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# Property Insurance, Portfolio Selection and their Interdependence

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# Property Insurance, Portfolio Selection and their Interdependence

## Abstract

This paper studies the interdependence between property insurance and portfolio selection. The insurance premium of property loss is shown to play the role of subsistence consumption in the analysis. Then, “security” becomes a necessity good and an increase in any insurance parameter would make the investor more “conservative.” The effect of a stock market parameter on the marginal propensity to insure is shown to be opposite that on the marginal propensity to consume. Consequently, an increase in volatility would encourage those with a greater-than-unity relative risk aversion to purchase more insurance at the expense of current consumption.

JEL Code: G11, G22, E21.

Keywords: insurance premium, subsistence consumption, portfolio substitution, optimal saving under uncertainty.

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## 1. Introduction

In general, insuring property loss and investing in risky assets are studied separately in the literature. While random shocks to each decision problem may not be related, the decisions usually are. For example, the stock market often reacts negatively to bad news according to conventional wisdom. Would investors adjust their investments when facing a graver danger of property loss? More specifically, would investors in New Orleans change their portfolio after Katrina? Similarly, insurance coverage may respond to the stock market fluctuation, especially when the market becomes more volatile. In what way is property insurance influenced by the stock market parameters? Would the economic agent reduce her insurance coverage during market downturns? To understand the cause and effect of such interdependence, we present a theory of optimal consumption, insurance and portfolio rules when shocks to property loss and shocks to capital markets are independent.

This paper is an extension of the theory of saving and insurance. In this theory, insuring properties and saving are two alternatives for current consumption. Major contributions include static model of Moffet (1977), discrete-time models of Dionne and Eeckhoudt (1984) and Gollier (2001), and continuous-time models of Briys (1986, 88), Gollier (1994) and Somerville (2004). The capital market therein is a simple risk-free bond market with a constant rate of return. As pointed out in Sandmo (1969), it is incomplete to analyze saving behavior without considering capital risk in the process. This paper sheds some light on this issue by including risky assets in a consumer's savings portfolio.

This paper is also an extension of the theory of consumption and portfolio se-

lection of Samuelson (1969) and Merton (1971). Major contributions in discrete-time models since the publication of their seminal papers are succinctly presented in Gollier (2001). The development in continuous time has a special place in finance, most notably, by Karatzas, Lehoczky, Sethi and Shreve (1986), Cox and Huang (1989, 1991), Merton (1990), Bodie, Merton and Samuelson (1992), Vila and Zariphopoulou (1997), and Duffie, Fleming, Soner and Zariphopoulou (1997). However, none of the aforementioned works consider property insurance in their analysis. Since investors also face the possibility of accidents and property damages in their lifetimes, the inclusion of the property insurance in the portfolio selection theory is yet another extension of this branch of the literature.

In order to examine the cause and effect of interdependence between portfolio selection and property insurance, we assume that respective shocks are independent. Indeed, the risks involved in property loss, e.g., fire hazards or auto accidents, are generally considered as having no direct impact on stock market fluctuations. Conversely, financial risks generally do not cause fire hazards or auto accidents either. Allowing dependence between the two shocks will no doubt make the model more general and add another dimension of interdependence. However, from modeling viewpoints, it would inevitably complicate and convolute the analysis. For this reason, we do not consider this variant in the paper.

It should be mentioned that there is a literature on the interdependence between insurance purchase and portfolio selection. However, most of the authors assume that insurance purchase and risky investments face the same risk. For example, Briys (1988) assumed risky assets are also subject to some damages in addition to their underlying fluctuations. Meyer and Ormiston (1995) allowed a consumer to purchase insurance on risky assets. Their findings are complements,

not substitutes, to this paper.

The exception is Mayers and Smith (1983) in which insurance contracts are treated as a subset of assets of an individual's portfolio. Their paper is static, and is cast in the mean-variance framework of portfolio selection. As such, they do not consider the probabilistic nature of property loss. Their objective is to find factors affecting insurance purchase. In contrast, we have a dynamic portfolio selection model that includes property insurance. In addition to finding factors influencing insurance purchase, we analyze the effect of insurance parameters on portfolio selection. This paper can thus be considered as a dynamic extension of Mayers and Smith (1983).

To make the model tractable, we assume the following: Shocks to assets are stationary and mutually independent. Property loss is modeled as an income risk, where the size and the probability of loss are constant over time. The utility function exhibits constant relative risk aversion (CRRA). Under these assumptions, we show that the insurance premium for property loss plays the role of subsistence consumption. Then, the problem of insurance and portfolio selection can be simplified as a standard consumption-portfolio problem when the utility function takes the intertemporal Stone-Geary form.

The presence of the subsistence consumption suggests that, to avoid a streak of bad luck, a consumer must, at any point in time, allocate sufficient resources to cover the current and future subsistence consumption. The wealth in excess of the sum of discounted insurance premiums of current and future losses is called the disposable wealth. A sufficient condition is imposed to ensure that the disposable wealth is positive at all times with probability one. Then we show that the dynamic optimization problem has a closed form solution and that the opti-

mal consumption in both states, investments in the risky assets, and insurance purchases are all linear functions of disposable wealth.

To disentangle the taste parameter from the market parameters, we apply the Arrow-Pratt approximation to the first order conditions of the dynamic optimization problem. Then the optimal policies are functions of the degree of relative risk aversion, the insurance parameters and the means and variances of risky assets. Using these closed form representations, we arrive at the following comparative dynamics.

An increase in the loading factor, the size or the probability of loss raises the subsistence consumption, which in turn makes the investor more conservative in the sense that they substitute the risk-free asset for risky assets. The theory thus predicts that people in the area prone to hazards would not be too cavalier in their investment. Furthermore, the presence of the subsistence consumption in the analysis also reverses a claim by Arrow (1965) that “security” (the risk-free asset) is a luxury good. Specifically, we show that the demand for the risk-free asset is linear in wealth with a positive intercept and hence is a necessity good. Similarly, risky assets are all luxury goods in this case.

Since the probability of loss and the loading factor enter the relative price of insurance, a change in either one of them would also produce a substitution effect between different states of consumption. The net effect is the aggregate of this substitution effect and the aforementioned negative wealth effect on consumption due to a reduction in disposable wealth. We show that an increased loading factor tends to lower the consumption in the hazardous state.

The effect of increased probability of loss on consumption has a third effect: it changes the marginal rate of substitution between the two states by increasing

the willingness to insure. We show that if the consumer is not very risk averse, the willingness-to-insure effect would be dominated by the other two effects. Consequently, the effect of increased probability would also lower the consumption in the hazardous state.

The stock market effect on property insurance turns out to be closely related to the classic theory of optimal savings under uncertainty. The reason is that stock market parameters affect property insurance solely through their effect on the marginal propensity to insure out of wealth. We show that these parametric effects on the marginal propensity to insure are exactly opposite of that on the marginal propensity to consume. Insurance behaves as an alternative to current consumption when stock market parameters are changed. According to Levhari and Srinivasan (1969), when the utility function is isoelastic, an increase in uncertainty increases current consumption if the degree of relative risk aversion is less than unity. Sandmo (1970) interpreted this result as when the substitution effect dominates the wealth effect of increasing risk. When the degree of relative risk aversion is greater than unity, the increasing risk effect on current consumption is negative; the effect is neutral if the degree is unitary.

We show that we can replace “an increase in uncertainty” with “an increase in the variance” or “a decrease in the mean” of any risky asset. Since the stock market parameters do not enter the relative price of insurance, the result applies to consumption in both states. Thus, in the case that the relative risk aversion is less than unity, a decrease in the mean or an increase in the variance of any risky asset would increase consumption in both states and decrease property insurance. The theory predicts that there would be cutbacks in insurance coverage in more volatile times if the consumer is not very risk averse.



In general, we find that property insurance and investments are interdependent, even though we assume their risks are independent. However, there are two separation theorems of insurance that should be mentioned. When the insurance market is actuarially fair, the consumer is fully insured and, hence, property insurance is independent of all parameters except the size of property loss. This is quite intuitive and fully expected. When the utility function is of logarithmic form, i.e., exhibiting unitary relative risk aversion, the means and variances of the risky assets have no effect on consumption or insurance. This is also intuitive because the wealth effect of increasing risk is exactly offset by the substitution effect.

## 2. Capital Risk and Property Loss

The structure of discrete-time portfolio selection is modeled after Samuelson (1969) and Gollier (2001). Assume there are  $n + 1$  assets in the consumer's portfolio with  $(n + 1)$ -th asset being risk-free. Let  $r$  and  $\varepsilon_{i,t}$  be, respectively, the risk-free rate of return and the random rate of return to asset  $i$  at time  $t$ ,  $i = 1, 2, \dots, n$ . Following Samuelson (1969), consumption  $x_t$  and investments are assumed to be taken place at the beginning of the time period. Let  $\alpha_{i,t}$  and  $m_t$  be, respectively, the investment in the  $i$ -th asset and the investment in the risk-free asset at time  $t$ ; together with  $x_t$  they exhaust the total resources, i.e.,

$$w_t = x_t + m_t + \sum_{i=1}^n \alpha_{i,t}.$$

Then the wealth passes on to the next period is

$$w_{t+1} = (1 + r) m_t + \sum_{i=1}^n (1 + \varepsilon_{i,t}) \alpha_{i,t}.$$

Hence the budget equation can be written as

$$w_{t+1} = (1 + r)(w_t - x_t) + \sum_{i=1}^n \alpha_{i,t} (\varepsilon_{i,t} - r). \quad (1)$$

In addition to capital risk, the consumer also faces the possibility of property loss. We assume property losses are identical and independent distributed (i.i.d.) over time and are also independent of the excess returns of risky assets  $\{\varepsilon_{i,t} - r\}$ ,  $i = 1, 2, \dots, n$ . The reason for the last independence is that the risk of property loss (such as fire hazards and auto accidents) is not related to capital risk. To make the model tractable, we assume a two-state framework for property loss as in the classic insurance literature so that, at any point in time, the world is either in the hazardous state suffering a loss of  $L$  with probability  $\pi$ , where  $0 < \pi < 1$ , or in the good state without loss with probability  $1 - \pi$ .

We assume that the consumer does not diversify her loss over time and that the insurance premium is funded by the allocation of  $x_t$  alone. See, for example, Gollier (2001, 2003) for a discussion and modeling of time diversification. In other words, property loss presents only income risk to the consumer. Modeling insurance financing this way, the insurance premium does not enter the budget equation (1). As such, the model does not automatically generate intertemporal substitution arising from property loss so that we can obtain some insight into the interdependence between the two markets without the said substitution effect.

Assume the loading factor of the insurance market is  $\ell \geq 1$ . If the face value of insurance coverage at time  $t$  is  $f_t$ , then the insurance premium is  $\ell\pi f_t$ . Then the consumption in the hazardous state is

$$c_{1,t} = x_t - \ell\pi f_t - L + f_t,$$

and the consumption in good state is

$$c_{2,t} = x_t - \ell\pi f_t,$$

while the budget equation (1) remains the same.

Assume the utility function  $u(c)$  is state independent, depending only on the amount of consumption and is strictly increasing and strictly concave in  $c$ . Then the consumer's problem is to maximize the expected lifetime utility by choosing the spending allocation  $\{x_t\}$ , the portfolio selection  $\{\alpha_{i,t}\}_{i=1}^n$ , and property insurance  $\{f_t\}$  subject to the aforementioned wealth constraint (1). Formally, the problem is this:

$$\begin{aligned} \max_{\{x_t, \alpha_{1,t}, \dots, \alpha_{n,t}, f_t\}} E_0 \sum_{t=0}^{\infty} \beta^t ([\pi u(c_{1,t}) + (1 - \pi) u(c_{2,t})]) \\ \text{s.t. (1), and given } w(0). \end{aligned} \quad (2)$$

### 3. Insurance Premium as Subsistence consumption

Rewrite Problem (2) as

$$\begin{aligned} \max_{\{x_t, \alpha_{1,t}, \dots, \alpha_{n,t}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \max_{f_t} [\pi u(c_{1,t}) + (1 - \pi) u(c_{2,t})] \right\}, \\ \text{s.t. (1), and given } w(0), \end{aligned}$$

i.e., the problem can be solved in two stages. In the first stage, for any feasible allocation of  $\{x_t\}$  and  $\{\alpha_{1,t}, \dots, \alpha_{n,t}\}$ , the consumer at any time  $t$  solves a "static" insurance problem

$$\max_{f_t} [\pi u(c_{1,t}) + (1 - \pi) u(c_{2,t})]. \quad (3)$$

Since the insurance premium is funded by  $x_t$  alone, we can denote the value function of (3) by  $v(x_t)$ . In the second stage, the consumer chooses  $\{x_t\}$  and  $\{\alpha_{1,t}, \dots, \alpha_{n,t}\}$  to maximize the expected lifetime utility, i.e.,

$$\max_{\{x_t, \alpha_{1,t}, \dots, \alpha_{n,t}\}} E_0 \sum_{t=0}^{\infty} \beta^t v(x_t), \text{ s.t. (1), and given } w(0). \quad (4)$$

This is a standard portfolio selection problem.

The first order condition for Problem (3) is

$$\frac{u'(c_{1,t})}{u'(c_{2,t})} = \frac{\ell(1-\pi)}{1-\ell\pi} = M. \quad (5)$$

To facilitate the analysis, assume  $u(c)$  exhibits constant relative risk aversion (CRRA), i.e.,

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad \gamma > 0, \gamma \neq 1.$$

When  $\gamma = 1$ , it is understood that  $u(c) = \log c$ . Then the solution to Problem (3) is

$$f_t = [M^{1/\gamma}L - (M^{1/\gamma} - 1)x_t] / B, \text{ for all } \gamma > 0,$$

where

$$B = \ell\pi + M^{1/\gamma}(1 - \ell\pi).$$

Consequently, the consumption in the hazardous state is

$$c_{1,t} = x_t - L + (1 - \ell\pi)f_t = (x_t - \ell\pi L) / B,$$

and the consumption in the good state is

$$c_{2,t} = M^{1/\gamma}c_{1,t} = M^{1/\gamma}(x_t - \ell\pi L) / B.$$

Substituting them into (3), the value function is

$$v(x_t) = \frac{B^\gamma (x_t - \ell\pi L)^{1-\gamma}}{\ell(1-\gamma)}, \quad \gamma > 0, \gamma \neq 1.$$

When  $\gamma = 1$ ,  $B = \ell$  and the value function is

$$v(x_t) = \log(x_t - \ell\pi L) - \log \ell + (1 - \pi) \log M.$$

To solve the second stage optimization problem (4) for  $\gamma > 0$  and  $\gamma \neq 1$ , we can instead solve the following portfolio selection problem

$$\max_{\{x_t, \alpha_{1,t}, \dots, \alpha_{n,t}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(x_t - \ell\pi L)^{1-\gamma}}{1-\gamma} \right\}, \text{ s.t. (1), and given } w_0, \quad (6)$$

because the difference between (4) and (6) is a constant multiplicative factor of  $B^\gamma/\ell$ . When  $\gamma = 1$ , we would solve

$$\max_{\{x_t, \alpha_{1,t}, \dots, \alpha_{n,t}\}} E_0 \sum_{t=0}^{\infty} \beta^t \log(x_t - \ell\pi L), \text{ s.t. (1), and given } w_0, \quad (7)$$

because  $(1 - \pi) \log M - \log \ell$  is a constant.

Since  $\ell\pi$  is the price of insurance,  $\ell\pi L$  is the insurance premium of property loss  $L$ . The utility function in (6) or (7) is defined only for  $x_t > \ell\pi L$ . In consumer theory, this minimum level of consumption is often referred to as the subsistence consumption. See, for example, Layard and Walters (1978). The objective function in (6) is referred to as the intertemporal Stone-Geary utility function. In portfolio theory, such a function exhibits hyperbolic absolute risk aversion (HARA). An important feature of Problem (6) is that Problem (2) has been reduced to a “pure” consumption-portfolio problem when the insurance premium of property loss,  $\ell\pi L$ , enters the utility function as the subsistence consumption.

#### 4. Closed Form Solution for the Portfolio Problem

Assume the excess returns of risky assets  $\{\varepsilon_{i,t} - r\}$ ,  $t = 0, 1, 2, \dots$ , are stationary and mutually independent. In the infinite horizon setting, the current value of

the value function of (6) at the any time  $t$

$$\max_{\{x_s, \alpha_{1,s}, \dots, \alpha_{n,s}\}} E_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \frac{(x_t - \ell\pi L)^{1-\gamma}}{1-\gamma} \right\}, \text{ s.t. (1), and given } w_t$$

is independent of the “initial” time  $t$ , dependant only on the “initial” wealth  $w_t$ . Henceforth, it is denoted by  $J(w_t)$ . This value function satisfies the Bellman equation

$$J(w_t) = \max_{x_t, \alpha_{1,t}, \dots, \alpha_{n,t}} \left\{ \frac{(x_t - \ell\pi L)^{1-\gamma}}{1-\gamma} + \beta E [J(w_{t+1})] \right\}. \quad (8)$$

The first-order conditions of the Bellman equation are

$$(x_t - \ell\pi L)^{-\gamma} = (1+r) \beta E [J^0(w_{t+1})], \quad (9)$$

and

$$E [J^0(w_{t+1}) (\varepsilon_{i,t} - r)] = 0, i = 1, 2, \dots, n, t = 0, 1, 2, \dots \quad (10)$$

The assumption of stationary excess returns allows us to drop the subscript  $t$  so that  $\varepsilon_{i,t}$  is denoted by  $\varepsilon_i$  to simplify the notation. It also reduces the system of equations (10) to a system of only  $n$  equations. Applying the envelope theorem to (8), we have

$$J^0(w_t) = (1+r) \beta E [J^0(w_{t+1})].$$

Then, the first order condition (9) can be written as

$$(x_t - \ell\pi L)^{-\gamma} = J^0(w_t). \quad (11)$$

Recall that  $\ell\pi L$  is the subsistence consumption at any point in time. Therefore, the total discounted subsistence consumption over the remaining lifetime including period  $t$ , evaluated at time  $t$ , is

$$\sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^{s-t} \ell\pi L = \left( \frac{1+r}{r} \right) \ell\pi L.$$

Call  $w_t - (1 + r) \ell\pi L/r$  the disposable wealth at time  $t$ . Following Gollier (2001), we guess the value function to be of the form

$$J(w_t) = K^{-\gamma} \frac{[w_t - (1 + r) \ell\pi L/r]^{1-\gamma}}{1 - \gamma}, \gamma > 0, \gamma \neq 1, \quad (12)$$

where  $K > 0$  is a constant to be specified below.

The motivation for guessing this form of value function is quite intuitive. From (11), we have

$$x_t - \ell\pi L = K [w_t - (1 + r) \ell\pi L/r]. \quad (13)$$

A simple algebraic manipulation shows that the disposable wealth at time  $t + 1$  is of the form

$$\begin{aligned} & w_{t+1} - (1 + r) \ell\pi L/r \\ &= (1 + r) (1 - K) [w_t - (1 + r) \ell\pi L/r] + \sum_{i=1}^n \alpha_{i,t} (\varepsilon_i - r). \end{aligned}$$

If the risky investments are also stationary and linear in the disposable wealth, i.e., if

$$\alpha_{i,t} = b_i [w_t - (1 + r) \ell\pi L/r], i = 1, 2, \dots, n, \quad (14)$$

for some coefficient  $b_i$  so that the Bellman equation is an equation expressed solely in the order of  $[w_t - (1 + r) \ell\pi L/r]^{1-\gamma}$ , then a closed form solution is possible.

Formally, let  $\{b_i\}_{i=1}^n$  be the solutions to the system of equations

$$E \left[ \left( (1 + r) (1 - K) + \sum_{i=1}^n b_i (\varepsilon_i - r) \right)^{-\gamma} (\varepsilon_i - r) \right] = 0, i = 1, 2, \dots, n. \quad (15)$$

Then the Bellman equation is reduced to

$$1 = K + \beta E \left[ \left( (1 + r) (1 - K) + \sum_{i=1}^n b_i (\varepsilon_i - r) \right)^{1-\gamma} \right]. \quad (16)$$

The coefficient  $K$  is chosen to be the solution of (16). In other words,  $b_i$  and  $K$  are the solutions to the system of  $n + 1$  equations, (15) and (16). By definition, the coefficient  $K$  is determined exclusively by the parameters in the capital market; the parameters of the insurance market do not affect this coefficient. It should be mentioned that, from (13), the coefficient  $K = dx_t/dw_t$  has the interpretation of marginal propensity to consumer out of wealth.

Using value function (12), the optimal spending allocation at time  $t$  is (13), the optimal consumption in the hazardous state is

$$c_{1,t} = K [w_t - (1 + r) \ell \pi L / r] / B, \quad (17)$$

and the optimal consumption in the good state is

$$c_{2,t} = M^{1/\gamma} K [w_t - (1 + r) \ell \pi L / r] / B, \quad (18)$$

the optimal investment in a risky asset is (14) and the optimal property insurance is

$$f_t = L - \frac{(M^{1/\gamma} - 1) K}{B} \left[ w_t - \left( \frac{1 + r}{r} \right) \ell \pi L \right]. \quad (19)$$

Notice that all optimal policy functions are linear in  $w_t - (1 + r) \ell \pi L / r$ .

Is the disposable wealth positive with probability one? If so, then, from (17) and (18), consumption in both states are positive at all times and with probability one. Similarly, from (19),  $f_t < L$ , if  $\ell > 1$  (and  $K > 0$ ), i.e., full insurance is not optimal if the insurance market is not actuarially fair. Moreover, from (13), we have  $x_t > \ell \pi L$  with probability one, i.e., the resource optimally allocated to each time period is more than enough to cover the subsistence consumption.

As discussed above, the disposable wealth satisfies the following stochastic



difference equation

$$w_{t+1} - (1+r)\ell\pi L/r = \left[ (1+r)(1-K) + \sum_{i=1}^n b_i(\varepsilon_i - r) \right] [w_t - (1+r)\ell\pi L/r], \quad (20)$$

with expected (gross) growth rate

$$g = E \left[ (1+r)(1-K) + \sum_{i=1}^n b_i(\varepsilon_i - r) \right] = (1+r)(1-K) + \sum_{i=1}^n b_i(\mu_i - r). \quad (21)$$

Even if the initial disposable wealth,  $w_0 - (1+r)\ell\pi L/r > 0$ , is large, a streak of bad luck (with some  $\varepsilon_i$  assuming sufficiently large negative values) would drive the disposable wealth into negative values at some time. That is, all  $\varepsilon_i$ 's must be bounded from below is a necessary condition for positive disposable wealth at all times with probability one. A sufficient condition is  $w_0 - (1+r)\ell\pi L/r > 0$  and

$$\sum_{i=1}^n b_i(r - h_i) < (1+r)(1-K), \quad (22)$$

where  $h_i = \min \varepsilon_i$  with  $r > h_i$ . The economic content of this condition would become clear once  $K$  and  $b_i$  are solved in closed form.

When  $\gamma = 1$ , the value function is of the form

$$J(w_t) = \frac{1}{1-\beta} \{ \log [w_t - (1+r)\ell\pi L/r] \} + K^0,$$

where

$$K^0 = \frac{1}{1-\beta} \left\{ \log(1-\beta) + \frac{\beta}{1-\beta} E \left[ \log \left( (1+r)\beta + \sum_{i=1}^n b_i(\varepsilon_i - r) \right) \right] \right\}.$$

In this case,  $K = 1 - \beta$ ,  $B = \ell$ , and

$$f_t = L - \frac{(\ell-1)(1-\beta)}{\ell(1-\ell\pi)} \left[ w_t - \left( \frac{1+r}{r} \right) \ell\pi L \right] \quad (23)$$

is completely independent of means and variances of risky assets.

## 5. Decoding Coefficients $K$ and $b_i$

To have a better understanding of the coefficients  $K$  and  $b_i$ 's, we apply the Arrow-Pratt approximation to equations (15) and (16). Specifically, let  $E[\varepsilon_i] = \mu_i > r$  and  $E[(\varepsilon_i - \mu_i)^2] = \sigma_i^2$ . By assumption,  $\sigma_i^2$  is small. Since  $\mu_i - r$  is the expected excess return on risky asset  $i$ , we can assume that it is fairly small, a fact supported by the findings of Kocherlakota (1996). Then we can ignore second and higher order terms that involve  $\mu_i - r$  and  $\sigma_i^2$  when we apply the Taylor series expansion about  $g$ , defined in (21), to  $[(1+r)(1-K) + \sum_{i=1}^n b_i (\varepsilon_i - r)]^{-\gamma}$  and  $[(1+r)(1-K) + \sum_{i=1}^n b_i (\varepsilon_i - r)]^{1-\gamma}$ , respectively. In particular, we repeatedly use

$$E[(\varepsilon_i - \mu_i)(\varepsilon_j - \mu_j)] = -(\mu_i - r)(\mu_j - r) \approx 0, \quad \text{if } i \neq j,$$

and

$$E\left[(\varepsilon_i - r) \sum_{j=1}^n b_j (\varepsilon_j - \mu_j)\right] = E\left[(\varepsilon_i - \mu_i + \mu_i - r) \sum_{j=1}^n b_j (\varepsilon_j - \mu_j)\right] \approx b_i \sigma_i^2,$$

since  $\varepsilon_i - r$  and  $\varepsilon_j - r$  are mutually independent.

The approximation allows us to find closed form representation of  $K$  and  $b_i$  in terms  $\mu_i$ ,  $\sigma_i^2$ ,  $r$ , and  $\gamma$ . The results are summarized below with detailed computation shown in the Appendix. Let

$$H = \sum_{i=1}^n \left( \frac{\mu_i - r}{\sigma_i} \right)^2.$$

The marginal propensity to consume is

$$K = 1 - \left\{ \frac{\beta (1+r)^{1-\gamma} [1 + (H/2)(1-1/\gamma)]}{(1-H/\gamma)^{1-\gamma}} \right\}^{\frac{1}{\gamma}}. \quad (24)$$

The expected growth rate of disposable wealth, and the point around which the Taylor theorem applies, is

$$g = \left\{ \frac{\beta (1 + r) [1 + (H/2) (1 - 1/\gamma)]}{1 - H/\gamma} \right\}^{\frac{1}{\gamma}}, \quad (25)$$

with  $g = \beta (1 + r) / (1 - H)$  when  $\gamma = 1$ . Obviously,  $K$  and  $g$  are related by

$$K = 1 - g \frac{1 - H/\gamma}{1 + r}. \quad (26)$$

Lastly, the investment coefficient  $b_i$  is

$$b_i = g \frac{\mu_i - r}{\gamma \sigma_i^2}. \quad (27)$$

Using the approximation, the condition (22) for ensuring that the disposable wealth is positive at all time and with probability one becomes

$$\sum_{i=1}^n \frac{(\mu_i - r) (\mu_i - h_i)}{\sigma_i^2} < \gamma. \quad (28)$$

It is well-known that the Sharpe ratio  $(\mu_i - r) / \sigma_i$  of a risky asset is usually small. The proposed inequality thus requires that the ratio  $(\mu_i - h_i) / \sigma_i$  would be relatively small as well.

An immediate corollary of (28) is  $H < \gamma$  because  $h_i < r$ . Substituting  $H < \gamma$  into (25), the expected growth rate of disposable wealth is positive,  $g > 0$ . It follows that, from (27),  $b_i > 0$  for all  $i$ , i.e., there is no short sale of any risky asset. Inequality (28) implies (22) because inequality (22) can be written as

$$g \left[ 1 - \sum_{i=1}^n \frac{(\mu_i - r) (\mu_i - h_i)}{\gamma \sigma_i^2} \right] > 0,$$

by substituting (24), (25), and (27) into (22).

Substituting  $H < \gamma$  into (24), we have  $K < 1$ , i.e., the marginal propensity to consume is less than one. If, in addition,

$$g < \frac{1+r}{1-H/\gamma}, \quad (29)$$

then, from (26), we have  $K > 0$  and the marginal propensity to consume is positive. When  $\gamma = 1$ , the above inequality is reduced to  $\beta < 1$ . Notice that  $K > 0$  also implies a positive marginal utility of wealth, from (12). The proposed inequality (29) is a joint condition on the subjective discount factor  $\beta$ , the market interest rate  $r$ , the risk aversion  $\gamma$  and the sum of squared Sharpe-ratios  $H$ . Such a condition on exogenous parameters that ensures a positive marginal utility of wealth is standard in continuous-time portfolio models. See, for example, Chang (2004, p.181, eq.(5.18)).

## 6. Insurance Effects

### 6.1. Portfolio Substitution

By definition, the demand for risk-free asset,  $m_t$ , is  $m_t = w_t - x_t - \sum_{i=1}^n \alpha_{i,t}$ . Substituting (13) and (27) into the definition, we arrive at

$$\begin{aligned} m_t = & g \left( \frac{1-H/\gamma}{1+r} - \sum_{i=1}^n \frac{\mu_i - r}{\gamma \sigma_i^2} \right) w_t \\ & + \ell \pi L g \frac{1+r}{r} \left[ \sum_{i=1}^n \frac{\mu_i - r}{\gamma \sigma_i^2} - \frac{1-H/\gamma}{1+r} + \frac{1}{g(1+r)} \right]. \end{aligned}$$

A sufficient condition for a positive investment in the risk-free asset is

$$\frac{\gamma - H}{1+r} - \frac{\gamma}{g(1+r)} < \sum_{i=1}^n \frac{\mu_i - r}{\sigma_i^2} < \frac{\gamma - H}{1+r}. \quad (30)$$

For example, when  $\gamma = 1$  and  $\beta = 1/(1+r)$ , the lower bound in (30) is zero.

What is  $\sum_{i=1}^n (\mu_i - r) / (\sigma_i^2)$ ? In continuous time models, if the utility function exhibits CRRA and asset prices follow geometric Brownian motions, then the share of wealth invested in the  $i$ -th risky asset is a constant  $s_i = (\mu_i - r) / (\gamma\sigma_i^2)$ . See, for example, Merton (1971, 1990) and Chang (2004). In that case, a positive investment in the risk-free asset requires  $\sum_{i=1}^n s_i < 1$ , i.e.,  $\sum_{i=1}^n (\mu_i - r) / (\sigma_i^2) < \gamma$ . The proposed condition (30) is a generalization of this continuous-time result.

Under the condition of (30),  $m_t$  rises with  $\ell$ ,  $\pi$ , or  $L$ . From (14), an increase in  $\ell$ ,  $\pi$ , or  $L$  would lower the disposable wealth and hence reduces the investment in the risky assets  $\alpha_{i,t}$ . In other words, an increase in any insurance parameter, be that the loading factor  $\ell$ , the probability of loss  $\pi$  or the loss  $L$ , would make the consumer more “conservative” in the sense of substituting the risk-free asset for risky assets. The theory thus predicts that, other things being equal, investors living in the areas prone to hazards would invest more in the risk-free asset.

## 6.2. Is Security a Luxury Good?

In the classic, static theory of portfolio selection, Arrow (1965) showed that the risk-free asset (“security”) can never be a necessity good if the utility function exhibits non-decreasing relative risk aversion. His luxury-good result was weakened by Sandmo (1969, p.595) in a two-period model to the conclusion that the wealth elasticity of the risk-free asset is at least as great as that of the risky asset.

In this paper we provide a counterexample to Arrow’s claim and reverse Sandmo’s inequality. Specifically, if property loss is present in the model and the utility function exhibits constant relative risk aversion, then security is a necessity good. This is because, as a function of wealth, the subsistence consumption

generates a positive intercept so that the demand for the risk-free asset is of “Keynesian” form. Moreover, the demand for a risky asset as shown in (14) is linear in wealth with a negative intercept, i.e., the wealth elasticity of any risky asset is greater than unity. Sandmo’s inequality is reversed because all risky assets are luxury goods, while security is a necessity good.

### 6.3. Consumption Falls as Hazard Rises

An increase in  $L$  lowers the disposable wealth, which produces a negative wealth effect on consumption in both states. This intuition is verified as  $\partial c_i / \partial L < 0$ ,  $i = 1, 2$ .

An increase in the loading factor  $\ell$  has two effects on consumption. The first is through disposable wealth, which lowers consumption in both states. The second is through the increase on the relative price of insurance,  $\ell\pi / (1 - \ell\pi)$ , which produces a substitution effect that decreases  $c_1$  and increases  $c_2$ . It suggests that the loading factor is likely to have a negative effect on  $c_1$ . A sufficient condition for this intuition of a negative loading-factor effect on  $c_1$  is  $\gamma\ell\pi < 1$ , because

$$\frac{\partial B}{\partial \ell} = \pi + \frac{M^{1/\gamma} (1 - \gamma\ell\pi)}{\gamma\ell} > 0.$$

Recall that  $\ell\pi$  is the insurance premium for \$1 coverage, which is fairly small. The condition  $\gamma\ell\pi < 1$  is therefore not very restrictive. Since

$$M^{1/\gamma} / B = [1 - \ell\pi (1 - M^{-1/\gamma})]^{-1}$$

rises with  $\ell$ , the net effect on  $c_2$  is ambiguous as expected.

The effect of an increased probability of loss on consumption is generally ambiguous because there are three effects on consumption. The first is the negative

wealth effect on consumption in both states. The second is the relative price effect, which tends to substitute  $c_2$  for  $c_1$ . Again, these two effects tend to lower the consumption in the hazardous state. The third effect is that an increased  $\pi$  alters the marginal rate of substitution between the two states in such a way that the willingness to insure is increased at all points. That is, the third effect on  $c_1$  works in the opposite direction of the first two, and the net effect could be ambiguous.

However, if the investor is not very risk averse, then the third effect, which arises from the willingness to insure, would be relatively small. Then an unambiguous result would emerge. We claim that if  $\gamma \leq 1$ , then  $c_1$  would fall as  $\pi$  rises. To prove it, we note that when  $\gamma = 1$ ,  $B = \ell$  and  $\partial B/\partial \pi = 0$ . In this case, only the (negative) wealth effect matters. When  $\gamma < 1$ , we have  $M^{1/\gamma-1} > 1$  and

$$\begin{aligned} \frac{\partial B}{\partial \pi} &= \ell + \frac{\ell M^{1/\gamma-1} (\ell - 1) - \gamma \ell (1 - \pi)}{\gamma (1 - \ell \pi)} > \ell + \frac{\ell (\ell - 1) - \gamma \ell (1 - \pi)}{\gamma (1 - \ell \pi)} \\ &= \frac{\ell (1 - \gamma) (\ell - 1)}{\gamma (1 - \ell \pi)} > 0. \end{aligned}$$

## 7. Capital Market Effects

A capital market parameter, except the interest rate  $r$ , affects consumption and insurance through the marginal propensity to consume out of wealth,  $K$ . A direct comparison of (17), (18), and (19) shows that the capital market effect on insurance is exactly opposite that on consumption. Insurance is an alternative to current consumption when responding to stock market changes. As such, the interdependence of property insurance on capital market is closely tied to the classic theory of optimal savings under uncertainty.

As shown in Levhari and Srinivasan (1969) and Sandmo (1970), if the utility exhibits decreasing absolute risk aversion (DARA), then an increase in capital risk

produces an income effect and a substitution effect on current consumption. The income effect decreases current consumption because it amounts to a reduction in income. The substitution effect increases current consumption so as to prevent earnings from losing to capital market fluctuation. They showed that the substitution effect dominates the income effect if  $0 < \gamma < 1$ . The result is reversed if  $\gamma > 1$ ; there is no capital risk on consumption if  $\gamma = 1$ .

### 7.1. Property insurance as Saving

The classic theory of optimal savings under uncertainty extends to the current model. It is straightforward to verify, from (24), that  $\partial K/\partial H > 0$  if  $\gamma > 1$ ,  $\partial K/\partial H < 0$  if  $0 < \gamma < 1$ , and  $\partial K/\partial H = 0$  if  $\gamma = 1$ . Since the Sharpe ratio of a risky asset is positively related to  $\mu_i$  and negatively related to  $\sigma_i$ , the effects of the means and variances of risky assets on consumption in either state and on insurance purchase are, for all  $i = 1, 2, \dots, n$ , and  $j = 1, 2$ ,

$$\left\{ \begin{array}{l} \frac{\partial c_j(t)}{\partial \mu_i} > 0 \text{ and } \frac{\partial c_j(t)}{\partial \sigma_i} < 0, \frac{\partial f}{\partial \mu_i} < 0 \text{ and } \frac{\partial f}{\partial \sigma_i} > 0, \text{ if } \gamma > 1, \\ \frac{\partial c_j(t)}{\partial \mu_i} = \frac{\partial c_j(t)}{\partial \sigma_i} = \frac{\partial f}{\partial \mu_i} = \frac{\partial f}{\partial \sigma_i} = 0, \text{ if } \gamma = 1, \\ \frac{\partial c_j(t)}{\partial \mu_i} < 0 \text{ and } \frac{\partial c_j(t)}{\partial \sigma_i} > 0, \frac{\partial f}{\partial \mu_i} > 0 \text{ and } \frac{\partial f}{\partial \sigma_i} < 0, \text{ if } 0 < \gamma < 1, \end{array} \right. \quad (31)$$

The theory predicts that an increase in stock market volatility would make people cut back on their property insurance provided that they are not very risk averse. For people who are somewhat risk averse in the sense of  $\gamma > 1$ , they would increase their protection of their property in a volatile stock market.

The interest rate effect on consumption and insurance has two components. One is the disposable wealth effect that an increase in the interest rate  $r$  increases the disposable wealth  $w_t - \ell\pi L/r$ , and hence consumption. The other is through its



effect on the marginal propensity to consume  $K$ , because it has a negative effect on the Sharpe ratio. These two effects could work in the opposite direction if  $\gamma > 1$ . The unambiguous case is that the interest rate would produce a positive effect on consumption in both states and a negative effect on insurance if  $0 < \gamma \leq 1$ .

## 7.2. Separation Results

There are two special cases in which insurance decision is independent of stock market parameters. One is the benchmark case of fair insurance  $\ell = 1$  (and  $M = 1$ ). The demand for insurance as given in (19) becomes  $f_t = L$ , and the consumer is fully insured. This is an extension of the separability result of, e.g., Dionne and Eeckhoudt (1984), Briys (1986, p.720), and Somerville (2004, p.1134). The other is the case of logarithmic utility function as shown in (23). It should be mentioned that Briys (1986), by introducing a Poisson process of property loss in the asset equation (1), showed that insurance is independent of consumption if  $0 < \gamma < 1$ . Obviously, such a separation result is sensitive to model specification.

## 8. Conclusion

Intuitively, for an investor who is determining the optimal consumption and portfolio rules, the presence of property loss would mean a loss in wealth. If, in addition, she does not diversify her losses over time, then she must allocate resources to each time period more than enough to cover the insurance premium. This paper formalizes this intuition by showing that the insurance premium of property loss plays the role of subsistence consumption in the investor's decision making. Consequently, the risk-free asset becomes a necessity good, and any insurance parameter that increases the premium would produce a portfolio substitution from

risky assets to the risk-free asset.

It is equally intuitive that, when risky assets are included in one's savings portfolio, a consumer who is making saving and insurance decisions must now solve the classic theory of optimal savings under uncertainty, especially its dependence on the attitude toward risk. This paper generalizes the classic result by showing that an increase in uncertainty means an increase in the variance or a decrease in the expected rates of return of a risky asset. Moreover, as an inferior good, the demand for insurance purchase responds to stock market parameters in exactly the opposite way of consumption.

There are at least three directions that we can extend this model. Assuming a general concave utility function instead of a CRRA utility function, we will have to replace the concept of subsistence consumption with the concept of certainty equivalent. Assuming shocks to risky assets are serially correlated, we may no longer have the closed form representation for the value function, which in turn would make the analysis more complicated and render some results ambiguous. Finally, assuming time diversification in the model, we may not have the closed form solution to the problem. We will need other solution methods, including numerical ones to tackle the problem.

## 9. Appendix: Derivation of the Value Function

Since the excess returns  $\{\varepsilon_i - r\}$  are mutually independent,  $\mu_i - r$ 's and  $\sigma_i^2$ 's are small, we have

$$E [(\varepsilon_i - \mu_i) (\varepsilon_j - \mu_j)] = -(\mu_i - r) (\mu_j - r) \approx 0, \quad \text{if } i \neq j,$$

and

$$E \left[ (\varepsilon_i - r) \sum_{j=1}^n b_j (\varepsilon_j - \mu_j) \right] = E \left[ (\varepsilon_i - \mu_i + \mu_i - r) \sum_{j=1}^n b_j (\varepsilon_j - \mu_j) \right] \approx b_i \sigma_i^2.$$

Applying the Arrow-Pratt approximation to  $[(1+r)(1-K) + \sum_{i=1}^n b_i (\varepsilon_i - r)]^{-\gamma}$  about the mean  $g = (1+r)(1-K) + \sum_{i=1}^n b_i (\mu_i - r)$  in (15), we have

$$\begin{aligned} 0 &= E \left[ \left( (1+r)(1-K) + \sum_{i=1}^n b_i (\varepsilon_i - r) \right)^{-\gamma} (\varepsilon_i - r) \right] \\ &= \left( (1+r)(1-K) + \sum_{i=1}^n b_i (\mu_i - r) \right)^{-\gamma} (\mu_i - r) \\ &\quad - \gamma \left( (1+r)(1-K) + \sum_{i=1}^n b_i (\mu_i - r) \right)^{-\gamma-1} b_i \sigma_i^2. \end{aligned}$$

Solving the above equation, we have (27). Using (27),

$$\sum_{i=1}^n b_i (\mu_i - r) = g \sum_{i=1}^n \frac{(\mu_i - r)^2}{\gamma \sigma_i^2} = g \frac{H}{\gamma} = \left[ (1+r)(1-K) + \sum_{i=1}^n b_i (\mu_i - r) \right] \frac{H}{\gamma}.$$

Solving for  $\sum_{i=1}^n b_i (\mu_i - r)$ , we have

$$\sum_{i=1}^n b_i (\mu_i - r) = \frac{(1+r)(1-K) H/\gamma}{1 - H/\gamma},$$

which in turn implies

$$g = (1+r)(1-K) + \sum_{i=1}^n b_i (\mu_i - r) = \frac{(1+r)(1-K)}{1 - H/\gamma}. \quad (32)$$

Next, rewrite (16) as

$$\frac{1-K}{\beta} = E \left[ \left( (1+r)(1-K) + \sum_{i=1}^n b_i (\varepsilon_i - r) \right)^{1-\gamma} \right].$$

Once again, we apply the Arrow-Pratt approximation about the same mean  $g$  to

$[(1+r)(1-K) + \sum_{i=1}^n b_i (\varepsilon_i - r)]^{1-\gamma}$  to reach

$$\begin{aligned} \frac{1-K}{\beta} &= \left( (1+r)(1-K) + \sum_{i=1}^n b_i (\mu_i - r) \right)^{1-\gamma} \\ &\quad - \frac{\gamma(1-\gamma)}{2} \left( (1+r)(1-K) + \sum_{i=1}^n b_i (\mu_i - r) \right)^{-\gamma-1} \sum_{i=1}^n b_i^2 \sigma_i^2 \\ &= g^{1-\gamma} - \frac{\gamma(1-\gamma)}{2} g^{-\gamma-1} \sum_{i=1}^n b_i^2 \sigma_i^2. \end{aligned}$$

Substituting (27) into the above equation, we have

$$\frac{1-K}{\beta} = g^{1-\gamma} - \frac{1-\gamma}{2} g^{1-\gamma} \frac{H}{\gamma} = \left( \frac{(1+r)(1-K)}{1-H/\gamma} \right)^{1-\gamma} \left( 1 + \frac{H}{2} - \frac{H}{2\gamma} \right),$$

using (32). Rewrite the above equation as

$$(1-K)^\gamma = \beta \left( \frac{1+r}{1-H/\gamma} \right)^{1-\gamma} \left( 1 + \frac{H}{2} - \frac{H}{2\gamma} \right),$$

which implies (24). Substituting (24) into (32),

$$\begin{aligned} g &= \frac{(1+r)(1-K)}{1-H/\gamma} = \frac{(1+r)}{1-H/\gamma} \left\{ \beta \left( \frac{1+r}{1-H/\gamma} \right)^{1-\gamma} \left( 1 + \frac{H}{2} - \frac{H}{2\gamma} \right) \right\}^{\frac{1}{\gamma}} \\ &= \left\{ \frac{\beta(1+r)[1+(H/2)(1-1/\gamma)]}{1-H/\gamma} \right\}^{\frac{1}{\gamma}}, \end{aligned}$$

which is (25).

The effect of  $H$  on  $K$  is dictated by

$$\frac{\partial}{\partial H} \left( \frac{1+(H/2)(1-1/\gamma)}{(1-H/\gamma)^{1-\gamma}} \right) = - \left( 1 - \frac{1}{\gamma} \right) \frac{1+H}{2} \left( 1 - \frac{H}{\gamma} \right)^{\gamma-2}.$$

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