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Price and death

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PRICE AND DEATH

Abstract

How does an artist’s death impact on the price of his or her works of art? We investigate this question in an infinite-horizon dynamic general equilibrium setting. Employing the open-loop Stackelberg equilibrium concept to describe the interactive behaviour of collectors and artists, we find that the art price remains at some well-defined "pseudo-competitive" level as long as the artist is alive. Only when the artist unexpectedly dies, the price increases on impact. This so-called death effect varies negatively with the artist’s age at death. If it is well known that an artist is ailing from some terminal illness and his or her death thus does not come as a surprise, the price of the ailing artist’s work increases when the news of the ailment is divulged; the price immediately jumps to the level which will prevail at the time when the artist dies.

JEL Code: D90, E31, E52.

Keywords: art prices, durable-goods monopoly, Stackelberg equilibrium.

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1 Introduction

In this paper we investigate how an artist’s death affects the price of his or her works of art. Our study thus contributes to the literature on art price formation. In contrast to the bulk of this literature we do, however, not restrict our analysis to the demand side of the art market; our study gives due weight to the channels of influence which work through the supply side. In particular, we explicitly make allowance for the fact that production ceases when the artist dies, implying that the collectors can be sure that in the future no new similar pieces of art will put pressure on the prices in this specific market segment. More specifically, we provide a microeconomic foundation for the so-called death effect hypothesis which maintains that an artist’s death causes a sudden change in the price of his or her works of art. Empirical studies that have identified death-related changes in art prices include Ekelund et al. (2000), Maddison and Jul-Pedersen (2007), and Ursprung and Wiermann (2008).

By insisting that the supply side of the art market is of crucial importance for understanding art price formation, we do, of course, not mean to denigrate the established literature which focuses on the demand side. Precisely because we believe that both sides of the market and, in particular, the interaction of demand and supply side effects are instrumental in the pricing of works of art, our analysis is closely connected to the established literature on art price formation.1

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1The literature on art price formation is multifaceted. Making use of auction price records (for a survey of the literature on art auctions, see Ashenfelter and Graddy, 2006), many studies start out with the construction of some kind of art price index (see Ginsburgh et al., 2006). These indices usually cover whole market segments – such as certain periods, media, and levels of artistic quality (for example master pieces versus lower end works). The classic 1974 paper by Anderson, for example, investigates the markets for Old Master paintings, English 18th and 19th century art, Impressionists and 20th century paintings. The media covered in the literature include paintings, prints, and sculptures. Price indices have, however, also been developed for selected famous individual artists such as Picasso (cf. Czajack, 1997). Two different methods have been used to construct art price indices: the hedonic regression method pioneered by Anderson (1974) and the repeated-sales method used in Baumol’s influential 1986 paper (for a comparison of the two approaches see Chanel et al., 1996). Art price indices reveal information about the efficiency of art markets and they are a crucial prerequisite for estimating rates of returns on investments in objects of visual art. The respective literature indicates that art markets are surprisingly efficient even though stable patterns of minor deviations from the law of one price can be identified (see, for example, Pesando, 1993) and estimates of pre-sale auction prices made by experts are slightly biased (see, for example, Bauwens and Ginsburgh, 2000). As far as the financial rates of return are concerned, there is general agreement that returns in the art market are low and risk is high. Not surprisingly, the rates of returns depend on trends in tastes, implying that certain genres and pieces from certain periods may fetch returns that are much higher than average returns. Interestingly, master pieces appear to underperform from a financial point of view (see Mei and Moses, 2002). This, however, is quite in line with the notion that master pieces provide a higher non-financial return, i.e. more pleasure of consumption. More recently, the correlation patterns of art and financial asset returns have
Even though economic investigations into the technology and perspicacious planning of art production are still grossly underrepresented in the cultural economics literature, there are notable exceptions. David Galenson, in particular, has made a valiant attempt at characterizing the working methods of visual artists (see, for example, Galenson, 2006). He distinguishes between experimental innovators who arrive at their major contributions by a time-consuming learning-by-doing process and conceptual innovators whose work is based on radical new ideas that can be turned into artistic output without a great deal of craftsmanship. To be sure, this dichotomous distinction oversimplifies matters and therefore exposes any resulting classification of artists to critical inquiries (see, for example, Ginsburgh and Weyers, 2006). Nevertheless, if it is fair to say that with the onset of impressionism in the 1860s experiment-based techniques gave more and more way to techniques relying on conceptual imagination, this switch in production technology may well be explained by the growing media attention given to artists and the increasing importance of the gallery system, both of which depend on a steady flow of innovation and an ever increasing number of solvent collectors. In any event, most scholars seem to agree that the supply of works of art and the technology of artistic production are intimately related to the nature of demand, implying that a well balanced portrait of the art market needs to model the demand as well as the supply side.

Our study embraces this integrated view and explicitly models the interaction of the agents on the supply and the demand side of the art market, i.e. the producing artists and the consuming collectors. We portray the artists as manufacturers who operate in an environment of monopolistic competition and set their output at a level commensurate with their objectives. This view of artistic production, which presupposes a great deal of flexibility in the quantity produced, corresponds to the self-evaluation of conceptual artists who appear to dominate today’s art market. Frank Stella, for example, thought that a good pictorial idea is worth more than a lot of manual dexterity, Sol LeWitt had his work often executed by draftsmen because he insisted that all the planning and decisions are made beforehand (which relegates

\footnote{been investigated with a view to detecting potential diversification gains deriving from the inclusion of art in traditional portfolios. Whether such diversification gains are indeed available appears, however, to be a still unresolved issue (see, for example, Worthington and Higgs, 2004, and Campbell, 2004, for two different results).}

\footnote{The monopoly position of artists is especially strong because the consumers (collectors) themselves set great value in the fact that the consumed works of art can indubitably be traced to the producer.}
the execution to a perfunctory affair), and Andy Warhol de-emphasized craftsmanship to such an extent that his paintings were manufactured in his "factory" in about four minutes a piece (all quoted directly or indirectly from Galenson and Weinberg, 2000, pp. 767-8). These examples may be extreme, but they indicate that art production is certainly more flexible than the traditional conception of the artist may convey. As has already been observed in the pioneering study by Ekelund et al. (2000), the artists are, in principle, in a position to flood their own market, and, since works of visual art are durable, find themselves in the tricky situation described by Coase (1972): either they are able to credibly commit to restrained production in the future which allows them to exploit their monopoly position, or, if no commitment device is available, they are forced to produce along the time-consistent production path which will exert a downward pressure on the prices of their works of art.

The general issue raised in this paper, even though motivated by the specificities of art production, clearly transcends the art market setting. From an industrial organization point of view, we deal here with precisely those circumstances that also characterize the interplay of consumer expectations and time consistent producer behavior in the classical problem faced by monopolistic durable goods producers. The distinguishing feature of our study is that we explicitly consider the fact that the producer may, with a certain probability, cease to exist at every point of time. In the case of art production, this additional feature is an obvious fact of life (or rather death), but the assumption of finitely lived producers may also portray common aspects of business life such as production cessation due to new regulatory constraints.

To analyze the pertinent market interactions, we present a dynamic general equilibrium model populated with perfectly rational producers and consumers. Standard models of durable goods monopolies employ in this context the concept of the perfect Markov equilibrium (PME) which is defined by a pair of continuous functions which only depend on the current state of the system; one of these functions describes the buyers’ expectations, the other one the monopolist’s sale strategy (see, e.g. Stokey, 1981, Kahn, 1986, Bond and Samuelson, 1987, Karp, 1996, and Driskill, 1997). In equilibrium, the buyers’ expectations are satisfied along the realized path of production and the seller’s strategy maximizes the present discounted value of profits, given the buyers’ expectations. Moreover, the PME solution entails the property of
time consistency.

A severe drawback of the standard approach is, however, that the PME approach, as a rule, presupposes that expectations and production depend only on the state of the system as described by the produced stock of the durable good. This assumption does not allow incorporating new information, since the produced stock of the durable good does not change if the system is unexpectedly perturbed. Since the state variable does not reflect perturbations, expectations are not affected either, and neither is production because the feedback function is autonomous, i.e. independent of time. Hence, we cannot directly apply this approach to analyze death effects. Abolishing the assumption of a univariate representation of the system’s state, i.e. allowing price expectations to depend not only on the stock of production but also on other factors which reflect news, unfortunately turns out not to be a passable escape because in such a framework it is, in general, impossible to derive closed solutions.

For these reasons we do not use the PME approach in our study, but rather resort to the open-loop Stackelberg equilibrium concept (see, e.g. Dockner et al., 2000). The open-loop Stackelberg equilibrium (OLSE) as applied to our setting implies that the art collectors (buyers) hold rational expectations and the artist acts as a Stackelberg leader, taking into account the intertemporal optimizing behavior of the collectors. Since this approach does not yield the time-consistent production path, we focus our analysis of the long-run dynamics of the system on the situation in which the artist can credibly commit to a production plan which he or she communicates to the potential collectors at the onset of his or her carrier. In the case of artistic production it may not be too farfetched to assume that artists indeed do stick to idiosyncratic production strategies which are dictated by their artistic temperament.

Using this setup, we derive several results. First, we show that a unique stable accumulation path of works of art exists. Second, even though artistic production is spread over time, the art price equals at any point of time some counterfactual price that would prevail if the artist were immortal. Having established these results, we then continue to show that the behavior of the linearized version of the dynamic system does not depend on the chosen control path. In other words, the linearized system captures the local behavior irrespective of whether the artist is able to commit to the optimal control path or is forced to follow the time-consistent
production plan. This insight allows us to draw the following universally valid conclusions with respect to the death effect: regardless of whether the death of the artist comes as a surprise or is anticipated (if, for example, the artist has been suffering from a terminal disease), the price of the artist’s oeuvre will jump to its final level at the moment when the news of the artist’s death or impending death is revealed. Moreover, we show that the size of the death effect varies negatively with the artist’s age at death.

The paper unfolds as follows. Section 2 presents the dynamic general equilibrium model which describes the behavior of the consumers (i.e. the collectors) and the behavior of the durable goods monopolist (i.e. the artist). In section 3, we analyze the steady-state and dynamic stability properties of our model. The influence of the model’s crucial structural parameters on the price and the production of works of art, as well as the associated comparative-dynamic properties of the model are discussed in section 4. In section 5, we then turn to the impact effect of an artist’s death, be it unexpected or expected. Section 6 concludes. Some technical details are relegated to the appendices.

2 The Model

2.1 The demand side: the collectors’ behavior

On the demand side, we identify the collectors as the final consumers of works of art. We thus sidestep the issue of potential market makers (galleries, museums, art fairs, the deceased artist’s estates, etc.) by assuming that these institutions, which admittedly play an important role in the art market, can be portrayed as intermediaries without any market power.

Assume a unity-mass of collectors who are identical in terms of preferences and income. Collectors are thus price takers in the sense that the behavior of any single collector has no influence on the market price, but in the aggregate the behavior of all collectors does, of course, have a noticeable impact on the price. At any time \( s \) the collectors have infinite planning horizons. We assume that the collectors cannot access the capital market in order to satisfy their artistic cravings. The collectors derive instantaneous utility from the consumption of a composite non-durable (numeraire) good \( x(s) \) and a flow of services proportional to the stock
$Q(s)$ of the artist’s works they own at time $s$, which captures aesthetic pleasure from holding works of art. The representative collector $j$’s utility function $U(x_j(s), Q_j(s))$ is assumed to be increasing in its respective arguments, continuously twice-differentiable and strictly concave. Let $q_j(s)$ be the representative collector’s flow purchases or sales of works of art. We assume that the stock (oeuvre) of the artist’s works takes a non-negative value and is not subject to depreciation, implying that $\dot{Q}_j(s) = q_j(s)$.

Notice, that $q_j(s)$ can be negative; i.e., the collector is always free to sell at the price $p(s)$ some pieces of art he owns at time $s$.

The representative collector receives a constant flow of income denoted by $y$. At any time his or her flow budget constraint is thus given by 

$$p(s) q_j(s) + x_j(s) = y.$$ (1)

The artist’s career begins at time $s = 0$, implying that $Q_j(0) = 0$.

Suppose the artist dies at time $t$, where $t$ is a random variable. We assume that the artist dies with probability $\theta dt$ in the unit interval $dt$. The Poisson arrival rate $\theta$ may take any nonnegative value: $\theta = 0$ indicates immortality, while $\theta \to \infty$ implies that death is imminent. Given the Poisson distribution, the probability that the artist is still alive at time $t$ amounts to $e^{-\theta t}$ and the probability that the artist dies exactly at time $t$ equals $\theta e^{-\theta t}$.

If the artist dies at time $t$, the price of his or her works of art will remain constant afterwards. This is so because the price depends (ceteris paribus) only on the accumulated stock $Q$, and this stock does not change anymore after the artist’s death if losses and faking are ruled out: $\dot{Q}(s) = 0$ for $s > t$. After the artist’s death no relevant new information appears since we disregard changes in preferences and substitution effects in the art market; the collectors’ portfolios will thus not change anymore ($q_j(s) = 0$ for $s > t$) if the rate of time preference equals the interest rate, and we have $x_j(s) = y$. Hence, the collectors’ total utility after time $t$ will depend on the holdings of the artist’s oeuvre $Q_j(t)$ at time $t$ and the exogenously given flow of income $y$. As a result, the present discounted value of total utility

---

3In the literature on durable-goods monopolies, several authors have pointed out that the presence of a positive depreciation rate profoundly affects the characteristics of the resulting equilibrium (see, for example, Bond and Samuelson, 1987, and Karp, 1996). Even though we admit that works of art do get destroyed or lost at times, our assumption does not appear to be immoderately restrictive.
after time $t$, $\int_t^\infty U(y, Q_j(t)) e^{-rs} ds = U(y, Q_j(t)) e^{-rt}/r$, turns out to be constant.

Using this fact and exploiting the properties of the Possion distribution, the expected utility of the collector can be expressed as follows:

$$
E \left\{ \int_0^t U(x_j(s), Q_j(s)) e^{-rs} ds + \int_t^\infty U(x_j(s), Q_j(s)) e^{-rs} ds \right\},
$$

$$
= E \left\{ \int_0^t U(x_j(s), Q_j(s)) e^{-rs} ds + U(y, Q_j(t)) e^{-rt} \right\},
$$

$$
= \int_0^\infty e^{-\theta s} U(x_j(s), Q_j(s)) e^{-rs} ds + \int_0^\infty \theta e^{-\theta s} U(x_j(s), Q_j(s)) e^{-rs} \frac{e^{-rt}}{r} ds,
$$

$$
= \int_0^\infty e^{-(r+\theta)s} \left[ U(x_j(s), Q_j(s)) + \frac{\theta}{r} U(x_j(s), Q_j(s)) \right] ds. \quad (2)
$$

The collector’s problem thus consists in choosing at each point in time $s < t$ the consumption bundle $(x_j(s), q_j(s))$ that maximizes his or her expected present value of total lifetime utility (2) subject to the budget constraint (1),

$$
\dot{Q}_j = q_j, \quad (3)
$$

$$
x_j \geq 0, \quad (4)
$$

where the symbol of time $s$ is omitted whenever misunderstandings can be ruled out.

The current-value Hamiltonian for the collector’s problem has the following appearance:

$$
H_j^C = \left( 1 + \frac{\theta}{r} \right) U(x_j, Q_j) + \lambda \frac{1}{p} [y - x_j].
$$

The first-order necessary conditions for an interior solution thus are:

$$
\frac{\partial H_j^C}{\partial x_j} = \frac{\partial U}{\partial x_j} - \frac{\lambda}{p} = 0, \quad (5)
$$

$$
\dot{\lambda} = (r + \theta) \lambda - \frac{\partial H_j^C}{\partial Q_j} = (r + \theta) \lambda - \left( 1 + \frac{\theta}{r} \right) \frac{\partial U}{\partial Q_j}, \quad (6)
$$

$$
\lim_{s \to \infty} e^{-(r+\theta)s} \lambda(s) Q_j(s) = 0, \quad (7)
$$
where $\lambda$ is the co-state variable associated with the state variable $Q$. Equation (5) states that in equilibrium the marginal utility of consumption of the non-durable good equals the marginal utility of holding the price-adjusted stock of art work. Equation (6) describes the evolution of the shadow price associated with the holdings of the artist’s oeuvre as long as the artist is alive.\(^4\)

Since it is our objective to derive comparative-static and comparative-dynamic results we need to arrive at a closed-form solution. We therefore further restrict our analysis to instantaneous utility functions that are separable in $Q_j$ and $x_j$, and quadratic in $Q_j$; that is:

$$U(x_j, Q_j) = \log x_j + aQ_j - \frac{Q_j^2}{2}.$$ 

Using this specification, equations (5) and (6), respectively, become

$$\frac{1}{x_j} = \lambda p,$$  \hspace{1cm} (8)

$$\frac{\dot{\lambda}}{\lambda} = (r + \theta) \left[ 1 - \frac{1}{\lambda} \frac{a - Q_j}{r} \right] = (r + \theta) \left[ 1 - \frac{x_j (a - Q_j)}{pr} \right].$$  \hspace{1cm} (9)

Differentiating the logarithmic transformation of (8) with respect to time and substituting the result into (9) yields

$$\frac{\dot{p}}{p} + \left( 1 + \frac{\theta}{r} \right) x_j \frac{(a - Q_j)}{p} = (r + \theta) + \frac{\dot{x}_j}{x_j}.$$  \hspace{1cm} (10)

Since the collector can buy or sell pieces of art in an efficient market at the market price $p(s)$, the marginal benefit from the last dollar spent on art at any time $s$ equals the marginal cost of doing so, whereby the marginal benefit corresponds to the expected marginal utility of the flow of art services per dollar ($[1 + (\theta/r)] \frac{(\partial U/\partial Q)}{p} = [1 + (\theta/r)] \frac{(a - Q_j)}{p}$) in terms of the marginal utility of non-durable consumption ($\partial U/\partial x_j = x_j^{-1}$) plus the percentage expected capital gain ($\dot{p}/p$). The marginal cost corresponds to the sum of the marginal cost of waiting, that is, the rate of time preference, $r$, plus the risk premium associated with the artist’s death, $\theta$.\(^4\)

\(^4\)If the artist were immortal (i.e. $\theta = 0$), our result would reduce to $\dot{p} = pr - \frac{\partial U/\partial Q}{\partial U/\partial x_j}$ which corresponds to the case of the standard durable-goods monopoly without depreciation (see, for example, Kahn, 1986).
\( \theta \), and the utility loss caused by a reduction in non-durable goods consumption. Exploiting symmetry and integrating (10) over the unit mass of all collectors yields the market demand:

\[
\frac{\dot{p}}{p} = (r + \theta) \left[ 1 - \frac{x(a - Q)}{pr} \right] + \frac{\dot{x}}{x},
\]

which also represents a rational expectation constraint for the artist.

2.2 The supply side: the artist’s behavior

Artists find themselves in the position of durable goods monopolists. Their market power is, however, limited by the fact that they are not able to make perfect contractual arrangements with the collectors. We assume the artist to maximize the present discounted return net of the disutility incurred from devoting labor effort to artistic production. Disutility is measured in monetary units and described by the standard quadratic cost function \( C(q) = \frac{b}{2} q^2 \).

The standard interpretation of the objective function (12) below presumes that the artist’s motives are purely commercial. If one believes that artists are rather driven by an urge to achieve eminence in the art world, one can easily re-interpret the objective function (12) to accommodate artistic pursuits. The price \( p \), after all, reflects the connoisseurs’ appreciation of the artist’s efforts, which implies that \( pQ \) represents a valid measure of artistic achievement.

Whichever the appropriate interpretation may be, the artist takes the inter-temporal optimizing behavior of the collectors as given and chooses a production pattern to maximize lifetime utility. The artist’s problem therefore consists in choosing \( q(s) \) over \( s \in [0, \infty) \) in order to maximize the present value of the net utility stream over the uncertain lifetime horizon:

\[
\max \int_0^\infty \left[ pq - \frac{b}{2} q^2 \right] e^{-(r+\theta)s} ds,
\]

subject to (1), (3) and (11), and \( Q(0) = 0 \). Note that the exponential probability of dying at time \( s \) simply increases the artist’s effective rate of time preference. After substitution of (1), the current-value Hamiltonian of the artist’s problem has the following appearance:

\[
H^A = (y - x) - \frac{b}{2} \frac{(y - x)^2}{p^2} + \mu_1 \frac{y - x}{p} + \mu_2 \left[ (r + \theta) \left\{ p - \frac{x(a - Q)}{r} \right\} + \frac{p}{x} \frac{m}{x} \right] + \mu_3 m,
\]
where \( m = \dot{x} \) is a slack variable. The necessary conditions for an interior optimum are

\[
\frac{\partial H^A}{\partial m} = \mu_2 \frac{p}{x} + \mu_3 = 0, \tag{13}
\]

\[
\dot{\mu}_1 = -\frac{\partial H^A}{\partial Q} + (r + \theta) \mu_1 = (r + \theta) \left[ -\mu_2 \frac{x}{r} + \mu_3 \right], \tag{14}
\]

\[
\dot{\mu}_2 = -\frac{\partial H^A}{\partial p} + (r + \theta) \mu_2 = \frac{y - x}{p^2} \left[ -\frac{b}{p} \frac{y - x}{p} + \mu_1 \right] - \mu_2 \frac{m}{x}, \tag{15}
\]

\[
\dot{\mu}_3 = -\frac{\partial H^A}{\partial x} + (r + \theta) \mu_3 = 1 - \frac{b}{p^2} \frac{y - x}{p} + \mu_1 + \mu_3 \left[ (r + \theta) \left\{ 1 - \frac{x(a - Q)}{pr} \right\} - \frac{m}{x} \right], \tag{16}
\]

and the transversality conditions have the following appearance:

\[
\lim_{s \to \infty} e^{-(r+\theta)s} \mu_1 (s) Q (s) = \lim_{s \to \infty} e^{-(r+\theta)s} \mu_2 (s) p (s) = \lim_{s \to \infty} e^{-(r+\theta)s} \mu_3 (s) x (s) = 0,
\]

where the \( \mu_i \) \((i = 1, 2, 3)\) denote the shadow prices associated with the stock \( Q (s) \), the price \( p \), and consumption \( x \), respectively. We assume that the artist is always active until he or she dies, i.e. the non-negativity constraint on \( q (s) \) is almost never binding.

### 3 Price and production during the artist’s lifetime

Combining and manipulating the optimality conditions of the artist’s and the collectors’ problems we obtain the following equations (17)-(20) which summarize our system of price and production dynamics. Equation (19) is simply a restatement of (11); the other three equations are derived in Appendix A.

\[
\dot{Q} = q = \frac{1}{p} (y - x), \tag{17}
\]
\[
\frac{\dot{x}}{x} = \Omega(x, p) \left[ \left\{ \mu_2 + \frac{x(a - Q)}{y} \right\} \frac{x}{r} - b \frac{y - x}{p} \left\{ 2 - \frac{x(a - Q)}{pr} \right\} \right], \quad (18)
\]

\[
\dot{p} = p (r + \theta) \left[ 1 - \frac{x(a - Q)}{pr} \right] + \frac{\dot{x}}{x}, \quad (19)
\]

\[
\dot{\mu}_2 = - \frac{y - x x}{p} - \frac{\dot{x}}{x}, \quad (20)
\]

where \( \Omega(x, p) \equiv (r + \theta) / \left\{ 2 \left( \frac{xp}{y} \right) + b \left( \frac{y}{p} \right) \right\} \).

### 3.1 The steady state

In the steady state of the dynamic system (17)-(20) that governs the development of the endogenous variables during the artist’s lifetime we have \( \dot{Q} = 0 \) in (17) and thus obtain \( \bar{q} = 0 \), which implies via the collectors’ budget constraint (1)

\[
\bar{x} = \bar{y}, \quad (21)
\]

where a bar indicates the steady state value of the corresponding variable. Using this result and setting \( \dot{x} = 0 \) and \( \dot{p} = 0 \) in (18) and (19), we obtain

\[
\bar{p} = \frac{y (a - \bar{Q})}{r}, \quad (22)
\]

\[
\bar{\mu}_2 = - (a - \bar{Q}). \quad (23)
\]

Note, that the steady state conditions (22) and (23) are not sufficient to uniquely determine \( \bar{Q}, \bar{p} \) and \( \bar{\mu}_2 \). This is because the equations of the dynamic system (17)-(20) (and thus the long run equilibrium conditions (21)-(23)) are linearly dependent on each other since the budget constraint (1) needs always to be met. This does however not imply that the adjustment path and the steady state of the model are indeterminate. After having derived the explicit time paths characterized by (17)-(20), we can use the initial conditions \( (\mu_2(0) = 0 \) and \( Q(0) = 0 \) and the steady state conditions (21)-(23) to identify the unique adjustment paths and the unique steady state. Since it is, in general, impossible to derive the adjustment paths of the
endogenous variables by solving the non-linear dynamic system (17)-(20), we focus on the linearized system. As shown in Appendix C, $\bar{p}$ is implicitly and uniquely determined (see equation C3).

Note, however, that the following proposition holds for the original non-linear system as well as for the linearized version thereof (The proof is to be found in Appendix C):

**Proposition 1**

(i) The steady-state stock $\bar{Q}$ of the artist’s works varies positively with the natural mortality rate $\theta$, the collectors’ income $y$ and their appreciation $a$ of the artist’s works of art, and negatively with the marginal cost $b$ of production and the interest rate $r$.

(ii) The steady-state price $\bar{p}$ of the artist’s works varies positively with the collectors’ income $y$, their appreciation $a$ of the artist’s works of art and the marginal cost $b$ of production, and negatively with the natural mortality rate $\theta$ and the interest rate $r$.

These results are quite intuitive. The larger the natural mortality rate, the more the artist is inclined to exploit the market in the short run. In other words, a shorter life expectancy induces the artist to be very productive and to sell more works of art at earlier stages of his life. This strategy results in a large oeuvre and low art prices if the artist is lucky enough to beat the odds and to grow to an old age. Production is also stimulated if the collectors have a high income or if they especially appreciate the artist’s work. This demand effect not only increases production but also the price of the art work. High interest rates, on the other hand, imply that the collectors heavily discount future utility streams associated with the art portfolio, thereby reducing demand for works of art. As a result, art prices decrease and so does production. If, finally, disutility associated with artistic effort is high (as captured by a high value of the parameter $b$), the supply of works of art is low and prices are high.

### 3.2 The dynamic adjustment

In order to describe how $p$ and $Q$ (or, equivalently, $q$) change during the artist’s career, we linearize our dynamic system (17)-(20) around the steady state and arrive at the following
where $\bar{\Omega} \equiv \Omega (y, \bar{p})$. The artist’s oeuvre $Q$ is, of course, by virtue of being a stock variable, predetermined. The shadow variable $\mu_2$ is associated with the evolution of $p$, which, via (10), dictates the inter-temporal behavior of the collectors. Since there is no restriction on the initial value of $p$, $\mu_2 (0)$ equals zero.\(^5\) In our model, the price is thus a jump variable which stands in sharp contrast with the standard models of durable-goods monopolies with a continuous price expectations function which does not allow discontinuous price changes.

To sum up, we have two jump variables, $p$ and $x$, in our system (24). In Appendix B, we show that there are indeed two unstable (i.e. positive) roots. According to Buiter (1984), the system exhibits saddle-point stability, implying the existence of a unique stable (non-explosive) solution of the dynamic system (24), which is given by:

$$Q (s) = (Q (0) - \bar{Q}) e^{\lambda_s} + \bar{Q} = (1 - e^{\lambda_s}) \bar{Q},$$

\(^5\)More precisely, if the initial value of $\lambda$ (alternatively, $x$ and $p$) in (9) is controllable, the leader (=artist) is free to choose any value $\lambda (0)$ (see Dockner et al., 2000), so that the optimal value of the associated co-state variable $\mu_2 (0)$ should be set equal to zero. Nevertheless, $\mu_2 (t_1) \neq 0$ for any $t_1 > 0$ due to the evolution of $\mu_2 (s)$ governed by (20) over time. This discrepancy gives rise to a time-inconsistent solution. To show this, we suppose that the original solution is expressed by $(Q^* (s), p^* (s), x^* (s), \mu_2^* (s))$ with $Q^* (0) = 0$ and $\mu_2^* (0) = 0$ for $s \in [0, \infty)$. When the artist is allowed to replan her optimal production plan at time $t_1$ in the sense that from $t_1 > 0$ onwards she is no longer bound to the initially announced production path $q^* (s)$ for $s \in [0, \infty)$, she will choose a new solution $(Q^{**} (s), p^{**} (s), x^{**} (s), \mu_2^{**} (s))$ which satisfies $\mu_2^{**} (s) = 0$, taking $Q^{**} (t_1) = Q^* (t_1)$ as given for $s \in [t_1, \infty)$. Since this new value $\mu_2^{**} (t_1) = 0$ is not equal to $\mu_2^* (t_1) \neq 0$, the artist will announce a new production path $q^{**} (s)$ for $s \in [t_1, \infty)$ which does not coincide with the remaining part of the original production path announced by her at time $Q, q^* (s)$.
\[ p(s) = p(0) = \bar{p}, \]  
\[ x(s) = (x(0) - \bar{x}) e^{\Lambda s} + \bar{x} = \bar{p} \Lambda e^{\Lambda s} + y, \]  
\[ \mu_2(s) = (\mu_2(0) - \bar{\mu}_2) e^{\Lambda s} + \bar{\mu}_2 = (a - \bar{Q}) \left( e^{\Lambda s} - 1 \right), \]

where \( \Lambda \) is the unique stable (negative) root (i.e. the negative root of \( \lambda \) in (B2) of Appendix B). The variables \( p(0) \) and \( x(0) \) are the endogenously determined initial values of \( p(s) \) and \( x(s) \). More precisely, since the variables \( p \) and \( x \) can freely jump, these initial values have to be chosen such that the adjustment path lies on the stable arm of the saddle-point. Indeed, it follows from (24) that \( x(s) - \bar{x} = -\bar{p} \Lambda \left[ Q(s) - \bar{Q} \right] \), which implies \( x(0) = y - \bar{p} \Lambda \bar{Q} \). Substituting this initial value into the middle expression in (27) yields the last expression.

Interestingly, the price \( p \) settles at once at the steady state level, so that \( p \) remains constant during the transition of the system towards the steady state as long as no new exogenous disturbance of the system occurs. To see this, we use the budget constraint (1) in conjunction with (25) and (27) to obtain

\[ p(s) = \frac{y - x(s)}{q(s)} = \frac{y - \left[ \bar{p} \Lambda \bar{Q} e^{\Lambda s} + y \right]}{-\Lambda Q e^{\Lambda s}} = \bar{p} \text{ for } s \in [0, \infty), \]

inferring from (25) that \( q(s) = \dot{Q}(s) = -\Lambda \bar{Q} e^{\Lambda s} \).

As \( \Lambda \ (< 0) \) gets smaller, the oeuvre \( Q(s) \) and consumption \( x(s) \) are approaching their steady state values at a more rapid rate. On the other hand, the production flow \( q(s) \) is gradually falling to zero as the artist grows older. These results are summarized in:

**Proposition 2** The market for works of visual art as portrayed by the linearized model (24) exhibits a unique and stable rational expectations equilibrium path. Along this path production gradually declines to zero as the artist grows older; the artist’s oeuvre thus converges to an upper limit \( \bar{Q} \). The art price remains constant at the steady state level during the artist’s entire career.
The proof immediately follows from (25)-(28) in conjunction with (C3).

Our Stackelberg solution characterized by (17)-(20) is not time consistent for the reason outlined in footnote 5. The artist thus has, in principle, an incentive to choose a new production plan at any time $t_1 > 0$. However, the time paths for the linearized system, i.e. (25) – (28), continue to hold for the linearized system associated with this new production plan that satisfies $\mu_2(t_1) = 0$. This is so because the optimal paths of $p(s), Q(s)$ and $x(s)$ characterized by (25) – (27) do not depend on the evolution of $\mu_2(s)$ given by (28). The linearized paths of $p(s), Q(s)$ and $x(s)$ can therefore be used to characterize the time paths associated with the time-consistent solution. This peculiarity depends of course on the special structure of our model. It allows us to arrive at very general conclusions with respect to the comparative-dynamic properties of our model which we will turn to in the following sections.

4 Comparative-dynamic properties

In this section we investigate how changes in the structural parameters of our model occurring during the artist’s lifetime affect the time paths of the model’s crucial endogenous variables. The effects depend of course on whether the disturbance is transitory or permanent, as well as on whether the respective shocks have been anticipated or not.\(^6\) Since the structural parameters of our model describe the socio-economic environment of the art scene, we focus on the effects of permanent and unanticipated changes.

A change of the model’s parameters at time $t$, gives rise to the following impact on flow production:

\(^6\)In the literature on macroeconomic dynamics (see, for example, Turnovsky, 2000), these distinctions play a key role in evaluating the effects of policy changes.
\[
\begin{align*}
\frac{dq(s)}{d\theta} \bigg|_{s=t} &= -\Lambda \frac{d\bar{Q}}{d\theta} - \left[ \frac{\partial \Lambda}{\partial \bar{p}} \frac{d\bar{p}}{d\theta} + \frac{\partial \Lambda}{\partial \theta} \right] (\bar{Q} - Q(t)) > 0, \quad (29) \\
\frac{dq(s)}{dy} \bigg|_{s=t} &= -\Lambda \frac{d\bar{Q}}{dy} - \left[ \frac{\partial \Lambda}{\partial \bar{p}} \frac{d\bar{p}}{dy} + \frac{\partial \Lambda}{\partial y} \right] (\bar{Q} - Q(t)) > 0, \quad (30) \\
\frac{dq(s)}{dr} \bigg|_{s=t} &= -\Lambda \frac{d\bar{Q}}{dr} - \left[ \frac{\partial \Lambda}{\partial \bar{p}} \frac{d\bar{p}}{dr} + \frac{\partial \Lambda}{\partial r} \right] (\bar{Q} - Q(t)) \geq 0, \quad (31) \\
\frac{dq(s)}{da} \bigg|_{s=t} &= -\Lambda \frac{d\bar{Q}}{da} - \frac{\partial \Lambda}{\partial \bar{p}} \left( \bar{Q} - Q(t) \right) \geq 0, \quad (32) \\
\frac{dq(s)}{db} \bigg|_{s=t} &= -\Lambda \frac{d\bar{Q}}{db} - \left[ \frac{\partial \Lambda}{\partial \bar{p}} \frac{d\bar{p}}{db} + \frac{\partial \Lambda}{\partial b} \right] (\bar{Q} - Q(t)) < 0, \quad (33)
\end{align*}
\]

where (+) and (-) indicate the sign of the respective terms (notice that \( \bar{Q} > Q(t) \) and \( \Lambda < 0 \)).

The impact effects on flow production \( q \) are governed by two effects: (1) the effect on the steady state stock (oeuvre) of the artist’s works, and (2) the effect on the speed of art production (i.e. \( d\Lambda/dk \) for \( k = \theta, y, r, a, b \)). The signs of \( d\bar{Q}/dk \) \( k = \theta, y, r, a, b \) have been described in Proposition 1. It is quite intuitive that an increase in the steady state level of the oeuvre induces an increase in flow production. The speed-effect works through two channels. On the one hand, the speed of adjustment is influenced via changes in the steady state price (i.e. \( (\partial \Lambda/\partial \bar{p}) (d\bar{p}/dk) \)). This adjustment reflects intertemporal substitution, i.e. higher future prices induce the artist to postpone production, while lower future prices induce him or her to expand current production. On the other hand, the speed of adjustment depends on specific effects which directly affect the rate of production (i.e. \( \partial \Lambda/\partial k, k = \theta, y, r, a, b \)). A higher mortality rate \( \theta \), for example, implies that future utility is discounted more heavily, thus rendering the artist more myopic which, in turn, stimulates production (i.e. \( \partial \Lambda/\partial \theta < 0 \)).

This is the direct adjustment-speed effect. The indirect adjustment-speed effect which works through price changes, operates in the same direction: A higher \( \theta \) decreases prices and therefore increases current production because the artist is induced to sell more at earlier stages of his
career \((\partial \Lambda/\partial \bar{p})(d\bar{p}/d\theta) < 0\). The steady-state effect, finally, is given by \(d\bar{Q}/d\theta < 0\). The impact effect on \(q(t)\) is therefore unambiguously positive, as illustrated in Figure 1. The impact effect of cost \(b\) on flow production \(q\) is also unambiguous: Higher values of \(b\) create an incentive to spread the artist’s production over time, thereby reducing his current production (see Figure 5). This reaction is consistent with the behavior of all durable-goods monopolists faced with increased marginal cost of production.

Turning to the other exogenous variables of the model, affairs become more complex. The increase in demand caused by the increase in income \(y\) stimulates production, and thus raises the price which in turn discourages current production. Although these two effects operate in opposite directions, the latter effect dominates (see Figure 2). The effect of \(r\) on the speed of adjustment \(\Lambda\) is ambiguous and thus induces an ambiguous total effect on flow production. Even though an increase in \(a\) unambiguously reduces the adjustment-speed effect, the steady-state effect on \(\bar{Q}\) is positive, resulting in an ambiguous net effect on production (see Figure 4). Nevertheless, when the path of \(Q(t)\) is sufficiently close to the steady state (i.e. \(Q(t) \simeq \bar{Q}\) for sufficiently large \(t\)), the steady state effects of \(r\) and \(a\) will dominate, implying that the signs of \(dq(t)/dr\) and \(dq(t)/da\) will be consistent with their respective steady-state effects. Note also that the respective impact effects on the price coincide with the corresponding steady-state effects, implying that "perverse" impact effects never occur. All these results (the respective proofs are to be found in Appendix D) are summarized in

**Proposition 3**

(i) An unanticipated permanent increase in the natural mortality rate \(\theta\) gives rise to a discontinuous increase in art production \(q\) and causes the price \(p\) of the artist’s works to drop;

(ii) An unanticipated permanent increase in income \(y\) gives rise to a discontinuous increase in production \(q\) and causes the price \(p\) to increase on impact;

(iii) An unanticipated permanent increase in the interest rate \(r\) (the appreciation \(a\) of the artist’s works) can give rise to either a decrease or an increase in art production \(q\); the price \(p\) however unambiguously drops (increases);

(iv) An unanticipated permanent increase in the cost \(b\) of art production gives rise to a decrease in production \(q\); whereas the price \(p\) increases on impact.
5 The death effect

In this section we show that the unanticipated death of a visual artist is followed by a discontinuous increase in the price of his or her works of art. If the artist’s death does not come as a surprise, the discontinuous price increase will occur when the public learns about the artist’s imminent death.

Suppose the artist unexpectedly dies at time $t$. The production of works of art that bear the deceased artist’s idiosyncratic way of expression thus ceases, and the stock of this kind of works of art remains constant thereafter. Since the late artist’s works are no longer traded in our model world in which nothing happens after the artist’s death, consumption of the non-durable composite good now equals the flow of income, that is, $x(t) = y$. As a consequence, the price of the late artist’s works will also remain constant after the death-induced impact effect (see Figure 5). Indeed, it follows from (11) that

$$p^{\text{death}}(s) = \frac{y(a - Q(t))}{r} \text{ for } s \geq t.$$  (34)
Figure 2: The effects of $y$

Figure 3: The effects of $r$
Figure 4: The effects of $a$

Figure 5: The effects of $b$
As long as the artist is alive and in good health, the price, denoted by $p_{\text{alive}}(s)$, is always equal to the steady state price level as we have shown in (26):

$$p_{\text{alive}}(s) = \frac{y(a - \bar{Q})}{r + \theta} = \bar{p}.$$  \hspace{1cm} (35)

Since for any finite $s$ we have $\bar{Q} > Q(s)$, it turns out that $p_{\text{death}} > p_{\text{alive}}(s)$. In other words, the price $p$ of the artist’s works increases on impact when the artist’s death is revealed to the public. The effect of the artist’s (unanticipated) death on the price $p$ is illustrated in the lower panel of Figure 6, while the effect on the time path of $q$ is depicted in the upper panel. Equations (34) and (35) demonstrate that the death effect \(\Delta p(t) \equiv p_{\text{death}}(t) - p_{\text{alive}}(t) = (y/r) (\bar{Q} - Q(t)) > 0 \) (for \(t < \infty\)) varies negatively with the final size $Q(t)$ of the artist’s oeuvre. Since an artist’s oeuvre increases with age, this implies that the death effect varies negatively with the artist’s age at death. This result is not due to the fact that the death of an old person is more likely (and thus less surprising) than the death of a young one. As a matter of fact, this effect is not captured in our model because we have assumed the natural death rate $\theta$ to be constant. It is remarkable that this negative age effect materializes even when one counterfactually assumes a constant natural death rate.

We now turn to investigating the effects of an artist’s death that does not come as a surprise. Consider the case in which an artist’s terminal illness is made public at time $t$, and assume that on the basis of this announcement the collectors are in a position to correctly anticipate the artist’s death at time $T (> t)$.\footnote{Keith Haring’s biography might serve as an example. Haring who was diagnosed with AIDS in 1988 enlisted during the last years of his life his artistic standing to generate activism and awareness about this decease. Via the establishment of the Haring Foundations these activities were well publicized with the consequence that nobody in the art scene was surprised by the early death of the artist in 1990.} Nevertheless, the announcement at time $t$ itself comes as a surprise and therefore does give rise to an impact effect on the price $p$ of the ailing artist’s works. On the basis of the announcement, the collectors re-optimize at time $t$:

$$\max \int_t^T e^{-rs}U(x(s), Q(s)) \, ds + e^{-r(T-t)}U(y, Q(T)),$$

subject to (3), (4), and the initial condition $Q(t)$. The first-order necessary condition and the terminal condition are given by 8:
\[
\frac{\dot{\lambda}}{\lambda} = r \left[ 1 - \frac{x(a - Q)}{pr} \right], \tag{36} \\
\lambda(T) = U_Q(y, Q(T)). \tag{37}
\]

Since the age at death, \( T \), is no longer a random variable after the news has been revealed, the parameter \( \theta \) does not appear anymore in (36) and (37). Solving the artist’s optimization problem on the basis of this modification results in the adjustment path given by (25)-(28) with \( r + \theta \) being replaced by \( r \). After the announcement the price, denoted by \( p^{\text{after}}(s) \), therefore immediately jumps to the price that will prevail at time \( T \):

\[
p^{\text{after}}(t) = p(T) = \frac{y(a - Q(T))}{r}.
\]

In order to establish whether the impact effect on the price is positive or negative, we compare \( p^{\text{after}}(t) \) with \( p^{\text{before}}(t) = \bar{p} = y(a - \bar{Q})/r \). This simply amounts to comparing the size of \( Q(T) \) and \( \bar{Q} \). Given the size of the artist’s oeuvre at time \( t \), \( Q(t) \), the path of \( Q(s) \) \((s \geq t)\)
is given by
\[
Q(s) = [Q(t) - Q(T)] \frac{1 - e^{\hat{\Lambda}(s-T)}}{1 - e^{\hat{\Lambda}(t-T)}} + Q(T),
\]
where
\[
\hat{\Lambda} \equiv \frac{1}{2} \left[ r - \sqrt{r^2 + \frac{8y^2}{2\bar{p}^2 + yb}} \right].
\]

It can now easily be seen from (38) that \( Q(s) = Q(t) \) at \( s = t \) and \( Q(s) = Q(T) \) at \( s = T \).

Setting \( t = 0 \) and taking \( T \to \infty \), \( Q(s) \) can be reduced to
\[
Q(s) = \left[ 1 - e^{s\hat{\Lambda}} \right] \hat{Q},
\]
where \( \lim_{T \to \infty} Q(T) = \hat{Q} \). Noting that by definition \( Q(T) < \hat{Q} \) for \( T < \infty \), we compare \( \hat{Q} \) with \( \hat{Q} \). Using the facts that the steady state level \( \hat{Q} \) is reached when \( \theta = 0 \) and that \( d\hat{Q}/d\theta > 0 \), it is immediate that \( \hat{Q} < \hat{Q} \). We therefore conclude that \( p_{after}(t) > \hat{p} > \bar{p} \), where \( \hat{p} = (a - \hat{Q})/r \), i.e. the price increases when the announcement is made. Moreover, note that the size of the price increase varies negatively with the lead time \( T - t \).

At time \( T \) no new information is revealed, implying that there will be no price jump at \( T \); \( q(s) \) therefore continuously converges to zero as \( s \) converges to \( T \). To measure the impact effect of the announcement on the flow supply \( q \) at time \( t \), we differentiate (38) with respect to time to obtain
\[
\frac{dQ(s)}{ds} = q(s) = -\hat{\Lambda} [Q(T) - Q(t)] \frac{e^{\hat{\Lambda}(s-T)}}{e^{\hat{\Lambda}(t-T)} - 1}.
\]

By comparing (40) with \( q(s) = -\Lambda\hat{Q} e^{\Lambda s} \), it turns out that the announcement effect on \( q(s) \) is ambiguous because \( 0 > \hat{\Lambda} > \Lambda \) and \( \hat{Q} > \hat{Q} > Q(T) - Q(t) \), but \( e^{\hat{\Lambda}(t-T)}/[e^{\hat{\Lambda}(t-T)} - 1] > e^{\Lambda t} \) when evaluated at time \( s = t \). This ambiguity stems from two conflicting effects: an intertemporal substitution effect caused by higher future prices which tends to reduce production, and an effect relating to the shorter lifetime which stimulates current production.

We summarize these results in our final

**Proposition 4**

(i) When an artist unexpectedly dies at time \( t \), the price \( p \) of his or her works of art increases...
Figure 7: The effects of anticipated death

on impact by the amount of $\Delta p(t) = (y/r)(Q - Q(t))$, where $Q(t)$ represents the final size of the artist’s oeuvre. The size of the impact effects on the price is inversely related to the artist’s age at death.

(ii) When, at time $t$, an artist’s impending death at time $T$ is announced, the price $p$ of the artist’s works immediately increases by $(y/r)(\bar{Q} - Q(T))$, and remains constant thereafter, although flow production may gradually rise or fall after the announcement. The size of the impact effect on the price $p$ varies negatively with (1) the age $t$ at which the artist’s impending death is announced and (2) the artist’s remaining lifetime $T - t$.

6 Concluding Remarks

Unless durable goods monopolists are in a position to make credible commitments to restrain their output, the consumers will always anticipate that the producers will have an incentive to increase the output in the future in order to skim the remaining willingness to pay. The con-
sumers’ time-consistent anticipation therefore exerts a downward pressure on the prices which will, as a consequence, immediately settle at the competitive level. Since credible commitment devices are hard to come by, Coase conjectured in his famous 1972 article that durable goods monopolists, as a rule, offer competitive prices. Artistic production may, however, follow different rules because the artistic temperament is liable to impose certain constraints on an artist’s production pattern. To what extent such an implicit commitment prevails, is an open question. In any event, the death of a visual artist can be interpreted to represent the ultimate device to restrain output. The fact that all artists are eventually, nolens volens, forced to make use of this device, provided the motivation for our study.

We arrive at the conclusion that all prices of visual works of art, whether the artist has been able to commit to an optimal production plan or not, are subject to a positive death effect. Moreover, if an artist is known to have an above-average mortality rate, or if the artist is known to suffer from a terminal illness and will die at a foreseeable point of time, his or her works of art will, ceteris paribus, fetch a higher price. Finally, we show that the death effect deceases with the artist’s age at death. This is so because older artists produce less than younger ones; the death of an old artist thus implies a relatively small reduction in the expected size of his or her oeuvre which, in turn, causes a relatively small price increase.

Our analysis has made use of several shortcuts imposed by analytical convenience. First of all, our analysis does not allow us to characterize the time consistent production plan of an artist who cannot credibly commit to an optimal production strategy. We have pointed out, however, that the linearized paths of our open-loop Stackelberg solution reflect time consistency in the sense that these time paths are independent of whether the artist does or does not reoptimize at any intermediate point of time.

More restrictive is probably our assumption that an artist’s reputation is constant. Reputation is clearly an endogenous variable and can only be acquired by a substantial track record of outstanding artistic achievements over an artist’s entire life cycle. Works of young promising artists may nevertheless be bought at a relatively high price by connoisseurs because they expect the artist to achieve eminence in the future. If such an aspiring artist dies at a relatively young age, the collectors’ expectations of future fame are frustrated and the
price of the artist’s works may, as a consequence, decrease after his death. In their empirical study, Ursprung and Wiermann (2008) have indeed shown that the death effect is negative if an artist dies an untimely death. The death effect of frustrated positive expectations thus countervails the death effect portrayed in our model, i.e. the death effect which derives from averted negative expectations of an excessively large future production.

A further important aspect of artistic production that we did not consider in our analysis is that the artist is confronted with a trade-off between quantity and quality. We rather assumed that works of art are reproducible at will, just as factory-produced goods. This assumption is, as we have pointed out above, not too far removed from a fair representation of how some late 20th century artists produce. Nevertheless, we do of course acknowledge that this is not true for all modern artists and certainly not for all periods. Moreover, we readily acknowledge that creativity may change over the life cycle which would imply a change of quality as the artists grows older. Since quality is the main determinant of prices, all these considerations need to be taken into account if one attempts to arrive at a well-proportioned portrait of art price formation in general, and the death effect in particular.

**Appendix A**

Equation (17) follows from substitution of (1) into (3). Take a time differentiation of the logarithm of (13) to obtain

\[
\frac{\dot{p}}{p} = \frac{\dot{\mu}_3}{\mu_3} - \frac{\dot{\mu}_2}{\mu_2} + \frac{x}{x}. \tag{A1}
\]

Comparing (A1) with (11) leads to

\[
\frac{\dot{\mu}_3}{\mu_3} - \frac{\dot{\mu}_2}{\mu_2} = (r + \theta) \left[1 - \frac{x(a - Q)}{pr}\right]. \tag{A2}
\]

Substituting (15) and (16) into (A2) and manipulating results in

\[
\mu_1 = b\frac{y - x}{p} - p\frac{x}{y}. \tag{A3}
\]
By taking time-differentiation of (A3) and substituting (A3) into the resultant expression, we obtain

\[- \left( \frac{bx}{p} + \frac{px}{y} \right) \frac{\dot{x}}{x} = \dot{\mu}_1 + \left[ \frac{y - x}{pr} + p \frac{x}{y} \right] \frac{\dot{p}}{p}. \]  

(A4)

Then substitution of (14) into $\dot{\mu}_1$ in (A4) results in

\[- \left( \frac{xb}{p} + \frac{xp}{y} \right) \frac{\dot{x}}{x} = (r + \theta) \left[ -\frac{x}{r} + \mu_1 \right] + \left[ \frac{y - x}{pr} + p \frac{x}{y} \right] \frac{\dot{p}}{p}. \]  

(A5)

Further substitution of (A3) and (11) into (A5) and manipulation yields (18).

Finally, we can easily show that substitution of (A3) into (15) leads to (20).

**Appendix B**

The characteristic equation for the Jacobian matrix in the dynamic system (24) is given by

\[(r + \theta - \lambda) \lambda \left[ \lambda^2 - (r + \theta) \lambda - \frac{2 (r + \theta) y^2}{2p^2 + yb} \right] = 0. \]  

(B1)

It is immediate that equation (B1) has two roots, 0 and $r + \theta$, while the other two roots are given by solving the quadratic equation $\lambda^2 - (r + \theta) \lambda - \left[ 2 (r + \theta) y^2 / (2p^2 + yb) \right] = 0$. The latter roots are of the form

\[\lambda_{3,4} = \frac{(r + \theta) \pm \sqrt{(r + \theta)^2 + \frac{8 (r + \theta) y^2}{2p^2 + yb} \frac{r}{r}}}{2}. \]  

(B2)

It is immediately seen that since the argument inside the square root function in (B2) is positive, the two roots are real. Moreover, the larger of the roots is positive, while the smaller of the roots is negative. Taken together, we have one negative root, two positive roots and the zero root. The zero root implies that the determinant of the Jacobian in (24) is equal to zero. It means that one of the differential equations is a multiple of the other (i.e., they are linearly dependent), so there are really only three equations characterizing the dynamics of this system. This happens because the dynamics characterized by (25) – (28) (or (24)) is
linked by the instantaneous budget constraint (1) at any moment in time.

Appendix C

Using the equation appearing in the third row of the matrix appearing on the left-hand side of (24) and noting that \( p(s) = \bar{p} \) for all times, we have

\[
\begin{align*}
(r + \theta) - \bar{p}\Omega \frac{y}{r} [Q(s) - \bar{Q}] + \left\{ \bar{p}\Omega - (r + \theta) \right\} \frac{a - \bar{Q}}{r} + b\bar{\Omega} \right] [x(s) - \bar{x}] \\
+ \bar{p}\Omega \frac{y}{r} [\mu_2(s) - \bar{\mu}_2] = 0.
\end{align*}
\]

(C1)

Using (25) and (27), we have \( x(s) - \bar{x} = -\bar{p}\Lambda [Q(s) - \bar{Q}] \). Then substitution of this expression into (C1) yields

\[
\begin{align*}
- \left\{ (r + \theta) - \bar{p}\Omega \right\} \frac{y}{r} + \left\{ (\bar{p}\Omega - (r + \theta)) \frac{a - \bar{Q}}{r} + b\bar{\Omega} \right\} (-\bar{p}\Lambda) \right] [Q(s) - \bar{Q}] \\
+ \bar{p}\Omega \frac{y}{r} [\mu_2(s) - \bar{\mu}_2] = 0,
\end{align*}
\]

which is, by substituting (28) and deleting the term \( Q(s) - \bar{Q} \), reduced into

\[
\begin{align*}
(r + \theta) - \bar{p}\Omega \frac{y}{r} + \left\{ \bar{p}\Omega - (r + \theta) \right\} \frac{a - \bar{Q}}{r} + b\bar{\Omega} \right\} (-\bar{p}\Lambda) + \bar{p}\Omega \frac{y}{r} \left( \frac{a - \bar{Q}}{Q} \right) = 0.
\end{align*}
\]

(C2)

Further manipulation of (C2) coupled with the definition of \( \bar{\Omega} \) results in

\[
2 + b\frac{y}{\bar{p}^2} + \bar{p}^2\frac{r}{y^2} - \frac{ay}{ay - r\bar{p}} = 0.
\]

(C3)

Let us denote the left-hand side of (C3) as \( f(p; \theta, y, r, a) \). Differentiate \( f(,) \) with respect to \( \bar{p} \) to obtain:

\[
\frac{\partial f}{\partial \bar{p}} = -b\frac{y}{\bar{p}^2}2\bar{p} + \bar{p}\frac{r}{y^2} \left( 2\Lambda + \bar{p}\frac{\partial \Lambda}{\partial \bar{p}} \right) + \frac{ay(-r)}{(ay - r\bar{p})^2} < 0,
\]

(C4)

which is signed by noting that \( 2\Lambda + \bar{p}(\partial \Lambda / \partial \bar{p}) < 0 \). By employing the implicit function theorem
in conjunction with (C4) to the function \( f(\cdot) \), we have

\[
\frac{d\bar{p}}{d\theta} = - \frac{\partial f / \partial \theta}{\partial f / \partial p} < 0,
\]

because \( \partial f / \partial \theta = \bar{p}^2 \left( r/y^2 \right) (\partial \Lambda / \partial \theta) < 0 \) (due to \( \partial \Lambda / \partial \theta < 0 \)). Moreover, differentiation of (22) with respect to \( \theta \) and using (C5) establishes

\[
\frac{d\bar{Q}}{d\theta} = - \frac{r \, d\bar{p}}{y \, d\theta} > 0.
\]

Similarly, we have

\[
\frac{d\bar{p}}{dy} = - \frac{\partial f / \partial y}{\partial f / \partial p} > 0 \quad \text{and} \quad \frac{d\bar{Q}}{dy} = - \frac{r}{y^2} \left( \frac{dp}{dy} y - \bar{p} \right) = - \frac{r}{y^2} \frac{1}{\partial f / \partial p} \left( \frac{\partial f}{\partial p} + \frac{\partial f}{\partial y} \right) > 0,
\]

because

\[
\frac{\partial f}{\partial y} = - \frac{b}{\bar{p}^2} + \frac{\bar{p}^2}{y^2} \left( -2\Lambda + y \frac{\partial \Lambda}{\partial y} \right) + \frac{ar\bar{p}}{(ay - r\bar{p})^2} > 0 \quad \text{(due to} -2\Lambda + y (\partial \Lambda / \partial y) > 0) \quad \text{and} \quad \bar{p} (\partial f / \partial p) + y (\partial f / \partial y) < 0.
\]

\[
\frac{d\bar{p}}{dr} = - \frac{\partial f / \partial r}{\partial f / \partial p} < 0 \quad \text{and} \quad \frac{d\bar{Q}}{dr} = - \frac{1}{y} \left( \frac{d\bar{p}}{dr} + \bar{p} \right) = - \frac{1}{y} \frac{1}{\partial f / \partial p} \left( - \frac{\partial f}{\partial r} + \bar{p} \frac{\partial f}{\partial p} \right) < 0,
\]

because

\[
\frac{\partial f}{\partial r} = \frac{\bar{p}^2}{y^2} \left( \Lambda + r \frac{\partial \Lambda}{\partial r} \right) + \frac{ay (-\bar{p})}{(ay - r\bar{p})^2} < 0 \quad \text{(due to} \Lambda + r (\partial \Lambda / \partial r) < 0) \quad \text{and} \quad -r (\partial f / \partial r) + \bar{p} (\partial f / \partial p) < 0.
\]

\[
\frac{d\bar{p}}{da} = - \frac{\partial f / \partial a}{\partial f / \partial p} > 0 \quad \text{and} \quad \frac{d\bar{Q}}{da} = 1 - \frac{r \, d\bar{p}}{y \, da} = \frac{1}{\partial f / \partial p} \left( \frac{\partial f}{\partial p} + \frac{r \, \partial f}{y \, \partial a} \right) > 0,
\]

because \( \partial f / \partial a = yr\bar{p} / (ay - r\bar{p})^2 > 0 \) and \( (\partial f / \partial p) + (r/y) (\partial f / \partial a) < 0 \Lambda \). Finally, we have

\[
\frac{d\bar{p}}{db} = - \frac{\partial f / \partial b}{\partial f / \partial p} > 0 \quad \text{and} \quad \frac{d\bar{Q}}{db} = - \frac{r \, d\bar{p}}{y \, db} < 0,
\]

because \( \partial f / \partial b = (y/\bar{p}^2) + p^2 \left( r/y^2 \right) (\partial \Lambda / \partial b) > 0 \) (due to \( \partial \Lambda / \partial b > 0 \)).
Appendix D

The instantaneous impact on a permanent change in, say the probability of death, $\theta$, on the rate of production at time $t$ is given by differentiating (1) with respect to $\theta$, given $p(t) = \bar{p}$:

$$\frac{dq(t)}{d\theta} = \frac{1}{\bar{p}^2} \left[ -\frac{dx(t)}{d\theta} \bar{p} - (y - x(t)) \frac{dp}{d\theta} \right].$$  \hfill (D1)

We differentiate $x(s) - \bar{x} = -\bar{p} \Lambda [Q(s) - \bar{Q}]$ with respect to $\theta$ at time $t$:

$$\frac{dx(t)}{d\theta} = \frac{dp}{d\theta} \Lambda (\bar{Q} - Q(t)) + \bar{p} \frac{d\Lambda}{d\theta} (\bar{Q} - Q(t)) + \bar{p} \Lambda \frac{d\bar{Q}}{d\theta}. \hfill (D2)$$

Substituting (D2) into $dx(t)/d\theta$ in (D1) results in

$$\frac{dq(t)}{d\theta} = \frac{1}{\bar{p}^2} \left[ -\frac{dp}{d\theta} \Lambda (\bar{Q} - Q(t)) + (y - x(t)) \right] - \bar{p}^2 \frac{d\Lambda}{d\theta} (\bar{Q} - Q(t)) - \bar{p}^2 \Lambda \frac{d\bar{Q}}{d\theta}. \hfill (D3)$$

Once again using $x(s) - \bar{x} = -\bar{p} \Lambda [Q(s) - \bar{Q}]$ with $y = \bar{x}$ and expressing $d\Lambda/d\theta \equiv (\partial \Lambda/\partial \bar{p})(dp/d\theta) + (\partial \Lambda/\partial \theta)$, (D3) can be rewritten as (29).

In an analogous way, we can show the effects of permanent changes in other parameters on $q(t)$ are given by (30)-(33).

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