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## Simple Analytics and Empirics of the Government Spending Multiplier and Other "Keynesian" Paradoxes

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**Working Paper No. 240**

**“Simple Analytics and Empirics of the Government  
Spending Multiplier and Other “Keynesian”  
Paradoxes”**

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# **Simple Analytics and Empirics of the Government Spending Multiplier and Other “Keynesian” Paradoxes\***

by Casey B. Mulligan

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## **Abstract**

Factor supply increases (depresses) output for many of the same reasons that the government spending multiplier might be less (greater) than one. Data from three 2008-9 recession episodes – the labor supply shifts associated with the seasonal cycle, the 2009 federal minimum wage hike, and the collapse of residential construction spending – clearly show that markets absorb an increased supply of factors of production by increasing output. The findings contradict the “paradox of toil” and suggest that government purchases, and marginal tax rates, reduce private consumption, even during the recession.

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## **I. Introduction**

The Great Recession of 2008-9 brought forth some intriguing claims about public policy and the nature of factor supply. Using “New Keynesian” models to guide the discussion, a number of economists suggested that government purchases might stimulate private spending, rather than crowd it out, thereby increasing total spending more than dollar-for-dollar (Christiano, Eichenbaum, and Rebelo, 2009; Eggertsson, 2009; Woodford, 2011). At the same time, few have evaluated current macroeconomic policies on the basis of the incentives they provide to supply labor and other factors of production. Is it possible that factor supply does not matter during a recession? Or even worse, that our economy suffers from a “paradox of toil:” expansions in factor supply actually reduce aggregate output (Eggertsson, 2010a)?

Economic theory suggests that the government spending multiplier – the marginal effect of government spending on total GDP – and the paradox of toil are related, because both involve the (general equilibrium) relationship between factor supply conditions and private sector factor demand. Models with crowding out through market mechanisms predict that a reduction in the supply of factors to the private sector – either because the government is using some of those factors or because a distortion causes some of the supply to be withheld – ultimately reduces private sector output and factor usage. One market mechanism achieving this result is that private sector employers pass on their factor costs into output prices, which causes their customers to demand less. In “Keynesian” models, this pass through doesn’t happen and perhaps even the high factor rental rates feed back to increased demand for private sector goods.

One approach to these questions would be to use historical data to measure the government spending multiplier (Barro, 1981; Alesina and Ardagna, 2009; Barro and Redlick, 2009; Mountford and Uhlig, 2009; Ramey, 2011) or to measure output effects of factor supply growth. But it has been claimed that historical output responses to government spending impulses ought to be atypical of those that occur today, because today output is far below potential, and monetary policy is fundamentally different than it was in the past (Christiano, Eichenbaum, and Rebelo, 2009; Eggertsson, 2009; Woodford, 2011).

Even without the added burden of estimating a separate multiplier for deep recessions, clear and significant shifts in government demand that are economically similar to the kinds of spending proposed in government “stimulus”

laws are difficult to find, and thereby difficult to translate into an accurate estimate of the government spending multiplier. The purpose of this paper is to exploit the ready availability of obvious factor supply shifts during this recession to test the Keynesian pass-through hypothesis that is at the heart of the paradox of toil and many of the government spending multiplier results. The empirical analysis can be interpreted as tests of whether government spending stimulates private spending that are admittedly indirect, but not reliant on the historical data.

Sections II and III use a variant of the Sidrauski (1967) and Calvo (1983) models to show how the crowding out of private consumption spending is related to the output effect of factor supply shifts, and how both of these differ according to whether output prices are “sticky” rather than clearing the market. As in Woodford (2011), for the purposes of illustration I focus on models in which the output multiplier is one minus the rate of crowding out of private consumption spending, and rule out the possibility that government and private consumption are close substitutes in utility. The “sticky price” model has exactly zero crowding out, as distinct from the “flexible price” case in which crowding out is strictly positive. Section IV explains how an economy with sticky output prices may nonetheless occasionally behave like one with flexible prices, as it might under particular monetary rules. Thus, under the sticky price hypothesis, the government purchases multiplier and the output effect of factor supply vary over time, and might be different during this recession than in previous years.

Section V examines three events that happened *during* this recession, for the purpose of determining whether the outcomes confirm the paradoxes rather than showing significant resource reallocation among competing uses of the economy’s output. Those events are: the labor supply shifts associated with the annual seasons, the minimum wage hike of July 24, 2009, and the collapse of residential construction spending. Section VI concludes.

## **II. A Simple Model for Comparing Flexible and Sticky Price Outcomes**

The economic mechanisms behind the government spending multiplier and other Keynesian paradoxes can be illustrated in a variant of the Sidrauski (1967) model without capital.<sup>1</sup> Because the economic issues to be examined relate to factor supply and nominal prices, my version of the model distinguishes

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<sup>1</sup> As regards the model tastes and technology, my model is a continuous-time extension of Eggertson (2009, 2010) and Woodford (2010) in which seasonal cycles and the money stock appear explicitly, a richer time pattern of taste shocks is modeled, and the production function is potentially concave. For simplicity, I omit one feature of Eggertson (2010): his paper has a rate of time preference that varies stochastically over time, which he analyzes by considering a linear approximation of his model in the neighborhood of the steady state.

leisure or work time from commodities, and includes a numeraire commodity called “money.”  $N_t$  denotes aggregate work hours over the  $t$ th time interval (an interval might, for example, be a month, or a week). In order to consider the effects of changes in one sector’s demand on the amount and composition of total output, I group the other goods in the economy into two categories:  $C$  and  $G$ . As my primary example,  $C_t$  denotes the real quantity of privately purchased goods during time interval  $t$ , and  $G_t$  the real quantity of public purchases.

There is a representative consumer and worker with preferences  $u(c_t, g_t, m_t/P_t, n_t, t)$  that potentially vary over time. In order to obtain sharper analytical results, I consider the limit of continuous time, so  $m_t$  denotes the quantity of money held by the consumer at moment  $t$ ,  $P_t$  the price level ( $m_t/P_t$  is the real money balance), lower case  $n_t$  denotes the consumer’s labor at moment  $t$ , and  $c_t$  and  $g_t$  denote the consumer’s rate of consumption of the two consumption goods. For simplicity, and to be transparent about the meaning of “demand shifts” over time, I assume that the utility flow is additively separable in all four goods:  $u(c, g, m/P, n, t) = \alpha_t u_c(c) + u_g(g) + u_m(m/P) - \gamma_t u_n(n)$ , with  $u_c', u_n', u_n'' > 0$  and  $u_c'', u_m'' < 0$ .<sup>2</sup> The flow is discounted over time at constant time preference rate  $\rho$ .

The preference parameter time paths  $\alpha_t \in [0,1]$  and  $\gamma_t \in [0,1]$  model fluctuations over time in the demand for  $c$  and supply of labor as, for example, they vary over the academic year and the Christmas season. Normalizing one unit of time to be a calendar year, we have  $\alpha_t = \alpha_{t-1}$  and  $\gamma_t = \gamma_{t-1}$  for all  $t$ , which means that the seasonal preference cycle is the same every year. I also assume that  $\alpha$  and  $\gamma$  are piecewise continuous functions of time. For simplicity, the preferences for  $m$  and  $g$  are constant.

$P_t$  denotes the time  $t$  nominal price of  $C_t$  and  $G_t$ . Aggregate consistency requires that  $c_t = C_t$ ,  $n_t = N_t$ ,  $g_t = G_t$ , and  $m_t = M_t$ , where  $M_t$  is the aggregate supply of money at date  $t$ . The details are not considered here, but  $C_t$  and  $G_t$  can be interpreted as composite commodities, produced by many independent firms (see Blanchard and Kiyotaki, 1987, and many subsequent papers using the Dixit-Stiglitz setup). Under this interpretation, the output of each firm contributes to the composite symmetrically, but as imperfect substitutes with each of the others.

Aggregate production  $F(N)$  is strictly increasing and weakly concave in the amount of labor  $N$ :

$$C_t + G_t = F(N_t) \tag{1}$$

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<sup>2</sup>  $u_m(x)$  does not have to be a monotone function: there could be a finite real balance  $x^*$  at which point consumers are satiated. Nor do I rule out the possibility that labor supply is, say, quite inelastic with respect to the real wage.

Firms hire date  $t$  labor at nominal rate  $W_t$ , and workers receive rate  $(1-\tau_t)W_t$ , with the amount  $\tau_t W_t N_t$  going to the government as nominal labor income tax revenue. The government finances the remainder of its spending with a lump sum tax  $L_t$  (or, if  $\tau_t W_t N_t > G_t$ , a lump sum transfer; government spending is the sum of transfers, if any, and government purchases).

With short duration loans available at date  $t$  at instantaneous nominal interest rate  $R_t$ , and money balances earning no interest, the representative consumer has the time zero intertemporal budget constraint:

$$\int_0^{\infty} e^{-\int_0^t R_s ds} [(1-\tau_t)W_t n_t - L_t - P_t c_t - R_t m_t] dt + m_0 = 0 \quad (2)$$

where  $R_t m_t$  is the nominal flow opportunity cost of holding money balances in the amount  $m_t$ . For now, I assume that the second consumption good  $g$  is publicly provided (financed with some combination of lump sum and labor income taxes), so that it does not appear directly in (2).

The consumer's demand for private commodities and supply of labor satisfy two first order conditions equating marginal rates of substitution in utility to  $R_t$  and  $(1-\tau_t)W_t/P_t$ , respectively. When combined with the aggregate consistency conditions, those conditions are:

$$\frac{u'_m(M_t/P_t)}{\alpha_t u'_c(C_t)} = R_t \quad (3)$$

$$\frac{\gamma_t u'_n(N_t)}{\alpha_t u'_c(C_t)} = (1-\tau_t) \frac{W_t}{P_t} \quad (4)$$

The intertemporal first order condition is:

$$-\frac{d}{dt} \ln [\alpha_t u'_c(C_t)] = \frac{u'_m(M_t/P_t)}{\alpha_t u'_c(C_t)} - \rho - \frac{\dot{P}_t}{P_t} \quad (5)$$

where dots indicate derivatives with respect to time. Equation (5) is the familiar consumption Euler equation because the left hand side is the growth rate of marginal utility (times minus one) and the right hand side subtracts the rate of time preference and the inflation rate from the nominal interest rate (see equation (3)).

The profits of the representative firm are nonnegative if and only if:

$$P_t F(N_t) - W_t N_t \geq 0 \quad (6)$$

I follow the New Keynesian literature and consider comparative statics for equilibria that are sufficiently close to a steady state that (6) does not bind. However, some of the empirical results might be interpreted in terms of entry and exit phenomenon for which (6) could be an important part of the analysis.

If each firm could change its price continuously, optimal price setting from the perspective of the representative firm would equate marginal revenue to marginal cost at each moment:

$$\mu = \frac{P_t}{W_t / F'(N_t)} \quad (7)$$

where  $\mu \geq 1$  is a constant reflecting the possibility that the representative firm may face a downward sloping demand for its product (that is, a gap between marginal revenue and  $P_t$ ) and mark up its price accordingly.

Definition (Flexible Price Equilibrium) Given piecewise continuous time paths  $\{G_t, M_t, \tau_t, \alpha_t, \gamma_t\}_{t=0}^{\infty}$ , and a pair of scalars  $(\mu, \rho)$ , a **flexible price equilibrium** is a list of time paths  $\{C_t, N_t, W_t, R_t, P_t\}_{t=0}^{\infty}$  satisfying equations (1), (3), (4), (5), and (7) for all  $t \geq 0$ .<sup>3</sup>

In order to capture the sluggish price response that might occur with staggered price setting, I adopt the variant of the Calvo (1983) setup that has become the workhorse in the New Keynesian macro literature (see Clarida, Gali, and Gertler, 1999, footnote 13, and the references cited therein for a full derivation).  $P_t$  is the aggregate geometric index of the prices set by individual producers facing identical demand curves. Each producer has its price fixed until it is randomly designated to update it. A constant fraction  $\lambda dt > 0$  of producers is so designated during any time interval  $dt$ , and all producers are equally likely to be designated (regardless of their history). As a result, all producers designated to set their price at date  $t$  update their price to the same amount, which I denote  $P_t^*$ . Aggregation tells us that the inflation rate is  $\lambda$  times the log of the ratio of the price updates  $P_t^*$  to the aggregate price index  $P_t$ :

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<sup>3</sup> Throughout the paper, I assume that  $G_t$  is close enough to zero that a flexible price equilibrium exists (that is, I rule out cases in which  $G_t$  ever exceeds the market economy's production capacity). See Baxter and King (1993) for an early quantitative study of fiscal policy in general equilibrium.



$$\frac{d}{dt} \ln P_t = \lambda (\ln P_t^* - \ln P_t) \quad (8)$$

Producers set their price understanding that it may be fixed for some time, and therefore equate the present value of marginal revenue to the present value of marginal cost. The producers setting price at time  $t$  face essentially the same costs and benefits as the producers setting price at time  $t + dt$ , with the exception that the marginal costs at time  $t$  are no longer relevant to the latter price setting group. It is straightforward, but cumbersome, to examine the exact expression for the evolution of  $P_t^*$ , so I follow much of the literature and display a first-order Taylor approximation to that expression that is log-linear in  $P_t^*$  and marginal cost  $W/F(N)$ :<sup>4</sup>

$$\frac{d}{dt} \ln P_t^* = (\lambda + \rho) \left[ \ln P_t^* - \ln \left( \mu \frac{W_t}{F'(N_t)} \right) \right] \quad (9)$$

The differential equations (8) and (9) together imply that the aggregate price index  $P_t$  gradually evolves in the direction of marginal cost (times the markup factor  $\mu$ ), with a speed of adjustment that increases with the density  $\lambda$  of price updates.

Having committed to their price, firms have the choice of whether to produce or not at any time  $t$ , but otherwise must produce whatever their time  $t$  customers demand.

Definition (Sticky Price Equilibrium) Given piecewise continuous time paths  $\{G_t, M_t, \tau_t, \alpha_t, \gamma_t\}_{t=0}^{\infty}$  and three scalars  $(P_0, \mu, \rho)$ , a **sticky price equilibrium** is a list of time paths  $\{C_t, N_t, W_t, R_t, P_t, P_t^*\}_{t=0}^{\infty}$  satisfying equations (1), (3), (4), (5), (8), (9), and the inequality (6) for all  $t \geq 0$ .

In order to examine the paradox of toil and the government spending multiplier, it helps to define a steady state seasonal cycle that would be an

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<sup>4</sup> For example, Woodford (2011, equation 18) has the discrete time version of this expression. Calvo (1983, equation 5a) ignores discounting, and has the continuous time version. The more cumbersome exact expression includes integrals of time paths for interest rates and aggregate output, and is less familiar from the literature. Readers will notice that none of my proofs rely on the linearity of (9); the same results would be found with the exact expression. Moreover, as noted further below, the qualitative results do not derive from the price-setting details, but rather the assumption that supply does not matter at the margin.

equilibrium for the sticky price model when the preference and policy impulses were themselves in a seasonal steady state.

**Definition (Steady State Seasonal Cycle)** Given time paths  $\{M_t, G_t, \tau_t\}_{t=0}^{\infty}$  for the money stock, government consumption, and the marginal tax rate that do not vary year-over-year, and given time paths  $\{\alpha_t, \gamma_t\}_{t=0}^{\infty}$  and a pair of scalars  $(\mu, \rho)$ , a **steady state seasonal cycle** is a list of time paths  $\{\bar{C}_t, \bar{N}_t, \bar{W}_t, \bar{R}_t, \bar{P}_t, \bar{P}_t^*\}_{t=0}^{\infty}$  that are a sticky price equilibrium and do not vary year-over-year. On the time interval  $[0,1]$ , the consumption, labor, and price paths solve the boundary value problem:

$$-\frac{d}{dt} \ln \left[ \alpha_t u_c'(\bar{C}_t) \right] = \frac{u_m'(M_t / \bar{P}_t)}{\alpha_t u_c'(\bar{C}_t)} - \rho - \frac{d}{dt} \ln \bar{P}_t$$

$$\frac{d}{dt} \ln \bar{P}_t = \lambda (\ln \bar{P}_t^* - \ln \bar{P}_t) \tag{10}$$

$$\frac{d}{dt} \ln \bar{P}_t^* = (\lambda + \rho) \left[ \ln u_c'(\bar{C}_t) + \ln \left( \frac{\alpha_t (1 - \tau_t)}{\gamma_t} \right) + \ln \left( \frac{F'(\bar{N}_t)}{\mu} \right) - \ln u_n'(\bar{N}_t) \right]$$

$$\text{s.t. } F(\bar{N}_t) = \bar{C}_t + G_t, \quad \bar{P}_1 = \bar{P}_0, \quad \bar{P}_1^* = \bar{P}_0^*, \quad \bar{C}_1 = \bar{C}_0$$

After substitution from the first order conditions (3) and (4), the three differential equations included in the definition are the sticky price equilibrium consumption Euler equation (5), the price aggregation equation (8), and the price adjustment equation (9). By assumption, none of the impulse variables vary year-over-year, so the state of the system (10) is exactly the same at time 1 as it was at time 0, so the steady state seasonal cycle beyond time 1 satisfies  $\bar{C}_t = \bar{C}_{t-1}, \bar{N}_t = \bar{N}_{t-1}$ , etc.<sup>5</sup>

This paper considers the effects of short run changes in the policy variables  $M, G, \tau$ , so I do not assume that they are constant year-over-year. However, for convenience I do assume that, for large  $t$ , the time paths  $\{G_t, M_t, \tau_t\}_{t=0}^{\infty}$  approach time paths that are constant year-over-year so that sticky price equilibrium paths eventually approach seasonal steady state paths. Thus, a sticky price equilibrium is the solution to the boundary value problem (11):

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<sup>5</sup> When the impulse variables are constant over time – both within and across years – a steady state seasonal cycle is just the more familiar “steady state”: a list of scalars  $(\bar{C}, \bar{N}, \bar{W}, \bar{R}, \bar{P}, \bar{P}^*)$  so that the time paths  $\{C_t = \bar{C}, N_t = \bar{N}, W_t = \bar{W}, R_t = \bar{R}, P_t = \bar{P}, P_t^* = \bar{P}^*\}_{t=0}^{\infty}$  are a sticky price equilibrium given  $\{G_t = \bar{G}, M_t = \bar{M}, \tau_t = \bar{\tau}, \alpha_t = \bar{\alpha}, \gamma_t = \bar{\gamma}\}_{t=0}^{\infty}$ .

$$\begin{aligned}
-\frac{d}{dt} \ln \left[ \alpha_t u_c'(C_t) \right] &= \frac{u_m'(M_t/P_t)}{\alpha_t u_c'(C_t)} - \rho - \frac{d}{dt} \ln P_t \\
\frac{d}{dt} \ln P_t &= \lambda (\ln P_t^* - \ln P_t) \\
\frac{d}{dt} \ln P_t^* &= (\lambda + \rho) \left[ \ln u_c'(C_t) + \ln \left( \frac{\alpha_t (1 - \tau_t)}{\gamma_t} \right) + \ln \left( \frac{F'(N_t)}{\mu} \right) - \ln u_n'(N_t) \right] \\
\text{s.t. } F(N_t) &= C_t + G_t, \quad \lim_{t \rightarrow \infty} \frac{u_c'(C_t)}{u_c'(\bar{C}_t)} = 1, \quad \lim_{t \rightarrow \infty} \frac{P_t}{\bar{P}_t} = 1, \quad P_0 \text{ given}
\end{aligned} \tag{11}$$

where  $\{\bar{C}_t\}_{t=0}^{\infty}$  is the steady state seasonal cycle for private consumption corresponding to the long run steady state seasonal cycle of the impulse variables  $\{G_t, M_t, \tau_t, \alpha_t, \gamma_t\}_{t=0}^{\infty}$ .

### III. Analytics of the Multiplier and Other Paradoxes

Prices are a means by which consumer purchases reflect factor market conditions: in the flexible price model (7) prices always reflect marginal cost, whereas the sticky price model has them move only gradually in the direction of marginal cost. Propositions 1-3 contrast comparative statics of flexible and sticky price equilibria, showing how government purchases and marginal tax rates affect total spending to degrees that depend on the price mechanism. Simply put, supply does not matter at the margin in the sticky price model, so that temporary changes in marginal tax rates do not affect outcomes, and temporary increases in government purchases do not run into supply constraints.

Seasonal cycles in the preference parameters induce seasonal cycles for labor and consumption that depend on whether prices are sticky or flexible, unless the cycles for tastes are exactly offset by cycles in monetary or fiscal impulses. Proposition 4 shows how the seasonal cycles in the flexible and sticky price models are different in the same way, and for exactly the same reason, that the fiscal policy comparative statics are different in the two models.

**Proposition 1 (Crowding Out)** For any  $t \geq 0$ , lump-sum tax financed government purchases  $G_t$ , holding constant government purchases at all other dates, reduce private spending  $C_t$  in the flexible price equilibrium and have no effect on private spending in the sticky price equilibrium.  $G_t$  increases total period  $t$  spending in both cases.

The comparative statics  $dC_t/dG_t$  in the flexible and sticky price cases are, respectively (the notation  $dP_t = 0$  indicates the sticky price comparative static):

$$\frac{dC_t}{dG_t} = \frac{\frac{F''(N_t)}{F'(N_t)} u_n'(N_t) - u_n''(N_t)}{u_n''(N_t) - \frac{F''(N_t)}{F'(N_t)} u_n'(N_t) - u_n'(N_t) F'(N_t) \frac{u_c'(C_t)}{u_c'(C_t)}} \in (-1, 0), \quad \left. \frac{dC_t}{dG_t} \right|_{dP_t=0} = 0 \quad (12)$$

**Proposition 2 (Labor Supply)** For any  $t \geq 0$ , a reduction in the labor income tax rate  $\tau_t$ , financed with a change in lump-sum taxes  $L_t$  and holding constant the tax rate at all other dates, increases labor usage  $N_t$  and private spending  $C_t$  in the flexible price equilibrium and has no effect on employment and private spending in the sticky price equilibrium.

The comparative static  $dC_t/d\tau_t$  in the flexible and sticky price cases are, respectively:

$$\frac{dC_t}{d\tau_t} = \frac{\frac{F'(N_t)}{1-\tau_t} u_n'(N_t)}{-\frac{F''(N_t)}{F'(N_t)} u_n'(N_t) + u_n''(N_t)} \frac{dC_t}{dG_t} < 0, \quad \left. \frac{dC_t}{d\tau_t} \right|_{dP_t=0} = 0 \quad (13)$$

Because  $F(N_t) = C_t + G_t$ , the comparative statics for labor usage are:

$$\frac{dN_t}{d\tau_t} = \frac{1}{F'(N_t)} \frac{dC_t}{d\tau_t} < 0, \quad \left. \frac{dN_t}{d\tau_t} \right|_{dP_t=0} = 0 \quad (14)$$

Proofs of all of the propositions are shown in the appendix. Flexible price equilibrium quantities depend only on contemporaneous tastes, technology, and public policy. A time  $t$  increase (in the comparative static sense) in government purchases has a income effect that reduces time  $t$  flexible price equilibrium consumption and leisure, and raises the time  $t$  price index  $P_t$ . The sticky price model has the price index as its only state variable, so the only way that  $G_t$  or  $\tau_t$  can affect  $P_t$  is by affecting the time path of marginal costs for an extended time interval. Marginal costs at each date depend on the amount consumed and worked at that time, and these behaviors are linked to date  $t$  fiscal policies through the household intertemporal budget constraint, but the lifetime wealth effects of  $G_t$  or  $\tau_t$  are negligible by assumption that the policy change is temporary.<sup>6</sup>

<sup>6</sup> As noted by Woodford (2010), a permanent increase in government purchases would markedly reduce private consumption, even when prices are sticky. A long-lived, but less than permanent, increase in government consumption would reduce private consumption in the short run by

The limiting case of constant consumption preferences  $\alpha = 1$  and extremely sluggish price adjustment  $\lambda \rightarrow 0$  helps to illustrate the proofs of Proposition 1 and 2. As  $\lambda$  gets small, the system (11) evolves more slowly so that the inflation rate and the consumption growth rates are essentially zero. The money demand equation becomes (15):

$$\frac{u_m'(M_t / P_t)}{u_c'(C_t)} = \rho \quad (15)$$

Equation (15) can be inverted to calculate the private “demand” for goods  $C(M/P)$ , which is decreasing in its own price  $P$  relative to money. The shape of this demand function depends only on three components of the utility function:  $\rho$ ,  $u_c'$ , and  $u_m'$ .

The demand for goods  $C(M/P)$  can be used to begin a calculation of the demand for labor at any point in time, because, as long as profits are non-negative, enough labor must be used to satisfy the demand  $C(M/P) + G$ :

$$N_t = \begin{cases} F^{-1}(C(M_t / P_t) + G_t) & \text{if } P_t F(N_t) - W_t N_t \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

Figure 1 graphs labor usage  $N_t$  on its horizontal axis, and the real (pre-tax) wage rate on the vertical axis. Over the range in which profits are non-negative, sticky price equilibrium labor demand does not depend on the real wage rate and the equilibrium amount of labor usage does not vary with labor supply preferences  $\gamma_t$ , or with the marginal tax rate  $\tau_t$ . The government can add to distortions by raising marginal tax rates, imposing minimum wage rules, etc., and, as long as the price level is fixed and profits are non-negative, have no effect on labor usage. Labor market conditions only affect wage rates paid by employers and received by employees.

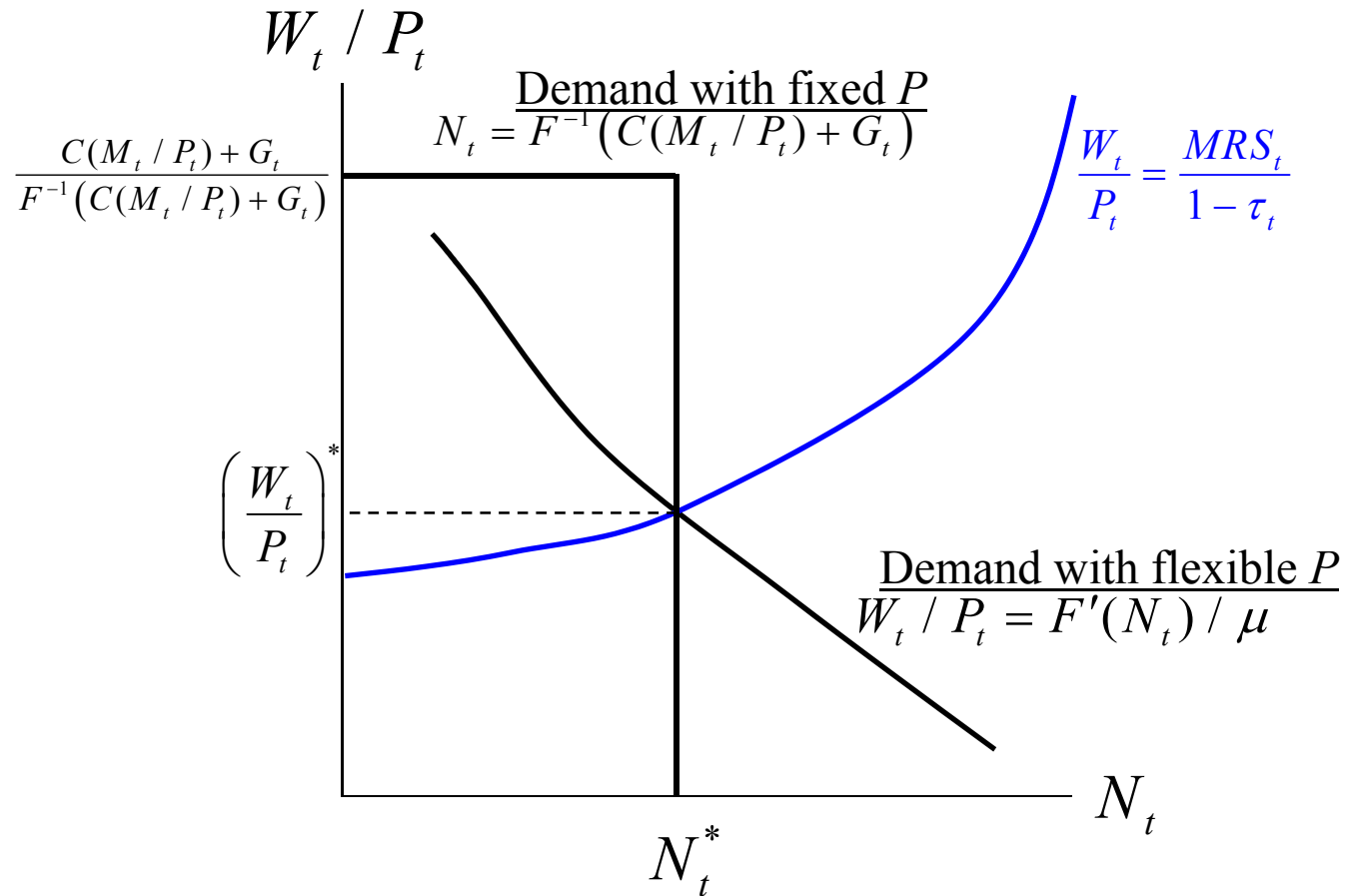
In the flexible price model, prices adjust in response to the various impulses. Producers raise prices in response to an increase in government demand, and this induces the private sector to economize on its spending. At a given employment level, an improvement in labor market distortions reduces the amount employers pay for their labor – regardless of whether output prices are fixed or flexible – and in the flexible price model producers pass on the cost

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something in between that amount and the zero crowd-out reported in Proposition 1 for a momentary increase in the sticky price model. Also note that the propositions describe both an unanticipated fiscal policy change (that is, one occurring at time zero) and an anticipated fiscal policy change (that is, one occurring well after time zero).

## Figure 1. Labor Market Equilibrium with Fixed and Flexible Output Prices

When output price  $P_t$  is fixed, employers demand the number of employees needed to produce the output demanded, shown as the vertical curve in the Figure. When  $P_t$  is flexible, employer demand for labor is elastic according to the marginal product of labor schedule, because wage costs are at least partly passed on to customers (who have elastic demand curves). The Figure displays a single labor “supply” curve that is common to both models.



savings to their customers in the form of lower prices. Producers lower their prices knowing that consumers will demand more, so the producers use more labor in order to have that additional production.

For the purposes of characterizing the flexible price equilibrium, the sticky price labor demand curve (16) is not particularly helpful, because the curve would have to be shifted for every instance of price adjustment. The flexible price analysis features a labor demand or marginal productivity schedule for which movements along include the output price adjustments. The flexible price labor demand curve drawn in Figure 1 is therefore the inverse of equation (7) rather than the inverse (16) of equation (1) used to represent the sticky price equilibrium. In this case, Figure 1 clearly shows that labor distortions reduce flexible price equilibrium employment (see also the proof of Proposition 2).

In both the flexible and sticky price models, Propositions 1 and 2 are closely related because the crowding out of activity in one sector as a result of demand in another is about factor supply. In both models, factors of production are needed to satisfy government demand, but only in the flexible price model does competition for factors cause the private sector to economize on its factor usage.

The aggregate effects of unemployment benefit payments financed with lump sum taxes are one application of Proposition 2, because unemployment benefits are a transfer payment with positive (implicit) marginal labor income tax rates.<sup>7</sup> Economists debate the magnitude of the incentive effects, but they generally agree that unemployment benefits normally reduce aggregate employment. But the 2008-9 recession has been said to be abnormal in this regard: as one economist put it, “Traditionally, many economists have been leery of prolonged unemployment benefits because they can reduce the incentive to seek work. But that should not be a concern now because jobs remain so scarce.”<sup>8</sup> One way to rationalize this view: the economy is often adequately described by the flexible price model, but during a recession the sticky price model offers the better description. Proposition 2 shows how, in this case, the payment of unemployment benefits would normally reduce aggregate employment, but would not reduce it during a recession. This is one reason why it is important to have empirical evidence on recession-era aggregate effects of labor supply.

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<sup>7</sup> The marginal tax rate is positive because unemployment benefits cease once the beneficiary becomes employed, which affects the beneficiary’s tradeoff between unemployment and employment (Meyer, 1990). Unemployment benefits are an example of government spending that is not government purchases: holding constant the marginal tax rate, transfer payments do not reduce private consumption even in the flexible price model.

<sup>8</sup> As quoted by Eckholm (2009).

Proposition 3 (Tax Contraction) Assuming that the economy is on the upward sloping part of the Laffer curve, labor income tax financed government purchases  $G_t$  can reduce labor usage  $N_t$  and total spending  $C_t + G_t$  in the flexible price equilibrium but necessarily increase them in the sticky price equilibrium.

By definition of “upward sloping part of the Laffer curve”, an increase in the labor income tax rate  $\tau$  for a given amount of government purchases requires budget balance via a decrease in lump sum taxes, rather than an increase or no change. In this case, a labor income tax financed increase in government purchases has an income effect (Proposition 1) and a substitution effect (Proposition 2). In the flexible price model, both effects are in the direction of reducing private consumption and the two effects on labor are in opposite directions. The Appendix offers an example in which the substitution effect dominates: labor income tax financed government purchases  $G_t$  reduce labor  $N_t$  and therefore reduce total spending  $C_t + G_t$ . In the sticky price model, neither the wealth nor substitution effects of  $G_t$  impact  $C_t$ , so total spending necessarily increases with  $G_t$ .

At first glance, a “stimulus” law that had the government purchase goods and services and finance those purchases with public debt would seem to be better described by Proposition 1 than Proposition 3, because the former holds marginal tax rates constant. However, in practice much “stimulus” spending raises the marginal tax rate because the government purchases are targeted toward persons with low incomes.<sup>9</sup> In this case, Proposition 3 helps frame the debate about the aggregate effects of stimulus laws: if the recession economy is described by the sticky price model, then the incentive effects of stimulus spending do not matter and that spending does not crowd out private spending. In the flexible price model, stimulus spending crowds out private spending and may ultimately reduce aggregate labor usage.

The government purchases multiplier of exactly one, and the exactly zero employment effect of labor market distortions help illustrate a number of paradoxes that arise in public policy discussions, but readers should recognize that other factors can push the multipliers up or down. For example, the government purchases multiplier would be lower in both models if those purchases were close substitutes with private purchases (Barro, 1981, p. 1091 has such a model). Depending on the future of government purchases and tax rates, the government

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<sup>9</sup> Formally, a means test can be modeled by writing the government transfer  $L$  as a lump sum minus a linear function of labor income  $Wn$ . Once substituted into the representative household’s intertemporal budget constraint (2), note that both the means test and labor income tax terms are linear functions of labor income, and interpret their combined coefficients as “the marginal tax rate.” See Mulligan (2010a) for an instance from this recession when a means-tested transfer created very large marginal tax rates.



purchases multiplier could be greater than one in both flexible price and sticky price models with capital to the degree that investment reacts in the short run to the anticipation of greater labor usage in the long run (Aiyagari, Christiano, and Eichenbaum, 1992), even while private consumption is crowded out. In a model of heterogeneous preferences, government spending could have the additional effect of redistributing purchasing power from households with a strong preference for money balances to households with a weak preference, which would increase consumption demand at a given price. In this way, additional government purchases or additional labor market distortions could actually increase private consumption spending in variations of the sticky price model.

Because the magnitude of the government spending multiplier depends on the importance of supply and demand at the margin, the seasonal cycle is related to the multiplier. In order to make the comparison more formally, it helps to define a “short season” analogous to the short duration fiscal policy shocks examined in Propositions 1-3:

Definition (Short Seasonal) Let the money stock, government consumption, and marginal tax rates be constant within and across years. There are two perpetually recurring alternating seasons of duration  $S$  and  $1-S$ , with the first season commencing at time zero. The preference parameters  $\alpha$  and  $\gamma$  vary over the seasons, and are constant within seasons and constant year-over-year. A **short seasonal** steady state is a list of on-season and off-season outcome values  $\bar{C}_{on}, \bar{N}_{on}, \bar{W}_{on}, \bar{R}_{on}, \bar{P}_{on}, \bar{C}_{off}, \bar{N}_{off}, \bar{W}_{off}, \bar{R}_{off}, \bar{P}_{off}$ , respectively, such that the seasonal steady state cycle corresponding to the assumed impulses approaches, in the limit as  $S \rightarrow 0$ , the time paths that take on the on-season values for the first fraction  $S$  of each year and the off-season values otherwise.

In the flexible price model, consumption and labor at any point in time depend on both the preferences for consumption and the preferences for labor supply, according to the condition relating the marginal product of labor to the marginal rate of substitution:  $\mu\gamma_t u_n'(N_t) = \alpha_t u_c'(F(N_t) - G_t)(1 - \tau_t)F'(N_t)$ . But Proposition 4 shows how seasonal fluctuations in the sticky price model depend only on the seasonal for demand:

Proposition 4 (Seasonal Supply and Demand) The short seasonal fluctuations in consumption and labor depend only on the seasonal fluctuations in the consumption preference parameter, and not on the seasonal fluctuations in the labor preference parameter.

The proof of Proposition 4 shown in the Appendix is related to the proofs of Propositions 1 and 2: labor supply can only affect consumption through prices, which do not change in the short run of the sticky price model. At the same time, prices are the means by which the output effects of consumption demand changes are mitigated by crowding out, but prices do not change in the short run in the sticky price model. As a result, the sticky price model predicts that labor usage is insensitive to labor supply and highly sensitive to demand.

Proposition 4 can be applied to a season like the summer when, for a short duration of time, a larger fraction of the population is available to work. If the summer surge in labor supply is not offset by monetary or fiscal policy – and thereby prices have to fall in order for the market to absorb the additional workers – then there will be no labor usage surge in the sticky price model. It can also be applied to a season like Christmas when the demand for goods is high for a short duration of time. If the Christmas demand surge is not offset by monetary or fiscal policy – and thereby prices have to rise in order for the market to voluntarily supply the extra demand – then the Christmas labor surge will be larger in the sticky price model than in the flexible price model.

#### **IV. Market Clearing Mechanisms**

A rich literature has examined historical data in order to determine whether government purchases crowd out private purchases, and whether labor market distortions like taxes, minimum wages, and the moral hazards of unemployment insurance, reduce labor usage and output (e.g., the studies cited in Moffitt, 2002). But it is claimed that these empirical results are neither informative about the effects of fiscal policy during the 2008-9 recession nor corresponding to comparative statics like those I examined in Propositions 1-3. In particular, the stock of money might “normally” respond to changes in tastes, technology, and fiscal policy in the direction of stabilizing prices, in large part because of deliberate actions by the monetary authority, but Christiano, Eichenbaum, and Rebelo (2009) and others suggest that the usual kinds of monetary adjustments could not occur during this recession because of the “zero interest lower bound.”<sup>10</sup>

It is true that monetary policy has historically responded to oil shocks (Bernanke, Gertler, and Watson, 1997), the seasonal cycle (Sharp, 1988), and other changes in economic fundamentals. “Taylor rules” for targeting the federal funds rate prescribe such responses, and are said to characterize actual postwar monetary policy in the United States (Clarida, Galí, and Gertler, 2000). Thus, the

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<sup>10</sup> In this regard, McGrattan and Ohanian’s (2008) analysis of the World War II multiplier is especially relevant because short term government securities also had near zero yields at that time.

historical effects of fiscal policy on private spending could be a combination of a direct effect – the comparative statics featured in my Proposition 1-2 are direct effects – and a possible indirect effect through changes in monetary policy. For example, while an increase in the marginal tax rate  $\tau_t$  does not directly reduce labor usage in the sticky price model (Proposition 2), it does create “inflationary pressures,” and the monetary authority might have reacted in the past to those pressures by reducing the money stock below what it would have been.<sup>11</sup> Because less money means less consumption and labor in the sticky price model, a higher marginal tax rate could reduce labor usage in the sticky price model through this mechanism. If the effects of fiscal policy on the money stock were enough to fully neutralize inflationary pressures created by fiscal policy, then fiscal policy would have the same ultimate labor effects in the fixed and flexible price models.

It is also true that money markets behaved quite differently during the 2008-9 recession than they had in the past. For example, the federal funds rate throughout 2009 was close to zero and, contrary to prior years, well above what the Taylor rule prescribed (Rudebusch, 2009). For years, the amount of reserves of depository institutions held with the Federal Reserve system corresponded closely to the amount of required reserves, but in late 2008 excess reserves increased by a factor of 400 in a matter of four months (at the same time the fed funds rate fell to zero, see <http://research.stlouisfed.org/>). During those same four months, consumer prices fell four percent (that is, deflation at a 12 percent annual rate, see [www.bls.gov](http://www.bls.gov)).

Thus, it is unlikely that either monetary policy or other money market events even approximately eliminated deflationary pressures during the 2008-9 recession, and unlikely that they would offset any change in those pressures that might have been created by fiscal policy, or changes in tastes and technology, during the recession. Perhaps this monetary state of affairs occurred because a zero lower bound on the fed funds rate,<sup>12</sup> but in any case it suggests that the money-stock constant comparative statics examined in my Propositions 1-3 would better describe the effects of shocks to labor market distortions and spending during the recession than it would during the previous years. It also suggests that evidence on the effects of changes in supply and demand during the recession would be especially valuable for determining whether the fixed or flexible price model better describes the recession-era effects of supply and demand shocks.

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<sup>11</sup> In the context of my model, inflationary (deflationary) “pressure” means a price level that is less (greater) than the representative firm’s expected future marginal revenue products of labor divided by the nominal wage rate, respectively. By equations (8) and (9), the price level is rising (falling), respectively.

<sup>12</sup> As noted by Curdia and Woodford (2010) and Ohanian (2010), the nominal interest rate in the model’s consumption Euler equation is not the same as the fed funds rate, and the gap between the two may well have changed during the recession because financial intermediaries were under stress.

The “sticky” versus “flexible” price dichotomy has received much attention in macroeconomic theory over the years, and that attention has spawned a number of empirical studies of whether actual prices are sticky (Davis and Hamilton, 2004; Nakamura and Steinsson, 2008). However, the real issue here is whether *something* in the economy operates to reallocate output among competing uses in response to changes in tastes, technology, or public policy. For the purpose of applying the paradoxes, the real question is whether government purchases somehow significantly crowd out private spending and whether consumers somehow consume significantly less when the producers of the consumer goods find it more difficult to hire.

Monetary policy is not the only such mechanism. For example, the U.S. government purchased military equipment during World War II, while it also put controls on consumer prices. In a flexible price world, one private sector response would be an increase in the prices of private sector goods (autos, refrigerators, etc.) that would be produced with many of the same resources used by the military equipment sector. In fact, the government ordered that former consumer durable factories be converted to military equipment production, and rationed many of the consumer durables that were put out of production. The end result was that government purchases significantly reduced private spending, which is a result that accords with the flexible price model rather than the sticky price model (Barro, 1987).

Or consider an increase in the minimum wage that raises employers’ costs of hiring. In the flexible output price model, consumers would ultimately purchase a lesser volume of goods because the producers of those goods pass their added employment costs into output prices. But in reality other mechanisms could produce this response, even (especially) if output prices were fixed. For example, prior to the minimum wage increase, a fraction of producers might have had a price that barely covered their variable costs, and the minimum wage increase pushes them to cease production all together. To the degree that the goods going out of production were imperfect substitutes in utility with the remaining goods, total production and labor usage would fall.

In summary, the government spending multiplier depends critically on whether factor market costs are somehow passed through to consumers, and whether this pass-through occurs during recessions. Pass-through can occur through a variety of mechanisms, so the empirical question is whether private consumption falls when tastes, technology, or public policy changes during a recession affect the factor costs of the firms producing those goods.

## V. The Great Recession Economy Resembles a Flexible Price Economy

A contribution of this paper is therefore to consider three events that happened *during* this recession, and examine whether the outcomes confirm the paradoxes rather than showing significant resource allocation among competing uses of the economy's output. Those events are: the labor supply shifts associated with the annual seasons, the minimum wage hike of July 24, 2009, and the collapse of residential construction spending. The events are used to test the hypothesis that factor supply expands output, both at the industry and aggregate levels.

### *V.A. The Seasonal Cycle Proceeded as Usual*

Gauti Eggertsson (2010a, p. 1) poses the hypothetical question "What happens [if] everyone wakes up [one day] with exactly the same idea: Let's go out and look for some more work?" He suggests that partial equilibrium answers to this question are highly misleading, and that the answer is that aggregate employment may *fall*, at least if the macroeconomy were caught in a liquidity trap, much like the one purported to characterize the U.S. economy during the recession of 2008-9.

The end of the academic year is remarkably similar to the question posed (Mulligan, 2009). Schools vary somewhat on the exact day that their academic years end, but during the month of May academic years end rather abruptly around the United States, and many of the teenage students storm into the job market to look for work. Academic years begin just as abruptly in late August and early September.

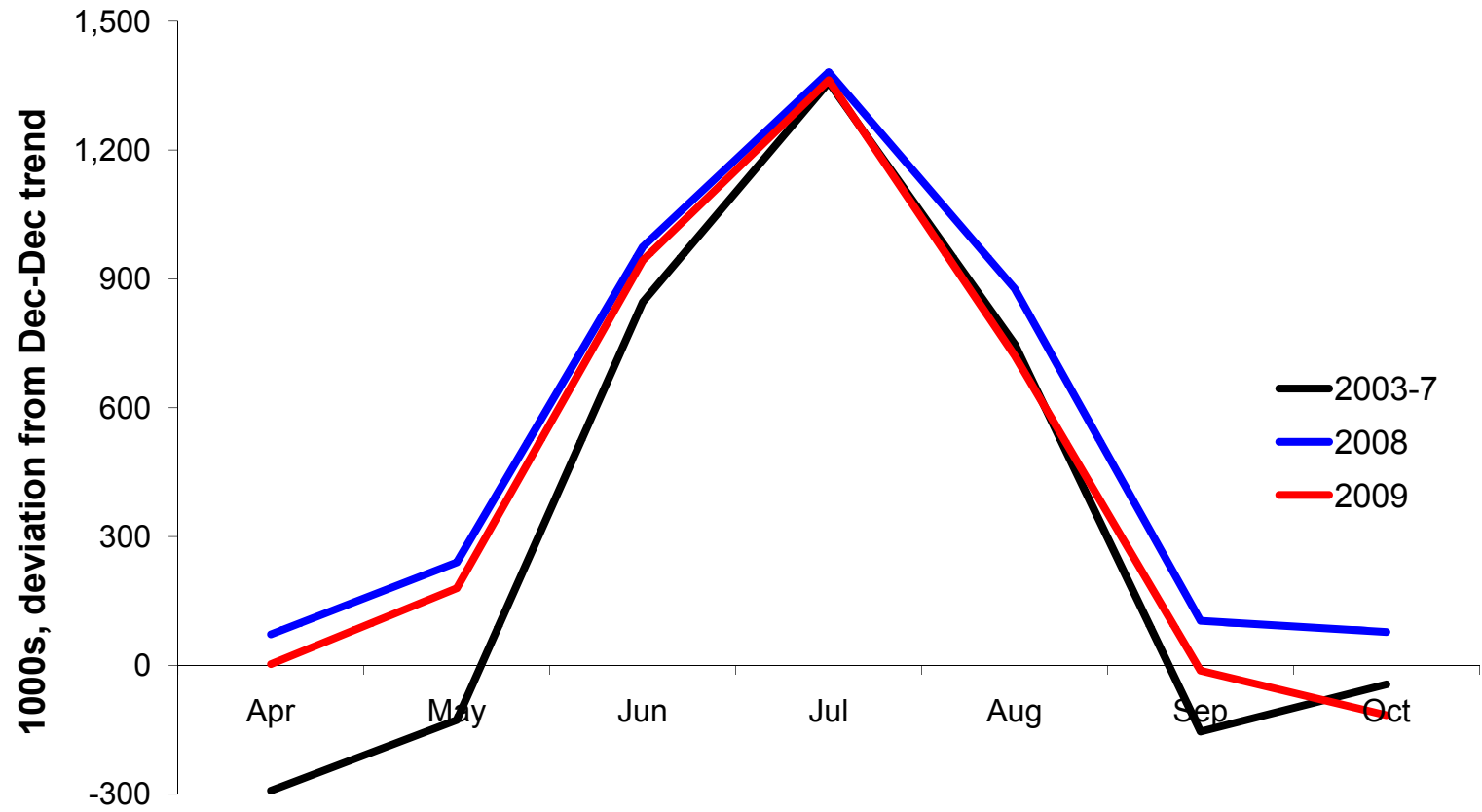
To see how the actual labor market responds to such an event, consider the United States Census Bureau's monthly household survey, whose employment totals have been summarized by the Bureau as national aggregates for each of several age groups. I have used their seasonally *unadjusted* series for persons aged 16-19 to calculate monthly employment deviations from each year's December to December trend. Those deviations are averaged for the five years 2003-7 prior to the current recession, and the April-October result displayed as the black series in Figure 2.<sup>13</sup> For example, a value of 1357 for July means that July teen employment was 1,357,000 above the December to December trend, on average for 2003-7. Figure 3 shows the same calculation for total employment (all ages 16+). Figure 4 shows teen unemployment.

Teen employment is sharply higher in June, and sharply higher again in July, for a total April-July increase of 1,649,000 teen employees. Figure 3 shows

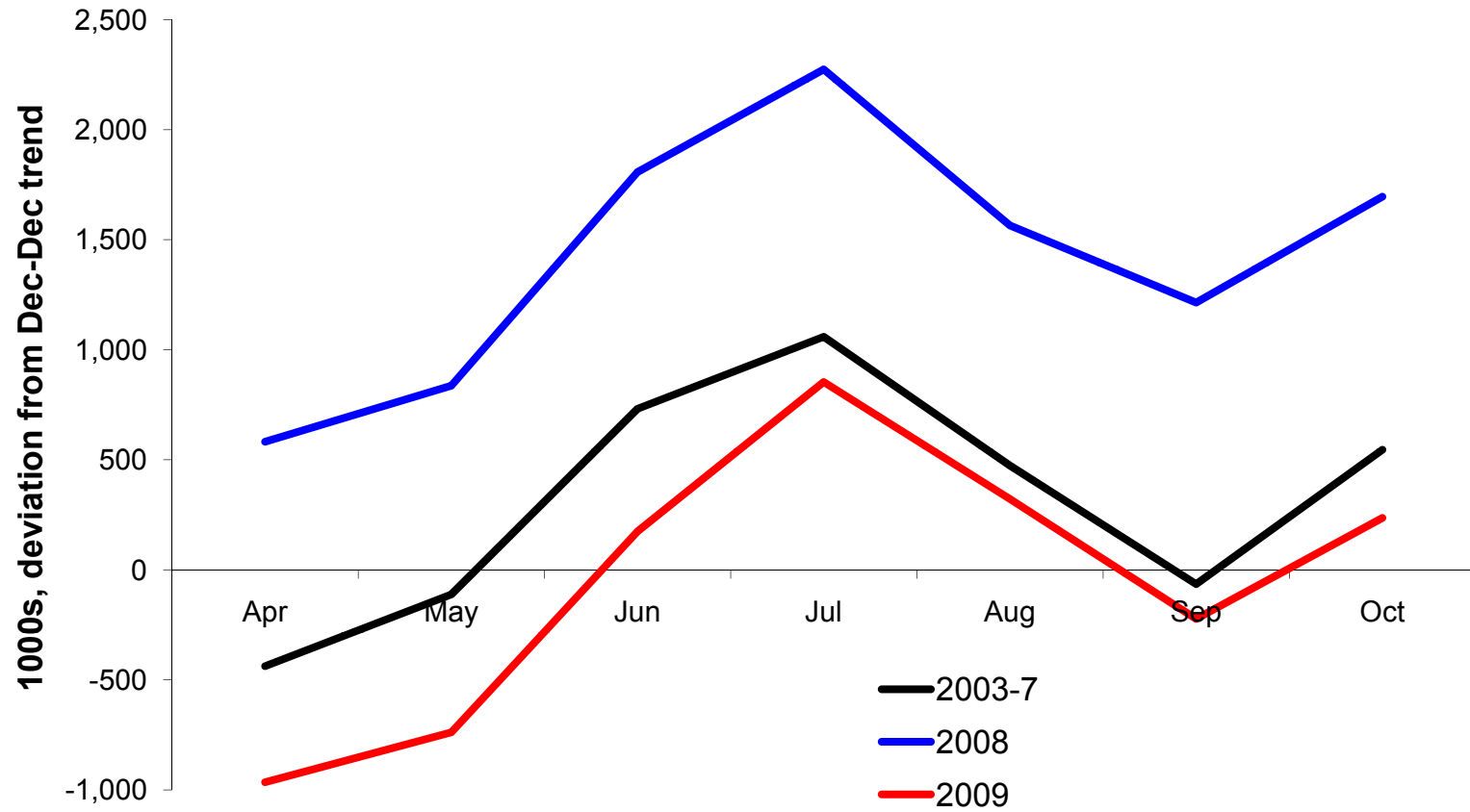
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<sup>13</sup> The prior literature on teen summer employment has used April as its academic year benchmark (U.S. Bureau of Labor Statistics, 2009).

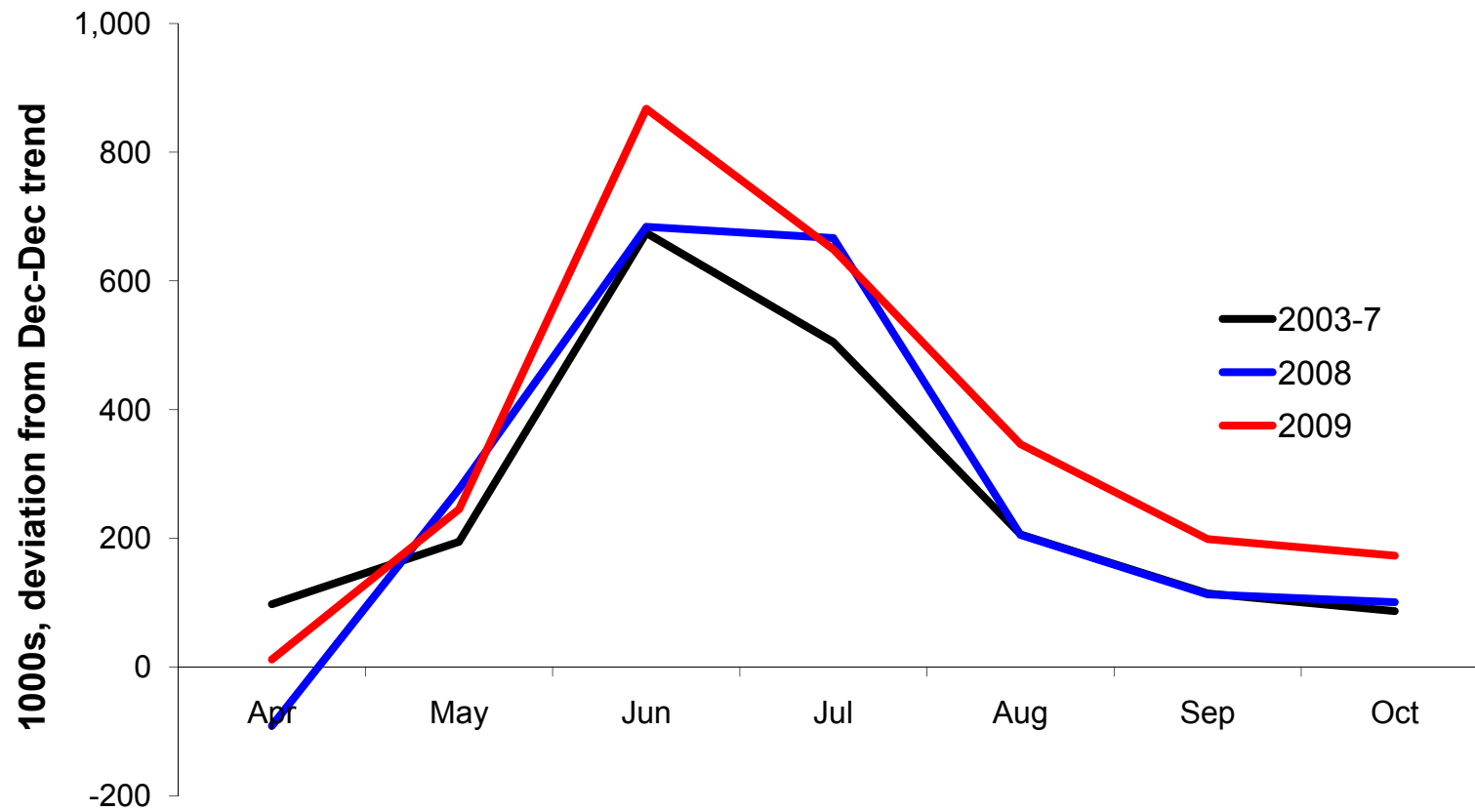
**Figure 2. Teen Employment by Month**



**Figure 3. Employment by Month, All Ages 16+**



**Figure 4. Teen Unemployment by Month**





how total employment also increases significantly, so the teen rush into the labor market does not merely reallocate jobs from older persons to teens.

The summer teen employment surge is largely a consequence of seasonality in supply, not demand.<sup>14</sup> To see this, note that a pure summer demand surge would draw teens into the labor market with low teen summer unemployment, high summer real wages, and low summer unemployment among persons not enrolled in school during the academic year.<sup>15</sup> Figure 4 shows that, in fact, teen unemployment spikes in June as the labor market absorbs more than one million teens. Unemployment of persons aged 25 and older (not shown in the figures) is high throughout the summer, peaking in July at almost 700,000 persons above trend. Median nominal and real weekly wages for teens are often at their lowest of the year in the third quarter (July – September), and presumably hourly wages are even lower due to longer teen summer work weeks. These patterns reverse when the academic year ends.

Also consistent with the supply interpretation, Mulligan (2010b) shows how age groups with the largest summer log employment and log unemployment spikes are those with the greatest school enrollment rates during the academic year, and the summer log employment spike may even be negative for groups with near zero school enrollment. Nor do many of the summer jobs for teens appear to be in industries that have a significant spike in labor demand, because 77% of those jobs are in industries that expand their employment of persons aged 25-34 less than two percent, if at all.<sup>16</sup>

The recession years of 2008 and 2009 were no different in this regard: the academic year came to an end as it usually does, and got started again in the fall. Figures 2, 3, and 4 display series for each of 2008 and 2009 in blue and red, respectively. Consistent with the flexible price model, both teen employment and total employment increased significantly at the beginning of the summer, and fell back to trend when summer ended. The summer teen employment spike is a bit smaller in 2008 and 2009 than it was in prior years, and the summer total

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<sup>14</sup> For the purposes of testing the fixed versus flexible price models, it is not necessary to assume that the summer surge is only the result of supply (see Mulligan, 2010b, for a formal analysis of this point).

<sup>15</sup> The hypothetical demand surge would also have to be quite large – about as large as doubling the size of the nation’s military in a mere two months – because the end result is about a million new jobs for teens.

<sup>16</sup> These are based on calculations using the May, July, and September 2005 Current Population survey. The top industry hiring teens in the summer was “arts, entertainment, and recreation” (accounting for 19 percent of the teen summer jobs), which had no change in the number of persons aged 25-34 employed. The second industry (also accounting for 19 percent) is “accommodation and food services,” which actually cut its employment of persons aged 25-34 by 4 percent during the summer.

employment spike is a bit larger in 2009.<sup>17</sup> These data provide no support for the sticky price model hypothesis that the annual rush of teens into a recession labor market would fail to increase employment, and no support for Eggertsson's hypothesis that it would decrease employment.

*V.B. The 2009 Minimum Wage Hike Reversed the Trend for Part-time Work, and Further Reduced Low-Skill Full-time Employment*

Taken literally, the variable  $\tau$  in my model is the labor income tax rate because it enters the government budget constraint. However, with an adjustment of the lump sum tax term  $L$  that does not affect any of the model's first order conditions,  $\tau$  can be interpreted as anything that drives a wedge between the marginal rate of substitution and the marginal product of labor, or anything that shifts the marginal rate of substitution in the direction of less labor supply. Under any of the interpretations, an increase in the labor market distortion  $\tau$ , such as an increase in a binding minimum wage rate, raises employer costs. The flexible price model says that the higher costs are passed on to consumers, who demand less product from those employers, and the employers reduce their labor usage. The sticky price model predicts no labor usage effect.

In July 2007, the federal minimum hourly wage was increased for the first time in 10 years, to \$5.85 from \$5.15. It was increased again a year later to \$6.55, and increased yet again on July 24, 2009 to \$7.25 (Dept. of Labor, 2009). Consumer prices were generally rising prior to the summer of 2008, but fell 2.1 percent from July 2008 to July 2009.<sup>18</sup> Thus, the real minimum wage hike was large in July 2009, and began from the highest base, and is therefore expected to have the largest effect on the costs of firms that employ low-hourly-wage workers.

Part-time and teen employees are especially likely to have hourly wages near the federal minimum. Table 1 displays the number of persons who are paid an hourly wage at or below the federal minimum wage, expressed as a percentage of employment, for selected demographic groups reported by the Census Bureau for 2008, the last full year that the federal minimum wage would be below \$7.25. The percentage is 6.0 for all employees aged 16-24, and is presumably even greater for the narrower group of teens (ages 16-19). Part-time employees also

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<sup>17</sup> Mulligan (2010b) analyzes these patterns over a longer time frame, and for various age groups, regressing the summer log employment spike on a smooth function of time and a dummy variable for recessions, finding no statistically or economically significant difference between the summer spike in recessions and the summer spike in other years. He also finds that, contrary to the sticky price theory, the Christmas demand shock does not have larger employment impacts during recessions.

<sup>18</sup> The July CPI (NSA) for all items was 219.964 and 215.351 in 2008 and 2009, respectively ([www.bls.gov/cpi](http://www.bls.gov/cpi)).

**Table 1. Hourly Workers at or Below the Federal Minimum Wage, 2008**

Selected Groups of Wage & Salary Workers

<u>Group</u>	<u>Percentage who work hourly at or below Fed. Minimum Wage</u>
Employed, Ages 16-24	6.0%
Employed Part-time, Ages 16+	6.0%
Employed, Ages 16+	1.7%
Employed Full-time Female, Ages 16+	1.1%
Employed Full-time Male, Ages 16+	0.6%

Source: Census Bureau. *Labor Force Statistics from the Current Population Survey* .

Numbers employed from Table A-18.

Numbers who work hourly at or below federal minimum wage from Table 44.

have about a six percent incidence of earning at or below the federal minimum. Full-time employees, especially men working full-time, were quite unlikely to earn minimum wage. Based on the patterns shown in the Table, the July 2009 federal minimum wage hike is expected to affect, if anything, the employment of teens and part-time workers, and to have little effect on the number of adults employed full-time.

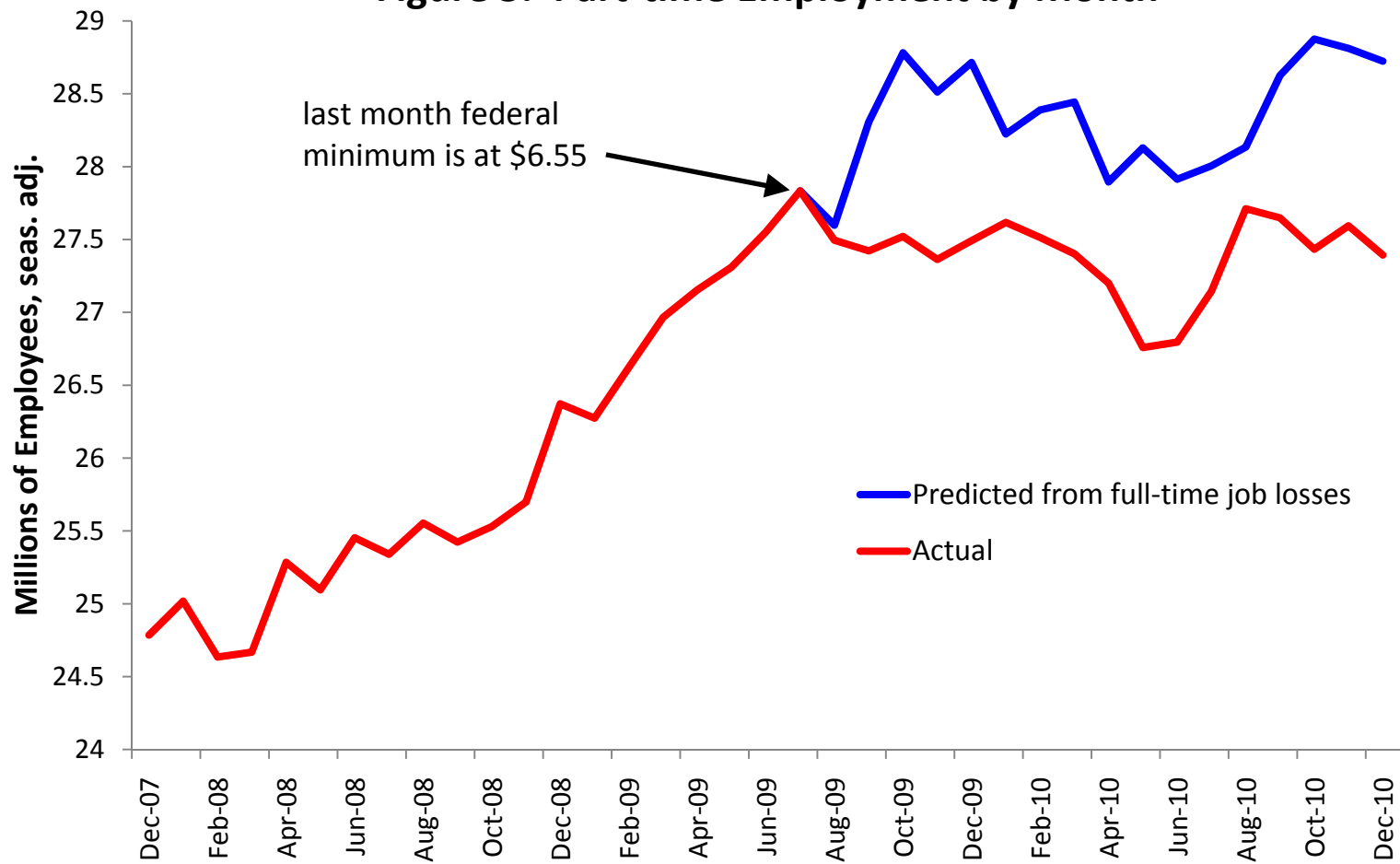
Figure 5's red series displays seasonally adjusted national part-time employment by month, from the Census Bureau's monthly household survey. Prior to July 2009, part-time employment increased by about 3 million during the recession. July 2009 was the peak level of part-time employment. In order to investigate the possibility that the July 2009 hike stopped further increases in part-time employment, and perhaps affected other employment categories, I estimated an auto-regressive monthly model of national part-time and full-time employment per capita for each of twelve demographic groups distinguished according to race, gender, and age (white vs. nonwhite, male vs. female, and 16-19 vs. 20-54 vs. 55 and over):

$$\begin{aligned}
 \left( \frac{PT_{i,t}}{POP_{i,t}} \right) &= a_i^{PT} + \beta_i^{PT} \left( \frac{PT_{i,t-1}}{POP_{i,t-1}} \right) + b_{0i}^{PT} \left( \frac{FT_{0,t}}{POP_{0,t}} \right) + b_{1i}^{PT} \left( \frac{FT_{0,t-1}}{POP_{0,t-1}} \right) \\
 &\quad + \delta_{0i}^{PT} \left( \frac{FT_{i,t}}{POP_{i,t}} \right) + \delta_{1i}^{PT} \left( \frac{FT_{i,t-1}}{POP_{i,t-1}} \right) + \varepsilon_{it}^{PT} \\
 \left( \frac{FT_{i,t}}{POP_{i,t}} \right) &= a_i^{FT} + \beta_i^{FT} \left( \frac{FT_{i,t-1}}{POP_{i,t-1}} \right) + b_{0i}^{FT} \left( \frac{FT_{0,t}}{POP_{0,t}} \right) + b_{1i}^{FT} \left( \frac{FT_{0,t-1}}{POP_{0,t-1}} \right) \\
 &\quad + \delta_{0i}^{FT} \left( \frac{PT_{i,t}}{POP_{i,t}} \right) + \delta_{1i}^{FT} \left( \frac{PT_{i,t-1}}{POP_{i,t-1}} \right) + \varepsilon_{it}^{FT}
 \end{aligned} \tag{17}$$

where  $PT_{it}$ ,  $FT_{it}$ , and  $POP_{it}$  denote demographic group  $i$ 's month  $t$  part-time employment, full-time employment, and population, respectively, and the  $a$ 's,  $\beta$ 's,  $b$ 's and  $\delta$ 's are regression coefficients. Demographic group  $i = 0$  is comprised of prime-aged (ages 20-54) white males. Each demographic group's two per capita not-seasonally-adjusted employment series<sup>19</sup> were separately seasonally adjusted by taking the residual from regressions of log per capita employment on twelve month dummies over the fifteen year period 1993 – 2007. Using the seasonally adjusted series, I estimated the model (17) over the period January 2004 – July 2009. Holding fixed the post-hike time series for population and the number of

<sup>19</sup> Census Bureau (various issues). CPS Monthly Tables A-13 and A-18.

### Figure 5. Part-time Employment by Month



full-time positions held by prime-aged males,<sup>20</sup> I used the model to dynamically forecast part-time and full-time employment for each demographic group for August 2009 through December 2010 (by construction, the full-time forecast for prime-aged males coincides with the actual). The aggregate deviation of the part-time predictions from the actual was added to the red series in Figure 5 to arrive at the aggregate part-time prediction shown as Figure 5's solid blue series.<sup>21</sup>

After falling 9.3 million during the recession through July 2009, aggregate full-time employment fell another 1.8 million by the end of the year, and still remained below July 2009 levels at the end of 2010. Some people laid off from their full-time jobs likely had trouble finding another suitable full-time job, and some of them had been working part-time while they searched.<sup>22</sup> Part-time jobs pay less than full-time jobs — even on an hourly basis — so some employers may also be using part-time employees to accomplish tasks where they previously might have used full-time jobs. Consistent with these stories, my estimates of the pre-hike regression model (17) predict that part-time employment would have continued to increase during the second half of 2009 because, prior to the hike, part-time employment tended to increase with full-time job losses. For example, the part-time regression (17) for prime-aged white women has estimates  $\beta = 0.18$ ,  $b_0 + b_1 = 0.07$ , and  $\delta_0 + \delta_1 = -0.50$ , so that a persistent reduction in their own (prime-aged male) full-time employment was associated with more (less) of their own part-time employment, respectively.<sup>23</sup>

The actual and predicted series depart dramatically beginning in September 2009, with actual part-time employment 1.2 million below predicted part-time employment by December, and averaging 975,000 part-time positions below predicted over the months August 2009 – December 2010. One reason to attribute much of the gap between actual and predicted part-time employment to the July 24, 2009 minimum wage hike is that, as noted above, the real federal minimum wage was substantially different after July 2009 than it was before, but not expected to significantly affect the full-time employment of prime-aged males

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<sup>20</sup> Table 1 reported that only 0.6 percent of all full-time employed men were hourly workers paid at or below the federal minimum; presumably the percentage among prime-aged (ages 20-54) full-time employed men was even lower.

<sup>21</sup> The aggregate of my seasonally adjusted actual part-time employment series is slightly different from the Bureau of Labor Statistics' seasonally-adjusted aggregate; BLS does not report seasonally adjusted series for specific demographic groups.

<sup>22</sup> To the extent that part-time employment rises during recessions as terminated full-time employees took part-time employment with different employers, this by itself suggests that, contrary to the sticky price model, part-time employers increase employment and output during a recession in response to an additional supply of part-time workers.

<sup>23</sup> An early version of this paper arrived at a similar aggregate part-time forecast merely by fitting an aggregate part-time employment model, with full-time employment as the only independent variable, over the months December 2007 through July 2009.

that are the basis for the forecasting model. Moreover, my forecasting model (17) permits me to decompose the aggregate gap by demographic group, with the findings matching the theory. Figure 6 is a scatter diagram comparing percentage gaps between predicted and actual after July 2009 to first quartile hourly earnings in 2008,<sup>24</sup> for six demographic groups distinguished by age and employment status. The three lowest wage groups – the groups expected to have the largest employment impact – are the ones with the largest (negative) estimated impacts, with their impact rank among groups identical to their rank in terms of first quartile earnings. The three highest wage groups have estimated employment impacts of essentially zero.

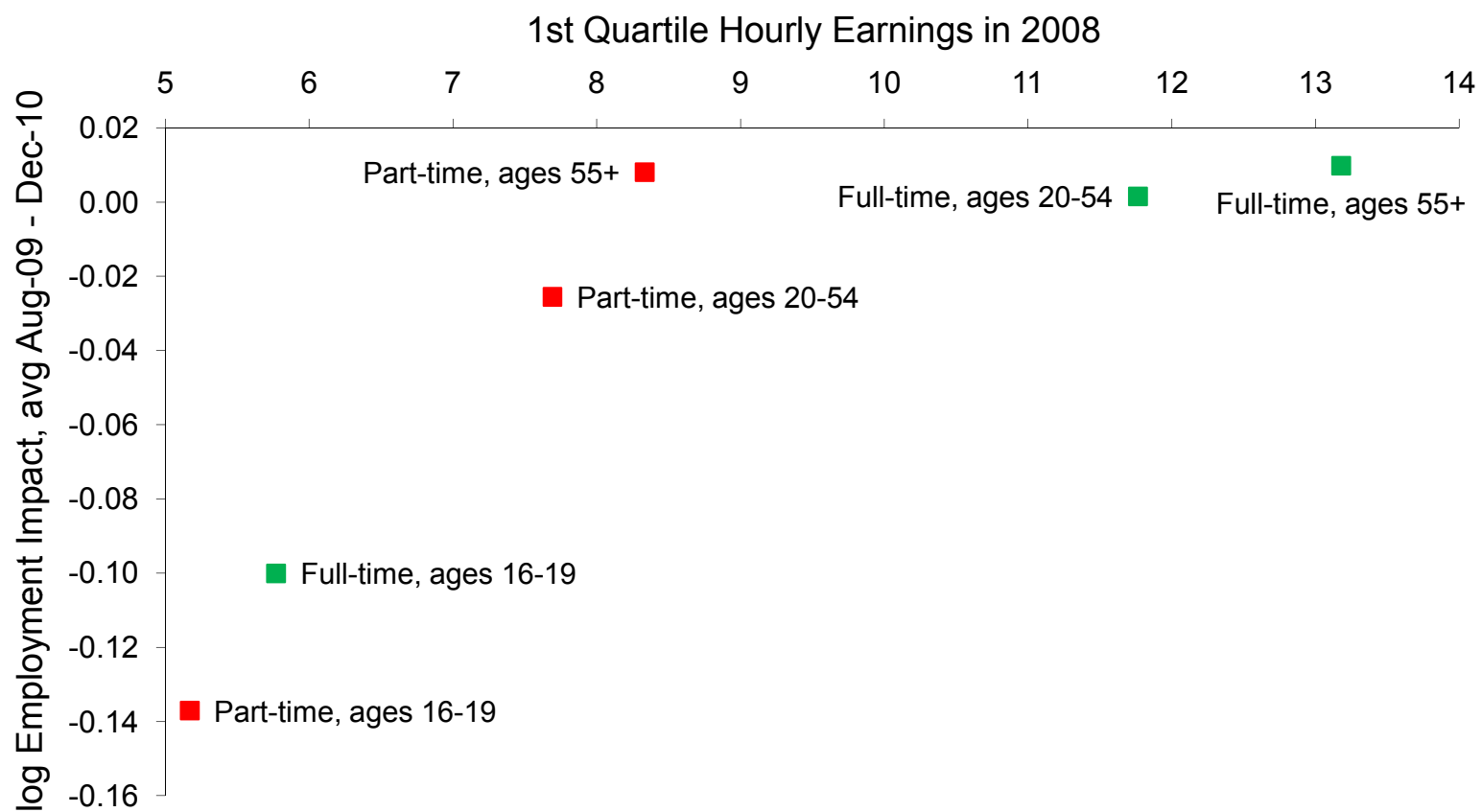
The 829,000 estimated employment effect of the federal minimum wage hike (975,000 part-time and -146,000 full-time positions) is consistent with the flexible price model. For a back-of-the-envelope calculation that elaborates on the model presented above, it helps to aggregate labor into two types: a fraction  $\omega \ll 1$  that would have worked at a wage between \$7.25 and \$6.55 if the minimum wage had remained constant, the fraction  $1 - \omega$  of all other labor. Given that  $\omega$  is small (and that the affected group's share of aggregate payroll is even smaller than  $\omega$ ), the magnitude of the minimum wage hike's short run log employment effect for the affected group is the product of the size  $\Delta$  of the effect on the log wages of the  $\omega$  and the elasticity of substitution between the two types of labor. As of July 2009, the Economic Policy Institute (2009) estimated that 2.8 million people – 2.0 percent of the July 2009 workforce of 139.8 million – earned less than \$7.25 per hour. The wages of some of those 2.8 million people would not be covered by the new minimum of \$7.25 because they did seasonal work, received tips, or worked in some other job not covered by the law. Other employers may succeed in changing the nature of the affected jobs (e.g., reducing the amount of “free” training provided) so that worker productivity would rise with the new minimum. For these reasons, the number of jobs ultimately covered by the hike would be more like 1 – 2 million rather than the full 2.8 million earning less than \$7.25. Assuming that the substitution elasticity is between 4 and 8, the flexible-price theoretical short run employment impact of raising the minimum wage from \$6.55 to \$7.25 would be between 0.3 million and 1.1 million.<sup>25</sup>

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<sup>24</sup> 2008 first quartile hourly earnings are calculated from respondents to the March 2009 CPS who reported positive hours, earnings, and weeks worked, and zero self-employment earnings in 2008. 2008 workers are considered “full-time” if in full-time positions for at least 75 percent of the weeks they worked in 2008, and part-time otherwise.

<sup>25</sup> The short run wage elasticity of total labor demand would be -4 if the aggregate production function were Cobb-Douglas with labor share equal to 0.75. Assuming that the cross-price elasticities of labor demand are positive, then the substitution elasticity has to be greater in magnitude than the short run elasticity of total labor demand, which is why I assume a substitution elasticity range of 4-8. I assume that most of the  $\omega$  workers with wage below \$7.25 had their wage at or below \$6.55, so that the log wage impact is 0.101.

Figure 6. Estimated Employment Impact of July 2009 Min. Wage Hike, by Demographic Group





### *V.C. Housing Investment Crowds Out Non-Residential Construction*

In the sticky price model, a demand increase in one sector increases output in that sector (say, the public sector), without reducing output in other sectors because the competition for factors of production is not passed into output prices that would otherwise cause production to be reallocated to the demanded sector (see Proposition 1). For the same reason, a reduction in demand in one sector would not cause the other sectors to produce more.

The private residential and nonresidential building sectors are an interesting case study, because the demand for housing surged 2000-2005, and collapsed thereafter. Admittedly, looking at the economy as residential versus nonresidential is not the same as looking at it as public versus private, but the former gives us some information about how different sectors are connected through factor markets, and this connection is at the heart of the crowding out hypothesis. In fact, measuring the effects of residential building on non-residential building offers a tough test of the crowding out hypothesis because the housing collapse left so many unemployed and, depending on the degree to which factors employed in building sectors (residential and non-residential) are substitutes for factors employed in the rest of the economy, crowding out by housing demand could largely occur in the rest of the economy, rather than the non-residential building sector that is my focus here.

Figure 7 displays quarterly real residential and real non-residential structures investment since 2000 Q1.<sup>26</sup> Non-residential investment remained low throughout the housing boom. Both residential and non-residential investment turned at almost exactly the same time, in opposite directions. Non-residential investment increased throughout 2006, 2007, and 2008, while residential investment was collapsing.

The large reduction in the workforce that became apparent by 2009, not to mention tight credit, likely reduced the desired stock of non-residential buildings and this by itself would cut non-residential investment activity, so it helps to separate the effect of an increased supply of resources for non-residential investment from reduced demand. I attempt to do so by examining the two investment series in a regression framework that includes measures of the business cycle.

I use quarterly data on male employment rates and per capita real structures investment from 1996Q1 through 2010Q3.<sup>27</sup> Each column of Table 2

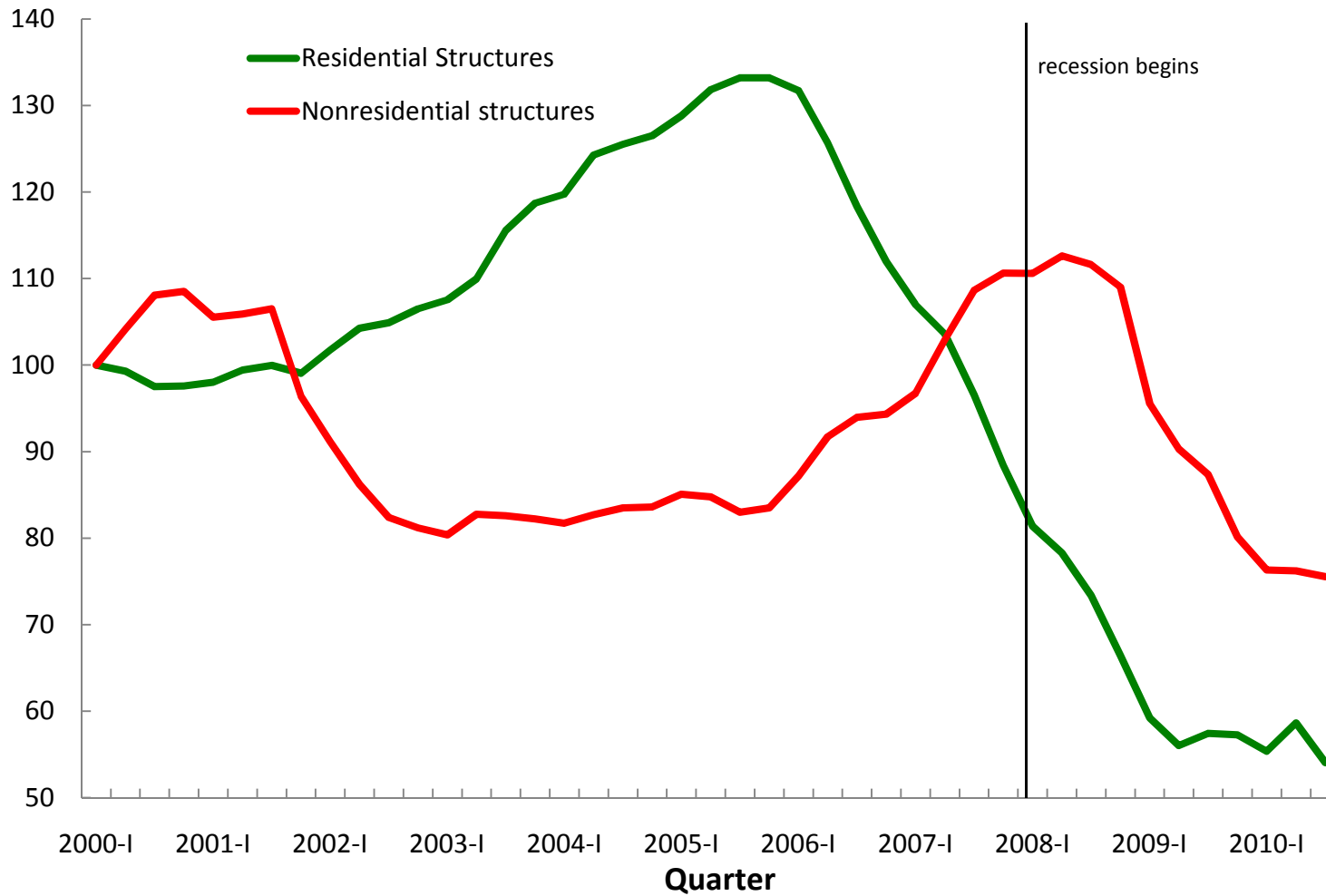
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<sup>26</sup> Figure 7 is reproduced and updated from Mulligan and Threinen (2008).

<sup>27</sup> I obtained quantity indices for residential and non-residential structures investment from the Bureau of Economic Analysis' NIPA Table 5.3.3 and mid-quarter population from their NIPA Table 2.1. Based on the assumption that building factors would move between the two sectors on

Real investment  
index, SA  
2000:Q1 = 100

Figure 7. Real Investment in Structures by Sector



**Table 2. Crowding Out of Real Inv. in Non-Res. Structures**

Each column of the Table reports results from a real per capita non-residential structures investment regression.

	(1)	(2)	(3)	(4)	(5)
end sample:	2007Q4	2007Q4	2007Q4	2010Q3	2010Q3
<u>estimation method:</u>	<u>levels</u>	<u>first diff.</u>	<u>levels</u>	<u>levels</u>	<u>levels</u>
no. of current & lagged terms:	4	4	1	1	1
sum of coef's on real per capita housing investment terms	-0.32 (0.09)	-0.30 (0.13)	-0.29 (0.09)	-0.29 (0.09)	-0.24 (0.07)
sum of coef's on log male emp. rate terms, converted to elasticity	8.05 (1.14)	8.05 (2.00)	2.23 (1.13)	2.22 (1.14)	4.80 (0.78)
recession measured as:	N/A	N/A	N/A	2008Q1- 2010Q3	NBER dates
recession indicator				-5.10 (2.95)	0.92 (2.05)
recession indicator * (real per capita housing investment)				0.06 (0.30)	0.11 (0.17)
recession indicator * (rescaled log male employment rate)				2.98 (1.79)	-0.57 (1.26)
time trend (constant in first diff.)	0.003 (0.001)	0.003 (0.002)	0.001 (0.001)	0.001 (0.001)	0.001 (0.001)
constant	-13.6 (2.0)	N/A	-3.4 (2.0)	-3.4 (2.0)	-7.8 (1.4)
Observations	48	48	48	59	59
Adjusted R-squared	0.88	0.26	0.73	0.74	0.78
s.e.	0.010	0.010	0.012	0.012	0.012

Notes: (a) quarterly observations beginning 1996Q1

(b) regressions are estimated with the Prais-Winsten correction for first-order serial

(c) standard errors in parentheses

(d) independent variables are current and lagged (up to 3 lags) real housing per capita housing inv. and log male emp. rate (rescaled by the dependent variable's mean) and, when applicable, a recession indicator interacted with the those terms.

(e) both structures investment variables are measured in 2005 dollars per capita

reports estimates of a time series regression with real per capita non-residential structures investment as the dependent variable. The independent variables include a linear time trend, real per capita residential structures investment, and the log of the male employment rate. In some of the specifications, lags of the independent variables are included in which case the table displays the sum of the coefficients estimated on all lags (including lag zero).<sup>28</sup> Both investment variables are measured in 2005 dollars, so a coefficient of -1 on the residential investment variable would be found if the only fluctuations in building activity were the substitution of building activity in one sector for the same amount of building activity in the other sector. The log employment rate series was rescaled by multiplying by the 1996Q1 – 2007Q4 average of the dependent variable: its coefficient can therefore be interpreted as an elasticity.

Columns (1) and (2) differ only in terms of the estimation method – levels versus first-differences – and both report an economically and statistically significant negative relationship between structures investment in the two sectors. The point estimates suggest that one hundred units more housing investment is associated with 30-32 units less non-residential structures investment, which is consistent with a significant amount of crowding out of one sector's building by building in the other sector. Column (3) omits any lags of the independent variables, but reports a similar negative relationship between the two sectors' structures investment.

Columns (1)-(3) use only data from before 2008, and it has been argued that crowding out would not occur during the recession, even while it occurred in years before. As one way to examine this possibility, I estimated two least squares versions of each of Table 2's columns (1) and (2), again using only the data prior to 2008, and then used those estimates to predict non-residential building through 2010Q3. One version has the same independent variables as used in columns (1) and (2) of the Table. The other version omits the housing investment variable, so that the difference between the two predictions can be interpreted as the expected effect of the housing crash on non-residential structures investment since 2007. The predictions for non-residential structures investment were made by using the estimated coefficients (and, for the level specifications, adjusting the constant term so that the models exactly fit 2007Q4

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roughly a dollar-for-dollar basis, I converted each sector's quantity index series (which were equal to 100 in 2005) to 2005 dollars by multiplying the series by the sector's 2005 nominal investment expenditure from BEA Table 5.3.5 (results are similar if investment is measured as the log of the per capita quantity index, rather than in chained dollars). Male employment rates are used rather than overall employment rates to focus on the business cycle rather than secular changes in the propensity of women to work.

<sup>28</sup> Time-to-build and price measurement errors are good reasons to include lagged price terms in the investment regressions.

non-residential structures investment<sup>29</sup>) and the actual data through 2010Q3 for housing investment and the male employment rate.

The predictions are shown in Figures 8a and 8b, together with the actual investment series, with the vertical line to the left of 2008Q1 indicating the quarters that were excluded from the regressions used to make the predictions. When the housing investment variable is ignored, non-residential building is predicted to drop all quarters (the black series in Figure 8a) or all quarters but one (the black series in Figure 8b) since 2007.<sup>30</sup> In fact, non-residential building peaked in 2008Q2 and remained pretty flat through the end of the year. The models including the housing investment variable (blue series in Figures 8a and 8b) correctly predict this pattern, as well as the actual sharp drop to begin 2009. Overall, the models without housing investment consistently under-predict non-residential building whereas the predictions based on the housing investment variable are closer, having predictions on both sides of the actual series. Figure 8a and 8b are inconsistent with the claim that crowding out disappeared during the recent recession.

Table 2's columns (4) and (5) further explore the possibility that crowding out is different in recessions than other times by interacting the independent variables with a recession indicator. One of the recession indicators is a dummy for the quarters since 2007, and the other is an indicator for the quarters coded as recession by the National Bureau of Economic Research. If crowding out were zero during a recession, then the coefficient on the housing investment interaction would be positive and equal in magnitude to the negative coefficient on the un-interacted housing investment term. Instead, the estimated coefficient on that interaction term is economically and statistically insignificant. Thus, Table 2 is inconsistent with the claim that recessions have significantly less crowding out.

## VI. Conclusions

From a partial equilibrium perspective, it would be surprising if government purchases did not crowd out at least some private consumption, and that a reduction in factor supply did not result in less output. Yet some “New

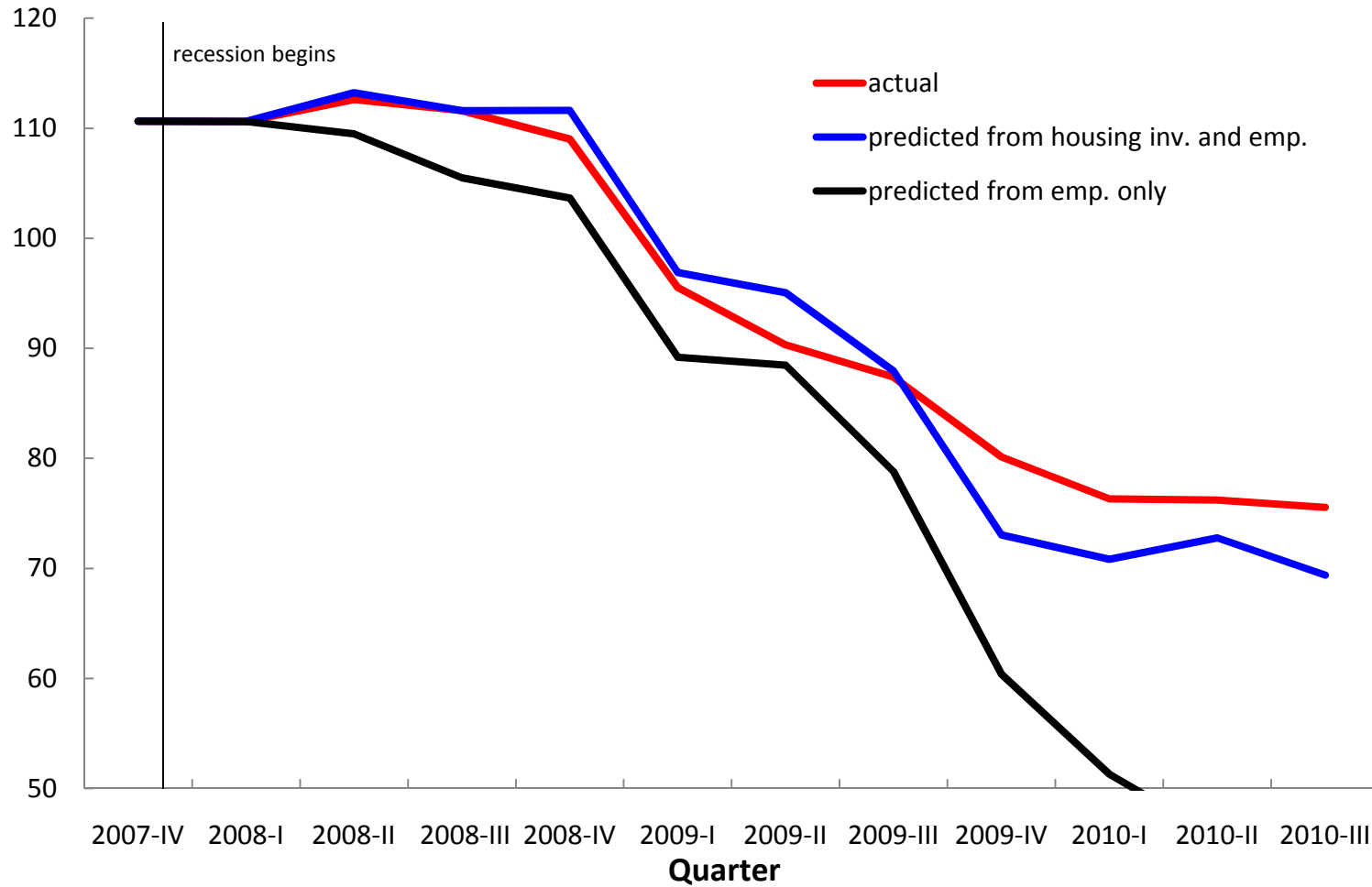
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<sup>29</sup> The specifications without housing investment ignore crowding out even before 2008 and thereby grossly under-predict non-residential investment for 2007Q4 – my procedure of adjusting the constant allows the model to fit 2007Q4 in order to see whether crowding out is needed to predict the non-residential investment changes during the recession (that is, since 2007Q4). The adjustment of the constant for the specification including housing investment is quite small, because that model predicts 2007Q4 well.

<sup>30</sup> Each Figure 8a, 8b is based on two regressions. Alternatively, each Figure could have been based on a single regression, with the “emp. only” prediction calculated by setting the employment coefficients to zero – this alternative calculation turns out to be very similar to the “emp. only” series shown.

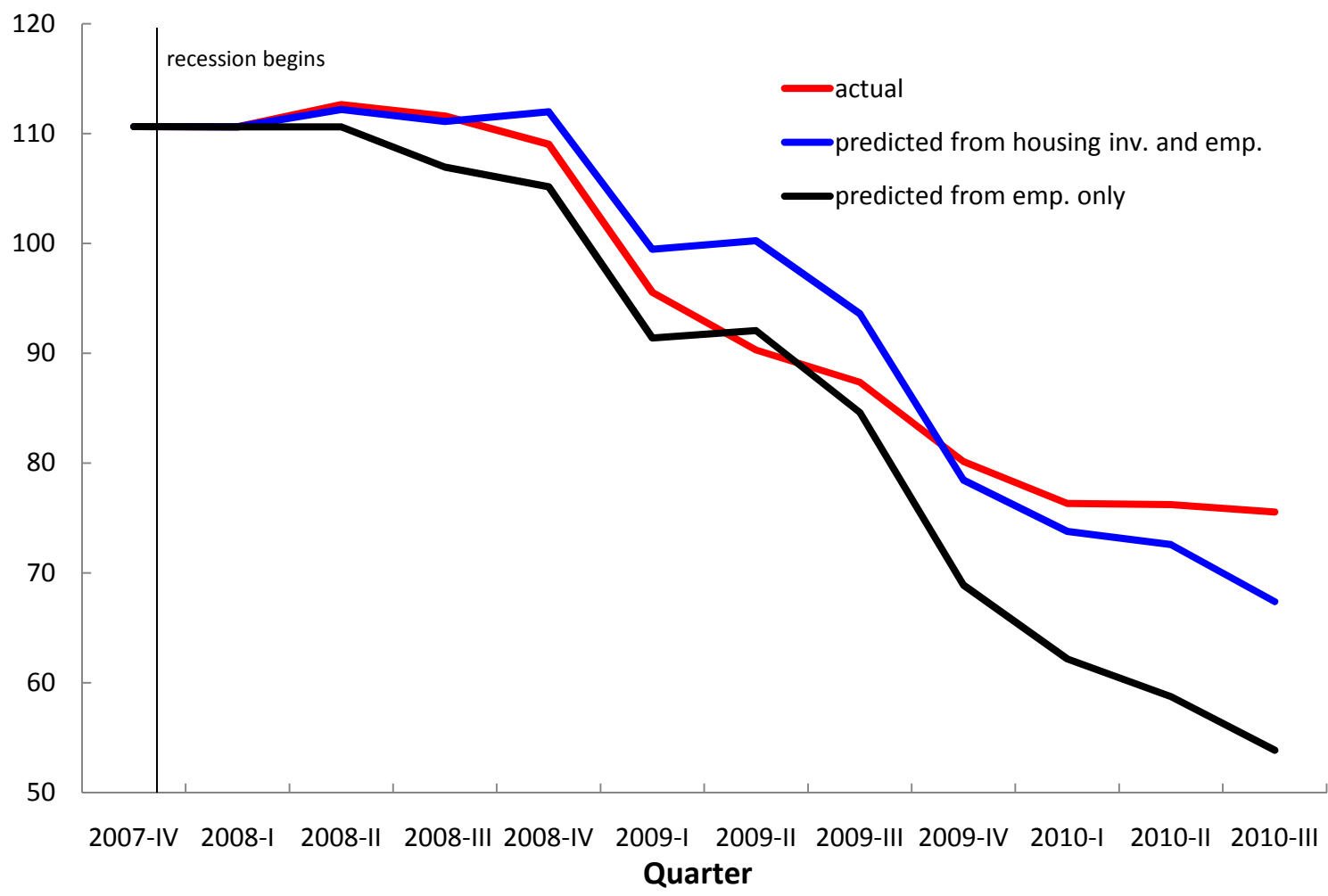
Real investment  
index, SA  
2000:Q1 = 100

Figure 8a. Real Investment in Non-Residential Structures: Crowding Out  
(out-of-sample predictions from level specifications)



Real investment index, SA  
2000:Q1 = 100

Figure 8b. Real Investment in Non-Residential Structures: Crowding Out  
(out-of-sample predictions from first diff. specifications)



Keynesian” models, not to mention much public policy commentary, claim that today’s economy has turned this partial equilibrium reasoning on its head, even while it might have been historically valid. Among other things, individual firms and the aggregate private sector are alleged to leave their production invariant to changes in factor supply conditions during this recession. This paper shows how the government spending multiplier and the “paradox of toil” are related in theory, and examines evidence *from this recession* on the output effects of factor supply.

The academic year concluded twice during this recession, and both times over a million teens entered the labor market. Well over a million of them found employment, and as a result total employment for the economy was significantly higher in July than it was in April. This pattern reversed itself the two times that the academic year resumed during this recession. The real federal minimum wage was hiked at the end of July 2009 from an already high level relative to the CPI. Employers of part-time workers appeared to respond by significantly cutting part-time employment after July 2009, despite the fact that part-time employment had trended strongly up prior to the hike. The hike appears to have reduced nationwide employment by about 800,000 on average between August 2009 and the end of 2010. Finally, the collapse of housing construction served to shift resources into non-residential building.

Despite the presence of perhaps the deepest recession of our lifetimes, and nominal interest rates on government securities that were essentially zero, these three episodes show how factor markets seemed to behave as if output prices were flexible at the margin. In particular, markets absorb an increased supply of factors of production – even during a recession like this one – and do so by increasing output. The seasonal patterns and minimum wage episode show this result at the aggregate level, while the minimum wage and housing episodes illustrate it at a sectoral level.

This paper does not contain a numerical estimate of the government purchases multiplier. However, its examination of data exclusively from the 2008-9 recession suggests that sectoral and aggregate employment and output vary with supply conditions in much the same way they did before the recession. The results contradict Keynesian claims that the government purchases multiplier would be significantly greater during the recession than it was before 2008, suggesting instead that historical estimates of the effects of fiscal policies are informative about fiscal policy effects in more recent years. Moreover, the supply incentives created by government spending cannot be ignored merely because 2008 and 2009 were recession years; rather incentives mattered as much as ever. Government purchases likely moved factors away from activities that would have supported private purchases. Unemployment insurance, food stamps, and other



expanding means-tested government programs likely reduced employment and output during this recession, in much the same way they did in years past.

Nothing about my results implies that this recession was efficient, or that government spending necessarily reduces efficiency. Indeed, my “flexible price model” includes a distortion in the output market (recall the parameter  $\mu$ ) and a distortion in the labor market (recall the parameter  $\tau$ ).<sup>31</sup> The presence of distortions by itself does not tell us whether government purchases stimulate private spending, or how output responds at the margin to factor supply shifts.

This is not to say that output prices were actually flexible during the recession, because producer entry and exit and a variety of other market mechanisms could have many of the qualitative effects of flexible prices. Moreover, even if it were shown that output prices actually were flexible during this recession, that does not preclude the possibility that those prices would be inflexible in response to smaller shocks. But, when it comes to this recession, models that feature sticky prices have been a poor description of actual events in the real economy.

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<sup>31</sup> Another theory is that labor market outcomes during this recession were inefficient as a result of rigid nominal wages. My minimum wage findings are consistent with that theory, but my results for the seasons and for non-residential construction caution against going so far as assuming, as in “old Keynesian” models, that rigid nominal wages rendered supply irrelevant for determining market outcomes during the recession.

## VII. Appendix – Proofs of Propositions

Each of the Propositions 1-3 contains both flexible price and sticky price results. To prove the flexible price parts of Propositions 1 and 2, note that the flexible price equilibrium conditions are (1), (3), (4), (5), and (7). The flexible price equilibrium labor usage  $N_t$  can be calculated from a single algebraic equation derived from (1), (4), and (7):

$$\mu\gamma_t u_n'(N_t) = \alpha_t u_c'(F(N_t) - G_t)(1 - \tau_t)F'(N_t).$$
 Total differentiation of this condition yields expressions for  $dN_t/dG_t$  (holding  $\tau_t$  fixed) and  $dN_t/d\tau_t$  (holding  $G_t$  fixed), which is the expression shown in equation (14). From these,  $dC_t/dG_t$  and  $dC_t/d\tau_t$  can be calculated from the resource constraint (1).

Proposition 3 considers the effects of labor income tax financed government purchases, and claims that  $G_t$  can reduce flexible price equilibrium total spending  $C_t + G_t$  when the labor income tax rate  $\tau_t$  is adjusted to balance the time  $t$  government budget constraint. Substituting the government budget constraint into the first order condition above, we have a single condition implicitly defining labor  $N_t$  as a function of government purchases  $G_t$ :

$$\mu\gamma_t u_n'(N_t) = \alpha_t u_c'(F(N_t) - G_t) \left( 1 - \frac{G_t}{N_t F'(N_t)} \right) F'(N_t).$$
 Government purchases

appear twice in this condition: once as a wealth effect increasing labor and the other as a substitution effect decreasing it. Differentiation of this condition shows that the substitution effect can dominate as it does, for example, when  $F''(N) < 0$ ,  $NF'(N)/F(N) < 1$ ,  $u_c(C) = \ln C$ , and  $G_t$  is sufficiently close to zero. The resource constraint requires that total spending equal  $F(N)$ , so  $dN_t/dG_t$  (adjusting  $\tau_t$  to balance the government budget constraint)  $< 0$  means that total spending declines with  $G_t$ .

To prove the sticky price parts of Propositions 1 and 2, note that, by definition, a sticky price equilibrium  $\{C_t, N_t, W_t, R_t, P_t, P_t^*\}_{t=0}^\infty$  corresponding to benchmark time paths  $\{G_t, M_t, \tau_t, \alpha_t, \gamma_t\}_{t=0}^\infty$  solves the boundary value problem (11). Propositions 1 and 2 perturb the benchmark time path for government purchases, or marginal tax rates, on a short time interval  $[t, t+\delta]$ :

**Lemma 1** If  $\{C_t, N_t, W_t, R_t, P_t, P_t^*\}_{t=0}^\infty$  are benchmark sticky price equilibrium time paths, then the same time paths, with the two exceptions that  $N_t$  is adjusted on the interval  $[t, t+S]$  to satisfy the resource constraint and  $W_t$  is adjusted to satisfy the labor supply condition (4), are, in the limit  $S \rightarrow 0$ , a sticky price

equilibrium corresponding to the perturbed paths for government purchases and marginal tax rates.

Proof By construction, the proposed time paths satisfy the resource constraint at all dates. Because the benchmark equilibrium satisfies the differential equations for the price index  $P$  and private consumption  $C$  at all points in time, the proposed time paths also solve them at all points in time. For any  $T > t$ , the proposed path for the price updates  $P^*$  satisfies:

$$\begin{aligned} \ln P_T^* &= \ln P_0^* + (\lambda + \rho) \int_0^T \left[ \ln u_c'(C_s) + \ln \left( \frac{\alpha_s(1-\tau_s)}{\gamma_s} \right) + \ln \left( \frac{F'(N_s)}{\mu} \right) - \ln u_n'(N_s) \right] ds \\ &= \ln P_0^* + (\lambda + \rho) \int_0^t \left[ \ln u_c'(C_s) + \ln \left( \frac{\alpha_s(1-\tau_s)}{\gamma_s} \right) + \ln \left( \frac{F'(N_s)}{\mu} \right) - \ln u_n'(N_s) \right] ds \\ &\quad + (\lambda + \rho) \int_t^{t+S} \left[ \ln u_c'(C_s) + \ln \left( \frac{\alpha_s(1-\tau_s)}{\gamma_s} \right) + \ln \left( \frac{F'(N_s)}{\mu} \right) - \ln u_n'(N_s) \right] ds \\ &\quad + (\lambda + \rho) \int_{t+S}^T \left[ \ln u_c'(C_s) + \ln \left( \frac{\alpha_s(1-\tau_s)}{\gamma_s} \right) + \ln \left( \frac{F'(N_s)}{\mu} \right) - \ln u_n'(N_s) \right] ds \end{aligned}$$

The first and third integrals are the same for the benchmark equilibrium as for the proposed path. Because the integrand is finite, the middle integral goes to zero as  $S$  goes to zero, which means that the proposed time path for price updates  $P^*$  satisfies its differential equation in the limit as  $S$  goes to zero.

Lemma 1 says that  $\left. \frac{dC_t}{dG_t} \right|_{dP_t=d\tau_t=0} = \left. \frac{dC_t}{d\tau_t} \right|_{dP_t=dG_t=0} = \left. \frac{dN_t}{d\tau_t} \right|_{dP_t=dG_t=0} = 0$ . The

consumption derivatives are zero because the proposed solution was constructed from the benchmark solution without changing consumption at any point in time. The zero labor derivative comes from the resource constraint.

Assuming that the economy is on the upward sloping part of the Laffer curve, Proposition 3's comparative static is equivalent to a labor income tax financed increase in lump sum transfers (Proposition 2) added to a lump sum tax financed increase in government purchases (Proposition 1) in an amount that has the lump sum taxes exactly offsetting the lump sum transfers. Neither component of the comparative static affects private consumption in the sticky price model, so the combination also has zero effect on private consumption. With private consumption held constant (in the sticky price model), total spending rises with government purchases.

Lemma 2 If  $\bar{C}, \bar{N}, \bar{W}, \rho, \bar{P}, \bar{C}, \bar{N}, \bar{W}, \rho, \bar{P}$  are short seasonal steady state values corresponding to a benchmark seasonal for the preference parameters  $\alpha$  and  $\gamma$  that are constant throughout the year, then  $\bar{C}_{on}, \bar{N}_{on}, \bar{W}_{on}, \rho, \bar{P}, \bar{C}, \bar{N}, \bar{W}, \rho, \bar{P}$  are short seasonal steady state values corresponding to the preference parameters  $\alpha_{on}, \gamma_{on}, \alpha, \gamma$ , where the on-season wage rate and quantities satisfy:

$$\alpha_{on} u'_c(\bar{C}_{on}) = \alpha u'_c(\bar{C})$$

$$F(\bar{N}_{on}) = \bar{C}_{on} + G$$

$$\frac{\gamma_{on} u'_n(\bar{N}_{on})}{\alpha u'_c(\bar{C})} = (1 - \tau) \frac{\bar{W}_{on}}{\bar{P}}$$

Proof The proposed paths for the price index and price updates are constant throughout the year, the latter of which is consistent with sticky price equilibrium only if:

$$\begin{aligned} 0 &= \int_0^1 \left[ \ln u'_c(C_t) + \ln \left( \frac{\alpha_t(1-\tau)}{\gamma_t} \right) + \ln \left( \frac{F'(N_t)}{\mu} \right) - \ln u'_n(N_t) \right] dt \\ &= \int_0^S \left[ \ln u'_c(\bar{C}_{on}) + \ln \left( \frac{\alpha_{on}(1-\tau)}{\gamma_{on}} \right) + \ln \left( \frac{F'(\bar{N}_{on})}{\mu} \right) - \ln u'_n(\bar{N}_{on}) \right] dt \\ &\quad + \int_S^1 \left[ \ln u'_c(\bar{C}) + \ln \left( \frac{\alpha(1-\tau)}{\gamma} \right) + \ln \left( \frac{F'(\bar{N})}{\mu} \right) - \ln u'_n(\bar{N}) \right] dt \\ &= \int_0^S \left[ \ln u'_c(\bar{C}) + \ln \left( \frac{\alpha(1-\tau)}{\gamma_{on}} \right) + \ln \left( \frac{F'(\bar{N}_{on})}{\mu} \right) - \ln u'_n(\bar{N}_{on}) \right] dt \\ &\quad + \int_S^1 \left[ \ln u'_c(\bar{C}) + \ln \left( \frac{\alpha(1-\tau)}{\gamma} \right) + \ln \left( \frac{F'(\bar{N})}{\mu} \right) - \ln u'_n(\bar{N}) \right] dt \end{aligned}$$

The first equality is just a decomposition of the integral into two components, and using the fact that the proposed paths are constant within season. The second equality reflects the fact that the proposed solution has the marginal utility constant throughout the year. The second integral's integrand is zero because  $\bar{C}, \bar{N}, \bar{W}, \rho, \bar{P}, \bar{C}, \bar{N}, \bar{W}, \rho, \bar{P}$  are short seasonal steady state values corresponding to a benchmark seasonal for the preference parameters  $\alpha$  and  $\gamma$  that are constant throughout the year. The first integral's integrand is not zero to the extent that  $\alpha_{on} \neq \alpha$  or  $\gamma_{on} \neq \gamma$ , but the integral itself goes to zero as  $S \rightarrow 0$ . It follows that the sum of the two integrals is zero as  $S \rightarrow 0$ .

Proposition 4's result that short seasonal fluctuations in consumption and labor are independent of the labor preference parameter  $\gamma$  follows from Lemma 2: with  $\alpha$  constant throughout the year, consumption must be constant throughout the year regardless of  $\gamma$ . The resource constraint requires that labor must also be constant throughout the year.

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