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Policy Reforms and the Amount of Checks \& Balances

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## Economics Working Paper Series

# Policy Reforms and the <br> Amount of Checks \& Balances* 

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#### Abstract

We examine how democracies choose their amount of checks and balances (C\&B). For this purpose, we consider a simple model of political competition with costly policy reforms. The cost of a marginal reform is determined endogenously at the constitutional phase - i.e. before policies are chosen-through the choice of (the amount of) C\&B. We characterize the set of stable C\&B for different constitutional rules which vary depending on (i) who has the power to propose changes to $\mathrm{C} \& \mathrm{~B}$ and (ii) on the qualified majority used for approving such changes. Our main results show that stable C\&B always exist, are never zero, lead to gridlock, and are higher if the proposal-maker is the party in government. We also find that higher majority requirements for constitutional changes and more polarized societies are conducive to larger sets of stable C\&B.


Keywords: elections; democracy; political polarization; reform costs; constitutions; checks and balances

JEL Classification: C72, D72, D78, H4

[^1]
## 1 Introduction

Checks and balances (in short, C\&B) are central to democracy and determine how difficult or costly policy reforms are. In this paper, we take a reduced-form approach and envision reform costs as a summary statistics for the many institutional features of the political system and various regulations that ultimately determine $C \& B$. For instance, reform costs arise when the legislative procedure to pass a law is long and legislators in favor and against the law incur opportunity costs that lower the provision of public goods in their constituencies, which may additionally endanger their reelection chances. Reform costs also arise when the vote threshold in parliament necessary to reform the status quo policy is high, and thus may require the vote of some parliament members who want to obtain pork barrel policies. ${ }^{1}$ On other occasions, the government must maintain a fraction of past spending programs (see Piguillem and Riboni, 2021), which also introduce inefficiencies in the reform process. No matter their source, reform costs may be effectively incurred if policy reforms take place, or they may simply influence policy by preventing reforms altogether, thereby leading to gridlock. In either case, reform costs reduce the ability of incumbents to dictate policy.

Our goal is to examine which amounts or levels of $C \mathscr{B} B$ are chosen in a democracy. For this, we build on and extend the model in Gersbach et al. (2019). They consider a two-party, twoperiod model of political competition in which governments choose one-dimensional policies under uncertainty about who will hold power in the future and in which reforming the policy in place is costly. We follow suit and assume that changing the first-period policy in the second period creates costs for the politicians and for the citizens that increase (linearly) with the extent of the policy change; these are called reform costs. By contrast, choosing a policy in the first period does not generate any costs. Examples of policies displaying these properties are introducing mandatory healthcare, joining an important international treaty, or building nuclear energy capacity for the first time. We can therefore conceive of the unidimensional policy dimension as capturing the initiation of the provision of some good (in the first period), the supplied amount of which can later be changed (in the second period).

Our main innovation is to allow for the marginal cost of reform - which is parametrized by a nonnegative number-to be determined endogenously by the political parties and/or by the citizens

[^2]through the choice of C\&B in the constitutional phase, i.e., before actual policies are chosen (see more details below). Since one party is in office, parties are not symmetric in the constitutional phase. This is a reflection of the premise that constitutional changes cannot usually happen de novo but have to happen - if at all-while a government already exists which controls policy. This lack of symmetry between parties together with polarization in the parties' preferences opens up the possibility for the amount of C\&B-viz. the marginal reform cost-to be subject to strategic considerations. If no new (amount of) C\&B is approved in the constitutional stage, the exogenously given status quo C\&B is maintained.

In our analysis, we look for subgame perfect equilibria of the whole constitutional game with no stage-dominated strategies to rule out implausible behavior; we just call them equilibria. We also introduce the notion of stable (amounts of) C\&B. C\&B are stable if they are not changed during the constitutional phase. Then we study the set of stable C\&B for different rules governing the constitutional phase. The latter consists of a proposal-making stage in which C\&B are proposed, followed by a vote where the proposed C\&B are pitted against the status quo C\&B. Each constitutional rule is then characterized by (i) which agent has the right to make a proposal for new $\mathrm{C} \& \mathrm{~B}$, and (ii) the share of votes needed for the proposed $\mathrm{C} \& \mathrm{~B}$ to replace the status quo $\mathrm{C} \& \mathrm{~B} .{ }^{2}$

In the constitutional phase, parties can make a proposal and vote on it, but so can a representative voter (also called the median voter). Adding this third agent allows us to capture the possibility that beyond the government and the legislative chambers, the citizenry can also participate in each stage of the constitutional phase, say via a popular initiative or referendum. In Switzerland, popular initiatives enable the citizens to propose changes to the federal constitution. In California, constitutional amendments must be submitted to the voters as mandatory referendum. The median voter could also represent a constitutional court that has a say in suggesting or interpreting constitutional rules, or it could capture the possibility that constitutional changes could be either proposed or upset at higher (national or international) levels.

We obtain several results. First, no matter the constitutional rule, the set of stable C\&B is nonempty and only contains strictly positive C\&B. Moreover, C\&B are always chosen so that they lead to gridlock, in the sense that no policy reform occurs (i.e., the policy in both periods is the same, no matter how far apart the policy in the first period is from the peak of the party in office in the second period). The persistence (or stability) of C\&B over time can be observed empirically in

[^3]

Figure 1: Evolution of C\&B over 45 years in Canada (left) and the UK (right).


Figure 2: Evolution of C\&B over 45 years in Spain (left) and France (right).
some countries. For instance, the case of Canada and the UK is shown in Figure 1, which depicts C\&B for the two countries over a time span of 45 years. ${ }^{3}$

Second, we also find that depending on the constitutional rules, many, if not most C\&B are not stable. If the model's parameters were to change for exogenous reasons, our theory would then predict changes in C\&B in most cases. For a given constitutional rule, the set of stable C\&B depends generically on features of the political system such as the degree of party rotation and the lack of symmetry of the median voter preferences with respect to the parties preferences, which can be subject to shocks. One interpretation of this insight is that democratic societies experimenting rapid changes in different political dimensions endure also volatile $\mathrm{C} \& \mathrm{~B}$, and hence, volatile political institutions. From an empirical perspective, high volatility of C\&B can be observed in another group of countries, as shown in Figure 2 for Spain and France.

Third, our analysis allows us to obtain comparative statics on the set of stable C\&B in terms of the constitutional rules by varying (i) the share of votes needed to approve a new C\&B (keeping

[^4]the proposal-maker fixed), and by varying (ii) the proposal-maker's identity (keeping the share of votes needed for approval of a new C\&B fixed). On the one hand, an increase in the share of votes needed for approval naturally leads to a larger set of stable C\&B, no matter the proposal-maker. However, more stringent vote share requirements do not necessarily lead to larger C\&B, as the opposite can occur. On the other hand, it matters who can propose constitutional changes. For instance, the incumbent party proposes larger C\&B-i.e., larger reform costs - than the challenger party and the median voter. The reason is that the incumbent party wants to make it costly for the challenger party to change the policies the former party will enact in the first period. This captures the situations where the parties forming the government have the right to propose constitutional changes, which is common in democracies. These observations may e.g. explain the preservation of the Filibuster rule in the US senate. ${ }^{4}$ Even if a party could abolish such a rule at some point in time, it may not do so as it would fear that its new policies would be undone once the other party obtains a simple majority in the senate.

The fact that different constitutional rules and different characteristics of the political system lead to different sets of stable C\&B may help to rationalize the differences in the amount of C\&B observed across democracies. For instance, our underlying model of political competition features a positive relation between party polarization and reform costs (i.e., amounts of C\&B). Such a relationship is consistent with empirical observations, as shown in Figure 3, which depicts the evolution of C\&B and party polarization over 40 years in 38 democratic countries chosen based on available data. ${ }^{5}$ Two observations are in order. First, the relation between C\&B and party polarization is positive throughout this time span. Second, one observes that C\&B and party polarization increase over time for many countries. Since our model also predicts a positive relationship between C\&B and policy polarization, we posit the existence of a channel through which higher party polarization may lead to higher policy polarization, namely, via larger amounts of C\&B.

The paper is organized as follows: In Section 2 we review the papers that are most closely connected to our work. In Section 3 we describe the model. In Section 4 we first analyze the political game and, second, different constitutional rules. Section 5 compares different constitutional rules. Section 6 concludes. The proofs are in the appendix.

[^5]

Figure 3: Comparison of 38 countries regarding checks and party polarization over 40 years. The list of all countries is in the Appendix.

## 2 Relation to the Literature

Our paper is part of a growing literature devoted to analyzing the effects of reform costs-also called costs of change - in elections (see Glazer et al., 1998; Gersbach and Tejada, 2018; Gersbach et al., 2019, 2020a,b; Eraslan and Piazza, 2020; Dziuda and Loeper, 2021). We contribute to this literature by being the first to endogenize such costs, which we interpret as a reduced form of (executive) C\&B, i.e., of all the institutions, laws, and regulations that reduce the incumbent's ability to dictate policy. By considering different constitutional rules to determine $\mathrm{C} \& \mathrm{~B}$, we can use our simple stylized framework to provide new insights on the design of democracy.

We are not the first to provide a theoretical account for how political institutions arise endogenously and later may survive over time. Barberà and Jackson (2004) examine the self-stability of majority rules and, similarly to us, obtain that such rules (or constitutions) always exist but that majority rules may not survive generically. Messner and Polborn (2004) characterize voting rules that arise endogenously. Our approach is novel in that we envision the amount of $\mathrm{C} \& \mathrm{~B}$ as the total costs associated with policy change and focus on its stability over time. This allows us to obtain new insights on how democratic societies choose their amount of C\&B and on how this is influenced by political characteristics (such as party polarization and party rotation) and by constitutional
features (such as the proposal-making rule and the majority rule).
A large literature investigates the effect of political institutions on policy-making and other aspects of elections, both from a theoretical and empirical viewpoint (see e.g. Alesina and Rosenthal, 1996; Persson and Tabellini, 2002; Tsebelis, 2002; Persson and Tabellini, 2005; De Sinopoli and Iannantuoni, 2007; Stephenson and Nzelibe, 2010; Iaryczower and Mattozzi, 2013; Matakos et al., 2016, among many others). Our main contribution to this literature is to emphasize the offequilibrium role of $\mathrm{C} \& \mathrm{~B}$ in shaping policies and how the choices of $\mathrm{C} \& \mathrm{~B}$ made in the constitutional phase lead to gridlock (i.e., to no reform) in the political stage regardless of how large or small party polarization is. In our model, reforms are neither intrinsically good nor bad, and our results are in keeping with the idea that enacting a new fundamental policy (e.g. joining an international organization like the EU) will often not be undone since the costs associated with such reforms will be made untenable for society.

Another strand of literature evaluates policy outcomes under different electoral systems (see e.g. Morelli, 2004; Bouton et al., 2018). We also consider different voting rules, but the novelty of our analysis is that we focus on how such constitutional rules influence policy through the choice of C\&B. Our analysis identifies a link between party extremism and policy extremism, which is mediated by (the amount of) C\&B, and more generally by the constitutional design. This is in line with Bordignon et al. (2016), who show that certain voting rules can moderate party extremism. Our model assumes only one policy (or project) dimension for which one party is initially the incumbent, but our insights extend to the case where multiple project dimensions are considered in different points in time and different parties are responsible for the initial decision. It suffices to assume that parties discount such future situations sufficiently enough.

Finally, C\&B are the subject of substantial empirical and theoretical work, the focus of which are e.g. veto players (see e.g. Tsebelis, 1999), bicameral systems (Riker, 1992; Diermeier and Myerson, 1999; Tsebelis and Money, 1997), and/or separating the authority over different policy dimensions (see e.g. Ashworth and Bueno de Mesquita, 2017; Besley and Coate, 2003; Nakaguma, 2015; Persson et al., 1997). Some papers have examined the link between C\&B and policy reforms. Aghion et al. (2004) investigate how C\&B in the form of the share of votes needed to block the incumbent's policy affect the implementation of efficient reforms (see also Acemoglu et al., 2013; Forteza and Pereyra, 2019; Forteza et al., 2019; Alesina and Rosenthal, 2000). In a recent paper, Gratton and Morelli (2021) study how C\&B should be set to reduce type-I errors, at the expense of type-II errors, in policy decision making. From an empirical perspective, Cox and Weingast
(2018) show the importance of political C\&B in preventing large economic declines. To the best of our knowledge, we are the first to model (executive) C\&B as reform costs. This reduced-form approach to $\mathrm{C} \& \mathrm{~B}$ has the advantage of subsuming in one (continuous) parameter the many features, institutional and political alike, that restrict the ability of incumbents to choose policy. Our model then allows us to obtain new insights on how stable amounts of $C \mathcal{B} B$ emerge endogenously for given constitutional rules. To quote Gratton and Morelli (2021), the amount of C\&B is "...at the center of the debate over the merits of a constitution." Our contribution can be best recognized through this premise.

## 3 Model

We consider a dynamic political game consisting of two periods $(t=1,2)$ in which one of two parties, $L$ and $R$, holds power and determines the policy. Later we augment this game by adding an initial constitutional stage, which is our main innovation. The set of policy choices is $[0,1]$, where 0 corresponds to the leftmost policy and 1 to the rightmost policy. Without loss of generality, we assume that party $R$ holds power in period $t=1$. As we will see below, this implies that both parties are not symmetric from a constitutional perspective. Then, in period $t=2$, party $R$ remains in power with probability $p \in[0,1]$. Hence, power shifts across periods with probability $1-p$. Besides the two parties there is also a median voter, denoted by $M$, whose role is visible only in the constitutional stage (we do not model elections explicitly).

In any period, agents have standard quadratic utilities over policies in $[0,1]$, which are characterized by the agents' peak. Moreover, reforming the policy is costly for all agents. Such reform costs accrue only in period $t=2$, as there is no status quo policy in place at the beginning of period $t=1$. Specifically, given a policy $i_{1} \in[0,1]$ chosen in period $t=1$, we assume that the policy choice $i_{2} \in[0,1]$ made in period $t=2$ imposes utility losses on the two parties and on the median voter that amount to $c_{1}$ per unit of reform, with $c_{1} \geq 0$. Therefore, if policy $i_{t} \in[0,1]$ is chosen in period $t \in\{1,2\}$, agent $K \in\{M, L, R\}$ with peak $\mu_{K}$ derives the following utility in this period:

$$
u_{K}^{t}\left(i_{t-1}, i_{t}\right):=-\left(i_{t}-\mu_{K}\right)^{2}-c_{1} \cdot\left|i_{t-1}-i_{t}\right|,
$$

where we write $i_{0}=i_{1}$ for the sake of notation. Hence, unless $c_{1}=0$, the costs of policy reform across the two periods increase linearly with the absolute difference between the policies adopted in both periods. Assuming that costs of change are linear is a first-order approximation to the general case where costs associated with policy changes increase more generally in the extent of
the policy shift. As is standard, the agents' peaks satisfy the following condition:

$$
\begin{equation*}
0 \leq \mu_{L}<\mu_{M}<\mu_{R} \leq 1 . \tag{1}
\end{equation*}
$$

That is, the median voter has a moderate peak compared to each of the two parties. ${ }^{6}$ Henceforth, we let $\Pi=\mu_{R}-\mu_{L}$ denote the degree of party polarization.

The game defined above, which we denote by $\mathcal{G}\left(c_{1}\right)$, has been studied in Gersbach et al. (2019). In the following we extend this game by considering a constitutional phase that takes place prior to period $t=1$, say in period $t=0$. In this phase, the society chooses the reform $\operatorname{costs} c_{1}$ that will be in place between the first and the second period, given the status quo reform $\operatorname{costs} c_{0} \geq 0$. The reform costs that are in place are called (the amount of) $C \mathcal{B} B$.

To sum up, the timeline of the constitutional game, which we generically denote by $\mathcal{G}^{+}\left(c_{0}\right)$, is the following:
$(t=0)$ The society chooses the value of $c_{1}$ according to some procedure (to be specified later), given the value of $c_{0}$.
$(t=1)$ Party $R$ chooses policy $i_{R 1} \in[0,1]$.
$(t=2)$ Party $R$ wins the election in $t=2$ with probability $p$, where $0<p<1$. Otherwise, Party $L$ wins the election. Party $K \in\{L, R\}$ who wins the election chooses $i_{K 2} \in[0,1]$.

We consider different variants for the constitutional stage that takes place in period $t=0$. These variants depend on who has proposal-making power and on the majority rule used. For each variant of the constitutional phase, we study the subgame perfect equilibria (or just equilibria) of the resulting dynamic game. As is standard, we impose the refinement that no agent uses stage weakly dominated strategies, so that in any voting stage all agents vote as if they were pivotal. ${ }^{7}$ As a non-essential tie-breaking rule we also assume that in case of indifference agents vote in favor of the proposed C\&B instead of the status quo C\&B.

Our main focus are the C\&B that the society does not change in the constitutional phase and thus will persist. This leads to the following definition:

[^6]
## Definition 1

For a given variant of the game $\mathcal{G}^{+}\left(c_{0}\right)$, the amount of $C \& B c_{0}$ is stable if $c_{1}=c_{0}$ in any equilibrium of the corresponding constitutional game.

That is, for a given variant of the constitutional phase, C\&B are stable if society does not change them and therefore remain in place. Our goal in the following section is to find the set of stable C\&B for several variants of the constitutional phase. This allows useful comparative statics across variants. We generically denote such a set by $\mathcal{S C}$.

## 4 Analysis

Our analysis is divided into two parts. First, we focus on (sub)game $\mathcal{G}\left(c_{1}\right)$. Second, we focus on the different variants of game $\mathcal{G}^{+}\left(c_{0}\right)$.

### 4.1 Analysis of the political game

The following result characterizes the policies chosen on the equilibrium path in game $\mathcal{G}\left(c_{1}\right)$ :

## Proposition 1 (Gersbach et al. (2019))

In the unique equilibrium of game $\mathcal{G}\left(c_{1}\right)$ :
(i) Party $R$ 's policy choice in period $t=1$ is

$$
i_{R 1}^{*}=i_{R 1}^{*}\left(c_{1}\right):= \begin{cases}\mu_{R}-\frac{c_{1}}{2} \cdot \frac{1-p}{1+p} & \text { if } c_{1} \leq(1+p) \Pi \\ \mu_{L}+\frac{c_{1}}{2} & \text { if }(1+p) \Pi<c_{1}<2 \Pi \\ \mu_{R} & \text { if } 2 \Pi \leq c_{1}\end{cases}
$$

(ii) Let $K \in\{L, R\}$ be the party that holds power in period $t=2$. Party $K$ 's best response to the policy $i_{1} \in[0,1]$ chosen in period $t=1$ is

$$
i_{K 2}^{*}=i_{K 2}^{*}\left(i_{1}, c_{1}\right):=\min \left\{\max \left\{\mu_{K}-\frac{c_{1}}{2}, i_{1}\right\}, \mu_{K}+\frac{c_{1}}{2}\right\} .
$$

According to Proposition 1, party $R$ never reforms policy if it keeps power. By contrast, if there is turnover, party $L$ chooses a policy that is (weakly) closer to its own peak than to policy chosen in the previous period by party $R$. Moreover, the extent of the policy reform carried out by party $L$, $\left|i_{L 2}^{*}-i_{R 1}^{*}\right|$, decreases with $c_{1}$, until the policies chosen in period $t=2$ by both parties are the same


Figure 4: The policy choices in periods $t=1$ and $t=2$.
if $c_{1} \geq(1+p) \Pi$. This is shown graphically in Figure 4, which replicates the result of Gersbach et al. (2019).

The next corollary follows immediately from Proposition 1 and states the relationship between $\mathrm{C} \& \mathrm{~B}$ and the expected reform size defined as $p \cdot\left|i_{R 1}^{*}-i_{R 2}^{*}\right|+(1-p) \cdot\left|i_{R 1}^{*}-i_{L 2}^{*}\right|$.

## Corollary 1

In the equilibrium of the game $\mathcal{G}\left(c_{1}\right)$, there is an inverse relation between $C \& B c_{1}$ and the expected reform size. Moreover, if $c_{1} \geq(1+p) \Pi$, no reform takes place regardless of who holds power, and a marginal increase of $c_{1}$ yields a more extreme policy.

As shown in Figure 4, starting from $c_{1}=0$ the policy chosen by party $R$ becomes more moderate until $c_{1}=(1+p) \Pi$ and it increases monotonically thereafter until $c_{1}=2 \Pi$. The effect of increasing reform costs above $c_{1} \geq(1+p) \Pi$ is then worth discussing as such CB will obtain in equilibrium once we allow CB to be determined endogenously in the constitutional phase. Larger C\&B do not affect the expected reform size, but they yield more extreme policies. Although no cost of reform is thus observed on the equilibrium path, the role of $\mathrm{C} \& \mathrm{~B}$ in such cases is to shape the policy via the off-equilibrium possibility of incurring costs if a marginal policy change is implemented.

Finally, note that for any $c_{1} \geq 2 \Pi$, all policies chosen by all parties are equal to $\mu_{R}$. Hence, values of $c_{1}$ equal or larger than $2 \Pi$ are equivalent in terms of outcomes - they lead to the same policies and to no reform costs. This yields the property that $c_{1}$, with $c_{1} \geq 2 \Pi$, is stable if and only if $c_{1}=2 \Pi$ is stable. Accordingly, for the sake of exposition we focus our subsequent analysis on
$c_{1} \in[0,2 \Pi]$.

### 4.2 Analysis of the constitutional game

We now investigate the constitutional stage, which is the bulk of our analysis, by building on the results of the previous section. The expected lifetime utility - henceforth simply called utility -of agent $K \in\{M, L, R\}$ in period $t=0$ is

$$
\begin{align*}
H\left(c_{1}, \mu_{K}, p\right): & =\mathbb{E}\left[u_{K}^{1}\left(i_{R 1}, i_{R 1}\right)+u_{K}^{2}\left(i_{R 1}, i_{K 2}\right)\right]  \tag{2}\\
& =p \cdot\left[-\left(i_{R 2}^{*}\left(i_{R 1}^{*}\left(c_{1}\right), c_{1}\right)-\mu_{K}\right)^{2}-c_{1}\left|i_{R 2}^{*}\left(i_{R 1}^{*}\left(c_{1}\right), c_{1}\right)-i_{R 1}^{*}(c)\right|\right] \\
& +(1-p) \cdot\left[-\left(i_{L 2}^{*}\left(i_{R 1}^{*}\left(c_{1}\right), c_{1}\right)-\mu_{K}\right)^{2}-c_{1}\left|i_{L 2}^{*}\left(i_{R 1}^{*}\left(c_{1}\right), c_{1}\right)-i_{R 1}^{*}\left(c_{1}\right)\right|\right]-\left(i_{R 1}^{*}\left(c_{1}\right)-\mu_{K}\right)^{2},
\end{align*}
$$

where the policy choices have been characterized in Proposition 1. If $c_{1} \in[0,(1+p) \Pi]$, Equation (2) can be written as

$$
\begin{align*}
\bar{H}\left(c_{1}, \mu_{K}, p\right)= & -(1+p) \cdot\left(\mu_{R}-\frac{c_{1}}{2} \cdot \frac{1-p}{1+p}-\mu_{K}\right)^{2}-(1-p) \cdot\left(\mu_{L}+\frac{c_{1}}{2}-\mu_{K}\right)^{2} \\
& -(1-p) \cdot c_{1} \cdot \Pi+\frac{1-p}{1+p} \cdot c_{1}^{2} . \tag{3}
\end{align*}
$$

In the following, we consider different variants for the constitutional phase in which the value of $c_{1}$ is determined. Each variant depends on the constitutional rules, i.e., (i) on who has the proposal-making power (incumbent party $R$, challenger party $L$, or median voter $M$ ) and (ii) on the majority rule used (simple majority, double majority, or unanimity). For either variant of the constitutional phase, the status quo $\mathrm{C} \& \mathrm{~B} c_{0}$ remains in place if the proposed $\mathrm{C} \& \mathrm{~B} c_{1}$ fails to gather the necessary votes. ${ }^{8}$

### 4.2.1 Simple majority rule

We start with the variant of the constitutional stage in which agent $K \in\{L, M, R\}$ can unilaterally decide on $\mathrm{C} \& \mathrm{~B} c_{1}$. The agent with the decision-making power-called the decision-maker-is therefore either one of the parties or the median voter. If $K \in\{L, R\}$ is the decision-maker, this means party $K$ 's has a parliamentary majority that suffices to approve $c_{1}$. If party $R$ has the majority in parliament, in particular, then both the executive power and the legislative power are in the same hands in period $t=1$. If party $L$ has the majority in parliament, by contrast, the

[^7]executive power and the legislative power are in different hands in period $t=1$. Assuming that the median voter is the decision-maker means that the decision about $\mathrm{C} \& \mathrm{~B} c_{1}$ can be taken by a simple majority of the electorate. In each of the three cases, the status quo $\mathrm{C} \& \mathrm{~B} c_{0}$ is immaterial for the outcomes as it can be simply overruled by the decision-maker. Alternatively, one can view this setup as equivalent to the case where agent $K$ has the proposal-making power and only one vote from party $R$, party $L$, and the median voter $M$ is required for approval of a new $\mathrm{C} \& \mathrm{~B}$. Thus the simple majority rule is in place in the constitutional phase in such a case. We denote the resulting game by $\mathcal{G}_{K, 1}^{+}\left(c_{0}\right)$ for $K \in\{M, L, R\}$.

For our analysis in this and the next sections, it is convenient to define

$$
\bar{\Pi}:=\frac{2}{1+p} \cdot\left(\mu_{M}-\mu_{L}\right)
$$

Parameter $\bar{\Pi}$ measures how polarized the median voter is compared to the challenger party $L$. Therefore it captures the asymmetry in preferences within the electorate. Note that

$$
\begin{equation*}
\frac{\overline{\bar{\Pi}}}{\bar{\Pi}}=\frac{2}{1+p} \cdot \frac{\mu_{M}-\mu_{L}}{\mu_{R}-\mu_{L}} \tag{4}
\end{equation*}
$$

The first term of the right-hand side of (4) increases with the turnover probability $1-p$ and is equal to one if $p=1$, i.e., if there is no turnover. The second term of the right-hand side of (4) increases in $\mu_{M}$ and is equal to one if $\mu_{M}=\mu_{R}$, i.e., if the median voter has the same peak as party $R$. To sum up, $\bar{\Pi} / \Pi$ is lower, the less likely turnover is and the less biased the median voter is in favor of the incumbent (relative to the challenger). As we show next, the relationship between $\bar{\Pi}$ and $\Pi$ determines the C\&B that the median voter $M$ would choose if they were the decision-maker in the constitutional stage.

If the simple majority rule is used in the constitutional phase, we obtain the following result:

## Theorem 1

In the unique equilibrium of game $\mathcal{G}_{K, 1}^{+}\left(c_{0}\right)$, the $C \& B$ chosen is

$$
\left(c_{1}\right)_{K, 1}^{*}:=\left(c_{1}\right)_{K, 1}^{*}\left(c_{0}\right)= \begin{cases}(1+p) \Pi & \text { if } K=L  \tag{5}\\ (1+p) \max \{\Pi, \bar{\Pi}\} & \text { if } K=M \\ 2 \Pi & \text { if } K=R\end{cases}
$$

Theorem 1 characterizes all the agents' optimal choice for the amount of $\mathrm{C} \& \mathrm{~B}$. To understand such choices, it is useful to plot the utilities of all agents as a function of $\mathrm{C} \& \mathrm{~B} c_{1}$. This is done in Figure 5. The first observation is that the decision-maker's utility increases for $c_{1} \in[0,(1+p) \Pi]$ no


Figure 5: Schematic representation of (expected) utilities for $\Pi<\bar{\Pi}$ (left) and $\bar{\Pi} \leq \Pi$ (right).
matter their peak. For party $R$, utility then increases for all $c_{1}$ and therefore reaches its maximum at $2 \Pi$. By contrast, party $L$ 's utility decreases for values of $c_{1}$ above $(1+p) \Pi$, so its peak is at $(1+p) \Pi$. For the median voter, utility is maximal at $(1+p) \max \{\Pi, \bar{\Pi}\}$. It increases to the left of such threshold and decreases to its right. Hence, all agents have single-peak preferences regarding C\&B. The relative order of their peaks is inherited from the relative order of their peaks regarding policies in $[0,1]$ and from the fact that party $R$ is the incumbent and gets to choose the policy in period $t=1$. If $\bar{\Pi} \leq \Pi$, in particular, the challenger party $L$ and the median voter $M$ are perfectly aligned in terms of their interests regarding C\&B, despite having different peaks for the optimal policy in $[0,1]$. Yet the median voter's peak must nonetheless be sufficiently close to that of party $L$ relative to incumbent party $R$ 's peak. Figure 6 plots the $\mathrm{C} \& \mathrm{~B} c_{1}$ chosen as a function of the decision-maker's peak, $\mu_{K}$. It shows that if we start from $\mu_{K}=\mu_{L}$, then $\left(c_{1}\right)_{K, 1}^{*}\left(\mu_{K}\right)$ does not change as we further increase $\mu_{K}$ until $\bar{\mu}_{K}=\mu_{L}+\frac{1+p}{2} \cdot \Pi$. From this cost threshold onward, $\left(c_{1}\right)_{K, 1}^{*}\left(\mu_{K}\right)$ increases linearly with $\mu_{K}$. The ranges above $\mu_{R}$ and below $\mu_{L}$ do not yield different results. This is because policy-making is in the hands of the two parties, which have peaks equal to $\mu_{L}$ and $\mu_{R}$, respectively, and because parties never choose policies that are more extreme than their peaks.

It also follows from Theorem 1 that if given the monopoly power to change $\mathrm{C} \& \mathrm{~B}$, party $R$ would choose C\&B that prevent any political reform and ensure that its peak is chosen in every period. This is the best outcome for party $R$. More interestingly, party $L$ and median voter $M$ also choose C\&B that prevent any policy reform. This is formalized in the next corollary.

## Corollary 2

If the simple majority rule is used in the constitutional phase, no policy reform occurs.


Figure 6: Choice $\left(c_{1}\right)_{K, 1}^{*}$ of agent $K$ with peak $\mu_{K}$, where $\bar{\mu}_{K}=\mu_{L}+\frac{1+p}{2} \Pi$.
The above corollary follows immediately from Corollary 1, since all C\&B chosen are at least as large as $(1+p) \Pi$ if the simple majority rule is used in the constitutional phase. The property that no reforms take place on the equilibrium path is stark and leads to gridlock (policy is not changed in period $t=2$ by neither party despite it does not generically coincide with their peaks). It obtains in our setup because costs of change are linear and losses from policies are quadratic. If costs of change were convex, albeit less convex than the quadratic utility loss function from policies, we would observe policy reforms in period $t=2$. However, such reforms would be small, particularly for moderately convex costs of change. Later we show that the result that no reform occurs on the equilibrium path also holds for all other variants of the constitutional phase.

Although no reform is carried out, the implemented policy does vary depending on the value of $\mathrm{C} \& \mathrm{~B}$, at least for the range of $\mathrm{C} \& \mathrm{~B}$ between $(1+p) \Pi$ and $2 \Pi$, as it affects the off-equilibrium incentives to carry out a reform. Hence, the decision-maker's peak matters for policy. Indeed, conditional on choosing $\mathrm{C} \& \mathrm{~B}$ that ensure the property that there will be no policy reform in period $t=2$, the challenger party $L$ chooses the smallest possible C\&B, while party $R$ chooses the largest possible $\mathrm{C} \& \mathrm{~B}$ (up to $2 \Pi$ ). This is because, as stated in Corollary 1, for the range of C\&B that lead to no reform there is a positive relationship between C\&B and how extreme the implemented policy is. The latter is a measure of policy polarization, which in our model arises endogenously as a function of the endogenously chosen C\&B.

Theorem 1 also shows that the probability of turnover, $1-p$, has an influence on the $\mathrm{C} \& \mathrm{~B} c_{1}$ chosen. The higher $p$, the higher $c_{1}$, at least weakly. This means that less policy turnover (i.e., higher $p$ ) leads to (weakly) higher C\&B, which in turn leads to more extreme policies. Such a
property is void for party $R$ since the C\&B chosen by the incumbent party is independent of $p$, but it has a bite for both the challenger party $L$ and median voter $M$. For the median voter, up to a certain probability threshold $\frac{2\left(\mu_{M}-\mu_{L}\right)}{\Pi}-1$, they propose C\&B that are independent of $p$. Above this threshold, the C\&B chosen increases linearly with $p$. The latter is always the case for the challenger party $L$. Note that the degree of instability, randomness, or rotation of the political system can be captured in our model by $1-p$. It therefore follows that if the simple majority is used in the constitutional phase, then instability matters for $C \& B$ only if there are divided institutions. That is, only if the decision-making power at the constitutional phase is in hands different from government. In such a case, more instability (i.e., lower $p$ ) translates into lower C\&B.

Finally, the next corollary follows trivially from Theorem 1.

## Corollary 3

Suppose that agent $K \in\{L, M, R\}$ is the decision-maker in the constitutional phase and that the simple majority is used. Then

$$
\mathcal{S C}_{K, 1}= \begin{cases}\{(1+p) \Pi\} & \text { if } K=L, \\ \{(1+p) \max \{\bar{\Pi}, \Pi\}\} & \text { if } K=M, \\ \{2 \Pi\} & \text { if } K=R .\end{cases}
$$

According to Corollary 3, most levels of C\&B are not stable if an agent-the decision-makerhas the monopoly power to choose C\&B in the constitutional phase. Moreover, higher party polarization translates into higher C\&B.

### 4.2.2 Double majority rule

In this section we assume that agent $K \in\{L, M, R\}$ has the right to make a proposal for a new $\mathrm{C} \& \mathrm{~B}$ that is later pitted against the status quo $\mathrm{C} \& \mathrm{~B} c_{0}$ in a (simultaneous) vote among all agents. The difference with respect to the previous section is that we now assume that two votes from the three agents (party $L$, party $R$, and median voter $M$ ) are needed to approve a new C\&B in the constitutional phase instead of just one vote. If the proposal for a new $\mathrm{C} \& \mathrm{~B}$ fails to gather at least two votes, the status quo $\mathrm{C} \& \mathrm{~B} c_{0}$ prevails. For instance, suppose that a party proposes a new C\&B that is later approved by such a party and the median voter. One interpretation is that the parliamentary majority of party $L$ or $R$ has to approve $c_{1}$ but so must median voter $M$. Therefore a double majority is required, from parliament (or government) and from the electorate. In actual democracies, it is common to require that certain constitutional changes that have been approved
by parliament must be additionally approved by the electorate through a referendum before they take effect. We denote the resulting game by $\mathcal{G}_{K, 2}^{+}\left(c_{0}\right)$.

If the double majority rule is used in the constitutional phase, we obtain the following result, where $\left(c_{1}\right)_{K, 2}^{*}:=\left(c_{1}\right)_{K, 2}^{*}\left(c_{0}\right)$ for each agent $K \in\{L, M, R\}$ :

## Theorem 2

In any equilibrium of game $\mathcal{G}_{K, 2}^{+}\left(c_{0}\right)$, the $C \& B$ chosen is the following:
(i) If $\Pi<\bar{\Pi}$,

$$
\begin{align*}
& \left(c_{1}\right)_{L, 2}^{*}= \begin{cases}(1+p) \Pi & \text { if } 0 \leq c_{0} \leq(1+p) \Pi, \\
c_{0} & \text { if }(1+p) \bar{\Pi}<c_{0} \leq(1+p) \bar{\Pi}, \\
\max \left\{(1+p) \Pi, 2(1+p) \bar{\Pi}-c_{0}\right\} & \text { if }(1+p) \bar{\Pi}<c_{0} \leq 2 \Pi,\end{cases}  \tag{6}\\
& \left(c_{1}\right)_{M, 2}^{*}=(1+p) \bar{\Pi}, \\
& \left(c_{1}\right)_{R, 2}^{*}= \begin{cases}\min \left\{2 \Pi, \sqrt{-2 \bar{H}\left(c_{0}, \mu_{M}, p\right)}+(1+p) \bar{\Pi}\right\} & \text { if } 0 \leq c_{0} \leq(1+p) \Pi, \\
\min \left\{2 \Pi, 2(1+p) \bar{\Pi}-c_{0}\right\} & \text { if }(1+p) \Pi<c_{0} \leq(1+p) \bar{\Pi}, \\
c_{0} & \text { if }(1+p) \bar{\Pi}<c_{0} \leq 2 \Pi .\end{cases} \tag{7}
\end{align*}
$$

(ii) If $\bar{\Pi} \leq \Pi$,

$$
\begin{align*}
\left(c_{1}\right)_{L, 2}^{*} & =(1+p) \Pi  \tag{8}\\
\left(c_{1}\right)_{M, 2}^{*} & =(1+p) \Pi \\
\left(c_{1}\right)_{R, 2}^{*} & =\max \left\{c_{0},(1+p) \Pi\right\} \tag{9}
\end{align*}
$$

Figures 7 and 8 illustrate the two parties' proposals (or choices) for $\mathrm{C} \& \mathrm{~B}$ described in Theorem 2 depending on the status quo C\&B. These proposals are voted in equilibrium by two agents and are best understood with the help of Figure 5. We recall that $\bar{H}\left(c_{0}, \mu_{M}, p\right)$ has been defined in Equation (3).

First, Figure 7 illustrates party $L^{\prime}$ s choice. On the left figure there is the case where $\Pi<\bar{\Pi}$, which on average yields higher C\&B than when $\bar{\Pi} \leq \Pi$. This is because in the former case the interests regarding the optimal $\mathrm{C} \& \mathrm{~B}$ diverge between the challenger party $L$ and the median voter $M$, and thus the median voter $M$ 's interests regarding the optimal C\&B approaches the incumbent party $R$ 's interests, which are to have as large C\&B as possible. By contrast, if $\bar{\Pi} \leq \Pi$, a case that is illustrated by the right figure, party $L$ 's and median voter $M$ 's proposal coincide. These choices


Figure 7: Choice $\left(c_{1}\right)_{L, 2}^{*}$ of party $L$ given by Equation (6) (left, case $\Pi<\bar{\Pi}$ ) and by Equation (8) (right, case $\bar{\Pi} \leq \Pi$ ).


Figure 8: Choice $\left(c_{1}\right)_{R, 2}^{*}$ of party $R$ given by Equation (7) (left, case $\Pi<\bar{\Pi}$ ) and by Equation (9) (right, case $\bar{\Pi} \leq \Pi$ ).
for a new C\&B are moreover independent of the status quo $\mathrm{C} \& \mathrm{~B} c_{0}$, since party $L$ and median voter $M$ have the necessary votes to implement their desired amount of C\&B.

Second, Figure 8 illustrates party $R$ 's choice. Similar to party $L$, the incumbent proposes a higher C\&B on average in the case where $\Pi<\bar{\Pi}$ compared to the case where $\bar{\Pi} \leq \Pi$. In the former case, party $R$ can rely on median voter $M$ to push $c_{1}$ further up. Moreover, the choices for $c_{1}$ are higher if the proposal-maker is party $R$ compared to party $L$.

There is a noteworthy feature of the parties' choices for C\&B if the double majority is used in the
constitutional phase which we do not obtain if the single majority is used: if $\Pi<\bar{\Pi}$, so that median voter $M$ 's and party $L$ 's interests regarding $c_{1}$ are not aligned, then the status quo $\mathrm{C} \& \mathrm{~B} c_{0}$ matters. Similar to the logic behind Romer and Rosenthal (1978), the further away $c_{0}$ is from the median voter's preferred amount of C\&B, the more leverage the party with the proposal-making power has. This implies the non-monotonicity of the choices described in Figures 7 and 8 (left cases). If the proposal-maker is the incumbent party $R$, its leverage increases as $c_{0}$ becomes smaller. In such a case, median voter $M$ is willing to accept a higher $\mathrm{C} \& \mathrm{~B}$ than their optimal one which gives them at least as much utility as the status quo $c_{0}$. If the proposal-maker is party $L$, the logic is reversed and the party's leverage is maximal if $c_{0}=2 \Pi$.

As in the case of a single decision-maker analyzed in Section 4.2.1, we obtain the following corollary:

## Corollary 4

If the double majority rule is used in the constitutional phase, no policy reform occurs.

According to Corollary 4, the proposal-maker proposes C\&B that yield no reform, no matter their identity and peak. In equilibrium, such C\&B must be approved by themselves and one other agent, and thus no proposals are made for $\mathrm{C} \& \mathrm{~B} c_{1}$ which are bound to be rejected. Conditional on satisfying the no-reform property, the proposal-maker chooses the C\&B that maximize their own utility. Due to Equation (1), if either party is the proposal-maker, then they seek the vote of the median voter. If the median voter is the proposal-maker, on the other hand, then they seek the support of one of the parties (party $R$ if the status quo $c_{0}$ is low, and party $L$ otherwise). The median voter has a peak between those of the parties, which then implies the property that they can always rely on one of the parties to change the status quo $\mathrm{C} \& \mathrm{~B} c_{0}$, no matter what the latter is. Then, under the double majority rule, the median voter always has their preferred amount of C\&B implemented.

The next corollary also follows trivially from Theorem 2:

## Corollary 5

Suppose that agent $K \in\{L, M, R\}$ proposes $c_{1}$ and the double majority rule is used. Then

$$
\mathcal{S C}_{K, 2}= \begin{cases}{[(1+p) \Pi,(1+p) \max \{\Pi, \bar{\Pi}\}]} & \text { if } K=L \\ \{(1+p) \max \{\Pi, \bar{\Pi}\}\} & \text { if } K=M \\ {[(1+p) \max \{\Pi, \bar{\Pi}\}, 2 \Pi]} & \text { if } K=R\end{cases}
$$

Because under the double majority rule only $\mathrm{C} \& \mathrm{~B}$ are proposed that ensure no reform in the political process, then status quo C\&B below $(1+p) \Pi$ cannot be stable. Graphically, stable $\mathrm{C} \& \mathrm{~B}$
can be easily seen in Figures 7 and 8 as they must lie on the $45^{\circ}$-line.
First, consider that the challenger party $L$ is the proposal-maker in the constitutional phase. Then the Lebesgue measure of the set of stable C\&B is larger if party polarization, $\Pi$, is low compared to $\bar{\Pi}$. In this case, the farther apart the peaks of party $L$ and of the median voter are, the larger the set of stable C\&B becomes. By contrast, if party polarization $\Pi$ is larger than $\bar{\Pi}$, the set of stable C\&B is a singleton. This level of reform costs is the lowest possible C\&B that guarantees no reform in the political process, and thus it is the most moderate policy conditional on no reform. The relationship between policy moderation and $\mathrm{C} \& \mathrm{~B}$ is stated in Corollary 1.

Second, consider that the median voter $M$ is the proposal-maker in the constitutional phase. Then, most C\&B are not stable no matter the relationship between $\Pi$ and $\bar{\Pi}$. As mentioned above, although the median voter never dictates policy, the fact that their peak lies between those of the parties allows them to obtain their preferred amount of $\mathrm{C} \& \mathrm{~B}$ approved. If a social planner would like to maximize welfare measured as the median voters' utility, the social planner should then give the proposal-making power for changing C\&B to the citizenry via referenda.

Third, consider that the incumbent party $R$ has proposal-making power in the constitutional phase. Then, in contrast with the case where the challenger party $L$ is the proposal-maker, assuming $\Pi \leq \bar{\Pi}$, the more the peaks of party $L$ and median voter $M$ are aligned, the larger the Lebesgue measure of the set of stable C\&B. If $\bar{\Pi}=\Pi$, then the set of stable C\&B is maximal for a fixed value of $\Pi$, as it contains the whole set of $C \& B$ that entail no policy reforms. This is shown in Figure 8.

Finally, it is worth noting that in general there is no monotonic relationship between the Lebesgue measure of the set of stable C\&B and the proposal-maker's identity.

### 4.2.3 Unanimity rule

In this section we assume that unanimity is required for changing the status quo C\&B. This means that the proposal-maker needs the votes of the remaining two agents to have a new $\mathrm{C} \& \mathrm{~B}$ approved. One interpretation is that such a change in the constitutional phase has to be approved by the government, by a qualified majority in parliament, and also by the electorate. This is the most stringent case. We denote the resulting game by $\mathcal{G}_{K, 3}^{+}\left(c_{0}\right)$.

If the unanimity rule is used in the constitutional phase, we obtain the following result, with $\left(c_{1}\right)_{K, 2}^{*}:=\left(c_{1}\right)_{K, 2}^{*}\left(c_{0}\right)$ for agent $K$ :

## Theorem 3

In any equilibrium of game $\mathcal{G}_{K, 3}^{+}\left(c_{0}\right)$, the $C \& B$ chosen is the following:
(i) If $\Pi<\bar{\Pi}$,

$$
\begin{align*}
& \left(c_{1}\right)_{L, 3}^{*}=\max \left\{c_{0},(1+p) \Pi\right\},  \tag{10}\\
& \left(c_{1}\right)_{M, 3}^{*}= \begin{cases}\min \left\{(1+p) \bar{\Pi}, \sqrt{-2 \bar{H}\left(c_{0}, \mu_{L}, p\right)}\right\} & \text { if } 0 \leq c_{0} \leq(1+p) \Pi, \\
c_{0} & \text { if }(1+p) \Pi<c_{0} \leq 2 \Pi,\end{cases}  \tag{11}\\
& \left(c_{1}\right)_{R, 3}^{*}= \begin{cases}\min \left\{\sqrt{-2 \bar{H}\left(c_{0}, \mu_{M}, p\right)}+(1+p) \bar{\Pi}, \sqrt{-2 \bar{H}\left(c_{0}, \mu_{L}, p\right)}\right\} & \text { if } 0 \leq c_{0} \leq(1+p) \Pi, \\
c_{0} & \text { if }(1+p) \Pi<c_{0} \leq 2 \Pi .\end{cases} \tag{12}
\end{align*}
$$

(ii) If $\bar{\Pi} \leq \Pi$,

$$
\begin{align*}
& \left(c_{1}\right)_{L, 3}^{*}=\max \left\{c_{0},(1+p) \Pi\right\}, \\
& \left(c_{1}\right)_{M, 3}^{*}=\max \left\{c_{0},(1+p) \Pi\right\},  \tag{13}\\
& \left(c_{1}\right)_{R, 3}^{*}= \begin{cases}\min \left\{\sqrt{-2 \bar{H}\left(c_{0}, \mu_{M}, p\right)}+(1+p) \bar{\Pi}, \sqrt{-2 \bar{H}\left(c_{0}, \mu_{L}, p\right)}\right\} & \text { if } 0 \leq c_{0} \leq(1+p) \Pi \\
c_{0} & \text { if }(1+p) \Pi<c_{0} \leq 2 \Pi\end{cases} \tag{14}
\end{align*}
$$

We recall that $\bar{H}\left(c_{0}, \mu_{M}, p\right)$ has been defined in Equation (3). If unanimity is used in the constitutional phase, every change on the status quo C\&B must be approved by all agents and the proposal-making rule is then paramount to outcomes. In equilibrium, it is not generically true that proposals are approved, as it is often the case that the status quo C\&B prevails. The reason is that any agent has a veto power in the voting stage over proposals that differ from such a status quo. As in the case of double majority, the status quo C\&B is also a source of leverage for the proposal-maker. We therefore observe again a non-monotonic behavior for the proposals of some agents, viz. party $R$ and median voter $M$. This is illustrated in Figures 10 and 11, which show voter $M$ 's and party $R$ 's proposal (or choice) for $\mathrm{C} \& \mathrm{~B}$ depending on the status quo $\mathrm{C} \& \mathrm{~B}$, as described in Theorem 3. For instance, consider that $\Pi<\bar{\Pi}$. Then both median voter $M$ and party $R$ can make party $L$ indifferent in utility terms between accepting the status quo $\mathrm{C} \& \mathrm{~B}$ and choosing a new, larger $\mathrm{C} \& \mathrm{~B}$. By contrast, party $L$ has no leverage against party $R$, as the latter prefers as high $\mathrm{C} \& \mathrm{~B}$ as possible. This leads to a monotonic relationship between $c_{0}$ and $c_{1}$, as shown in Figures 9.

As with the previous constitutional rules, the following corollary also holds:


Figure 9: Choice $\left(c_{1}\right)_{L, 3}^{*}$ of party $L$ given by Equation (10).


Figure 10: Choice $\left(c_{1}\right)_{M, 3}^{*}$ of median voter $M$ given by Equation (11) (left, case $\Pi<\bar{\Pi}$ ) and by Equation (13) (right, case $\bar{\Pi} \leq \Pi$ ).

## Corollary 6

If unanimity is used in the constitutional phase, no policy reform occurs regardless of the proposalmaker's identity.

According to the above corollary, if the proposal-maker wants to change the status quo $\mathrm{C} \& \mathrm{~B}$, they must propose C\&B that yield no reform which must then be approved additionally by all agents. To do so, the proposal-maker needs to choose C\&B that maximize their own utility conditional on C\&B satisfying the requirement that C\&B must be between $(1+p) \Pi$ and $2 \Pi$. Since all agents agree that reforms should be avoided, we obtain gridlock in equilibrium. Other than this property, the three agents never agree on anything else unanimously. This is because in the range from $(1+p) \Pi$


Figure 11: Choice $\left(c_{1}\right)_{R, 3}^{*}$ of party $R$ given by Equation (12) (left, case $\Pi<\bar{\Pi}$ ) and by Equation (14) (right, case $\bar{\Pi} \leq \Pi$ ).
to $2 \Pi$ both parties have opposed interests regarding the amount of $C \& B$. Hence they never agree. Finally, the following corollary states what the set of stable C\&B is if the unanimity rule is used.

## Corollary 7

Suppose that agent $K \in\{L, M, R\}$ proposes $c_{1}$ and the unanimity voting rule is used. Then

$$
\mathcal{S C}_{K, 3}=[(1+p) \Pi, 2 \Pi] .
$$

Accordingly, the set of the stable $\mathrm{C} \& \mathrm{~B}$ is maximal if unanimity is required at the constitutional phase, and, moreover, it coincides with the set of C\&B that ensure no reform in the political process. The Lebesgue measure of either set increases with turnover probability $1-p$ and party polarization $\Pi$. Another noteworthy property of the unanimity rule is that the proposal-maker's identity is irrelevant for the set of stable C\&B.

## 5 Comparing constitutions

In our setup, a constitution consists of (i) C\&B in place, (ii) a majority rule to change the status quo C\&B in the constitutional phase, and (iii) a rule that determines who has the power to make proposals for a new C\&B in the constitutional phase. In the previous sections, we have analyzed endogenous levels of (i) for given combinations of (ii) and (iii). In this section, we compare the constitutions along the single dimension (ii), keeping the other dimensions fixed.

The results in the previous sections lead to the following theorem:

## Theorem 4

The Lebesgue measure of the set of stable $C \& B$ increases for
(i) higher majority requirements for constitutional changes, and
(ii) higher party polarization $\Pi$ if the double majority rule or unanimity rule is used.

First, Theorem (i) follows from the comparison of stable C\&B for each voting rule, as described in the Corollaries 3,5 and 7 . In the case of simple majority, most levels of C\&B are not stable, whereas under unanimity voting, the set of stable C\&B is maximal. The intuition behind this result is that for higher majority requirements, it is more difficult to obtain the necessary votes to change the status quo level of C\&B.

Second, Theorem (ii) states that the set of stable C\&B increases for higher party polarization $\Pi$ if more than one agent is required to approve the constitutional change. In the case of simple majority rule, the set of stable C\&B is a singleton, despite party polarization. For the double majority rule and the unanimity rule, it is more difficult to find an agreement to change C\&B between the parties, if polarization is high. Therefore, a larger set of C\&B is stable. Hence, increased polarization is not a threat for C\&B, but implies policies that diverge more from the median position.

## 6 Conclusion

We have extended an existing model of political competition with reform costs by including a constitutional phase in which the level of such costs is endogenously determined. We have argued that reform costs can be interpreted as (executive) C\&B, which enables our paper to yield new insights on the question which amount of C\&B democratic societies choose. In our analysis we consider different constitutional rules to determine the level of reform costs. These rules vary depending on the majority rule they use and on to whom they allocate the power to make proposals. We find that stable C\&B always exist and are never zero, but that in general, many, if not most, $\mathrm{C} \& \mathrm{~B}$ will not survive in society. We also find that endogenous $\mathrm{C} \& \mathrm{~B}$ lead to gridlock.

Many avenues for future research can be pursued and our model can be enriched in several ways. For instance, one could add shocks to the future distribution of parties' peaks. This would induce policy reforms on the equilibrium path. One could also allow for uncertainty about the consequences of policies. In such cases, more stringent C\&B may produce more information about the consequences
of policies and thus may add a further rationale why C\&B should be in place. These and other conceivable extensions of the simple model may produce further insights about which amount of C\&B democracies choose or should choose.

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## Appendix

## List of Countries for Figure 3:

We use the discrete variables CHECKS and POLARIZ from the Database of Political Institutions [2020], available from Inter-American Development Bank, Cruz et al. (2021). The list was chosen based on the available data. Since we want to depict a picture of a large set of democratic countries regarding the relation of $\mathrm{C} \& \mathrm{~B}$ to polarization, we decide to use the countries with the most available data. These are: Albania, Bulgaria, Brazil, Botswana, Canada, Chile, Costa Rica, Czech Republic, Denmark, Dominican Republic, Ecuador, Spain, France, UK, Ghana, Greece, Guyana, Hungary, India, Ireland, Iceland, Israel, Republic of (South) Korea, Sri Lanka, Luxembourg, Malaysia, Netherlands, Norway, Peru, Philippines, Poland, Portugal, Paraguay, Sweden, Thailand, Taiwan, Uruguay, South Africa.

## Proof of Corollary 1:

Our goal is to show that the expected reform size $p \cdot\left|i_{R 1}^{*}-i_{R 2}^{*}\right|+(1-p) \cdot\left|i_{R 1}^{*}-i_{L 2}^{*}\right|$ decreases in $c_{1}$, if it changes at all.

First, let $0 \leq c_{1} \leq(1+p) \Pi$. From Proposition 1 we know that the expected reform size equals

$$
\begin{aligned}
(1-p) \cdot\left|i_{R 1}^{*}-i_{L 2}^{*}\right| & =(1-p) \cdot\left|\mu_{R}-\frac{c_{1}}{2} \cdot \frac{1-p}{1+p}-\left(\mu_{L}+\frac{c_{1}}{2}\right)\right| \\
& =(1-p) \cdot\left(\Pi-\frac{c_{1}}{2} \cdot\left(\frac{1-p}{1+p}+1\right)\right)=(1-p) \cdot\left(\Pi-\frac{c_{1}}{2} \cdot \frac{2}{1+p}\right) \\
& =(1-p) \cdot\left(\Pi-\frac{c_{1}}{1+p}\right),
\end{aligned}
$$

and hence decreases in $c_{1}$. Second, from Proposition 1 we also know that for $c_{1} \geq(1+p) \Pi$ no reform takes place. Moreover, for $c_{1} \geq(1+p) \Pi$ policies $i_{R 1}^{*}$ and $i_{K 2}^{*}$ increase in $c_{1}$ for $K \in\{L, R\}$.

## Proof of Theorem 1:

Our goal is to maximize utility in terms of $c_{1}$ as defined in Equation (2) by inserting the policy choices in the first and second period given by Proposition 1. We distinguish two ranges.

Case 1: $0 \leq c_{1} \leq(1+p) \Pi$
For this range, the policies for the first and second period are $i_{R 1}^{*}\left(c_{1}\right)=\mu_{R}-\frac{c_{1}}{2} \cdot \frac{1-p}{1+p}, i_{R 2}^{*}\left(c_{1}\right)=$
$i_{R 1}^{*}\left(c_{1}\right)$, and $i_{L 2}^{*}\left(c_{1}\right)=\mu_{L}+\frac{c_{1}}{2}$, as stated in Proposition 1. Then agent $K$ 's utility is given by

$$
\begin{align*}
\bar{H}\left(c_{1}, \mu_{K}\right)= & -p \cdot\left(\mu_{R}-\frac{c_{1}}{2} \cdot \frac{1-p}{1+p}-\mu_{K}\right)^{2} \\
& -(1-p) \cdot\left[\left(\mu_{L}+\frac{c_{1}}{2}-\mu_{K}\right)^{2}+c_{1}\left(\left(\mu_{R}-\frac{c_{1}}{2} \cdot \frac{1-p}{1+p}\right)-\left(\mu_{L}+\frac{c_{1}}{2}\right)\right)\right] \\
& -\left(\mu_{R}-\frac{c_{1}}{2} \cdot \frac{1-p}{1+p}-\mu_{K}\right)^{2} \\
= & -(1+p) \cdot\left(\mu_{R}-\frac{c_{1}}{2} \cdot \frac{1-p}{1+p}-\mu_{K}\right)^{2}-(1-p) \cdot\left(\mu_{L}+\frac{c_{1}}{2}-\mu_{K}\right)^{2} \\
& -(1-p) \cdot c_{1} \cdot\left(\mu_{R}-\mu_{L}\right)+\frac{1-p}{1+p} \cdot c_{1}^{2} . \tag{15}
\end{align*}
$$

Then the first order partial derivative of $H\left(c_{1}, \mu_{K}\right)$ w.r.t. $c_{1}$ is

$$
\begin{align*}
\frac{\partial \bar{H}\left(c_{1}, \mu_{K}\right)}{\partial c_{1}}= & (1-p) \cdot\left(\mu_{R}-\frac{c_{1}}{2} \cdot \frac{1-p}{1+p}-\mu_{K}\right)-(1-p) \cdot\left(\mu_{L}+\frac{c_{1}}{2}-\mu_{K}\right) \\
& -(1-p) \cdot\left(\mu_{R}-\mu_{L}\right)+2 c_{1} \cdot \frac{1-p}{1+p} \\
= & -(1-p) \cdot \frac{c_{1}}{2}-\frac{(1-p)^{2}}{1+p} \cdot \frac{c_{1}}{2}+2 c_{1} \cdot \frac{1-p}{1+p}=\frac{(1-p) c_{1}}{1+p} \geq 0 . \tag{16}
\end{align*}
$$

Hence, $H\left(c_{1}, \mu_{K}\right)$ is an increasing function in $c_{1}$ independent of $\mu_{K}$.
Case 2: $(1+p) \Pi<c_{1}<2 \Pi$
For this range, the policies for the first and second period are $i_{R 1}^{*}\left(c_{1}\right)=\mu_{L}+\frac{c_{1}}{2}$ and $i_{K 2}^{*}\left(c_{1}\right)=i_{R 1}^{*}$ as stated in Proposition 1. Then utility as well as its first and second order partial derivative are

$$
\begin{align*}
H\left(c_{1}, \mu_{K}\right) & =-p \cdot\left(\mu_{L}+\frac{c_{1}}{2}-\mu_{K}\right)^{2}-(1-p) \cdot\left(\mu_{L}+\frac{c_{1}}{2}-\mu_{K}\right)^{2}-\left(\mu_{L}+\frac{c_{1}}{2}-\mu_{K}\right)^{2} \\
& =-2\left(\mu_{L}+\frac{c_{1}}{2}-\mu_{K}\right)^{2},  \tag{17}\\
\frac{\partial H\left(c_{1}, \mu_{K}\right)}{\partial c_{1}} & =-2\left(\mu_{L}+\frac{c_{1}}{2}-\mu_{K}\right),  \tag{18}\\
\frac{\partial^{2} H\left(c_{1}, \mu_{K}\right)}{\partial c_{1}^{2}} & =-1 .
\end{align*}
$$

Equation (18) yields the unrestricted utility-maximizing C\&B

$$
\left(c_{1}^{*}\right)_{\mathrm{Max}}:=2\left(\mu_{K}-\mu_{L}\right) \leq 2 \Pi .
$$

One can then verify that

$$
(1+p) \Pi<\left(c_{1}^{*}\right)_{\mathrm{Max}}
$$

if and only if

$$
\Pi<\frac{2\left(\mu_{K}-\mu_{L}\right)}{(1+p)}:=\bar{\Pi}_{K} .
$$

Hence, if $\bar{\Pi}_{K} \leq \Pi$, utility is decreasing for all $c_{1}$ in this range. Note that $\bar{\Pi}_{L}=0, \bar{\Pi}_{M}=\bar{\Pi}$, and $\bar{\Pi}_{R}=\frac{2}{(1+p)} \Pi$. Note also that

$$
H\left(2 \Pi, \mu_{M}\right)=-2\left(\mu_{L}+\Pi-\mu_{M}\right)^{2}=-2\left(\mu_{R}-\mu_{M}\right)^{2}
$$

Finally, we determine the preferred level of C\&B-i.e., the level of C\&B that each agent would choose if they have proposal-making power-for each agent.

For party $L$ utility increases for $0 \leq c_{1} \leq(1+p) \Pi$, which can be directly derived from Equation (16), and decreases for $(1+p) \Pi<c_{1}<2 \Pi$ as shown in Equation (18). Hence party $L$ 's ideal C\&B is $c_{1}^{*}=(1+p) \Pi$.

For the median voter $M$, utility increases for $0 \leq c_{1} \leq(1+p) \Pi$, which can be directly derived from Equation (16). The exact level of party polarization impacts the shape of their utility for $(1+p) \Pi<c_{1}<2 \Pi$. For low party polarization, i.e. if $\Pi<\bar{\Pi}$, the median voter's utility increases up to their utility-maximizing $\mathrm{C} \& \mathrm{~B} c_{1}^{*}=2\left(\mu_{M}-\mu_{L}\right)=(1+p) \bar{\Pi}$ and then decreases for $c_{1}^{*}<c_{1}<2 \Pi$. For high party polarization, i.e. $\bar{\Pi} \leq \Pi$, the median voter's utility decreases in the full interval $(1+p) \Pi<c_{1}<2 \Pi$. Hence the median voter's preferred C\&B is $c_{1}^{*}=(1+p) \bar{\Pi}$ for $0 \leq \Pi<\bar{\Pi}$ and $c_{1}^{*}=(1+p) \Pi$ for $\bar{\Pi} \leq \Pi \leq 1$.

For party $R$, utility increases for $0 \leq c_{1} \leq(1+p) \Pi$, which can be directly derived from Equation (16), and increases further for $(1+p) \Pi<c_{1}<2 \Pi$ as shown in Equation (18) until it reaches its maximum at $\mathrm{C} \& \mathrm{~B} c_{1}^{*}=2 \Pi$. Hence party $R$ 's preferred C\&B is $c_{1}^{*}=2 \Pi$.

## Proof of Corollary 2:

The corollary follows from Theorem 1. Since all chosen levels of C\&B $c_{1}$ are at least $(1+p) \Pi$, no policy reform occurs.

## Proof of Corollary 3:

The corollary follows from Theorem 1. Since $c_{1}$ is unilaterally chosen, stable levels can only occur if the status quo level coincides with the preferred level of each agent.

## Proof of Theorem 2:

We distinguish two cases.
Case I: $\Pi<\bar{\Pi}$
From the proof of Theorem 1, we know that (i) party $L$ 's utility as a function of $c_{1}$ increases for $c_{1} \in[0,(1+p) \Pi)$ and decreases for $c_{1} \in((1+p) \Pi, 2 \Pi]$, that (ii) median voter $M$ 's utility as a function of $c_{1}$ increases for $c_{1} \in[0,(1+p) \bar{\Pi})$ and decreases for $c_{2} \in((1+p) \bar{\Pi}, 2 \Pi]$, and that (iii) party $R$ 's utility as a function of $c_{1}$ increases for $c_{1} \in[0,2 \Pi]$.

First, suppose that party $L$ is the proposal-maker. If $c_{0} \in[0,(1+p) \Pi]$, then party $L$ proposes $c_{1}^{*}=(1+p) \Pi$ since all agents prefer $c_{1}^{*}$ to $c_{0}$ and $c_{1}^{*}$ is party $L$ 's optimal choice for $c_{1}$. If $c_{0} \in$ $[(1+p) \Pi,(1+p) \bar{\Pi}]$, then party $L$ proposes $c_{0}$, as both median voter $M$ and party $R$ dislike choices of $c_{1}$ further below $c_{0}$ and party $L$ dislikes choices of $c_{1}$ further above $c_{0}$. It therefore remains to consider the case where $c_{0} \in[(1+p) \bar{\Pi}, 2 \Pi]$. In such a case, due to Equation (4) and Corollary 1, one can easily verify that in equilibrium party $L$ will propose the lowest C\&B that can be accepted by median voter $M$. This is because party $R$ will not accept any $c_{1}$ below $c_{0}$. The $\mathrm{C} \& \mathrm{~B} c_{1}$ proposed by party $L$ will then be at least $(1+p) \Pi$ and at most $(1+p) \bar{\Pi}$, due to the shape of party $L$ 's and median voter $M$ 's utilities as described above, and is therefore pinned down by the following equation:

$$
\begin{aligned}
& \quad H\left(c_{1}, \mu_{M}, p\right)=H\left(c_{0}, \mu_{M}, p\right) \\
& \text { s.t. }(1+p) \Pi \leq c_{1} \leq(1+p) \bar{\Pi} .
\end{aligned}
$$

Using Equation (17) one can see that solving the above problem is equivalent to solving the
following equation in $c_{1}$ :

$$
\mu_{L}+\frac{c_{1}}{2}-\mu_{M}=-\mu_{L}-\frac{c_{0}}{2}+\mu_{M}
$$

subject to $(1+p) \Pi \leq c_{1} \leq(1+p) \bar{\Pi}$. Solving the latter equation with the restriction yields the unique solution $c_{1}^{*}$ to the above problem, namely

$$
c_{1}^{*}=\max \left\{(1+p) \Pi, 2(1+p) \bar{\Pi}-c_{0}\right\} .
$$

Second, suppose that median voter $M$ is the proposal-maker. If $c_{0} \in[0,(1+p) \bar{\Pi}]$, then median voter $M$ proposes $c_{1}^{*}=(1+p) \bar{\Pi}$ since both median voter $M$ and party $R$ prefer $c_{1}^{*}$ to $c_{0}$ and $c_{1}^{*}$ is median voter $M$ 's optimal choice for $c_{1}$. If $c_{0} \in[(1+p) \bar{\Pi}, 2 \Pi]$, then median voter $M$ proposes $c_{1}^{*}=(1+p) \bar{\Pi}$, as both median voter $M$ and party $L$ prefer $c_{1}^{*}$ to $c_{0}$ and $c_{1}^{*}$ is median voter $M^{\prime}$ s optimal choice for $c_{1}$. In either case, the median voter obtains their most preferred C\&B.

Third and last, suppose that party $R$ is the proposal-maker. Let $c_{0} \in[0,(1+p) \bar{\Pi}]$. Due to Equation (4) and Corollary 1, one can easily verify that in equilibrium party $R$ will propose the largest C\&B that can be accepted by median voter $M$. This C\&B will be at least $(1+p) \bar{\Pi}$ (and at most $2 \Pi$ ). The reason is that both median voter $M$ and party $R$ prefer $(1+p) \bar{\Pi}$ to any $c_{0}$ lower than $(1+p) \bar{\Pi}$. The desired $\mathrm{C} \& \mathrm{~B}$ is therefore pinned down by the following equation:

$$
\begin{align*}
& \quad H\left(c_{0}, \mu_{M}, p\right)=H\left(c_{1}, \mu_{M}, p\right)  \tag{19}\\
& \text { s.t. }(1+p) \bar{\Pi} \leq c_{1} \leq 2 \Pi .
\end{align*}
$$

To solve the above equation, one needs to distinguish two cases. On the one hand, suppose that $c_{0} \in[0,(1+p) \Pi]$. Then, using Equations (3) and (17), one obtains that Equation (19) can be rewritten as:

$$
\bar{H}\left(c_{0}, \mu_{M}, p\right)=-2\left(\mu_{L}+\frac{c_{1}}{2}-\mu_{M}\right)^{2},
$$

with the condition that the solution must lie between $(1+p) \Pi$ and $2 \Pi$. Together with the restriction, the above equation yields the following unique solution to the above problem:

$$
c_{1}^{*}=\sqrt{-2 \bar{H}\left(c_{0}, \mu_{M}, p\right)}+(1+p) \bar{\Pi} .
$$

The negative solution $c_{1}^{*}=-\sqrt{-2 \bar{H}\left(c_{0}, \mu_{M}, p\right)}+(1+p) \bar{\Pi}$ is not feasible for $(1+p) \bar{\Pi} \leq c_{1} \leq 2 \Pi$. Note that as one can see from definition of the (expected) utility, as stated in Equation (2), the utility of all agents is negative for all $c_{0} \in[0,2 \Pi]$. On the other hand, suppose that $c_{0} \in$ $[(1+p) \Pi,(1+p) \bar{\Pi}]$. Then, using Equation (17), one obtains that Equation (19) can be rewritten as

$$
\mu_{L}+\frac{c_{1}}{2}-\mu_{M}=\mu_{M}-\frac{c_{0}}{2}-\mu_{L},
$$

with the condition that the solution must lie between $(1+p) \Pi$ and $2 \Pi$. This yields the following unique solution:

$$
c_{1}^{*}=\min \left\{2 \Pi, 2(1+p) \bar{\Pi}-c_{0}\right\} .
$$

Finally, it remains to analyze the case where $c_{0} \in((1+p) \bar{\Pi}, 2 \Pi]$. In such a case, party $R$ proposes $c_{0}$, as it cannot have any larger $\mathrm{C} \& \mathrm{~B}$ approved by median voter $M$ (or party $L$ ) against $c_{0}$. This is because both median voter $M$ 's and party $L$ 's utility are decreasing as functions of $c_{1}$ if $c_{0} \in((1+p) \bar{\Pi}, 2 \Pi]$.

Case II: $\bar{\Pi} \leq \Pi$
From the proof of Theorem 1 we know that (i) party $L$ 's utility as a function of $c_{1}$ increases for $c_{1} \in[0,(1+p) \Pi)$ and decreases for $c_{1} \in((1+p) \Pi, 2 \Pi]$, that (ii) median voter $M$ 's utility as a function of $c_{1}$ increases for $c_{1} \in[0,(1+p) \Pi)$ and decreases for $c_{1} \in((1+p) \Pi, 2 \Pi]$, and that (iii) party $R$ 's utility as a function of $c_{1}$ increases for $c_{1} \in[0,2 \Pi]$.

First, suppose that either party $L$ or median voter $M$ is the proposal-maker. Since their utilities are perfectly aligned, both agents make the same proposal for C\&B and approve them. Then, regardless of $c_{0}$, either agent proposes $c_{1}^{*}=(1+p) \Pi$ since $c_{1}^{*}$ is their optimal choice for $c_{1}$.

Second and last, suppose that party $R$ is the proposal-maker. Since both party $L$ and median voter $M$ 's utilities are perfectly aligned, in equilibrium party $R$ will propose the largest C\&B that can be accepted by either median voter $M$ or party $L$. This level of C\&B will be at least as much as $(1+p) \Pi$. This is because all agents prefer $(1+p) \Pi$ to any $c_{0}$ lower than $(1+p) \Pi$. For $c_{1} \in((1+p) \Pi, 2 \Pi]$ party $R$ proposes $c_{0}$, as both median voter $M$ and party $L$ dislike choices of $c_{1}$ further above $c_{0}$ and party $R$ dislikes choices of $c_{1}$ further below $c_{0}$.

## Proof of Corollary 4:

The corollary follows from Theorem 2. Since all chosen levels of C\&B $c_{1}$ are at least $(1+p) \Pi$, no policy reform occurs.

## Proof of Corollary 5:

The corollary follows directly from Theorem 2.

## Proof of Theorem 3:

We distinguish two cases.
Case I: $\Pi<\bar{\Pi}$
From the proof of Theorem 1, we know that (i) party L's utility as a function of $c_{1}$ increases for $c_{1} \in[0,(1+p) \Pi)$ and decreases for $c_{1} \in((1+p) \Pi, 2 \Pi]$, that (ii) median voter $M$ 's utility as a function of $c_{1}$ increases for $c_{1} \in[0,(1+p) \bar{\Pi})$ and decreases for $c_{2} \in((1+p) \bar{\Pi}, 2 \Pi]$, and that (iii) party $R$ 's utility as a function of $c_{1}$ increases for $c_{1} \in[0,2 \Pi]$.

First, suppose that party $L$ is the proposal-maker. If $c_{0} \in[0,(1+p) \Pi]$, then party $L$ proposes $c_{1}^{*}=(1+p) \Pi$ since all agents prefer $c_{1}^{*}$ to $c_{0}$ and $c_{1}^{*}$ is party $L$ 's optimal choice for $c_{1}$. If $c_{0} \in$ $[(1+p) \Pi, 2 \Pi]$, then party $L$ proposes $c_{0}$, as party $R$ dislike choices of $c_{1}$ further below $c_{0}$ and party $L$ dislikes choices of $c_{1}$ further above $c_{0}$.

Second, suppose that median voter $M$ is the proposal-maker. If $c_{0} \in[0,(1+p) \Pi]$, then median voter $M$ must propose C\&B that give party $L$ at least as much utility as under $c_{0}$. For its part, party $R$ simply wants to have the largest possible C\&B up to $2 \Pi$. Due to the shape of the utilities of party $L$ and median voter $M$, one can see that the proposed C\&B will be in the range from $(1+p) \Pi$ to $(1+p) \bar{\Pi}$. Hence, the median voter $M$ proposes $c_{1}$, such that

$$
\begin{gathered}
H\left(c_{0}, \mu_{L}\right)=H\left(c_{1}, \mu_{L}\right) \\
\text { s.t. }(1+p) \Pi<c_{1} \leq(1+p) \bar{\Pi}
\end{gathered}
$$

Using Equations (3) and (17) one can see that solving the above problem is equivalent to solving the following equation in $c_{1}$ :

$$
\begin{equation*}
\bar{H}\left(c_{0}, \mu_{L}, p\right)=-\frac{c_{1}^{2}}{2} \tag{20}
\end{equation*}
$$

subject to $(1+p) \Pi \leq c_{1} \leq(1+p) \bar{\Pi}$. The negative solution $c_{1}=-\sqrt{-2 \bar{H}\left(c_{0}, \mu_{L}, p\right)}$ of Equation (20) is not feasible for $(1+p) \Pi<c_{1} \leq(1+p) \bar{\Pi}$. Solving Equation (20) with the restriction yields the unique solution $c_{1}^{*}$ to the above problem, namely

$$
c_{1}^{*}=\min \left\{(1+p) \bar{\Pi}, \sqrt{-2 \bar{H}\left(c_{0}, \mu_{L}, p\right)}\right\} .
$$

If $c_{0} \in[(1+p) \Pi, 2 \Pi]$ then the median voter $M$ will propose $c_{0}$, as party $L$ dislike choices of $c_{1}$ further above $c_{0}$ and party $R$ dislike choices of $c_{1}$ further below $c_{0}$.

Third and last, suppose that party $R$ is the proposal-maker. If $c_{0} \in[0,(1+p) \Pi]$, then party $R$ proposes will propose the largest C\&B that will be accepted by median voter $M$ and party $L$. Hence, party $R^{\prime}$ s proposal must solve the following maximization problem:

$$
\begin{align*}
& \max _{c_{1} \in[(1+p) \Pi, 2 \Pi]} c_{1} \\
& \text { s.t. } H\left(c_{0}, \mu_{K}\right) \leq H\left(c_{1}, \mu_{K}\right) \text { for all } K \in\{L, M\} \text {. } \tag{21}
\end{align*}
$$

For $K=L$, using Equation (3) the Constraint (21) can be rewritten as

$$
\bar{H}\left(c_{0}, \mu_{L}, p\right)=-\frac{c_{1}^{2}}{2} .
$$

The negative solution $c_{1}=-\sqrt{-2 \bar{H}\left(c_{0}, \mu_{L}, p\right)}$ of the equation above is not feasible for $(1+p) \Pi<$ $c_{1} \leq 2 \Pi$. Together with the restriction, one obtains the following unique solution to the above problem:

$$
\begin{equation*}
c_{1}^{*}=\sqrt{-2 \bar{H}\left(c_{0}, \mu_{L}, p\right)} \tag{22}
\end{equation*}
$$

For $K=M$, using Equation (3) the Constraint (21) can be rewritten as

$$
\bar{H}\left(c_{0}, \mu_{M}, p\right)=-2\left(\mu_{L}+\frac{c_{1}}{2}-\mu_{M}\right)^{2}
$$

The negative solution $c_{1}=-\sqrt{-2 \bar{H}\left(c_{0}, \mu_{M}, p\right)}+(1+p) \bar{\Pi}$ of the equation above is not feasible for $(1+p) \Pi<c_{1} \leq 2 \Pi$. Together with the restriction, the above equation yields the following unique solution to the above problem:

$$
\begin{equation*}
c_{1}^{*}=\sqrt{-2 \bar{H}\left(c_{0}, \mu_{M}, p\right)}+(1+p) \bar{\Pi} \tag{23}
\end{equation*}
$$

Then party $R$ proposes the minimum of the two solutions stated in Equation (22) and (23), since then party $L$ and the median voter $M$ both agree.

If $c_{0} \in[(1+p) \Pi, 2 \Pi]$ then the party $R$ will propose $c_{0}$, as party $L$ dislike choices of $c_{1}$ further above $c_{0}$ and party $R$ dislike choices of $c_{1}$ further below $c_{0}$.

Case II: $\bar{\Pi} \leq \Pi$
From the proof of Theorem 1, we know that (i) party $L$ 's utility as a function of $c_{1}$ increases for $c_{1} \in[0,(1+p) \Pi)$ and decreases for $c_{1} \in((1+p) \Pi, 2 \Pi]$, that (ii) median voter $M$ 's utility as a function of $c_{1}$ increases for $c_{1} \in[0,(1+p) \Pi)$ and decreases for $c_{2} \in((1+p) \Pi, 2 \Pi]$, and that (iii) party $R$ 's utility as a function of $c_{1}$ increases for $c_{1} \in[0,2 \Pi]$.

First, suppose that either party $L$ or median voter $M$ is the proposal-maker. Since their utilities are perfectly aligned, both agents make the same proposal for C\&B. If $c_{0} \in[0,(1+p) \Pi]$, then party $L$ and median voter $M$ propose $c_{1}^{*}=(1+p) \Pi$ since all agents prefer $c_{1}^{*}$ to $c_{0}$ and $c_{1}^{*}$ is party $L$ 's and median voter $M$ 's optimal choice for $c_{1}$. If $c_{0} \in[(1+p) \Pi, 2 \Pi]$, then party $L$ and median voter $M$ propose $c_{0}$, as party $R$ dislike choices of $c_{1}$ further below $c_{0}$ and party $L$ and the median voter dislike choices of $c_{1}$ further above $c_{0}$.

Second and last, suppose that party $R$ is the proposal-maker. Since both party $L$ and median voter $M$ 's utilities are perfectly aligned, in equilibrium party $R$ will propose the largest $\mathrm{C} \& \mathrm{~B}$ that can be accepted by median voter $M$ and party $L$. If $c_{0} \in[0,(1+p) \Pi]$, then party $R$ proposes will propose the largest C\&B that will be accepted by median voter $M$ and party $L$. Hence, party $R$ must solve the following maximization problem:

$$
\begin{align*}
& \max _{c_{1} \in[(1+p) \Pi, 2 \Pi]} c_{1} \\
& \text { s.t. } H\left(c_{0}, \mu_{K}\right) \leq H\left(c_{1}, \mu_{K}\right) \text { for all } K \in\{L, M\} \text {. } \tag{24}
\end{align*}
$$

For $K=L$, using Equation (3) one can write Equation (24) as

$$
\bar{H}\left(c_{0}, \mu_{L}, p\right)=-\frac{c_{1}^{2}}{2}
$$

The negative solution $c_{1}=-\sqrt{-2 \bar{H}\left(c_{0}, \mu_{L}, p\right)}$ of the equation above is not feasible for $(1+p) \Pi<$ $c_{1} \leq 2 \Pi$. Together with the restriction, one obtains the following unique solution to the above problem:

$$
\begin{equation*}
c_{1}^{*}=\sqrt{-2 \bar{H}\left(c_{0}, \mu_{L}, p\right)} \tag{25}
\end{equation*}
$$

For $K=M$, using Equation (3) one can write Equation (24) as

$$
\bar{H}\left(c_{0}, \mu_{M}, p\right)=-\left(\mu_{L}+\frac{c_{1}}{2}-\mu_{M}\right)^{2} .
$$

Together with the restriction, the above equation yields the following unique solution to the above problem:

$$
\begin{equation*}
c_{1}^{*}=\sqrt{-2 \bar{H}\left(c_{0}, \mu_{M}, p\right)}+(1+p) \bar{\Pi} \tag{26}
\end{equation*}
$$

Then party $R$ proposes the minimum of the two solutions stated in Equation (25) and (26), since then party $L$ and the median voter $M$ both agree. If $c_{0} \in((1+p) \Pi, 2 \Pi]$, then median party $R$ proposes $c_{0}$ as median voter $M$ and party $L$ dislike choices of $c_{1}$ above $c_{0}$.

## Proof of Corollary 6

The corollary follows from Theorem 3. Since all chosen levels of C\&B $c_{1}$ are at least $(1+p) \Pi$, no policy reform occurs.

## Proof of Theorem 4

First, consider ( $i$ ). The part follows from Corollaries 3,5 and 7 . In the case of the simple majority rule the set of stable C\&B is a singleton. If the unanimity voting rule is in place the Lebesgue measure of the set of stable C\&B is maximal. Second, consider (ii). As stated in Corollaries 5 and 7 the set of stable C\&B increases in party polarization $\Pi$.

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[^2]:    ${ }^{1}$ One recent example is the use of the filibuster by the Republicans in the US to block a bill to launch an independent commission investigating the January 62021 insurrection. See
    https://edition.cnn.com/2021/05/28/opinions/republican-threat-to-democracy-filibusterzupnick/index.html, retrieved 7 July 2021. The filibuster is a procedural tool of the Senate that can prevent or delay a bill, as the vote needed for approval moves from 51 to 60 votes.

[^3]:    ${ }^{2}$ Both the voting rule and the proposal-making rule used in the constitutional phase can be interpreted as C\&B for constitutional changes. Our analysis then sheds light on how C\&B at the constitutional level affect executive C\&B.

[^4]:    ${ }^{3}$ CHECKS is a discrete variable from the Database of Political Institutions provided by the Inter-American Development Bank, which is defined based on Legislative and Executive Indices of Electoral Competitiveness.

[^5]:    ${ }^{4}$ See e.g. https://www.brookings.edu/policy2020/votervital/what-is-the-senate-filibuster-and-what-would-it-take-to-eliminate-it/, retrieved 28 September 2021.
    ${ }^{5}$ POLARIZ is a discrete variable from the Database of Political Institutions [2020], available from Inter-American Development Bank, Cruz et al. (2021). We consider the variable POLARIZ, which is defined as the maximum polarization between the executive party and the four principal parties of the legislature.

[^6]:    ${ }^{6}$ In standard models, the positions of political parties are determined by the expected position of the median voter. In the citizen-candidate model-see Besley and Coate (1997) and Osborne and Slivinski (1996)—, the two political actors locate at equidistant positions on opposite sides of the median voter's preferred position. We relax this condition and simply assume that the median voter's peak lies in between.
    ${ }^{7}$ This is only relevant if parties $L$ and $R$ as well as the median voter participate in the voting procedure during the constitutional stage.

[^7]:    ${ }^{8}$ In total there are nine different variants of the constitutional phase.

