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UNCERTAIN AVAILABILITY, SPATIAL LOCATION AND REGULATION: AN APPLICATION TO THE AIRLINE INDUSTRY

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UNCERTAIN AVAILABILITY, SPATIAL LOCATION AND REGULATION: AN APPLICATION TO THE AIRLINE INDUSTRY

I. INTRODUCTION

This paper presents an analysis of industries characterized by both uncertain availability and spatial location of the product. The analysis is applied to the U.S. airline industry and is used to analyze the differential effect of price regulation on different markets and firms. From this analysis we obtain insights into which firms and markets may have benefited most from the deregulation of the industry in 1978.

Previous works on the regulation of the airline industry (see Caves [1962], Jordan [1970], Keeler [1972] and Douglas and Miller [1973] among others) implicitly assume that markets have the same demand structure. This work takes into account different demand structures in analyzing the effect of price regulation[1] (i.e., the difference between the regulated and the unregulated solution).

If airline services are of homogeneous quality, then we do not expect the unregulated solution to depend on demand structures. If on the other hand airline services vary in their quality component, then different markets may have different equilibrium price-quality combinations depending on the characteristics of demand.[2] Some markets may be characterized by low price-low quality, while others by high price-high quality.

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[1] In this sense, this paper extends and generalizes the analysis done in Panzar (1979).

[2] Like in a hedonic model à la Rosen (1974).

Minimum price regulation will affect mainly the low price-low quality market. The loss in consumer surplus is larger for that market because the difference between the regulated and the unregulated price is larger. Customers (as well as firms in the short run) in this kind of market would benefit the most from deregulation. This paper explores the implications of different demand structures on the impact of (de)regulation.

The analysis of industries with spatial location and uncertain availability of the product provides a framework in which to analyze the implications of different demand structures.

The airline industry (as well as many other industries like retailing, tourism and transportation) can be characterized by the following features: a) demand has a spatial aspect (that is a 3 a.m. flight is completely different from a 3 p.m. flight), b) demand is random, c) capacity per location (size of aircraft for each flight time) and number of locations (flights) are determined before demand is realized and d) demand depends on the probability of obtaining the good (a seat) at a specific location (flight time).

Firms have to decide how many flights and what type of aircraft to supply before demand is realized. Since demand for each specific flight time is a random variable, there is a positive probability that at each flight time capacity may be rationed. Customers therefore face a probability of not obtaining a seat at a specific flight time. Demand will depend on that probability in the sense that the larger the probability

it is the way to achieve required levels of availability and frequency. Moreover, the optimal level of excess capacity will depend on demand characteristics like unpredictability of the demand (i.e., the variance of the stochastic term in the demand), and availability and frequency elasticities of demand.

The optimal price will exceed the marginal cost of passengers by a factor that depends on the availability elasticity of demand and on the variability of demand. That factor will be exactly equal to the marginal cost of capacity divided by the expected utilization rate (i.e., the expected load factor). This would also imply that if there are no fixed costs and capacity average costs are constant, profits would be zero and there would be a finite optimal number of firms (and flights). This result is in contradiction to non-stochastic spatial models that require fixed costs in order to get a finite number of firms. The reason for this discrepancy is that uncertain demand with capacity being determined before demand is realized creates short run fixed costs therefore limiting the optimal number of firms.

We analyze how the optimal solution changes when price is regulated. This analysis is performed in two stages. In the first stage it is assumed that either total flights (i.e., frequency) is given or that available seats per flight are given. This partial analysis provides straightforward results. Minimum price regulation implies an increase in seats per flight in the first case, and in the second case it implies

an increase in frequency. Moreover, the larger the availability elasticity of demand the smaller the increase in capacity and in flight frequency due to the minimum price regulation. Once we allow for both frequency and capacity per flight to be endogenously determined the results are not as straightforward as in the partial cases. The reason is that quality is multidimensional and availability can be produced by increasing either capacity or frequency. An increase in the regulated price may imply an increase in only some of the dimensions of quality--for example an increase in availability and a reduction in frequency coupled with an increase in capacity per flight.

The analysis presented in this paper is related to previous works in the area of uncertain demand, availability and spatial location. For example, Carlton [1977], Crew and Kleindorfer [1978] and Visscher [1973] analyze optimal solutions to non-spatial models characterized by random demand and capacity being determined before demand is realized. Gould [1978] and De Vany and Savings [1977] introduce availability as a factor in the demand. Carlton [1978] analyzes availability in a competitive market with demand being stochastic. Finally De Vany [1975] analyzes availability as affecting the total (non-stochastic) industry demand.[1]

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[1]Availability in a non-stochastic framework like De Vany's has a different meaning than in a stochastic one. In the case of airlines availability in a non-stochastic framework it would imply that people dislike crowded airplanes even when the probability of obtaining a seat is always one.

In the area of spatial location our work is related to optimal models like Telser [1978] and Sharkey [1976]. Telser [1978] deals only with the spatial location problem. Sharkey [1976] introduces a random demand but it is not spatially located and it is independent of the availability of the good. He deals with the optimal distribution of firm size.[1]

In this paper the theoretical analysis is applied to the airline industry; therefore, location means flight time and capacity supplied at each flight time is the number of available seats.

Section II is organized as follows. First we describe demand and cost conditions. We introduce the concepts of rationing and availability and their relationships to demand and supply. We later develop the 'Total Expected Net Benefit' which is the objective function to be maximized. The optimal solution is described next and compared with previous results. Finally we analyze how the optimal solution depends on demand characteristics.

In Section III we present preliminary results of estimating demand characteristics for different airlines. It is found that airlines differ in the average type of market they serve according to the relevant demand characteristics discussed in Section II. This result implies that different firms should be affected differently by the deregulation of the industry. Moreover, we can rank firms according to the predicted effect of deregulation. In another paper (Spiller [1980a]) we found that there is a high correlation between the predicted and the actual effects of past

[1] It differs from monopolistic competition models in spatial location frameworks (like Prescott and Visscher (1977) or Salop (1979)) in analyzing the optimal (and therefore fully cooperative) solution. For a non-cooperative solution to this problem see Spiller (1980c).

regulatory changes, providing support to the idea that differences in demand characteristics are relevant in explaining the differential impact of deregulation. Finally, in a study of the reaction of individual stock prices to the news of deregulation (Spiller [1980b]) it is shown that the firms found in this paper to be more constrained by the regulation of the industry had significant increases in their relative risk and in their profitability due to deregulation. In Section IV we present the conclusions and ideas for future research.

II. THE THEORETICAL MODEL

1. The Demand and Cost Structures

a. The Demand Side

Let $G(|k - t|, P, v, \underline{Z}, \underline{R}, \underline{V})$ be the maximum number of potential customers located at time k (i.e., that their preferred flight time is k) who would fly at time t when the price of the flight is P and the probability of getting an empty seat on that flight is v. $\underline{Z}$ is a vector of variables that includes the price of alternative modes of transportation, the income of the population with desired time k and other relevant exogenous variables. $\underline{R}$ and $\underline{V}$ are vectors containing the prices and availability levels of all other flights.

The function G(.) is derived as follows. Assume that there is a continuum of flights all around the day, that all the flights charge the same price and have the same availability levels. In this situation some customers would like to fly at k. That is, k is their desired flight time. Assume

that we have a finite number of flights. It may be that those customers who wanted to fly at time k find that there is no flight at that time but that there is a flight at time t. Then $G(|t - k|, P, v, \underline{Z}, \underline{R}, \underline{V})$ is the maximum number of customers that would take the flight at t with time k being their most desired flight time.[1,2]

Since the model is a stationary-stochastic one[3] (i.e., the distribution of passengers over time does not depend on the specific time and the cost structure will be independent of time of day), prices and availability levels at each flight would be the same.

For the same prices and availability, customers at k would be willing to take flight t if there is no other flight t' such that $|k - t| > |k - t'|$.

Since the optimal solution is a symmetric solution, we can express the demand for flight t by customers located at k as depending only on $|k - t|$, P and v.

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[1]For convenience we shall not express all of the arguments that appear in functional forms.

[2]It is assumed that customers are indifferent between two flights that are equally distant from their most desired flight, but one is an earlier, and the other is a later flight. This symmetry assumption may not always hold. Some customers may value much less a later flight than an earlier one. For simplicity of exposition we shall assume symmetry in customers' preferences.

[3]We are excluding from the analysis deterministic peak load pricing considerations. These arise from periods of time having a higher (mean) demand than others, like day versus night. If the cost of any one flight is independent of the existence of other flights (i.e., there are no economies of scope), then time differences in the distribution of the demand do not affect the main conclusions of this section, it will only create price differences over time.

The function $G(|k - t|, P, v, \underline{Z}, \underline{R}, \underline{V})$ is derived from a simple model of consumer behavior.[1] For the sake of simplicity I will assume G(.) to be non-stochastic. The stochastic aspect of the demand is introduced by assuming that only a certain fraction s of the maximum number of customers show up. Let s be a random variable, $s \varepsilon (0,1)$, with F(s) and f(s) being the probability distribution and the density functions respectively. I assume that the distribution of s is independent of time of day.[2]

The signs of the derivatives of G(.) are: $G_p < 0$, $G_v > 0$ and $G_z < 0$ where $z = |k - t|$.

In order to get the expected demand for flight t it is necessary to know who would like to fly at time t. Given all other flights, we can say that the customers that eventually show up will come from a time interval around t, being the lower and upper boundaries k_t and k_{t+1} respectively.

Let's call $N(t,P,v,k_t,k_{t+1})$ the expected number of customers that show up for flight t. Then

$$(2.1) \quad N(t,P,v,k_t,k_{t+1}) = \int_{k_t}^{k_{t+1}} \int_0^1 sG(|k-t|,P,v) f(s)\,ds\,dk$$

$$= \bar{s} \int_{k_t}^{k_{t+1}} G(|k-t|,P,v)\,dk = \bar{s}g(k_t,k_{t+1},t,P,v)$$

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[1] See Spiller (1980c, Appendix C).

[2] A more sophisticated way would be to let G(.) be stochastic, then we may have different sources of variation, a market specific source of variation and an economy-wide one. I do not pursue this line of research here.

where $\bar{s} \equiv \int_0^1 sf(s)ds$

and $g(k_t, k_{t+1}, t, P, v) \equiv \int_{k_t}^{k_{t+1}} G(|k - t|, P, v)dk.$

$N_{k_{t+1}}{}^1$ is the increase in the expected demand for flight t if we increase its upper boundary. $N_{k_{t+1}} = \bar{s}g_{k_{t+1}} = \bar{s}G(|k_{t+1}-t|, P, v)$. The expected demand for flight t will increase if the customers located exactly at the boundaries have a positive demand. Their demand depends on how much they value proximity of a

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[1] $N_{k_{t+1}} = \frac{\partial N}{\partial k_{t+1}}$. For convenience, partial derivatives will be expressed by subscripts.

flight to their most desired flight. As we will see later if n is the number of flights, $k_{t+1} = t + 1/2n$ and $k_t = t - 1/2n$, then an increase in the number of flights reduces (increases) the upper (lower) boundary and therefore reduces the demand for flight t. The change in the demand for flight t arising from a change in the number of flights is: $\bar{s}g_n = -\bar{s}g_{k_{t+1}}/n^2$. The expression g_n is the basis for what will be called the frequency elasticity of demand. It is the percentage change in total expected demand from a percentage change in the number of flights. If $E_{i,j}$ is the elasticity of i with respect to j, then:

$$E_{\bar{s}gn,n} = 1 + ng_n/g = 1 - g_{k_{t+1}}/(ng).$$

If $g_{k_{t+1}} = 0$, that is the customers at the boundaries have zero demand, then the elasticity of demand with respect to number of flights will be one; otherwise the frequency elasticity of demand will always be less than one (it can be seen from the second order conditions for the optimal solution that the frequency elasticity of demand has to be less than one). In the same way, g_v is the basis for the availability elasticity of demand:

$$E_{\bar{s}gn,v} = \frac{g_v}{g} v.$$

Following Savings and De Vany [1978] we can define the probability of getting a seat on flight t in the following

way. v is the unconditional probability of getting a seat. There are just two relevant future states of the world, either there will be an empty seat at t or there will be no empty seats at time t. In the case of no empty seats, it is necessary to arrange some way of rationing seats. I will assume that if more customers than available seats show up at t, seats will be rationed randomly with the probability of a customer getting a seat equal to the number of seats divided by the actual number of customers that showed up.

Define Q_t as available seats at time t. Then if (realized demand) $s.g(k_t,k_{t+1},t,P,v) \geq Q_t$ the probability of getting a seat is $Q_t/[s.g(k_t,k_{t+1},t,P,v)]$. If on the other hand, $s.g(k_t,k_{t+1},t,P,v) \leq Q_t$ the probability of getting a seat is one.

Therefore, the unconditional probability is:

$$(2.2) \qquad v = \int_0^{Q_t/g(v)} f(s)ds + \int_{Q_t/g(v)}^{1} [Q_t/sg(v)]f(s)ds.$$

If v_1 satisfies (2.2), then v_1 is a rational expectations equilibrium value of the probability of getting a seat. It is an equilibrium in the sense that for a given capacity Q_t, if customers think that v_1 is the expected value of the probability of getting a seat, they will behave in such a way that v_1 will turn out to be the expected value of the probability of getting the seat. Since the r.h.s. of (2.2)

is a contraction mapping[1] on v, equation (2.2) implies that v is an implicit function of k_t, k_{t+1}, t, P and Q_t.

In the r.h.s. of (2.2), g is a function of v. For any v, customers will demand g(v). If v_1 satisfies (2.2), then customers are demanding $g(v_1)$, that is they are behaving with the knowledge that v_1 is the probability of obtaining a seat. In this sense, a solution to (2.2) is a rational expectations solution.

From (2.2) we can get the implicit partial derivatives of v with respect to each one of the parameters. They are stated in equations (2.3), (2.4), (2.5) and (2.6).

$$\frac{\partial v}{\partial Q} = \frac{\frac{1}{g}\int_{Q/g}^{1}\frac{1}{s}f(s)ds}{1-\phi_v} > 0. \tag{2.3}$$

[2]

$$\frac{\partial v}{\partial k_t} = -\frac{1}{1-\phi_v} g_{k_t} \frac{Q}{g^2}\int_{Q/g}^{1}\frac{1}{s}f(s)ds. \tag{2.4}$$

Further, from the definition of g(.),

$$\frac{\partial v}{\partial k_t} = \frac{1}{1-\phi_v} G(|k_t - t|,P,v)\ \frac{Q}{g^2}\int_{Q/g}^{1}\frac{1}{s}f(s)ds > 0.$$

$$\frac{\partial v}{\partial k_{t+1}} = -\frac{1}{1-\phi_v} G(|k_{t+1} - t|,P,v)\ \frac{Q}{g^2}\int_{Q/g}^{1}\frac{1}{s}f(s)ds < 0 \tag{2.5}$$

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[1]The absolute value of the derivative of the r.h.s. of (2.2) is $\frac{Q}{g}\int_{Q/g}^{1} f(s)/sds.g_v/g$ that is smaller than 1 if $\int_{Q/g}^{1} f(s)/sds$ is smaller than 1.

[2]The function $\phi(k_t,k_{t+1},P,t,v)$ is the r.h.s. of equation (2.2).

and,

$$(2.6) \qquad \frac{\partial v}{\partial P} = -\frac{1}{1-\phi_v} g_P \frac{Q}{g^2} \int_{Q/g}^{1} \frac{1}{s} f(s)\,ds > 0.$$

b. The Expected Social Gross Benefit

We shall now turn to analyze how much the public values flying at t when v is the probability of getting a seat and Q is the capacity supplied. Both the probability of an empty seat and the capacity level are important since the number of customers that show up at time t depends directly on P and v, but the number of those who actually fly depends on the capacity supplied Q.

Assume that for a price of P and a probability of v, the random variable s turns out to be s_0. Then as in Figure 2 all of those customers will fly and their consumer benefit is:[1]

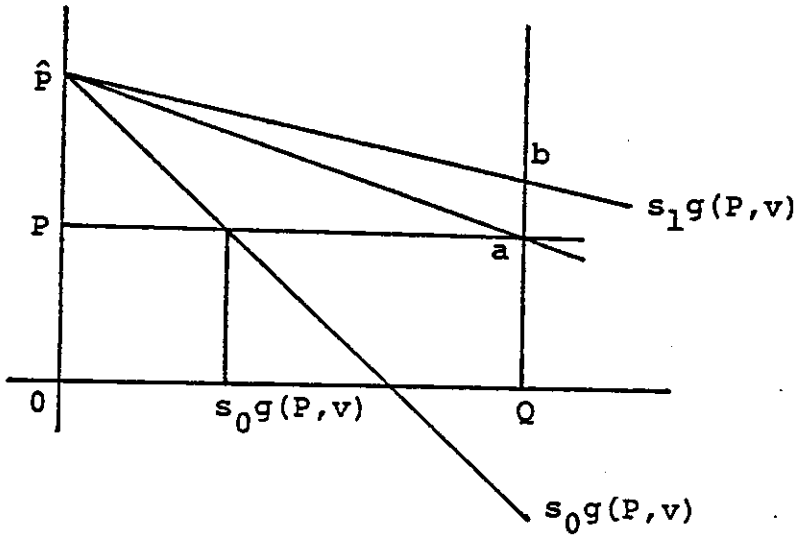

Figure 1

[1]Several works such as Carlton [1977] and Caves and Pazner [1975] have mentioned that when there is rationing

$$\int_{P}^{\hat{P}} s_0 g(p,v)\,dp + Ps_0(P,v)$$

where $\hat{P}$ is such that $g(\hat{P},v) = 0$.

On the other hand, if s turns out to be s_1, only Q will fly. In order to calculate the social gross benefit for the Q passengers that fly, we have to introduce rationing in the way that was stated before.

I assumed that rationing is a random process, that is, the probability of getting a seat is independent of the customer's valuation. This assumption is not free of criticism because actually there are ways to influence the probability of getting a seat.

As a first approximation I will assume that rationing is random as stated above--therefore disregarding the possibility of more than one class of service, that is unsatisfied coach passengers are not able to buy a first class seat.

In order to calculate the social gross benefit I will use a method proposed by Harberger [1973]. If the distribution of customers according to their valuation is the same as the distribution of actual passengers,[1] the consumer

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the relationship between the area under the demand curve and consumer benefit is not necessarily a one to one relationship, even if there are no other distortions in the economy. It can be shown (see Spiller [1980c], Appendix C) that if customers differ only on their valuation of the flight but not on income or in the cost of not flying, the condition for a one to one relationship is the same as in the normal case, i.e., the marginal utility of income is constant over the relevant range.

[1]Alternatively each customer gets an equal proportion of its demand in the case of rationing.

benefit of those that travel is the area $Qa\hat{P}0$[1] which is equal to:

$$\int_P^{\hat{P}} \frac{Q}{g} g(p,v)\, dp + QP$$

Then the expected social gross benefit is:

$$(2.7) \quad B(k_t,k_{t+1},t,P,v) = \int_0^{Q/g} \left[s\int_P^{\hat{P}} g(p)dp + sg(P)P \right] f(s)ds + \int_{Q/g}^{1} \left[QP + \frac{Q}{g}\int_P^{\hat{P}} g(p)dp \right] f(s)ds.$$

If we define $\hat{N}$ as the expected output, then

$$(2.8) \quad \hat{N} = \int_0^{Q/g} sf(s)ds.g(P) + Q\int_{Q/g}^{1} f(s)ds$$

and we can express the expected social gross benefit as:

$$(2.7a) \quad B(k_t,k_{t+1},t,P,v) = (P + \int_P^{\hat{P}} g(p)/g(P)dp).\hat{N}$$

$$(2.7b) \quad \partial B/\partial P = \int_0^{Q/g} sf(s)ds(g_P + g_v v_P)\left(1 - \int_P^{\hat{P}} \frac{g(p)}{g(P)}dp \frac{\int_{Q/g}^{1} f(s)dsQ}{\int_0^{Q/g} sf(s)dsg}\right) < 0.$$

From (2.7b) it can be seen that the change in expected gross benefit from a change in price depend on the price elasticity of the demand but also on the extent of rationing. The larger the proportion of rationed to non-rationed customers

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[1] If rationing were done according to willingness to pay, the consumer benefit of those that travel would be the area $0Qb\hat{P}$ (Brown and Johnson [1969]).

the smaller the reduction in expected gross benefit from an increase in price.

c. The Cost Structure

I will assume a simple cost structure: the cost of one flight is independent of the existence of other flights: that is there are constant returns to flights. Total cost will be composed of two components. The first is a capacity cost and it is assumed to depend only on the size of the airplane chosen: $C(Q)$. $C(Q)$ in a short run horizon is a fixed cost because capacity is supplied before demand is realized, but in the long run $C(Q)$ is a variable cost since by changing aircraft size we may change the capacity costs. The second component depends on the realized number of passengers. I will assume it to be proportional to the realized number of passengers. Let's call the average and marginal variable cost per passenger c. Thus for values of s such that $s.g(k_t,k_{t+1},t,P,v) < Q$.

$$(2.8a) \quad TC(k_t,k_{t+1},t,P,v,Q,s) = C(Q) + sg(k_t,k_{t+1},t,P,v).c$$

and when $s.g(k_t,k_{t+1},t,P,v) \geq Q$.

$$(2.8b) \quad TC(k_t,k_{t+1},t,P,v,Q,s) = C(Q) + sQc$$

when TC(.)--total cost--is a random variable.

The expected value of total cost is:

$$(2.9) \quad \overline{TC}(k_t,k_{t+1},t,P,v,Q) = C(Q) + c\int_0^{Q/g} s.g(k_t,k_{t+1},t,P,v)f(s)ds + cQ\int_{Q/g}^{1} f(s)ds.$$

From (2.9) we see that the expected total marginal cost of capacity $\partial\overline{TC}/\partial Q$ is different from the marginal cost of capacity C_Q, since the former includes the increase in expected costs from an increase in expected output due to the increase in availability. $\partial\overline{TC}/\partial Q$ is expressed in (2.9a).

$$(2.9a) \qquad \partial\overline{TC}/\partial Q = C_Q + c\frac{g_v}{g^2}\int_{Q/g}^{1}\frac{f(s)}{s}\,ds\,\hat{N}/(1-\phi_v) + c\int_{Q/g}^{1} f(s)ds/(1-\phi_v).$$

It will be assumed that $C_Q > 0$ and $C_{QQ} > 0$.

2. The Optimum Solution

The optimum solution will be the one that maximizes the sum of the expected net benefits of the different flights, which we call the total Expected Net Benefit.

Let us assume that n is the optimal number of flights per period. Since the solution is symmetric, all the flights will be equally spaced. Moreover, all flights will charge equal prices and provide equal capacity and availability. The boundary between two flights will be at the mid-point between them.

Call $H(k_t,k_{t+1},P,t,Q)$ the expected net benefit of a flight at t with capacity Q and price P, with customers coming from the interval (k_t,k_{t+1}), that is:

$$(2.10) \qquad H(k_t,k_{t+1},t,P,Q) = B(k_t,k_{t+1},P,v,Q) - \overline{TC}(k_t,k_{t+1},P,v,Q).$$

If n is the optimal number of flights (and flights are located in a circle with a circumference of length one), $k_{t+1} = t + 1/(2n)$ and $k_t = t - 1/2n$. Therefore, the Total Expected Net Benefit is:

$$TH(n,P,Q) = nH(t - \tfrac{1}{2}/n, t + \tfrac{1}{2}/n, P, t, Q).$$

Since t can be any point in the circle (since the distribution of the demand is the same over the circle) we are going to maximize TH with respect to n,P,Q.

The problem is to solve:

$$\text{Max}\{TH(n,P,Q) = n[B(t,v,P,t - \tfrac{1}{2}/n, t + \tfrac{1}{2}/n) - \overline{TC}(t,v,P,Q,t - \tfrac{1}{2}/n, t + \tfrac{1}{2}/n)]\}.$$

$$\text{s.t. } v = v(t - \tfrac{1}{2}/n, t + \tfrac{1}{2}/n, t, P, Q).$$

After substituting v(.) into B(.) and $\overline{TC}(.)$ the first order conditions are:

$$B_Q - \overline{TC}_Q + (B_v - \overline{TC}_v)v_Q = 0 \tag{2.11}$$

$$B_P - \overline{TC}_P + (B_v - \overline{TC}_v)v_P = 0 \tag{2.12}$$

$$B(.) - \overline{TC}(.) + [B_{k_t} - B_{k_{t+1}} - \overline{TC}_{k_t} + \overline{TC}_{k_{t+1}} + (B_v - \overline{TC}_v)(v_{k_t} - v_{k_{t+1}})]\tfrac{1}{2}/n = 0. \tag{2.13}$$

From the definition of B(.) in (2.7), $B_{k_t} = -B_{k_{t+1}}$ and $TC_{k_t} = -TC_{k_{t+1}}$ therefore (2.13) can be expressed as

$$(2.13a)\quad B(.) - \overline{TC}(.) = [B_{k_{t+1}} - \overline{TC}_{k_{t+1}} + (B_v - TC_v)v_{k_{t+1}}]/n.$$

The first result is related to the pricing policy. It says that if there is a positive probability of rationing, then the optimal price has to exceed the marginal cost of a passenger. The different is a congestion fee that is exactly equal to the marginal cost of increasing capacity divided by the expected load factor.[1]

From the definition of $B(.)$ and $\overline{TC}(.)$ we have

$$B_P - \overline{TC}_P = (P - c)g_P \int_0^{Q/g} sf(s)ds - Q/(g^2).g_P \int_P^{\hat{P}} g(p)dp \int_{Q/g}^1 f(s)ds$$

and

$$B_Q - \overline{TC}_Q = (P - c)\int_{Q/g}^1 f(s)ds + 1/g \int_{Q/g}^1 f(s)ds \int_P^{\hat{P}} g(p)dp - C_Q.$$

Substituting these expressions in (2.11) and (2.12) and equation (2.11) into (2.12), and recalling that $v_P/v_Q = -g_P Q/g$ we get:

$$(2.14)\qquad P - c = C_Q\, Q/\hat{N}$$

where $\hat{N} = \int_0^{Q/g} sf(s)dsg(P) + Q\int_{Q/g}^1 f(s)ds$ is expected output and $\hat{N}/Q$ is expected load factor.

Proposition 1: Optimal price exceeds short run marginal cost by a congestion toll exactly equal to the marginal cost of

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[1]This result generalizes Carlton's [1977]. He gets the same result for a non-spatial peak load pricing with uncertain demand that does not depend on availability.

capacity divided by the expected load factor.

Expression (2.14) says that there is a positive relationship between price and availability. $\hat{N}/Q$ is the expected load factor. In the case of $f(s) = 1$ (i.e., s is uniformly distributed) from (2.2), the probability of getting a seat is exactly $Q/g.[1 - \text{Log}(Q/g)]$ and the expected load factor is $1 - \frac{Q}{2g}$. Therefore, when the probability of getting a seat goes up, the expected load factor goes down and price is increased.

Therefore:

Corollary: If s is uniformly distributed, and the marginal cost of capacity is constant, then optimal price and availability are positively related.

By substituting expression (2.14) into (2.11) we get:

$$(2.15)\quad P - c = \left(1 + \frac{Q}{g}\frac{\int_{Q/g}^{1} f(s)ds}{\int_{0}^{Q/g} sf(s)ds}\right)\frac{Q}{g}\int_{P}^{\hat{P}} g_v(p)/g(P)dp\int_{Q/g}^{1} f(s)/sds$$

$$+ \frac{Q}{g}\frac{\int_{Q/g}^{1} f(s)ds}{\int_{0}^{Q/g} sf(s)ds}\int_{P}^{\hat{P}} g(p)/g(P)dp$$

Expression (2.15) suggests that the optimal difference between price and short run marginal cost depends positively on the availability elasticity of demand (which is related to the expression g_v/g) and on the proportion of expected rationed to non-rationed customers. The first implication is intuitively

clear. Markets with larger availability elasticity of demand will have larger availability and higher prices (see Appendix A for a proof of this statement).

Proposition 2: The optimal difference between price and short run marginal cost will depend positively on the availability elasticity of demand.

The second implication can be explained in the following way. The ratio

$$\frac{Q\int_{Q/g}^{1} f(s)ds}{g\int_{0}^{Q/g} sf(s)ds}$$

is a measure of the extent of rationing in the market. The numerator is the expected number of customers given that there is rationing, while the denominator is the expected number of customers given that there is no rationing. In the case of $f(s) = 1$, the ratio is just $\frac{g}{Q} - 1$. The probability of obtaining a seat is $Q/g[1 - \text{Log}(Q/g)]$. An increase in g/Q increases the ratio of rationed to non-rationed customers and decreases the probability of obtaining a seat. That is availability and the ratio of rationed to non-rationed customers are negatively related. We may expect this negative correlation to hold also for more general density functions.

There are many reasons for changes in the proportion of rationed to non-rationed customers. One possible reason is an increase in the variance of the demand (if $f(s)$ is symmetric).

For given prices and capacity levels, an increase in the variance of the demand reduces the probability of obtaining a seat. The optimal response may be to increase price and capacity per flight. We may expect the increase in price and capacity not to restore the probability v to its former level. We may end up therefore with a larger proportion of rationed to non-rationed customers and with a higher price.[1]

The next result shows that the optimal size of firm[2] also depends on the stochastic process generating the demand. This is shown by looking at the optimal number of firms when there are no fixed costs and when marginal and average capacity costs are constant. In a non-stochastic framework, these conditions imply an infinite number of firms. In a stochastic framework the optimal number of firms is finite. Uncertain demand with capacity being determined before demand is realized provides an extra source of returns to scale, over and beyond those implied by the existence of fixed capacity costs. That is, the firms have a larger market area--the distance between the lower and upper boundaries increases. This result provides some more intuitive support to the claim that an increase in the variance of the demand will increase optimal prices.

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[1] We have been unable to provide a proof to this statement. In the meantime, this implication is a conjecture.

[2] In this model firms and flights are interchangeable terms. Since there are no returns of scope each firm is assumed to supply one flight.

Expected net benefit is:

$$(2.10a)\quad B(.) - \overline{TC}(.) = (P - c)\hat{N} + \int_{P}^{\hat{P}} g(p)/g(P)\,dp\hat{N} - c(Q).$$

Substituting (2.14) into (2.10a) we get:

$$(2.16)\quad B(.) - \overline{TC}(.) = C_Q Q - C(Q) + \int_{P}^{\hat{P}} g(p)/g(P)\,dp\hat{N}.$$

By substituting (2.14) and (2.16) into the first order condition with respect to number of flights (equation (2.13)) we get:

$$(2.17)\quad C_Q Q - C(Q) + \int_{P}^{\hat{P}} g(p)/g(P)\,dp\hat{N}$$

$$= \frac{\hat{N}}{ng}\left(\int_{P}^{\hat{P}} g_{k_{t+1}}\,dp + g_{k_{t+1}}\frac{Q}{g^2}\int_{P}^{\hat{P}} g_v\,dp\int_{Q/g}^{1}\frac{f(s)}{s}ds\right)(1-\phi_v).$$

Equation (2.17) shows that if there are no fixed costs and the marginal cost of capacity is constant [$C_Q Q = C(Q)$], then the optimal number of firms (flights) n is finite and is equal to:

$$(2.18)\quad n = \frac{\int_{P}^{\hat{P}} g_{k_{t+1}}\,dp + g_{k_{t+1}}\frac{Q}{g^2}\int_{P}^{\hat{P}} g_v\,dp\int_{Q/g}^{1}\frac{f(s)}{s}ds}{\int_{P}^{\hat{P}} g(p)\,dp(1-\phi_v)}$$

This result then shows that uncertain demand is a source of increasing returns to scale when capacity has to be supplied before demand is realized.

Proposition 3: The optimal number of firms (or flights) is

finite, even with constant marginal (and average) cost of capacity and no fixed costs.

In the next subsection we will analyze the effects of (minimum) price regulation and their dependence on demand characteristics.

3. A Diagrammatical Exposition of the Effect of Price Regulation

In this section we will analyze the effect of regulating prices in three stages. The first two are partial analyses when we allow the firm to decide only on one variable (either size of aircraft or frequency). This analysis gives straightforward results. The third stage is a general one in which firms decide both on frequency and capacity. In this stage sufficient conditions are given for the partial results to hold in the general framework.[1]

a. Effect of Price Regulation with Given Frequency

Let us first assume that frequency is given. Therefore, the only two decision variables are price (P) and aircraft size (Q). The two first order conditions are:

$$-(B_P - \overline{TC}_P)/v_P + B_V - \overline{TC}_V = 0 \qquad (2.12)$$

$$B_Q/v_Q - \overline{TC}_Q/v_Q + B_V - \overline{TC}_V = 0. \qquad (2.11)$$

Equations (2.11) and (2.12) can be represented in a (P,Q) axis, and we get two schedules that can be called "zero

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[1] See Appendix B for analytical proofs of the results of this subsection.

marginal consumer surplus" schedules. From the second order conditions it can be seen that around the equilibrium point the slope of $ZMCS_Q$ (the schedule derived from equation (2.11)) is steeper than the slope of $ZMCS_P$. Rearranging (2.12) and

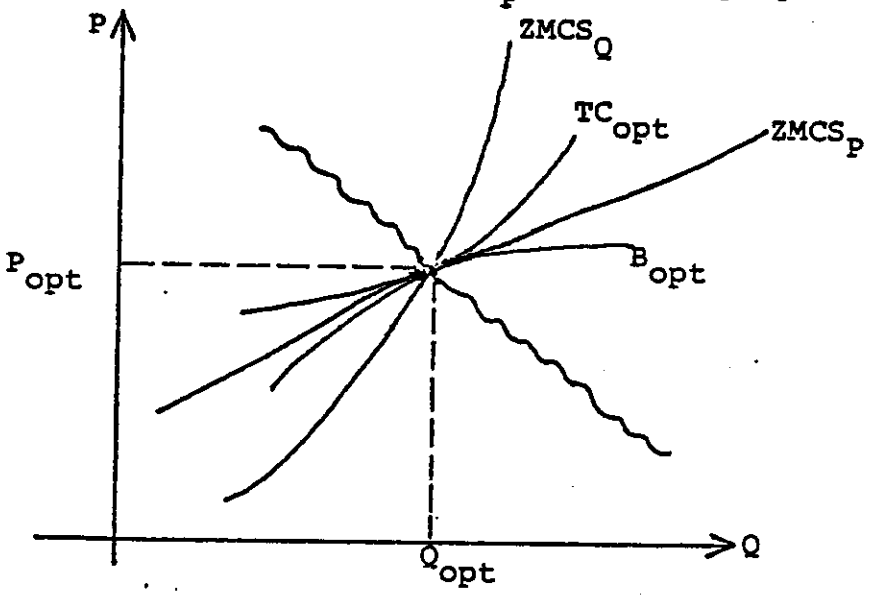

Figure 2

(2.11) and dividing one by the other we get:

$$(2.19)\qquad \left.\frac{dQ}{dP}\right|_{\overline{B}} = -\frac{B_P + B_v v_P}{B_v v_Q + B_Q} = -\frac{\overline{TC}_v v_P + \overline{TC}_P}{\overline{TC}_v v_Q + \overline{TC}_Q} = \left.\frac{dQ}{dP}\right|_{\overline{TC}}$$

That is, the equilibrium point is a tangency point between an iso-expected cost schedule (TC) and an iso-expected consumer benefit one (B). There are many tangency points in the (P,Q) axis, but only the one that coincides with the intersection of the zero marginal consumer benefit schedules is the optimum one. In fact, the schedule of all the tangencies

can be shown to be represented by the equilibrium price schedule:

$$(2.14) \qquad (P - c)\frac{\hat{N}}{Q} = \overline{TC}_Q$$

The right hand side of (2.11) and (2.12) can be expressed by:

$$(2.20) \qquad B_v - \overline{TC}_v = \frac{\hat{N}}{g}\int_P^{\hat{P}} g_v(p)\,dp + \frac{g_v}{g}\left[\int_0^{Q/g} gsf(s)\,ds\,(P - c) - \frac{Q}{g}\int_{Q/g}^{1} f(s)\,ds \int_P^{\hat{P}} g(p)\,dp\right]$$

which in equilibrium has to be positive.

If g_v is independent of P then $B_v - \overline{TC}_v$ is proportional to the semi-elasticity of the demand with respect to seat availability $(g_v)/g$. An increase in g_v (holding g constant) shifts both ZMCS schedules to the northeast, bringing therefore a new equilibrium with higher price and bigger aircrafts. This is represented in Figure 3. (See Appendix B.1).

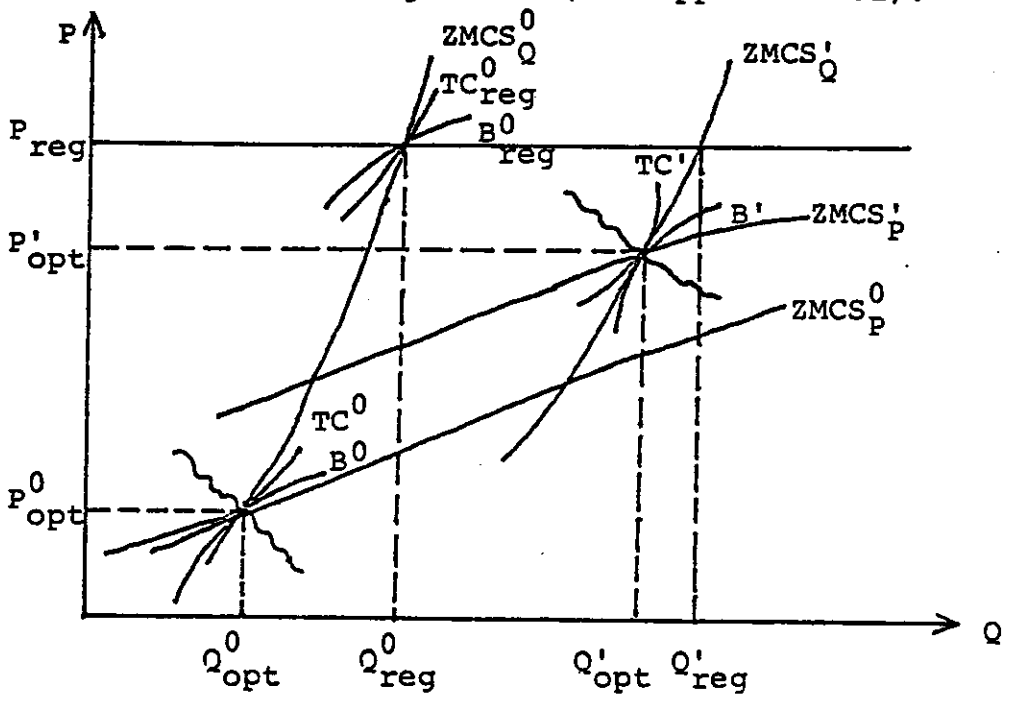

Figure 3

We can now analyze the effect of regulating the price of flights. When prices are regulated there is only one decision variable, aircraft size. Therefore, the market will move along the $ZMCS_Q$ schedule. We do not get any tangency condition and the equilibrium level of seats is determined by the intersection of the regulated price and the relevant zero marginal consumer benefit schedule ($ZMCS_Q$).

From Figure 3 we see that markets with low availability elasticity of demand are more affected by price regulation (when the regulated price is above the optimal one) than those markets with high availability elasticity of demand.

Regulation therefore, will imply more seats per aircraft--i.e., more capacity--and therefore a level of excess capacity or availability that is larger than the optimal one.

b. The Effect of Minimum Price Regulation When Aircraft Size is Exogenous

When aircraft size (or capacity per location) is given, the two decision variables are price and number of flights. The first order conditions are therefore:

$$(2.12) \qquad B_P - \overline{TC}_P + (B_v - \overline{TC}_v)v_P = 0$$

$$(2.13) \qquad B(.) - \overline{TC}(.) + n(B_n - \overline{TC}_n + (B_v - \overline{TC}_v)v_n) = 0$$

Both equations can be represented in a (P,n) plane and the optimal price-frequency combination is the crossing point of both schedules. This is shown in Figure 4.

Equation (2.12) is the set of price and frequency combinations for which the change in expected consumer surplus from a change in price is zero. We call this schedule the Zero Marginal Consumer Surplus with respect to price ($ZMCS_p$). In the same way we can call equation (2.13) the $ZMCS_n$ schedule, since (2.13) is the set of price and frequency combinations for which the expected consumer surplus does not change with small changes in frequency. From the second order conditions we get that in Figure 4, $ZMCS_n$ has to be steeper than $ZMCS_p$.

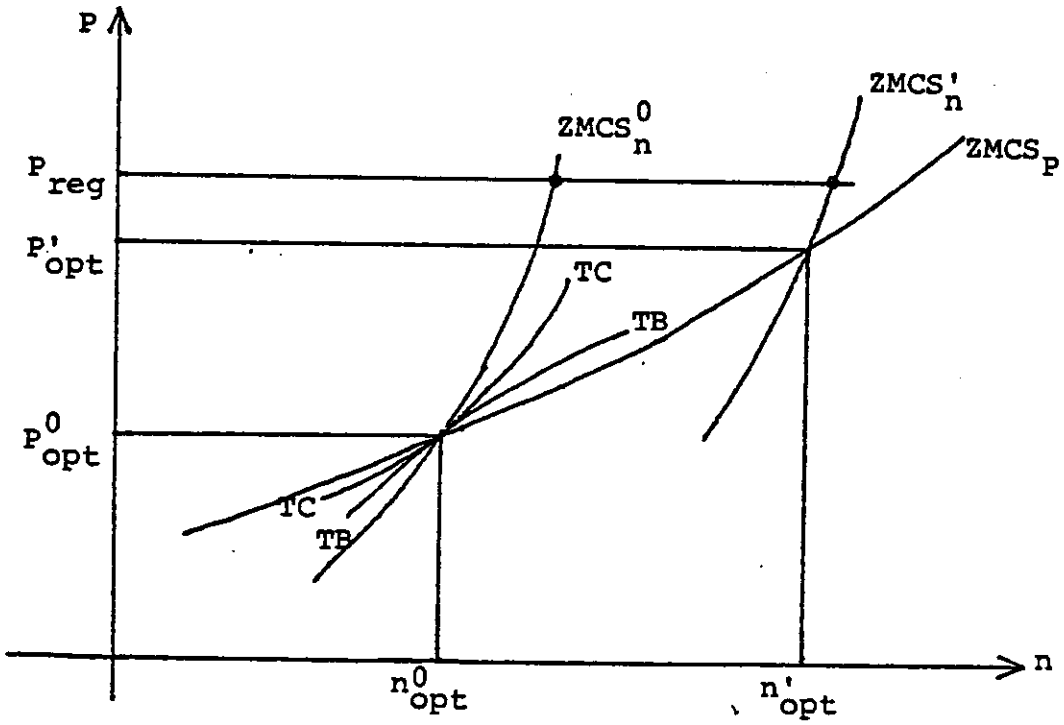

Figure 4

The optimal solution (P_{opt},n_{opt}) is also a tangency point between an iso-expected cost schedule (TC) and an iso-expected total gross benefit (TB) one. This can be seen by rearranging (2.12) and (2.13) and dividing one by the other.

Equation (2.13)--the $ZMCS_n$ schedule--depends on the

frequency elasticity of demand. For a larger frequency elasticity the $ZMCS_n$ shifts to the east (for the same price more frequency flights are required) crossing the $ZMCS_P$ schedule at a higher price and larger number of flights (P'_{opt}, n'_{opt}).

On the other hand, a change in the availability elasticity of demand shifts both schedules. The $ZMCS_n$ (equation (2.13)) shifts to the east from an increase in the availability elasticity and the $ZMCS_p$ (equation (2.12)) shifts to the north if availability and frequency are substitutes (i.e, if $g_{vn} < 0$). This result is presented in Figure 5 with (n'_{opt}, P'_{opt}) being the new optimal point.

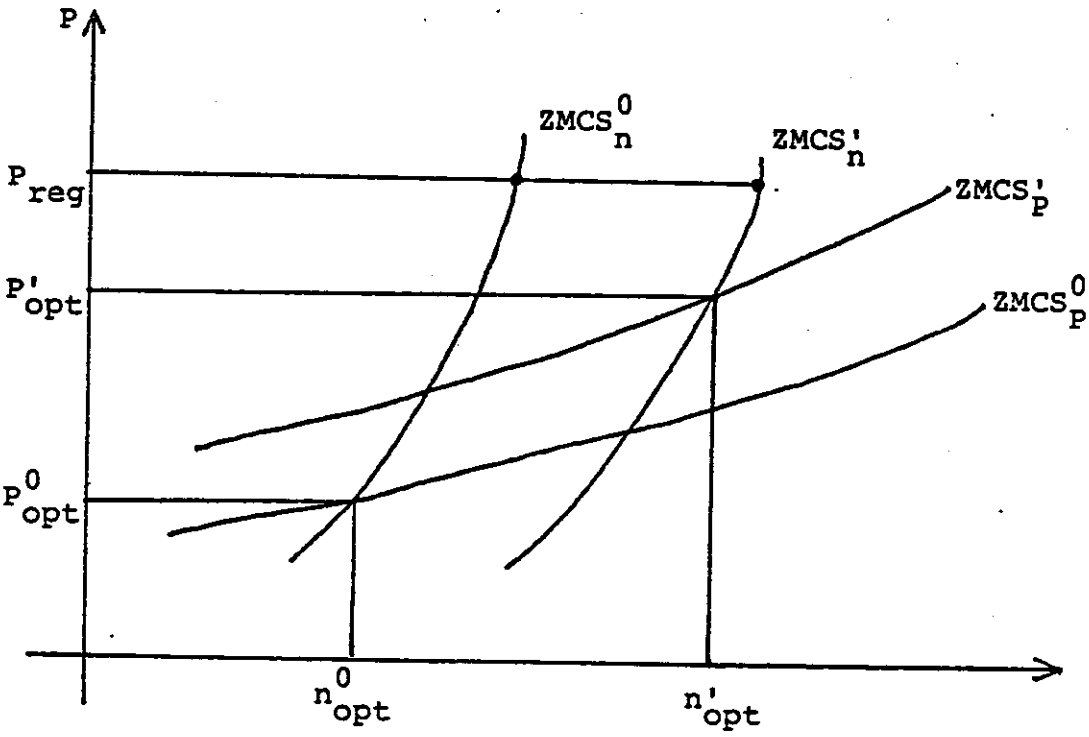

Figure 5

If we now impose a minimum price regulation the only decision variable is the number of flights. The "second best" solution will be the number of flights at which the $ZMCS_n$ schedule crosses the regulated price. As can be seen from Figures 4 and 5 the smaller the availability and frequency elasticities the larger the difference between the optimal and the actual frequency (See Appendix B.2 for proof).

c. Effect of Minimum Price Regulation with Frequency and Capacity Being Endogenous

The results of the two previous subsections are partial results and in this section we will try to generalize them when both frequency and capacity are endogenous.

The relevant first order conditions are:

$$(2.11) \qquad B_Q - \overline{TC}_Q + (B_v - \overline{TC}_v)v_Q = 0$$

$$(2.13) \qquad B(.) - \overline{TC}(.) + n(B_n - \overline{TC}_n + (B_v - \overline{TC}_v)v_n) = 0$$

Equations (2.11) and (2.13) are the Zero Marginal Consumer Surplus equations with respect to capacity and flight frequency and were used in the previous subsections. Since price is regulated we can plot those equations in an (n,Q) plane. This is done in Figure 6.

Q_0 and n_0 are the "second best" solutions for capacity and frequency when price is fixed at P_0. An increase in the frequency elasticity will shift $ZMCS_n$ upwards while leaving

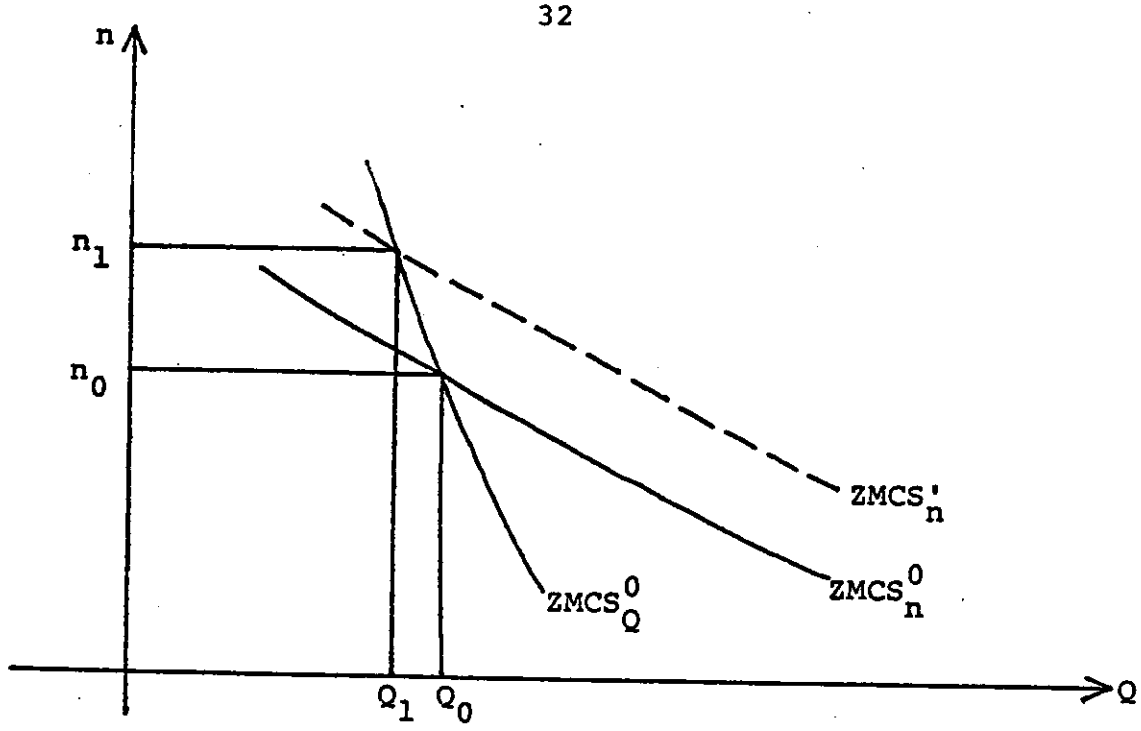

Figure 6

unchanged the $ZMCS_Q$ schedule. It will imply an increase in flight frequency but a reduction in capacity per flight. This result is expected since an increase in frequency brings an increase in availability that was not required by the increase in the frequency elasticity.

On the other hand, an increase in the availability elasticity will shift both schedules to the right implying that capacity or frequency may decrease if the other increases sufficiently. It may also imply that both capacity and frequency are increased. It cannot be that both are reduced. This result is shown in Figure 7. We can conclude that: (See Appendix B.3 for an analytical proof).

Proposition 4: With minimum regulation markets with larger availability elasticity of demand will provide more flights

or more capacity per flight (or both) than markets with lower availability elasticity. On the other hand, markets with larger frequency elasticity of demand will provide more flights and lower capacity per flight.

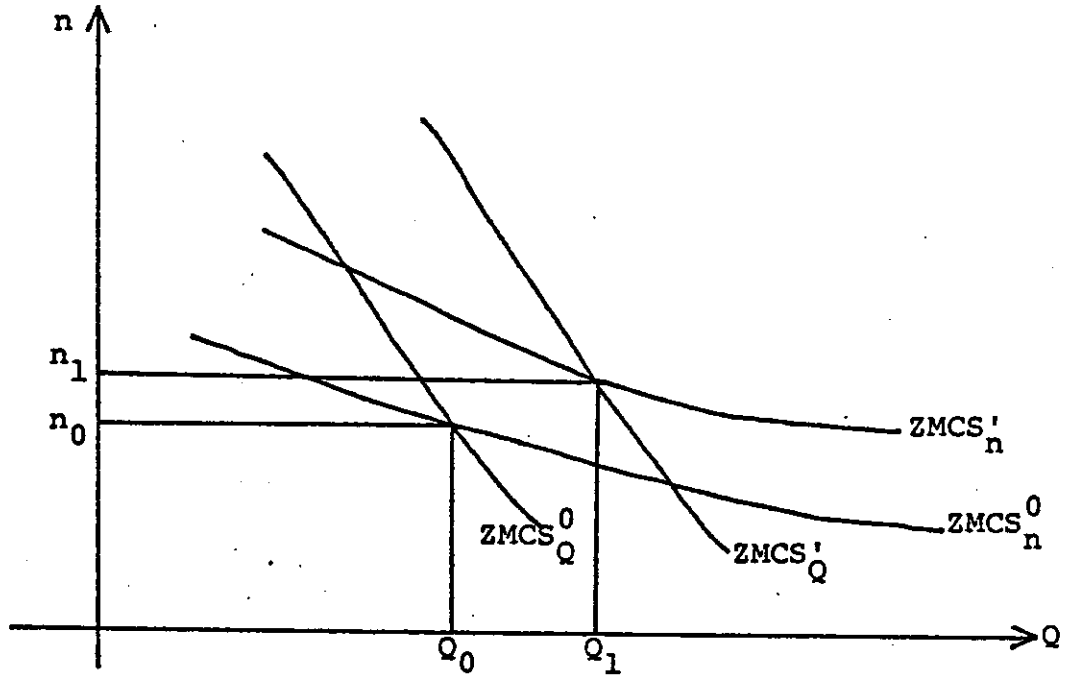

Figure 7

Figure 7 is done for a given price. We can assume that (Q_0, n_0) and (Q_1, n_1) represent the respective equilibrium points in two different markets. Let us further assume that the regulated price is exactly the optimal price for the market with the larger availability elasticity of demand. (Q_1, n_1) will then be the optimal combination of capacity and frequency for that market. Deregulation will have no effect on this market but will shift $ZMCS^0_n$ and $ZMCS^0_Q$ to the left implying a reduction in either frequency or capacity (or both) for the market with the lower availability

elasticity of demand.

In the same way, price deregulation should have a larger effect on markets with lower frequency elasticity of demand.[1]

Proposition 5: Price deregulation will imply a larger reduction in frequency and/or in capacity per flight (and therefore in price) the lower the availability and the frequency elasticity of demand.

This proposition will be used in the next chapter when analyzing the differential effect on individual firms of several regulatory price changes.

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[1]See Appendix A.

III. AN ANALYSIS OF THE DEMAND FOR AIRLINE SERVICES

1. Introduction

In Section II it is shown how demand characteristics like elasticities with respect to availability and frequency are relevant in assessing the effect of deregulation.

Previous works on the demand for air travel, either did not take those factors into account (see Verleger [1972] for an extensive analysis of demand using city-pair data) or did not differentiate between the two factors (like De Vany [1975] or Olson and Trapani [1979]).

This Section presents an estimation of demand functions for airlines' coach services that takes both availability and frequency into account.

2. The Data[1]

The data used in this Appendix is yearly data for individual airlines for the period 1959/1975. That is, for each airline an observation is a sum over all its routes for the whole year.

For the price variable we used average revenue per coach passenger per mile. This measure would be a good proxy for

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[1]For sources and description of the data see Appendix C.

price if all the customers had paid the same price. Since different customers paid different prices, this measure may bias downwards the estimate of its coefficient. (See Verleger [1972] for a discussion of average revenue per mile as a proxy for price). In the demand equation average revenue per passenger mile is divided by the cost of living index. In the supply equations it is divided by the wholesale price index; this is the reason for two different price variables $PRICE^1$ and $PRICE^2$, the former in the demand and the latter in the supply equation.

The cost variable used in the supply equations is the average operating cost per flight divided by the wholesale price index.

For the income variable in the demand equation, we used either the industrial production index or the real GNP. Real GNP turned out to be more useful in the regressions of EAL, TWA, UAL and WAL, while the industrial production index was more useful in the other equations. This result may be related to the national characteristic of the big four trunk airlines (AMR, EAL, TWA and UAL). The demand for these airlines' services may depend on the aggregate movement of the economy and not on industrial output alone.

Load factor in this model is a measure of availability, for a given level of flight frequency. An increase in load factors (holding constant flight frequency) would reduce availability. On the other hand, an increase in frequency

(for a given level of availability) would reduce the discrepancy between actual and desired flight time. Since aircraft size is a variable in the model, availability cannot be identified by departures (or frequency) alone. That would be the case if aircraft size was a constant. Some works have used total available seats (that is, departures times aircraft size) as the relevant variable for availability (see Olson and Trapani [1979]), but availability may change even if total available seats remains constant (for example if we double the number of flights but reduce to half the size of aircraft, total available seats will remain constant but availability will go down).

For the distance variable we used the average trip length of passengers.

For a measure of frequency of service we used total number of departures.

3. The Model

Each of the demand equations that we are interested in estimating belongs to a simultaneous equations model. Demand depends on availability and on frequency. Frequency is a firm's decision variable, while availability is determined by the decisions of the firms on frequency and capacity per flight, and on the distribution of the demand. The model is formed by a demand equation, an equation describing the firm's decision on flight frequency, another equation describing the decision on size of aircraft and an identity defining availability.

Load factor in coach is used as a proxy for availability.

We postulate the following model[1] for each airline:

- The Demand Equation:

$$(3.1)\quad PASSENGERS_t = a_1 + b_1 DEPARTURES_t + c_1 LOAD\ FACTOR_t + d_1 PRICE^1_t + e_1 TIME + f_1 DISTANCE_t + g_1 INCOME_t + error\ term_t.$$

- The Supply of Frequency Equation:

$$(3.2)\quad DEPARTURES_t = a_2 + b_2 PASSENGERS_t + c_2 LOAD\ FACTOR_t + d_2 PRICE^2_t + e_2 COST_t + f_2 TIME + error\ term_t.$$

- The Aircraft Size Equation:

$$(3.3)\quad AIRCRAFT\ SIZE_t = a_3 + b_3 AIRCRAFT\ SIZE_{t-1} + c_3 EXPECTED\ FUTURE\ LOAD\ FACTOR_t + d_3 PRICE^2_t + e_3 COST_t + f_3 TIME + g_3 DISTANCE_t + error\ term_t.$$

- Definition of Load Factor:

$$(3.4)\quad LOAD\ FACTOR_t = PASSENGERS_t - DEPARTURES_t - AVAILABLE\ SEATS_t.$$

The demand equation postulates that the annual number of passengers is a function of actual values of price, frequency, availability (load factor), distance, trend and income (different

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[1] All the variables--except for the time trend--are in logarithms.

lags of income were used to capture persistent income effects of more than one year). We would expect the coefficients to have the following signs:

$$b_1 > 0, \quad c_1 < 0,^{1} \quad d_1 < 0, \quad f_1 > 0, \quad g_1 > 0, \quad e_1 \gtrless 0.$$

The supply of frequency suggests that an increase in passengers implies an increase in departures supplied, load factor and price should also increase the supply of departures; an increase in cost per flight should reduce frequency.

Finally, the supply of aircraft size depends on previous average aircraft size, since supposedly airlines have to incur significant adjustment costs if they desire to change drastically their fleet size. Expected future load factor is included since decisions on aircraft pruchases are not--in general--for a one year term.[2] Distance is included in this equation since different aircrafts are suited for different distance ranges.

Since this is a regulated industry we did not include an equation for price. It is assumed that price is exogenously determined by the CAB.[3]

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[1] c_1 has to be negative, since load factor is negatively related to availability, and an increase in availability should increase demand.

[2] Since we do not estimate equations (3.2) and (3.3), but use them to obtain the instrumental variables to estimate equation (3.1), we did not attempt to formulate a precise way to form the expectations on future load factors. Sufficient for our purposes that it be based on previous values of the exogenous variables of this model.

[3] In future work I intend to perform exogeneity tests using those developed by Wu [1973] and [1974].

4. The Estimation

The above model can be estimated either using simple equation or full information methods. Since we are mainly interested in the demand equation, it was estimated using two stages least squares.[1]

The endogenous variables in the estimation are DEPARTURES and LOAD FACTOR. The instrumental variables used were those exogenous variables not appearing in equation (3.1), that is: the cost variable, the price variable (divided by the wholesale price index), and lagged variables (lagged aircraft size, lagged departures, lagged cost). The exogenous variables are the price variable divided by the consumer price index, the time trend, distance and income (including lagged terms).

The basic functional form was linear in the natural logarithm. We also tested for non-constant elasticities for departures and load factors--we used a regression linear in their logs and in the squared values of their logs. For four firms these specifications significantly reduced the serial correlation of the disturbance term. The time trend was introduced linearly or in its log forms.

The results of the two stages estimation are in Table 2. The results of Table 2 show us an industry where availability is a relevant factor. In almost all the equations the point

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[1]For single equations estimation, two stages least squares is asymptotically efficient, but in small samples the results from two stages may differ from the ordinary least squares and we may not have a clear justification for using one or the other.

TABLE 1

GLOSSARY OF TERMS USED IN THE DEMAND STUDY[a]

C	- Constant
LFY	- Load Factor in Coach
LFY^2	- Load Factor in Coach squared
DEP	- Departures
DEP^2	- Departures squared
PRICE	- Average Revenue per Passenger Mile divided by the cost of living index (1970=100)
DIST	- Distance: average stage length of passengers' flights
INDP	- Industrial Production Index (1970=100)
INDP1	- INDP lagged one year
RGNP	- Real GNP
RGP1	- RGNP lagged one year
RGP2	- RGNP lagged two years
ID	- Trend
LID	- Log of ID

[a]See Appendix C for a description of the variables and the sources used.

TABLE 2

DEMAND FOR AIRLINES' PASSENGER SERVICES (COACH)--TWO STAGES LEAST SQUARES ESTIMATION (1959/1975)[a]

	AMR	EAL	TWA	UAL	BNF
C	-199.64	86.78	28.66	-3.96	-251.53
	(-2.21)	(1.47)	(.46)	(-.04)	(-1.86)
LFY	-.77	.00	-.45	-.24	-3.45
	(-.76)	(.00)	(-1.46)	(-.52)	(-2.29)
LFY^2	-----	-----	-----	-----	-----
	-----	-----	-----	-----	-----
DEP	.30	1.42	.85	-17.06	.25
	(.46)	(3.18)	(1.88)	(-.95)	(.28)
DEP^2	-----	-----	-----	.69	-----
	-----	-----	-----	(1.00)	-----
PRICE	.89	2.45	.76	.48	2.98
	(.60)	(2.37)	(.81)	(.88)	(1.36)
DIST	-3.61	-.35	-1.83	.25	-1.27
	(-2.07)	(-.55)	(-3.03)	(.59)	(-.67)
INDP	1.65	-----	-----	-----	2.07
	(3.64)	-----	-----	-----	(1.72)
INDP1	-----	-----	-----	-----	-----
	-----	-----	-----	-----	-----
RGNP	-----	1.20	1.58	.44	-----
	-----	(1.66)	(2.58)	(.76)	-----
RGP1	-----	1.17	.16	.82	-----
	-----	(1.70)	(.29)	(1.52)	-----
RGP2	-----	2.08	1.36	-----	-----
	-----	(3.30)	(2.52)	-----	-----
ID	.11	-.04	.00	.05	.13
	(2.20)	(-1.40)	(-.29)	(2.05)	(1.87)
LID	-----	-----	-----	-----	-----
	-----	-----	-----	-----	-----
R^2	.98	.99	.99	.99	.96
DW	1.84	2.30	1.95	1.86	2.32

[a]The left hand side variable in this equation is the log of coach passengers, the right hand side endogenous variables are load factor and departures (LFY, LFY^2, DEP, DEP^2). t-statistics are in parentheses. See Table 1 for definition of variable.

TABLE 2 (CONT.)

CAL	DAL	NAL	NWL	WAL
-150.78 (-.99)	-174.56 (-4.16)	-99.79 (-4.29)	-1164.27 (-2.24)	-3220.13 (-3.25)
-1.67 (-1.69)	-.33 (-.94)	.28 (2.18)	.11 (.30)	-13.77 (-1.48)
----- -----	----- -----	----- -----	----- -----	-10.73 (-1.34)
.96 (.03)	.04 (.12)	1.06 (9.60)	1.07 (4.13)	.63 (.80)
-.03 (-.02)	----- -----	----- -----	----- -----	----- -----
-.17 (-.12)	-1.19 (-2.27)	1.02 (3.80)	.51 (.69)	.66 (.42)
1.92 (1.22)	2.17 (4.60)	2.20 (6.26)	-.47 (-.75)	-2.60 (-3.49)
1.07 (1.44)	1.49 (4.27)	.77 (2.04)	.25 (.40)	----- -----
----- -----	-.16 (-.34)	-.11 (-.36)	.33 (.48)	----- -----
----- -----	----- -----	----- -----	----- -----	-.06 (-.04)
----- -----	----- -----	----- -----	----- -----	----- -----
----- -----	----- -----	----- -----	----- -----	----- -----
.06 (1.63)	.07 (3.54)	.04 (3.28)	----- -----	----- -----
----- -----	----- -----	----- -----	153.43 (2.22)	426.99 (3.26)
.98	.99	.99	.99	.98
1.73	2.41	2.27	1.42	2.30

estimate of the coefficient of load factor is negative, but its statistical significance is relatively weak. The coefficient of departures is positive in almost all cases and is statistically significant (at normal levels) in three equations.[1] The price variable which is the average revenue per passenger mile turned out to be insignificant (and on the wrong sign for two significant cases).

The frequency elasticity estimates found in this analysis are lower than those found in previous works such as De Vany [1975] or Olson and Trapani [1979]. The difference in the estimates is derived from performing different underlying experiments when estimating the demand function.

De Vany's [1975] experiment is as follows. Increase frequency while not changing aircraft size, therefore increasing total available seats proportionally. The increase in frequency will imply an increase in demand, load factors would go down further stimulating the demand. The total effect of increasing frequency is therefore to increase availability as well and the estimate of the frequency elasticity does include a term related to the availability elasticity of demand. In our case since we are holding availability constant, we are performing the following experiment. Increase frequency and

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[1]Frequency of service has a positive coefficient in almost all the equations, but for those with t-statistics larger than one, the point estimate is very close to one. It can be shown (Spiller [1980c], Appendix B) that in a spatial framework a monopoly would never operate at a point on the demand with unity frequency elasticity since at that point his profits would be zero. If these firms (EAL, NAL, NWL) had a significant market power then we would be facing a "second best" regulatory price.

decrease aircraft size such that the increase in demand induced by the increase in frequency will not imply a change in availability. The above discussion shows that our estimates should be lower than De Vany's [1975] or Olson and Trapani's [1979].

This demand analysis shows large differences among firms in their availability elasticities. Braniff, Continental, American and Western have availabilities elasticities larger than .5 while the others have lower elasticities. The elasticities are presented in Table 3. These two groups will be affected very differently by price deregulation. Those firms with large availability elasticities would not be affected as much as those with smaller elasticities.

TABLE 3

RANKING OF AIRLINES ACCORDING TO THEIR POINT ESTIMATES OF THE AVAILABILITY ELASTICITIES OF DEMAND[a]

Ranking	Firm	Point Estimate
1	BNF	-3.45
2	CAL	-1.67
3	WAL	-1.32[b]
4	AMR	-.77
5	TWA	-.45
6	DAL	-.33
7	UAL	-.24
8	EAL	.00
9	NWL	.11[b]
10	NAL	.28

[a]See Table 29 for the demand equations.

[b]Elasticities calculated at mean values.

IV. SUMMARY

We developed a theoretical framework to analyze industries characterized by uncertain availability and spatial location of the product. This framework was applied to the airline industry and used to analyze the differential impact of price regulation on different markets and firms. It was shown that the effect depends on demand characteristics such as demand elasticities with respect to availability and frequency.

In the empirical part a demand analysis for each airline was performed and used to assess the willingness to pay for availability. It was found that firms differ in their customers' willingness to pay for availability. These differences were found elsewhere to be highly correlated to the differential impact deregulation had on relative risk and profitability of the domestic trunks.

This work fits into the Economic Theory of Regulation as developed by Stigler [1971] and Peltzman [1976] when trying to explain the deregulation of the airline industry. It suggests that the simple capture theory of regulation may not be valid in the airline case, since regulation may have had a different impact across firms. This result has relevance for regulation studies in general. When analyzing the effect of a regulatory decision on an industry, it is necessary to analyze whether we expect a differential impact. If it is expected, pooling firms not according to the predicted effect may significantly bias the estimated effect of the regulatory change.

The main thrust of this work is that different firms were affected differently by the price regulations of the CAB. We pointed to the theoretical and empirical relevance of willingness to pay for quality (availability) as a discriminating factor in analyzing the differential impact of price regulation. Little is known about the sources of the differences across markets in their valuation of availability.

Several factors may be of relevance. First, different routes may differ in the type of passenger they serve. Business versus pleasure may be a relevant distinction. We will expect, for example, Florida routes to be routes with--relatively--low willingness to pay for availability.[1]

Second, passengers' value of time may also affect the willingness to pay for availability. Since the firms' networks are in different regional areas (with some degree of overlapping), differences in average values of time may have an effect on the differences in valuation of quality across markets.

Third, the degree of competition may also be an important factor.

Fourth, the type of network may affect the willingness to pay for availability. Compare a non-stop flight with a multi-stop one. The probaiblity of obtaining a seat for the whole flight in a multi-stop flight is the product of the probabilities in the different segments of the flight. If

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[1]It is interesting to note that Florida routes were among those to have significant entry and price reductions (Keeler [1980]).

all the different segments have the same probability--and the same load factor--a non-stop flight with the same probability--and load factor--as each one of the segments of the multi-stop flight will have a higher probability than the multi-stop flight. Customers of multi-stop flights will require for a given price a lower load factor (to keep the same utility level constant). Moreover, the difference in the required load factor increases with the level of load factor (since for low load factors, the probability is close to one). This implies that customers of firms with mostly multi-stop flights will show a larger willingness to pay for load factor than those using airlines with mostly non-stop flights.[1]

This list of potentially relevant factors is not intended to be exhaustive. An exhaustive analysis of the determinants of the differences in the willingness to pay for availability is the topic of a future research project.

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[1]This seems to be a relevant factor. The ratio Average Stage Length (ASL) to Overall Stage Length (OSL) is (approximately) the average number of stops each firms' flight have (ASL is the average length of customers' flights, OSL is the average length of airplanes' flights). The average ratio for the 1959/78 period ranges from 1.41 (TWA) to 1.74 (CAL) and 1.88 (NAL). The rank correlation between the ranking of firms according to the effect of availability on the probability of selecting first class over coach--our measure of relevance of availability on demand--and the ranking according to number of stops is .33 (and it is .7 if we exclude NAL, the firm with the largest number of stops).

APPENDIX A

THE EFFECT OF THE AVAILABILITY AND FREQUENCY ELASTICITIES OF DEMAND ON OPTIMAL PRICE, FREQUENCY AND CAPACITY

The first order conditions are:

$$H_P = B_P - \overline{TC}_P + (B_v - \overline{TC}_v)v_P = 0, \tag{A.1}$$

$$H_Q = B_Q - \overline{TC}_Q + (B_v - \overline{TC}_v)v_Q = 0, \tag{A.2}$$

$$TH_n = B(.) - \overline{TC}(.) - (B_{k_{t+1}} - \overline{TC}_{k_{t+1}} + (B_v - \overline{TC}_v)v_{k_{t+1}})/n = 0. \tag{A.3}$$

If 'a' is a shift variable, then the comparative statistics are obtained in the following way:

$$\begin{pmatrix} dQ/da \\ dP/da \\ dn/da \end{pmatrix} = \frac{1}{D}\begin{bmatrix} H_{PP}TH_{nn}-H_{Pn}TH_{nP} & TH_{nQ}H_{Pn}-H_{QP}H_{nn} & H_{PQ}TH_{nP}-H_{PP}TH_{nQ} \\ H_{Qn}H_{nP}-H_{QP}TH_{nn} & H_{QQ}TH_{nn}-H_{Qn}TH_{nQ} & H_{QP}TH_{nQ}-H_{QQ}TH_{nP} \\ H_{PQ}H_{Pn}-H_{PP}H_{Qn} & H_{QP}H_{Qn}-H_{QQ}H_{Pn} & H_{QQ}H_{PP}-H_{QP}H_{PQ} \end{bmatrix} \times \begin{pmatrix} -H_{Qa} \\ -H_{Pa} \\ -H_{na} \end{pmatrix} . \tag{A.4}$$

where D is the determinant of the Hessian matrix ($D < 0$).

If 'a' is the change in the demand for each individual

flight from an increase in the number of flights (i.e., 'a' is g_n),[1] then (the approximations assume g_v and g_p constant)

$$H_{Qg_n} = H_{Pg_n} = 0$$

and

$$-TH_{ng_n} \cong 2n\left((P-c)\int_0^{Q/g} sf(s)\,ds - \frac{Q}{g^2}\int_{Q/g}^{1} f(s)\,ds\int_{\hat{P}}^{P} g(p)\,dp\right).$$

If $-TH_{ng_n} > 0$, then $dn/dg_n < 0$ and the sign of dQ/dg_n and dP/dg_n depend on the signs of $H_{PQ}TH_{nP}-H_{PP}TH_{nQ}$ and $H_{QP}TH_{nQ}-H_{QQ}TH_{nP}$ respectively. This is a plausible assumption, since it means that an increase in the absolute value of g_n (a reduction in g_n) implies an increase in number of flights if the first order condition (A 3) is to hold. Since TH_{nQ} (and H_{Qn}) is negative both signs are indeterminate. On the other hand, the second order conditions constrain the signs of both terms. A sufficient condition for the second order conditions to hold is that the first term be negative and the second positive (the opposite would violate the second order conditions).[2] If that is the case, then

$$dQ/dg_n > 0 \quad \text{and}$$

$$dP/dg_n < 0.$$

---

[1] g_n is negative, therefore an increase in g_n means a reduction in its absolute value.

[2] The implications of these signs in terms of characteristics of demand functions will be explored in future work.

That is, an increase in the absolute value of g_n (i.e., and increase in the frequency elasticity of demand) would increase frequency, decrease capacity per flight and increase price.

If $H_{PQ}TH_{nP}-H_{PP}TH_{nQ}$ is negative, then in order for the second conditions to hold it is necessary that $H_{Qn}TH_{nP}-H_{QP}TH_{nn}$ be positive. In this case all the terms of the matrix in the right hand side of (A.4) have a definite sign. In particular, all the terms of the second row are of positive sign.

If the shift variable 'a' is g_v (the increase in demand from an increase in availability), then

$$-H_{Qa} = -H_{Qg_v} \cong -\frac{\hat{N}}{g^2}\int_{Q/g}^{1} f(s)\,sds\,(P - c) < 0$$

$$-H_{Pa} = -H_{Pg_v} \cong -\frac{\hat{N}}{g}(\hat{P} - P) < 0 \qquad \text{(A.5)}$$

$$-TH_{na} = -TH_{ng_v} \cong -(B - \overline{TC})\int_{Q/g}^{1} f(s)/sds\frac{Q}{g^2} < 0.$$

From (.4) and (.5) we obtain that if we assume the sufficient conditions just described to hold, the only straightforward answer is that optimal prices will rise.

APPENDIX B

THE EFFECT OF MINIMUM PRICE REGULATION

1. Effect of Price Regulation with Given Frequency

The first order conditions[1] are:

$$H_P = B_P - \overline{TC}_P + (B_V - \overline{TC}_V)v_P = 0 \tag{B.1}$$

$$H_Q = B_Q - \overline{TC}_Q + (B_V - \overline{TC}_V)v_Q = 0. \tag{B.2}$$

If we define 'a' as a shift variable, and 'da' as the change in a, the comparative statistics can be performed in the following way:

$$\begin{pmatrix} dP/da \\ dQ/da \end{pmatrix} = \frac{1}{W} \begin{bmatrix} H_{QQ} & -H_{QP} \\ -H_{QP} & H_{PP} \end{bmatrix} \begin{pmatrix} -H_{Pa} \\ -H_{Qa} \end{pmatrix}$$

where $W = (H_{PP}H_{QQ} - H_{QP}H_{PQ}) > 0$.

If we let 'a' be g_v (that is, the derivative of the expected demand with respect to availability), then:[2]

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[1] When price is regulated equation (B.1) does not hold anymore.

[2] The approximations were performed by assuming g_v and g_p constant.

$$-H_{Pa} = -H_{Pg_v} \cong -\frac{\hat{N}}{g^2}\int_{Q/g}^{1} f(s)/s\,ds(P - c) < 0$$

$$-H_{Qa} = -H_{Qg_v} \cong -\frac{\hat{N}}{g}(\hat{P} - P) < 0;$$

therefore,

$$\begin{pmatrix} dP/da \\ dQ/da \end{pmatrix} = \begin{pmatrix} dP/dg_v \\ dQ/dg_v \end{pmatrix} > 0.$$

2. Effect of Price Regulation when Aircraft Size is Exogenous

The first order conditions[1] in this case are:

(B.1) $$H_P = 0$$

(B.3) $$TH_n = 0 = B(.) - \overline{TC}(.) + n(B_n - \overline{TC}_n + (B_v - TC_v)v_n).$$

Defining 'a' as the shift variable, the comparative statistics are:

$$\begin{pmatrix} dP/da \\ dn/da \end{pmatrix} = \frac{1}{W_1}\begin{bmatrix} TH_{nn} & -TH_{nP} \\ -H_{Pn} & H_{PP} \end{bmatrix}\begin{pmatrix} -H_{Pa} \\ -TH_{na} \end{pmatrix}$$

where $W_1 = H_{PP}TH_{nn} - TH_{nP}H_{Pn} > 0$.

If we let 'a' be g_n (that is the change in each flight's demand as a consequence of an increase in the total number of flights $-g_n < 0$-), then,

---

[1] When price is regulated equation (B.1) does not hold anymore.

$$H_{Pg_n} = 0$$

$$-TH_{ng_n} \cong 2n\left((P - c)\int_0^{Q/g} sf(s)ds - \frac{Q}{g^2}\int_{Q/g}^{1} f(s)ds\int_P^{\hat{P}} g(p)dp\right).$$

If $-TH_{ng_n} > 0$, then we have that

$$\begin{pmatrix} dP/dg_n \\ dn/dg_n \end{pmatrix} < 0.$$

If 'a' is g_v, then from Section 1, $-H_{Pg_v} < 0$, and we have that:

$$-H_{ng_v} = -(B - TC)\int_{Q/g}^{1} f(s)/sds\frac{Q}{g^2} < 0$$

so that

$$\begin{pmatrix} dP/dg_v \\ dn/dg_v \end{pmatrix} > 0.$$

3. Effect of Minimum Price Regulation with Frequency and Capacity Being Endogenous

If price is regulated the two relevant first order conditions are:

(B.2) $$H_Q = 0$$

(B.3) $$TH_n = 0.$$

If 'a' is the shift variable, then the comparative statistics

can be obtained from:

$$\begin{pmatrix} dQ/da \\ dn/da \end{pmatrix} = \frac{1}{W_2} \begin{bmatrix} TH_{nn} & -TH_{nQ} \\ -H_{Qn} & H_{QQ} \end{bmatrix} \begin{pmatrix} -H_{Qa} \\ -TH_{na} \end{pmatrix}$$

where $W_2 = H_{QQ}TH_{nn} - H_{Qn}TH_{nQ} > 0$.

If 'a' is g_n then from Section 2 we know that $-TH_{ng_n} > 0$, and since $-H_{Qg_n} = 0$ and $H_{nQ} < 0$ we obtain that

$$dQ/dg_n > 0 \quad \text{and}$$

$$dn/dg_n < 0.$$

If 'a' is g_v we have that

$$(B.4) \quad \begin{pmatrix} dQ/dg_v \\ dn/dg_v \end{pmatrix} = \frac{1}{W_2} \begin{pmatrix} [TH_{nn}(-H_{Qg_v}) - TH_{nQ}(-TH_{ng_v})] \\ [-H_{Qn}(-H_{Qg_v}) + H_{QQ}(-TH_{ng_v})] \end{pmatrix}.$$

From expression (B.4) we obtain that dQ/dg_v and dn/dg_v cannot both be negative since that will contradict the second order condition $W_2 > 0$. If $dQ/dg_v < 0$, then

$$(B.5) \qquad TH_{nn}(-H_{Qg_v}) - TH_{nQ}(-TH_{ng_v}) < 0 \quad \text{or}$$

$$(B.5a) \qquad TH_{nn}/TH_{nQ} < (-TH_{ng_v})/(-H_{Qg_v}).$$

If $dn/dg_v < 0$, then

(B.6) $$-H_{Qn}(-H_{Qg_v}) + H_{QQ}(-TH_{ng_v}) < 0 \quad \text{or}$$

(B.6a) $$H_{Qn}/H_{QQ} > (-TH_{ng_v})/(-H_{Qg_v}).$$

From (E.5a) and (E.6a) we obtain

(B.7) $$TH_{nn}H_{QQ} - H_{Qn}TH_{nQ} < 0,$$

therefore contradicting the second order condition $W_2 > 0$.

APPENDIX C

DATA DESCRIPTION AND SOURCES

1. Data Sources

There are two types of data used in Section III: airlines' data (traffic and financial), and economy-wide information (price levels, income). The sources for airlines' data are the CAB reports: "Handbook of Airline Statistics", "Air Carrier Financial Statistics" and "Air Carrier Traffic Statistics" (issues 1958 to 1978). The source for economy-wide variables is the IMF publication: "International Financial Statistics" (various issues, 1958/1978). All data is yearly.

2. Data Description and Glossary

- Load Factor: is the proportion of occupied seats (average). It is obtained by dividing total passenger miles by available seat miles (in each category of service: coach or first class).
- Average Revenue per Passenger Mile: is obtained by dividing Revenue of Passengers (in $000) by Revenue Passenger Miles (in thousand miles). The ratio is in $ value.
- Departures: is the total number of departures performed by the airline during the year.
- Distance: There are two measures of distance. a) average stage length of passengers' flights (in miles): is the average

distance of each passenger's flight; b) average stage length of aircraft (in miles): is the average distance of each aircraft flight. The latter measure is smaller than the former. Average stage length of passengers divided by average stage length of aircraft is the average number of stops each passenger's flight has. In the demand analysis, the measure of distance used is average stage length of passengers' flights.

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