

Reshidi, Edona

Working Paper

Vertical bargaining and obfuscation

Bank of Canada Staff Working Paper, No. 2022-13

Provided in Cooperation with:

Bank of Canada, Ottawa

Suggested Citation: Reshidi, Edona (2022) : Vertical bargaining and obfuscation, Bank of Canada Staff Working Paper, No. 2022-13, Bank of Canada, Ottawa, <https://doi.org/10.34989/swp-2022-13>

This Version is available at:

<https://hdl.handle.net/10419/261266>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

Vertical Bargaining and Obfuscation

by Edona Reshidi

Banking and Payments Department
Bank of Canada
ereshidi@bankofcanada.ca



Bank of Canada staff working papers provide a forum for staff to publish work-in-progress research independently from the Bank's Governing Council. This research may support or challenge prevailing policy orthodoxy. Therefore, the views expressed in this paper are solely those of the authors and may differ from official Bank of Canada views. No responsibility for them should be attributed to the Bank.

DOI: <https://doi.org/10.34989/swp-2022-13> | ISSN 1701-9397

©2022 Bank of Canada

Acknowledgements

I thank Maarten Janssen, Karl Schlag, Heski Bar-Isaac, Philipp Schmidt-Dengler, Eeva Muring, Jason Allen, Shota Ichihashi, Marcel Preuss, Daniel Garcia, Juha Tolvanen, Marc Goñi, Matan Tsur, Clement Minaudier, Simon Martin, Anton Sobolev and participants at the 10th Consumer Search and Switching Costs Workshop (UCLA), University of Vienna and University of Toronto for helpful comments.

Abstract

Manufacturers often engage in practices that impede consumer search. Examples include proliferating product varieties, imposing vertical informational restraints, and banning online sales to make it more difficult for consumers to compare prices. This paper models vertical bargaining over wholesale prices and obfuscation levels and finds that obfuscation arises in equilibrium whenever retailers have some bargaining power. Once the bargaining power rests with the manufacturer, the equilibrium involves no obfuscation. The final consumers, however, are worse off compared with settings when retailers have all the bargaining power. We show that in vertical markets, policies that impose caps on obfuscation may induce higher wholesale and retail prices. Instead, we propose caps on wholesale prices as an effective consumer protection policy.

Topics: Economic models; Market structure and pricing

JEL codes: C70, L42, L13

1 Introduction

Obfuscation practices, defined as actions taken by firms that increase consumers’ costs of finding product information, are prevalent in many markets. In this paper, we analyze settings where product manufacturers engage in obfuscation. According to Ellison and Ellison [2018], one way manufacturers obfuscate is by “proliferating product varieties, even along dimensions that customers do not care about, so that comparing prices becomes a complicated and tedious process.” For instance, Richards, Klein, Bonnet, and Bouamra-Mechemache [2020] show that soft-drink manufacturers offer retail-specific variants of their products, which differ only slightly on their multi-pack or container sizes. Vertical restraints are another form of obfuscation manufacturers often use. Asker and Bar-Isaac [2020] show that informational restraints, such as minimum advertised prices (MAPs), limit the information that retailers can provide to consumers, therefore making it more difficult for consumers to compare products. According to a recent report of the European Commission [2017], retailers are faced with different informational restraints, such as not being allowed to freely advertise prices, sell online, or participate in price comparison websites.

Despite the widespread use of such practices by manufacturers, the literature on obfuscation has largely ignored vertical markets. This paper seeks to fill this gap by developing a model that incorporates a vertical market and enables the analysis of manufacturer obfuscation. We analyze a vertical bargaining framework where a monopolist manufacturer bargains with retailers over wholesale prices and obfuscation levels. In vertical markets, the issue of who sets prices and who imposes vertical restraints is subtle. The vertical contracting literature has mainly worked under the assumption that the bargaining power rests upstream. However, given the dramatic developments in retail markets, such as scanner devices and the introduction of discounters, downstream power has increased. In many markets, the general perception is that bargaining power has shifted towards retailers.¹ Retailers with high bargaining power have also been known to impose restraints on their suppliers. Such practices are known as “buyer-driven” restraints.² Therefore, a framework of vertical bargaining over wholesale prices and obfuscation levels seems reasonable to use when analyzing such settings.³

To analyze the drivers and welfare effects of obfuscation and bargaining, we consider a setting where a monopolist manufacturer produces a homogeneous product and sells it to two downstream retailers who then compete in prices. We study a situation where the

¹Inderst and Wey [2007], Competition Commission [2000], Competition Commission [2008], and OECD [2009] provide evidence on the growing bargaining power of retailers across Europe and in the US.

²Examples of buyer-driven restraints include most-favoured customer clauses, additional payment requirements, conditional purchase behaviour, deliberate risk shifting, etc. For a survey on such restraints see Dobson [2008].

³Draganska, Klapper, and Villas-Boas [2010] provide evidence of vertical bargaining in the coffee industry, Crawford and Yurukoglu [2012] show that distributors bargain over input prices in the cable TV industry, and Ho and Lee [2017] analyze bargaining in health care markets.

manufacturer cannot discriminate between its retailers, which is in line with most of the legislation regulating wholesale price discrimination.⁴ In the first stage, the manufacturer bargains with the retailers over a linear wholesale price and over the search cost, or obfuscation level. Afterwards, retailers set their prices. Lastly, consumers engage in sequential search. The consumers have unit demands and are modelled à la Stahl [1989]. Thus a fraction of consumers are shoppers and can search freely, while a fraction are non-shoppers and incur a search cost to learn a firm's price. The novelty here is that the search cost faced by the non-shoppers is an endogenous outcome of the vertical bargaining process between the manufacturer and retailers.

We show that in equilibrium the downstream market exhibits price dispersion. Retailers face a trade-off between charging high prices to extract profit from the non-shoppers and charging low prices to attract shoppers (Stahl [1989]). Retailers have an incentive to engage in obfuscation, while the manufacturer does not. The reasoning goes as follows. First, an increase in obfuscation means that the non-shoppers face a higher search cost, which in turn implies that retailers have more market power and can thus charge higher prices. Second, given that the expected retail price is increasing in both wholesale price and search cost, a higher search cost restricts the manufacturer's ability to set a high wholesale price without losing any consumers. Therefore, obfuscation increases retailers' profits by increasing their market power and also restricting the wholesale prices charged by the manufacturer. The monopolist manufacturer can achieve maximum profits if he sets the monopoly wholesale price equal to the consumers' valuation. This is only possible if no consumer incurs a positive search cost. Under obfuscation, however, if the manufacturer charges the monopoly wholesale price, then the consumers that have to incur the search cost would drop out from the market. In order for the manufacturer not to lose any consumers, he has to charge a lower price. In contrast to retailers, the manufacturer has no incentive to increase non-shoppers' search cost.

This paper makes three contributions. First, our analysis highlights the role of bargaining power by showing that it is the retailers' bargaining power that gives rise to obfuscation. Thus, it provides a new rationale for the widespread use of obfuscation practices in vertical markets. As mentioned, most of the literature on vertical markets assumes that the bargaining power lies with the manufacturer. Yet over the last few years, the bargaining power in many markets has shifted to large retailers. Empirical evidence indicates that the strong position of retailers is positively correlated with buyer-driven vertical restraints (see, e.g., Dobson [2008]). Furthermore, there is anecdotal evidence that many retailers pressure their manufacturers in imposing informational vertical restraints such as MAPs. Our analysis confirms these observations. If the bargaining power is upstream, the manufacturer charges the monopoly price and the retailers set retail prices

⁴See, e.g., European Union's Article 102 (c) of the treaty, which forbids dominant firms from applying "dissimilar conditions to equivalent transactions with other trading parties, thereby placing them at a competitive disadvantage."

equal to their marginal costs. Therefore, the industry monopoly outcome is achieved without obfuscation. If retailers have some bargaining power, however, obfuscation arises in equilibrium.

Second, we show that once a vertical structure is considered in a search model with obfuscation, qualitatively different properties arise as compared to settings that disregard upstream arrangements. More specifically, we show that when the production costs of retailers are not exogenously fixed but set strategically by an upstream manufacturer, an increase in the bargaining power of retailers, while leading to an increased obfuscation level, results in lower prices for final consumers. Therefore, consumers are better off when faced with higher search costs. This happens because an increase in the bargaining power of the retailers not only affects the obfuscation or search level that consumers incur but also the input price that the manufacturer sets to the retailers.

The mechanism works as follows. An increase in the bargaining power of retailers has two distinct effects. First, it enables the retailers to bargain a higher search cost, which gives them more market power and increases their profits. We call this the *obfuscation effect*. Second, it allows them to obtain better deals in terms of wholesale prices from the manufacturer; we name this the *input effect*. The *obfuscation effect* puts upward pressure on retail prices, while the *input effect* drives them down. We show that in our setting, as long as the market features a positive share of shoppers, the *input effect* dominates and thus consumers are better off when retailers have higher bargaining power. We find that if the manufacturer has all the bargaining power, no obfuscation occurs in equilibrium and thus the downstream market is perfectly competitive. Consumers, however, are worse off compared to settings where retailers have some bargaining power. This is because the manufacturer acts as a monopolist, charges all retailers the monopoly price, and gains monopoly profits by driving both retailers' profits and consumer surplus down to zero.

Finally, as a third contribution, the paper shows that taking vertical markets into account makes a difference when considering the effectiveness of regulation. We discuss the effects that different policy interventions might have in settings where obfuscation and vertical markets coexist. We show that a policy that puts a cap on obfuscation may not be effective in protecting consumers. For instance, regulators can ask firms to disclose their fees, display their prices in such a way that they include all possible add-on prices or taxes, or limit the length of complicated contracts. Such a policy has the direct effect of limiting obfuscation, but it also has an indirect unintended effect of inducing higher wholesale prices. We show that, under a binding obfuscation cap, the manufacturer bargains a higher wholesale price. Therefore, any reduction in obfuscation would be outweighed by a higher wholesale price, which would in turn be passed down from retailers to the final consumers. We propose that in cases where the upstream market is either monopolistic or where not enough supplier rivalry exists, consumer protection policies that instead impose a cap on the wholesale price could be effective. Such a policy intervention is followed, for instance, by Ofgem, the government regulator for gas and electricity markets in the

UK. Since 2019, the regulator has started imposing a cap on energy prices and plans to remove it once enough evidence of supplier rivalry exists.

In Section 5 we show that the findings are robust to a number of extensions. If there is an oligopoly in the downstream market rather than a duopoly, we find that wholesale prices increase with the number of retailers. This implies that an additional countervailing buyer effect arises. We also show that the analysis is robust to the use of two-part tariffs, where the manufacturer and retailers bargain over a search cost, a wholesale price, and a fixed fee. Additionally, the results do not change if we think of obfuscation as a decrease in the share of shoppers in the market instead of an increase in the search cost of non-shoppers, nor if retailers differ in terms of their bargaining power.

Related Literature. This paper contributes to the expanding obfuscation literature that analyzes firms' incentives to impede consumer search (see, e.g., Carlin [2009], Wilson [2010], Ellison and Wolinsky [2012], Piccione and Speigler [2012], Gamp [2016], and Petrikaitė [2018]). The focus of many of the papers in this literature is the so-called "collective action" problem, which notes that while it may be collectively rational for firms to obfuscate, it might not be individually rational for them to do so. This is true especially if the obfuscation level is observed ex-ante by consumers. This issue disappears when analyzing a setting with an upstream manufacturer, as in our paper, since the manufacturer partakes in obfuscation.

Unlike the present paper, existing studies do not consider a vertical setting and thus take the firms' production costs as exogenously given. A notable exception is Asker and Bar-Isaac [2020], who focus on the pro- and anti-competitive effects of MAPs. MAPs are seen as restrictions used by an upstream manufacturer to obfuscate actual rather than advertised prices. The authors assume that the manufacturer has all the bargaining power and makes take-it-or-leave-it offers to the retailers. The paper makes use of differences either in consumer valuations or retailers' marginal costs, or considers upstream competition to provide either a price discrimination, service provision, or collusion rationale for MAPs. We differ from this paper in several ways. First, we draw attention to environments where the bargaining power is not solely with the upstream firm. Second, we study a setup where both upstream and downstream firms are able to endogenously affect consumers' search costs. Lastly, we provide a different rationale for upstream obfuscation that is not driven by differences either in consumers' valuations, retailers marginal costs, or upstream competition, but simply by the bargaining power of retailers and the manufacturer.

This paper also adds to the literature on search in vertically related markets (see, e.g., Janssen and Shelegia [2015], Lubensky [2017], Garcia, Honda, and Janssen [2017], Garcia and Janssen [2018], Janssen and Reshidi [2019], Janssen and Shelegia [2020], and Rhodes, Watanabe, and Zhou [2021]). All of these papers assume that the bargaining power lies either with the upstream manufacturer or, in special instances, with a monopolist intermediary. Therefore, none of them considers the possibility of bargaining between

firms in the supply chain. Furthermore, they take the cost of search as exogenously given and do not allow the possibility of obfuscation. This paper differs from the rest of the literature on vertical markets with search by incorporating vertical bargaining over wholesale prices and search costs, thus allowing for the possibility of search costs being endogenously affected.

The remainder of the paper is organized as follows. First, in Section 2, we describe the model and the vertical bargaining protocol between the manufacturer and the retailers. Then, in Section 3, we characterize the equilibrium, first by analyzing the retail market and then by looking at the outcome of the bargaining stage and show comparative static results. Section 4 discusses policy implications, while extensions are provided in Section 5. Finally, Section 6 concludes.

2 The Model

A monopolist manufacturer, M , produces a homogeneous good and sells it to two competing downstream retailers, R_1 and R_2 .⁵ For simplicity, the manufacturer's production costs are normalized to zero, and this is assumed to be common knowledge to all market participants. Retailers compete in prices and the wholesale price is the only cost they face. There is a unit mass of final consumers. Each consumer has unit demand and a maximum willingness to pay of v . Consumers differ in their search costs and are indistinguishable to retailers. A share $\lambda \in (0, 1)$ are shoppers and have zero search costs, while a share $(1 - \lambda)$ of final consumers are non-shoppers and have to pay a search cost $s > 0$ for every search they make, including the first one. Therefore, the model considered in this paper is close to the one first used in Janssen and Shelegia [2015], given that it adds a wholesale level to the model analyzed in Stahl [1989]. There are, however, three main differences from the Janssen and Shelegia [2015] setting. First, to incorporate the fact that manufacturers can engage in obfuscation, we enable the manufacturer to endogenously affect consumers' search cost, not only the wholesale prices. Second, we allow for vertical bargaining between the manufacturer and the retailers over these two choice variables.⁶ Thus, the wholesale price and the search cost that the non-shoppers face are endogenous outcomes of the bargaining process between the manufacturer and retailers. Finally, to simplify the analysis and be able to study such settings, we focus on the case of unit demand, where we are able to explicitly solve for the reservation price.

The timing of the game is as follows. First, the manufacturer bargains with retailers over the wholesale price and the level of search cost. The bargaining process can be over different wholesale prices w_i and different levels of search costs s_i . We focus on an equilibrium that is uniform in wholesale prices and search costs. The retailers and the

⁵The case of $N \geq 2$ is considered as an extension.

⁶In an extension, we also consider the case where the manufacturer bargains over wholesale prices and the share of shoppers λ .

manufacturer can influence the search cost at no cost. We work under the assumption of observable wholesale prices and search costs.⁷ Then, the retailers compete in the downstream market and set retail prices. The retail price distribution is denoted by $F(p)$ and its density by $f(p)$. Finally, after observing the wholesale price w and the level of the search cost s , but not knowing retail prices, consumers engage in sequential search with perfect recall. We use SPE as the solution concept, given that the wholesale price and search cost are observed.

2.1 Bargaining Protocol

In the first stage, the manufacturer bargains with the retailers over the wholesale price and the obfuscation level. We denote the bargaining power of the manufacturer by β , while the bargaining power of each retailer is $(1 - \beta)$, with $\beta \in (0, 1)$. When discrimination is forbidden, the two retailers pay the same wholesale price to the upstream manufacturer and also negotiate the same obfuscation levels or search costs. In these scenarios, it is not clear what role each retailer plays in determining the wholesale price and search costs.

I follow O'Brien [2014] and allow the manufacturer to randomly select one of the two retailers to negotiate a wholesale price and search cost.⁸ Given that retailers are symmetric in terms of their bargaining power, they are indifferent about which one of them is chosen to bargain with the upstream manufacturer.⁹ Therefore, we assume that the bargaining stage goes as follows. The manufacturer randomly chooses one of the two retailers to bargain with over the contract terms, and after a successful bargaining he then makes a take-it-or-leave-it offer to the remaining retailer. Following Nash [1950], the generalized bargaining process between the manufacturer and the chosen retailer solves:

$$\max_{w,s} [\pi_M(w, s) - \pi_M^0(\hat{w}, \hat{s})]^\beta [\pi_R(w, s) - \pi_R^0]^{1-\beta}$$

$$s.t. \quad \pi_M(w, s) \geq \pi_M^0(\hat{w}, \hat{s}) \quad \text{and} \quad \pi_R(w, s) \geq \pi_R^0$$

where (w, s) is the bargaining outcome; $\pi_M(w, s)$ and $\pi_R(w, s)$ are the profits of the manufacturer and the chosen retailer, respectively, and \hat{w} and \hat{s} are the wholesale price and the search cost negotiated with the remaining retailer. Thus, π_M^0 and π_R^0 are the disagreement profits in case the negotiation with the chosen retailer breaks down. Given that the manufacturer is a monopolist, we normalize the retailers' disagreement profits π_R^0 to zero, while the manufacturer's disagreement profit $\pi_M^0(\hat{w}, \hat{s})$ is determined endogenously.

⁷In a setting without vertical markets, Ellison and Wolinsky [2012] assume that consumers will be aware of the value of the search cost s_i only once they visit firm i . They show that in that case consumers must view firms as ex-ante identical and that the search order will not matter.

⁸Another form of bargaining would be for M to negotiate jointly with both retailers; we discuss this in Section 6.

⁹The setting would be more complicated to analyze if the retailers were asymmetric in terms of their bargaining power. We abstract from such asymmetries for now, but refer the reader to Section 5 for an extension of the model to asymmetric retailers.

3 Equilibrium Analysis

In this section, we solve the model by initially considering the retail market and analyzing consumers' and retailers' behaviour for a given wholesale price w and a given search cost s . Afterwards, we analyze the outcomes of bargaining and also provide comparative static results with respect to the bargaining power parameter β .

3.1 The Retail Market

In a setting with shoppers, $\lambda \in (0, 1)$, and non-shoppers, retailers face a trade-off between charging high prices to extract profit from the non-shoppers and charging low prices to attract shoppers. In such cases there exists no pure strategy equilibrium and there are no mass points in the equilibrium price distribution. Stahl [1989] has shown that there is, however, a unique symmetric equilibrium in mixed strategies where consumers' behaviour satisfies a reservation price property. In this equilibrium, retailers have to be indifferent between charging any price in support $[\underline{p}, \bar{p}]$ of the equilibrium price distribution $F(p)$. Given a mixed strategy chosen by the competitor, a retailer's profit from charging any price p in support of $F(p)$ will be:

$$\pi_R(p, F(p), w) = (p - w) \left[\frac{(1 - \lambda)}{2} + \lambda(1 - F(p)) \right].$$

The first term represents the profit the retailer makes over the non-shoppers, while the second term corresponds to the profit made from the shoppers, whom the retailer serves with probability $(1 - F(p))$. This profit must equal the profit that the retailer makes if it charged the upper bound of the price distribution \bar{p} , which equals:

$$\pi_R(\bar{p}, w) = \frac{(1 - \lambda)}{2}(\bar{p} - w) \tag{1}$$

Janssen, Moraga-González, and Wildenbeest [2005] have shown that in a setting where the first search is costly, the upper bound of the support \bar{p} must be equal to the consumers' reservation price ρ . Therefore, in a symmetric equilibrium, no retailer will have an incentive to charge a price higher than the consumers' reservation price. The equilibrium retail price distribution, shown in Stahl [1989], is characterized in Proposition 1.

Proposition 1 *For $\lambda \in (0, 1)$, the equilibrium price distribution for the subgame starting with a given w and s is given by:*

$$F(p, w) = 1 - \frac{1 - \lambda}{2\lambda} \frac{\bar{p} - p}{p - w} \tag{2}$$

with density

$$f(p, w) = \frac{\bar{p} - w}{(p - w)^2} \frac{1 - \lambda}{2\lambda} \tag{3}$$

and support $[\underline{p}, \bar{p}]$ where $\underline{p} = \frac{(1-\lambda)\bar{p}+2\lambda w}{1+\lambda}$ and $\bar{p} = \rho$.

Proof. See Stahl [1989]. ■

Proposition 1 gives the equilibrium retail price distribution for a given wholesale price w and search cost s , where both are assumed to be observed by the final consumers. Now, we analyze optimal consumer behaviour. The reservation price ρ is the price that makes the non-shoppers indifferent between purchasing at ρ and paying an extra search cost to receive a new price quote from the equilibrium price distribution. Thus, given a distribution of prices $F(p)$ and an observed price p' , the non-shoppers' reservation price, ρ , is determined by solving the following equality:

$$v - \rho = v - s - \int_{\underline{p}}^{\rho} p' f(p) dp$$

Given that in equilibrium $\bar{p} = \rho$, the above expression becomes:

$$\rho = s + E(p) \tag{4}$$

Janssen, Pichler, and Weidenholzer [2011] have shown that the expected price paid by the shoppers, who observe all prices in the market and buy at the lowest price, denoted by $E(p_l)$, with $p_l = \min\{p_1, p_2\}$, can be expressed as:

$$E(p_l) = w + \frac{1-\lambda}{\lambda} s \tag{5}$$

On the other hand, the expected price paid by the non-shoppers $E(p)$ can be written as:

$$E(p) = w + \frac{\alpha}{1-\alpha} s \tag{6}$$

where $\alpha = \int_0^1 \frac{1}{1+\frac{2\lambda}{1-\lambda}z} dz \in [0, 1)$. Note that α goes to one as the fraction of shoppers shrinks, $\lambda \rightarrow 0$.

We make use of these results to simplify the expressions needed when analyzing the first-stage bargaining process. Equations (4) and (6) imply that we can rewrite ρ as:

$$\rho = \bar{p} = w + \frac{s}{1-\alpha} \tag{7}$$

Furthermore, the non-shoppers must find it worthwhile to search once rather than not at all. Therefore, in equilibrium the following full participation condition needs to be satisfied:

$$v - E(p) - s \geq 0$$

which by making use of equation (6) can be rewritten as:

$$v - w - \frac{s}{(1-\alpha)} \geq 0 \tag{8}$$

Finally, by using equation (7) we can rewrite the retail profit given in equation (1) as:

$$\pi_R(w, s) = \frac{s}{(1-\alpha)} \frac{(1-\lambda)}{2} \quad (9)$$

Note that because the upper bound of the price distribution \bar{p} simply adds a mark-up over the wholesale price w , the retail profit does not depend on w ; however, it is decreasing in the fraction of shoppers λ and increasing in non-shoppers' search cost s . This summarizes the behaviour of retailers and consumers for a given w and s . Next, we focus on characterizing the bargaining process outcome between the monopolist manufacturer and the downstream retailers.

3.2 Bargaining over the Wholesale Price and Obfuscation

Suppose the manufacturer bargains with R_1 and makes a take-it-or-leave-it offer to R_2 . In order to determine the outcome of the bargaining process between M and R_1 , we first need to determine the disagreement profit $\pi_{M_1}^0$, which is the profit the manufacturer would obtain if the negotiations with R_1 break down. In case the negotiation with R_1 breaks down, the manufacturer will have to bargain with the last remaining retailer, R_2 . If the negotiations with R_2 fail, then given that there is no other retailer to bargain with, the manufacturer's disagreement profit when bargaining with R_2 is $\pi_{M_2}^0 = 0$. In this instance, R_2 is a monopolist in the market and his profit will be $\pi_{R_2}(w, s) = v - s - w$. The manufacturer's profit will be $\pi_M = w$, since there is a unit mass of final consumers and his production costs are normalized to zero. Therefore, the generalized bargaining process between M and R_2 solves the following problem:

$$\begin{aligned} \max_{w,s} & [(w)^\beta (v - s - w)^{(1-\beta)}] \\ \text{s.t.} & \quad w \geq 0 \quad \text{and} \quad v - s - w \geq 0 \end{aligned} \quad (10)$$

Solving, we obtain $w^* = \beta v$ and $s^* = 0$. Therefore, the manufacturer's profit in case of a successful negotiation with R_2 is $\pi_{M_2} = \beta v$. This profit, which is endogenously determined by negotiations between M and R_2 , serves as the manufacturer's disagreement profit when bargaining with the chosen retailer R_1 . Thus, we can write $\pi_{M_1}^0 = \beta v$. We have calculated and simplified the profit of a given retailer in the retail market analysis above. This profit is given in equation (9) and will now serve as the profit of the chosen retailer R_1 . Furthermore, note that the wholesale price and obfuscation level outcomes are subject to the full participation constraint explained and simplified in equation (8).

Therefore, the generalized Nash bargaining problem between M and R_1 is:

$$\begin{aligned} \max_{w,s} & \left[(w - \beta v)^\beta \left(\frac{s}{(1-\alpha)} \frac{(1-\lambda)}{2} \right)^{(1-\beta)} \right] \\ \text{s.t.} & \quad v - w - \frac{s}{1-\alpha} \geq 0 \end{aligned} \quad (11)$$

Proposition 2 is one of the main results of this analysis and characterizes the equilibrium outcome of the bargaining stage.

Proposition 2 *Under uniform wholesale prices and search costs, the equilibrium wholesale price and search cost are given by:*

$$w^* = v - (1 - \beta)^2 v \quad (12)$$

$$s^* = v(1 - \alpha)(1 - \beta)^2 \quad (13)$$

where $\alpha = \int_0^1 \frac{1}{1 + \frac{2\lambda}{(1-\lambda)z}} dz \in [0, 1)$. The wholesale price is increasing in β , while the search cost is decreasing in β . The profit of the manufacturer increases in β , while the retail profits decrease in β .

The above result shows that if the bargaining power rests downstream, i.e., when $\beta = 0$, the manufacturer sets the wholesale price equal to his marginal cost, which we have normalized to zero, and chooses the highest value of obfuscation that can be set without losing any consumers. By contrast, if the bargaining power lies entirely with the manufacturer, i.e., when $\beta = 1$, the manufacturer does not engage in obfuscation and sets the wholesale price at the monopoly level. The obfuscation level decreases in the manufacturer's bargaining power and is positive for any β smaller than 1. Therefore, this result supports the view that higher obfuscation levels are associated with higher bargaining power of retailers. The manufacturer's profit increases in w and decreases in the obfuscation level faced by the final consumers in the downstream market, while the opposite holds true for the retailers. Therefore, unsurprisingly, the profit of the manufacturer increases in β , while the retailers' profits decrease in β . Figure 1 depicts the equilibrium wholesale price and search cost for different values of the bargaining power parameter.

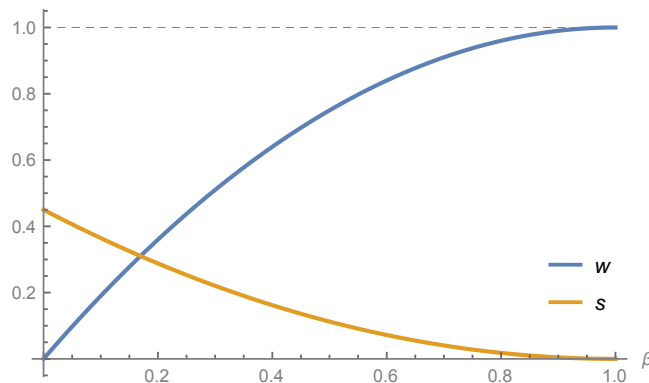


Figure 1: Wholesale price and search cost for different values of β when $v = 1$ and $\lambda = 0.5$

3.3 Comparative Statics

In our model, a decrease in the manufacturer's bargaining power, denoted by β , has two different effects on the expected retail prices: that paid by the shoppers and that paid

by the non-shoppers. First, it decreases the wholesale price charged by the manufacturer to the retailers, thus putting downward pressure on expected retail prices, which makes consumers better off. We call this the *input effect*. Second, a decrease in β increases the search cost that the final consumers face. We call this the *obfuscation effect*. The following proposition shows that in our setting the *input effect* dominates the *obfuscation effect*. Thus, expected prices increase and consumer surplus decreases with an increase of the bargaining power of the manufacturer.

Proposition 3 *The expected price paid by the non-shoppers $E(p)$ and the expected price paid by the shoppers $E(p_I)$ are both increasing in β . Consumer surplus decreases in β . In the limit, as $\beta \rightarrow 1$, $E(p)$, $E(p_I)$, and w^* converge to the monopoly price v .*

In order to understand the mechanism that is driving this result, let us first substitute the optimal bargained values of the wholesale price w^* and search cost s^* into the expected price that the non-shoppers pay given in equation (6). Doing so, we obtain:

$$E(p) = v - (1 - \beta)^2 v + \alpha v (1 - \beta)^2 \quad (14)$$

Taking the derivative of equation (14) with respect to β gives:

$$\frac{\partial E(p)}{\partial \beta} = \underbrace{2v(1 - \beta)}_{\text{input effect}} - \underbrace{2v(1 - \beta)\alpha}_{\text{obfuscation effect}} \quad (15)$$

From equation (15), one can see that, because $\alpha \in [0, 1)$, the input effect dominates the obfuscation effect. As the fraction of shoppers shrinks, $\lambda \rightarrow 0$, α goes to one. In this case, the effects cancel out, and thus the expected price paid by the non-shoppers does not change with β . However, as long as there are some shoppers in the market, α is smaller than one, and thus an increase in the bargaining power of the manufacturer leads to a higher expected retail price. Intuitively, obfuscation affects competition less because it changes the way the non-shoppers search while it does not affect the search process of shoppers. On the other hand, a change in the wholesale price, given that it changes the marginal cost of all units sold, affects both types of consumers.

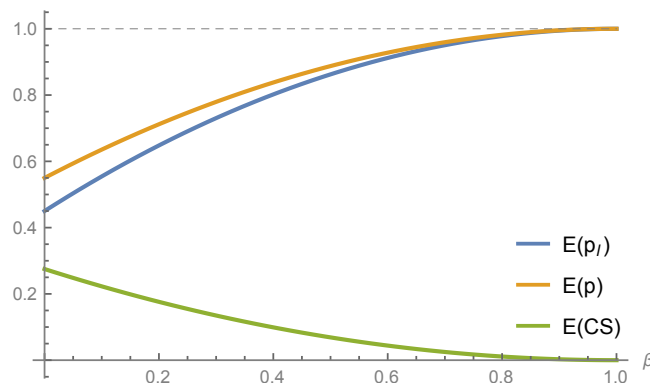


Figure 2: $E(p_I)$, $E(p)$, and CS for different values of β when $v = 1$ and $\lambda = 0.5$

Figure 2 depicts both expected prices and the consumer surplus for different values of β . The expected consumer surplus $E(CS)$ is calculated using the following expression:

$$E(CS) = \lambda(v - E(p_l)) + (1 - \lambda)(v - E(p) - s)$$

The first term on the right denotes the surplus of the shoppers, of which there is a λ share in the market, while the second term denotes the surplus from the non-shoppers, of which there is a $(1 - \lambda)$ share. When the bargaining power rests completely with the manufacturer, there is no obfuscation and thus the downstream market is perfectly competitive. However, while consumers face no search cost, they get no surplus. This is because the manufacturer acts as a monopolist and sets the wholesale price equal to the consumers' valuation. Retailers in turn set retail prices equal to their marginal cost of v . The comparative static results in Proposition 2 and Proposition 3 generate new testable predictions. Specifically, the model predicts that we should observe higher obfuscation levels when retailers have more bargaining power and that expected retail prices will be higher under lower search costs.

In our model, an increase in the share of shoppers increases the search cost s through its effect on α , which unambiguously puts downward pressure on w . Furthermore, an increase in λ decreases the expected retail price that the non-shoppers pay, again through its effect on α , which puts upward pressure on w . Proposition 4 shows that the net result of these two effects on w is zero. In Figure 3, we depict the wholesale price and the search cost for different values of the share of shoppers, while in Figure 4 we show how the expected retail prices and the expected consumer surplus change with λ .

Proposition 4 *The expected price paid by the non-shoppers $E(p)$ and the expected price paid by the shoppers $E(p_l)$ are both decreasing in λ . Consumer surplus increases in λ . In the limit, as $\lambda \rightarrow 1$, $E(p)$ and $E(p_l)$ converge to the wholesale price w^* . The wholesale price w is independent of λ , while the search cost s is increasing in λ .*

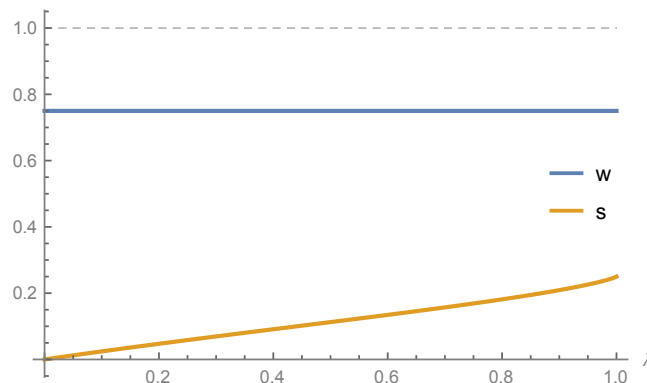


Figure 3: w and s for different values of λ when $v = 1$ and $\beta = 0.5$

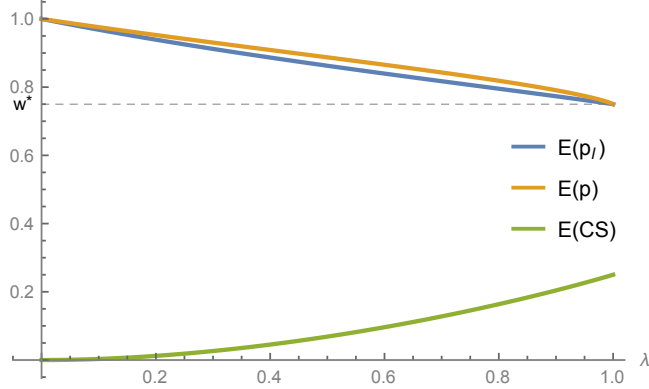


Figure 4: $E(p_l)$, $E(p)$, and $E(CS)$ for different values of λ when $v = 1$ and $\beta = 0.5$

4 Policy Implications

In this section we discuss potential effects of different regulations and show that some of them may have undesired effects in vertical markets.

First, suppose that a regulator imposes a cap of $\bar{s} \geq 0$ on the search cost. Then, the bargained search cost should be below the cap. If the cap is not binding then the bargained outcomes s^* and w^* would not be affected and still be as in equations (12) and (13). However, if the imposed cap \bar{s} is binding, then $s = \bar{s}$ is the optimal search cost and $\bar{w} = v - \frac{\bar{s}}{1-\alpha}$. Given that $\bar{s} < s^*$, this implies that $\bar{w} > w^*$. As we have shown before, such an increase in the wholesale price outweighs the decrease in the search cost and results in a higher expected retail price. So a regulation that would impose a (binding) cap on s would first have the desired effect of limiting obfuscation. However, such a regulation would also have an indirect undesired effect of inducing higher input (wholesale) prices. Such an intervention would lead to higher expected retail prices and thus make final consumers worse off. So we find that while policies that limit obfuscation may be effective in retail markets, they can backfire when imposed in vertical markets.

One can also lower obfuscation in vertical markets by reducing retailers' bargaining power. Recently, with an increase in the buying power of retailers, regulators have been interested in implementing policies of this type. For instance, Hayashida [2019] shows that policy makers in Japan are trying to equalize the bargaining power between suppliers and retailers in the Japanese grocery supply chains. He finds that such a policy translates to higher wholesale prices, which in turn result in higher retail prices and thus lower consumer welfare. In our model, such a policy can be interpreted as an exogenous decrease in β . According to our comparative static result with respect to β , a decrease in retailers' bargaining power leads to higher expected retail prices and thus lower consumer welfare. Therefore, policies that try to reduce retailers' bargaining power may yield undesired effects.

Alternatively, suppose that a regulator implements a policy that increases the share of shoppers in the market. This could be done, for instance, by offering educational

programs or by promoting price comparison websites. In order to analyze if such an intervention would have the desired effects, it suffices to consider what happens to the expected consumer surplus with an increase in λ . In the previous section we showed that an increase in the share of shoppers, λ , leads to lower expected retail prices and thus higher expected consumer surplus. Thus, in these markets, instead of limiting obfuscation, an effective policy on the consumers side could be an intervention that increases the share of shoppers.

Finally, suppose that a regulator imposes a cap $\bar{w} > 0$ on the wholesale price that the manufacturer charges. This would imply that the wholesale price bargained would be $\in [0, \bar{w}]$. If the cap is binding, then in equilibrium $w^* = \bar{w}$ while the search cost would be $\bar{s} = (v - \bar{w})(1 - \alpha) > (v - w)(1 - \alpha) = s^*$. Depending on the increase in the search cost, such a policy could be effective in protecting consumers. An example of such a policy intervention is Ofgem, the government regulator for gas and electricity markets in the UK. Since 2019, the regulator imposes a cap on energy prices and plans to remove it once there is enough evidence of supplier rivalry. We propose that a combination of both caps, one on s and another on w , would be ideal in protecting final consumers. This is because a combination of caps removes the undesired indirect effects of imposing only one type of cap. For instance, we showed that a cap on obfuscation leads to higher wholesale prices; however, if there is a binding cap on the wholesale price as well then this undesired effect is eliminated. Thus, consumers are directly protected from both higher wholesale prices and higher search costs.

5 Extensions

In this section I discuss some extensions of the model.

5.1 Many Retailers

Until now, we have looked at a duopoly setting in the downstream market. This was done with the aim of making the bargaining protocol process between the manufacturer and retailers easy to understand and follow. Here, we analyze the robustness of our results in an oligopoly setting. We find that an additional countervailing buyer power arises in these markets. A larger number of retailers downstream leads to higher wholesale and retail prices. We will show how this mechanism works.

First, assume that there are $N \geq 2$ retailers in the downstream market. As we have shown in Section 3, there exists a unique symmetric equilibrium in mixed strategies. A retailer's profit from charging any price p in support $[\underline{p}, \bar{p}]$ of the equilibrium price distribution $F(p)$ will be:

$$\pi_R(p, F(p), w) = \left[\frac{1 - \lambda}{N} + \lambda(1 - F(p))^{N-1} \right] (p - w)$$

The first term gives the profit that the retailer makes from the non-shoppers, while the second term shows the profit a retailer makes over the shoppers (which he serves with probability $(1 - F(p))^{N-1}$). In a mixed strategy equilibrium, this profit has to be equal to the profit that a retailer makes if it charges the upper bound of the price distribution, which is $\frac{1-\lambda}{N}(\bar{p} - w)$. The equilibrium price distribution is characterized below.

Proposition 5 *For $\lambda \in (0, 1)$, the equilibrium price distribution for the subgame starting with a given w and s is given by:*

$$F(p, w) = 1 - \left(\frac{1 - \lambda}{N\lambda} \frac{\bar{p} - p}{p - w} \right)^{\frac{1}{N-1}} \quad (16)$$

with density

$$f(p, w) = \frac{1}{N-1} \frac{\bar{p} - w}{(p - w)^2} \left(\frac{1 - \lambda}{N\lambda} \right)^{\frac{1}{N-1}} \left(\frac{\bar{p} - p}{p - w} \right)^{\frac{2-N}{N-1}} \quad (17)$$

and support $[\underline{p}, \bar{p}]$ where $\underline{p} = \frac{\lambda N}{\lambda N + 1 - \lambda} w + \frac{1 - \lambda}{\lambda N + 1 - \lambda} \bar{p}$ and $\bar{p} = \rho$.

The optimal consumer behaviour does not change and thus the reservation price ρ and the expected prices $E(p_l)$ and $E(p)$ are as given in equations (4), (5), and (6). Thus, the generalized Nash bargaining problem between M and R_1 is:

$$\max_{w, s} \left[(w - (v - (1 - \beta)^{(N-1)}v))^\beta \left(\frac{s}{(1 - \alpha)} \frac{(1 - \lambda)}{N} \right)^{(1-\beta)} \right] \quad (18)$$

$$s.t. \quad v - w - \frac{s}{1 - \alpha} \geq 0$$

The following proposition characterizes the equilibrium outcome of the bargaining stage for the case of oligopoly instead of the duopoly case that we characterized before.

Proposition 6 *When there are $N \geq 2$ retailers in the downstream market, the wholesale price and search cost are given by:*

$$w^* = v - (1 - \beta)^N v \quad (19)$$

$$s^* = v(1 - \alpha)(1 - \beta)^N \quad (20)$$

where $\alpha = \int_0^1 \frac{1}{1 + \frac{N\lambda}{(1-\lambda)} z^{N-1}} dz \in [0, 1)$. The wholesale price is increasing in β , while the search cost is decreasing in β . The profit of the manufacturer increases in β , while the retail profits decreases in β .

The comparative static result with respect to the number of retailers is given in the following proposition. This result shows that the optimal wholesale price is increasing in the number of retailers, while the opposite holds true for the search cost bargained.

Proposition 7 *The wholesale price increases in N , while the search cost decreases in N . As $N \rightarrow \infty$, w^* converges to the monopoly price v while s^* converges to 0.*

This relates to the concept of countervailing buyer power, which says that greater retail concentration not only increases retailers' market power, but also increases their bargaining power and in this way it can lead to lower input prices. We see that, in our setting, a countervailing effect arises since wholesale prices are indeed lower for smaller N . The Stahl [1989] type models have the interesting feature that the equilibrium expected price $E(p)$ increases with the number of firms. As N increases, competition increases, which pushes firms to charge lower prices; but with an increase in N , the probability of being the cheapest firm also decreases, which puts upward pressure on prices. It has been shown that, overall, the second effect dominates. Here, however, we are showing that under bargaining another force will also drive equilibrium expected retail prices upwards, and this is the change in the input price that the retailers will face. So, a higher N will also mean higher input or wholesale prices.

In this paper, we are showing another way in which countervailing buyer power may arise—by increasing consumers' search costs and not retail concentration. We have seen that an increase in search costs, while leading to higher market power of retailers, also leads to an increase in their bargaining power and lower input prices. It is important, however, to note the difference in the countervailing effects coming from these two distinct situations. An increase in retail concentration increases retailers' bargaining power through decreasing the manufacturer's disagreement profit that the manufacturer obtains in case of a negotiation breakdown. However, an increase in search costs increases the bargaining power of retailers by limiting the scope of the wholesale price that the manufacturer can set without losing final consumers.

5.2 Two-Part Tariffs

Until now our model has worked under the assumption that the vertical contracts between the manufacturer and the retailers are linear in wholesale prices. Once we have positive search costs in the downstream market, such contracts lead to double marginalization problems. Such types of contracts are used extensively in practice.¹⁰ However, there are also markets where firms engage in either optimal or suboptimal non-linear contracts, which enable firms to maximize their joint profits. In this section we extend the model to two-part tariffs. More specifically, we analyze cases in which the manufacturer and retailers do not bargain only over a linear wholesale price w and a search cost s , but also over a fixed fee F . We show that the manufacturer is not better off under two-part tariffs.

¹⁰For evidence on linear contracts see, e.g., Crawford and Yurukoglu [2012] on arrangements between TV channels and cable TV distributors, Grennan [2013] on medical device manufacturers and hospitals, and Gilbert [2015] on book publishers and resellers.

Let us assume that the manufacturer chooses to bargain with R_1 . As we have done before, we first have to determine what the disagreement profit is in case of a negotiation breakdown. This is determined endogenously, by a separate bargaining between M and R_2 . Thus, the generalized bargaining process between M and R_2 solves:

$$\begin{aligned} & \max_{w,F,s} [(w + F)^\beta (v - w - F)^{(1-\beta)}] \\ & \text{s.t. } w + F \geq 0 \quad \text{and} \quad v - w - F \geq 0 \end{aligned} \quad (21)$$

Solving we obtain $w^* = \beta v$, $F^* = 0$, and $s^* = 0$. So, we have determined that the profit in case of negotiation failure equals $\pi_M^0 = \beta v$. Thus, the generalized Nash bargaining problem between M and R_1 is:

$$\begin{aligned} & \max_{w,F,s} \left[(w + F - \beta v)^\beta \left(\frac{s}{(1-\alpha)} \frac{(1-\lambda)}{2} - F \right)^{(1-\beta)} \right] \\ & \text{s.t. } v - w - \frac{s}{1-\alpha} \geq 0 \end{aligned} \quad (22)$$

The proposition below characterizes the equilibrium outcome of this bargaining stage.

Proposition 8 *Under two-part tariffs, the wholesale price, the fixed fee, and the search cost are given by:*

$$w^* = \frac{(-1 + \lambda + 2\beta)v}{(1 + \lambda)} \quad (23)$$

$$F^* = \frac{(1 - \lambda)(1 - \beta)v}{(1 + \lambda)} \quad (24)$$

$$s^* = \frac{2(1 - \alpha)(1 - \beta)v}{(1 + \lambda)} \quad (25)$$

where $\alpha = \int_0^1 \frac{1}{1 + \frac{2\lambda}{1-\lambda}z} dz \in [0, 1)$. The wholesale price w increases in β , while the fixed fee F and the search cost s decrease in β .

Proposition 8 shows that the results we have obtained in the case of linear tariffs are robust even if the manufacturer and retailers bargain over two-part tariffs. More specifically, we find that the wholesale price increases in the manufacturer's bargaining power, while the fixed fee and the search cost decrease in β . It is interesting to point out that the manufacturer is not better off under two-part tariffs. This is because he has to first bargain over the fixed fee and thus cannot simply extract the retail profit completely and second because the search cost under two-part tariffs is higher compared to the case of linear contracts.

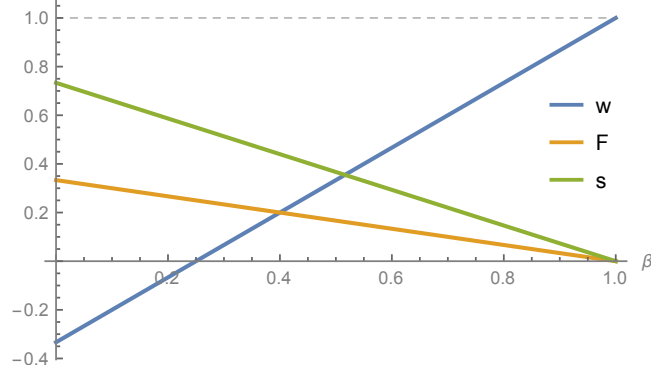


Figure 5: Wholesale price, fixed fee, and search cost for different values of β when $v = 1$ and $\lambda = 0.5$

5.3 Asymmetric Retailers

Until now we have worked under the assumption that retailers are symmetric in terms of their bargaining power. However, it is natural to think that some retailers may have stronger bargaining power compared to others. In such settings, the retailers might prefer that the one with the stronger bargaining power is chosen to bargain with the manufacturer, since better terms could be negotiated for both of them. On the other hand, the manufacturer might prefer to negotiate with the weaker retailer. Therefore, the choice of the negotiating retailer may be more complicated compared to settings with symmetric retailers.

In order to provide answers to such questions, we now analyze a setting where the retailers in the downstream market differ in their bargaining power and have to negotiate with the upstream manufacturer. Let us denote the bargaining power of R_1 by $(1 - \beta_1)$ and the bargaining power of R_2 by $(1 - \beta_2)$, and suppose that $\beta_1 < \beta_2$. Therefore, we are assuming that retailer R_1 is stronger in bargaining than retailer R_2 . Suppose that the manufacturer chooses to negotiate with the weaker retailer R_2 . We know from our previous analysis that if this bargain fails, the manufacturer would have to negotiate with R_1 . So the disagreement profit when M negotiates with R_2 is $\pi_M^0 = \beta_1 v$. So, the Nash bargaining problem between M and R_2 would be:

$$\max_{w,s} \left[(w - \beta_1 v)^{\beta_2} \left(\frac{s}{(1-\alpha)} \frac{(1-\lambda)}{2} \right)^{(1-\beta_2)} \right] \quad (26)$$

$$s.t. \quad v - w - \frac{s}{1-\alpha} \geq 0$$

Alternatively, suppose that M decides to first bargain with the stronger retailer R_1 . In this case, the disagreement profit in case of negotiation failure equals $\pi_M^0 = \beta_2 v$ and thus the Nash bargaining problem between M and R_1 solves:

$$\max_{w,s} \left[(w - \beta_2 v)^{\beta_1} \left(\frac{s}{(1-\alpha)} \frac{(1-\lambda)}{2} \right)^{(1-\beta_1)} \right] \quad (27)$$

$$s.t. \quad v - w - \frac{s}{1-\alpha} \geq 0$$

In the following proposition we characterize the equilibrium outcomes of both of these Nash bargaining problems. We show that, under endogenous disagreement profit calculations, it does not matter which retailer is chosen as the "representative" retailer to bargain with the manufacturer since the equilibrium outcomes of both of these problems coincide.¹¹ In addition, we show that, as long as at least one of the retailers has some bargaining power, the equilibrium will exhibit obfuscation.

Proposition 9 *Let $N = 2$, and let the bargaining power of R_1 and R_2 be denoted by $(1 - \beta_1)$ and $(1 - \beta_2)$, respectively, where $\beta_1 < \beta_2$. No matter which retailer is chosen by the manufacturer to bargain with, the wholesale price and search cost are given by:*

$$w^* = v - (1 - \beta_1)(1 - \beta_2)v$$

$$s^* = (1 - \alpha)(1 - \beta_1)(1 - \beta_2)v$$

where $\alpha = \int_0^1 \frac{1}{1 + \frac{2\lambda}{1-\lambda}z} dz \in [0, 1)$. The wholesale price w^* increases in β_1 and β_2 , while the search cost s^* decreases in β_1 and β_2 .

Thus, as long as the bargaining protocol remains the same, where one retailer is chosen at random and the other receives a take-it-or-leave-it offer from the manufacturer, the results of this paper are robust to retailers having different bargaining powers. This is because no matter which retailer is chosen as a representative, the tension between the manufacturer wanting higher wholesale prices and lower obfuscation levels and the retailers preferring the opposite does not disappear, nor does it depend on differences in the retailers' bargaining powers.

5.4 Bargaining over λ Instead of s

Up to this point, we have thought of obfuscation as an action that increases consumers' search cost. However, we can also think of obfuscation as an action that leads to a

¹¹The issue would be different if the legal regulation on wholesale price discrimination imposed on the manufacturer would also have to hold for the disagreement profits. This would imply that in case of negotiation failure with either one of the retailers, the manufacturer's disagreement profit would be set to zero. In this case, if M chose to bargain with R_1 , then we would have $w^* = \beta_1 v$ and $s^* = (1 - \alpha)(1 - \beta_1)v$, while if M were to bargain with R_2 we would have $w^* = \beta_2 v$ and $s^* = (1 - \alpha)(1 - \beta_2)v$. In such scenarios, coordination issues may arise if retailers are able to coordinate their actions, and thus the weaker retailer could simply refuse to negotiate with the manufacturer. If we abstract away such coordination possibilities in our analysis, we could think that M would choose to bargain with R_2 , which would lead to an equilibrium with higher wholesale prices and lower search costs under asymmetric retailers.

smaller share of shoppers in the market. Thus, we would then have a setting in which the manufacturer and retailers bargain over the wholesale price w and the share of shoppers λ , while the search cost s would be exogenously given. In that case, if the bargaining with R_1 fails, the manufacturer bargains with the remaining retailer R_2 . Thus, the generalized bargaining process between M and R_2 solves the following problem:

$$\begin{aligned} \max_{w,\lambda} & [(\lambda w)^\beta (\lambda(v-w))^{(1-\beta)}] \\ \text{s.t.} & \quad w \geq 0 \quad \text{and} \quad v-w \geq 0 \end{aligned} \tag{28}$$

Solving we obtain $w^* = \beta v$ and $\lambda^* = 1$. Therefore, the manufacturer's profit in case of a successful negotiation with R_2 is $\pi_{M_2} = \beta v$. This profit, which is endogenously determined by negotiations between M and R_2 , serves as the manufacturer's disagreement profit when bargaining with the chosen retailer R_1 . Thus, we can write $\pi_{M_1}^0 = \beta v$. We have calculated and simplified the profit of a given retailer in the retail market analysis above. This profit is given in equation (9) and will now serve as the profit of the chosen retailer R_1 . Furthermore, note that the wholesale price and obfuscation level outcomes are subject to the full participation constraint explained and simplified in equation (8). Therefore, the generalized Nash bargaining problem between M and R_1 in this case is:

$$\begin{aligned} \max_{w,\lambda} & \left[(w - \beta v)^\beta \left(\frac{s}{(1-\alpha)} \frac{(1-\lambda)}{2} \right)^{(1-\beta)} \right] \\ \text{s.t.} & \quad v - w - \frac{s}{1-\alpha} \geq 0 \end{aligned} \tag{29}$$

where $\alpha = \int_0^1 \frac{1}{1+\frac{2\lambda}{1-\lambda}z} dz \in [0, 1)$.

The proposition below characterizes the equilibrium outcome of this bargaining stage.

Proposition 10 *When bargaining over λ and w , the wholesale price is given by:*

$$w^* = v - 2sf(\lambda^*)$$

where $f(\lambda^*) = \frac{\lambda}{2\lambda - (1-\lambda)\log[\frac{1+\lambda}{1-\lambda}]}$ and λ^* is the solution to:

$$(1 - \lambda^*)(v(1 - \beta)^2 - 2sf(\lambda^*))f'(\lambda^*) - (1 - \beta)f(\lambda^*)(v(1 - \beta) - 2sf(\lambda^*)) = 0$$

The wholesale price w and the share of shoppers λ are increasing in β . In the limit, as $\beta \rightarrow 1$, λ^ goes to 1, while $f(\lambda^*) \rightarrow \frac{1}{2}$ and thus w^* converges to $v - s$.*

6 Conclusion

In this paper, we analyze obfuscation practices that come from upstream manufacturers rather than downstream firms. Such practices, which increase consumers' costs of searching for prices, are widespread in many different markets. Manufacturers can obfuscate by

imposing different vertical restraints that limit the information consumers have on prices or products. We show that obfuscation will arise once retailers have some bargaining power. On the other hand, when the bargaining power lies entirely with the monopolist manufacturer, no obfuscation occurs in equilibrium and thus the downstream market is perfectly competitive. The fact that there is no obfuscation does not imply, however, that consumers are better off, since the manufacturer acts as a monopolist and charges monopoly prices to its retailers, which then charge monopoly prices to the final consumers.

The findings suggest that regulators should take into account the market structure when designing consumer protection policies. For instance, we find that policies that put caps on obfuscation may backfire in vertical markets. In addition to the desired effect of limiting obfuscation, they also have an undesired effect of inducing higher wholesale prices. The findings suggest that policies that put caps on wholesale prices or that induce an increase in the share of shoppers may be effective instead. Recently, such a policy that limits wholesale prices is being used by the regulator of gas and electricity markets in the UK.

The bargaining protocol used relates to the “delegation approach” method used in the theoretical bargaining literature. In many applied fields of economics, such as labour, international, and financial economics, a group of individuals is considered as a single bargainer. This approach is suitable especially in settings where the group members are symmetric. Thus, in our case, given that we consider symmetric retailers in terms of marginal costs and in terms of their bargaining power, this seems to be a simplified and reasonable protocol to follow. We have also shown that the findings are robust even if retailers were to differ in their bargaining powers, even though the issue of choosing the retailer with whom to negotiate becomes more subtle. Another form of bargaining would be for manufacturers to negotiate jointly with retailers. Our findings are robust even under such a bargaining protocol, however we do not focus on this here since such forms of bargaining seem improbable and might be dubious from an antitrust perspective given that retailers have to compete in the downstream market.

References

- John Asker and Heski Bar-Isaac. Vertical Information Restraints: Pro- and Anti-Competitive Impacts of Minimum Advertised Price Restrictions. *Journal of Law and Economics*, 63(1):111–148, 2020.
- B.I. Carlin. Strategic Price Complexity in Retail Financial Markets. *Journal of Financial Economics*, vol. 91, pages 278–287, 2009.
- Competition Commission. Supermarkets: A Report on the Supply of Groceries from Multiple Stores in the United Kingdom. *London*, 2000.

- Competition Commission. The Supply of Groceries in the UK Market Investigation. *London*, 2008.
- G.S. Crawford and A. Yurukoglu. The Welfare Effects of Bundling in Multichannel Television Markets. *American Economic Review*, vol. 102(2), pages 643–685, 2012.
- Paul W. Dobson. Buyer-Driven Vertical Restraints. *Pros and Cons of Vertical Restraints*, Stockholm, Konkurrensverket, 2008.
- M. Draganska, D. Klapper, and S.B. Villas-Boas. A Larger Slice or a Larger Pie? An Empirical Investigation of Bargaining Power in the Distribution Channel. *Marketing Science*, vol. 29(1), pages 57–74, 2010.
- Glen Ellison and Sara Fisher Ellison. Search and Obfuscation in a Technologically Changing Retail Environment: Some Thoughts on Innovation and Policy. *Innov. Policy Econ*, pages 1–25, 2018.
- Glen Ellison and Alexander Wolinsky. A Search Cost Model of Obfuscation. *The RAND Journal of Economics*, (43)3, pages 417–441, 2012.
- European Commission. Final Report on the E-commerce Sector Inquiry. http://ec.europa.eu/competition/antitrust/sector_inquiry_sw_d_en.pdf, 2017.
- Tobias Gamp. Search, Differentiated Products and Obfuscation. *Working Paper*, University of Bonn, 2016.
- Daniel Garcia and Maarten Janssen. Retail Channel Management in Consumer Search Markets. *International Journal of Industrial Organization*, 58:162–182, 2018.
- Daniel Garcia, Jun Honda, and Maarten Janssen. The Double Diamond Paradox. *American Economic Journal: Microeconomics*, 9(3):63–99, 2017.
- R.J. Gilbert. E-books: A Tale of Digital Disruption. *Journal of Economic Perspectives*, vol. 29(3), pages 165–84, 2015.
- M. Grennan. Price Discrimination and Bargaining: Empirical Evidence from Medical Devices. *American Economic Review*, vol. 103(1), pages 145–77, 2013.
- Kohei Hayashida. Bargaining Power between Food Processors and Retailers: Evidence from Japanese Milk Transactions. *Working Paper*, 2019.
- K. Ho and R.S. Lee. Insurer Competition in Health Care Markets. *Econometrica*, vol. 85(2), pages 379–417, 2017.
- R. Inderst and C. Wey. Buyer Power and Supplier Incentives. *European Economic Review*, 51, pages 647–667, 2007.

- Maarten Janssen and Edona Reshidi. Retail Discrimination in Search Markets. *University of Vienna, mimeo*, 2019.
- Maarten Janssen and Sandro Shelegia. Consumer Search and Double Marginalization. *The American Economic Review*, 105(6):1683–1710, 2015.
- Maarten Janssen and Sandro Shelegia. Beliefs and Consumer Search in a Vertical Industry. *Journal of the European Economic Association*, 18(5):2359–2393, 2020.
- Maarten Janssen, José Luis Moraga-González, and Matthijs R. Wildenbeest. Truly Costly Sequential Search and Oligopolistic Pricing. *International Journal of Industrial Organization*, pages 451–466, 2005.
- Maarten Janssen, Paul Pichler, and Simon Weidenholzer. Oligopolistic Markets with Sequential Search and Production Cost Uncertainty. *The RAND Journal of Economics*, vol. 42(3), pages 444–470, 2011.
- Dmitry Lubensky. A model of recommended retail prices. *The RAND Journal of Economics*, 48(2):358–386, 2017.
- J.F. Nash, Jr. The Bargaining Problem. *Econometrica* 18(2), pages 155–162, 1950.
- Daniel O’Brien. The Welfare Effects of Third-Degree Price Discrimination in Intermediate Good Markets: The Case of Bargaining. *The RAND Journal of Economics*, vol. 45(1), pages 92–115, 2014.
- OECD. Policy Roundtables: Monopsony and Buyer Power, DAF/COMP. *Paris*, 2009.
- Vaiva Petrikaitė. Consumer Obfuscation by a Multiproduct Firm. *The RAND Journal of Economics*, vol. 49, pages 206–223, 2018.
- Michele Piccione and Ran Spiegler. Price Competition Under Limited Comparability. *Quarterly Journal of Economics*, 127, pages 97–135, 2012.
- A. Rhodes, M. Watanabe, and J. Zhou. Multiproduct Intermediaries. *Journal of Political Economy*, 129(2):421–464, 2021.
- Timothy Richards, Gordon Klein, Celine Bonnet, and Zohra Bouamra-Mechemache. Strategic obfuscation and retail pricing. *Review of Industrial Organization*, 57, 12 2020.
- Dale O Stahl. Oligopolistic pricing with sequential consumer search. *The American Economic Review*, pages 700–712, 1989.
- Chris Wilson. Ordered Search and Equilibrium Obfuscation. *International Journal of Industrial Organization*, 28, pages 496–506, 2010.

7 Appendix

Proof of Proposition 2: The first order conditions for equation (11) are:

$$2^{(-1+\beta)}\beta\left(\frac{s(1-\lambda)}{(1-\alpha)}\right)^{(1-\beta)}(w-\beta v)^{(-1+\beta)}-\mu=0 \quad (30)$$

$$\frac{2\mu-2^\beta(-1+\beta)(-1+\lambda)\left(\frac{s(-1+\lambda)}{(-1+\alpha)}\right)^{-\beta}(w-\beta v)^\beta}{2(-1+\alpha)}=0 \quad (31)$$

$$\mu\geq 0, \quad \mu\left(v-w-\frac{s}{1-\alpha}\right)=0 \quad (32)$$

where μ is the Lagrangian multiplier. We obtain equations (12) and (13) by solving equations (30), (31), and 32. The second order conditions for equation (11) are:

$$\begin{aligned} -2^{(-1+\beta)}\beta(1-\beta)\left(\frac{s(1-\lambda)}{(1-\alpha)}\right)^{(1-\beta)}(w-\beta v)^{(-2+\beta)} &< 0 \\ \frac{-2^{(-1+\beta)}\beta(1-\beta)\left(\frac{s(1-\lambda)}{(1-\alpha)}\right)^{(1-\beta)}(w-\beta v)^\beta}{s^2} &< 0 \end{aligned}$$

We now derive the comparative static result. Taking the derivative of equation (12) with respect to β , we obtain:

$$\frac{\partial w^*}{\partial \beta}=2v(1-\beta)>0$$

On the other hand, taking the derivative of equation (13) with respect to β gives:

$$\frac{\partial s^*}{\partial \beta}=-2v(1-\alpha)(1-\beta)<0$$

Substituting equations (12) and (13) into the manufacturer's profit function and into the retailer's profit functions gives: $\pi_M^*=\beta v(2-\beta)$ and $\pi_{R_i}^*=v(1-\beta)^2\frac{(1-\lambda)}{2}$, where $i=1,2$. Taking the derivative with respect to β gives: $\frac{\partial \pi_M^*}{\partial \beta}=2v(1-\beta)>0$ and $\frac{\partial \pi_{R_i}^*}{\partial \beta}=-v(1-\alpha)(1-\beta)<0$.

Proof of Proposition 3: Substituting the equilibrium wholesale price w^* given in equation (12) and the equilibrium search cost s^* given in equation (13) into equations (5) and (6) gives:

$$E(p_i)=\beta v(2-\beta)+\frac{(1-\lambda)v(1-\alpha)(1-\beta)^2}{\lambda} \quad (33)$$

$$E(p)=\beta v(2-\beta)+\alpha v(1-\beta)^2 \quad (34)$$

Taking the derivative of equations (33) and (34) with respect to β , we obtain:

$$\frac{\partial E(p_i)}{\partial \beta} = \frac{2v(1-\beta)[\lambda(1-\alpha) + \alpha]}{\lambda} > 0$$

$$\frac{\partial E(p)}{\partial \beta} = 2v(1-\alpha)(1-\beta) > 0$$

On the other hand, the expected consumer surplus becomes:

$$E(CS) = v\lambda(1-\beta)^2 - s(1-\lambda) \quad (35)$$

Taking the derivative of equation (35) with respect to β , we obtain:

$$\frac{\partial E(CS)}{\partial \beta} = -2\lambda v(1-\beta) < 0$$

Proof of Proposition 4: Taking the derivative of equations (33) and (34) with respect to λ , we obtain:

$$\frac{\partial E(p_i)}{\partial \lambda} = \frac{2v(1-\beta)[\lambda(1-\alpha) + \alpha]}{\lambda} < 0$$

$$\frac{\partial E(p)}{\partial \lambda} = 2v(1-\alpha)(1-\beta) < 0$$

Finally, taking the derivative of equation (35) with respect to λ , we obtain:

$$\frac{\partial E(CS)}{\partial \lambda} = v(1-\beta)^2 + s > 0$$

On the other hand, the derivatives of equations (12) and (13) with respect to the share of shoppers are as follows: $\frac{\partial w}{\partial \lambda} = 0$ and $\frac{\partial s}{\partial \lambda} = -v(1-\beta)^2 \frac{\partial \alpha}{\partial \lambda} > 0$ because $\frac{\partial \alpha}{\partial \lambda} < 0$.

Proof of Proposition 6: It is easy to observe that the manufacturer's disagreement profit $\pi_{M,N}^0$ satisfies the following recursive relation:

$$\pi_{M,N}^0 = \begin{cases} \beta(v - \pi_{M,N-1}^0) + \pi_{M,N-1}^0 & N \geq 1 \\ 0 & N = 0 \end{cases} \quad (36)$$

We claim that $\pi_{M,N}^0 = v - (1-\beta)^N v$. We proceed by induction. The base case holds trivially. Now, assume that this holds for $(N-1)$, i.e., $\pi_{M,N-1}^0 = v - (1-\beta)^{N-1} v$. We now show that $\pi_{M,N}^0 = v - (1-\beta)^N v$. Indeed,

$$\begin{aligned} \pi_{M,N}^0 &= \beta(v - w_{N-1}) + w_{N-1} \\ &= \beta(v - (v - (1-\beta)^{N-1} v)) + v - (1-\beta)^{N-1} v \\ &= v - (1-\beta)^N v, \end{aligned}$$

which proves our claim.

We can thus write the disagreement profit when M bargains with one out of N retailers as $\pi_M^0 = v - (1-\beta)^{N-1} v$, and so in this case the Nash bargaining product between M and

R_1 becomes the one given in equation (18). Substituting the binding full participation condition $w = v - \frac{s}{1-\alpha}$ into equation (18) we have:

$$\max_{w,s} \left[\left(v - \frac{s}{1-\alpha} - (v - (1-\beta)^{N-1}v) \right)^\beta \left(\frac{s}{(1-\alpha)} \frac{(1-\lambda)}{N} \right)^{(1-\beta)} \right] \quad (37)$$

Maximizing equation (37) with respect to s gives:

$$s = v(1-\alpha)(1-\beta)^N$$

Substituting this into $w = v - \frac{s}{1-\alpha}$, we obtain: $w = v - (1-\beta)^N v$.

Now we can derive the comparative static results. Taking the derivative of equation (19) with respect to β we get $\frac{\partial w}{\partial \beta} = N(1-\beta)^{N-1}v > 0$, while taking the derivative of equation (20) with respect to β we have $\frac{\partial s}{\partial \beta} = -Nv(1-\alpha)(1-\beta)^{N-1} < 0$.

Proof of Proposition 7: Taking the derivative of equation (19) with respect to N , we obtain:

$$\frac{\partial w^*}{\partial N} = -v(1-\beta)^N \ln(1-\beta) > 0$$

since $\ln(1-\beta) < 1$ given $\beta \in [0, 1]$.

On the other hand, taking the derivative of equation (20) with respect to N gives:

$$\frac{\partial s^*}{\partial N} = v(1-\beta)^N \left[(1-\alpha) \ln(1-\beta) - \frac{\partial \alpha}{\partial N} \right] < 0$$

This is because $[(1-\alpha) \ln(1-\beta) - \frac{\partial \alpha}{\partial N}] < 0$ given $\ln(1-\beta) < 0$, and since $\frac{\partial \alpha}{\partial N} > 0$.

Proof of Proposition 8: Rewriting the binding full participation constraint $w = v - \frac{s}{1-\alpha}$ and substituting it into equation (22) gives:

$$\max_{w,F,s} \left[\left(v - \frac{s}{1-\alpha} + F - \beta v \right)^\beta \left(\frac{s}{(1-\alpha)} \frac{(1-\lambda)}{2} - F \right)^{(1-\beta)} \right] \quad (38)$$

Maximizing equation (38) with respect to s yields:

$$s = \frac{(1-\alpha) [F((1+\beta) - (1-\beta)\lambda) + (1-\beta)^2(1-\lambda)v]}{(1-\lambda)} \quad (39)$$

Substituting equation (39) into (38) and then maximizing equation (38) with respect to F gives:

$$F^* = \frac{(1-\beta)(1-\lambda)v}{(1+\lambda)} \quad (40)$$

Substituting equation (40) into (39) gives:

$$s^* = \frac{2(1-\alpha)(1-\beta)v}{(1+\lambda)} \quad (41)$$

Finally, substituting equation (41) into $w = v - \frac{s}{1-\alpha}$ gives:

$$w^* = \frac{(-1 + \lambda + 2\beta)v}{(1 + \lambda)} \quad (42)$$

Taking the derivative of equation (42) with respect to β gives: $\frac{2v}{(1+\lambda)} > 0$, while the derivative of equation (41) with respect to β gives $\frac{-2(1-\alpha)v}{(1+\lambda)} < 0$. Finally, taking the derivative of equation (40) with respect to β equals $\frac{-(1-\lambda)v}{(1+\lambda)} < 0$.

Proof of Proposition 9: We can rewrite the full participation constraint as $w = v - \frac{s}{1-\alpha}$, and since it is binding we can substitute it into equation (26) to obtain:

$$\max_{w,s} \left[\left(v - \frac{s}{1-\alpha} - \beta_1 v \right)^{\beta_2} \left(\frac{s}{(1-\alpha)} \frac{(1-\lambda)}{2} \right)^{(1-\beta_2)} \right] \quad (43)$$

Taking the first order condition of equation (43) with respect to s and solving for s , we obtain:

$$s^* = (1-\alpha)(1-\beta_1)(1-\beta_2)v \quad (44)$$

Substituting equation (44) into w , we obtain:

$$w^* = v - (1-\beta_1)(1-\beta_2)v \quad (45)$$

The optimal values of w^* and s^* when the manufacturer bargains with R_1 are obtained in the same manner. Now we derive the comparative static results. Taking the derivative of equation (44) with respect to β_1 and β_2 , we obtain: $\frac{\partial s}{\partial \beta_1} = -(1-\alpha)(1-\beta_2)v < 0$ and $\frac{\partial s}{\partial \beta_2} = -(1-\alpha)(1-\beta_1)v < 0$, respectively. On the other hand, taking the derivative of equation (45) with respect to β_1 and β_2 , we obtain: $\frac{\partial w}{\partial \beta_1} = (1-\beta_2)v > 0$ and $\frac{\partial w}{\partial \beta_2} = (1-\beta_1)v > 0$, respectively.

Proof of Proposition 10: By making use of the fact that $\alpha = \int_0^1 \frac{1}{1+\frac{2\lambda}{1-\lambda}z} dz$ and the binding constraint $w = v - \frac{s}{1-\alpha}$, we can rewrite equation (29) as:

$$\max_{w,\lambda} \left[\left((1-\beta)v - \frac{2s\lambda}{2\lambda - (1-\lambda)\log[\frac{1+\lambda}{1-\lambda}]} \right)^\beta \left(\frac{s(1-\lambda)\lambda}{2\lambda - (1-\lambda)\log[\frac{1+\lambda}{1-\lambda}]} \right)^{(1-\beta)} \right] \quad (46)$$

Define the following function:

$$f(\lambda) = \frac{\lambda}{2\lambda - (1-\lambda)\log[\frac{1+\lambda}{1-\lambda}]}$$

It is easy to see that this function is always positive and its derivative is:

$$f'(\lambda) = \frac{2\lambda - (1+\lambda)\log[\frac{1+\lambda}{1-\lambda}]}{(1+\lambda)(2\lambda - (1-\lambda)\log[\frac{1+\lambda}{1-\lambda}])^2} \quad (47)$$

which is always negative. Thus, this function is always positive, but as λ increases, its value decreases. We can write the initial problem in equation (46) as follows:

$$(s(1 - \lambda)f(\lambda))^{1-\beta} (v(1 - \beta) - 2sf(\lambda))^\beta \quad (48)$$

To ensure that this value is positive, we assume that the following holds:

$$v(1 - \beta) - 2sf(\lambda) > 0$$

This implicitly defines a lower value for λ , call it $\tilde{\lambda}$. Thus, we are considering $\lambda \in (\tilde{\lambda}, 1]$. Now, notice that if $\lambda = \tilde{\lambda}$, the right side of equation (48) would equal 0, so the whole value equals 0. On the other hand, if $\lambda = 1$, the left side of equation (48) will equal 0 and thus once again the whole value would be 0. Below I argue that the function is maximized for some interior value of $\lambda \in (\tilde{\lambda}, 1]$. Taking the derivative of equation (48) with respect to λ , we obtain:

$$\frac{(s(s(1-\lambda)f(\lambda))^{-\beta}(v(1-\beta)-2sf(\lambda))^\beta((-1+\beta)f(\lambda)(v(-1+\beta)+2sf(\lambda))+(-1+\lambda)(v(-1+\beta)^2-2sf(\lambda))f'(\lambda))}{-(v(1-\beta)-2sf(\lambda))}$$

Since we have assumed that $(v(1 - \beta) - 2sf(\lambda)) > 0$, it must be the case that the sign of the denominator is negative. Furthermore, $(s(s(1 - \lambda)f(\lambda))$ is clearly positive, as is $(v(1 - \beta) - 2sf(\lambda))$. Thus, the sign of the derivative, taking into account the sign of the denominator, is determined by:

$$(1 - \beta)f(\lambda)(v(-1 + \beta) + 2sf(\lambda)) + (1 - \lambda)(v(-1 + \beta)^2 - 2sf(\lambda))f'(\lambda)$$

which we can write as:

$$(1 - \lambda)(v(1 - \beta)^2 - 2sf(\lambda))f'(\lambda) - (1 - \beta)f(\lambda)(v(1 - \beta) - 2sf(\lambda))$$

Now, when $\lambda = \tilde{\lambda}$, the above reduces to $(1 - \lambda)(v(1 - \beta)^2 - 2sf(\lambda))f'(\lambda)$. If $v(1 - \beta) - 2sf(\lambda) = 0$, then clearly $(1 - \beta)^2 - 2sf(\lambda) < 0$ and since $f'(\lambda)$ is negative, the derivative is initially positive. Now, as we increase λ further, $-(1 - \beta)f(\lambda)(v(1 - \beta) - 2sf(\lambda))$ is continuously becoming more negative. As λ goes to 1, $f(\lambda)$ converges to $\frac{1}{2}$, and $-(1 - \beta)f(\lambda)(v(1 - \beta) - 2sf(\lambda))$ converges to $\frac{1}{2}(v(1 - \beta) - s)(1 - \beta)$, while the left part, $(1 - \lambda)(v(1 - \beta)^2 - 2sf(\lambda))f'(\lambda)$, converges to 0 as λ goes to 1. Thus, to ensure that the derivative eventually becomes negative, it must be that $v(1 - \beta) > s$. To recap, if $\lambda = \tilde{\lambda}$, the initial derivative is positive, it is monotonically decreasing as λ increases, and if $v(1 - \beta) > s$, then it eventually becomes negative. Thus, there must exist some λ^* that sets the derivative equal to 0. We define λ^* implicitly by setting the derivative equal to 0:

$$(1 - \lambda^*)(v(1 - \beta)^2 - 2sf(\lambda^*))f'(\lambda^*) - (1 - \beta)f(\lambda^*)(v(1 - \beta) - 2sf(\lambda^*)) = 0 \quad (49)$$

Now we derive the comparative static results. Our implicit function is:

$$F(v, s, \beta, \lambda) = (1 - \lambda^*)(v(1 - \beta)^2 - 2sf(\lambda^*))f'(\lambda^*) - (1 - \beta)f(\lambda^*)(v(1 - \beta) - 2sf(\lambda^*)) = 0 \quad (50)$$

From the implicit function theorem $\frac{\partial \lambda}{\partial \beta} = -\frac{\frac{\partial F(v, s, \beta, \lambda)}{\partial \beta}}{\frac{\partial F(v, s, \beta, \lambda)}{\partial \lambda}}$. We know from above that the denominator is negative and the first order derivative is monotonically decreasing in λ . So the sign of $\frac{\partial \lambda}{\partial \beta}$ is determined by $\frac{\partial F(v, s, \beta, \lambda)}{\partial \beta}$. Taking the derivative of equation (49) with respect to β , we obtain:

$$2f(\lambda^*)(v(1 - \beta) - sf(\lambda^*)) - 2v(1 - \beta)(1 - \lambda^*)f'(\lambda^*)$$

$2f(\lambda^*)(v(1 - \beta) - sf(\lambda^*))$ is positive, $2v(1 - \beta)(1 - \lambda^*)$ is positive, and $f'(\lambda^*)$ is negative, thus the expression is positive. As a consequence, $\frac{\partial \lambda^*}{\partial \beta} > 0$. On the other hand, we have $w^* = v - 2sf(\lambda^*)$. Taking the first derivative with respect to β we obtain: $-2sf'(\lambda) \frac{\partial \lambda}{\partial \beta}$. We showed that $\frac{\partial \lambda}{\partial \beta} > 0$ and we know that $f'(\lambda) < 0$, thus $\frac{\partial w}{\partial \beta} > 0$.