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Not your average firm: A quantile regression approach to the firm level investment

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**Not Your Average Firm: a quantile regression approach to the firm
level investment**

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Working Paper No: 2021-02

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Abstract

The large majority of the work published on firm investment is done in the neoclassical frame of a rational optimizing firm attempting to achieve optimal size. While this frame addresses one important consideration in firm investment, it has two important shortcomings that this paper will address. First, it doesn't have a clear interpretation of how the cash-flows are affecting the firm investment decisions. Second, the standard approach operates on an "average firm," which in fact is significantly different from a firm with modal investment behavior. This study employs a Bayesian quantile regression model that yields two significant results. First concerning the relative responsiveness of these two neglected factors, it determines that the firms with higher investment rates have higher responsiveness to the valuation ratio and lower responsiveness to the profit rate. Second and of broader political economic note, it finds a decline in the responsiveness of firm investment to these factors that is consistent with the widely discussed macroeconomic "secular stagnation" of the US economy, and within that consistency, that the decline varies across sectors, and is more pronounced in firms with higher investment rates.

Keywords: Tobin's Q; Investment Rate; Profit Rate; Finance Constraint; Secular Stagnation; Bayesian Econometrics; Bayesian Quantile Regression

JEL Classification: D22; D24; E12; E22; G11

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1 Introduction

Theories explaining the investment decision of the firms consider the valuation ratio as one of the key variables. The empirical research investigating future expected profits and the investment rate relationship either did not indicate a high explanatory power of the valuation ratio or have shown the relevance of other factors which proved to be of equal importance (Abel and Blanchard (1983), Fazzari et al. (1987), Gilchrist and Himmelberg (1995)) while another branch empirical of research point out to the shortcomings of the estimation methods (Schaller (1990), Erickson and Whited (2000), Erickson and Whited (2006)). Disparities in the firm level investment rates are far from being symmetric, in other words, investment rate distribution is not Gaussian. Yet, the investment rate distribution show significant persistence in its skewed shape. Arguing against the plausibility of the neo-classical theory of investment, the evolutionary theory of firm investment/growth do not assume profit maximizing-rationally forecasting firms but a growth process characterized with replicator dynamics, suggesting a domain of investment rates as equilibrium outcomes (Coad (2010)). This paper aims to show the implications of these disparities to the firms' responsiveness to the measures of profitability and valuation while revisiting the widely documented secular slowdown in the investments.

2 Literature Review

"The *rate* of investment" Tobin (1969) states "should be related, if anything, to q , the value of capital relative to its replacement cost". While studying the separation of ownership and management Keynes (1936) have pointed out to the real impacts of valuations for corporate securities on real economic activity. Keynes (1936) points out with the emergence of stock exchange a mechanism that revalues the investments every day have appeared and without this mechanism there is no object frequently attempting to do this task. Keynes famously says:

But the daily revaluations of the Stock Exchange (...) inevitably exert a decisive influence on the rate of current investment. For there is no sense in building up a new enterprise at a cost greater than that at which a similar existing enterprise can be purchased; whilst there is an inducement to spend on a new project what may seem an extravagant sum, if it can be floated off on the Stock Exchange at an immediate profit.

Keynes, therefore, points out not only marginal efficiency of capital but also state of confidence determines the investments. The uncertainty in financial markets, having a direct impact on the firms' market value, now has a direct impact on the real investment decisions as well.

Crotty (1990) points out in Keynes' view, when main indicators for revaluing the investment offer conflicting signals, investors would follow the stock market's views and thus as ownership and management are separated, the managers could only follow the expectations of stockholders. Foley et al. (2019) study the effects of separation of ownership

and management in a corporate capitalism model where they identify “rentier capitalism”, “managerial capitalism” and “hybrid capitalism” regimes. The separation of ownership and management show itself in managers’ sensitivity to the q -ratio where this sensitivity constitutes regimes where either investment or consumption adjust, or a spectrum of cases where adjustment from both sides occur. [Foley and Sidrauski \(1970\)](#) abstracting away from ownership-management conflicts, studied portfolio choice, investment rates and the impact of the monetary policy using the rental rate per unit of value of capital as a key indicator of investments. Later, [Foley \(2020\)](#) pointed out this line of research have suggested, when asset markets reach to both stock and flow equilibrium conditions, expectations of the agents need to be fulfilled. However, majority of the literature in the following years, according to [Foley \(2020\)](#), have assumed fulfilled expectations at the very beginning. Indeed, [Lucas Jr and Prescott \(1971\)](#) assumes a competitive environment with many small firms and distinguishes between current value of a unit of capital and value per unit of capital expected to prevail next period and according to [Lucas Jr and Prescott \(1971\)](#):

(...) the firm need not even form its own estimation of the future, beyond forecasting the value placed on assets in its industry next period. Of course, entrepreneurs, in common with other agents in securities markets, form judgments on the income streams of their own and other firms. The point is that these judgments are apart from, and irrelevant to, the investment goods demand decision.

In [Lucas Jr and Prescott \(1971\)](#) the relevant variable is the marginal contribution of new capital goods to future profits. With [Hayashi \(1982\)](#) showing in a competitive economy, average q (what we observe) and marginal q (the key variable in this framework) are equal to each other, it was up to formal empirical tests to show the existence of the relationship.

[Summers et al. \(1981\)](#) studying the impacts of tax policy, found out the q -theory holds mostly for what is called tax-adjusted Q . When q is not adjusted for the tax rate, it mostly has small power in explaining the investments. [Fazzari et al. \(1987\)](#) put emphasis on internal and external funds and for some firms’ these funds not being perfect substitutes. Due to a financing hierarchy [Fazzari et al. \(1987\)](#) say, firms’ internal cash flow may matter for their investments. As it assumes capital markets that are not perfect, this goes against [Modigliani and Miller \(1958\)](#) where it is claimed in perfect capital markets firms’ financial structure will not affect their market value and real firm decisions. According to [Fazzari et al. \(1987\)](#) firms that are retaining all their earnings are more sensitive in their investments to cash flows and liquidity.

According to [Erickson and Whited \(2000\)](#) the relationship between investment rates and the q is hard to identify since i) there could be possible non-linearities pointed out by [Abel and Eberly \(2002\)](#), ii) possible endogeneity in the estimation equation and state mismeasurement of marginal q could generate all the pathologies afflicting empirical q models. Suggesting GMM estimates [Erickson and Whited \(2000\)](#) show cash flows do not have an important impact on the investment rates of the firms. [Erickson and Whited \(2006\)](#) point out the measurement error occurs as the market’s valuation of the capital and manager’s valuation of capital may be different due to information asymmetries or different incentives. Yet

adapting an errors-in-variables model, the true q proxies generated by [Erickson and Whited \(2006\)](#) could explain very little of the variation. Recently, [Andrei et al. \(2019\)](#) adapted a model in which the decision to invest is endogenized. Defining research-intensive industries in accordance with [Kile and Phillips \(2009\)](#) and [Brown et al. \(2009\)](#), [Andrei et al. \(2019\)](#) show q -theory of investment works well within research-intensive industries.

This paper follows the main intuition of [Coad and Rao \(2008\)](#) in which it is pointed out a firm, on average, experiences moderate growth rates. [Coad and Rao \(2008\)](#) also point out returns to innovation are highly skewed and that growth rates distributions are heavy-tailed. Under such circumstances, regression methods concentrating on the average firm would be misleading. The empirical literature on the neo-classical theory of investment, although acknowledging measurement errors of the Tobin's q , rely heavily on methods that describe the average firms' behavior whereas investment rate has a highly skewed, fat-tailed distribution. The standard least-squares assumption of normally distributed errors may not hold for the key variables, likewise, as documented by [Scharfenaker and dos Santos \(2015\)](#), [dos Santos and Scharfenaker \(2019\)](#) and [Scharfenaker and Semieniuk \(2017\)](#) Tobin's q and profit rates follow a double-exponential rather than a Gaussian distribution. As most empirical papers try to establish theoretically expected relationship between the investment rate, profit rate and valuation rate by separating the firms in accordance with their size or sector, the estimated firm responsiveness is far from being representative of the typical firm even under these categories.

On a similar account [Coad \(2010\)](#) argue there are significant limitations of the neoclassical theory of investment itself where models are built not to be reflective of firm behavior but to have mathematical tractability. Regarding the relevance of cash-flows, [Coad \(2010\)](#) state most of the empirical studies on the investment equation, if they find any relevance of cash-flows to the investments, present this as 'bad news' since this sensitivity is supposed to be caused by information asymmetries and imperfections in capital markets. Thus, an evolutionary perspective adapted in explaining firm investment and growth behavior could present an alternative framework in which relevance of the firm profits is discussed. According to [Coad \(2010\)](#) the evolutionary view suggest the growth of the firm is not necessarily the growth of the 'fittest' firm but is actually a domain of 'fitter' firms; firms show considerable heterogeneity in their productivity even under narrowly defined industries ([Dosi and Grazzi \(2006\)](#)). In this framework [Coad \(2010\)](#) says, a firm's future can not be necessarily known since factors that can not be rationally anticipated play a role in it and thus the firms could as well be assumed to be following some behavioral rules in their decision to invest such as considering their current financial performance. Similarly, the evolutionary perspective differs in the optimal decisions' implications for the firm: the neoclassical view on firm investments identify an optimal level of assets at which the firm will not change their behavior anymore while the evolutionary view assumes firms 'exist to grow' and compete with each other for growth opportunities. Evolutionary view sees firm growth as always bounded by available liquidity and doesn't take the viability of growth opportunities for granted as the responsiveness to cash-flows indicate a healthy workings of an economy as opposed to existence of information asymmetries or imperfect borrowing conditions ([Coad \(2010\)](#)).

In this paper, by utilizing a Bayesian quantile regression model, the estimates are expected to be robust to outliers and heavy-tailed distributions we observe in random variables such as the investment rate. Thus, the parameter estimates for conventional measurements of determinants of the investment will not only be more reliable, but also the estimates will uncover responsiveness of firms on different quantiles of a persistent and stable distribution of investment rate.

3 Data and Estimation

In order to estimate the parameters reflecting the relationship between investment rate, Tobin's Q and the profit rate, annual data for the US firms from COMPUSTAT database is used. Firms that are in regulated utilities, government and finance sectors are dropped from the dataset. To calculate the q, common share price (PRCC), amount of common shares (CSHO), short and long term debt (DLC and DLTT) and total assets (AT) are used.

Investment equation estimation is done both with lagged variables and variables that are all at the same period of time (Summers et al. (1981), Gutiérrez and Philippon (2016), Erickson and Whited (2000) as some examples) yet some important points need to be noted. The baseline neo-classical theory of investment doesn't suggest a particular lag structure (Lucas Jr and Prescott (1971), Hayashi (1982)) and the lagged values, although conceptually appealing, may not be the theoretically relevant values such as the average valuation rate or profit rate of past quarters. Regarding the treatment of data, Griliches (1984), Hirsch and Seaks (1993) and Jaffe (1986) utilize $\log[q]$ as an independent variable in the regression analysis and show the empirical validity as well as theoretical relevance as the exponential pricing function is a credible functional form, thus throughout the empirical analysis I will be using $\log[q]$ which will capture the future looking aspects of investment decision.

In order to calculate the investment rate, simply investment (CAPX) and plant, property and equipment (PPEGT) and for the profit rate operating income before depreciation (OIBDP) and total assets are used.¹ Davis and de Souza (2021) point out the profit rates show important variation depending on the definition followed. Although the broad definition of the profit rate -including all assets and all income- and less broad definition of the profit rate -including only tangible capital and operating income- seem to have similar time trends at different quantiles, the firms that are on the top quantile in different definitions seem to be overlapping less and less in recent years. Similarly the ratio of financial assets and intangible assets to all assets seem to affect this overlap. The implications of these findings go beyond the scope of this paper, yet, it is important to point out the inclusion of

¹Using the COMPUSTAT database the corresponding variables would define Tobin's Q, profit rate and investment rate as $q = \frac{PRCC * CSHO + DLC + DLTT}{AT}$, $r = \frac{OIBDP}{AT}$ and $inv = \frac{CAPX}{PPEGT}$. Data is comprised of firms under the standard industrial classification (SIC) numbers 1,000–6,000 and 7,000–9,000. After assigning zero to missing debt data the observations with negative values for total assets, debt and investment in the dataset are dropped. High tech (HT) is defined according to Kile and Phillips (2009) and intangible capital intensive firms (INT) are defined as firms with more than %10 intangible assets in their total assets. All variables have their 1% tail observations dropped, a total of 238313 observations left after above described procedure adapted

the profit rate, as defined in this paper, into the regression equation has aimed to reflect the financial constraints debated in the relevant literature, which requires all assets and types of income to be considered for the firm (Fazzari et al. (1987)). Lastly, Scharfenaker and dos Santos (2015) found the sectoral impacts and asset composition to have small statistical impact, the impacts are discussed in more details in sections discussing the estimation results.

For the estimation of the parameters a Bayesian approach to the quantile regression is adapted. **Figure 1** show probability distribution for the pooled investment rate and stacked log-probability distributions for investment rates in selected years. The dependent variable in our analysis, the investment rate, seems to be highly asymmetric and heavy tailed as summarized in **Table 1**. Thus the asymmetric laplace distribution could be a better candidate than a normal distribution as a likelihood function for the investment rate. **Figure 2** presents the autocorrelation coefficients violin plots for the investment rate, profit rate and the $\log[q]$ for firms who survived for more than 10 years. Mundt et al. (2016) point out the autocorrelation coefficient estimates demonstrate a stronger significance for the profit rate than the growth rate of the firms, however, the investment rate show a very strong autocorrelation, for the firms who lived more than 10 years, as well. As expected, the autocorrelation coefficient violin plots show very similar shapes and median values for all three variables for these surviving firms. Thus, I claim identifying firms at different investment rates can reveal information on persistent behavior for the firms as the stacked log-probability plots also indicate the heavily skewed character of the investment rates is also a persistent feature.

3.1 Characterization of the Firms at Different Quantiles

It has been shown the typical firm has a very modest investment rate and the firm with average rate of investment is far from being representative to be called the typical firm. We have seen the typical firm experiences lower investment rates than the average and almost all the effects are less pronounced for the firms with lower investment rates. This section will describe firms at the different quantiles of the investment rates².

Figure 3 presents the median size and median growth rates of the firms at the different quantiles of the investment rates. The typical investment rate seem to be representing the largest firms in our data, yet, the “average firm” experiences a moderate growth rate (Coad and Rao (2008)) in comparison with high investment rate firms. Coad (2007b) and Coad (2007a) state the firm growth rate depend on its’ size and its’ lagged growth rate, yet, **Figure 3** point out to a very important aspect of firm growth that is missing in these explanations: the investment rate. Firms with higher investment rates consistently experiencing higher growth rates should motivate investigating the rest of the firm growth rate determinants in their relationship to the investment rates. As pointed out by Distante et al. (2018) size

²Firm size is defined as $\log(AT)$ and the growth rate is defined as the annual log difference of total assets. The debt size is defined as $\log[DLC + DLTT]$, the repurchase rate is defined as $\frac{PRSTKC}{AT}$ and the research and development spending size is defined as $\log[XRD]$. The percent of institutional owned shares comes from Thomson-Reuters’ Institutional Holdings database. Red dashed lines represent median rate of variable of interest, black dashed line represent modal rate of investment

pushes both low and high growth rate firms to a central rate, consistent with modal rate of investment firms, amongst the largest in size in the dataset, experiencing modest growth rates. Re-emphasizing the importance of investment rates in this picture: a relatively smaller firm could be both a firm with high investment rate (high growth) firm or low investment rate (low growth) firm. Thus the empirical research exploring the size-growth rate relationship on a firm level following [Gibrat \(1931\)](#) could be extended by studying investment decisions.

Figure 4 presents the median profit rate and $\log[q]$ at different quantiles of the investment rate. Theory of investment under uncertainty suggests the firms are maximizing their profits on an infinite horizon, out of which one can obtain marginal and average q ([Lucas Jr and Prescott \(1971\)](#), [Hayashi \(1982\)](#)) yet the empirical research on this question show the q has very little explanatory power for investment rates, whereas variables like lagged q or cash flow can be significant. The idea of financial constraints as introduced in [Fazzari et al. \(1987\)](#) and [Fazzari et al. \(2000\)](#) has been seen as a motivation to use cash flows as a relevant variable for understanding the variation in the investment rates yet [Kaplan and Zingales \(1997\)](#) and [Kaplan and Zingales \(2000\)](#) state investment-cash flow sensitivity is neither theoretically nor empirically is a good measure for financial constraints firms face. In our data the profit rate seem to be lower -even negative- for firms with very high or low investment rates yet for the firms with modal rate of investment, and quite a few other firms, the median profit rate seems to be very similar. For the reasons discussed at the beginning of section 3, when we observe the investment rate and the $\log[q]$ relationship in different investment rate quantiles, we observe supporting the q -theory of investment, the firms with higher investment rates seem to be the ones with higher $\log[q]$ values as well.

Figure 5 present median firm age and debt size of the firms. The firms with a modal rate of investment seem to be relatively older. We expect highest growing firms to be the firms with highest investment rates and the firms that are relatively younger out of **Figure 3** and **Figure 5** consistent with the summary by [Coad \(2007b\)](#) that the growth rate and firm age appears to be negatively dependent. Debt size and investment rate relationship could be interpreted in a similar way with the profit rate in which the firms with modal rate of investment seem to be among the top with their debt size yet a small debt size could belong to a low or high investment rate firm, indicating the financial constraint argument could be less informative then it seems.

Figure 6 presents median percentage of institutional owned common shares and HHI of the institutional ownership, indicating the firms with modal rate of investment do not differ too much with high investment rate firms in their institutional ownership ratios or the HHI of these institutional ownerships. It is worth pointing out [Gutiérrez and Philippon \(2016\)](#) suggest the increase in the index and quasi-index fund ownership is related with the decline in the investments. The very low investment rates indeed are characterized with high HHI of the institutional ownership yet the lowest investment rate firms seem to be far away from representing the typical investment rates.

Figure 7 documents the median R&D expense size and mean stock repurchase rate at different investment rate quantiles. Innovation is seen as a central factor in firm growth and

indeed the firms with higher investment rates, tend to have large R&D expenditures. Yet the hardship of converting the valuable knowledge from R&D expenditures into economic growth indicate not all firms with similar sizes of R&D expenditures grow at the same rate. Similar findings are documented by [Coad and Rao \(2008\)](#) where it is shown the high growth firms benefit from and depend on innovation. The left panel in **Figure 7** indicate the firms with high R&D expenditures could as well be high investment rate firms which allow high growth. Lastly, we see firms with very small and large investment rates tend to have small stock repurchase rates yet the variation seems to be small overall as well. [Gutiérrez and Philippon \(2016\)](#) document an increase in repurchase rate has started in 1982 with SEC rule 10b-18. We see the highest rate of repurchase corresponds to the modal rate of investment firms, yet, the amount spent on stock-repurchases is a very small share of the total assets, for even the highest cases, on average.

Thus we observe high investment rate firms, relatively small in size and their profit rate, relatively young, having relatively small debt and not so different than other firms' R&D expenditures (yet still with high R&D expenditures) having the highest rates of growth. At different rates of investments, there is very little difference in firms' median profit rates, their institutional ownership ratios, R&D expenditures and mean stock repurchases. Yet the firms responsiveness to the profit rate and valuation ratio varies. A possible explanation could be the stable form investment rate distribution reflects. The stable character investment rate distribution presents an equilibrium behavior that could be a result of different firm decision making problems. A firm may have a revenue maximizing behavior, as opposed to profit maximizing ([Baumol \(1959\)](#)) and it may prioritize balanced growth over maximum profits ([Marris \(1963\)](#)) or managers while exploiting growth opportunities may get their attention diverted from profit maximization ([Penrose \(2009\)](#)). All these are reasons to categorize firms in accordance with their investment rates, as opposed to their financial situation, and analyze the responsiveness to the key determinants of investment at different levels of investment rates.

3.2 Estimation Process

To address the asymmetric character of the key variables [Yu and Zhang \(2005\)](#) is followed, where a three-parameter asymmetric laplace distribution is proposed with the skewness parameter used to model the quantile of interest:

$$f(x|\mu, T, \tau) = \frac{\tau(1-\tau)}{T} e^{-\rho_\tau(\frac{x-\mu}{T})} \quad (1)$$

where $\rho_\tau(x) = x(\tau - I(x < 0))$ and $I(\cdot)$ denotes the indicator function. Maximize a regression likelihood using asymmetric laplace distribution, errors with $\mu = X^T_i \beta$. Specifying the quantile of interest, τ , and priors for β and T , the resulting posterior distribution can be represented as follows:

$$\psi(\beta, T|Y, X, \tau) \propto \pi(\beta, T) \prod_{i=1}^n f(Y_i|X^T_i \beta, T, \tau) \quad (2)$$

where $\pi(\beta, T)$ is the joint prior on the regression parameters. For $\mu = X^T_i \beta$, use

$$\mu = \alpha_\tau + \beta_\tau \log(q) + \theta_\tau r + \zeta_{1,\tau} YEAR + \zeta_{2,\tau} SECTOR \quad (3)$$

where $\tau = \{0.05, 0.25, 0.50, 0.75, 0.95\}$. Thus the quantile regression model is using sector and year fixed effects. To compare these results, a cross sectional model is estimated where

$$\mu = \alpha_\tau + \beta_\tau \log(q) + \theta_\tau r + \zeta_\tau SECTOR \quad (4)$$

with $\tau = \{0.05, 0.25, 0.50, 0.75, 0.95\}$, which will bring light on the time evolution of the relationship between investment rate, q and the profit rate.

Figures 8 present the traceplots of the posterior distributions for parameters in different quantiles. **Figures 9-11** present the pooled quantile estimations and pooled OLS plotted together for different parameters.

3.3 Pooled Estimation Results

Figure 9 presents the pooled quantile estimates and OLS estimate for the intercept term. The estimate values are varying in different quantiles and increasing through quantiles as expected and all estimates are positive. The results indicate firms with lower investment rates have less than average intercept terms.

Figure 10 presents the pooled quantile regression estimates and OLS estimates for β . The average firm's response to an increase in $\log[q]$, indicated by the OLS estimate, is well above than the response of the firms with the median and below investment rate. The firms with high investment rates tend to respond to an increase in the $\log[q]$ stronger than the firms with average rates of investment.

Figure 11 represent the pooled quantile regression estimates and OLS estimates for θ . Firms with median and below investment rate seem to respond to an increase in the profit rate stronger than the firms with average rate of profit. However, the firms with above median rate of investment, not only show a weaker response to profit rate, a very strong decline in their investment rates can be observed as they see an increase in their profit rate.

3.4 Cross Sectional Estimation Results

[Gutiérrez and Philippon \(2016\)](#) conduct a comprehensive analysis of the investment rate in the US and show the investment rate is relatively weak against profit rate and the Tobin's Q . Their analysis consider financial constraints, competitiveness, governance and measurement errors in key variables as the main possible reasons for relative weakness in the investment rate. **Figure 12** summarizes my findings for the autonomous investment for the firms at the 25th quantile of the investment rates, closest to the firms with modal rate of investments. The firms with typical rates of investments have considerably lower rates of autonomous

investments then the average firm, although the time path is not necessarily telling. **Figure 13** summarizes the result for 5th to 95th quantiles where point estimates are lower for the firms below the average rate of investments and higher for the firms with above average rate of investments.

One could see the strong decline in responsiveness to $\log[q]$ in **Figure 14**. The firms with typical investment rates point estimates for $\log[q]$ are well below the point estimates for the average firms. Yet **Figure 15** shows us the gap is much bigger for the firms with even smaller investment rates. The decline in the responsiveness is most pronounced at the higher quantiles of the investment rate. The firms with lower rate of investment had consistently smaller responsiveness to an increase in $\log[q]$ than the firms with average rate of investment, as could be observed in the distance between the cross sectional estimates for lower quantiles and the OLS estimates. Point estimates could be found in **Table 2-4** and they are close to the findings of [Strauss and Yang \(2020\)](#), however, only for the firms with relatively high rates of investment. Although the decline is observed for all firms in different rates of investments, the responsiveness is still higher with firms with higher rates of investment.

In **Figure 16** we see the impact of an increase in the profit rate on the investment rate for the typical firm, which has declined since 1980s although remained positive and above the responsiveness of the average firm. The impact of the profit rate on the investment rate seem to be similar between low investment rate firms and the average investment rate firms as could be observed in **Figure 17**. However, higher investment rate firms respond to increases in the profit rate much weaker than the firms with the average rate of investment. There is a strong and consistent negative response by the firms on the upper quantiles of the investment rate to an increase in the profit rate. Similar with [Strauss and Yang \(2020\)](#) these findings indicate a relatively small impact of the profit rate on the investment rate for all but firms with high investment rates, however, my findings are not indicating a “secular stagnation” through a decline in autonomous investment but a decline in responsiveness.

[Andrei et al. \(2019\)](#) point out contrasting with earlier years, the relationship between Tobin’s Q and the aggregate investments has been strong lately and they argue the growth in the number of firms in research-intensive sectors can account for this change. I extend the cross sectional analysis to include fixed effect models for high-tech, intangible intensive and manufacturing sectors. The quantile regression model is estimated with following location parameters:

$$\mu = \alpha_\tau + \beta_\tau \log(q) + \theta_\tau r + \zeta_\tau HT \quad (5)$$

for the firms in high tech sector,

$$\mu = \alpha_\tau + \beta_\tau \log(q) + \theta_\tau r + \zeta_\tau INT \quad (6)$$

for the intangible-intensive firms and,

$$\mu = \alpha_\tau + \beta_\tau \log(q) + \theta_\tau r + \zeta_\tau MFG \quad (7)$$

for the firms in the manufacturing sector, where $\tau = \{0.05, 0.25, 0.50, 0.75, 0.95\}$ and the estimates are presented in **Figure 18-21**. The presented estimates include the parameter estimates and fixed effects for the corresponding sectors. There is a persistent pattern of the cross sectional estimates which none of the fixed effect models are showing any strong deviation from. **Figure 18-19** point out the movement of the autonomous investments in manufacturing, high-tech and intangible intensive firms were similar with the overall pattern.

Figure 20 shows The decline in responsiveness to $\log[q]$ in manufacturing firms is the strongest. Not only in the typical investment rate firms but for all investment rates I considered in this analysis the decline in responsiveness is largest in the manufacturing sector. Firms in high-tech sector responds to an increase in $\log[q]$ stronger than the firms in the manufacturing sector yet the point estimates are not necessarily very different than cross sectional estimates when all firms considered. The intangible capital intensive firms show significant variations in their responsiveness. Firms with high ratios of intangible capital to all assets, with typical investment rates, have a higher than average responsiveness to $\log[q]$ all sectors considered. The same is observed in low investment rates as well and disappears as we move along the investment rates. The response of different firms in different sectors to the profit rate is summarized in **Figure 22-23** and seems to be indistinguishable across sectors.

4 Discussion and Conclusions

The theoretical and empirical research on the investment emphasize the importance of the future expected profits and as firms may face financial constraints/capital market imperfections, the current cash flows. The empirical research have brought contrasting results regarding the relevance of the measures of firm valuation and profit as well as their strength in explaining investment rates and the magnitude of their impact. This paper claims much of the empirical relations observed in between investment and firm characteristics could be well understood by characterizing firms with their investment rates themselves. While most empirical studies on the firm investment consider the firms as profit maximizing entities which are supposed to find an optimal size of the firm as a result of their profit maximizing actions, the evolutionary perspective considers the firms to be growing agents, following some behavioral rules. Using the persistent pattern of skewed investment rate distribution, this paper tried to describe the firm responsiveness in different quantiles of the investment rate domain. Conducting an empirical analysis on the different quantiles of the investment rate this paper shows the disparities in the responsiveness of the firms and relevance and magnitude of the key variables. Findings are consistent with the observed secular slowdown of the investments, within different sectors as well. A striking finding of this paper is the typical firm's responsiveness to the profit rate and the valuation rate are persistently comparable, while high investment rate firms respond to the valuation ratio more than the profit rate.

I documented the point estimates of the responsiveness to the profit rate are larger for the firms with smaller investment rates. Firms with lower investment rates show above aver-

age responsiveness to changes in the profits yet the firms with larger investment rates show below average and negative response. Lower investment rate firms' responsiveness to the profit and valuation measures are similar. While lower investment rate firms' responsiveness to an increase in profit rate shows an expected sign, the response by the high investment rate firms suggest an almost mean-reverting mechanism that pushes firms to a modal rate of investment.

Typical firms demonstrate a decline in their responsiveness to measures of valuation, yet the decline in responsiveness to the valuation ratio is stronger in firms with already larger investment rates. Although their responsiveness to the valuation ratio has declined, firms with higher investment rates still have higher responsiveness than firms with lower investment rates.

Manufacturing sector firms has shown significant decline in responsiveness to profits and valuation at all quantiles of investment rate. The responsiveness of manufacturing is below the all firms considered cases at almost all quantiles. This finding is concerning as the manufacturing constitute an important part of the economy and is a relatively mobile form of investment. The firms with higher ratios of intangible capital show variation in their responsiveness at different investment levels: at lower investment rates the firms with high ratios of intangible capital are more responsive then other firms to the valuation ratio. Responsiveness of the firms in the high-tech sector seem to be very similar to the responsiveness when all firms considered.

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5 Tables

Table 1: Moments, IQR and Median of Investment Rates

	Mean	Variance	Skewness	Kurtosis	IQR	Median
Pooled	0.14	0.02	2.08	8.49	0.12	0.10
1963	0.11	0.01	3.00	17.56	0.07	0.09
1964	0.11	0.01	2.52	13.52	0.07	0.10
1965	0.13	0.01	2.68	16.13	0.07	0.11
1966	0.14	0.01	1.94	9.10	0.09	0.12
1967	0.14	0.01	2.26	11.62	0.09	0.11
1968	0.14	0.01	2.13	10.20	0.09	0.11
1969	0.15	0.01	1.85	7.38	0.11	0.12
1970	0.14	0.01	2.14	9.28	0.10	0.11
1971	0.12	0.01	2.37	11.49	0.09	0.09
1972	0.14	0.01	2.07	8.98	0.11	0.11
1973	0.16	0.01	1.80	7.23	0.12	0.13
1974	0.15	0.01	1.85	8.00	0.11	0.13
1975	0.12	0.01	1.94	8.61	0.10	0.10
1976	0.13	0.01	2.12	9.67	0.09	0.11
1977	0.14	0.01	1.99	8.96	0.10	0.12
1978	0.15	0.01	1.77	7.65	0.10	0.13
1979	0.16	0.01	1.95	8.94	0.11	0.14
1980	0.17	0.01	1.82	7.45	0.12	0.14
1981	0.18	0.02	1.77	6.64	0.14	0.14
1982	0.16	0.02	1.78	6.80	0.13	0.12
1983	0.15	0.02	2.00	7.74	0.13	0.11
1984	0.18	0.02	1.67	5.93	0.16	0.13
1985	0.16	0.02	1.90	7.38	0.13	0.12
1986	0.15	0.02	1.89	7.42	0.13	0.11
1987	0.16	0.02	1.92	7.24	0.13	0.12
1988	0.15	0.02	2.03	8.25	0.12	0.11
1989	0.14	0.01	2.00	8.29	0.12	0.11
1990	0.13	0.01	2.13	8.99	0.10	0.10
1991	0.12	0.01	2.14	9.63	0.10	0.09
1992	0.13	0.01	2.14	8.92	0.11	0.09
1993	0.14	0.02	1.97	7.58	0.13	0.10
1994	0.16	0.02	1.78	6.52	0.14	0.12
1995	0.16	0.02	1.77	6.67	0.14	0.12
1996	0.16	0.02	1.72	6.35	0.14	0.12
1997	0.17	0.02	1.60	5.81	0.15	0.13
1998	0.16	0.02	1.70	6.60	0.14	0.13
1999	0.15	0.02	1.79	7.00	0.13	0.11
2000	0.16	0.02	1.80	6.61	0.15	0.12
2001	0.14	0.02	1.95	7.84	0.13	0.10
2002	0.11	0.01	2.40	10.78	0.09	0.08
2003	0.10	0.01	2.62	12.44	0.09	0.08
2004	0.12	0.01	2.48	10.79	0.10	0.09
2005	0.14	0.02	2.16	8.54	0.11	0.09
2006	0.15	0.02	2.01	7.89	0.13	0.10
2007	0.15	0.02	1.95	7.27	0.13	0.11
2008	0.14	0.02	1.95	7.56	0.12	0.11
2009	0.10	0.01	2.59	12.70	0.09	0.07
2010	0.11	0.01	2.45	11.08	0.10	0.08
2011	0.13	0.02	2.21	8.96	0.11	0.09
2012	0.13	0.01	2.23	9.55	0.11	0.09
2013	0.12	0.01	2.32	9.91	0.10	0.09
2014	0.13	0.01	2.41	10.48	0.10	0.09
2015	0.12	0.01	2.46	10.82	0.10	0.09
2016	0.12	0.01	2.60	11.89	0.10	0.08
2017	0.12	0.02	2.58	11.24	0.10	0.08
2018	0.12	0.01	2.47	10.80	0.09	0.08
2019	0.10	0.01	2.96	15.37	0.07	0.07

Note: Mean, Variance, Skewness, Kurtosis, IQR and Median of Investment Rates

Table 2: α (intercept) Estimates

	$\alpha_{0.05}$	l-95% CI	u-95% CI	$\alpha_{0.25}$	l-95% CI	u-95% CI	$\alpha_{0.50}$	l-95% CI	u-95% CI	$\alpha_{0.75}$	l-95% CI	u-95% CI	$\alpha_{0.95}$	l-95% CI	u-95% CI
Pooled	0.023	0.022	0.023	0.049	0.049	0.050	0.071	0.071	0.072	0.126	0.125	0.126	0.336	0.335	0.338
1963	0.033	-0.014	0.050	0.039	0.005	0.079	0.071	0.027	0.119	0.108	0.069	0.188	0.251	0.119	0.423
1964	0.036	-0.007	0.053	0.050	0.010	0.092	0.092	0.041	0.146	0.118	0.069	0.196	0.176	0.086	0.331
1965	0.022	-0.031	0.038	0.032	-0.009	0.079	0.065	0.021	0.136	0.135	0.072	0.200	0.214	0.156	0.355
1966	0.022	-0.040	0.043	0.033	-0.010	0.084	0.075	0.022	0.125	0.109	0.071	0.196	0.234	0.184	0.379
1967	0.023	-0.061	0.046	0.051	-0.011	0.094	0.072	0.028	0.145	0.143	0.066	0.216	0.195	0.151	0.380
1968	0.023	-0.005	0.036	0.029	0.005	0.065	0.059	0.027	0.090	0.082	0.057	0.134	0.200	0.141	0.276
1969	0.039	0.007	0.056	0.049	0.017	0.089	0.094	0.059	0.135	0.146	0.103	0.184	0.296	0.217	0.377
1970	0.037	0.019	0.052	0.064	0.040	0.105	0.150	0.120	0.171	0.202	0.159	0.222	0.333	0.297	0.382
1971	0.018	0.006	0.045	0.071	0.045	0.103	0.130	0.105	0.165	0.199	0.177	0.223	0.315	0.271	0.355
1972	0.018	0.003	0.029	0.030	0.009	0.080	0.149	0.128	0.169	0.194	0.167	0.231	0.496	0.433	0.539
1973	0.030	0.014	0.054	0.065	0.045	0.090	0.122	0.094	0.148	0.207	0.178	0.238	0.481	0.345	0.528
1974	0.043	0.026	0.063	0.128	0.090	0.151	0.161	0.142	0.186	0.212	0.184	0.240	0.501	0.396	0.551
1975	0.036	0.021	0.045	0.060	0.036	0.078	0.105	0.078	0.123	0.135	0.119	0.176	0.288	0.254	0.320
1976	0.049	0.035	0.062	0.073	0.058	0.096	0.118	0.090	0.134	0.164	0.145	0.212	0.277	0.236	0.309
1977	0.014	-0.002	0.025	0.050	0.016	0.071	0.095	0.072	0.113	0.122	0.103	0.144	0.261	0.202	0.296
1978	0.015	-0.001	0.030	0.054	0.027	0.083	0.105	0.083	0.124	0.204	0.161	0.239	0.462	0.374	0.522
1979	0.000	-0.016	0.018	0.044	0.025	0.080	0.135	0.113	0.155	0.207	0.180	0.235	0.333	0.311	0.378
1980	-0.004	-0.018	0.038	0.068	0.043	0.091	0.123	0.101	0.151	0.227	0.178	0.255	0.917	0.540	0.984
1981	0.025	0.002	0.047	0.086	0.062	0.110	0.151	0.124	0.180	0.294	0.259	0.319	0.504	0.381	0.528
1982	0.033	0.010	0.046	0.061	0.045	0.079	0.107	0.091	0.130	0.245	0.217	0.265	0.371	0.327	0.540
1983	0.010	-0.013	0.023	0.027	0.009	0.045	0.085	0.070	0.100	0.211	0.185	0.252	0.652	0.628	0.736
1984	0.011	-0.007	0.032	0.055	0.040	0.070	0.090	0.074	0.115	0.283	0.249	0.301	0.653	0.613	0.686
1985	0.006	-0.008	0.025	0.058	0.039	0.069	0.078	0.066	0.095	0.145	0.125	0.176	0.458	0.420	0.698
1986	0.006	-0.014	0.015	0.043	0.014	0.061	0.081	0.060	0.116	0.158	0.138	0.183	0.460	0.391	0.513
1987	0.008	-0.010	0.019	0.034	0.012	0.052	0.096	0.083	0.120	0.128	0.108	0.150	0.227	0.204	0.333
1988	0.014	-0.006	0.030	0.034	0.017	0.053	0.064	0.038	0.094	0.154	0.121	0.189	0.307	0.260	0.347
1989	0.015	0.001	0.022	0.022	0.007	0.056	0.085	0.074	0.114	0.172	0.140	0.196	0.264	0.227	0.296
1990	0.018	-0.000	0.032	0.043	0.015	0.065	0.108	0.082	0.146	0.212	0.156	0.240	0.290	0.253	0.338
1991	0.006	-0.004	0.025	0.032	0.017	0.046	0.048	0.035	0.066	0.122	0.071	0.155	1.048	0.443	1.135
1992	0.011	-0.007	0.024	0.043	0.017	0.071	0.087	0.062	0.107	0.148	0.122	0.215	0.555	0.380	0.590
1993	0.008	-0.007	0.047	0.071	0.051	0.084	0.081	0.069	0.097	0.114	0.095	0.144	0.332	0.277	0.361
1994	-0.003	-0.019	0.019	0.040	0.030	0.063	0.080	0.062	0.095	0.141	0.121	0.193	0.300	0.278	0.326
1995	0.008	-0.011	0.020	0.044	0.030	0.053	0.067	0.054	0.084	0.101	0.081	0.118	0.319	0.276	0.456
1996	0.009	-0.006	0.026	0.048	0.029	0.059	0.080	0.064	0.090	0.134	0.120	0.150	0.280	0.252	0.373
1997	0.013	-0.001	0.032	0.045	0.037	0.058	0.065	0.055	0.082	0.127	0.100	0.142	0.336	0.311	0.377
1998	0.012	-0.000	0.024	0.054	0.039	0.066	0.081	0.072	0.097	0.132	0.112	0.161	0.374	0.340	0.460
1999	0.020	0.001	0.034	0.050	0.039	0.059	0.098	0.090	0.108	0.158	0.141	0.180	0.258	0.242	0.306
2000	0.015	-0.003	0.028	0.042	0.033	0.056	0.080	0.072	0.092	0.139	0.117	0.155	0.196	0.178	0.215
2001	0.007	-0.007	0.023	0.052	0.042	0.061	0.076	0.066	0.082	0.110	0.096	0.123	0.178	0.156	0.255
2002	0.008	-0.006	0.014	0.042	0.031	0.049	0.058	0.051	0.065	0.082	0.069	0.102	0.171	0.131	0.314
2003	0.016	0.004	0.026	0.030	0.022	0.043	0.047	0.039	0.061	0.090	0.073	0.103	0.324	0.220	0.347
2004	0.004	-0.010	0.018	0.033	0.018	0.041	0.042	0.032	0.055	0.100	0.068	0.126	0.247	0.199	0.262
2005	0.002	-0.013	0.021	0.035	0.018	0.044	0.058	0.045	0.069	0.080	0.061	0.091	0.316	0.221	0.341
2006	0.007	-0.009	0.019	0.026	0.014	0.045	0.057	0.043	0.083	0.108	0.093	0.123	0.377	0.252	0.410
2007	0.009	-0.008	0.033	0.038	0.023	0.052	0.055	0.044	0.077	0.128	0.107	0.157	0.805	0.639	0.844
2008	0.004	-0.009	0.013	0.027	0.017	0.041	0.072	0.055	0.103	0.178	0.163	0.196	0.555	0.517	0.661
2009	0.009	-0.005	0.017	0.031	0.021	0.040	0.054	0.039	0.065	0.098	0.081	0.129	0.254	0.216	0.458
2010	0.014	-0.001	0.028	0.037	0.025	0.050	0.076	0.067	0.100	0.194	0.172	0.221	0.534	0.502	0.570
2011	0.024	-0.003	0.033	0.050	0.042	0.066	0.128	0.105	0.149	0.305	0.280	0.334	0.493	0.456	0.522
2012	0.016	-0.002	0.034	0.053	0.040	0.071	0.105	0.086	0.114	0.302	0.224	0.323	0.513	0.471	0.663
2013	0.021	0.001	0.032	0.050	0.038	0.068	0.084	0.072	0.102	0.145	0.128	0.181	0.668	0.644	0.749
2014	0.033	-0.001	0.043	0.064	0.050	0.075	0.076	0.064	0.087	0.220	0.193	0.258	0.393	0.368	0.420
2015	0.013	-0.004	0.026	0.047	0.024	0.066	0.078	0.062	0.092	0.119	0.097	0.142	0.417	0.272	0.455
2016	0.015	-0.001	0.035	0.064	0.045	0.078	0.095	0.069	0.119	0.176	0.148	0.210	0.404	0.327	0.446
2017	0.008	-0.009	0.023	0.036	0.020	0.057	0.068	0.050	0.096	0.204	0.164	0.236	0.343	0.298	0.387
2018	0.021	0.005	0.043	0.057	0.033	0.071	0.077	0.060	0.099	0.156	0.130	0.178	0.284	0.254	0.329
2019	0.016	0.001	0.032	0.049	0.027	0.064	0.089	0.049	0.111	0.162	0.149	0.204	0.624	0.519	0.685

Note: Results are for the models in equations (3) and (4). Each estimate is presented with 90% credibility interval

Table 3: β (log[q]) Estimates

	$\beta_{0.05}$	l-95% CI	u-95% CI	$\beta_{0.25}$	l-95% CI	u-95% CI	$\beta_{0.50}$	l-95% CI	u-95% CI	$\beta_{0.75}$	l-95% CI	u-95% CI	$\beta_{0.95}$	l-95% CI	u-95% CI
Pooled	0.007	0.007	0.007	0.024	0.024	0.024	0.045	0.045	0.045	0.077	0.077	0.077	0.121	0.121	0.122
1963	0.016	0.009	0.022	0.022	0.012	0.032	0.033	0.023	0.044	0.056	0.044	0.069	0.098	0.073	0.135
1964	0.016	0.010	0.021	0.018	0.010	0.029	0.029	0.019	0.041	0.039	0.026	0.055	0.077	0.056	0.109
1965	0.018	0.014	0.023	0.022	0.014	0.029	0.033	0.023	0.043	0.057	0.044	0.068	0.115	0.097	0.138
1966	0.021	0.018	0.026	0.030	0.025	0.036	0.044	0.036	0.051	0.069	0.058	0.081	0.186	0.169	0.204
1967	0.017	0.012	0.021	0.037	0.032	0.042	0.062	0.057	0.068	0.096	0.086	0.104	0.168	0.159	0.177
1968	0.007	0.004	0.011	0.028	0.023	0.033	0.054	0.048	0.061	0.111	0.101	0.120	0.226	0.215	0.239
1969	0.012	0.009	0.015	0.034	0.029	0.039	0.059	0.054	0.066	0.112	0.104	0.120	0.208	0.193	0.223
1970	0.014	0.011	0.017	0.030	0.026	0.033	0.058	0.053	0.062	0.103	0.097	0.110	0.191	0.180	0.205
1971	0.012	0.011	0.014	0.029	0.026	0.033	0.054	0.051	0.058	0.098	0.092	0.103	0.204	0.192	0.215
1972	0.015	0.013	0.018	0.029	0.026	0.032	0.053	0.049	0.057	0.103	0.097	0.108	0.208	0.200	0.219
1973	0.018	0.015	0.020	0.032	0.029	0.036	0.051	0.048	0.056	0.096	0.090	0.103	0.191	0.181	0.200
1974	0.018	0.015	0.022	0.040	0.035	0.044	0.054	0.050	0.058	0.085	0.079	0.091	0.154	0.144	0.163
1975	0.009	0.007	0.012	0.022	0.019	0.026	0.040	0.036	0.044	0.068	0.063	0.073	0.134	0.127	0.141
1976	0.008	0.004	0.011	0.022	0.019	0.026	0.042	0.037	0.046	0.077	0.070	0.084	0.146	0.137	0.154
1977	0.006	0.003	0.009	0.026	0.020	0.030	0.052	0.046	0.055	0.069	0.065	0.076	0.161	0.150	0.173
1978	0.008	0.005	0.011	0.028	0.024	0.032	0.057	0.052	0.062	0.101	0.095	0.110	0.208	0.197	0.217
1979	0.013	0.010	0.016	0.035	0.031	0.039	0.069	0.064	0.074	0.122	0.118	0.127	0.193	0.180	0.204
1980	0.011	0.009	0.014	0.050	0.047	0.053	0.083	0.079	0.087	0.133	0.129	0.137	0.204	0.196	0.210
1981	0.017	0.013	0.019	0.056	0.053	0.059	0.103	0.098	0.106	0.157	0.153	0.161	0.208	0.199	0.216
1982	0.011	0.008	0.013	0.035	0.032	0.038	0.065	0.062	0.068	0.103	0.100	0.107	0.153	0.147	0.158
1983	0.014	0.012	0.017	0.042	0.039	0.044	0.084	0.081	0.087	0.146	0.143	0.150	0.236	0.229	0.243
1984	0.012	0.010	0.015	0.054	0.051	0.057	0.099	0.095	0.101	0.162	0.158	0.166	0.208	0.195	0.220
1985	0.011	0.009	0.014	0.040	0.037	0.043	0.075	0.072	0.078	0.124	0.120	0.128	0.182	0.175	0.188
1986	0.010	0.008	0.012	0.034	0.032	0.036	0.071	0.068	0.073	0.127	0.124	0.131	0.170	0.163	0.177
1987	0.010	0.007	0.012	0.029	0.027	0.032	0.061	0.059	0.064	0.100	0.097	0.104	0.155	0.147	0.164
1988	0.007	0.005	0.009	0.027	0.025	0.030	0.049	0.047	0.052	0.101	0.098	0.104	0.149	0.141	0.156
1989	0.008	0.006	0.010	0.025	0.023	0.027	0.042	0.039	0.044	0.074	0.071	0.077	0.129	0.124	0.135
1990	0.008	0.007	0.010	0.020	0.018	0.023	0.036	0.034	0.039	0.064	0.061	0.068	0.114	0.108	0.121
1991	0.005	0.004	0.007	0.024	0.022	0.026	0.045	0.043	0.047	0.070	0.068	0.073	0.110	0.105	0.116
1992	0.010	0.008	0.011	0.032	0.030	0.035	0.059	0.057	0.061	0.092	0.088	0.095	0.140	0.134	0.144
1993	0.008	0.006	0.010	0.035	0.033	0.038	0.067	0.064	0.069	0.120	0.118	0.123	0.183	0.177	0.188
1994	0.014	0.012	0.016	0.037	0.035	0.039	0.072	0.070	0.074	0.125	0.123	0.128	0.165	0.159	0.173
1995	0.009	0.008	0.011	0.032	0.030	0.034	0.058	0.056	0.060	0.101	0.099	0.104	0.140	0.135	0.145
1996	0.012	0.010	0.013	0.036	0.034	0.038	0.070	0.068	0.072	0.116	0.114	0.119	0.136	0.132	0.142
1997	0.013	0.012	0.015	0.033	0.031	0.034	0.060	0.058	0.061	0.095	0.093	0.097	0.112	0.106	0.117
1998	0.010	0.008	0.012	0.026	0.025	0.028	0.044	0.042	0.046	0.071	0.069	0.073	0.102	0.097	0.107
1999	0.008	0.007	0.010	0.029	0.028	0.031	0.053	0.052	0.055	0.088	0.087	0.090	0.124	0.122	0.128
2000	0.007	0.005	0.008	0.024	0.023	0.025	0.045	0.044	0.046	0.074	0.072	0.075	0.103	0.099	0.107
2001	0.005	0.004	0.006	0.023	0.022	0.025	0.039	0.037	0.040	0.060	0.058	0.061	0.090	0.087	0.093
2002	0.003	0.003	0.004	0.014	0.013	0.015	0.027	0.025	0.028	0.042	0.040	0.044	0.053	0.051	0.057
2003	0.002	0.001	0.002	0.011	0.010	0.012	0.029	0.028	0.031	0.048	0.046	0.050	0.085	0.080	0.089
2004	0.005	0.004	0.006	0.017	0.015	0.018	0.037	0.036	0.038	0.064	0.062	0.066	0.107	0.103	0.111
2005	0.004	0.003	0.006	0.018	0.017	0.020	0.037	0.036	0.039	0.065	0.064	0.068	0.100	0.097	0.104
2006	0.006	0.004	0.007	0.021	0.020	0.023	0.042	0.040	0.044	0.071	0.068	0.073	0.113	0.107	0.118
2007	0.006	0.004	0.007	0.019	0.018	0.021	0.038	0.036	0.039	0.070	0.068	0.072	0.119	0.114	0.122
2008	0.003	0.002	0.004	0.011	0.010	0.013	0.018	0.016	0.019	0.038	0.036	0.040	0.056	0.050	0.062
2009	0.002	0.001	0.003	0.014	0.013	0.016	0.028	0.027	0.030	0.046	0.044	0.048	0.065	0.061	0.070
2010	0.003	0.002	0.004	0.016	0.015	0.018	0.030	0.029	0.032	0.057	0.055	0.059	0.089	0.084	0.095
2011	0.003	0.002	0.005	0.015	0.014	0.017	0.031	0.030	0.033	0.051	0.049	0.053	0.059	0.053	0.064
2012	0.003	0.002	0.004	0.019	0.018	0.020	0.035	0.034	0.037	0.061	0.059	0.063	0.077	0.072	0.081
2013	0.004	0.003	0.005	0.017	0.016	0.019	0.035	0.033	0.036	0.062	0.060	0.064	0.113	0.109	0.118
2014	0.002	0.001	0.003	0.016	0.014	0.017	0.034	0.033	0.036	0.064	0.062	0.066	0.115	0.110	0.123
2015	0.003	0.002	0.004	0.015	0.014	0.016	0.031	0.029	0.032	0.058	0.056	0.061	0.098	0.093	0.102
2016	0.004	0.002	0.005	0.015	0.014	0.017	0.029	0.028	0.031	0.051	0.049	0.054	0.080	0.076	0.085
2017	0.004	0.003	0.005	0.016	0.015	0.018	0.032	0.030	0.033	0.059	0.056	0.062	0.098	0.093	0.103
2018	0.003	0.002	0.004	0.016	0.015	0.017	0.028	0.026	0.029	0.054	0.052	0.056	0.087	0.082	0.093
2019	0.002	0.001	0.003	0.012	0.011	0.013	0.022	0.020	0.023	0.040	0.037	0.042	0.074	0.068	0.078

Note: Results are for the models in equations (3) and (4). Each estimate is presented with 90% credibility interval

Table 4: θ (Profit Rate) Estimates

	$\theta_{0.05}$	l-95% CI	u-95% CI	$\theta_{0.25}$	l-95% CI	u-95% CI	$\theta_{0.50}$	l-95% CI	u-95% CI	$\theta_{0.75}$	l-95% CI	u-95% CI	$\theta_{0.95}$	l-95% CI	u-95% CI
Pooled	0.016	0.016	0.017	0.019	0.019	0.020	0.010	0.010	0.011	-0.037	-0.037	-0.036	-0.193	-0.194	-0.193
1963	-0.006	-0.050	0.033	0.036	-0.028	0.105	0.028	-0.052	0.107	-0.047	-0.130	0.036	-0.289	-0.514	-0.021
1964	0.014	-0.019	0.052	0.045	-0.014	0.116	0.032	-0.030	0.111	0.058	-0.040	0.154	0.148	-0.112	0.298
1965	0.034	-0.006	0.058	0.077	0.018	0.135	0.043	-0.027	0.129	0.019	-0.055	0.101	-0.114	-0.273	-0.020
1966	0.075	0.052	0.091	0.080	0.053	0.122	0.087	0.040	0.130	0.029	-0.067	0.115	-0.175	-0.236	-0.090
1967	0.088	0.065	0.111	0.010	-0.019	0.047	-0.059	-0.102	-0.026	-0.178	-0.235	-0.115	-0.379	-0.451	-0.299
1968	0.054	0.040	0.072	0.051	0.022	0.078	0.005	-0.038	0.050	-0.177	-0.239	-0.108	-0.563	-0.624	-0.485
1969	0.058	0.040	0.072	0.047	0.012	0.081	0.042	0.011	0.071	-0.135	-0.180	-0.091	-0.454	-0.560	-0.363
1970	0.083	0.065	0.102	0.068	0.036	0.088	-0.021	-0.047	0.004	-0.184	-0.224	-0.146	-0.606	-0.659	-0.538
1971	0.062	0.054	0.072	0.063	0.041	0.082	-0.003	-0.025	0.020	-0.127	-0.150	-0.098	-0.428	-0.481	-0.387
1972	0.072	0.059	0.087	0.081	0.055	0.102	0.039	0.010	0.070	-0.061	-0.104	-0.018	-0.368	-0.429	-0.284
1973	0.066	0.044	0.081	0.096	0.076	0.120	0.087	0.064	0.113	-0.036	-0.065	-0.005	-0.162	-0.210	-0.117
1974	0.087	0.073	0.101	0.086	0.068	0.107	0.076	0.052	0.095	0.063	0.032	0.091	-0.094	-0.125	-0.066
1975	0.085	0.076	0.096	0.122	0.106	0.139	0.149	0.130	0.169	0.145	0.116	0.169	0.059	0.024	0.098
1976	0.096	0.081	0.109	0.133	0.114	0.150	0.135	0.111	0.160	0.131	0.102	0.158	0.127	0.080	0.158
1977	0.111	0.098	0.122	0.145	0.124	0.166	0.113	0.098	0.131	0.141	0.126	0.161	0.002	-0.038	0.046
1978	0.125	0.108	0.138	0.146	0.131	0.166	0.120	0.094	0.141	0.017	-0.016	0.061	-0.316	-0.382	-0.249
1979	0.105	0.091	0.119	0.090	0.074	0.107	0.001	-0.011	0.017	-0.170	-0.191	-0.149	-0.569	-0.608	-0.529
1980	0.079	0.064	0.093	0.071	0.060	0.082	-0.015	-0.031	-0.003	-0.200	-0.219	-0.182	-0.419	-0.457	-0.373
1981	0.065	0.052	0.077	0.049	0.040	0.058	-0.071	-0.083	-0.052	-0.262	-0.283	-0.246	-0.578	-0.629	-0.538
1982	0.047	0.037	0.059	0.045	0.036	0.054	0.003	-0.008	0.015	-0.090	-0.105	-0.084	-0.348	-0.387	-0.299
1983	0.062	0.053	0.072	0.060	0.051	0.069	-0.052	-0.066	-0.038	-0.199	-0.224	-0.178	-0.565	-0.601	-0.532
1984	0.049	0.039	0.060	0.037	0.029	0.043	-0.003	-0.010	0.000	-0.129	-0.143	-0.117	-0.374	-0.416	-0.348
1985	0.049	0.041	0.056	0.040	0.033	0.047	0.028	0.022	0.033	-0.030	-0.041	-0.018	-0.192	-0.216	-0.167
1986	0.028	0.023	0.034	0.045	0.038	0.050	0.024	0.019	0.026	-0.062	-0.073	-0.049	-0.287	-0.317	-0.264
1987	0.030	0.025	0.035	0.029	0.024	0.035	-0.013	-0.019	-0.004	-0.111	-0.120	-0.100	-0.334	-0.368	-0.299
1988	0.027	0.021	0.033	0.039	0.032	0.045	0.019	0.012	0.024	-0.059	-0.067	-0.054	-0.255	-0.276	-0.234
1989	0.028	0.023	0.034	0.035	0.030	0.041	0.012	0.004	0.019	-0.031	-0.043	-0.020	-0.282	-0.300	-0.263
1990	0.037	0.031	0.042	0.053	0.045	0.060	0.032	0.024	0.039	-0.053	-0.058	-0.044	-0.221	-0.278	-0.189
1991	0.029	0.023	0.035	0.055	0.047	0.062	0.022	0.018	0.028	-0.010	-0.019	0.008	-0.169	-0.184	-0.156
1992	0.023	0.019	0.029	0.037	0.029	0.045	0.016	0.010	0.020	-0.074	-0.085	-0.063	-0.280	-0.316	-0.251
1993	0.031	0.025	0.038	0.037	0.031	0.042	0.009	0.004	0.017	-0.059	-0.068	-0.051	-0.260	-0.281	-0.237
1994	0.031	0.025	0.039	0.018	0.015	0.024	-0.007	-0.012	-0.001	-0.057	-0.066	-0.047	-0.283	-0.302	-0.269
1995	0.030	0.025	0.035	0.048	0.043	0.053	0.023	0.017	0.030	-0.022	-0.030	-0.016	-0.213	-0.240	-0.189
1996	0.029	0.025	0.034	0.033	0.031	0.037	0.021	0.014	0.025	-0.102	-0.109	-0.093	-0.408	-0.425	-0.386
1997	0.030	0.026	0.034	0.027	0.024	0.030	0.009	0.003	0.012	-0.048	-0.055	-0.044	-0.222	-0.233	-0.208
1998	0.027	0.022	0.031	0.027	0.024	0.029	0.016	0.014	0.019	-0.017	-0.021	-0.014	-0.195	-0.210	-0.179
1999	0.018	0.015	0.021	0.021	0.019	0.023	0.006	0.003	0.008	-0.036	-0.040	-0.034	-0.165	-0.172	-0.155
2000	0.010	0.007	0.012	0.002	-0.000	0.005	-0.018	-0.022	-0.017	-0.074	-0.078	-0.068	-0.201	-0.216	-0.191
2001	0.007	0.005	0.009	0.012	0.011	0.014	0.009	0.007	0.010	-0.027	-0.029	-0.026	-0.126	-0.133	-0.118
2002	0.006	0.005	0.008	0.009	0.008	0.010	0.009	0.008	0.011	-0.020	-0.022	-0.018	-0.123	-0.133	-0.120
2003	0.008	0.006	0.010	0.012	0.010	0.014	0.009	0.008	0.010	-0.013	-0.017	-0.011	-0.106	-0.116	-0.099
2004	0.009	0.007	0.010	0.013	0.011	0.014	0.004	0.001	0.005	-0.048	-0.050	-0.044	-0.196	-0.207	-0.189
2005	0.012	0.009	0.015	0.015	0.012	0.017	0.006	0.003	0.009	-0.044	-0.046	-0.041	-0.186	-0.193	-0.177
2006	0.020	0.017	0.023	0.022	0.020	0.024	0.013	0.011	0.015	-0.015	-0.017	-0.012	-0.150	-0.161	-0.138
2007	0.012	0.009	0.014	0.014	0.012	0.016	0.009	0.008	0.011	-0.024	-0.027	-0.021	-0.152	-0.156	-0.141
2008	0.010	0.008	0.012	0.012	0.010	0.013	0.001	-0.000	0.004	-0.030	-0.034	-0.026	-0.120	-0.143	-0.111
2009	0.008	0.007	0.011	0.016	0.014	0.018	0.012	0.011	0.014	-0.002	-0.004	0.000	-0.094	-0.108	-0.084
2010	0.010	0.007	0.013	0.016	0.014	0.019	0.006	0.004	0.008	-0.023	-0.026	-0.021	-0.123	-0.136	-0.113
2011	0.011	0.009	0.014	0.014	0.012	0.017	0.009	0.008	0.011	-0.033	-0.035	-0.027	-0.224	-0.232	-0.214
2012	0.009	0.007	0.011	0.016	0.015	0.018	0.006	0.005	0.008	-0.042	-0.045	-0.039	-0.138	-0.154	-0.131
2013	0.007	0.005	0.010	0.012	0.010	0.013	0.004	0.002	0.006	-0.039	-0.044	-0.036	-0.128	-0.139	-0.112
2014	0.005	0.003	0.008	0.010	0.009	0.013	0.000	-0.002	0.002	-0.058	-0.065	-0.056	-0.234	-0.246	-0.229
2015	0.006	0.005	0.008	0.010	0.009	0.012	0.008	0.007	0.009	-0.028	-0.033	-0.021	-0.199	-0.210	-0.196
2016	0.006	0.004	0.009	0.014	0.012	0.016	0.006	0.004	0.008	-0.049	-0.057	-0.043	-0.196	-0.213	-0.185
2017	0.010	0.007	0.014	0.012	0.010	0.014	0.001	-0.001	0.003	-0.051	-0.058	-0.046	-0.200	-0.222	-0.181
2018	0.008	0.006	0.010	0.014	0.013	0.017	0.008	0.005	0.011	-0.074	-0.081	-0.069	-0.327	-0.348	-0.313
2019	0.007	0.005	0.010	0.016	0.013	0.017	0.009	0.008	0.011	-0.038	-0.043	-0.030	-0.273	-0.289	-0.248

Note: Results are for the models in equations (3) and (4). Each estimate is presented with 90% credibility interval

6 Figures

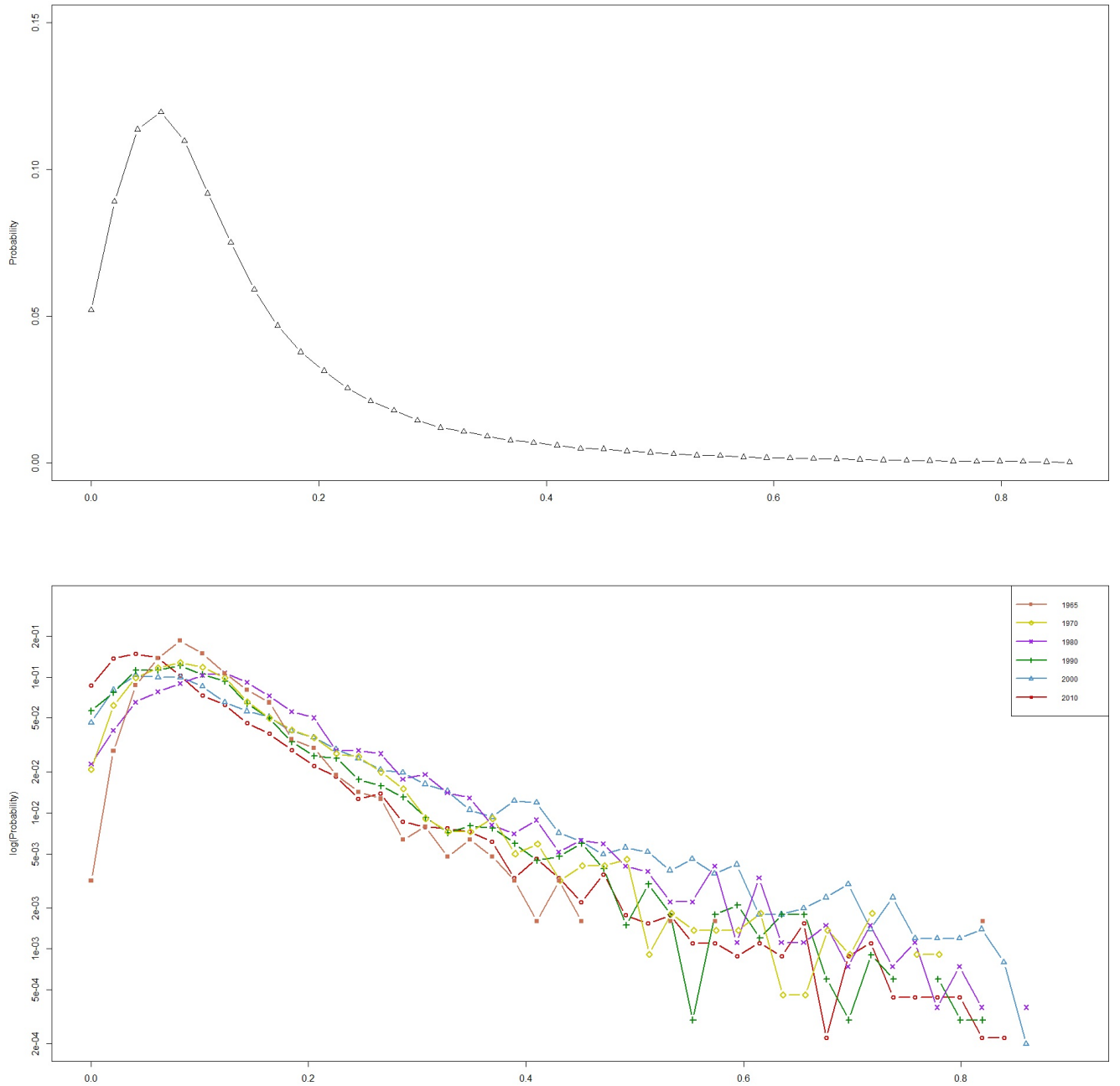


Figure 1: Pooled Investment Rate and Stacked Investment Rate Probability Distributions

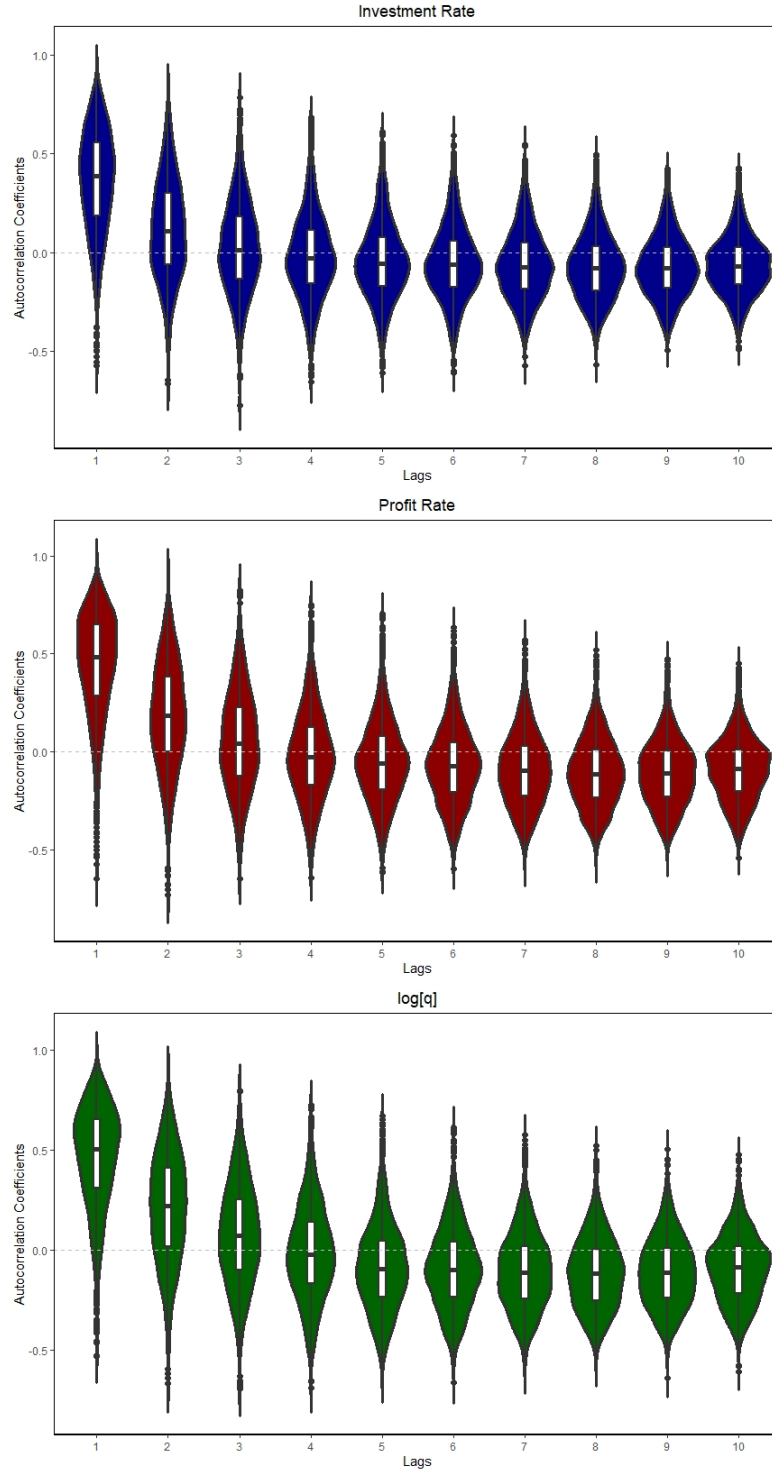


Figure 2: Autocorrelation coefficients for Investment Rate, Profit Rate and $\log[q]$

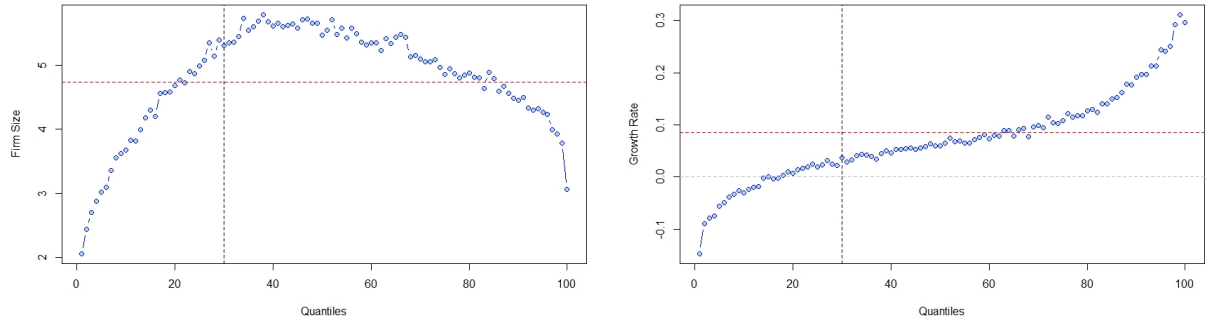


Figure 3: Median Size and Growth rate of the firms at Investment Rate quantiles

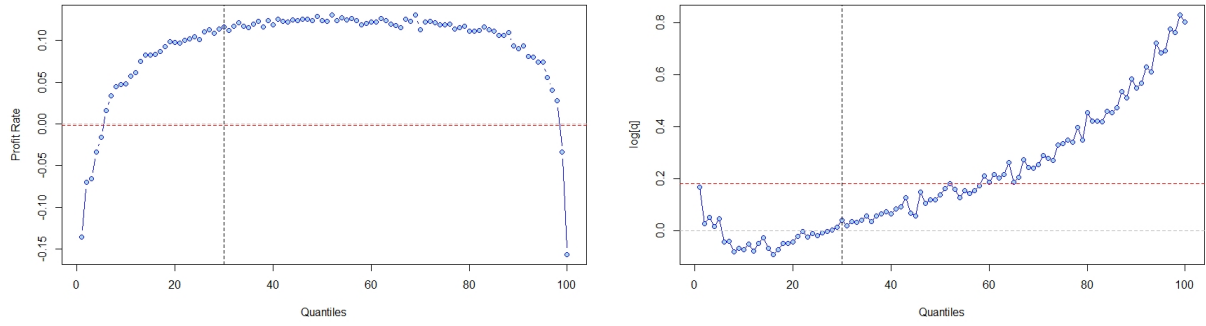


Figure 4: Median Profit Rate and $\log[q]$ of the firms at Investment Rate quantiles

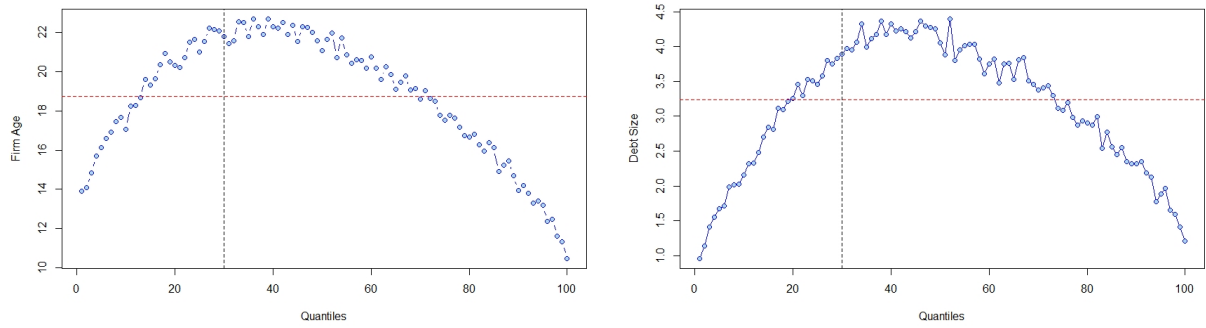


Figure 5: Mean Age and Median Debt Size of the firms at Investment Rate quantiles

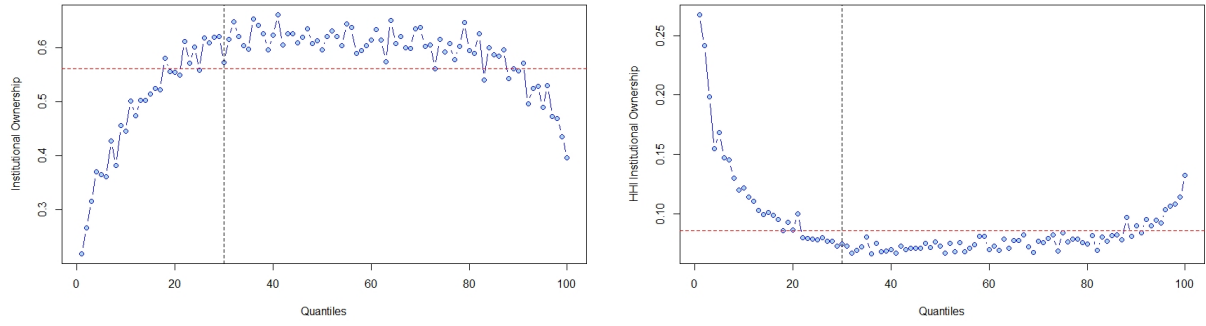


Figure 6: Median Institutional Ownership (%) and HHI of the Institutional Ownership of the firms at Investment Rate quantiles

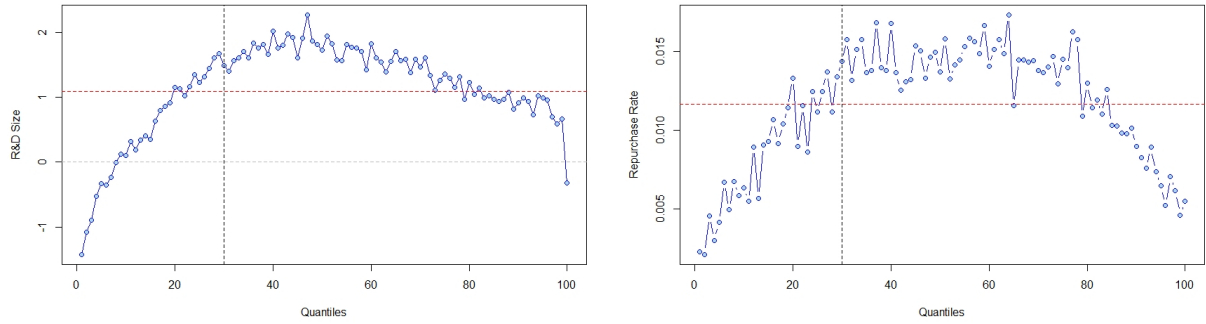


Figure 7: Median Research and Development expenses size and Mean Stock Repurchase Rate of the firms at Investment Rate quantiles

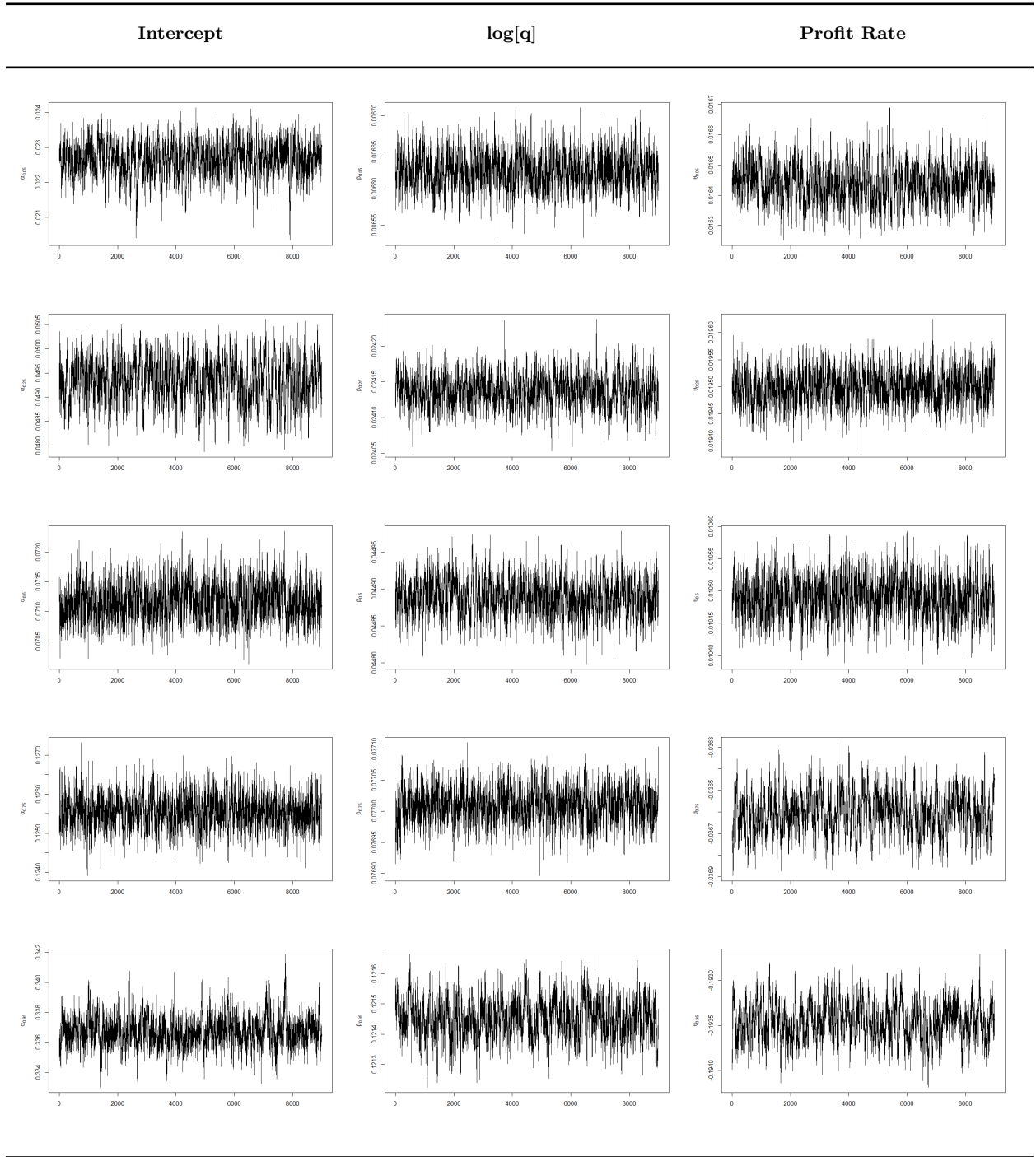


Figure 8: Traceplots of posterior samples from pooled quantile regression model, $\tau = \{0.05, 0.25, 0.50, 0.75, 0.95\}$

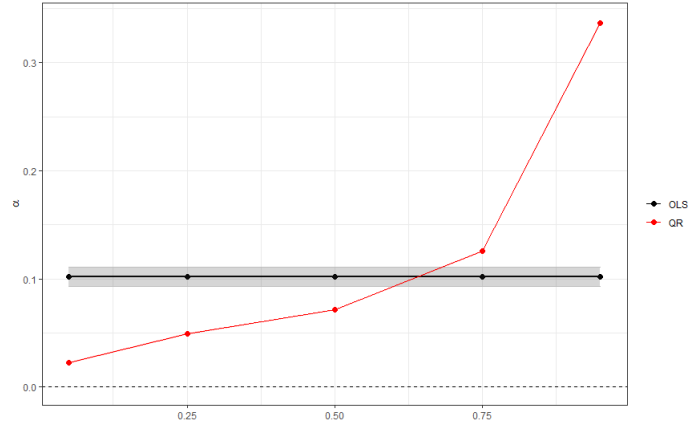


Figure 9: Pooled Quantile Regression estimates of α (intercept) with %90 Credibility Intervals

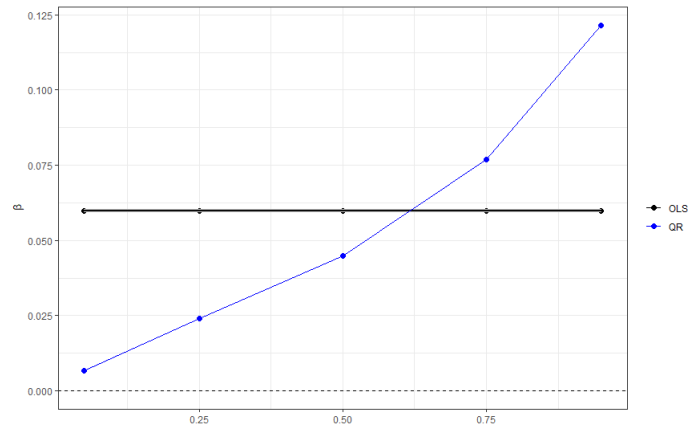


Figure 10: Pooled Quantile Regression estimates of β ($\log[q]$) with %90 Credibility Intervals

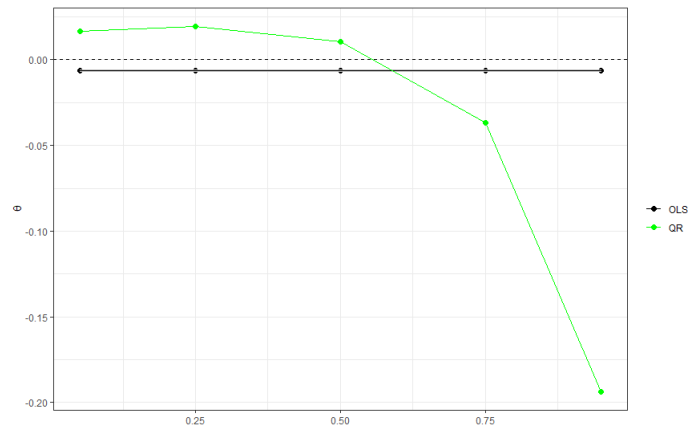


Figure 11: Pooled Quantile Regression estimates of θ (profit rate) with %90 Credibility Intervals

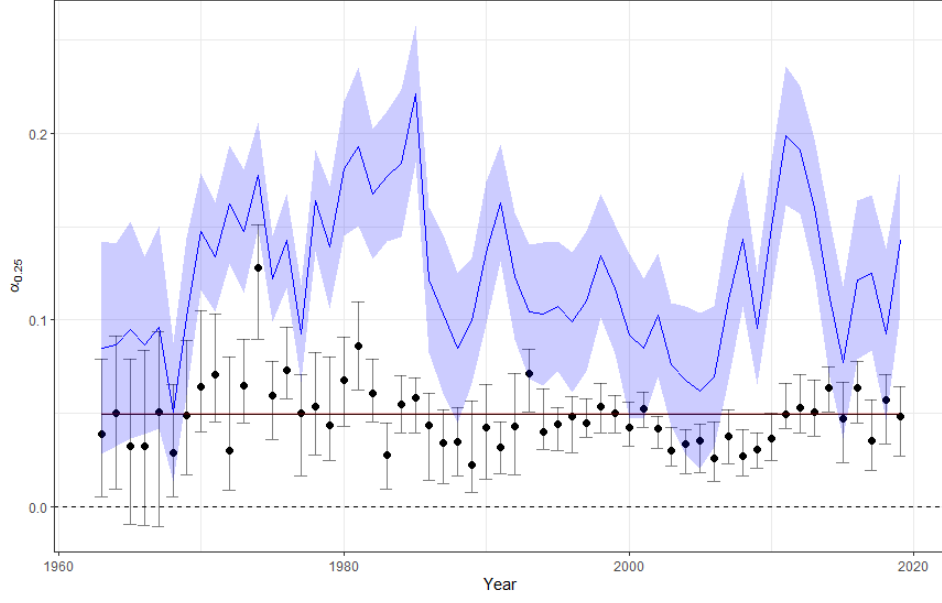


Figure 12: Point estimates time evaluation, blue lines represent cross sectional OLS estimates, black dots represent cross sectional quantile regression estimates and straight line represent pooled quantile regression estimate, $\tau = \{0.25\}$

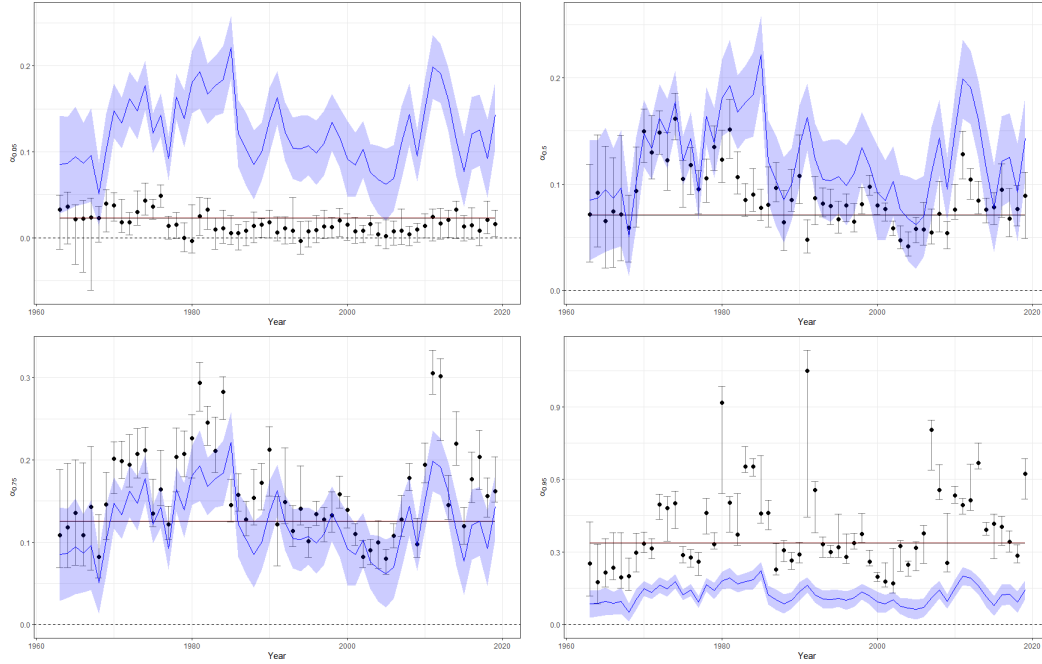


Figure 13: Point estimates time evaluation, blue lines represent cross sectional OLS estimates, black dots represent cross sectional quantile regression estimates and straight line represent pooled quantile regression estimate, $\tau = \{0.05, 0.50, 0.75, 0.95\}$

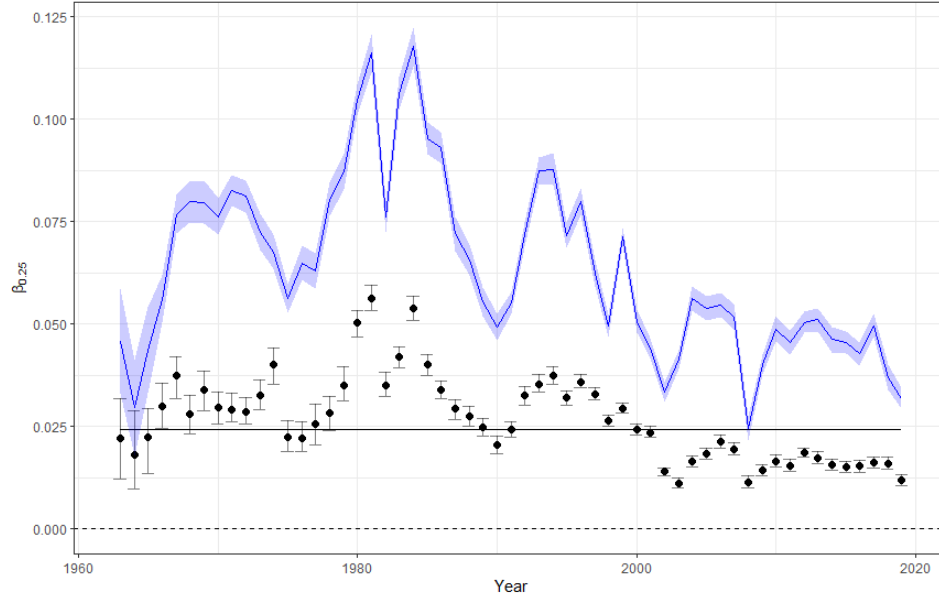


Figure 14: Point estimates time evaluation, blue lines represent cross sectional OLS estimates, black dots represent cross sectional quantile regression estimates and straight line represent pooled quantile regression estimate, $\tau = \{0.25\}$

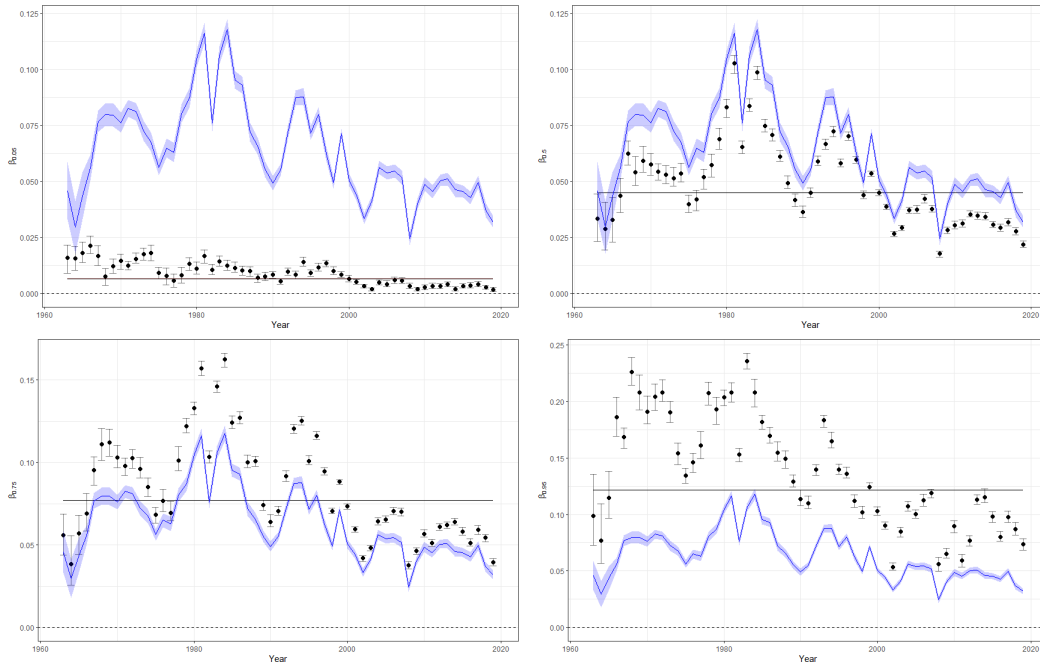


Figure 15: Point estimates time evaluation, blue lines represent cross sectional OLS estimates, black dots represent cross sectional quantile regression estimates and straight line represent pooled quantile regression estimate, $\tau = \{0.05, 0.50, 0.75, 0.95\}$

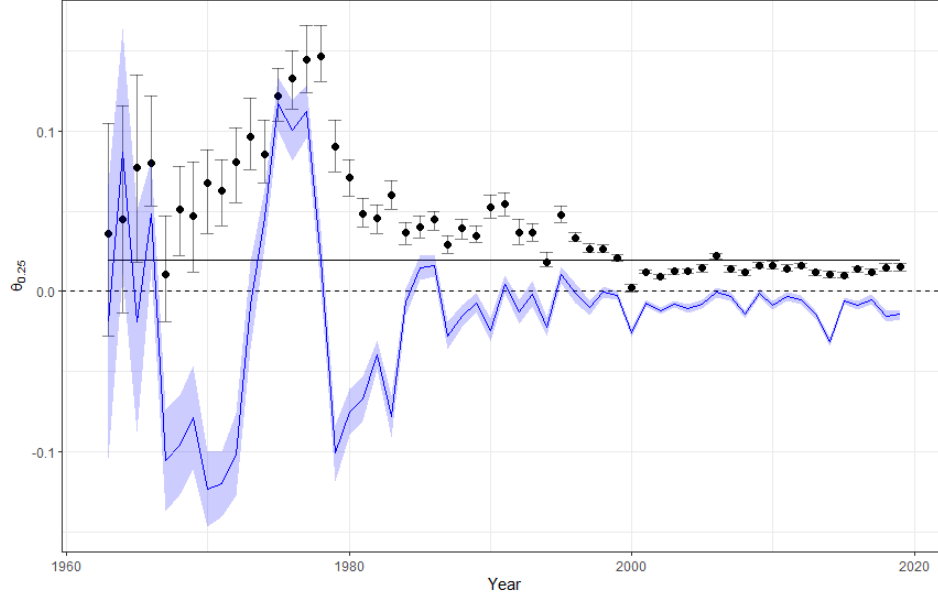


Figure 16: Point estimates time evaluation, blue lines represent cross sectional OLS estimates, black dots represent cross sectional quantile regression estimates and straight line represent pooled quantile regression estimate, $\tau = \{0.25\}$

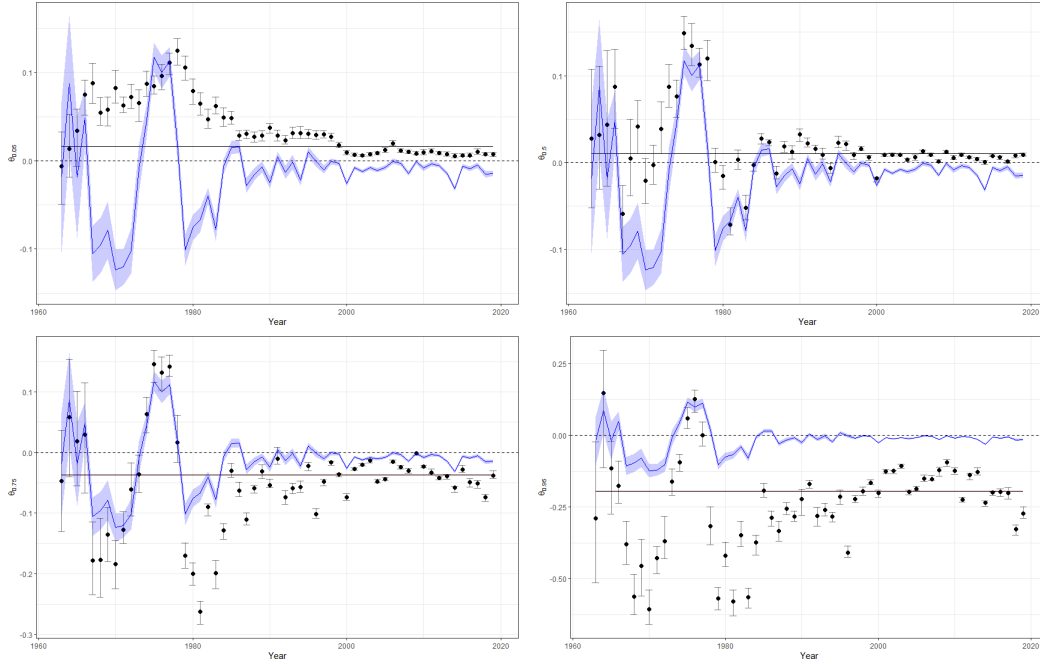


Figure 17: Point estimates time evaluation, blue lines represent cross sectional OLS estimates, black dots represent cross sectional quantile regression estimates and straight line represent pooled quantile regression estimate, $\tau = \{0.05, 0.50, 0.75, 0.95\}$

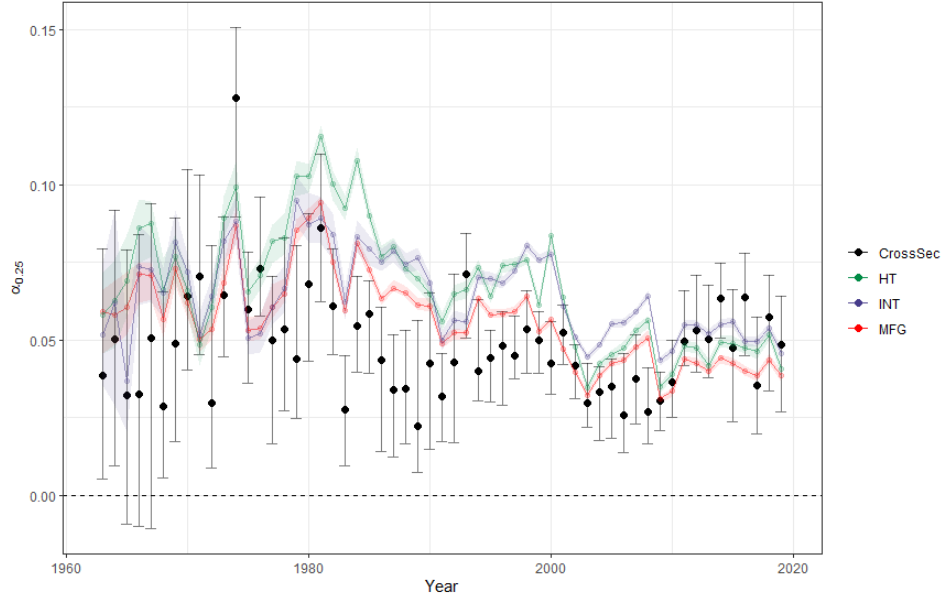


Figure 18: Point estimates time evaluation, sectoral fixed effects, intangible intensive fixed effects, manufacturing fixed effects and high-tech fixed effects presented, $\tau = \{0.25\}$



Figure 19: Point estimates time evaluation, sectoral fixed effects, intangible intensive fixed effects, manufacturing fixed effects and high-tech fixed effects presented, $\tau = \{0.05, 0.50, 0.75, 0.95\}$

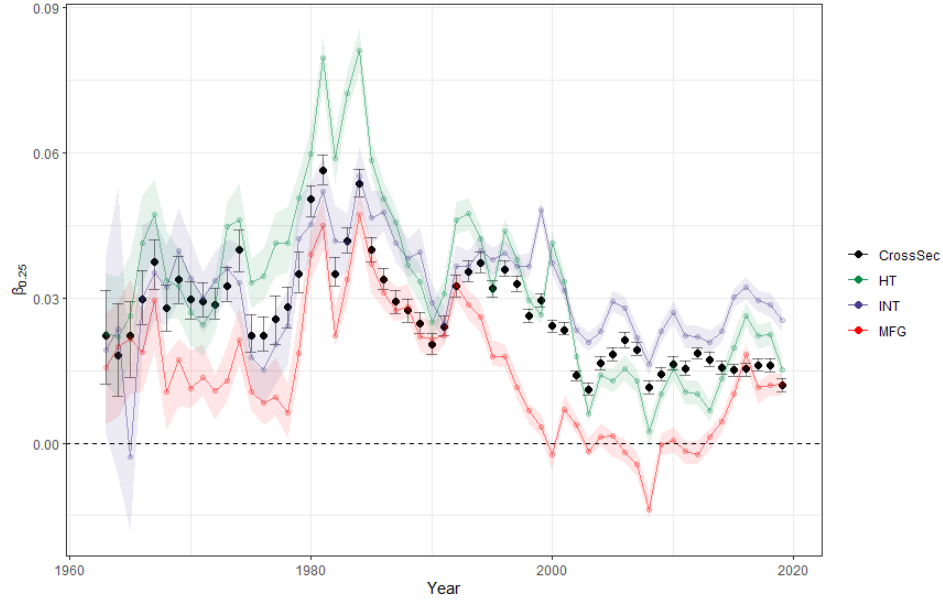


Figure 20: Point estimates time evaluation, sectoral fixed effects, intangible intensive fixed effects, manufacturing fixed effects and high-tech fixed effects presented, $\tau = \{0.25\}$

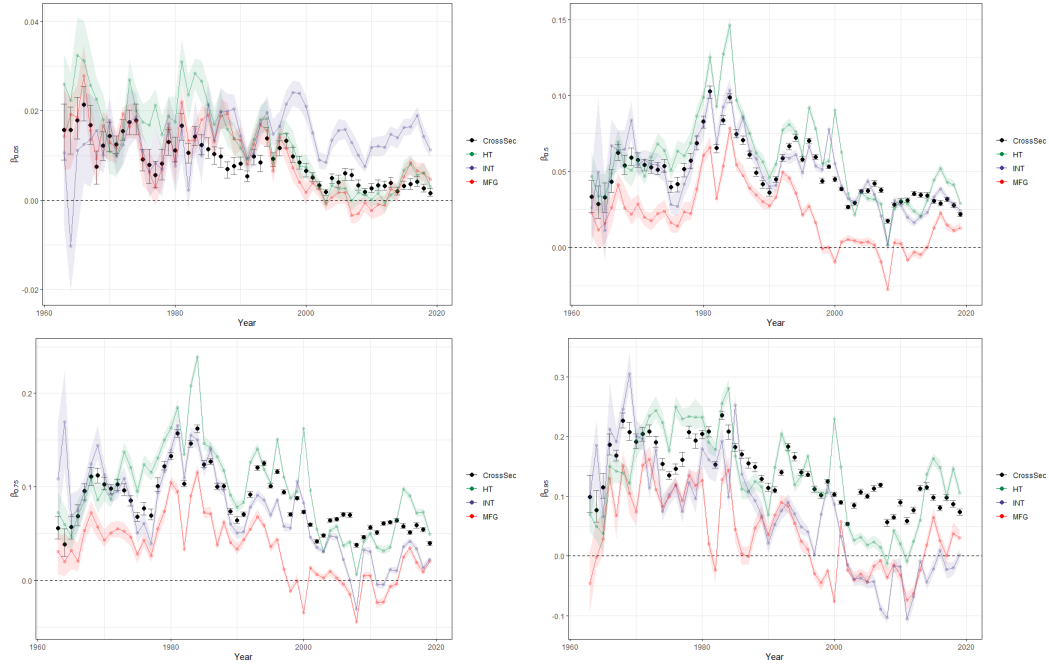


Figure 21: Point estimates time evaluation, sectoral fixed effects, intangible intensive fixed effects, manufacturing fixed effects and high-tech fixed effects presented, $\tau = \{0.05, 0.50, 0.75, 0.95\}$

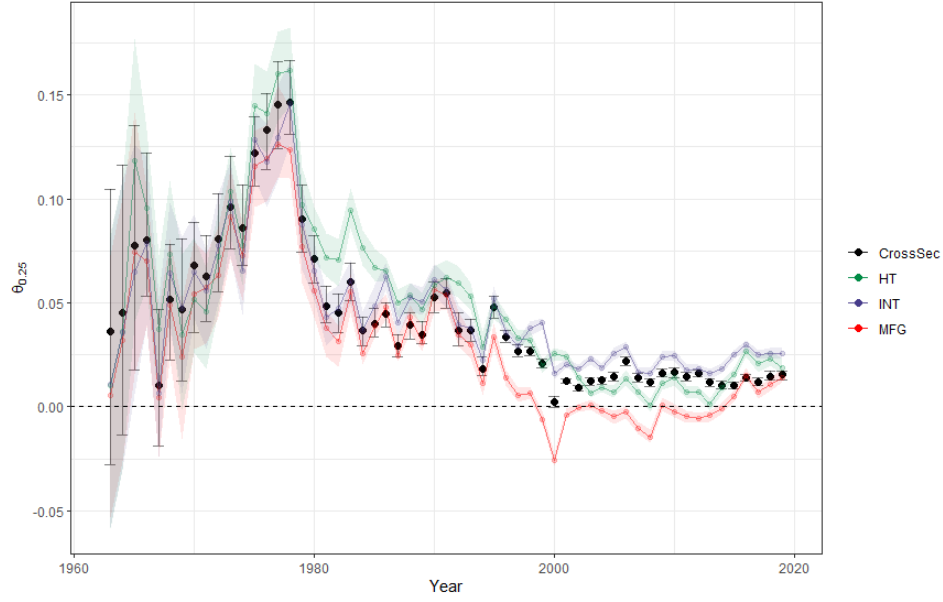


Figure 22: Point estimates time evaluation, sectoral fixed effects, intangible intensive fixed effects, manufacturing fixed effects and high-tech fixed effects presented, $\tau = \{0.25\}$

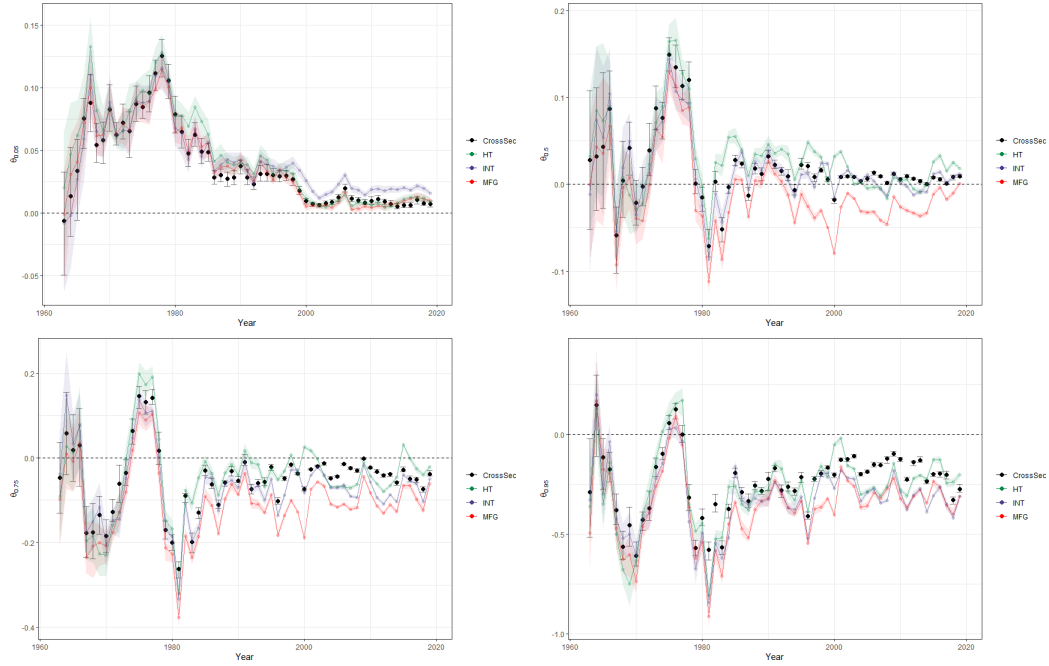


Figure 23: Point estimates time evaluation, sectoral fixed effects, intangible intensive fixed effects, manufacturing fixed effects and high-tech fixed effects presented, $\tau = \{0.05, 0.50, 0.75, 0.95\}$