

A Service of

PRIII

Leibniz-Informationszentrum Wirtschaft Leibniz Information Centre for Economics

Bühn, Andreas; Schneider, Friedrich G.

Working Paper MIMIC models, cointegration and error correction: an application to the French shadow economy

CESifo Working Paper, No. 2200

Provided in Cooperation with: Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

Suggested Citation: Bühn, Andreas; Schneider, Friedrich G. (2008) : MIMIC models, cointegration and error correction: an application to the French shadow economy, CESifo Working Paper, No. 2200, Center for Economic Studies and ifo Institute (CESifo), Munich

This Version is available at: <https://hdl.handle.net/10419/26245>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

WWW.ECONSTOR.EU

MIMIC MODELS, COINTEGRATION AND ERROR CORRECTION: AN APPLICATION TO THE FRENCH SHADOW ECONOMY

ANDREAS BÜHN FRIEDRICH SCHNEIDER

CESIFO WORKING PAPER NO. 2200 CATEGORY 1: PUBLIC FINANCE JANUARY 2008

An electronic version of the paper may be downloaded • *from the SSRN website: www.SSRN.com* • *from the RePEc website: www.RePEc.org* $• from the CES if o website:$ *www.CESifo-group.org/wp*

MIMIC MODELS, COINTEGRATION AND ERROR CORRECTION: AN APPLICATION TO THE FRENCH SHADOW ECONOMY

Abstract

The analysis of economic loss attributed to the shadow economy has attracted much attention in recent years by both academics and policy makers. Often, multiple indicators multiple causes (MIMIC) models are applied to time series data estimating the size and development of the shadow economy for a particular country. This type of model derives information about the relationship between cause and indicator variables and a latent variable, here the shadow economy, from covariance structures. As most macroeconomic variables do not satisfy stationarity, long run information is lost when employing first differences. Arguably, this shortcoming is rooted in the lack of an appropriate MIMIC model which considers cointegration among variables. This paper develops a MIMIC model which estimates the cointegration equilibrium relationship and the error correction short run dynamics, thereby retaining information for the long run. Using France as our example, we demonstrate that this approach allows researchers to obtain more accurate estimates about the size and development of the shadow economy.

JEL Code: O17, O5, D78, H2, H11, H26.

Keywords: shadow economy, tax burden, regulation, unemployment, cointegration, error correction models, MIMIC models.

Andreas Bühn Department of Economics TU Dresden 01062 Dresden Germany andreas.buehn@tu-dresden.de

Friedrich Schneider Department of Economics Johannes Kepler University of Linz 4040 Linz-Auhof Austria friedrich.schneider@jku.at

1. Introduction

This paper presents a first attempt to econometrically improve the multiple indicators multiple causes (MIMIC) model in order to gather more precise information about the size and development of the shadow economy over time. Its main contribution is the first ever analysis of the cointegration of the shadow economy's causes and indictors and the consideration of their long and short run relationships.

The shadow economy is still a controversial issue. Every day, many people around the world are engaged in black market activities. These people usually go underground in order to avoid taxation and save money as products and services are considerably cheaper. Policymakers are concerned about rising shadow economic activities because they weaken tax and social security bases. As a result, they may make ineffective policy decisions based on erroneous official indicators, which in turn give rise to crowding-out effects of official economic activity (Schneider and Enste, 2000). Economists focus on the shadow economy to improve economic theory. They also try to obtain accurate measures of the shadow economy for more effective formulation of economic policies. Specifically, the size and development of the shadow economy is a fundamental requirement. Our paper presents an improved estimation procedure that provides more precise figures than previous attempts to quantify the shadow economy.

Theoretical and empirical research in the field of shadow economics has gained much attention¹. Although some progress has been made, it is still a difficult task because individuals engaged in such activities do not wish to be identified. Consequently, the estimation of the shadow economy becomes a scientific passion to know the unknown. Today, a variety of techniques has been employed to measure the size and development of the shadow economy. This includes direct approaches such as surveys and discrepancy methods as well as indirect methodologies like the transaction or the currency demand approach.² Recently, the MIMIC model was derived as a special type of structural equation model with latent variables.³ It analyzes the covariance structures between observable cause and indicator variables to derive information about the relationship between them and an unobservable

 \overline{a}

¹ See for example Schneider and Enste (2000 and 2002).

 2^2 See Schneider and Enste (2000) for a reliable survey about diverse approaches.

³ The MIMIC model approach traces back to Weck (1983), Frey and Weck (1983), and Frey and Weck-Hannemann (1984) and was enhanced by Aigner *et al*. (1988).

latent variable. Because it is generally agreed that the shadow economy can be treated as a latent variable, the MIMIC model rightly supplements existing direct and indirect approaches. In contrast to the latter, its major advantage is that it differentiates between causes and indicators and, most notably, when estimating the shadow economy considers its various causes.

Often, MIMIC models are applied to time series data to derive estimates of the size and development of the shadow economy over time. As most macroeconomic variables do not satisfy the underlying assumption of stationarity, the problem of spurious regressions may arise. Researchers usually overcome this problem by transforming the time series into stationary ones, employing a difference operator. Alternatively, one could estimate an error correction model (ECM) if the variables were cointegrated and a stationary long run relationship existed between them. This approach has become popular in applied economics in recent years. The former approach, however, is still used in MIMIC model investigations of the shadow economy. The latter MIMIC model in first differences is referred to as the DYMIMIC model (Aigner *et al*., 1988). In the DYMIMIC model, on the other hand, the data's long run information is lost if the variables are used in their first differences, albeit they are cointegrated. Retaining this information may help to improve estimates of the shadow economy. The authors are not aware of any study that tests for cointegration in the variables and makes use of their long run equilibrium relationships in a MIMIC model estimation.

The paper is organized as follows. Section 2 provides a short summary of the basics of cointegration and the error correction mechanism. The MIMIC model is presented in Section 3. Section 4 widens the traditional MIMIC model, allowing for cointegration of the variables, and presents the covariance matrices for the new model's long run and short run equations. In Section 5 we employ our model to France and present the results. Section 6 concludes.

2. Cointegration and Error Correction Models

Empirical investigations in macroeconomics using time series data mostly involve variables that do not fulfill the assumed statistical properties. Specifically, they are not stationary and integrated to an order d different from zero $(I(d), d > 0)$. In the past, such variables were differenced to remove random walk and/or trend components and then analyzed using the Box and Jenkins method. The drawback of this approach is that valuable information about the variables' potential long run relationship is lost. Granger and Weiss (1983) later showed that two variables, x_t and y_t , each I(1), may have a linear combination, $u_t = y_t - \beta x_t$, that is, I(0). We would normally expect the standard linear regression model's error term, u_t , to be I(1). But if u_t is still a white noise series, x_t and y_t are said to be cointegrated with the cointegration vector $[1,-\beta]$. Generally, the cointegrated relation between variables is interpreted as their long run equilibrium. This concept of cointegration immediately enjoyed great popularity because it finally introduced the equilibrium concept of economics to econometrics.

For any two cointegrated I(1) variables x_t and y_t with cointegration vector [1,- β], we know that their first differences Δx_t and Δy_t as well as their long-run relation $y_t - \beta x_t$ are I(0). We can thus formulate the following equation:

$$
\Delta y_t = \gamma \Delta x_t + \lambda u_{t-1} + w_t, \qquad (1)
$$

where w_t is a white noise error term and all expressions are I(0). The residual u_{t-1} is the one-period lagged equilibrium error of the cointegrated long run equation and is used as an error correction term in the dynamic, first difference regression of equation (1). Engle and Granger (1987) proved that this error correction model (ECM) is the data generating process of any two cointegrated variables x_t and y_t and that ECMs generate cointegrated variables.⁴ Unlike the examination of the long run equilibrium, the ECM studies the relationship between the deviations of x_t and y_t from their respective long run trends. The expansion of equation (1) to the multivariate case where y_t is a vector of variables that are individually I(1), and x_t is a vector of variables containing a constant – variables that are assumed to be I(0) and where a time trend might also apply (Greene, 2007).

The concepts of cointegration and error correction enable researchers to study both the long run relationship between variables and the deviations from their respective long run trends and gather a better understanding of the economy's major features. Before showing how to utilize the idea of cointegration in an analysis of structural equation relations, we briefly introduce the standard MIMIC model.

 \overline{a}

⁴ This class of models traces back to Sargan (1964), who introduced a model that retained long run information in a non-integrated specification for the first time.

3. The MIMIC Model

The MIMIC model explains the relationship between observable variables and an unobservable variable by minimizing the distance between the sample covariance matrix and the covariance matrix predicted by the model. The observable variables are divided into causes of the latent variable and its indicators. Formally, the MIMIC model consists of two parts: the structural equation model and the measurement model. The structural equation model is given by:

$$
\eta_t = \gamma' \mathbf{x}_t + \varsigma_t \,,\tag{2}
$$

where $\mathbf{x}'_t = (x_{1t}, x_{2t},..., x_{qt})$ is a $(1 \times q)$ vector of time series variables as indicated by the subscript t. Each time series x_{it} , $i = 1,...,q$ is a potential cause of the latent variable η_t . $\gamma' = (\gamma_1, \gamma_2, ..., \gamma_q)$, a (1×q) vector of coefficients in the structural model describing the "causal" relationships between the latent variable and its causes. Since the structural equation model only partially explains the latent variable η_t , the error term ς_t represents the unexplained component. The MIMIC model assumes that the variables are measured as deviations from their means and that the error term does not correlate to the causes, i.e. $\mathbf{E}(\eta_t) = \mathbf{E}(\mathbf{x}_t) = \mathbf{E}(\zeta_t) = 0$ and $\mathbf{E}(\mathbf{x}_t \zeta_t') = \mathbf{E}(\zeta_t \mathbf{x}_t') = 0$. The variance of ζ_t is abbreviated by ψ and Φ is the (q × q) covariance matrix of the causes x_t .

The measurement model represents the link between the latent variable and its indicators, i.e. the latent unobservable variable is expressed in terms of observable variables. It is specified by:

$$
\mathbf{y}_t = \lambda \eta_t + \varepsilon_t \quad , \tag{3}
$$

where $y'_t = (y_{1t}, y_{2t},..., y_{pt})$ is a $(1 \times p)$ vector of individual time series variables y_{jt} , $j = 1,...,p$. $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t},..., \varepsilon_{pt})$ is a ($p \times 1$) vector of disturbances where every ε_{jt} , $j = 1,...,p$ is a white noise error term. Their ($p \times p$) covariance matrix is given by Θ_{ε} . The single λ_j , j = 1,..., p in the (p × 1) vector of regression coefficients λ , represents the magnitude of the expected change of the respective indicator for a unit change in the latent variable. Like the MIMIC model's causes, the indicators are directly measurable and expressed as deviations from their means, that is, $E(y_t) = E(\epsilon_t) = 0$. Moreover, it is assumed that the error terms in the measurement model do not correlate either to the causes \mathbf{x}_t or to

the latent variable η_t , hence, $E(x_t \varepsilon'_t) = E(\varepsilon_t x'_t) = 0$ and $E(\eta_t \varepsilon'_t) = E(\varepsilon_t \eta'_t) = 0$. A final assumption is that the ϵ_t s do not correlate to ζ_t , i.e. $\mathbf{E}(\epsilon_t \zeta_t') = \mathbf{E}(\zeta_t \epsilon_t') = 0$. Figure 3.1 shows the general structure of the MIMIC model.

Fig. 3.1 *General Structure of a MIMIC Model*

From equation (2) and (3) and making use of the definitions, we can derive the MIMIC model's covariance matrix Σ (see Appendix A). This matrix describes the relationship between the observed variables in terms of their covariances. Decomposing the matrix derives the structure between the observed variables and the unobservable, latent variable, here, the shadow economy. The model's covariance matrix is given by:

$$
\Sigma = \begin{pmatrix} \lambda (\gamma' \Phi \gamma + \psi) + \Theta_{\varepsilon} & \lambda \gamma' \Phi \\ \Phi \gamma \lambda' & \Phi \end{pmatrix},
$$
 (4)

where Σ is a function of the parameters λ , γ and the covariances contained in Φ , Θ_{ϵ} , and ψ . Since the latent variable is not observable, its size is unknown, and the parameters of the model must be estimated using the links between the observed variables' variances and covariances. Thus, the goal of the estimation procedure is to find values for the parameters and covariances that produce an estimate for Σ that is as close as possible to the sample covariance matrix for the observed causes and indicators, i.e. the x_t s and y_t s. Now we extend this model to include the concept of cointegration.

4. The EMIMIC Model

4.1. The MIMIC Model and Co-integration

The first step in deriving the error correction MIMIC (EMIMIC) model is to put equation (2) into equation (3). This yields:

$$
y_t = \Pi x_t + z_t \t\t(5)
$$

where $\Pi = \lambda \gamma'$ and $\mathbf{z}_t = \lambda \zeta_t + \varepsilon_t$. The error term \mathbf{z}_t in equation (5) is a ($p \times 1$) vector of linear combinations of the white noise error terms ς_t and $\boldsymbol{\varepsilon}_t$ from the structural equation and the measurement model, i.e. $z_t \sim (0, \Omega)$. The covariance matrix Ω is given as **Cov**(z_t) = $\lambda \lambda' \psi + \Theta_s$. Equation (5) is comparable to a simultaneous regression model where the endogenous variables y_{jt} , $j = 1,...,p$ are the latent variable η 's indicators and the exogenous variables x_{it} , $i = 1,...,q$ its causes. The application of cointegration theory to the MIMIC model is thus possible.

From Section 2 we already know that the presumption that every element z_j , $j = 1,..., p$ of **z** is a stationary white noise series is probably untrue if some of the causes and indicators x_{it} , i = 1,..., q and y_{it} , j = 1,..., p are I(1) series. Thus, if x_t and y_t are vectors of I(1) variables, one would normally expect every linear combination $y_{jt} - \pi_j x_t$ to be I(1), i.e. a trend over time. Here, π_j is the jth (1×q) row vector of matrix **Π** in equation (5); therefore, the jth long run equilibrium error z_{it} will also be I(1), which implies inconsistency in equation (5) of the model. On the other hand, we know that if a linear combination $z_{jt} = y_{jt} - \pi_j x_t$ exists, where z_{jt} is I(0), the variables are said to be cointegrated (Engle and Granger, 1987). The vector $[1, -\pi_j]$, where π_j itself is a $(1 \times q)$ row vector, is a cointegration vector. Generally, since every linear combination z_{jt} , $j = 1,...,p$ consists of $q+1$ variables, there could be more than one cointegration vector – in principle up to q linearly independent cointegration vectors (Greene, 2007). If p indicators are I(1), the number of linearly independent cointegration vectors is $(p \cdot q)$.

Of course, not every macroeconomic variable is I(1): there may also be some I(0) time series. We therefore generalize the assumptions of the previous paragraph and denote these causes as a vector $\mathbf{v}'_t = (v_{1t}, v_{2t}, \dots, v_{rt})$. Equation (5) then becomes:

$$
\mathbf{y}_t = \mathbf{\Pi} \mathbf{x}_t + \mathbf{T} \mathbf{v}_t + \mathbf{z}_t, \tag{6}
$$

where $T = \lambda \beta'$ and $\tau' = (\tau_1, \tau_2, ..., \tau_r)$ is the (1× r) vector of coefficients of the I(0) variables in the structural relationship. Given that r of the causes are I(0), the dimensions of $\Pi = \lambda \gamma'$ and \mathbf{x}_t are $[1 \times (q - r)]$ and $[(q - r) \times 1]$, respectively, and the dimensions of **T** and \mathbf{v}_t are $(p \times r)$ and $(r \times 1)$, respectively. Consequently, if r of the causes are I(0), the maximum number of linearly independent cointegration vectors for every disturbance term z_{jt} , $j = 1,...,p$ in equation (6) is $(q - r)$. Moreover, if $s \le p$ of the p indicator variables are also individually I(0), the maximum number of linearly independent cointegration vectors decreases to $(q - r) - s$.

4.2. Error Correction Representation of the EMIMIC Model

As equation (1) shows, every cointegration relationship has an error correction mechanism where the long run relationship leads to equilibrium and the short run relationship contains a dynamic mechanism (Engle and Granger, 1987). Thus, equation (6) can be written as:

$$
\Delta y_t = A \Delta x_t + T v_t + K z_{t-1} + w_t. \tag{7}
$$

where $\Delta y_t = y_t - y_{t-1}$, $\Delta x_t = x_t - x_{t-1}$, $z_{t-1} = y_{t-1} - \Pi x_{t-1}$, and A, B, and K are coefficient matrices in this dynamic, short run model specification. Furthermore, in this specification $A = \lambda \alpha'$ is the $[p \times (q - r)]$ coefficient matrix of the first differences of the I(1) causes, and $\mathbf{B} = \lambda \beta'$ is the ($p \times r$) coefficient matrix of the I(0) causes. The matrix $\mathbf{K} = \lambda \kappa'$ is the $(p \times p)$ coefficient matrix for the long run disequilibrium's error correction term and $w_t \sim (0, \Omega)$ is a white noise disturbance.⁵ As long as x_t and y_t are vectors of I(1) variables, their changes will be I(0). Thus, every term in equation (7) – given that y_t and x_t in equation (6) are cointegrated, leading to z_t that is also individually $I(0) -$ is $I(0)$. Together, equation (6) and (7) define the EMIMIC model. Since the fundamental idea of the MIMIC model is to

⁵ Both separate equations of the MIMIC model in first differences are $\Delta y_t = \lambda \Delta \eta_t + \epsilon_t$ and $\Delta \eta_t = \alpha' \Delta x_t + \beta' y_t + \kappa' z_{t-1} + \zeta_t$. Putting the first equation into the second and using the definitions of \vec{A} , \vec{B} , \vec{K} , and $\vec{w}_t = \lambda \varsigma_t + \varepsilon_t$ gives, at first, $\Delta y_t = \lambda \alpha' \Delta x_t + \lambda \beta' v_t + \lambda \kappa' z_{t-1} + \vec{w}_t$, and, finally, equation (7).

minimize the distance between the sample covariance matrix and the covariance matrix predicted by the model, the next step is to derive the underlying structural equation system, i.e. the covariance matrices of equations (6) and (7) for the EMIMIC model.

To begin, we define (6) in terms of **Σ** . Its general structure is:

$$
\Sigma = \begin{pmatrix} \text{Var}(y_t) \\ \text{Cov}(x_t, y_t) & \text{Var}(x_t) \\ \text{Cov}(v_t, y_t) & \text{Cov}(v_t, x_t) & \text{Var}(v_t) \end{pmatrix}.
$$

Next, we formulate this covariance matrix in terms of the observed variables as a function of the model parameters using the assumption established in Section 3 (see Appendix B). From this, it follows that:

$$
\Sigma = \begin{pmatrix} \lambda (\gamma' \Phi_1 \gamma + 2\gamma' N \tau + \tau' \Phi_2 \tau) \lambda' + \Theta_{\varepsilon} \\ (\Phi_1 \gamma + N \tau) \lambda' & \Phi_1 \\ (N' \gamma + \Phi_2 \tau) \lambda' & N' & \Phi_2 \end{pmatrix},
$$
\n(8)

where $N = E(v_t x'_t)$. Φ_1 is the covariance matrix for the I(1) causes and Φ_2 for the I(0) causes. For all other variables, the definitions introduced in the previous sections still hold. The covariance matrix of the short run error correction mechanism for equation (7) is:

$$
\Sigma = \begin{pmatrix} Var(\Delta y_t) & & & \\ Cov(\Delta x_t, \Delta y_t) & Var(\Delta x_t) & & \\ Cov(v_t, \Delta y_t) & Cov(v_t, \Delta x_t) & Var(v_t) & \\ Cov(z_{t-1}, \Delta y_t) & Cov(z_{t-1}, \Delta x_t) & Cov(z_{t-1}, v_t) & Var(z_{t-1}) \end{pmatrix}.
$$

Taking $z_t \sim (0, \Omega)$ into account, defining $M = E(v_t \Delta x_t)$, and making use of the assumptions established, we can also derive the short run equation's covariance matrix in terms of the model's parameters and covariances (see Appendix C). Thus, the covariance structure of equation (7) is:

$$
\Sigma = \begin{pmatrix} \lambda (\alpha' \Phi_3 \alpha + 2 \alpha' M \beta + \beta' \Phi_2 \beta + \kappa' \Omega \kappa) & & \\ (\Phi_3 \alpha + M' \beta) \lambda' & \Phi_3 & \\ (M \alpha + \Phi_2 \beta) \lambda' & M & \Phi_2 \\ \lambda (\psi \lambda' \kappa + \psi) \lambda' + \Theta_{\varepsilon} & 0 & 0 & \Omega \end{pmatrix} . \tag{9}
$$

A closer examination of equations (8) and (9) and a comparison with the MIMIC model's covariance matrix in equation (4) shows the effect of cointegration. In contrast to the latter, the EMIMIC model's covariance matrix in (9) is adjusted by the long run equilibrium error term's covariance matrix **Ω** and the error correction term's parameter vector **κ** . The further

modifications are needed to separate the $I(0)$ and $I(1)$ causes forming sub-matrices $(\Phi_3 \alpha + M' \beta) \lambda'$, $(M\alpha + \Phi_2 \beta) \lambda'$, Φ_2 , Φ_3 , and M. However, because Σ is still a function of the model parameters α , β , κ , and λ and of covariances, the estimation procedure ensures that an estimate for the EMIMIC model's covariance matrix can be adapted. This model is now applied to estimate the size and development of the French shadow economy.

5. Applying the EMIMIC Model

5.1 The Concrete EMIMIC Model

 \overline{a}

Most analyses of the shadow economy come to the conclusion that tax and social security burdens and the intensity of regulation are the two main causes affecting the size and development of the shadow economy.⁶ Taxes affect labor-leisure choices and stimulate labor supply in the shadow economy since the greater the difference between the total cost of labor in the official economy and the after-tax earnings from work, the greater the incentive to avoid this difference and to work in the shadow economy. An increase in the intensity of regulations, such as trade barriers and labor restrictions for foreigners, reduces the freedom (of choice) for individuals engaged in the official economy and leads to a substantial increase in labor costs in the official economy. Since most of these costs can be shifted onto employees, there is further incentive to work in the shadow economy – where they can be avoided. Statistically significant empirical evidence of the influence of taxation and the intensity of regulation on the shadow economy is provided in studies by Schneider (1994, 2005) and Johnson *et al*. (1998a, 1998b).

Unemployment and the hours worked per employee in the official economy also affect the shadow economy. While it is clear that a reduction in working hours in the official economy increases hours spent working in the shadow economy, unemployment's effect on the shadow economy is ambiguous. Whether the unemployment variable exhibits a positive or negative relationship depends on income and substitution effect. Income losses due to unemployment reduce demand in the shadow as well as the official economy. A substitution takes place as

⁶ See Thomas (1992), Lippert and Walker (1997), Schneider (1994, 1997, 2003, 2005), Johnson et al. (1998a,b), Tanzi (1999), Giles (1999), Mummert and Schneider (2001), Giles and Tedds (2002), Giles et al. (2002), and Dell'Anno and Schneider (2003).

unemployed workers turn to the shadow economy where cheaper goods make it easier to countervail utility losses. This may stimulate additional demand there. If the income effect exceeds the substitution effect, a negative relationship develops. Likewise, if the substitution effect exceeds the income effect, the relationship is positive.

To mirror activities in the shadow economy, we use the monetary aggregate M1 and a GDP volume index. These variables are particularly suitable for this purpose as a result of the following considerations. Transactions in the shadow economy are typically carried out using cash or money that is drawn from a current account at a moment's notice. We therefore expect a positive relationship between the shadow economy and M1. The lower the officially measured GDP, the fewer possibilities people have to earn money in the official economy, and the likelier they are to be driven into the shadow economy. In the short run, we expect this negative relationship to exist. In the long run, however, the official and the shadow economy are complements rather than substitutes, and the variables will thus exhibit a positive relationship. One possible explanation is that when the official economy grows, the shadow economy grows as well since favorable economic conditions do not discern between the official and unofficial economy. The demand for maintenance and other services in the shadow economy in particular increase in the long run as a result of higher consumption (e.g. cars) in the official economy. Based on these theoretical considerations, we employ tax and social security contribution burdens, the intensity of regulation (measured by the ratio of government employment to the total labor force), the unemployment rate, and an index of hours worked per employee as causal variables for the shadow economy in the long run equilibrium estimation. A GDP volume index and the monetary aggregate M1 are used as indicators. Figure 5.1 illustrates these relationships. The small squares attached to the arrows indicate the expected signs in the empirical analysis.

Fig. 5.1 *Hypothesized Relationships in the MIMIC Model*

5.2 Data and the Analysis of Stationarity and Co-integration

Our data cover each quarter between 1981 and 2006: the number of observations is 104. Data sources and in-depth definitions of the variables are summarized in Table D.1 in the Appendix. We begin our empirical analysis by pre-testing the data. In the first step, each series is individually examined under the null hypothesis of a unit root against the alternative of stationarity. As shown in Table 5.1 we find that all variables are I(1) using conventional unit root tests.

Variable	Test Equation	Level			Test Equation	First Difference		
Causes		ADF	PP	KPSS		ADF	PP	KPSS
				*** 0.103				0.052 ^{***}
Tax	C & T	0.2707	0.4633		C	0.0000	0.0000	
Unemp	C & T	0.5072	0.6582	0.196^*	\mathcal{C}	0.0106	0.0006	$0.304***$
Reg	C & T	0.2000	0.3190	0.276	C	0.0023	0.0012	0.539^{*}
Work	C & T	0.4872	0.2755	$0.145***$	\mathcal{C}	0.0286	0.1712	$0.138***$
Indicators		ADF	PP	KPSS		ADF	PP	KPSS
GDP	C & T	0.1696	0.5827	0.118 [*]	\mathcal{C}	0.0009	0.0000	0.109 ^{***}
M1	C & T	0.8536	0.0882	0.188	\mathcal{C}	0.0208	0.0000	*** 0.179
** ∗ *** Stationority of 10/ $Stationority of 50/$ $Stationority$ of $100/$								

Table 5.1: *Analysis of* S*tationarity*

 Stationarity at 1%. ** Stationarity at 5%. *** Stationarity at 10%. *Note:* For the Augmented Dickey-Fuller (ADF) test and the Phillips-Perron (PP) test, the MacKinnon (1996) one-sided *p* values are given whereas test statistics are reported for the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test. Its critical values are taken from Kwiatkowski *et al*. (1992). For a test equation with constant (C) the critical values are: 0.347 (10% level), 0.463 (5% level), and 0.739 (1% level) whereas for a test equation with constant and trend $(C & T)$ the critical values are: 0.119 (10% level), 0.146 (5% level), and 0.216 (1% level). For the order of the autoregressive correction for the ADF test, we use the Akaike Information Criterion (AIC). For the PP and KPSS tests, we use the Bartlett kernel estimator and the Newey-West (1994) data-based automatic bandwidth parameter method.

Next, we use the Engle and Granger two-step approach to see if all four causes are cointegrated with each indicator variable and therefore exhibit a valid error correction representation (Engle and Granger, 1987). To do this, we first estimate least square regressions with variables in levels, where the particular indicator is the dependent variable and the causes are the independent variables. Thus, the regression equations are:

 $GDP = \alpha_1 \cdot Taxes + \alpha_2 \cdot Unemp + \alpha_3 \cdot Reg + \alpha_4 \cdot Work + u_1$ and

 $M1 = \alpha_1 \cdot \text{Taxes} + \alpha_2 \cdot \text{Unemp} + \alpha_3 \cdot \text{Reg} + \alpha_4 \cdot \text{Work} + u_2$.

Because all variables are deviations from their means, no constant is included in the regression equations. Next we analyze the assumed cointegration relationship's residuals u_1 and u_2 using the ADF test. If the causes are cointegrated with the indicators, we expect the ADF test to reject the null hypothesis of a unit root against the alternative for both error terms u_1 and u_2 . As presented in Table 5.2, we can in fact reject the null hypothesis for both residuals at conventional significance levels. We therefore conclude that the causes are cointegrated with each indicator.

		\cdot				
Indicators	Causes				<i>t</i> -statistic for Residual	Jarque Bera Probability
GDP	Tax (0.0006)	Unemp (0.0000) (0.0007)	Reg	Work (0.0000)	-4.0569	0.55
M1	Tax (0.0452)	Unemp (0.0119)	Reg (0.0039)	Work (0.0000)	-3.8725	0.62

Table 5.2: *Analysis of Cointegration Between Causes and Indicators*

Note: The critical values of the ADF test's *t*-statistic are taken from Engle and Yoo (1987). For a sample with 100 observations, they are: 4.61 (1% level), 4.02 (5% level), and 3.71 (10% level). The order of the autoregressive correction has been chosen using the AIC as suggested by Engle and Yoo (1987). Thus, the null hypothesis of a unit root is rejected at the 10% level for residual u_1 and at the 5% level for residual u_2 . The *p*values of the parameter estimators are given in parenthesis.

The confirmation of both cointegration relationships permits the estimation of a long run equilibrium MIMIC model for the size and development of the shadow economy according to equation (5). The next step is to estimate the short run MIMIC model of equation (7) employing first differences of all causes and indicators. The estimation also includes the long run error correction terms u_1 and u_2 from both cointegration relationships. Here, the number of observations is 103. Table 5.3 presents the long run equilibrium model's parameter estimates and primary test statistics as well as those for the short run model. Supplementary summary statistics indicating the overall fit of both MIMIC models are provided in Table D.2 in the Appendix.

	Long-run	Short-run
	MIMIC Model	MIMIC Model
Causes		
Taxes	$0.17***$	$0.50***$
	(5.74)	(6.84)
Unemp	-0.20 ***	$0.26***$
	(-5.14)	(3.47)
Reg	$0.22***$	$0.21***$
	(5.39)	(2.86)
Work	$-0.75***$	-0.03
	(-21.47)	(-0.40)
u_1		-0.04
		(-0.47)
u_2		$-0.27***$
		(-3.50)
Indicators		
GDP	1.00	-1.00
M1	$0.99***$	0.19
	(45.58)	$(1.43)^{*}$
Statistics		
Chi-square	2.54	17.17
Degrees of Freedom	13	26
P -value	0.9989	0.9073
Root Mean Squared Error of	0.000	0.000
Approximation (RMSEA)		

Table 5.3: *MIMIC Models and Parameter Estimates*

T-statistics are reported in parenthesis. * Significance at 10% level. ** Significance at 5 % level. *** Significance at 1% level.

The estimated coefficients of all variables in the long run equilibrium relationship are highly statistically significant at the 1% significance level and have the theoretically expected sign. In the short run, the MIMIC model estimates for the independent variable "Work" and the residual u_1 from the cointegration relationship between the causes and the GDP are not statistically significantly different from zero. As in the long run equilibrium relationship, all remaining variables – with the exception of M1, which is statistically significant only at the 10% level – are statistically significant at the 1% level. Overall, the various test statistics point to a close fit.

In order to estimate not only the relative size of the parameters but also their levels, it is necessary to fix a scale for the unobservable latent variable. A convenient way to determine the relative magnitude of the variables is to set the coefficient of one of the measurement model's indicator variables to non-zero.⁷ Here, we fix the coefficient of the variable GDP for both the long run and the short run MIMIC estimation.

We now summarize our findings from the estimations. First, we can confirm our theoretical considerations. Second, tax and social security contribution burdens and the intensity of regulation are important causes of the size of the shadow economy. These findings confirm our hypothesis as well as previous empirical findings. As expected, the relation between hours worked per employee and the shadow economy is negative, i.e. decreasing working hours encourage people to engage in informal economic activities. With regard to unemployment, it turns out that the overall long run effect is negative. That is, the income effect exceeds the substitution effect. This finding is supported by the positive relation between the shadow economy and official GDP, suggesting that in the long run the two are complements rather than substitutes. As expected, however, the short run relationship is negative, i.e. people who face unemployment switch to the shadow economy thereby negatively affecting official GDP. The other causes also show the expected sign in the short run estimation. That is, high tax and social security contribution burdens and a high intensity of regulation force people into the shadow economy. Declining working hours create the required freedom for those activities.

Both the long run equilibrium relationship and the short run dynamic error correction representation of the MIMIC model represent our EMIMIC model. With it, we can now estimate the size and development of France's shadow economy. The first step uses the long run part of our model to calculate the ordinal index. This index is then transformed into a cardinal series using the average of the estimates from Dell' Anno et al. (2007) and Schneider (2005), which is 14.25% of official GDP in 1995. Next, the short run deviations from equilibrium are calculated. Finally, taking these into account, estimates for the French shadow economy are derived using Bajada and Schneider's (2005) calibration methodology. Figure 5.2 illustrates these results. The thick line is the long run equilibrium, and the thin line represents the final estimates for the French shadow economy taking short run dynamic fluctuations into account.

 $\overline{7}$ Giles and Tedds (2002), 109.

Fig. 5.2 *Size of the French Shadow Economy in % of Official GDP*

Applying the EMIMIC model to the French shadow economy produces an estimate of 12.88% in Q1 1982, which increases to 15.93% in Q4 2006. All quarterly estimates for 1982 to 2006 are shown in Table D.3 in the Appendix. Table D.4 compares these estimates with those of the long run equilibrium relationship and illustrates this deviation for France's shadow economy. Thus, our findings correspond to other recent studies of the French shadow economy with 13.80% for 1990-93 (Schneider and Enste, 2000) and 15.30% for 2000 (Schneider, 2005). Our approach is nonetheless quite different from previous investigations because we use the long run equilibrium estimation for the initial calculation of the cardinal time series index. The calibration methodology is then only used to correct for deviations from equilibrium in the short run. Previous studies, on the other hand, derive the cardinal index of the shadow economy in a particular country from their DYMIMIC estimates using some type of calibration methodology. Our EMIMIC model thus estimates the size and development of the shadow economy more precisely.

6. Summary and Conclusions

In this paper we consider cointegration and error correction techniques within a latent variable approach. First, we demonstrate the possibility of controlling for long run equilibrium relationships in a MIMIC model by using the standard econometrics of cointegration and error correction mechanisms. Next, we present the cointegration equation and the error correction equation of a MIMIC model where indicators and causes are cointegrated. Finally, we derive the covariance matrices for the EMIMIC model and employ this model to estimate the size and development of the French shadow economy. We demonstrate that the EMIMIC model better quantifies the size of the shadow economy because it considers both the long run equilibrium relationships and the short run dynamic error corrections at the same time. This is particularly advantageous for economists trying to gather more precise figures for and track the development of the shadow economy and, of course, for the improvement of research in this problematical field of statistics. The preciseness of our estimates also benefit policymakers' efforts to deal with the shadow economy and their formulation of economic policy strategies.

Literature

- Aigner, Dennis; Schneider, Friedrich and Damayanti Ghosh (1988): Me and my shadow: estimating the size of the US hidden economy from time series data, in W. A. Barnett; E. R. Berndt and H. White (eds.)*: Dynamic econometric modeling*, Cambridge (Mass.): Cambridge University Press, pp. 224-243.
- Bajada, Christopher and Friedrich Schneider (2005): The Shadow Economies of the Asia-Pacific, Pacific Economic Review 10/3, pp. 379-401.
- Dell'Anno, Roberto and Friedrich Schneider (2003): The shadow economy of Italy and other OECD countries: What do we know?, *Journal of Public Finance and Public Choice*, 21/2-3, pp. 97-120.
- Dell' Anno, Roberto, Miguel Gómez-Antonio and Angel Pardo (2007): The shadow economy in three Mediterranean countries: France, Spain and Greece. A MIMIC approach, *Empirical Economics*, 33, pp. 51-84.
- Engle, Robert F. and Clive W. J. Granger (1987): Cointegration and Error Correction: Representation, Estimation, and Testing, *Econometrica*, 55/2, March, pp. 251-276.
- Engle, Robert F. and Byung Sam Yoo (1987): Forecasting and Testing in Cointegrated Systems, *Journal of Econometrics*, 35, pp. 143-159.
- Frey, Bruno S. and Hannelore Weck (1983): Estimating the Shadow Economy: A 'Naive' Approach, *Oxford Economic Papers*, 35, pp. 23-44.
- Frey, Bruno S. and Weck-Hannemann, Hannelore (1984): The hidden economy as an "unobserved" variable, *European Economic Review*, 26/1, pp.33-53.
- Giles, David, E.A. (1999): Measuring the hidden economy: Implications for econometric modelling, *The Economic Journal*, 109/456, pp. 370-380.
- Giles, David, E.A. and Lindsay M. Tedds (2002): *Taxes and the Canadian Underground Economy*, Canadian Tax Paper No. 106, Toronto: Canadian Tax Foundation.
- Giles, David, E.A., Tedds, Lindsay, M. and Werkneh, Gugsa (2002): The Canadian underground and measured economies, *Applied Economics*, 34/4, pp. 2347-2352.
- Granger, Clive W. J. and A. A. Weiss (1983): Time Series Analysis of Error-Correcting Models, in *Studies in Econometrics, Time Series, and Multivariate Statistics*, New York: Academic Press, pp. 255-278.
- Greene W. H. (2007): Econometric Analysis, Prentice-Hall: London.
- Johnson, Simon; Kaufmann, Daniel and Pablo Zoido-Lobatón (1998a): Regulatory discretion and the unofficial economy. *The American Economic Review*, 88/ 2, pp. 387-392.
- Johnson, Simon; Kaufmann, Daniel and Pablo Zoido-Lobatón (1998b): Corruption, public finances and the unofficial economy*. Discussion paper*, The World Bank, Washington, D.C.
- Kwiatkowski, Denis, Peter C. B. Phillips, Peter Schmidt and Yongcheol Shin (1992): *Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root*', Journal of Econometrics, 54, 159-178.
- Lippert, Owen and Michael Walker (eds.) (1997): *The Underground Economy: Global Evidences of its Size and Impact*, Vancouver: The Frazer Institute.
- MacKinnon, James G. (1996): Numerical Distribution Functions for Unit Root and Cointegration Tests, *Journal of Applied Econometrics*, 11, 601-618.
- Mummert, Annette and Friedrich Schneider (2002): The German shadow economy: Parted in a united Germany?, *Finanzarchiv*, 58/3, pp. 286-316.
- Newey, Whitney and Kenneth West (1994): Automatic Lag Selection in Covariance Matrix Estimation, *Review of Economic Studies*, 61, 631-653.
- Sargan, J. D. (1964): Wages and prices in the United Kingdom: A study in economic methodology, in P. Hart, G. Mills, and J. N. Whittaker (eds.), *Econometric Analysis for National Economic Planning*, Butterworths: London.
- Schneider, Friedrich (1994): Can the shadow economy be reduced through major tax reforms? An empirical investigation for Austria, *Supplement to Public Finance/ Finances Publiques*, 49, pp. 137-152.
- Schneider, Friedrich (1997): The shadow economies of Western Europe, *Economic Affairs*, 17/3, pp. 42-48.
- Schneider, Friedrich (2003): The shadow economy, in: Charles K. Rowley and Friedrich Schneider (eds.), *Encyclopedia of Public Choice*, Dordrecht: Kluwer Academic Publishers.
- Schneider, Friedrich (2005): Shadow economies around the world: what do we really know?, *European Journal of Political Economy*, 21(3), September, pp. 598-642.
- Schneider, Friedrich and Dominik Enste (2000): Shadow economies: Size, causes, and consequences, *The Journal of Economic Literature*, 38/1, pp. 77-114.
- Schneider, Friedrich and Dominik Enste (2002): *The Shadow Economy: Theoretical Approaches, Empirical Studies, and Political Implications*, Cambridge: Cambridge University Press.
- Tanzi, Vito (1999): Uses and abuses of estimates of the underground economy, *The Economic Journal*, 109/456, pp. 338-340.
- Thomas, Jim J. (1992): *Informal Economic Activity*, LSE, Handbooks in Economics, London: Harvester Wheatsheaf.
- Weck, Hannelore (1983): *Schattenwirtschaft: Eine Möglichkeit zur Einschränkung der öffentlichen Verwaltung? Eine ökonomische Analyse*, Frankfurt/Main: Lang.

Appendix A: Deriving the MIMIC Model's Covariance Matrix

The MIMIC model's structural and measurement equations are $\eta_t = \gamma' x_t + \varsigma_t$ and $y_t = \lambda \eta_t + \varepsilon_t$, respectively. Expressing this model in terms of covariances gives:

$$
\Sigma = \begin{pmatrix} \mathbf{Var}(\mathbf{y}_t) & \mathbf{Cov}(\mathbf{y}_t, \mathbf{x}_t) \\ \mathbf{Cov}(\mathbf{x}_t, \mathbf{y}_t) & \mathbf{Var}(\mathbf{x}_t) \end{pmatrix} = \mathbf{E} \begin{pmatrix} \mathbf{y}_t \\ \mathbf{x}_t \end{pmatrix} \begin{bmatrix} \mathbf{y}_t \\ \mathbf{x}_t \end{bmatrix}^T \begin{bmatrix} \mathbf{y}_t \\ \mathbf{x}_t \end{bmatrix}.
$$

After taking the transposes, multiplications, and making use of the assumptions that:

- 1. the variables are measured as deviations from mean, i.e. $E(\eta_t) = E(\mathbf{x}_t) = E(\zeta_t) = E(\mathbf{y}_t) = E(\mathbf{\varepsilon}_t) = 0;$
- 2. the error terms do not correlate to the causes, i.e. $\mathbf{E}(\mathbf{x}_t \zeta_t') = \mathbf{E}(\zeta_t \mathbf{x}_t') = 0$ and $E(\mathbf{x}_t \mathbf{\varepsilon}'_t) = E(\mathbf{\varepsilon}_t \mathbf{x}'_t) = 0$;
- 3. the error terms do not correlate across equations, $\mathbf{E}(\epsilon_t \zeta_t') = \mathbf{E}(\zeta_t \epsilon_t') = 0$; and,
- 4. the errors of the measurement model do not correlate to the latent variable, i.e. $\mathbf{E}(\eta_t \mathbf{\varepsilon}'_t) = \mathbf{E}(\mathbf{\varepsilon}_t \eta'_t) = 0;$

we distribute the expectation operator and can thus derive both the variance and covariance between the observable variables. By doing this, it follows that:

$$
E(y_t y_t') = E[(\lambda \eta_t + \varepsilon_t)(\lambda \eta_t + \varepsilon_t')]
$$

\n
$$
= E(\lambda \eta_t \eta_t' \lambda' + \lambda \eta_t \varepsilon_t' + \varepsilon_t \eta_t' \lambda' + \varepsilon_t \varepsilon_t')
$$

\n
$$
= \lambda E(\eta_t \eta_t') \lambda' + \Theta_{\varepsilon}
$$

\n
$$
= \lambda E[(\gamma' x_t + \varsigma_t)(\gamma' x_t + \varsigma_t')'] \lambda' + \Theta_{\varepsilon}
$$

\n
$$
= \lambda E(\gamma' x_t x_t' \gamma + \gamma' x_t \varsigma_t' + \varsigma_t x_t' \gamma + \varsigma_t \varsigma_t') \lambda' + \Theta_{\varepsilon}
$$

\n
$$
= \lambda (\gamma' \Phi \gamma + \psi) \lambda' + \Theta_{\varepsilon}
$$

\n
$$
E(x_t y_t') = E[x_t (\lambda \eta_t + \varepsilon_t)']
$$

\n
$$
= E(x_t \eta_t' \lambda' + x_t \varepsilon_t' + \varepsilon_t \eta_t' \lambda' + \varepsilon_t \varepsilon_t')
$$

\n
$$
= E(x_t \eta_t') \lambda'
$$

\n
$$
= E(x_t x_t' \gamma + x_t \varsigma_t') \lambda'
$$

\n
$$
= \Phi \gamma \lambda'
$$

\n
$$
E(y_t x_t') = (\Phi \gamma \lambda')'
$$

\n
$$
= \lambda \gamma' \Phi
$$

 $\mathbf{E}(\mathbf{x}_t \mathbf{x}'_t) = \mathbf{\Phi}.$

Thus, $\mathbf{\Theta}_{\varepsilon}$ is the covariance matrix of the error terms in the measurement model; ψ is the variance of the error term in the structural equation; and, **Φ** is the covariance matrix of the causes. Finally, the covariance matrix of the MIMIC model is:

$$
\Sigma = \begin{pmatrix} \lambda \big(\gamma' \Phi \gamma + \psi \big) \lambda' + \Theta_{\epsilon} & \lambda \gamma' \Phi \big) \\ \Phi \gamma \lambda' & \Phi \end{pmatrix}.
$$

Appendix B: Covariance Matrix of the Long Run Part

The EMIMIC model's long run equations with I(0) and I(1) causes are $y_t = \lambda \eta_t + \varepsilon_t$ and $\eta_t = \gamma' \mathbf{x}_t + \tau' \mathbf{v}_t + \zeta_t$. As a result, the covariance matrix in its general form is given as:

$$
\Sigma = \begin{pmatrix} \mathbf{Var}(\mathbf{y}_t) \\ \mathbf{Cov}(\mathbf{x}_t, \mathbf{y}_t) & \mathbf{Var}(\mathbf{x}_t) \\ \mathbf{Cov}(\mathbf{v}_t, \mathbf{y}_t) & \mathbf{Cov}(\mathbf{v}_t, \mathbf{x}_t) & \mathbf{Var}(\mathbf{v}_t) \end{pmatrix} = \mathbf{E} \begin{bmatrix} \mathbf{y}_t \\ \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix} \begin{bmatrix} \mathbf{y}_t \\ \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix}^T \begin{bmatrix} \mathbf{y}_t \\ \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix}^T
$$

After multiplication, distributing the expectations operator, and using the necessary assumptions, we get the covariance matrix in terms of model parameters and covariances. Its sub-matrices are:

,

$$
\mathbf{E}(\mathbf{y}_{t}\mathbf{y}_{t}') = \mathbf{E}\Big[(\lambda\eta_{t} + \varepsilon_{t})(\lambda\eta_{t} + \varepsilon_{t})'\Big]
$$
\n
$$
= \mathbf{E}(\lambda\eta_{t}\eta_{t}'\lambda' + \lambda\eta_{t}\varepsilon_{t}' + \varepsilon_{t}\eta_{t}'\lambda' + \varepsilon_{t}\varepsilon_{t}')
$$
\n
$$
= \lambda\mathbf{E}(\eta_{t}\eta_{t}')\lambda' + \mathbf{\Theta}_{\varepsilon}
$$
\n
$$
= \lambda\mathbf{E}\Big[(\gamma'\mathbf{x}_{t} + \tau'\mathbf{v}_{t} + \varsigma_{t})(\gamma'\mathbf{x}_{t} + \tau'\mathbf{v}_{t} + \varsigma_{t})'\Big]\lambda' + \mathbf{\Theta}_{\varepsilon}
$$
\n
$$
= \lambda\mathbf{E}(\gamma'\mathbf{x}_{t}\mathbf{x}_{t}'\gamma + \gamma'\mathbf{x}_{t}\mathbf{v}_{t}'\tau + \tau'\mathbf{v}_{t}\mathbf{x}_{t}'\gamma + \tau'\mathbf{v}_{t}\mathbf{v}_{t}'\tau)\lambda' + \mathbf{\Theta}_{\varepsilon}
$$
\n
$$
= \lambda(\gamma'\mathbf{\Phi}_{1}\gamma + 2\gamma'\mathbf{N}\tau + \tau'\mathbf{\Phi}_{2}\tau)\lambda' + \mathbf{\Theta}_{\varepsilon}
$$

 $E(x_t x_t') = \Phi_1, E(v_t v_t') = \Phi_2, E(v_t x_t') = N',$

$$
E(x_t y_t') = E\left[x_t (\lambda \eta_t + \varepsilon_t)' \right]
$$

\n
$$
= E(x_t \eta_t' \lambda' + x_t \varepsilon_t')
$$

\n
$$
= E(x_t \eta_t') \lambda'
$$

\n
$$
= E\left[(x_t)(\gamma' x_t + \tau' v_t + \varsigma_t)' \right] \lambda'
$$

\n
$$
= E(x_t x_t' \gamma + x_t v_t' \tau) \lambda'
$$

\n
$$
= (\Phi_1 \gamma + N\tau) \lambda'
$$

\n
$$
E(v_t y_t') = E\left[v_t (\lambda \eta_t + \varepsilon_t)'\right]
$$

\n
$$
= E(v_t \eta_t') \lambda'
$$

\n
$$
= E\left[v_t \gamma(\gamma' x_t + \tau' v_t + \varsigma_t)' \right] \lambda',
$$

\n
$$
= E(v_t x_t' \gamma + v_t v_t' \tau) \lambda'
$$

\n
$$
= (N' \gamma + \Phi_2 \tau) \lambda'
$$

where Φ_1 and Φ_2 are sub-covariance matrices of the I(0) and I(1) causes, respectively. We further define $N = E(v_t x_t')$. Finally, we obtain:

$$
\Sigma = \begin{pmatrix} \lambda (\gamma' \Phi_1 \gamma + 2 \gamma' N \tau + \tau' \Phi_2 \tau) \lambda' + \Theta_{\varepsilon} \\ (\Phi_1 \gamma + N \tau) \lambda' & \Phi_1 \\ (N' \gamma + \Phi_2 \tau) \lambda' & N' & \Phi_2 \end{pmatrix}.
$$

Appendix C: Covariance Matrix of the Short Run Part

Both of the short run part of the EMIMIC model's equations are $\Delta y_t = \lambda \Delta \eta_t + \epsilon_t$ and $\Delta \eta_t = \alpha \Delta x_t + \beta v_t + \kappa z_{t-1} + \zeta_t$. As a result, the model's general covariance matrix is given as:

$$
\Sigma = \begin{pmatrix} \text{Var}(\Delta y_t) \\ \text{Cov}(\Delta x_t, \Delta y_t) & \text{Var}(\Delta x_t) \\ \text{Cov}(v_t, \Delta y_t) & \text{Cov}(v_t, \Delta x_t) & \text{Var}(v_t) \\ \text{Cov}(z_{t-1}, \Delta y_t) & \text{Cov}(z_{t-1}, \Delta x_t) & \text{Cov}(z_{t-1}, v_t) & \text{Var}(z_{t-1}) \end{pmatrix} = E \begin{bmatrix} \frac{\Delta y_t}{\Delta x_t} \\ \frac{\Delta x_t}{\Delta x_t} \\ \frac{\Delta x_t}{\Delta x_{t-1}} \end{bmatrix} \begin{bmatrix} \frac{\Delta y_t}{\Delta x_t} \\ \frac{\Delta x_t}{\Delta x_{t-1}} \end{bmatrix}.
$$

In terms of model parameters and covariances, it becomes:

$$
\Sigma = \begin{pmatrix} \lambda(\alpha' \Phi_3 \alpha + 2\alpha' M \beta + \beta' \Phi_2 \beta + \kappa' \Omega \kappa) \lambda' + \Theta_{\varepsilon} \\ (\Phi_3 \alpha + M' \beta) \lambda' & \Phi_3 \\ (M\alpha + \Phi_2 \beta) \lambda' & M & \Phi_2 \\ \lambda(\psi \lambda' \kappa + \psi) \lambda' + \Theta_{\varepsilon} & 0 & 0 & \Omega \end{pmatrix}.
$$

In addition to previous definitions, we define $\mathbf{M} = \mathbf{E}(\mathbf{v}_t, \Delta \mathbf{x}'_t)$, $\mathbf{\Phi}_3 = \mathbf{E}(\Delta \mathbf{x}_t \Delta \mathbf{x}'_t)$, and assume that the long run part's error term is a stationary white noise series. Consequently, its covariance matrix should not change with time, i.e. $Cov(z_t) = Cov(z_{t-1}) = \Omega$. We further assume that $\mathbf{E}(\mathbf{x}_t \mathbf{\zeta}_{t-1}') = 0$, $\mathbf{E}(\mathbf{\zeta}_{t-1} \mathbf{x}_t') = 0$, $\mathbf{E}(\mathbf{x}_t \mathbf{\zeta}_{t-1}') = 0$, and $\mathbf{E}(\mathbf{\varepsilon}_{t-1} \mathbf{x}_t') = 0$. In addition, $\mathbf{E}(\eta_1 \mathbf{\varepsilon}'_{t-1}) = 0$ and $\mathbf{E}(\mathbf{\varepsilon}_{t-1} \eta'_t) = 0$. As a result, the sub-matrices are derived as follows:

$$
E(\Delta y_t \Delta y'_t) = E[(\lambda \Delta \eta_t + \epsilon_t)(\lambda \Delta \eta_t + \epsilon_t)']
$$

\n
$$
= E(\lambda \eta_t \Delta \eta'_t \lambda' + \lambda \Delta \eta_t \epsilon'_t + \epsilon_t \Delta \eta'_t \lambda' + \epsilon_t \epsilon'_t)
$$

\n
$$
= \lambda E(\eta_t \eta'_t) \lambda' + \Theta_{\epsilon}
$$

\n
$$
= \lambda E[(\alpha' \Delta x_t + \beta' v_t + \kappa' z_{t-1} + \varsigma_t)(\alpha' \Delta x_t + \beta' v_t + \kappa' z_{t-1} + \varsigma_t)']\lambda' + \Theta_{\epsilon} ,
$$

\n
$$
= \lambda [\alpha' \Phi_3 \alpha + \alpha' M \beta + \alpha' E(\Delta x_t z'_{t-1}) \kappa + \beta' M' \alpha + \beta' \Phi_2 \beta +
$$

\n
$$
+ \beta' E(v_t z'_{t-1}) \kappa + \kappa' E(z_{t-1} \Delta x'_t) \alpha + \kappa' E(z_{t-1} v'_t) \beta + \kappa' \Omega \kappa \lambda' + \Theta_{\epsilon}
$$

\n
$$
= \lambda (\alpha' \Phi_3 \alpha + 2 \alpha' M \beta + \beta' \Phi_2 \beta + \kappa' \Phi_2 \kappa) \lambda' + \Theta_{\epsilon}
$$

\n
$$
E(\Delta x_t y'_t) = E[\Delta x_t (\lambda \Delta \eta_t + \epsilon_t)']
$$

\n
$$
= E[(\Delta x_t \Delta \eta'_t) \lambda'
$$

\n
$$
= E[(\Delta x_t \Delta \eta'_t) \lambda'
$$

\n
$$
= (\Phi_3 \alpha + M' \beta + E[(\Delta x_t)(\lambda \varsigma_{t-1} + \varepsilon_{t-1})] \kappa) \lambda'
$$

\n
$$
= (\Phi_3 \alpha + M' \beta) \lambda'
$$

\n
$$
E(v_t \Delta y'_t) = E[\nu_t (\lambda \Delta \eta_t + \varepsilon_t)']
$$

\n
$$
= E[(\nu_t) (\alpha' \Delta x_t + \beta' v_t + \kappa' z_{t-1} + \varsigma_t)'] \lambda' ,
$$

\n
$$
= E(v_t \Delta x'_t \alpha + v_t v'_t \beta + v_t z'_{t-1
$$

 $= (\mathbf{M}\boldsymbol{\alpha} + \boldsymbol{\Phi}_2\boldsymbol{\beta})\boldsymbol{\lambda}'$

$$
\mathbf{E}(z_{t-1}\Delta y'_{t}) = \mathbf{E}\Big[(\lambda \zeta_{t-1} + \varepsilon_{t-1})(\lambda \Delta \eta_{t} + \varepsilon_{t})'\Big]
$$
\n
$$
= \lambda \mathbf{E}(\zeta_{t-1}\Delta \eta'_{t})\lambda' + \mathbf{\Theta}_{\varepsilon}
$$
\n
$$
= \lambda \mathbf{E}\Big[(\zeta_{t-1})(\alpha' \Delta x_{t} + \beta' v_{t} + \kappa' z_{t-1} + \zeta_{t})'\Big]\lambda' + \mathbf{\Theta}_{\varepsilon},
$$
\n
$$
= \lambda \mathbf{E}\Big((\zeta_{t-1})(\lambda \zeta_{t-1} + \varepsilon_{t-1})' \kappa + \psi\Big)\lambda' + \mathbf{\Theta}_{\varepsilon}
$$
\n
$$
= \lambda(\psi \lambda' \kappa + \psi)\lambda' + \mathbf{\Theta}_{\varepsilon},
$$
\n
$$
\mathbf{E}(z_{t-1}\Delta x'_{t}) = \mathbf{E}\Big[(\lambda \zeta_{t-1} + \varepsilon_{t-1})(\Delta x_{t})'\Big] = 0,
$$
\n
$$
\mathbf{E}(z_{t-1}v'_{t}) = \mathbf{E}\Big[(\lambda \zeta_{t-1} + \varepsilon_{t-1})(v_{t})'\Big] = 0.
$$

rabic D.2. I armer Summary Statistics of the MINITO Models				
	Long Run	Short Run		
	MIMIC Model	MIMIC Model		
P-value for Test of Close				
Fit				
(RMSEA < 0.05)	1.0000	0.9800		
Root Mean Square				
Residual (RMR)	0.0035	0.0540		
Standardized RMR	0.0035	0.0550		
Goodness of Fit Index				
(GFI)	0.99	0.96		
Adjusted Goodness of				
Fit Index (AGFI)	0.99	0.94		
Parsimony Goodness of				
Fit Index (PGFI)	0.61	0.69		

Table D.2: *Further* S*ummary Statistics of the MIMIC Models*

Year	Q1	Q2	Q ₃	Q4
1982	12.88	12.86	13.11	13.06
1983	12.96	13.00	13.08	13.20
1984	13.30	13.47	13.32	13.47
1985	13.28	13.28	13.27	13.27
1986	13.39	13.29	13.39	13.58
1987	13.68	13.52	13.60	13.41
1988	13.29	13.40	13.32	13.51
1989	13.57	13.83	13.82	13.62
1990	13.81	13.98	13.99	14.14
1991	13.89	13.99	13.99	13.97
1992	13.71	13.98	14.01	14.13
1993	13.99	14.06	14.04	14.07
1994	14.03	13.96	14.02	14.12
1995	14.30	14.23	14.58	14.66
1996	14.58	14.49	14.54	14.52
1997	14.34	14.30	14.28	14.31
1998	14.33	14.35	14.56	14.68
1999	14.66	14.64	14.69	14.84
2000	14.83	15.09	15.27	15.25
2001	15.31	15.48	15.45	15.69
2002	15.52	15.68	15.77	15.89
2003	15.73	15.82	15.61	15.57
2004	15.52	15.35	15.42	15.34
2005	15.43	15.62	15.56	15.76
2006	15.72	15.72	15.93	15.93

Table D.3: *Size of the French Shadow Economy in % of Official GDP*

Year	EMIMIC	Long Run Equilibrium Relationship	Deviation from Long Run Equilibrium
1982	13.06	12.99	0.06
1983	13.20	13.03	0.18
1984	13.47	13.34	0.13
1985	13.27	13.36	-0.10
1986	13.58	13.49	0.09
1987	13.41	13.56	-0.15
1988	13.51	13.57	-0.06
1989	13.62	13.83	-0.21
1990	14.14	14.03	0.11
1991	13.97	13.93	0.04
1992	14.13	13.96	0.17
1993	14.07	13.96	0.12
1994	14.12	14.19	-0.07
1995	14.66	14.48	0.18
1996	14.52	14.47	0.05
1997	14.31	14.43	-0.12
1998	14.68	14.62	0.06
1999	14.84	14.97	-0.13
2000	15.25	15.35	-0.10
2001	15.69	15.55	0.15
2002	15.89	15.76	0.14
2003	15.57	15.53	0.04
2004	15.34	15.43	-0.09
2005	15.76	15.78	-0.03
2006	15.93	16.01	-0.09

Table D.4: *EMIMIC vs. Long Run Equilibrium (% of Official GDP)*

CESifo Working Paper Series

for full list see www.cesifo-group.org/wp (address: Poschingerstr. 5, 81679 Munich, Germany, office@cesifo.de)

2136 Xavier Freixas and Bruno M. Parigi, Banking Regulation and Prompt Corrective Action, November 2007

 $\mathcal{L}_\mathcal{L} = \{ \mathcal{L}_\mathcal{L} = \{ \mathcal{L}_\mathcal{$

- 2137 Jan K. Brueckner, Partial Fiscal Decentralization, November 2007
- 2138 Silvia Console Battilana, Uncovered Power: External Agenda Setting, Sophisticated Voting, and Transnational Lobbying, November 2007
- 2139 Alan J. Auerbach, Michael P. Devereux and Helen Simpson, Taxing Corporate Income, November 2007
- 2140 Lorenzo Cappellari, Paolo Ghinetti and Gilberto Turati, On Time and Money Donations, November 2007
- 2141 Roel Beetsma and Heikki Oksanen, Pension Systems, Ageing and the Stability and Growth Pact, November 2007
- 2142 Hikaru Ogawa and David E. Wildasin, Think Locally, Act Locally: Spillovers, Spillbacks, and Efficient Decentralized Policymaking, November 2007
- 2143 Alessandro Cigno, A Theoretical Analysis of the Effects of Legislation on Marriage, Fertility, Domestic Division of Labour, and the Education of Children, November 2007
- 2144 Kai A. Konrad, Mobile Tax Base as a Global Common, November 2007
- 2145 Ola Kvaløy and Trond E. Olsen, The Rise of Individual Performance Pay, November 2007
- 2146 Guglielmo Maria Caporale, Yannis Georgellis, Nicholas Tsitsianis and Ya Ping Yin, Income and Happiness across Europe: Do Reference Values Matter?, November 2007
- 2147 Dan Anderberg, Tax Credits, Income Support and Partnership Decisions, November 2007
- 2148 Andreas Irmen and Rainer Klump, Factor Substitution, Income Distribution, and Growth in a Generalized Neoclassical Model, November 2007
- 2149 Lorenz Blume, Jens Müller and Stefan Voigt, The Economic Effects of Direct Democracy – A First Global Assessment, November 2007
- 2150 Axel Dreher, Pierre-Guillaume Méon and Friedrich Schneider, The Devil is in the Shadow – Do Institutions Affect Income and Productivity or only Official Income and Official Productivity?, November 2007
- 2151 Valentina Bosetti, Carlo Carraro, Emanuele Massetti and Massimo Tavoni, International Energy R&D Spillovers and the Economics of Greenhouse Gas Atmospheric Stabilization, November 2007
- 2152 Balázs Égert and Dubravko Mihaljek, Determinants of House Prices in Central and Eastern Europe, November 2007
- 2153 Christa Hainz and Hendrik Hakenes, The Politician and his Banker, November 2007
- 2154 Josef Falkinger, Distribution and Use of Knowledge under the "Laws of the Web", December 2007
- 2155 Thorvaldur Gylfason and Eduard Hochreiter, Growing Apart? A Tale of Two Republics: Estonia and Georgia, December 2007
- 2156 Morris A. Davis and François Ortalo-Magné, Household Expenditures, Wages, Rents, December 2007
- 2157 Andreas Haufler and Christian Schulte, Merger Policy and Tax Competition, December 2007
- 2158 Marko Köthenbürger and Panu Poutvaara, Rent Taxation and its Intertemporal Welfare Effects in a Small Open Economy, December 2007
- 2159 Betsey Stevenson, Title IX and the Evolution of High School Sports, December 2007
- 2160 Stergios Skaperdas and Samarth Vaidya, Persuasion as a Contest, December 2007
- 2161 Morten Bennedsen and Christian Schultz, Arm's Length Provision of Public Services, December 2007
- 2162 Bas Jacobs, Optimal Redistributive Tax and Education Policies in General Equilibrium, December 2007
- 2163 Christian Jaag, Christian Keuschnigg and Mirela Keuschnigg, Pension Reform, Retirement and Life-Cycle Unemployment, December 2007
- 2164 Dieter M. Urban, Terms of Trade, Catch-up, and Home Market Effect: The Example of Japan, December 2007
- 2165 Marcelo Resende and Rodrigo M. Zeidan, Lionel Robbins: A Methodological Reappraisal, December 2007
- 2166 Samuel Bentolila, Juan J. Dolado and Juan F. Jimeno, Does Immigration Affect the Phillips Curve? Some Evidence for Spain, December 2007
- 2167 Rainald Borck, Federalism, Fertility and Growth, December 2007
- 2168 Erkki Koskela and Jan König, Strategic Outsourcing, Profit Sharing and Equilibrium Unemployment, December 2007
- 2169 Egil Matsen and Øystein Thøgersen, Habit Formation, Strategic Extremism and Debt Policy, December 2007
- 2170 Torben M. Andersen and Allan Sørensen, Product Market Integration and Income Taxation: Distortions and Gains from Trade, December 2007
- 2171 J. Atsu Amegashie, American Idol: Should it be a Singing Contest or a Popularity Contest?, December 2007
- 2172 Patricia Apps and Ray Rees, Household Models: An Historical Perspective, December 2007
- 2173 Ben Greiner, Axel Ockenfels and Peter Werner, The Dynamic Interplay of Inequality and Trust – An Experimental Study, December 2007
- 2174 Michael Melvin and Magali Valero, The Dark Side of International Cross-Listing: Effects on Rival Firms at Home, December 2007
- 2175 Gebhard Flaig and Horst Rottmann, Labour Market Institutions and the Employment Intensity of Output Growth. An International Comparison, December 2007
- 2176 Alexander Chudik and M. Hashem Pesaran, Infinite Dimensional VARs and Factor Models, December 2007
- 2177 Christoph Moser and Axel Dreher, Do Markets Care about Central Bank Governor Changes? Evidence from Emerging Markets, December 2007
- 2178 Alessandra Sgobbi and Carlo Carraro, A Stochastic Multiple Players Multi-Issues Bargaining Model for the Piave River Basin, December 2007
- 2179 Christa Hainz, Creditor Passivity: The Effects of Bank Competition and Institutions on the Strategic Use of Bankruptcy Filings, December 2007
- 2180 Emilia Del Bono, Andrea Weber and Rudolf Winter-Ebmer, Clash of Career and Family: Fertility Decisions after Job Displacement, January 2008
- 2181 Harald Badinger and Peter Egger, Intra- and Inter-Industry Productivity Spillovers in OECD Manufacturing: A Spatial Econometric Perspective, January 2008
- 2182 María del Carmen Boado-Penas, Salvador Valdés-Prieto and Carlos Vidal-Meliá, the Actuarial Balance Sheet for Pay-As-You-Go Finance: Solvency Indicators for Spain and Sweden, January 2008
- 2183 Assar Lindbeck, Economic-Social Interaction in China, January 2008
- 2184 Pierre Dubois, Bruno Jullien and Thierry Magnac, Formal and Informal Risk Sharing in LDCs: Theory and Empirical Evidence, January 2008
- 2185 Roel M. W. J. Beetsma, Ward E. Romp and Siert J. Vos, Intergenerational Risk Sharing, Pensions and Endogenous Labor Supply in General Equilibrium, January 2008
- 2186 Lans Bovenberg and Coen Teulings, Rhineland Exit?, January 2008
- 2187 Wolfgang Leininger and Axel Ockenfels, The Penalty-Duel and Institutional Design: Is there a Neeskens-Effect?, January 2008
- 2188 Sándor Csengődi and Dieter M. Urban, Foreign Takeovers and Wage Dispersion in Hungary, January 2008
- 2189 Joerg Baten and Andreas Böhm, Trends of Children's Height and Parental Unemployment: A Large-Scale Anthropometric Study on Eastern Germany, 1994 – 2006, January 2008
- 2190 Chris van Klaveren, Bernard van Praag and Henriette Maassen van den Brink, A Public Good Version of the Collective Household Model: An Empirical Approach with an Application to British Household Data, January 2008
- 2191 Harry Garretsen and Jolanda Peeters, FDI and the Relevance of Spatial Linkages: Do third Country Effects Matter for Dutch FDI?, January 2008
- 2192 Jan Bouckaert, Hans Degryse and Theon van Dijk, Price Discrimination Bans on Dominant Firms, January 2008
- 2193 M. Hashem Pesaran, L. Vanessa Smith and Takashi Yamagata, Panel Unit Root Tests in the Presence of a Multifactor Error Structure, January 2008
- 2194 Tomer Blumkin, Bradley J. Ruffle and Yosef Ganun, Are Income and Consumption Taxes ever really Equivalent? Evidence from a Real-Effort Experiment with Real Goods, January 2008
- 2195 Mika Widgrén, The Impact of Council's Internal Decision-Making Rules on the Future EU, January 2008
- 2196 Antonis Adam, Margarita Katsimi and Thomas Moutos, Inequality and the Import Demand Function, January 2008
- 2197 Helmut Seitz, Democratic Participation and the Size of Regions: An Empirical Study Using Data on German Counties, January 2008
- 2198 Theresa Fahrenberger and Hans Gersbach, Minority Voting and Long-term Decisions, January 2008
- 2199 Chiara Dalle Nogare and Roberto Ricciuti, Term Limits: Do they really Affect Fiscal Policy Choices?, January 2008
- 2200 Andreas Bühn and Friedrich Schneider, MIMIC Models, Cointegration and Error Correction: An Application to the French Shadow Economy, January 2008