

A Service of

ZBW

Leibniz-Informationszentrum Wirtschaft Leibniz Information Centre for Economics

Direr, Alexis

Working Paper Flexible life annuities

CESifo Working Paper, No. 2125

Provided in Cooperation with: Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

Suggested Citation: Direr, Alexis (2007) : Flexible life annuities, CESifo Working Paper, No. 2125, Center for Economic Studies and ifo Institute (CESifo), Munich

This Version is available at: https://hdl.handle.net/10419/26170

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



WWW.ECONSTOR.EU

FLEXIBLE LIFE ANNUITIES

ALEXIS DIRER

CESIFO WORKING PAPER NO. 2125 CATEGORY 3: SOCIAL PROTECTION OCTOBER 2007

PRESENTED AT CESIFO VENICE SUMMER INSTITUTE, WORKSHOP ON 'LONGEVITY AND ANNUITIZATION', JULY 2007

An electronic version of the paper may be downloaded • from the SSRN website: www.SSRN.com • from the RePEc website: www.RePEc.org • from the CESifo website: www.CESifo-group.org/wp

FLEXIBLE LIFE ANNUITIES

Abstract

Annuity contracts typically deliver a stream of income at a predetermined level in order to insure against the risk of longevity. This paper explores whether flexible annuities, which give subscribers the possibility to choose between different levels for their annuities, may be welfare enhancing. In the case where agents gradually discover their actual probability of survival, a predetermined and "one-size-fits-all" annuity plan is optimal. If an expenditure risk is added along with the longevity risk, a flexible annuity plan is better even though the consumption path cannot be isolated from uninsured expenses anymore.

JEL Code: E21, D82.

Alexis Direr LEA (INRA, PSE) University of Grenoble 48 Bd Jourdan 75014 Paris France direr@ens.fr

September, 24th 2007

This paper has benefited from useful comments by participants of the Paris School of Economics workshop "Health and Insurance" and the CESifo Venice Summer Institute Workshop on "Longevity and Annuitization". Any errors or omissions are my own.

It is a well established fact that very few assets are converted into life annuities outside of social security and traditional defined benefit pension plans. In the United States for example, 401(k) pension plans are now the dominant form of private pension. But very few 401(k) plans offer the option to annuitize. More than 50% of households do not expect to even partly convert their defined contribution account balances into an annuity (Brown, 2001). The United Kingdom has a long history of mandatory annuitization during retirement. In particular, defined contribution pension funds are currently required to provide 75-year-old retirees with 75 percent of pension assets in the form of annuities. However, less and less insurers are willing to offer such products. In France, a voluntary and fully funded personal savings account called PERP has been recently launched where buyers are compelled to fully annuitize the wealth accumulated in the plan at the date of retirement. Up to now, this constraint seems to hinder its commercial extension.

The underinvestment in life annuities contrasts with the benefits such annuities are expected to deliver to retirees. Yaari (1965) shows that full annuitization of assets is optimal in a standard model of saving without a bequest motive. Davidoff, Brown, and Diamond (2005) show an identical result for a large set of preferences and environments. A central question is therefore why the annuity market is so small. A preliminary answer is that even though life annuities provide unequalled insurance against longevity, they also have particular disadvantages. Most obviously, the purchaser loses control over his assets, as most annuity plans deliver a predetermined lifelong income. Hence, once annuitized, wealth cannot serve to absorb unexpected income shocks, where most of such shocks for the retirees are health-related. The risk of facing lower consumption following a period of unexpected health expenditures is recognized as being detrimental to the demand of annuities (Brown, 2004). It has also a significant impact on saving behavior. For instance, several articles argue that the existence of out-of-pocket medical expense risk explains why the elderly run down their assets so slowly (Palumbo 1999, De Nardi et al. 2006)

At first sight, this drawback could be minimized by providing the annuitant with some flexibility over the annuity profile. A flexible contract can be defined as a contract that gives the annuitant the possibility to withdraw a higher annuity in case of an expenses shock. The absence of such an option in existing contracts is commonly justified by an argument of adverse selection as stressed by Brown and Warshawsky (2001, p.14): "Insurance companies do not allow individuals to cancel an annuity agreement once it is in place. Otherwise adverse selection would obviously occur as individuals acquire information about their expected longevity." If individuals update their actual chance of longevity, annuitants who expect to live longer would indeed like to increase their financial stake in the plan while the shorter-lived consumers would like to consume greater amounts earlier, thereby reducing their annuitized wealth. If the insurer gives the subscribers the flexibility to choose between different levels at which they consume their annuity, the expected return of a pool of saving can be greatly reduced by an adverse shift in the composition of the population of subscribers. At the extreme limit, if agents were always informed sufficiently in advance of the day they die, they would have the time to close their personal account, thereby escaping the redistribution scheme. This would cause the collapse of the longevity insurance mechanism.

The objective of this paper is to explore this issue in depth and to ask

whether the adverse selection argument in itself prevents any flexibility in annuity plans from being feasible. I begin by presenting a simple model where life expectancy is the only source of uncertainty. Agents update their mortality risk during the retirement phase. This raises the question whether they should modify the level of their annuity according to their more precise estimate of mortality. It is shown that in this simple model, they should not. A fixed annuity plan which provides a predetermined stream of annuities independent of future information about longevity is optimal because it prevents annuitants to draw their wealth down in case of bad news about their mortality. This is a clear case where flexible life annuities are not welfare enhancing in line with the argument stressed by Brown and Warshawsky (2001).

Next, the previous model is extended by including a liquidity risk in addition to the longevity risk. Some savers face a liquidity need due to adverse shocks to their health. Some flexibility through a choice between different levels of annuities then becomes optimal. However, the risk of increased expenses cannot be fully insured, due to the adverse selection mechanism just outlined.

The main result of the analysis is that adverse selection *per se* does not justify the complete absence of flexibility. The choice of raising an annuity during one period up to a limit may cover some additional expenses, and thereby provide insurance. Flexibility is then optimal, even though a reduction in the rate of return arises due to adverse selection. This result is important, given the general lack of annuitization observed in most developed countries. Indeed, a minimal degree of flexibility may reduce the mismatch between the desired consumption path and the annuity income stream, thereby promoting wealth annuitization.

The literature on the annuity market has developed rapidly for two decades.

Most of the studies focus on the "annuity puzzle" by examining why households do not buy more annuity contracts when they retire. The studies generally assume an exogenous annuity stream and examine which type of consumers' preferences or which market imperfections allow to explain why the theoretical prediction of full annuitization (Yaari, 1965) does not seem to hold empirically (e.g. Kotlikoff and Spivak 1981, Walliser 1999, Milevsky and Young 2002, Davidoff, Brown, and Diamond 2005). Those articles differ in perspective from the present work by taking the annuity stream offered by the insurer as given. In contrast, the annuity plan is endogenous here in the sense that its optimal design depends on the presence of various types of uncertainty.

The issue addressed by the present article is formally close to Brugiavini (1993). He studies at which stage of their life individuals should purchase annuities given that they learn about their survival probability as they get older. His model predicts a purchase in an early stage of the life cycle. The present model displays similar implications if the period during which individuals update their survival probability is reinterpreted as a retirement period instead of an early working period. However, a different question is addressed in the two papers. His paper asks whether workers should buy an annuity contract at an early stage of their life and if they do, whether they could recontract when they retire. She finds that workers could not purchase more annuities later in life since it would reveal a high risk for insurers. In the present model, the early purchase of a fixed annuity works as a commitment device that prevents savers to consume more in case of bad news about their longevity. A formal connection between the two setups is explored further at the end of Section 2.

In section 1 a simple model of annuity demand when longevity is uncertain

is introduced. The basic argument of why a fixed annuity plan may be optimal is provided. Section 2 incorporates a liquidity shock and studies to what extent this additional source of uncertainty may be optimally insured by a flexible annuity plan. The last section concludes.

1 A model with uncertainty about longevity

In this section, a formal case against flexible annuities is presented in a basic setting with a longevity risk only. Subscribers are not allowed to choose their annuity profile on a period by period basis. This case will serve as a benchmark model when a second source of uncertainty will be introduced in the next section.

Consider an environment where agents allocate their wealth w between consumption at two dates t = 1, 2. The gross interest rate of the economy is denoted by R. Life expectancy is uncertain so that individuals face a probability to die before their last period consumption. The uncertainty of survival calls for the purchase of annuities which pay at a premium when alive in exchange for the subscribers' wealth upon death. In the absence of a bequest motive or uninsurable risk, full annuitization prevails (Yaari, 1965). The role of the insurer is to pool individuals with similar longevity expectations but varying longevity outcomes as a means to protect them against the longevity risk.

In the model, uncertainty about the mortality risk gradually resolves itself between a preliminary date 0 and date 1. Individuals have the same life expectancy at t = 0. At t = 1, they obtain more precise information about their actual probability to survive the last period t = 2. A fraction p of annuitants learn that their probability to survive is π^h . The remaining annuitants are characterized by a lower survival probability $\pi^l < \pi^h$. c_{ti} denotes consumption at date t = 1, 2 of agents with the longevity type i = h, l. Utility of consumption per period is denoted u(c), with u'(c) > 0, u''(c) < 0 and $\lim u'(c) = \infty$ when $c \to 0$. Intertemporal utility is additive and β is the subjective discount rate attached to the last period utility:

$$v(c_{1i}, c_{2i}; \pi_i) = u(c_{1i}) + \pi_i \beta u(c_{2i})$$

Information about longevity is private. Hence, insurers are unable to separate the annuitants by risk classes. Insurance firms cannot monitor whether costumers hold annuities also from other firms. They compete by offering the most attractive rate of return for saving. Individuals can purchase as many annuities as they want at the prevailing rate of return. This leads to the definition of an asymmetric information equilibrium (e.g. Abel, 1986) characterized by the annuity plan $\{(\hat{c}_{1i}, \hat{c}_{2i}); i = h, l\}$ which satisfies:

(i)
$$(\hat{c}_{1i}, \hat{c}_{2i}) = \arg \left\{ \begin{array}{c} \max v(c_{1i}, c_{2i}; \pi_i) \\ \text{s.t. } c_{1i} + \pi c_{2i}/R = w \end{array} \right\} \qquad i = h, l \qquad (1)$$

(*ii*)
$$\overline{\pi} = \frac{pc_{2h}}{p\widehat{c}_{2h} + (1-p)\widehat{c}_{2l}}\pi_h + \frac{(1-p)c_{2l}}{p\widehat{c}_{2h} + (1-p)\widehat{c}_{2l}}\pi_l$$

Insurers provide annuitants with the highest actuarially feasible rate of return $R/\overline{\pi}$. This rate takes as given the average mortality risk $\overline{\pi}$, weighted by the participation rate of each type in the annuity plan. Given this rate of return, consumers choose the best annuity profile according to the above definition.

Now, assume that the consumers have the possibility to sign a binding contract at date 0 at a time when they have not yet updated their information about their longevity. They leave their saving to the insurers, which maximizes their date 0 expected utility stream:

$$\begin{cases} \max pv(c_{1h}, c_{2h}; \pi_h) + (1-p)v(c_{1l}, c_{2l}; \pi_l) \\ \text{s.t. } p(w - c_{1h} - \pi_h c_{2h}/R) + (1-p)(w - c_{1l} - \pi_l c_{2l}/R) = 0 \end{cases}$$
(2)

The resulting consumption path $\{(c_{1i}^*,c_{2i}^*);i=h,l\}$ is defined by:

$$\begin{cases} c_{th}^* = c_{tl}^* & t = 1, 2\\ u'(c_{1i}) = \beta R u'(c_{2i}) & i = 1, 2 \end{cases}$$
(3)

The allocation is Pareto optimal since information is symmetric at date 0. It equalizes consumption across risks and makes the consumption path independent of the mortality risk.

In this equilibrium, insurers compete for savings as soon as date 0 by promising an annuity sequence $\{(c_{1i}^*, c_{2i}^*); i = h, l\}$. It remains to see whether individuals are willing to immediately buy such a contract or prefer to postpone their consumption decision. Indeed, even though this contract is optimal from an ex ante perspective, this is obviously not a contract that retirees would buy at date 1 once their individual longevity is better assessed. Proposition 1 states that the date 0 expected utility associated with buying the ex ante optimal contract is higher than the expected utility derived from postponing the consumption decision (proof in Appendix):

Proposition 1. $pv(c_{1h}^*, c_{2h}^*; \pi_h) + (1-p)v(c_{1l}^*, c_{2l}^*; \pi_l) > pv(\widehat{c}_{1h}, \widehat{c}_{2h}; \pi_h) + (1-p)v(\widehat{c}_{1l}, \widehat{c}_{2l}; \pi_l).$

There is a basic intuition behind this result, first outlined by Hirshleifer (1971). The reduction of uncertainty at date 1 prevents any transfer from the low risk agents to the high risk agents, which undermines the insurance scheme.

Proposition 1 is a clear case against flexible life annuities since it establishes the superior efficiency of an unconditional annuity plan that delivers a single level of annuity at dates 1 and 2. By not providing agents with the possibility to receive a higher annuity in case of a bad signal about their longevity, a fixed saving plan preserves a higher rate of return, and allows to mutualize the risk of longevity more efficiently.

Even though it is optimal to buy a predetermined annuity plan at date 0 according to proposition 1, it remains to check that agents cannot improve their situation by recontracting at date 1 by buying or selling new annuities. Let us denote by b_i the additional amount of annuities that they buy (or sell short) at time 1 for the last period consumption. Because they have invested all their saving at date 0 in the predetermined annuity plan, they use the payment c_1^* to consume and save at date 1: $\tilde{c}_{1i} + b_i = c_1^*$ so that their last date consumption if alive is $\tilde{c}_{2i} = c_2^* + Rb_i/\pi_i$. The rate of return is adjusted to each specific risk since buying new annuities reveals a low risk and short selling reveals a high risk¹. Hence, the revised consumption plan is determined by:

$$\left\{\begin{array}{c} \max v(c_{1i}, c_{2i}; \pi_i) \\ \text{s.t.} \ c_{1i} + \pi_i c_{2i}/R = c_1^* + \pi_i c_2^*/R \end{array}\right\} \qquad i = h, l$$

and does not deviate from the ex ante optimal allocation, which means that $b_i = 0$. This sums up the argument put forth by Brugiavini (1993). Agents do not recontract later since the effect of new information about longevity on consumption is exactly offset by the revision of the actuarially fair interest rate.

This section has shown the optimality of a fixed annuity plan. The next section adds uninsurable expenses, which leads to a different result.

¹Short selling could be ruled out without changing the revelation mechanism outlined.

2 Adding uninsurable expense shocks

The previous framework is preserved except that some uninsurable expenses, possibly related to health, may be incurred at date 1. There are now three sorts of consumers who discover their type at date 1. The first one, represented in proportion p, is a long-lived consumer who survives date 2 with probability $\pi > 0$. Such agents are denoted as of type h below. Agents of a second type m are represented with a proportion q and also survive with probability π , but incur an additional cost m > 0 at date 1. This cost is interpreted as an out-ofpocket medical expenditure. Last, type l agents, in proportion s = 1 - p - q, are short-lived consumers who die with certainty at the end of period 1.

Insurers offer an annuity plan at date 0, at a time when agents do not know their type and some insurance is still possible. An optimal plan is a set of annuities $\{(c_{1h}, c_{2h}); (c_{1m}, c_{2m}); (c_{1l}, 0)\}$ dedicated to types h, m and lrespectively.

It is informative to start the analysis with the hypothesis of complete information, where the three types are publicly known at date 1. Annuity contracts can be tailored to each type and full insurance between the insured prevails: $c_{1h}^* = c_{1m}^* - m = c_{1l}^*$ and $c_{2h}^* = c_{2m}^*$. The risk of longevity and the expense shock are perfectly mutualized across agents. Moreover, consumption evolves in accordance with the first best allocation: $u'(c_{1i}^*) = \beta Ru'(c_{2i}^*), i = h, l.$

In the more realistic case in which an agent's type is private information, the insurer has to make sure that each annuity plan is chosen by the type for whom it is designed. A first constraint checks that type h agents actually choose the annuity plan (c_{1h}, c_{2h}) instead of (c_{1m}, c_{2m}) designed for type mindividuals. This happens if the first type does not achieve a higher utility by selecting the annuity c_{1m} , saving the amount $x = c_{1m} - c_{1h}$ and then consuming $c_{2h} = c_{2m} + xR/\pi$ in the last period. The possibility to switch to a different annuity profile leads to a reservation level of utility denoted by v_h :

$$v_h(z) = \{\max u(c_1) + \pi\beta u(c_2); c_1 + c_2\pi/R = z\}$$
(4)

where $z = c_{1m} + \pi c_{2m}/R$ is the intertemporal value of the annuity plan (c_{1m}, c_{2m}) . The incentive compatibility constraint is then:

$$u(c_{1h}) + \pi \beta u(c_{2h}) \ge v_h(z) \tag{5}$$

Note that a first best policy with full information consists of a redistribution of resources to the agents who incur the expense shock because $c_{1h} = c_{1m} - m < c_{1m}$ and $c_{2h} = c_{2m}$. When the type is private information, the insurer is constrained to give the same actuarial value to both types (see the resource constraint in (4)). For the same reason, the reverse constraint in which a type m would prefer (c_{1h}, c_{2h}) to (c_{1m}, c_{2m}) is not operative in equilibrium as the insurance scheme implies transferring more resources to type m, not less.

A second incentive compatibility constraint recognizes that type l agents always choose the highest annuity available at date 1. It follows that the annuity c_{1l} cannot be less than the same period annuity designed for long-lived agents: $c_{1l} \ge \max(c_{1h}, c_{1m}).$

An equilibrium set of contracts maximizes the consumers' expected utility given a global resource constraint and the two information constraints:

$$\max p(u(c_{1h}) + \pi\beta u(c_{2h})) + q(u(c_{1m} - m) + \pi\beta u(c_{2m})) + su(c_{1l})$$

$$p(c_{1h} + \pi c_{2h}/R) + q(c_{1m} + \pi c_{2m}/R) + sc_{1l} = w$$
s.t.
$$u(c_{1h}) + \pi\beta u(c_{2h}) \ge v_h(z)$$

$$c_{1l} \ge \max(c_{1h}, c_{1m})$$
(6)

It is straightforward to show that $c_{1m} \ge c_{1h}$ in equilibrium. If this were not the case, the insurer could marginally reduce the date 1 consumption of type 1 and increase the consumption of type 2 by a factor p/q yielding a net utility increase of $u'(c_{1m} - m) - u'(c_{1h}) > 0$. Hence, it follows that the last information constraint simplifies to

$$c_{1l} \ge c_{1m} \tag{7}$$

The program (6) can be rewritten in a more convenient way:

$$\max p(u(c_{1h}(z)) + \pi\beta u(c_{2h}(z))) + q(u(c_{1m} - m) + \pi\beta u(c_{2m})) + su(c_{1l})$$

s.t.
$$(1 - s)(c_{1m} + \pi c_{2m}/R) + sc_{1l} = w$$

$$c_{1l} \ge c_{1m}$$

where $c_{1h}(z)$ and $c_{2h}(z)$ are agents' optimal demands of annuities solving (4). The second constraint merges with the budget constraint of (4) into the broader budget constraint of the insurer in (6). We can next examine the consequences for the optimal annuity plan.

Proposition 2. The incentive compatibility constraint (7) is binding: $c_{1l} = c_{1m}$.

The basic reason behind this inequality is that an expense shock is a legitimate reason to let the saver withdraw a higher annuity whereas a bad signal about survival is not. As a result, the insurer would like to offer a smaller annuity to a type l compared to the one proposed to a type m. As distinguishing between the two types is impossible, the insurer is compelled to offer a single annuity level for both types. A natural question that arises is whether the annuitants suffering an expense shock benefit at all from an insurance mechanism, given the adverse selection mechanism discussed in the introduction. The following proposition gives a positive answer:

Proposition 3. $c_{1m} > c_{1h}$.

This proposition shows that individuals can rely on a higher annuity when they must meet additional expenditures. This option comes at the cost of a diminished annuity in the last period since the information constraint (5) binds. Proposition 3 addresses to what extent this increases the insurance mechanism.

Proposition 4. The annuity plan (c_{1m}, c_{2m}) satisfies $u'(c_{1m}-m) > R\beta u'(c_{2m})$.

This proposition shows that annuitants who experience an expense shock cannot withdraw a sufficiently high annuity in order to fully smooth their consumption profile compared to the first best environment. Indeed, the insurer is prevented from raising the annuity because it magnifies the adverse selection effect and undermines the longevity insurance mechanism as previously discussed. On the contrary, perfect consumption smoothing could be restored if the adverse selection effect were absent:

Proposition 5. If s = 0, $u'(c_{1m} - m) = R\beta u'(c_{2m})$.

To sum up, the insurer proposes three annuity plans $\{(c_{1h}, c_{2h}); (c_{1m}, c_{2m}); (c_{1m}, c_{2m})\}$

 $(c_{1l}, 0)$, one for each type. A certain degree of flexibility is optimal, as agents are free to choose between consuming a high annuity c_{1m} and a low one c_{1h} . Choosing the high annuity reduces the last period income but improves the well-being of the agents who face an expense shock at date 1. Short-lived agents choose to "close" their annuity plan by choosing the high level $c_{1l} > c_{1h}$ as well. However, they face a high penalty as they cannot consume more than the level of the annuity that agents with an expense shock are allowed to withdraw. This penalty is optimal as it limits the scope of the adverse selection effect. Since insurers do not allow short-lived agents to deplete their account at date 1, agents in need of additional income cannot fully smooth their consumption profile.

3 Conclusion

This paper studies the optimal annuity contract when agents face a liquidity risk and a longevity risk. Simultaneously analyzing the effects of both types of risk is relevant. Taken separately, they produce conflicting predictions about the characteristics of the contract. With both types of risk, flexible annuities which provide annuitants with the possibility to choose between different withdrawal levels are welfare enhancing as they allow agents suffering the expense shock to better smooth their consumption path.

It is worth noting that flexibility does not mean total freedom for the annuitants. Indeed, the high pay-out level cannot be raised too much, as it leads the short-lived individuals to consume too much, thereby undermining the longevity insurance scheme. This constraint prevents agents suffering the expense shock from perfectly smoothing their consumption profile.

The main result of the paper is that the adverse selection effect sometimes

cited against flexible annuities is not sufficient for the optimality of a fixed and "one size fits all" annuity scheme. The option to increase withdrawals in one period up to a limited level may achieve a good deal of consumption smoothing while limiting the size of the rate of return reduction due to the adverse selection effect. A practical conclusion is that a minimal degree of flexibility could well promote wealth annuitization by reducing the mismatch between the desired consumption path and the annuity income stream.

References

Abel A. B. (1986) "Capital Accumulation and Uncertain Lifetimes with Adverse Selection" *Econometrica*, 54 (5) 1079-1097.

Brown J. R. (2001) "Private Pensions, Mortality Risk, and the Decision to Annuitize" *Journal of Public Economics* 82 (1) 29-62.

Brown, J. R. (2004) "Life Annuities and Uncertain Lifetimes" NBER Reporter: Research Summary Spring.

Brown, J. R., and M. J. Warshawsky (2001) "Longevity-Insured Retirement Distributions from Pension Plans: Market and Regulatory Issues", NBER Working Paper 8064.

Brugiavini, A. (1993) "Uncertainty Resolution and the Timing of Annuity Purchases", *Journal of Public Economics* 50 (1): 31-62.

De Nardi M., French E., and J. B. John (2006) "Differential Mortality, Uncertain Medical Expenses, and the saving of Ederly Singles" NBER Working Paper 12554.

Hirshleifer, J. (1971) "The Private and Social Value of Information and the Reward to Inventive Activity", *American Economic Review*, volume 61, no. 4, 561–574. Kotlikoff Laurence J., and A. Spivak (1981) "The Family as Incomplete Annuity Market", *Journal of Political Economy* 89, 372-391.

Milevsky, M. A. and V. R. Young (2002) "Optimal Asset Allocation and the Real Option to Delay Annuitization: It's Not Now-or-Never". Working Paper, York University.

Palumbo M. G. (1999) "Uncertain Medical Expenses and Precautionary Saving Near the End of the Life Cycle" *Review of Economic Studies*, 66 395-421.

Sheshinski E. "Annuities and Retirement", in D. Aliprantis, K. Arrow, P. Hammond, F. Kubler, H-M Wu and N. Yannelis (eds) Assets, Beliefs, and Equilibria in Economic Dynamics (Springer, 2003), 27-54.

Walliser, J. (1999) "Regulation of Withdrawals in Individual Account Systems" in New Ideas about Old Age Secirity.

Yaari, M.E. (1965) "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer", *Review of Economic Studies* 32, 2, 137-150.

Appendix

Proof of Proposition 1. By way of comparison, the date 0 problem in (2)

can be recast into:

$$\begin{cases} \max pv(c_{1h}, c_{2h}; \pi_h) + (1-p)v(c_{1l}, c_{2l}; \pi_l) \\ \text{s.t.} \ p(w - c_{1h} - \overline{\pi}c_{2h}/R) + (1-p)(w - c_{1l} - \overline{\pi}c_{2l}/R) = 0 \\ \overline{\pi} = \frac{pc_{2h}}{pc_{2h} + (1-p)c_{2l}} \pi_h + \frac{(1-p)c_{2l}}{pc_{2h} + (1-p)c_{2l}} \pi_l \end{cases}$$

This program closely resembles the ex post problem with asymmetric information defined in (1):

(i)
$$\{(\hat{c}_{1i}, \hat{c}_{2i}); i = h, l\} = \arg \left\{ \begin{array}{ll} \max pv(c_{1h}, c_{2h}; \pi_h) + (1-p)v(c_{1l}, c_{2l}; \pi_l) \\ \text{s.t. } c_{1i} + \overline{\pi}c_{2i}/R = w \quad i = h, l \end{array} \right\}$$

(ii) $\overline{\pi} = \frac{p\hat{c}_{2h}}{p\hat{c}_{2h} + (1-p)\hat{c}_{2l}}\pi_h + \frac{(1-p)\hat{c}_{2l}}{p\hat{c}_{2h} + (1-p)\hat{c}_{2l}}\pi_l$

except that the budget constraint is now split into two separate constraints instead of one and that the impact of the consumption choice on the rate of return $R/\overline{\pi}$ is not internalized. Hence, despite the objective being the same, the ex post problem includes additional constraints. As a result, it delivers a less efficient consumption stream from the the perspective of date 0.

Proof of Proposition 2.

If $c_{1l} \neq c_{1m}$, the program of the insurer is:

$$\begin{cases} \max p(u(c_{1h}(z)) + \pi\beta u(c_{2h}(z))) + q(u(c_{1m} - m) + \pi\beta u(c_{2m})) + su(c_{1l}) \\ \text{s.t.} \quad (p+q)(c_{1m} + \pi c_{2m}/R) + sc_{1l} = w \end{cases}$$

Forming the Lagrangian function:

$$L(c_{1m}, c_{2m}, c_{1l}) = p(u(c_{1h}(z)) + \pi\beta u(c_{2h}(z))) + q(u(c_{1m} - m) + \pi\beta u(c_{2m})) + su(c_{1l}) + \lambda[w - (p+q)(c_{1m} + \pi c_{2m}/R) - sc_{1l}]$$

and setting the partial derivatives to zero with respect to c_{1m} and c_{1l} :

$$\frac{\partial L}{\partial c_{1m}} = p[u'(c_{1h})\frac{\partial c_{1h}}{\partial z} + \pi\beta u'(c_{2h})\frac{\partial c_{2h}}{\partial z}] + qu'(c_{1m} - m) - (p+q)\lambda = 0$$
$$\frac{\partial L}{\partial c_{1l}} = su'(c_{1l}) - s\lambda = 0$$

Then, the following inequalities can be derived:

$$u'(c_{1l}) = \frac{p}{p+q} [u'(c_{1h})\frac{\partial c_{1h}}{\partial z} + \pi\beta u'(c_{2h})\frac{\partial c_{2h}}{\partial z}] + \frac{q}{p+q}u'(c_{1m}-m)$$
$$= \frac{p}{p+q}u'(c_{1h})[\frac{\partial c_{1h}}{\partial z} + \frac{\pi}{R}\frac{\partial c_{2h}}{\partial z}] + \frac{q}{p+q}u'(c_{1m}-m)$$
(i)

$$= \frac{p}{p+q}u'(c_{1h}) + \frac{q}{p+q}u'(c_{1m} - m)$$
(ii)

$$\geq \frac{p}{p+q}u'(c_{1m}) + \frac{q}{p+q}u'(c_{1m}) = u'(c_{1m})$$
(iii)

Equality (i) is obtained by using the Euler equation $u'(c_{1h}) = \beta R u'(c_{2h})$ derived from (4), (ii) by differentiating the budget constraint (4): $c_{1h}(z)$ + $c_{2h}(z)\pi/R = z$ and substituting for $\partial c_{1h}/\partial z$. Inequality (iii) exploits the fact that $c_{1m} \ge c_{1h}$ at equilibrium. The result is that $c_{1l} \le c_{1m}$ or $c_{1l} = c_{1m}$ when the information constraint (7) is taken into account.

Proof of Proposition 3.

Let us prove that $c_{1m} = c_{1h}$ cannot hold at the equilibrium. This implies $c_{2m} = c_{2h}$ from (4). The insurer's program with $c_{1h} = c_{1m} = c_{1l}$ is then:

$$\begin{cases} \max(1-q)u(c_{1m}) + qu(c_{1m}-m) + (1-s)\pi\beta u(c_{2m}) \\ \text{s.t. } c_{1m} + (1-s)c_{2m}\pi/R = w \end{cases}$$

Forming the Lagrangian function and setting the partial derivatives to zero leads to the Euler equation: $(1-q)u'(c_{1m}) + qu'(c_{1m} - m) = R\beta u'(c_{2m})$. Hence $u'(c_{1m}) < R\beta u'(c_{2m})$. But $c_{1m} = c_{1h}$ also implies $c_{2m} = c_{2h}$ and $u'(c_{1m}) = R\beta u'(c_{2m})$ from (4), which contradicts the former equation.

Proof of Proposition 4.

The insurer's program with $c_{1l} = c_{1m}$ is:

$$\begin{cases} \max p(u(c_{1h}(z)) + \pi\beta u(c_{2h}(z))) + q(u(c_{1m} - m) + \pi\beta u(c_{2m})) + su(c_{1m}) \\ \text{s.t. } c_{1m} + (1 - s)c_{2m}\pi/R = w \end{cases}$$

Forming the Lagrangian function and setting the partial derivatives to zero:

$$\frac{\partial L}{\partial c_{1m}} = pu'(c_{1h})\frac{\partial c_{1h}}{\partial z} + p\pi\beta u'(c_{2h})\frac{\partial c_{2h}}{\partial z} + qu'(c_{1m} - m) + su'(c_{1m}) - \lambda = 0$$

$$\frac{\partial L}{\partial c_{2m}} = pu'(c_{1h})\frac{\partial c_{1h}}{\partial z}\frac{\pi}{R} + p\pi\beta u'(c_{2h})\frac{\partial c_{2h}}{\partial z}\frac{\pi}{R} + q\pi\beta u'(c_{2m}) - (1 - s)\frac{\pi}{R}\lambda = 0$$

Substituting for the multiplier λ :

$$\frac{p}{1-s}u'(c_{1h})\frac{\partial c_{1h}}{\partial z} + \frac{p}{1-s}\pi\beta u'(c_{2h})\frac{\partial c_{2h}}{\partial z} + \frac{q}{1-s}\beta Ru'(c_{2m})$$
$$= pu'(c_{1h})\frac{\partial c_{1h}}{\partial z} + p\pi\beta u'(c_{2h})\frac{\partial c_{2h}}{\partial z} + qu'(c_{1m}-m) + su'(c_{1m})$$

Using $u'(c_{1h}) = R\beta u'(c_{2h})$:

$$\frac{q}{1-s}R\beta u'(c_{2m}) = qu'(c_{1m}-m) + su'(c_{1m}) + u'(c_{1h}) \left[p\frac{\partial c_{1h}}{\partial z} + p\frac{\pi}{R}\frac{\partial c_{2h}}{\partial z} - \frac{p}{1-s}\frac{\partial c_{1h}}{\partial z} - \frac{p}{1-s}\frac{\pi}{R}\frac{\partial c_{2h}}{\partial z} \right]$$

Differentiating $c_{1h}(z) + c_{2h}(z)\pi/R = z$ and substituting for $\partial c_{1h}/\partial z$ leads to the constrained Euler equation:

$$R\beta u'(c_{2m}) = (1-s)u'(c_{1m}-m) + \frac{s(1-s)}{q}u'(c_{1m}) - \frac{ps}{q}u'(c_{1h})$$
(8)
$$< (1-s)u'(c_{1m}-m) + \frac{s(1-s)}{q}u'(c_{1h}) - \frac{ps}{q}u'(c_{1h})$$
(i)
$$= (1-s)u'(c_{1m}-m) + su'(c_{1h})$$
(ii)
$$\leq (1-s)u'(c_{1m}-m) + su'(c_{1m}-m)$$
(ii)
$$= u'(c_{1m}-m)$$

Inequality (i) uses the fact that $c_{1m} > c_{1h}$ and (ii) $c_{1m} - m \leq c_{1h}$. Indeed, $c_{1m} - m > c_{1h}$ would lead to over-insurance. In this case, $u'(c_{1m} - m) < u'(c_{1h}) = R\beta u'(c_{2h})$. Moreover, $c_{1m} > c_{1h}$ implies $c_{2m} < c_{2h}$ from (4) and therefore $R\beta u'(c_{2m}) > R\beta u'(c_{2h})$ implying $u'(c_{1m} - m) < R\beta u'(c_{2m})$. That is, type *m* agents consume too much at date 1 despite additional expenses. It follows that insurers could improve the position of type *m* by reducing c_{1m} and raising c_{2m} without violating the type *m* resource constraint $c_{1m} + \pi c_{2m}/R = w$ and incentive compatibility constraints.

Proof of Proposition 5.

Take the constrained Euler **equation** (8) found in the previous proof and set s = 0 and p + q = 1.

CESifo Working Paper Series

for full list see www.cesifo-group.org/wp (address: Poschingerstr. 5, 81679 Munich, Germany, office@cesifo.de)

- 2064 Jan Hanousek, Dana Hajkova and Randall K. Filer, A Rise by Any Other Name? Sensitivity of Growth Regressions to Data Source, July 2007
- 2065 Yin-Wong Cheung and Xing Wang Qian, Hoarding of International Reserves: Mrs Machlup's Wardrobe and the Joneses, July 2007
- 2066 Sheilagh Ogilvie, 'Whatever Is, Is Right'?, Economic Institutions in Pre-Industrial Europe (Tawney Lecture 2006), August 2007
- 2067 Floriana Cerniglia and Laura Pagani, The European Union and the Member States: Which Level of Government Should Do what? An Empirical Analysis of Europeans' Preferences, August 2007
- 2068 Alessandro Balestrino and Cinzia Ciardi, Social Norms, Cognitive Dissonance and the Timing of Marriage, August 2007
- 2069 Massimo Bordignon, Exit and Voice. Yardstick versus Fiscal Competition across Governments, August 2007
- 2070 Emily Blanchard and Gerald Willmann, Political Stasis or Protectionist Rut? Policy Mechanisms for Trade Reform in a Democracy, August 2007
- 2071 Maarten Bosker and Harry Garretsen, Trade Costs, Market Access and Economic Geography: Why the Empirical Specification of Trade Costs Matters, August 2007
- 2072 Marco Runkel and Guttorm Schjelderup, The Choice of Apportionment Factors under Formula Apportionment, August 2007
- 2073 Jay Pil Choi, Tying in Two-Sided Markets with Multi-Homing, August 2007
- 2074 Marcella Nicolini, Institutions and Offshoring Decision, August 2007
- 2075 Rainer Niemann, The Impact of Tax Uncertainty on Irreversible Investment, August 2007
- 2076 Nikitas Konstantinidis, Gradualism and Uncertainty in International Union Formation, August 2007
- 2077 Maria Bas and Ivan Ledezma, Market Access and the Evolution of within Plant Productivity in Chile, August 2007
- 2078 Friedrich Breyer and Stefan Hupfeld, On the Fairness of Early Retirement Provisions, August 2007

- 2079 Scott Alan Carson, Black and White Labor Market Outcomes in the 19th Century American South, August 2007
- 2080 Christian Bauer, Paul De Grauwe and Stefan Reitz, Exchange Rates Dynamics in a Target Zone A Heterogeneous Expectations Approach, August 2007
- 2081 Ana Rute Cardoso, Miguel Portela, Carla Sá and Fernando Alexandre, Demand for Higher Education Programs: The Impact of the Bologna Process, August 2007
- 2082 Christian Hopp and Axel Dreher, Do Differences in Institutional and Legal Environments Explain Cross-Country Variations in IPO Underpricing?, August 2007
- 2083 Hans-Werner Sinn, Pareto Optimality in the Extraction of Fossil Fuels and the Greenhouse Effect: A Note, August 2007
- 2084 Robert Fenge, Maximilian von Ehrlich and Matthias Wrede, Fiscal Competition, Convergence and Agglomeration, August 2007
- 2085 Volker Nitsch, Die Another Day: Duration in German Import Trade, August 2007
- 2086 Kam Ki Tang and Jie Zhang, Morbidity, Mortality, Health Expenditures and Annuitization, August 2007
- 2087 Hans-Werner Sinn, Public Policies against Global Warming, August 2007
- 2088 Arti Grover, International Outsourcing and the Supply Side Productivity Determinants, September 2007
- 2089 M. Alejandra Cattaneo and Stefan C. Wolter, Are the Elderly a Threat to Educational Expenditures?, September 2007
- 2090 Ted Bergstrom, Rod Garratt and Damien Sheehan-Connor, One Chance in a Million: Altruism and the Bone Marrow Registry, September 2007
- 2091 Geraldo Cerqueiro, Hans Degryse and Steven Ongena, Rules versus Discretion in Loan Rate Setting, September 2007
- 2092 Henrik Jacobsen Kleven, Claus Thustrup Kreiner and Emmanuel Saez, The Optimal Income Taxation of Couples as a Multi-Dimensional Screening Problem, September 2007
- 2093 Michael Rauber and Heinrich W. Ursprung, Life Cycle and Cohort Productivity in Economic Research: The Case of Germany, September 2007
- 2094 David B. Audretsch, Oliver Falck and Stephan Heblich, It's All in Marshall: The Impact of External Economies on Regional Dynamics, September 2007
- 2095 Michael Binder and Christian J. Offermanns, International Investment Positions and Exchange Rate Dynamics: A Dynamic Panel Analysis, September 2007

2096 Louis N. Christofides and Amy Chen Peng, Real Wage Chronologies, September 2007

- 2097 Martin Kolmar and Andreas Wagener, Tax Competition with Formula Apportionment: The Interaction between Tax Base and Sharing Mechanism, September 2007
- 2098 Daniela Treutlein, What actually Happens to EU Directives in the Member States? A Cross-Country Cross-Sector View on National Transposition Instruments, September 2007
- 2099 Emmanuel C. Mamatzakis, An Analysis of the Impact of Public Infrastructure on Productivity Performance of Mexican Industry, September 2007
- 2100 Gunther Schnabl and Andreas Hoffmann, Monetary Policy, Vagabonding Liquidity and Bursting Bubbles in New and Emerging Markets – An Overinvestment View, September 2007
- 2101 Panu Poutvaara, The Expansion of Higher Education and Time-Consistent Taxation, September 2007
- 2102 Marko Koethenbuerger and Ben Lockwood, Does Tax Competition Really Promote Growth?, September 2007
- 2103 M. Hashem Pesaran and Elisa Tosetti, Large Panels with Common Factors and Spatial Correlations, September 2007
- 2104 Laszlo Goerke and Marco Runkel, Tax Evasion and Competition, September 2007
- 2105 Scott Alan Carson, Slave Prices, Geography and Insolation in 19th Century African-American Stature, September 2007
- 2106 Wolfram F. Richter, Efficient Tax Policy Ranks Education Higher than Saving, October 2007
- 2107 Jarko Fidrmuc and Roman Horváth, Volatility of Exchange Rates in Selected New EU Members: Evidence from Daily Data, October 2007
- 2108 Torben M. Andersen and Michael Svarer, Flexicurity Labour Market Performance in Denmark, October 2007
- 2109 Jonathan P. Thomas and Tim Worrall, Limited Commitment Models of the Labor Market, October 2007
- 2110 Carlos Pestana Barros, Guglielmo Maria Caporale and Luis A. Gil-Alana, Identification of Segments of European Banks with a Latent Class Frontier Model, October 2007
- 2111 Felicitas Nowak-Lehmann D., Sebastian Vollmer and Immaculada Martínez-Zarzoso, Competitiveness – A Comparison of China and Mexico, October 2007
- 2112 Mark Mink, Jan P.A.M. Jacobs and Jakob de Haan, Measuring Synchronicity and Comovement of Business Cycles with an Application to the Euro Area, October 2007

- 2113 Ossip Hühnerbein and Tobias Seidel, Intra-regional Tax Competition and Economic Geography, October 2007
- 2114 Christian Keuschnigg, Exports, Foreign Direct Investment and the Costs of Corporate Taxation, October 2007
- 2115 Werner Bönte, Oliver Falck and Stephan Heblich, Demography and Innovative Entrepreneurship, October 2007
- 2116 Katrin Assenmacher-Wesche and M. Hashem Pesaran, Assessing Forecast Uncertainties in a VECX Model for Switzerland: An Exercise in Forecast Combination across Models and Observation Windows, October 2007
- 2117 Ben Lockwood, Voting, Lobbying, and the Decentralization Theorem, October 2007
- 2118 Andrea Ichino, Guido Schwerdt, Rudolf Winter-Ebmer and Josef Zweimüller, Too Old to Work, too Young to Retire?, October 2007
- 2119 Wolfgang Eggert, Tim Krieger and Volker Meier, Education, Unemployment and Migration, October 2007
- 2120 Stefan Napel and Mika Widgrén, The European Commission Appointment, Preferences, and Institutional Relations, October 2007
- 2121 Bertil Holmlund and Martin Söderström, Estimating Income Responses to Tax Changes: A Dynamic Panel Data Approach, October 2007
- 2122 Doina Maria Radulescu, From Separate Accounting to Formula Apportionment: Analysis in a Dynamic Framework, October 2007
- 2123 Jelle Brouwer, Richard Paap and Jean-Marie Viaene, The Trade and FDI Effects of EMU Enlargement, October 2007
- 2124 Kurt R. Brekke, Luigi Siciliani and Odd Rune Straume, Competition and Waiting Times in Hospital Markets, October 2007
- 2125 Alexis Direr, Flexible Life Annuities, October 2007