s] \\
&= \sum_{k \in S_b} \sum_{l \in S_c} \mathbb{E}[\tilde{d}_a | \tilde{z} = c, x_{lk} = 1, s] \mathbb{P}[x_{lk} = 1 | \tilde{z} = c, s] \\
&= \sum_{l \in S_c} \sum_{k \in S_b} \eta_{alk} \mathbb{P}[x_{lk} = 1 | \tilde{z} = c, s] + \tilde{\pi}_{acb} + \psi_{ab}s,
\end{aligned} \tag{101}$$

where we first use that $\mathbb{P}[x_{lk} = 1 | \tilde{z} = c, s] = 0$ if $l \notin S_c$ and second that $\sum_{k \in S_b} \sum_{l \in S_c} \mathbb{P}[x_{lk} = 1 | \tilde{z} = c, s] = 1$. We take the limit of s towards zero:

$$\begin{aligned}
\lim_{\nu \rightarrow +0} \mathbb{E}[\tilde{d}_a | \tilde{z} = c, x, s] &= \lim_{\nu \rightarrow +0} \sum_{l \in S_c} \sum_{k \in S_b} \mathbb{E}[\tilde{d}_a | \tilde{z} = c, x_{lk} = 1, s = \nu] \lim_{\nu \rightarrow +0} \mathbb{P}[x_{lk} = 1 | \tilde{z} = c, s = \nu] \\
&= \sum_{l \in S_c} \sum_{k \in S_b} \eta_{alk} \lim_{\nu \rightarrow +0} \mathbb{P}[x_{lk} = 1 | \tilde{z} = c, s = \nu] + \tilde{\pi}_{acb}.
\end{aligned} \tag{102}$$

For $\tilde{z} = b$, we can write

$$\begin{aligned}
\sum_{l \in S_c} \sum_{k \in S_b} \mathbb{E}[\tilde{d}_a | \tilde{z} = b, x_{lk} = 1, s] \lim_{\nu \rightarrow +0} \mathbb{P}[x_{lk} = 1 | \tilde{z} = c, s = \nu] &= \\
\sum_{l \in S_c} \sum_{k \in S_b} \eta_{alk} \lim_{\nu \rightarrow +0} \mathbb{P}[x_{lk} = 1 | \tilde{z} = c, s = \nu] + \psi_{ab}s,
\end{aligned} \tag{103}$$

after which we take the limit of s towards zero:

$$\lim_{\nu \rightarrow -0} \sum_{l \in S_c} \sum_{k \in S_b} \mathbb{E}[\tilde{d}_a | \tilde{z} = b, x_{lk} = 1, s = \nu] \lim_{\nu \rightarrow +0} \mathbb{P}[x_{lk} = 1 | \tilde{z} = c, s = \nu] = \sum_{l \in S_c} \sum_{k \in S_b} \eta_{alk} \lim_{\nu \rightarrow +0} \mathbb{P}[x_{lk} = 1 | \tilde{z} = c, s = \nu],$$

from which it follows that $\tilde{\pi}_{acb}$ equals

$$\sum_{l \in S_c} \sum_{k \in S_b} \left(\lim_{\nu \rightarrow +0} \mathbb{E}[\tilde{d}_a | \tilde{z} = c, x_{lk} = 1, s = \nu] - \lim_{\nu \rightarrow -0} \mathbb{E}[\tilde{d}_a | \tilde{z} = b, x_{lk} = 1, s = \nu] \right) \lim_{\nu \rightarrow +0} \mathbb{P}[x_{lk} = 1 | \tilde{z} = c, s = \nu].$$

Next, we write the expectations in terms of potential outcomes. For $c \neq b$ we have that

$$\begin{aligned}
\mathbb{E}[\tilde{d}_a | \tilde{z} = c, x_{lk} = 1, s = \nu] &= \sum_{n \in S_a} \mathbb{E}[d_n | \tilde{z} = c, x_{lk} = 1, s = \nu] \\
&= \sum_{n \in S_a} \sum_{j \in S_c} \mathbb{E}[d_n | z = j, x_{lk} = 1, s = \nu] \mathbb{P}[z = j | z \in S_c, x_{lk} = 1, s = \nu] \\
&= \sum_{n \in S_a} \mathbb{E}[d_n^l | x_{lk} = 1, s = \nu],
\end{aligned} \tag{104}$$

which follows from the fact that given $z \in S_c$, and therefore $z \neq k$, we have that $z = x = l$. For $c = b$ holds that

$$\begin{aligned}
\mathbb{E}[\tilde{d}_a | \tilde{z} = b, x_{lk} = 1, s = \nu] &= \sum_{n \in S_a} \mathbb{E}[d_n | \tilde{z} = b, x_{lk} = 1, s = \nu] \\
&= \sum_{n \in S_a} \sum_{j \in S_b} \mathbb{E}[d_n | z = j, x_{lk} = 1, s = \nu] \mathbb{P}[z = j | z \in S_b, x_{lk} = 1, s = \nu] \\
&= \sum_{n \in S_a} \mathbb{E}[d_n^k | x_{lk} = 1, s = \nu],
\end{aligned} \tag{105}$$

which follows from the fact that given $z \in S_b$, and therefore $z \neq l$, we have that $z = x = k$.

Substitute the expressions for the expectations into the expression for $\tilde{\pi}_{acb}$:

$$\begin{aligned} \tilde{\pi}_{acb} &= \sum_{l \in S_c} \sum_{k \in S_b} \sum_{n \in S_a} \left(\lim_{\nu \rightarrow +0} \mathbb{E} \left[d_n^l | x_{lk} = 1, s = \nu \right] - \lim_{\nu \rightarrow -0} \mathbb{E} \left[d_n^k | x_{lk} = 1, s = \nu \right] \right) \lim_{\nu \rightarrow +0} \mathbb{P} [x_{lk} = 1 | \tilde{z} = c, s = \nu] \\ &= \sum_{l \in S_c} \sum_{k \in S_b} \sum_{n \in S_a} \mathbb{E} \left[d_n^l - d_n^k | x_{lk} = 1, s = 0 \right] \lim_{\nu \rightarrow +0} \mathbb{P} [x_{lk} = 1 | \tilde{z} = c, s = \nu], \end{aligned} \quad (106)$$

which follows from Assumption 1^D. Now the result follows from Lemma 1 and Theorem 2.

L Additional empirical results

Figure 11: LATE first stage estimates with next-best Agriculture, forestry and fisheries

Architecture and construction																				
Arts		-0.003	-0.002	-0.003	-0.002					-0.002	-0.004					-0.003				-0.002
Business sciences		1.014	0.061	0.015	0.013					0.031	0.020				0.016					-0.000
Commercial services		-0.004	0.901	-0.005	0.029					-0.004	-0.006				-0.005					-0.004
Engineering		-0.005	-0.005	0.974	-0.088					-0.005	-0.007				-0.006					-0.004
Health		0.004	0.004	0.005	0.854					-0.079	0.006				0.005					0.004
Humanities																				
IT																				
Information and journalism																				
Law																				
Life sciences		-0.013	-0.012	-0.014	-0.012					0.849	-0.019				-0.015					-0.047
Manufacturing		-0.001	-0.001	-0.001	-0.001					-0.001	0.960				-0.001					-0.001
Mathematics and statistics																				
Physics		-0.001	-0.001	-0.001	-0.001					-0.001	-0.001				-0.001					-0.001
Social and behavioral sciences		0.011	0.010	0.012	0.010					0.010	0.016				0.762					0.009
Social services																				
Teacher																				
Veterinary		-0.065	-0.061	-0.072	-0.062					-0.039	-0.094				-0.074					0.739

Figure 13: LATE first stage estimates with next-best Business sciences

Agriculture, forestry and fisheries	1.000	-0.001	-0.000	-0.000	-0.016	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
Architecture and construction	0.000	0.875	0.000	0.000	-0.007	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.001	0.000	0.000
Arts	0.000	-0.124	0.888	-0.004	0.004	0.004	0.000	0.000	-0.007	0.005	0.000			0.011	0.000	-0.000	0.000
Commercial services	-0.001	0.040	-0.012	0.885	-0.007	-0.001	0.004	-0.001	-0.015	-0.003	0.025			-0.001	0.062	-0.003	-0.020
Engineering	-0.003	-0.003	-0.003	0.001	0.915	-0.011	-0.003	0.016	-0.002	-0.002	-0.002			0.018	-0.113	-0.003	-0.002
Health	-0.001	-0.001	-0.001	0.002	0.005	0.851	-0.008	-0.001	-0.008	-0.001	0.050			-0.001	-0.001	0.001	-0.013
Humanities	-0.001	-0.001	0.005	-0.004	0.001	-0.013	0.805	-0.002	0.002	0.004	-0.001			-0.001	-0.001	-0.002	-0.001
IT	0.001	0.001	0.001	0.001	-0.013	0.001	0.001	0.964	-0.001	0.001	0.001			0.001	0.001	-0.000	0.001
Information and journalism	0.000	0.000	0.006	-0.002	0.002	0.000	-0.007	0.000	0.926	-0.002	0.000			0.000	-0.111	-0.003	0.000
Law	-0.003	-0.003	-0.003	-0.001	-0.011	-0.002	-0.005	-0.002	-0.009	0.921	0.023			-0.003	-0.002	-0.001	0.005
Life sciences	0.001	0.002	0.001	0.001	-0.007	0.001	-0.006	0.001	0.001	0.001	0.822			0.001	0.001	0.000	0.001
Manufacturing	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001			-0.001	-0.001	-0.001	-0.001
Mathematics and statistics	-0.001	-0.001	-0.001	-0.001	-0.009	-0.001	-0.001	-0.014	-0.001	-0.005	-0.001			0.904	-0.001	-0.000	-0.001
Physics	-0.000	-0.001	-0.000	-0.000	-0.000	-0.013	-0.000	-0.000	-0.000	-0.000	-0.000			-0.000	0.764	-0.002	-0.019
Social and behavioral sciences	-0.001	-0.002	0.004	-0.010	-0.010	-0.006	0.004	-0.001	-0.015	-0.010	-0.001			-0.018	-0.001	0.930	-0.001
Social services	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.004	0.000			0.000	0.000	-0.001	0.938
Teacher	-0.000	-0.000	-0.000	-0.000	-0.000	-0.012	-0.003	-0.000	-0.004	-0.004	-0.000			-0.000	-0.000	-0.000	-0.012
Veterinary	-0.001	-0.001	-0.001	-0.001	-0.001	0.003	-0.001	-0.001	-0.001	-0.001	-0.001			-0.017	-0.001	-0.000	-0.001
																	-0.001
																	0.908

Figure 14: LATE first stage estimates with next-best Commercial services

Agriculture, forestry and fisheries	0.785	0.044	0.001	0.000	0.004	-0.002	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	
Architecture and construction	-0.001	0.867	-0.002	-0.001	-0.017	-0.001	-0.001	-0.001	-0.001	-0.002	0.008	-0.001	-0.002	-0.047	-0.001	-0.001	-0.001	-0.001	
Arts	0.001	0.001	0.847	-0.002	-0.001	0.001	-0.033	0.001	0.001	0.001	0.010	0.001	0.001	0.001	-0.017	0.001	0.001	0.001	
Business sciences	0.009	-0.179	-0.064	0.918	-0.005	-0.002	0.152	0.010	0.021	0.004	0.005	0.011	0.019	0.010	-0.033	0.030	0.033	0.010	
Engineering	-0.008	-0.018	-0.012	-0.015	0.661	-0.009	-0.011	0.116	0.009	-0.012	-0.078	-0.232	-0.516	-0.035	-0.026	-0.008	-0.009	-0.009	
Health	-0.002	-0.204	-0.003	-0.005	-0.009	0.908	-0.002	-0.002	-0.002	-0.003	-0.020	0.057	-0.004	-0.002	-0.011	-0.107	-0.093	-0.002	
Humanities	-0.106	-0.014	-0.009	-0.011	-0.004	-0.007	0.812	-0.006	-0.033	-0.041	-0.020	-0.007	-0.012	-0.006	-0.015	-0.006	-0.007	-0.006	
IT	0.001	0.001	0.001	0.002	-0.007	0.001	0.001	0.876	-0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	
Information and journalism	-0.005	-0.012	-0.008	-0.010	-0.006	-0.006	-0.007	-0.006	0.893	-0.017	-0.005	-0.006	-0.011	-0.006	-0.014	-0.006	-0.006	-0.006	
Law	-0.002	-0.003	-0.002	-0.003	-0.007	-0.002	-0.002	-0.002	-0.002	0.897	-0.002	-0.002	-0.003	-0.002	-0.002	-0.002	-0.002	-0.002	
Life sciences	-0.105	-0.010	-0.007	-0.006	-0.023	-0.011	-0.006	-0.005	-0.005	-0.007	0.872	-0.005	-0.009	-0.005	-0.005	-0.005	-0.005	-0.005	
Manufacturing	-0.100	-0.001	-0.001	-0.001	-0.008	-0.000	-0.001	-0.000	-0.000	-0.001	-0.000	0.941	-0.001	-0.000	-0.000	-0.000	-0.000	-0.000	
Mathematics and statistics	0.002	0.047	0.002	0.005	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	1.003	0.002	0.002	0.002	0.002	0.002	
Physics	0.016	0.002	0.001	0.001	-0.007	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.002	0.918	0.001	0.001	0.001	0.001	
Social and behavioral sciences	-0.002	-0.004	-0.002	-0.008	-0.002	0.005	0.018	-0.002	-0.028	-0.035	0.008	-0.002	-0.003	0.017	0.953	-0.002	-0.002	-0.002	
Social services	-0.002	-0.005	-0.003	-0.001	-0.002	0.001	-0.003	-0.002	-0.002	-0.003	-0.002	-0.002	-0.004	-0.002	-0.011	0.958	0.002	-0.002	
Teacher	0.001	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.020	0.024	0.001	0.001	0.002	0.001	-0.008	0.001	0.978	0.001	
Veterinary	-0.001	-0.003	-0.002	-0.002	0.002	0.002	-0.002	-0.001	-0.001	-0.002	-0.001	-0.001	-0.003	-0.001	-0.001	-0.001	-0.001	-0.001	
																			0.665

Figure 15: LATE first stage estimates with next-best Engineering

Agriculture, forestry and fisheries	0.976	0.003	0.002	-0.005	0.001	0.001	0.002	0.002	0.001	0.002	0.002	0.017	0.002	0.005	0.002		0.002	-0.028
Architecture and construction	-0.003	0.807	-0.003	-0.003	-0.025	-0.002	-0.004	-0.012	-0.003	-0.003	-0.006	-0.038	-0.009	-0.008	-0.003		-0.003	-0.002
Arts	0.001	0.001	0.914	-0.006	0.009	-0.003	0.001	-0.004	0.000	0.000	-0.006	0.000	0.001	0.002	0.000		0.167	0.000
Business sciences	-0.004	0.018	-0.004	0.883	-0.040	-0.015	-0.005	-0.038	-0.003	-0.004	-0.014	-0.004	-0.002	-0.012	-0.049		-0.004	-0.032
Commercial services	0.002	0.003	0.002	-0.017	0.890	-0.001	0.002	-0.008	0.002	0.002	0.002	0.002	0.011	0.003	-0.021		0.002	0.001
Health	-0.007	-0.010	-0.007	-0.006	-0.020	0.829	-0.008	-0.015	-0.006	-0.007	0.004	-0.006	0.008	-0.004	-0.004		-0.007	0.002
Humanities	0.000	0.000	0.000	0.000	0.000	0.000	1.000	-0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000		0.000	0.000
IT	-0.003	-0.004	-0.003	-0.009	-0.002	-0.002	-0.004	0.858	-0.003	-0.003	-0.006	-0.003	0.004	-0.005	-0.003		-0.003	-0.002
Information and journalism	-0.000	-0.001	-0.044	-0.007	-0.023	0.001	-0.001	-0.000	0.666	-0.000	-0.000	-0.000	-0.001	-0.001	-0.012		-0.000	-0.000
Law	0.002	0.002	0.002	0.001	0.001	-0.003	0.002	0.007	0.001	0.763	-0.002	0.001	0.002	0.004	0.014		0.002	0.001
Life sciences	0.005	0.007	0.005	0.004	0.004	-0.003	0.006	0.009	0.004	0.004	0.936	0.020	0.006	0.013	0.004		0.005	-0.026
Manufacturing	-0.003	-0.004	-0.003	-0.002	-0.002	-0.001	-0.003	0.003	-0.002	-0.002	-0.002	0.921	-0.003	-0.003	-0.002		-0.003	-0.031
Mathematics and statistics	0.001	0.002	0.001	-0.005	0.001	0.000	0.002	0.003	0.001	0.001	0.001	0.001	0.855	-0.007	-0.010		0.001	0.001
Physics	-0.006	-0.008	-0.006	-0.005	-0.005	-0.004	-0.007	-0.019	-0.005	-0.005	-0.004	-0.005	-0.008	0.817	-0.005		-0.006	-0.004
Social and behavioral sciences	0.001	0.017	0.001	-0.037	0.001	0.003	0.001	-0.002	0.001	0.001	-0.001	0.016	0.009	0.006	0.869		0.001	0.001
Social services	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.003	0.000	0.000	0.000	0.000		0.000	0.000
Teacher	-0.001	-0.001	-0.001	-0.001	-0.001	-0.000	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001		0.832	-0.001
Veterinary	-0.175	-0.000	-0.000	-0.000	-0.000	-0.002	-0.000	-0.000	-0.000	-0.000	-0.004	-0.000	-0.000	-0.000	-0.000		-0.000	0.935

Figure 16: LATE first stage estimates with next-best Health

Agriculture, forestry and fisheries	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.004	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
Architecture and construction	1.003	0.002	0.002	0.001	-0.002	0.003	0.002	0.001	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.002	0.001	
Arts	0.001	0.917	0.000	0.000	-0.004	0.001	0.001	0.000	0.000	-0.001	0.001	0.000	0.001	0.000	0.000	0.001	0.006	
Business sciences	0.002	0.001	0.928	0.001	0.003	0.002	0.001	-0.199	-0.047	0.011	0.078	0.001	0.001	0.001	0.001	0.001	-0.006	
Commercial services	0.001	0.001	0.001	0.867	0.003	0.001	0.001	0.001	0.001	0.002	0.001	0.072	0.001	-0.027	0.251	0.001	-0.006	
Engineering	-0.021	-0.011	-0.015	-0.002	0.857	-0.020	0.129	-0.009	-0.059	-0.022	-0.014	-0.012	-0.016	-0.010	-0.010	-0.016	-0.006	
Humanities	-0.002	-0.001	-0.001	-0.001	-0.001	0.998	-0.001	-0.001	-0.049	-0.004	-0.001	-0.001	-0.002	-0.001	-0.001	-0.002	-0.001	
IT	0.007	0.004	0.003	0.003	0.003	0.006	0.719	0.003	0.004	0.005	0.004	0.004	0.005	0.003	0.003	0.005	0.003	
Information and journalism	0.001	0.000	0.000	0.000	0.000	0.001	0.000	1.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
Law	0.002	0.001	0.001	0.001	0.005	0.002	0.001	0.001	1.001	-0.007	0.001	0.001	0.001	0.001	0.001	0.001	-0.006	
Life sciences	-0.020	0.073	-0.010	-0.002	-0.016	-0.019	-0.013	-0.008	-0.011	0.879	-0.013	-0.011	-0.015	-0.009	-0.009	-0.015	-0.022	
Manufacturing	0.004	0.002	0.002	0.009	0.002	0.004	0.002	0.002	0.002	0.003	0.925	0.002	0.003	0.002	0.002	0.003	0.002	
Mathematics and statistics	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.857	-0.000	-0.000	-0.000	-0.000	-0.000	
Physics	0.002	0.001	0.001	0.001	-0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.955	0.001	0.001	0.001	0.001	
Social and behavioral sciences	0.006	0.003	0.003	0.003	-0.003	0.006	0.004	-0.198	0.003	0.007	0.004	0.075	0.004	0.960	0.003	0.004	-0.012	
Social services	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.501	0.001	0.001	
Teacher	0.002	0.001	0.001	0.001	0.001	0.002	0.002	0.001	0.001	0.003	0.002	0.001	0.002	0.001	0.001	1.002	-0.006	
Veterinary	-0.006	-0.003	-0.003	-0.003	-0.011	-0.006	-0.004	-0.003	-0.003	0.007	-0.004	-0.004	-0.005	-0.003	-0.003	-0.005	0.818	

Figure 18: LATE first stage estimates with next-best IT

Agriculture, forestry and fisheries	0.001	0.001	0.000	0.001	0.003	0.000		0.000		0.000		0.001	0.000	0.001			
Architecture and construction	1.003	0.003	-0.004	0.002	0.004	0.002		0.002		0.002		0.004	0.001	0.002			
Arts	0.004	0.684	0.004	0.003	0.001	0.002		0.003		0.003		0.005	0.002	0.003			
Business sciences	0.002	0.002	0.810	0.002	0.006	0.001		0.001		0.092		-0.164	0.001	-0.072			
Commercial services	0.002	0.002	-0.003	0.668	0.002	0.001		0.001		0.001		0.002	0.001	0.001			
Engineering	-1.055	-0.312	-0.035	-0.046	0.810	0.111		-0.038		-0.037		-0.073	-0.361	-0.048			
Health	0.003	0.003	0.002	0.003	0.003	0.716		0.002		0.002		0.004	0.002	0.003			
Humanities																	
Information and journalism	0.002	0.002	0.009	0.002	0.002	0.001		1.001		0.001		0.003	0.001	0.002			
Law																	
Life sciences	0.001	0.001	0.000	0.000	-0.004	0.000		0.000		0.819		0.001	0.000	0.000			
Manufacturing	0.001	0.001	0.000	0.000	0.002	0.000		0.000		0.000		0.001	0.000	0.001			
Mathematics and statistics	0.005	0.005	0.003	0.004	0.005	0.003		0.003		0.003		0.948	0.003	0.004			
Physics	0.003	0.003	0.002	0.002	0.003	0.002		0.002		0.002		0.004	1.001	0.003			
Social and behavioral sciences	0.007	0.007	-0.014	0.006	0.006	0.004		0.005		0.095		0.009	0.003	0.932			
Social services																	
Teacher	0.004	0.004	0.003	0.003	0.002	0.002		0.003		0.002		0.005	0.002	0.003			
Veterinary	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001		-0.001		-0.001		-0.001	-0.000	-0.001			

Figure 19: LATE first stage estimates with next-best Information and journalism

Agriculture, forestry and fisheries																			
Architecture and construction		0.003	0.003	0.003	0.003	0.003	0.004		0.003	0.003					0.003	0.003			
Arts		0.955	0.004	0.004	0.004	0.004	0.015		0.013	0.004					-0.011	0.004			
Business sciences		-0.033	0.908	0.002	0.002	0.002	-0.014		0.021	0.003					-0.008	-0.141			
Commercial services		0.013	0.007	0.891	0.003	0.003	0.014		0.012	0.003					0.012	0.003			
Engineering		0.001	0.001	0.015	0.934	0.001	0.002		-0.020	-0.249					0.001	0.001			
Health		0.011	0.018	0.009	0.009	0.978	0.014		0.019	0.011					0.010	0.009			
Humanities		-0.012	-0.004	-0.010	-0.010	-0.010	0.857		-0.007	-0.012					-0.002	0.018			
IT		-0.002	-0.002	-0.002	-0.002	-0.002	-0.003		-0.002	-0.002					-0.002	-0.002			
Law		-0.012	-0.007	-0.010	-0.010	-0.010	-0.005		0.894	-0.012					-0.002	-0.010			
Life sciences		0.000	0.000	0.000	0.000	0.000	0.000		0.000	0.750					0.000	0.000			
Manufacturing																			
Mathematics and statistics																			
Physics																			
Social and behavioral sciences		-0.006	-0.005	0.010	-0.005	-0.005	0.003		-0.005	-0.006					0.911	0.024			
Social services		0.003	0.003	0.002	0.002	0.002	0.003		0.002	0.003					0.002	0.945			
Teacher		-0.011	-0.010	-0.009	-0.009	-0.009	-0.014		-0.010	-0.011					-0.010	-0.009			
Veterinary																			

Figure 20: LATE first stage estimates with next-best Law

Agriculture, forestry and fisheries			-0.001	-0.001	-0.001	-0.001	-0.001	-0.001		-0.021	-0.001				-0.001	-0.001	-0.001	
Architecture and construction			0.062	0.002	0.003	0.003	0.002	0.004		0.002	0.003				0.002	0.002	0.002	
Arts			0.937	-0.004	-0.004	-0.005	-0.003	-0.006		-0.003	-0.004				-0.010	-0.004	-0.003	
Business sciences			0.005	0.858	0.005	0.131	0.003	0.035		0.013	0.104				-0.035	0.004	0.004	
Commercial services			0.002	0.002	1.002	0.003	0.001	0.003		-0.019	0.002				0.001	0.002	0.002	
Engineering			-0.002	0.001	-0.002	0.747	-0.001	-0.003		-0.002	0.098				0.002	-0.002	-0.002	
Health			0.002	0.002	0.002	0.002	0.875	0.003		0.001	0.002				0.004	0.002	0.001	
Humanities			-0.009	-0.010	-0.009	-0.010	-0.006	0.787		0.003	-0.008				0.006	-0.008	-0.007	
IT			0.001	-0.007	0.001	0.002	0.001	0.002		0.001	-0.249				0.001	0.001	0.001	
Information and journalism			-0.006	-0.021	-0.006	-0.007	-0.004	-0.008		0.816	-0.255				-0.001	-0.076	-0.004	
Life sciences			0.001	0.001	0.001	0.001	0.001	0.002		0.001	0.801				0.004	0.001	0.001	
Manufacturing																		
Mathematics and statistics																		
Physics																		
Social and behavioral sciences			-0.009	-0.017	-0.119	-0.010	-0.005	0.002		0.003	-0.008				0.865	-0.007	-0.006	
Social services			0.003	0.002	0.003	0.003	0.002	0.004		0.002	0.003				-0.012	1.003	0.048	
Teacher			-0.008	-0.009	-0.008	-0.009	-0.005	-0.011		-0.026	-0.007				-0.013	-0.007	0.837	
Veterinary			-0.001	-0.001	-0.001	-0.001	-0.001	-0.001		-0.001	-0.001				-0.001	-0.001	-0.001	

Figure 21: LATE first stage estimates with next-best Life sciences

Agriculture, forestry and fisheries	0.930	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
Architecture and construction	0.001	0.001	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
Arts	-0.000	0.916	-0.000	-0.000	-0.011	-0.004	-0.000	-0.001	-0.000	-0.001	-0.001	-0.001	-0.001	0.011			-0.000
Business sciences	0.002	0.003	0.895	0.017	0.003	0.001	0.001	0.003	0.002	0.002	0.055	0.009	-0.021				0.002
Commercial services	-0.000	-0.000	-0.000	0.932	-0.002	0.001	-0.334	-0.000	-0.000	-0.000	-0.000	0.006	-0.000				-0.000
Engineering	0.011	0.074	-0.007	-0.008	0.875	-0.012	-0.005	-0.009	-0.006	-0.008	-0.008	-0.032	0.004				-0.017
Health	0.014	-0.004	0.026	-0.025	-0.007	0.803	-0.002	-0.004	-0.080	-0.003	-0.004	0.022	-0.048				-0.015
Humanities	-0.000	-0.000	-0.000	-0.000	-0.000	0.001	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000				-0.000
IT	0.001	0.001	0.001	0.001	-0.002	0.001	1.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
Information and journalism	-0.002	-0.003	-0.002	-0.003	-0.003	-0.001	-0.002	0.997	-0.079	-0.003	-0.003	0.010	-0.002				0.005
Law	-0.002	-0.003	-0.002	-0.002	-0.001	-0.004	-0.002	-0.003	0.998	-0.003	-0.003	-0.003	-0.002				-0.002
Manufacturing	0.001	0.001	0.029	-0.036	-0.002	0.002	0.000	0.001	0.001	0.961	0.001	0.001	0.001				0.000
Mathematics and statistics	-0.001	-0.001	-0.001	-0.001	0.001	-0.003	-0.000	-0.001	-0.001	-0.001	0.947	-0.001	-0.001				-0.001
Physics	0.003	0.004	0.003	0.003	0.000	0.002	0.002	0.004	0.002	0.003	0.003	0.913	0.003				0.002
Social and behavioral sciences	0.001	0.002	-0.099	0.001	0.008	0.004	0.001	0.002	0.001	0.002	0.002	0.002	0.854				0.001
Social services																	
Teacher																	
Veterinary	0.030	-0.006	-0.005	-0.005	-0.020	-0.014	-0.003	-0.006	-0.004	-0.005	-0.005	-0.029	-0.005				0.807

Figure 22: LATE first stage estimates with next-best Manufacturing

Agriculture, forestry and fisheries	0.786	0.001	0.002	0.001	0.022	0.005	0.001		0.001		0.002	0.001	0.001	0.001	0.001			0.001
Architecture and construction	0.004	0.929	0.007	0.004	0.005	-0.001	0.006		0.004		0.009	0.006	0.006	0.005	0.005			0.004
Arts	-0.000	-0.000	1.000	-0.000	-0.000	-0.020	0.014		-0.000		-0.000	-0.000	-0.000	0.029	-0.000			-0.000
Business sciences	-0.005	-0.007	-0.007	0.929	-0.055	-0.016	-0.006		-0.004		-0.010	-0.006	-0.007	-0.006	-0.005			-0.004
Commercial services	0.000	0.001	0.001	-0.066	0.844	0.002	0.000		0.000		0.001	-0.060	0.001	0.000	0.000			0.000
Engineering	-0.005	-0.302	-0.008	-0.005	-0.084	0.850	-0.007		-0.004		-0.010	-0.018	-0.007	0.023	-0.005			-0.004
Health	-0.000	-0.000	-0.000	-0.000	0.042	-0.004	0.955		-0.000		-0.000	0.057	-0.000	-0.000	-0.000			-0.000
Humanities																		
IT	0.001	0.001	0.001	0.001	0.001	-0.005	0.001		0.501		0.001	0.001	0.001	-0.199	0.001			0.000
Information and journalism																		
Law	0.002	0.003	0.003	0.002	0.002	-0.004	0.002		0.002		1.004	0.002	0.003	0.002	0.002			0.002
Life sciences	-0.004	-0.005	-0.005	-0.003	-0.004	-0.008	-0.005		-0.003		-0.007	0.880	-0.005	-0.004	-0.004			-0.003
Mathematics and statistics	-0.004	-0.005	-0.006	-0.003	-0.004	-0.002	-0.005		-0.003		-0.007	-0.005	0.995	-0.004	-0.004			-0.003
Physics	-0.006	-0.008	-0.008	-0.005	-0.006	-0.012	-0.007		-0.005		-0.011	-0.007	-0.008	0.905	-0.006			-0.004
Social and behavioral sciences	0.009	-0.009	-0.010	-0.006	0.014	-0.012	-0.039		-0.006		-0.014	0.010	-0.009	-0.008	0.993			-0.005
Social services	-0.006	0.030	-0.009	-0.005	-0.006	-0.007	-0.008		-0.005		-0.012	-0.007	-0.008	-0.007	-0.006			-0.005
Teacher	0.002	0.002	0.003	0.002	0.002	0.002	0.002		0.001		0.004	0.002	0.002	0.002	0.002			0.001
Veterinary	0.031	-0.002	-0.002	-0.001	-0.001	0.001	-0.032		-0.001		-0.002	0.018	-0.001	-0.001	-0.001			0.999

Figure 24: LATE first stage estimates with next-best Physics

Agriculture, forestry and fisheries	0.999	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.000			0.038
Architecture and construction	-0.001	0.860	-0.001	-0.001	-0.001	0.003	-0.001	-0.001	-0.001	-0.001	-0.001	-0.005	0.029	-0.001	-0.001			-0.001
Arts	-0.002	-0.001	0.799	0.017	-0.001	0.001	-0.012	-0.001	-0.001	-0.001	-0.001	-0.006	-0.001	-0.002	-0.015			-0.001
Business sciences	0.005	0.058	0.002	0.925	-0.038	-0.006	0.008	0.003	-0.122	0.003	0.003	-0.017	0.002	0.005	-0.025			0.002
Commercial services	0.002	0.029	0.001	-0.027	0.919	-0.014	0.001	0.001	-0.124	0.001	0.001	0.001	0.001	0.022	0.013			0.001
Engineering	-0.025	0.013	-0.013	-0.038	-0.015	0.781	-0.040	-0.016	0.315	-0.017	-0.019	-0.037	-0.148	-0.124	-0.014			-0.010
Health	0.003	0.002	-0.198	-0.022	0.002	-0.003	0.873	0.002	0.002	0.002	-0.109	-0.008	-0.165	0.003	0.014			0.001
Humanities	0.000	0.000	-0.200	0.000	-0.027	0.002	0.000	0.900	0.000	-0.055	0.000	-0.004	0.000	0.021	0.011			0.000
IT	0.000	0.000	0.000	-0.023	0.000	-0.011	0.000	-0.050	0.667	0.000	0.000	-0.007	0.000	0.000	0.025			0.000
Information and journalism	0.003	0.002	0.001	-0.022	0.002	0.005	0.001	0.002	0.002	1.002	0.002	0.004	0.001	0.003	-0.012			0.001
Law	0.001	0.001	0.001	-0.023	0.001	0.001	0.001	0.001	0.001	-0.055	0.723	-0.006	0.001	0.001	-0.013			0.000
Life sciences	-0.007	-0.004	-0.003	-0.004	-0.058	-0.024	-0.052	-0.054	-0.005	-0.004	-0.005	0.827	-0.003	-0.006	-0.004			-0.003
Manufacturing	-0.002	-0.001	-0.001	-0.001	-0.028	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	0.774	-0.002	-0.001			-0.001
Mathematics and statistics	0.002	0.001	0.001	0.001	0.001	0.003	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.961	0.001			0.001
Social and behavioral sciences	0.003	0.029	0.001	-0.022	0.029	-0.012	-0.026	0.002	0.002	0.002	0.085	0.001	0.001	0.003	0.860			0.001
Social services	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	-0.055	0.001	0.001	0.001	0.001	0.013			0.000
Teacher	-0.002	-0.001	-0.001	-0.001	-0.001	-0.002	-0.001	-0.001	-0.002	-0.002	-0.002	-0.001	-0.001	-0.002	-0.001			-0.001
Veterinary	0.003	0.002	0.001	0.002	-0.025	-0.004	-0.015	0.002	0.002	0.002	0.002	0.001	0.001	0.003	0.002			0.363

Figure 26: LATE first stage estimates with next-best Teacher

Agriculture, forestry and fisheries																				
Architecture and construction																				
Arts		0.824	0.000	0.000		0.000	0.000		0.000	0.029							0.000	0.000		
Business sciences		0.032	0.850	-0.041		0.007	0.023		-0.017	0.007							0.008	0.013		
Commercial services		-0.003	-0.012	0.953		-0.003	-0.003		-0.003	-0.003							-0.003	-0.003		
Engineering		0.000	0.000	0.000		0.000	0.000		0.000	0.000							0.000	0.000		
Health		-0.002	-0.021	-0.015		0.913	-0.003		-0.026	-0.003							-0.023	0.004		
Humanities		0.002	0.002	0.002		0.002	0.889		0.003	0.003							0.002	0.002		
IT		-0.002	-0.002	-0.002		-0.002	-0.003		-0.003	-0.003							-0.002	-0.002		
Information and journalism		-0.003	-0.003	-0.003		-0.003	-0.004		0.877	-0.003							-0.002	-0.003		
Law		-0.000	0.009	-0.000		-0.000	-0.001		-0.000	0.891							0.004	-0.011		
Life sciences		-0.021	-0.021	-0.018		-0.021	-0.025		-0.023	-0.023							-0.019	-0.019		
Manufacturing																				
Mathematics and statistics		-0.005	-0.005	-0.005		-0.005	-0.006		-0.006	-0.006							-0.005	-0.005		
Physics		-0.004	-0.004	-0.004		-0.004	-0.005		-0.005	-0.005							-0.004	-0.004		
Social and behavioral sciences		-0.077	-0.042	-0.026		-0.015	-0.002		-0.040	-0.042							0.885	0.014		
Social services		0.007	0.007	0.006		-0.041	0.008		0.008	0.008							0.008	0.929		
Veterinary																				

Figure 27: LATE first stage estimates with next-best Veterinary

Agriculture, forestry and fisheries	0.966					0.003	0.003						0.005			0.504	0.005		
Architecture and construction																			
Arts																			
Business sciences	-0.021					-0.067	-0.017						-0.020			-0.016	0.103		
Commercial services																			
Engineering	0.024					0.833	-0.002						0.022			0.017	0.024		
Health	-0.047					-0.146	0.551						-0.026			-0.035	-0.049		
Humanities																			
IT																			
Information and journalism	0.001					0.001	0.003						0.001			0.001	0.001		
Law																			
Life sciences	0.031					-0.057	0.001						0.975			-0.005	-0.007		
Manufacturing																			
Mathematics and statistics	-0.010					0.036	-0.003						-0.010			-0.008	-0.011		
Physics	-0.008					-0.005	-0.004						-0.007			0.494	-0.008		
Social and behavioral sciences	0.001					-0.052	0.008						0.001			0.000	0.876		
Social services																			
Teacher																			

Figure 28: Second stage LATE estimates

Agriculture, forestry and fisheries	X			5.018	-4.034	1.019	-1.209					-1.613	-1.351			6.339			-1.959
Architecture and construction	4.589	X	3.915	2.654	14.557	8.574	18.743	-0.491	13.399	15.335	14.754	6.154	5.925	6.195	1.928	14.898		-3.790	
Arts		-6.449	X	-2.684	2.770	-2.232	-6.921	-4.893	-27.257	2.415	-2.274	-23.320			-9.844	-0.430	7.098	6.246	
Business sciences	-7.800	-7.804	1.076	X	1.565	-5.393	1.367	-1.580	0.630	0.548	-2.478	3.187		-2.764	2.775	-2.056	-0.088	-0.249	-11.918
Commercial services	-2.319	-8.266	-7.744	-3.962	X	-4.935	-1.963	1.639	-4.102	1.396	-6.445	0.034	-3.524	-9.187	1.512	1.379	11.138	0.561	-18.105
Engineering	1.480	-10.218	-2.862	1.253	4.430	X	1.146	-16.455	2.169	34.301	10.033	-0.592	5.953	-5.309	-7.706	1.080		-5.609	-4.373
Health		-5.125	4.470	-2.120	-6.197	-3.565	X	-5.628	-12.268	4.695	-0.898	-1.529	3.855	7.975	-7.030	-0.835	-19.486	-3.269	2.379
Humanities		-8.134	0.516	-0.742	3.714	-13.135	3.190	X		3.350	-5.753	-2.294		10.032	7.423	1.455	2.553	-3.066	
IT		39.086	-14.161	3.100	9.047	-0.772	-16.152		X	5.877		16.740		-0.133	-13.819	0.179			
Information and journalism			-5.262	-3.098	-0.677	-16.334	0.927	1.630		X	-4.570	-22.148				-4.257	4.127		
Law			5.828	3.961	11.013	23.330	6.602	15.208		8.995	X	-9.504				7.098	12.993	9.043	
Life sciences	5.640		17.040	-3.171	3.518	-1.845	0.249		-13.544	-5.400	0.131	X	-5.434	-0.868	-4.990	4.519			-1.752
Manufacturing	-8.698	15.489	-9.160	2.563	6.366	-3.894	-2.678		10.015		-21.596	0.832	X	-36.500	-14.436	-7.643			6.686
Mathematics and statistics				7.109	-5.457	-1.735	5.171		-22.400			-4.910	-3.732	X	-19.373	10.840		-10.791	
Physics	-17.229	10.480	-0.516	-8.292	6.416	1.780	6.299	3.584	-6.874	9.515	-11.779	3.861	-6.440	-1.675	X	3.428			33.060
Social and behavioral sciences		-9.670	-4.789	0.844	3.312	2.192	0.401	-1.845	4.080	1.016	-3.953	-2.965		-7.188	-15.233	X	0.241	2.606	-0.868
Social services			-7.936	-5.983	2.289		-4.167	-5.991		-1.989	-9.770				0.829	-5.777	X	-1.589	
Teacher			-4.110	-5.743	-2.556		-0.368	-4.246		0.579	-5.336					-1.648	0.636	X	
Veterinary	-8.991					-1.286	-2.056								19.969	-6.540			X