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THE EFFICIENCY LOSS OF CAPITAL INCOME TAXATION UNDER IMPERFECT LOSS OFFSET PROVISIONS

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Abstract

The importance of capital loss offset provisions in a world of risk is well documented in the tax literature. However, the potential deadweight losses owing to imperfect offset has not been fully explored. This paper develops a framework whereby that investigation can be carried out and utilizes numerical simulations to investigate the size of potential losses. Results show that when the government and private sector are equally efficient in handling market risk, welfare losses owing to the absence of offset provisions could be substantial. Under plausible assumptions about attitudes towards risk and time preference, and with a capital income tax rate of forty percent, over sixty cents per dollar of tax revenue raised would be dissipated. In contrast, full loss offset would reduce that loss to approximately fourteen cents.

JEL Code: H00, H21, H22.

Keywords: capital income taxation, uncertainty, deadweight loss, loss offset provisions.

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1 Introduction

Industrial countries generally allow losses from the sale of capital assets as offsets against taxable capital gains. Allowing a portion of any deficit as a deduction from other income and permitting net losses to be carried forward or back to previous tax years are common tax code rules. Whatever the rules, offset provisions reduce tax liability for investors incurring capital losses and encourage risky capital investment.

Atkinson and Stiglitz (1980) pointed out that partial rather than full loss offset provisions might be instituted because of a government’s inability to distinguish between consumption and production activities and also could be desirable if individuals were able to influence relevant states of nature. Auerbach (1986) provided a dynamic analysis of taxation impacts on investment decisions and proposed a number of rationales for asymmetry, including the desire to limit firms’ deduction of fictional expenses and the wish to discourage operation of unprofitable firms.

Connections between loss offset provisions and behaviour in the face of risk have been well noted in the literature, beginning with the analysis of Domar and Musgrave (1944). They observed that in the absence of offset, tax increases lessen risk-taking. Stiglitz (1969), using a more general expected utility model, confirmed the disincentive effect and it has been observed in similar models under various tax scenarios, including those of Mossin (1968), Feldstein (1969) and Ahsan (1974).

Most studies of this genre had ignored how the public sector would deal with the revenue risk and assumed in effect the state to be risk-neutral. When dealing with capital risks, the latter assumption may well be untenable. Bulow and Summers (1984) and Gordon (1985) have argued that, especially for corporate tax revenue, government need not be any more able to bear risk than the private sector. A key implication is that the costs of and benefits from risk-taking remain entirely in the private sector.

Early investigations of deadweight loss (DWL) owing to capital income taxation, including work by Boskin (1978), Feldstein (1978) and Summers (1981) ignored risk. Fullerton and Gordon (1983) and Slemrod (1983) incorporated risk into their analysis by replacing the tax-induced change in

---

1 The current United States tax code allows up to $3000 per year in capital losses to be written off against other income. Canadian losses in any one year can be applied only against gains.


3 In order to accomplish the latter, as detailed in the next section, we assume that tax revenues are returned to each taxpayer in amounts identical to revenues collected.
behaviour in Harberger’s classic formula by its expected value. Gordon and Wilson (1989), examining DWL under uncertainty with full loss offset, argued that in a dynamic multi-period setting the correct measure was the certainty equivalent of the lottery and concluded that in neglecting risk-aversion, previous studies had over-estimated efficiency costs.

The present analysis employs a simple two-period, two-asset, consumption-savings model to identify and measure efficiency losses in a risky world, comparing impacts under full and imperfect loss offsets. To allow focus on the effects of alternative tax rules we assume that the public sector is no more or less efficient than the private in handling risk, that there is no asymmetric information between individuals and the government, and that all risk is borne by the private sector.

Under these conditions and assuming non-expected utility preferences, we carry out numerical simulations of impacts and welfare costs. Starting from an initial tax rate of forty percent, we determine that under plausible assumptions about investor attitudes towards risk and consumption versus savings, the welfare loss from capital income taxation without offset would be substantial, amounting to approximately sixty two cents per dollar of tax revenue or some 18 percent of savings. In contrast, the welfare loss from capital income taxation with full loss offset would be approximately fourteen cents per dollar, or just 5 percent of savings.

The contribution of the present research is twofold. First, it presents a framework within which to calculate deadweight losses generated by capital income taxation under alternative offset rules and under the plausible rule that all risks remain in the private sector. Second, it examines the impacts that attitudes towards risk and consumption substitutability each have on tax-induced behaviour, and thence on welfare, under a given offset regime. In each case, simulations are employed to demonstrate outcomes.

The remainder of the paper is organized as follows. Section II models investor decision-making. Section III uses non-expected utility preferences to value the deadweight loss from distortions owing to capital income taxation. Section IV evaluates outcomes when capital asset selling prices, and hence tax revenues, are risky. Section V examines the sensitivity of DWL to alternative parameter values and distribution assumptions. Section VI concludes. Much of the technical derivation is relegated to the Appendix.
2 The Portfolio - Savings Decision

Consider an individual who works in the first period and earns a non-stochastic wage income $Y_1$. The household allocates this income to current consumption, denoted by $C_1$, and savings, $S_1$. Savings, $S_1$, can be invested in a risky asset $a_1$, and a riskless investment $m_1$ in order to provide consumption for retirement. Thus in the first period:

$$C_1 = Y_1 - S_1 = Y_1 - (a_1 + m_1) \quad (1)$$

In the second period, the safe asset yields an after tax return of $(1 - \tau)r$ where $\tau$ is the tax on capital income and $r$ the before tax return per unit of investment. The state of nature $i$ determines the return of the risky asset, $x_{2i}$. A good and a bad state of nature are modeled. In the good state of nature, the risky asset yields an after tax return $x_{2g}(1 - \tau) > 0$ with probability $p$ and in the bad state of nature $x_{2b}(1 - \varphi \tau) < 0$ with probability $(1 - p)$ per unit of investment.\(^4\) The parameter $0 \leq \varphi \leq 1$ indicates the level of offset provision. Full loss offset (FLO) occurs when $\varphi = 1$. When FLO is in effect, the household pays $x_{2g} \tau$ in taxes per unit of investment if the good state of nature occurs and is allowed a loss offset in the bad state of nature equal to $-x_{2b} \tau$. In the case of no loss offset (NLO), $\varphi = 0$, the household still pays $x_{2g} \tau$ in taxes in the good state of nature but is not allowed to offset any losses if the bad state materializes. A second period stochastic lump-sum transfer from the state, denoted by $G_{2i}$, is also provided.

During retirement the household consumes $C_{2b}$ if the bad state of nature materializes and $C_{2g}$ if the good state occurs.\(^5\)

$$C_{2b} = (1 + r(1 - \tau))m_1 + (1 + x_{2b}(1 - \varphi \tau))a_1 + G_{2b} \quad (2a)$$
$$C_{2g} = (1 + r(1 - \tau))m_1 + (1 + (1 - \tau)x_{2g})a_1 + G_{2g} \quad (2b)$$

The household’s preferences are described by the class of non-expected utility preferences as formulated in a two period setting by Selden (1978, 79). These preferences include the corresponding expected cardinal utility function as a special case. The household is assumed to make choices between current and certainty equivalent future consumption, $CE(C_2)$. The preferences are: \(^6\)

\(^4\)Limited liability requires that $x_{2b} > -1$.
\(^5\)There are no bequests in the model.
\(^6\)See Selden (1978, 79) for details.
\[ U(C_1, CE(C_2)) = [C_1^\delta + \delta CE(C_2)^\theta]^ {1/\theta} \]  

(3)

where the consumer’s rate of time preference is reflected in the discount factor \( \delta = 1/(1+\rho) \) with \( \rho \) measuring the rate of time preference and the inter-temporal substitutability between current and certainty equivalent future consumption is measured by \( \sigma = \frac{1}{1-\gamma} \).

The certainty equivalent of future consumption is given as follows:

\[ CE(C_2) = E(C_1^{1-\gamma}) \]  

(4)

where, \( \gamma \) is the relative relative risk aversion parameter measuring aversion to risk taking activity. The household computes the certainty equivalent future consumption given its risk preferences, and then relying on the inter-temporal substitutability combines current consumption with the certainty equivalent future consumption.

For a given \( G_2 \), the first-order condition with respect to the choice of current consumption is:

\[ C_1^{\theta-1} = \delta(1 + r(1 - \tau))CE(C_2)^{\theta+\gamma-1}E(C_2^{-\gamma}) \]  

(5)

The household sets the marginal utility of current consumption equal to the future value of the marginal utility of future consumption adjusted for its risk preference. Turning to the first order condition with respect to risk taking:

\[ E(C_2^{-\gamma}z_2) = 0 \]  

(6)

where \( z_2 \) is the after tax excess return of the risky asset relative to the safe asset. In the good state of nature \( z_{2g} = (x_{2g} - r)(1 - \tau) > 0 \), while in the bad state of nature \( z_{2b} = (x_{2b}(1 - \varphi \tau) - (1 - \tau)r) < 0 \). Equation (6) states that at the optimum the expected marginal gain from risk taking is equal to that of the riskless investment in terms of their contribution to future consumption.\(^8\)

\(^{7}\)The utility function takes on the familiar von-Neumann Morgenstern form when \( 1 - \gamma = \theta \). Risk neutral constant elasticity of inter-temporal substitution preferences (RINCE) developed by Farmer (1990) is observed when \( \gamma = 0 \).\(^8\) An interior solution obtains so long as the expected return on the risky asset exceeds the return on the riskless investment.
Since it is assumed that the government is no more (or less) efficient in handling risk we assume that tax revenue is returned to the taxpayer in the amount identical to revenues collected. This implies that the risk remains within the private sector, and hence investors ultimately bear the entire risk. Gordon justifies such an assumption as follows:

"Given that the government absorbs a sizable fraction of the risk as a result of the taxes on corporate income, one might have expected the market risk premium to fall. However, the government cannot freely dispose of the risk it bears. Individuals must ultimately bear this risk, whether through random tax rates on other income, random government expenditures, or random government deficits" (1985, p5).

Thus given optimal choices with taxes and offset provisions, the government transfers:

\[
G_{2g} = \tau r(Y_1 - C_1^*) + \tau(x_{2g} - r)a_1^* \quad (7a)
\]
\[
G_{2b} = \tau r(Y_1 - C_1^*) + \tau(\varphi x_{2b} - r)a_1^* \quad (7b)
\]

where \( G_{2i} \ (i = g, b) \) is precisely the tax paid by a typical individual. Under this assumption appendix 1 derives the explicit optimal values of asset choice and consumption decision. The optimal values are presented below in a general form:

\[
C_1^* = c_1(\tau, \varphi)Y_1 \quad \text{and} \quad a_1^* = a_1(\tau, \varphi)Y_1 \quad ((8))
\]

The optimal values of consumption and risky asset holdings are linear functions of the individual’s endowment, \( Y_1 \). Appendix 2 proceeds to derive the optimal response of current consumption and risky asset to an increase in capital income taxation assuming all risk remain with the private sector. The responses are as follows: 9

\[
\frac{\partial C_1^*}{\partial \tau} \bigg|_{NRS} > 0 \quad \text{if} \quad \varphi = 1 \quad (9a)
\]
\[
\frac{\partial a_1^*}{\partial \tau} \bigg|_{NRS} < 0 \quad \text{for all values of} \ \varphi \quad (9b)
\]

\[9\]NRS stands for no risk sharing by the government. The stochastic lump sum transfer is given to the individual in the same period it was collected but at the new inter-temporal price of consumption and asset choice. This eliminates all income effects.
These two expressions measure the expected change in current consumption due to a change in the tax rate and that of the risky asset under the assumption of no risk sharing by the government (NRS). Current consumption is encouraged, while at the same time risky asset choice is discouraged, because the increase in the capital income tax alters the relative price of current and future consumption distorting intertemporal decisions (i.e., see equation 5 which still holds). In the case of imperfect loss offset, risky asset holdings are discouraged even more because the relative asset returns are also distorted (i.e., see equation 6). The return of the stochastic tax revenue back to the household eliminates all income effects and only substitution effects remain. Taxation has no stimulating effects on risk taking activity.

3 The Deadweight Loss of Capital Income Taxation

The stochastic lump sum rebate of taxes paid would not be sufficient to hold the investor on the same indifference level as prior to the imposition of the tax, because the capital income tax would entail an efficiency loss by creating a distortion in the inter-temporal price of future consumption as well as to the assets’ after tax returns given partial loss offset provisions. In what follows we use the Diamond-McFadden (1974) approach to measuring the marginal deadweight loss (MDL) of capital income taxation, which quantifies the additional income (consumption in our case) required by the investor in the current period in order to remain just as well off after the tax increase and the consequent transfer payment. MDL is equal to \( \frac{dL_1}{d\tau} \bigg|_{v=c} \) where \( L_1 \) is the additional consumption needed in the first period to make the household indifferent to the tax increase. Appendix 3 shows the MDL as follows:

\[
MDL = \frac{dL_1}{d\tau} \bigg|_{v=c} = \frac{r\tau}{(1 + r(1 - \tau))} \frac{\partial C_1^*}{\partial \tau \ NRS} + \frac{\tau x_{2b}(1 - \varphi)}{(1 - \tau)(1 + r(1 - \tau))} \frac{(1 - p)C_{2b}^{-\gamma}}{E(C_2^{-\gamma})} \frac{\partial a_1^*}{\partial \tau \ NRS} \tag{10}
\]

\(^{10}\)In an important extension, Gordon and Wilson (1989) argue that in a multi-period context riskiness in future decision variables cannot be measured merely by the expected value. The correct procedure would be to use the certainty equivalent of the tax-induced change in Xi. However, in the present context, all decisions are made in the current period, and hence, are unaffected by future risk considerations.
The MDL is equal to the value of the tax outstanding per unit of the i-th activity multiplied by the change in the i-th activity due to the tax change.\textsuperscript{11} MDL is positive ($\frac{\partial L_1}{\partial \tau} v_{mc} > 0$) since $\frac{\partial C_1}{\partial \tau} NRS > 0$ and with imperfect loss offset provisions the second term is also positive given that $x_{2b} < 0$ as well as $\frac{\partial a_1}{\partial \tau} NRS < 0$.\textsuperscript{12} The value of the tax outstanding per unit of consumption activity is measured by the term $\frac{\tau x_{2b}(1-\rho)}{(1-\tau)(1+r(1-\tau))}$ and that of a risky asset choice by the term $\frac{\tau x_{2b}(1-\rho)}{(1-\tau)(1+r(1-\tau))}$. For the former, the quantity $(r\tau)$ is precisely the amount of tax paid on a unit of risk-less investment. This is entirely consistent with the approach of Gordon and Wilson, who explained that the "the size of the tax distortion is the same on each asset, and can be measured most simply by the taxes paid on the riskless investment" (Gordon and Wilson, p427).

The term $\frac{\tau x_{2b}(1-\rho)}{(1-\tau)(1+r(1-\tau))}$ reflects the distortion in the relative asset return that is associated with the partial loss offset provision of the tax code. The value of tax outstanding is equal to zero under full loss offset provision and negative under partial loss offset provisions. With full loss offset provisions there is no deadweight loss associated with the portfolio choice since the value of the tax outstanding per unit of the risky activity is equal to zero. With full loss offset provisions the deadweight loss consists only of the first right-hand term in equation (10). Furthermore, under full loss offset provisions it is the elasticity of substitution that establishes the existence of deadweight loss due to capital income taxation (given a non-zero EIS) as in the case of certainty. The relative risk aversion determines the magnitude and not the direction of the DWL figure under full loss offset provisions. In the more general case of partial loss offset provisions both the elasticity of inter-temporal substitution and the attitude towards risk play important roles.

The MDL measure above does not take into account the incremental revenue raised from an increase in capital income taxation ($MTR = \frac{\partial TR}{\partial \tau}$). In order to adjust the MDL for $MTR$, the Marginal cost of public funds ($MCF$) expression is used which is given by: \textsuperscript{13}

\textsuperscript{11}This measure of the marginal efficiency cost of capital income taxation under uncertainty is related the one derived by Arnold Harberger (1971). He expressed DWL as the product of a tax distortion term and a quantity measuring the tax-induced change in behaviour. The Harberger measure was developed under conditions of certainty, and later authors have interpreted the measure as applicable to behaviour under uncertainty by qualifying the tax-induced change in behavior by its expected value. The parallel between our result and that of the traditional Harberger formula is made transparent in what follows in the text.

\textsuperscript{12}MDL will be negative if the government is more efficient in handling risks than the private sector, leading to a marginal welfare benefit.

8
\[ MCF = 1 + \frac{MDL}{MTR} \]  

By increasing marginally the capital income tax rate, additional revenue is raised to finance public expenditures (i.e., \( MTR \)) but this increase also results in an additional cost, in terms of efficiency (i.e., \( MDL \)). Thus the \( MCF \) is the efficiency cost (in cents) of raising a dollar of revenue through distortionary taxation beyond that of the dollar being raised. If \( MDL = 0 \) then this ratio is 1. This would be valid if we had included lump sum taxes in the model. If \( MDL > 0 \), as is the case in this model, this ratio is greater than unity.

4 Revenue Valuation

In this section, an expression for \( MTR \) is provided which is consistent with the no risk sharing assumption. We argue that under uncertainty, the \( MTR = \frac{\partial CE(\text{TR})}{\partial \tau} \) is the additional value (certainty equivalent) of the tax revenue raised by increasing marginally the capital income tax. To be consistent with the assumption that the government is just as efficient in handling aggregate risk as the private sector the standard security market line of the capital asset pricing model developed by Lintner (1965), Mossin (1966) and Sharpe (1964) is used to value the tax revenue.\(^{14}\) Assume that the government issues securities that have a claim on (7a) and (7b) as reproduced below:

\[
\begin{align*}
G_{2_g} &= \tau r(Y_1 - C_1^*) + \tau(x_{2g} - r)\alpha_1^* \\
G_{2_b} &= \tau r(Y_1 - C_1^*) + \tau(\varphi x_{2b} - r)\alpha_1^*
\end{align*}
\]  

(12a)  

(12b)

where \( G_{2_i} (i = g, b) \) is precisely the tax paid by a typical individual. The market value of the above stochastic revenue flow would be equal to:

\[
CE(G_2) = E(G_2) - E(\bar{z}_{2m}) \frac{\text{cov}(G_2, \bar{z}_{2m})}{\text{var}(\bar{z}_{2m})}
\]  

(13)

\(^{13}\)Sandmo (1998) defines the MCF “as the multiplier to be applied to the direct resource cost in order to arrive at the socially relevant shadow price of resources to be used in the public sector.”

\(^{14}\)This is also consistent with the household’s portfolio allocation choice as represented by equation 6.
where $z_{2m}$ is the after tax excess return of the market portfolio, $cov(G_2, z_{2m})$ is the covariance of the revenue flow with the excess return of the market portfolio and $var(z_{2m})$ is the variance. The second part of (13) is the risk premium of the uncertain tax revenue flow as determined by the asset pricing model. Substituting in (13) the lump sum transfer and after simple manipulations yields:

\[
CE(G_2) = \tau r(Y_1 - C_1^*) + \tau a_1^* \left[ E(z_2) - E(z_{2m}) \frac{cov(z_2, z_{2m})}{var(z_{2m})} \right]
\]

(14)

\[
CE(G_2) = \tau r(Y_1 - C_1^*)
\]

(15)

where $E(z_2) = p(x_{2g} - r) + (1 - p)(\varphi x_{2b} - r)$ is the pre-tax excess return of the household’s portfolio. The certainty equivalent of the tax revenue from capital income taxation is equal to the tax revenue generated from risk free interest on total savings, i.e., $\tau r(Y - C_1^*)$. The certainty equivalent of the tax revenue generated from the excess return held by the household is zero. The excess return of the security held by the investor is equal to $E(z_2) = E(z_{2m}) \frac{cov(z_2, z_{2m})}{var(z_{2m})}$ according to the security market line. Thus the market value of the revenue stream from the excess return equals zero. The market prices all investments. As a result the government’s financial assets, which have a claim on the revenue of the excess returns, is of no value to the market because the claim does not offer to the investor any additional diversification possibilities other than those already offered by the market. Therefore, the additional revenue raised in the second period is:

\[
MTR = \frac{\partial CE(G_2)}{\partial \tau} = r(Y_1 - C_1^*) - \tau r \frac{\partial C_1^*}{\partial r} NRS
\]

(16)

Substituting the present value of (16) into (11) results in:

\[
MCF = \frac{r(Y_1 - C_1^*) + \tau x_{2g}(1-\varphi)}{(1-\tau)(1+\tau(1-\tau))} \frac{(1-p)C_2^{-\gamma}}{E(C_2^{-\gamma})} \frac{\partial C_1^*}{\partial r} NRS
\]

(17)

The loss is the smallest when there is full loss offset provisions since the second term in the numerator is absent in this case. MCF increases as loss offset provisions are removed. The next section provides a numerical illustration of the magnitude of the deadweight loss of capital income taxation under various parameter values.

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5 Numerical Simulations

The parameters of the model are largely based on empirical regularities. In addition some extreme cases are examined. The relative risk aversion parameter is assumed to vary between 0 (risk neutrality) to a maximum of value of 4.\textsuperscript{16} A zero RRA value will give results for a risk neutral agent whose behaviour is governed by RINCE preferences developed by Farmer (1990). A risk neutral investor is seen to allocate the entire savings to the (on average) higher yielding risky asset. She is also allowed to borrow the safe asset. The other extreme occurs when the relative risk aversion of an investor is at the other end of the spectrum in which case she invests all savings in the safe asset. Under this case the investor would still choose the consumption stream based on the inter-temporal rate of substitution. The welfare results in this latter case will correspond to the two period model life cycle model under certainty (eg, Feldstein, 1978). The elasticity of substitution is allowed to vary between 0.00 and 2.5.

The distribution of the asset returns is as follows. The value of the safe return is assumed to be 50 percent over a life cycle of say 25 years, which would translate to an annual compounded rate of return of 1.64 percent. This annual rate closely corresponds to the real yield on long government bonds. The rate of return of the risky asset is chosen initially to yield an average real annual rate of 5.80 percent, which is slightly higher than the real return on the S & P 500 index of approximately 5.17 percent over the past forty years. The probability of the good state of nature is set at 80 percent. The good state of nature yields an annual return of return equal to 6.6 percent, while the bad state of nature yields an annual loss of approximately 2.7 percent. The tax rate is set initially at 40 percent.

Numerical simulations follow. Figure 1 presents the marginal tax revenue generated by increasing the capital income tax rate under the assumption of full loss-offset provision. Figure 2 illustrates the no loss-offset provision.

\textsuperscript{16}Although higher RRA values can be simulated they do not provide any more insights.
MTR under loss offset is above the no loss offset function under the same relative risk aversion and EIS parameters. Thus the government is not capable of generating more additional revenue by imposing loss offset restrictions.
This phenomenon arises because under no loss offset provision savings are more severely discouraged than in the case of loss offset. Furthermore, MTR increases with the elasticity of intertemporal substitution, more so for low levels of risk aversion parameter. Holding the elasticity of substitution constant (and less than unity), MTR increases the with relative risk aversion parameter. In the case where the elasticity of substitution is greater than unity MTR declines as relative risk aversion parameter increases.

MCF ranges from the value of 1 to a maximum of 1.25 approximately. Figure 3 indicates that the largest MCF occurs for a person with the highest elasticity of substitution and the highest risk aversion parameter. For a representative investor with an elasticity of substitution of 1.67 and a relative risk aversion of around 2, the efficiency cost would equal fourteen cents to a dollar of revenue raised.

MCF varies with respect to EIS and RRA values.

With respect to EIS, MCF falls as the inter-temporal elasticity of substitution decreases for risk aversion parameters greater than unity. However,

\[\text{In terms of MDL relative to savings, the losses range from zero to 6.6 percent of savings in the same range of parameters. The largest deadweight losses, as a fraction of savings, occur for a person with the highest elasticity of substitution and the highest risk aversion parameter (i.e., 6.6 percent of savings). An investor with an elasticity of substitution of 1.67 and a relative risk aversion of around 2 would require compensation equal to 4.8 percent of his savings in order to be as well off as in the pre-tax position.}\]
for an individual that has a relative risk aversion less than unity, the losses peak at an inter-temporal elasticity of substitution greater than unity and then fall continuously as EIS falls.

With respect to risk aversion, MCF rises as RRA increases when the elasticity of substitution is greater than unity. MCF falls as RRA increases when the elasticity of substitution is less than or equal to unity. With a very high value of 4 for the relative risk aversion, individuals hold most of the entire fund in the safe asset, MCF is equal to 1.15 given an elasticity of inter-temporal substitution of 1.67. For the same inter-temporal substitution value, the MCF drops to 1.10 given risk neutrality. Thus under full loss offset provisions, the less risk averse an investor is, the lower the efficiency cost of capital income taxation for a given EIS.

Figure 4 below presents the results under the other extreme assumption of having no loss offset provisions. Under no loss offset the less risk averse an investor is, the higher the efficiency cost of capital income taxation.

The welfare losses range from a value of unity to a maximum of 2.48 within the given range. Under no loss offset, the welfare loss is highest for

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18 The same MCF shape (although lower corresponding MCF values) is observed for partial loss offset provision levels up to an approximate value of $\varphi =0.80$ level beyond which the Figure 4 shape takes effect.
a person that combines a high elasticity of inter-temporal substitution but is not very averse to risk taking activity. For example, a person with an elasticity of substitution of 1.67 and a relative risk aversion parameter of 0.5 will have a welfare loss equal to $2.48 for every dollar of revenue raised. But a risk aversion parameter of 2 results in a welfare loss of capital income taxation equal to $1.62. Contrasting this latter example with the case of full loss offset provision, the welfare loss under no loss offset increases by a magnitude of forty-eight additional cents to the dollar of revenue raised.19

Finally, Figure 5 shows the deadweight losses for a representative agent with an intertemporal elasticity of substitution of 1.67 and a relative risk aversion parameter of 2 over various tax rates and loss offset levels. As expected, the deadweight losses increase with the tax rate and with less provision to loss offsets. The DWL are at a minimum when there is full loss offset provisions and the capital income tax rate is as low as possible.

![Figure 5: Welfare Losses for a "Typical" Agent](image)

19The welfare losses as a fraction of savings range from zero to a maximum of thirty eight percent of savings over these same parameter values. For example, a person with an elasticity of substitution of 1.67 and a relative risk aversion parameter of .5 will have a welfare loss of 38 percent! While a risk aversion parameter of 2 will have a welfare loss of capital income taxation equal to 18 percent of savings. Contrasting this later example, with the case of full loss offset provision, the welfare loss under no loss offset increases by a magnitude of four times.
6 Conclusion

This paper has investigated deadweight losses stemming from capital income taxation in a two period model, comparing the impacts of complete and incomplete loss offset provisions. Key assumptions are that all risk remains within the private sector and that non-expected utility preferences prevail. The results indicate that capital income taxation under less than full loss offset deters risky investment activity and adds substantially to the inefficiency of taxing capital income.

Efficiency cost estimates without loss offset, under plausible assumptions about attitudes towards risk and time preference and given an initial capital income tax rate of forty percent, were shown to be on the order of sixty two cents per dollar of tax revenue. In contrast, under the same attitudes towards consumption and risk but with full loss offset, the dead weight loss was approximately fourteen cents per dollar of revenue raised.

In the absence of loss offset, efficiency costs are found to be greatest where agents have a low relative risk aversion and a high elasticity of inter-temporal substitution. With loss offset provisions in place, the welfare loss is greatest under high relative risk aversion. Finally, deadweight losses are found to increase not only with the tax rate, holding offset provisions constant, but also inversely with loss offset levels at any given tax rate.

Future research might fruitfully explore the consequences of asymmetric information between the government and investors, of the government’s being more (or less) efficient in the handling risk than the private sector, and of extending the analysis to a multi-period decision framework.

7 References


8 Appendices

8.1 Appendix 1: Calculations leading to optimal choices:

Given the preference structure:

\[ U(C_1, CE(C_2)) = [C_1^\theta + \delta E(C_2^{(1-\gamma)})]^{\frac{\theta}{1-\gamma}} \]  
(A1.1)

and the constraints:

\[
  C_{2b} = (1 + r(1 - \tau))(Y_1 - C_1) + (x_{2b}(1 - \varphi) - (1 - \tau)r)a_1 + G_{2b} \quad \text{(A1.2a)}
\]

\[
  C_{2g} = (1 + r(1 - \tau))(Y_1 - C_1) + (1 - \tau)(x_{2g} - r)a_1 + G_{2g} \quad \text{(A1.2b)}
\]

Maximization of A1.1 with respect to current consumption \((C_1)\), and the amount invested in the risky asset \((a_1)\) yields the first order conditions 5 and 6 in the main text.

\[
  [C_1^*]^{\theta-1} = \delta(1 + r(1 - \tau))CE(C_2)^{(\theta+\gamma-1)}E(C_2^{-\gamma}) \quad \text{(A1.3)}
\]

\[
  E(C_2^{-\gamma}z) = 0 \quad \text{(A1.4)}
\]

Utilizing the two states of nature approach these optimal conditions can be re-written as follow:

\[
  C_1^{\theta-1} = \delta(1 + \tau)(pC_{2g}^{1-\gamma} + (1 - p)C_{2b}^{1-\gamma})^{(\theta+\gamma-1)} E \left( pC_{2g}^{-\gamma} + (1 - p)C_{2b}^{-\gamma} \right) \quad \text{(A1.5)}
\]

\[
  pC_{2g}^{-\gamma}(x_{2g} - r)(1 - \tau) + (1 - p)C_{2b}^{-\gamma}(x_{2b}(1 - \tau\varphi) - (1 - \tau)r) = 0 \quad \text{(A1.6)}
\]

where \(\tau = r(1 - \tau)\). Using A1.6 to solve for \(C_{2g}\) yields:

\[
  C_{2g} = \left[ -\frac{p\bar{x}_{2g}}{(1 - p)\bar{z}_{2b}} \right]^{1/\gamma} \quad C_{2b} = B_0C_{2b} \quad \text{(A1.7)}
\]
Inserting A1.7 into using A1.5 and solving for $C_1$ as a function of $C_{2g}$ yields:

$$C_1 = A_0 C_{2g}$$  \hspace{1cm} (A1.8)

where $A_0 = \left[ \delta(1 + r(1 - \tau))p \left( \frac{z_2 - z_{2b}}{z_{2b}} \right) \left[ p + (1 - p)B_0^{\gamma - 1} \right] \right]^{g/(a - g)}$

The following system of equation composed of A1.7 and A1.8 along with the pre-budget constraints can be used to solve for $C_1, C_{2g}, C_{2b}, a_1$ and $m_1$:

$$
\begin{align*}
C_1 &= A_0 C_{2g} \\
C_{2g} &= B_0 C_{2b} \\
C_{2b} &= (1 + r)(Y_1 - C_1) + z_{2b}a_1 \\
C_{2g} &= (1 + r)(Y_1 - C_1) + z_{2g}a_1 \\
C_1 &= Y_1 - (a_1 + m_1)
\end{align*}
$$

After simple manipulations and defining $B_1 = B_0(z_{2g} - z_{2b})(1 + r)$ and $A_1 = A_0^{-1}(z_{2g} - B_0 z_{2b})$ current consumption is:

$$C_1^* = c_1(\tau, \varphi)Y_1 = \frac{B_1}{(A_1 + B_1)}Y_1$$  \hspace{1cm} (A1.9)

while risky asset allocation can be represented as follow:

$$a_1^* = a_1(\tau, \varphi)Y_1 = \frac{(1 + r)(B_0 - 1)}{(A_1 + B_1)A_0}Y_1$$  \hspace{1cm} (A1.10)

### 8.2 Appendix 2. Derivation of $\frac{\partial C_1^*}{\partial \tau} NRS$ and $\frac{\partial a_1^*}{\partial \tau} NRS$

Differentiating A1.9 with respect to the capital income tax yields:

$$\frac{\partial C_1^*}{\partial \tau} NRS = \frac{c_1(\tau, \varphi)^2}{B_1} \left[ \frac{\partial A_1}{\partial \tau} - \frac{A_1}{B_1} \frac{\partial B_1}{\partial \tau} \right] Y_1 =
\frac{c_1(\tau, \varphi)^2}{A_0 B_1} \left[ A_1 \frac{\partial A_0}{\partial \tau} + \frac{z_{2g}}{B_0} \frac{\partial B_0}{\partial \tau} \right] Y_1$$  \hspace{1cm} (A2.1)
where

\[
\frac{\partial A_0}{\partial \tau} = \sigma \frac{r A_0}{(1 + r(1 - \tau))} - \left(1 - \varphi\right) \sigma \frac{x_{2b} A_0}{(1 - \tau)\gamma} \left[ p + (1 - p) B_0^{-(1-\gamma)} \right] \left[ \frac{\sigma \gamma(1 - B_0) + \frac{z_{2g}}{z_{2b}-z_{2g}}}{z_{2b}-z_{2g}} \right] > 0 \quad (A2.2)
\]

and

\[
\frac{\partial B_0}{\partial \tau} = -(1 - \varphi) \frac{x_{2b} B_0}{(1 - \tau)\bar{z}_{2b}\gamma} \leq 0 \quad (A2.3)
\]

Under full loss offset provision \(\frac{\partial B_0}{\partial \tau} = 0\) and \(\frac{\partial A_0}{\partial \tau} = \frac{\sigma r A_0}{(1 + r(1 - \tau))}\). Thus under full loss offset provisions, current consumption is stimulated by a tax increase as follow:

\[
\frac{\partial C^*_1}{\partial \tau_{FLO, NRS}} = \frac{\sigma r S^*_1}{(1 + r(1 - \tau))} \frac{C^*_1}{Y_1} > 0 \quad (20a)
\]

The effect gets stronger with increases in EIS \(\sigma\). The RRA operates through its influence on average consumption and savings behavior. However under no loss offset current consumption may be discouraged in some cases since there are two opposing effects.

The effect of a tax increase on risky investment activity is given by differentiating \(A1.10\) with respect to \(\tau\):

\[
\frac{\partial a_1^*}{\partial \tau_{NRS}} = \frac{a_1^*}{(B_0 - 1)} \frac{\partial B_0}{\partial \tau} - \frac{a_1^*}{(A_1 + B_1)} \left[ \frac{\partial A_1}{\partial \tau} + \frac{\partial B_1}{\partial \tau} \right] - \frac{a_1(\tau, \varphi)}{A_0} \frac{\partial A_0}{\partial \tau} \quad (A2.4)
\]

This can be simplified further into the following two components:

\[
\frac{\partial a_1^*}{\partial \tau_{NRS}} = -\frac{a_1^*}{A_0} \frac{\partial A_0}{\partial \tau} \frac{C_1^*}{Y_1} + \frac{a_1^*}{A_0} \frac{\partial B_0}{\partial \tau} \left[ \frac{B_0(1 - \frac{C_1^*}{Y_1}) + \frac{B_0 C_1^*}{Y_1}}{B_0(B_0 - 1)A_1} z_{2g} - B_0 z_{2b} \right] < 0 \quad (A2.5)
\]

Risky asset choice is discouraged under full loss offset provisions

\[
\frac{\partial a_1^*}{\partial \tau_{FLO, NRS}} = -\frac{\sigma r a_1^*}{(1 + r(1 - \tau))} \frac{C_1^*}{Y_1} < 0 \quad (A2.6)
\]

and even more so under imperfect loss offset due to the presence of the the second term in \(A2.5\).
8.3 Appendix 3. Calculations leading to MDL = \frac{\partial L_1}{\partial \tau}:

The standard measure of marginal deadweight loss is the additional transfer the individual would have to receive in the first period to compensate her for the effects of the tax increase. To find the deadweight loss of capital income taxation, we differentiate the value function with respect to the tax rate and substitute the first-order conditions. The value function:

\[ V = U(C_1^* + L_1, CE(C_2)^*) = [(C_1^* + L_1)^\theta + \delta E(C_2^{(1-\gamma)})^{\frac{\theta}{\gamma}}]\frac{1}{\theta} \quad (A3.1) \]

where the optimal choices are given in section 8.1 of the appendix and where \(L_1\) represents the first period consumption transfer a household would require in order to be as well off as before the capital income tax. Therefore define \(MDL = \frac{\partial L_1}{\partial \tau v = c}\) as the marginal deadweight loss. The first order condition for consumption allocation given the stochastic lump sum transfer and the first period transfer is:

\[ [C_1^* + L_1]^{\theta-1} = \delta(1 + r(1 - \tau))CE(C_2)^{(\theta+\gamma-1)}E(C_2^{-\gamma}) \quad (A3.2) \]

while, that of asset allocation is:

\[ E(C_2^{-\gamma}) = 0 \quad (A3.3) \]

Utilizing the budget constraint after re-distribution of the tax revenue \(C_2 = (1 + r)(Y_1 - C_1^*) + za_1^*\) and differentiating the value function \(A3.1\) with respect to \(\tau\) and setting the result equal to zero yields:

\[
\begin{align*}
[C_1^* + L_1]^{\theta-1} & \left[ \frac{\partial C_1^*}{\partial \tau \text{ NRS}} + \frac{\partial L_1^*}{\partial \tau \text{ v = c}} \right] \\
& + \delta CE(C_2)^{(\theta+\gamma-1)}E \left[ C_2^{-\gamma} \left[ -(1 + r) \frac{\partial C_1^*}{\partial \tau \text{ NRS}} + z \frac{\partial a_1^*}{\partial \tau \text{ NRS}} \right] \right] = 0 \quad (A3.4)
\end{align*}
\]

After simple manipulation, and utilizing the first order conditions the efficiency loss quantity can be written in terms of compensating variation as in the text.
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