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**Working Paper**

## Inflation and output dynamics in a model with labor market search and capital accumulation

CESifo Working Paper, No. 2036

**Provided in Cooperation with:**

Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

*Suggested Citation:* Heer, Burkhard; Maußner, Alfred (2007) : Inflation and output dynamics in a model with labor market search and capital accumulation, CESifo Working Paper, No. 2036, Center for Economic Studies and ifo Institute (CESifo), Munich

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WITH LABOR MARKET SEARCH AND  
CAPITAL ACCUMULATION

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CESIFO WORKING PAPER NO. 2036  
CATEGORY 6: MONETARY POLICY AND INTERNATIONAL FINANCE  
JUNE 2007

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# INFLATION AND OUTPUT DYNAMICS IN A MODEL WITH LABOR MARKET SEARCH AND CAPITAL ACCUMULATION

## Abstract

In a sticky-price model with labor market search and habit persistence, Walsh (2005) shows that inertia in the interest rate policy helps to reconcile the inflation and output persistence with empirical observations for the US economy. We show that this finding is sensitive with regard to the introduction of capital formation. While we are able to replicate the findings for the inflation inertia in a model with capital adjustment costs and variable capacity utilization, the output response to an interest shock is found to be too large and no longer hump-shaped in this case. In addition we find that the response of output to a technology shock can only be reconciled with empirical findings if either the adjustment of the utilization rate is very costly or there is only a modest amount of nominal rigidity in the economy.

JEL Code: E52, E32, J64.

Keywords: interest rate policy, labor market search, business cycles, inflation, capital adjustment costs.

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# 1 Introduction

There is ample evidence from structural vector autoregressions using different identification schemes and data sets that a sudden increase of the short term nominal interest rate produces a persistent and hump-shaped response of output and inflation.<sup>1</sup> In recent studies, labor market imperfections have been introduced into monetary business cycle models in order to replicate these findings. Christiano et al. (2005) model nominal rigidities in the form of both price and wage staggering in order to explain the observed inertia in inflation after a monetary expansion. Walsh (2005) and Trigari (2004) consider search and matching frictions in the labor market. Walsh (2005) finds that the inertia of the interest rate policy itself is an important contributing factor for the explanation of the inflation and output inertia, while Trigari (2004) considers the effects of the wage bargaining process on the variation of both inflation and real wages following a monetary shock.<sup>2</sup>

In this paper, we consider the sensitivity of the latter studies with respect to the introduction of capital. Our economy is based upon the model of Walsh (2005). In addition, we introduce capital as a second production factor besides labor. The reasoning why capital may introduce a different dynamic response of inflation and output to a monetary shock is as follows: In the model of Walsh (2005) the marginal costs of price setters equal the relative price of intermediate goods in terms of the final good. Intermediate good firms adjust their nominal price immediately while wholesale firms respond only sluggishly to a demand or supply shock. Thus, marginal costs of price setters decrease in response to a negative demand shock. The size of this shock depends on the response of the household sector to an increase of the nominal interest rate. Without capital and with habit persistence in consumption this effect is small. However, if capital allows for intertemporal substitution, overall demand can decrease significantly. Obviously, the adjustment of capital as a second factor of production also affects the dynamics of output.

As one of our main results, our model with capital is able to generate inflation dynamics following an interest rate shock that is in accordance with empirical observations. Therefore, we are able to confirm this finding of Walsh (2005) who considers a model without

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<sup>1</sup>See, among others, Sims (1992), Leeper et al. (1996), and Christiano et al. (1999, 2005).

<sup>2</sup>Subsequently, the labor market search model has also been prominently applied to the analysis of the Ramsey policy as, e.g., in Faia (2007), or the study of the business-cycle dynamics of wages as in Rotemberg (2006).

capital. Similar to Christiano et al. (2005), we also find that the introduction of variable capital utilization is an important factor for the modelling of the inertia in the inflation dynamics. In this case, rather the capacity than the investment demand increases after a fall in the interest rate so that the real interest rate displays a smaller variation. In the model with capital, however, an unexpected rise in the nominal interest rate does not trigger a hump-shaped response of output, quite contrary to the model without capital.

In addition, we also analyze the effects of a technology shock on the output-inflation dynamics. Most studies including Walsh (2005), Christiano et al. (2005), or Trigari (2004) neglect this question. We consider it an interesting problem because a researcher is ultimately aiming for a monetary general equilibrium model that is able to match the empirical responses to various kinds of supply, demand, and policy shocks simultaneously. As one prominent example, consider the analysis of optimal monetary policy and to what extent the monetary authority should respond to a productivity shock. Here, too, we find that while the inflation dynamics is insensitive to the assumption of fixed capital services the output dynamics is not. In line with empirical evidence, we get a protracted hump-shaped decline of the rate of inflation in response to a productivity shock in our model with capital accumulation and a variable utilization rate of capital. However, this model also implies a significant immediate decrease of output that is not observed in estimated impulse response functions. We can reconcile the model with empirical evidence if we either assume that it is very costly to adjust the utilization rate of capital or that the degree of nominal rigidity in our model economy is small.

The remainder of the paper is structured as follows. Section 2 introduces the model. In Section 3, we describe the calibration and computation of the model. Section 4 presents our results, and Section 5 concludes. In the Appendix, we provide the log-linearized version of the model.

## 2 The model economy

In this section, we describe our model that is based upon Walsh (2005). Three different sectors are depicted: firms, households, and the monetary authority.

## 2.1 Firms

### 2.1.1 Retail sector

A final goods or retail sector buys differentiated goods  $Y_{jt}$  distributed over the unit interval,  $j \in [0, 1]$ , from wholesale firms and assembles the final output  $Y_t$  according to

$$Y_t = \left( \int_0^1 Y_{jt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1. \quad (1)$$

Profit maximization of retail firms,

$$\max_{\{Y_{jt}\}_{j=0}^1} Y_t - \int_0^1 P_{jt} Y_{jt} dj,$$

implies the demand function

$$Y_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\theta} Y_t, \quad (2)$$

where  $P_{jt}$  is the nominal price of good  $j \in [0, 1]$  and  $P_t$  is the price level. The zero profit condition for the retail sector implies that  $P_t$  is given by

$$P_t = \left( \int_0^1 P_{jt}^{1-\theta} dj \right)^{1/(1-\theta)}. \quad (3)$$

### 2.1.2 Wholesale sector

Firms in the wholesale sector purchase intermediate goods  $y_{jt}$ ,  $j \in [0, 1]$  from the production sector that is described below. The profit of a wholesaler in terms of the final output is given by

$$\left( \frac{P_{jt}}{P_t} - g_t \right) Y_{jt}, \quad (4)$$

where

$$g_t = \frac{P_t^W}{P_t} \quad (5)$$

is the price of the output of the production sector in terms of the final good. From the perspective of the wholesale sector  $g_t$  are the real marginal costs faced by any firm in this sector.

Prices are set according to the mechanism set out in Calvo (1983). In each period  $(1 - \omega)$  of the wholesale firms are allowed to set their relative price  $P_{jt}/P_t$  optimally. Henceforth we use the index  $A$  to refer to these firms. Walsh (2005) follows Christiano et al. (2005) and assumes that prices must be set before the monetary shock is realized. The remaining fraction of the wholesale firms, indexed by  $N$ , adjusts their price according to a rule of thumb: They increase their price according to the inflation factor (one plus the rate of inflation) of the previous period  $\pi_{t-1}$ :

$$P_{Nt} = \pi_{t-1}P_{Nt-1}, \quad \pi_t := \frac{P_t}{P_{t-1}}. \quad (6)$$

This price setting behavior implies the following log-linear Phillips curve equation:

$$\hat{\pi}_t = \frac{1}{1 + \beta} \hat{\pi}_{t-1} + \frac{\beta}{1 + \beta} E_{t-1} \hat{\pi}_{t+1} + \Gamma E_{t-1} \hat{g}_t \quad (7)$$

with  $\Gamma = \frac{(1-\omega)(1-\beta\omega)}{(1+\beta)\omega}$  and where a hat over a variable denotes its percentage deviation from its steady-state value.  $\beta$  denotes the discount factor of the household that will be introduced below. We also consider the effect of a monetary policy shock if the price setting firms  $A$  choose their price after they have observed the shock. This implies the following Phillips curve:

$$\hat{\pi}_t = \frac{1}{1 + \beta} \hat{\pi}_{t-1} + \frac{\beta}{1 + \beta} E_t \hat{\pi}_{t+1} + \Gamma \hat{g}_t. \quad (8)$$

### 2.1.3 Intermediate goods sector

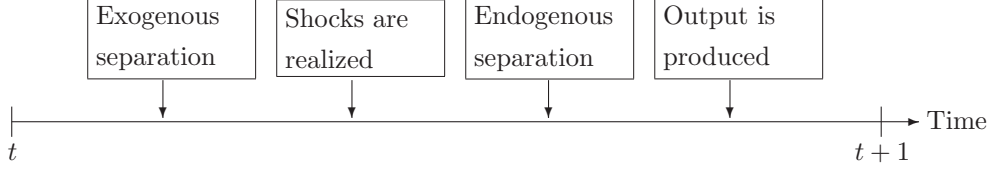
Employment relationships consist of a worker and a firm. At the beginning of each period there are  $N_t$  employed workers and, thus,  $N_t$  worker-firm pairs indexed by  $i$ . For reasons outside of the model, the fraction of  $\rho^x$  of those pairs separate. The remaining pairs observe the current state of the environment and decide whether or not to continue their relationship. Those that do not separate produce output. Figure 1 depicts the timing of events in this model.

Output produced by a worker-firm pair  $i$  is given by

$$y_{it} = Z_t a_{it} k_{it}^\alpha, \quad \alpha \in (0, 1). \quad (9)$$

$k_{it}$  are capital services,  $Z_t$  is a random productivity disturbance that is common to all firms, and  $a_{it}$  is a random productivity disturbance that is specific to relationship  $i \in [0, 1]$ . The

**Figure 1:** Timing of events within a period



worker and the firm observe both shocks and choose  $k_{jt}$  to maximize their joint payoff:

$$\max_{k_{it}} g_t Z_t a_{it} k_{it}^\alpha - r_t k_{it} - l,$$

where  $l$  denotes the disutility of work. Firm  $i$  pays real interest  $r_t$  on its capital services  $k_{it}$ . Profit maximization implies

$$k(a_{it}) = \left( \frac{\alpha g_t Z_t a_{it}}{r_t} \right)^{1/(1-\alpha)}. \quad (10)$$

**Job creation.** The decision to sever the relationship depends on the outside options of the worker and the firm and on the present value of continuing the relationship into the next period  $v_{it}$ . Note that except for the realization of  $a_{it}$  all employment relationships face the same conditions. Thus, if  $a_{it}$  is distributed identically and independently over time,  $v_{it}$  must be equal for all worker-firm pairs and we can drop the index  $i$  from this variable and all others as well. In equilibrium, the present value of the firm's outside opportunities is zero, and the value of the worker's outside opportunities equals the present value of being unemployed  $w_t^u$ . The surplus of an employment relationship thus can be written as

$$s_t = g_t Z_t a_t k(a_t)^\alpha - r_t k(a_t) - l + v_t - w_t^u. \quad (11)$$

The firm and the worker will terminate their relationship if  $a_{it} < \underline{a}_t$ , where  $\underline{a}_t$  is determined as solution to

$$g_t Z_t \underline{a}_t k(\underline{a}_t)^\alpha - r_t k(\underline{a}_t) - l + v_t - w_t^u = 0. \quad (12)$$

Note that due to (10) and (12) the surplus of an employment relationship can also be written as

$$s_t = (1 - \alpha) \left( \frac{\alpha}{r_t} \right)^{\alpha/(1-\alpha)} g_t Z_t^{1/(1-\alpha)} \left[ a_t^{1/(1-\alpha)} - \underline{a}_t^{1/(1-\alpha)} \right]. \quad (13)$$



Given the job destruction margin  $\underline{a}_t$  the endogenous job destruction rate  $\rho_t^n$  is obtained from

$$\rho_t^n = \int_0^{\underline{a}_t} f(a) da = F(\underline{a}_t), \quad (14)$$

where  $f(a)$  and  $F(a)$  denote the probability density function and the distribution function of  $a_{it}$ , respectively. Since new matches from period  $t$  will not produce before period  $t + 1$  the mass of workers that are unemployed during period  $t$  equals

$$\begin{aligned} U_t &= \underbrace{1 - N_t}_{\text{stock of unemployed}} + \underbrace{\rho^x N_t}_{\text{exogenous separations}} + \underbrace{(1 - \rho^x)\rho_t^n N_t}_{\text{endogenous separations}}, \\ &= 1 - (1 - \rho^x)(1 - \rho_t^n)N_t. \end{aligned} \quad (15)$$

**Matching technology.** Given the number of unemployed persons  $U_t$  and the number of firms that offer jobs  $V_t$ , employment evolves according to

$$N_{t+1} = (1 - \rho^x)(1 - \rho_t^n)N_t + m(U_t, V_t), \quad (16)$$

where  $m(U_t, V_t)$  is the number of aggregate matches. The matching function is assumed to be Cobb-Douglas:

$$m(U_t, V_t) = \psi U_t^\chi V_t^{1-\chi}, \quad \chi \in (0, 1). \quad (17)$$

The probability that a firm offering a job in period  $t$  will find a worker is given by

$$\kappa_t^f = \frac{m(U_t, V_t)}{V_t} = \psi \left( \frac{V_t}{U_t} \right)^{-\chi}. \quad (18)$$

Similarly, the probability that the unemployed worker is finding a job is given by

$$\kappa_t^w = \psi \left( \frac{V_t}{U_t} \right)^{1-\chi}. \quad (19)$$

**Job creation.** We assume that the firm obtains the share  $1 - \eta \in (0, 1)$  from an employment relationship that produces in period  $t$ . The probability that a worker-firm pair that is matched in period  $t$  will produce in period  $t + 1$  is  $(1 - \rho^x)(1 - \rho_{t+1}^n)$ . The expected value of this match in period  $t + 1$  equals

$$\int_{\underline{a}_{t+1}}^{\infty} s_{t+1} \frac{f(a)}{1 - \rho_{t+1}^n} da,$$

where  $f(a)/(1 - \rho_{t+1}^n)$  is the conditional density of the event  $a|a \geq \underline{a}_{t+1}$ . We assume free entry of firms and a cost of  $\gamma$  for offering a job. Thus, the number of vacancies is determined by the condition

$$\gamma = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ (1 - \eta) \kappa_t^f (1 - \rho^x) \int_{\underline{a}_{t+1}}^{\infty} s_{t+1} f(a) da \right\}, \quad (20)$$

where  $\beta(\lambda_{t+1}/\lambda_t)$  is the stochastic discount factor and  $\lambda_t$  the marginal utility of consumption that we will introduce in a moment. Equation (20) establishes that the outside value of a firm equals zero.

**The present value of unemployment.** In period  $t$  an unemployed worker faces the probability  $\kappa^w$  in (19) to find a job. The probability that he will not lose this job in the next period is  $(1 - \rho^x)(1 - \rho_{t+1}^n)$ . Since the worker always receives the value of his outside option, the present value of being unemployed is determined by

$$w_t^u = b + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ \eta \kappa_t^w (1 - \rho^x) \int_{\underline{a}_{t+1}}^{\infty} s_{t+1} f(a) da + w_{t+1}^u \right\}, \quad (21)$$

where  $b$  is the worker's valuation of leisure time.

**The present value of a continuing employment relationship.** A worker-firm pair that produces in the next period receives the expected value of its surplus. Since the worker always receives the value of its outside option  $w_{t+1}^u$  and since the value of the firm's outside option equals zero, the present discounted value of a match that continues to produce in  $t + 1$  is given by

$$v_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ (1 - \rho^x) \int_{\underline{a}_{t+1}}^{\infty} s_{t+1} f(a) da + w_{t+1}^u \right\}. \quad (22)$$

Note that equations (22) and (21) imply

$$q_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ (1 - \rho^x)(1 - \eta \kappa_t^w) \int_{\underline{a}_{t+1}}^{\infty} s_{t+1} f(a) da \right\}, \quad (23)$$

where

$$q_t = v_t - w_t^u + b.$$

Division of (23) by equation (20) yields

$$q_t = \frac{\gamma(1 - \eta\kappa_t^w)}{(1 - \eta)\kappa_t^f}. \quad (24)$$

## 2.2 Households

Employed and unemployed workers pool their income so that we can ignore distributional issues. Employed workers supply one unit of labor inelastically with disutility  $l$  while unemployed workers enjoy leisure at value  $b$ . As in Walsh (2005), we introduce habit formation in the utility function. In addition, the household obtains utility from real money  $M_t/P_t$ . The household's current-period utility function is given by:

$$u(C_t, C_{t-1}, \zeta_t, M_t/P_t) := \frac{(C_t - hC_{t-1})^{1-\sigma} - 1}{1-\sigma} + (1 - \zeta_t)b - \zeta_t l + \phi(M_t/P_t),$$

$$h \in [0, 1), \quad \zeta_t = \begin{cases} 1 & \text{if employed} \\ 0 & \text{if unemployed.} \end{cases}$$

According to this specification the household's marginal utility of consumption also depends upon his level of consumption in the previous period. In particular, the marginal utility of consumption is higher if  $C_t$  is closer to  $C_{t-1}$ .

Income received from employment relationships that are not severed at the beginning of period  $t$  is given by

$$Inc_t = (1 - \rho^x)(1 - \rho_t^n)N_t \int_{a_t}^{\infty} \left[ g_t Z_t a_t k(a_t)^\alpha - r_t k(a_t) \frac{f(a_t)}{1 - \rho_t^n} \right] da_t. \quad (25)$$

In addition, the household receives profits  $\Omega_t$  from the wholesale sector and transfers  $T_t$  from the monetary authority.

The household holds beginning-of-period nominal money  $M_t$  and bonds  $B_t$ , as well as real physical capital stock  $\bar{K}_t$ . Bonds are issued by other households and pay a nominal rate of interest  $i_t$ . The nominal interest rate factor is denoted by  $R_t := 1 + i_t$ . Following Christiano et al. (2005), capital services  $K_t$  are related to the physical stock of capital  $\bar{K}_t$  by  $K_t = u_t \bar{K}_t$ , where  $u_t$  denotes the utilization rate of capital.<sup>3</sup> The household's budget constraint is given by :

$$\frac{B_{t+1} + M_{t+1}}{P_t} \leq Inc_t + \Omega_t + r_t u_t \bar{K}_t + T_t + R_t \frac{B_t}{P_t} + \frac{M_t}{P_t} - \gamma V_t - C_t - I_t - \iota(u_t) \bar{K}_t, \quad (26)$$

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<sup>3</sup>For reason of modelling simplicity, the household rather than the firm chooses  $u_t$ .

where  $I_t$  and  $\iota(u_t)$  denotes investment and the costs of setting the utilization rate to  $u_t$ , respectively. In the non-stochastic steady state,  $\bar{u} = 1$  and  $\iota(\bar{u}) = 0$ .

The stock of capital evolves according to

$$\bar{K}_{t+1} = \Phi\left(\frac{I_t}{\bar{K}_t}\right) \bar{K}_t + (1 - \delta)\bar{K}_t. \quad (27)$$

We assume that the concave function  $\Phi(\cdot)$  does not change the non-stochastic steady state of the model. Thus,  $I = \delta K$  implying  $\Phi(\delta) = \delta$  and  $\Phi'(\delta) = 1$ . The absolute value of the elasticity of  $\Phi'$  with respect to its argument  $I/K$  is given by the parameter  $\sigma_\Phi$ .

Households maximize

$$E_t \sum_{s=0}^{\infty} \beta^s u\left(C_{t+s}, \zeta_{t+s}, \frac{M_{t+s}}{P_{t+s}}\right)$$

with regard to  $M_{t+1}$ ,  $B_{t+1}$ ,  $\bar{K}_{t+1}$ ,  $C_t$ ,  $I_t$ , and  $u_t$  subject to (26) and (27). The first-order conditions of the household are given by:

$$\lambda_t = (C_t - hC_{t-1})^{-\sigma} - \beta h E_t (C_{t+1} - hC_t)^{-\sigma} \quad (28a)$$

$$\iota'(u_t) = r_t \quad (28b)$$

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{R_{t+1}}{\pi_{t+1}}, \quad (28c)$$

$$\lambda_t = \beta E_t \left\{ \frac{\phi'(M_{t+1}/P_{t+1}) + \lambda_{t+1}}{\pi_{t+1}} \right\}, \quad (28d)$$

$$\xi_t = \frac{1}{\Phi'(I_t/K_t)}, \quad (28e)$$

$$\xi_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[ r_{t+1} u_{t+1} - \iota(u_{t+1}) - \frac{I_{t+1}}{K_{t+1}} + \xi_{t+1} (1 - \delta + \Phi(I_{t+1}/K_{t+1})) \right]. \quad (28f)$$

Equations (28a) and (28b) are the optimal conditions for the current-period consumption level  $C_t$  and utilization rate  $u_t$ , respectively. Condition (28c) ensures that bonds have the same expected rate of return as capital. Note, that  $B_t \equiv 0$  in equilibrium, since we are aggregating the holdings of bonds over the members of the representative household. Equation (28d) induces a money demand function. Since the central bank will pursue an interest rate policy, we can disregard this equation. In (28e), the variable  $\xi_t$  is Tobin's  $q$  and gives the number of units of output which must be forgone to increase the stock of capital by one unit (this equals  $\Theta_t/\lambda_t$ , where  $\Theta_t$  is the Lagrange multiplier of the constraint (27) in the household's optimization problem).

## 2.3 Monetary authority

The central bank targets the nominal interest rate and supplies the amount of money necessary to achieve its target rate. We use the following rule:

$$R_{t+1} = \bar{\pi}^{(1-\rho_R)(1-\phi_\pi)} \beta^{-(1-\rho_R)} R_t^{\rho_R} \pi_t^{\phi_\pi(1-\rho_R)} e^{\phi_t}, \quad \phi_t \sim N(0, \sigma_\phi). \quad (29)$$

It is well-known that the exponent of the inflation factor  $\phi_\pi$  must be greater than one to ensure a determinate equilibrium. In the non-stochastic stationary equilibrium of the model the Euler equation (28c) implies  $\pi = \beta R$  and the Taylor rule delivers  $\pi = \bar{\pi}$ .<sup>4</sup>

Given the monetary policy, the nominal quantity of money adjust so that the money market is in equilibrium. Seignorage  $T_t$  is transferred to the households:

$$T_t = \frac{M_{t+1} - M_t}{P_t}. \quad (30)$$

## 2.4 Equilibrium

In equilibrium,

$$K_t = u_t \bar{K}_t,$$

and the aggregate amount of capital services,  $K_t$ , is given by the sum of the individual capital services

$$K_t = (1 - \rho^x)(1 - \rho_t^n) N_t \int_{\underline{a}_t}^{\infty} k(a_t) \frac{f(a_t)}{1 - \rho_t^n} da_t,$$

implying

$$K_t = (1 - \rho^x) N_t H(\underline{a}_t) \left( \frac{\alpha g_t Z_t}{r_t} \right)^{1/(1-\alpha)}, \quad H(\underline{a}_t) := \int_{\underline{a}_t}^{\infty} a_t^{(1/(1-\alpha))} f(a_t) da_t. \quad (31)$$

Aggregating  $y_{it}$  in (9) over all productive worker-firm pairs using this definition of capital yields the aggregate production function

$$Y_t = Z_t \left[ (1 - \rho^x) N_t H(\underline{a}_t) \right]^{1-\alpha} K_t^\alpha. \quad (32)$$

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<sup>4</sup>The policy rule of Walsh (2005) is only consistent with zero inflation,  $\bar{\pi} = 1$ .

Firms redistribute all profits to the households, and the monetary authority transfers the seignorage. In equilibrium and using the definition of income from (25), the resource constraint of the economy is given by

$$Y_t = C_t + I_t + \gamma V_t + \iota(u_t)\bar{K}_t. \quad (33)$$

### 3 Calibration and computation

If not mentioned otherwise, the choice of the functional forms and the parameterization follows Walsh (2005).

#### 3.1 Functional form assumptions

We assume that the firm-specific productivity shock  $a$  is log-normally distributed with mean zero and standard deviation  $\sigma_a = 0.13$ :

$$f(a) = \frac{1}{a\sigma_a\sqrt{2\pi}}e^{-0.5(\ln a/\sigma_a)^2}.$$

Thus,

$$z := \frac{\ln a}{\sigma_a}$$

has a standard normal distribution, and we get  $z$  from the inverse of the cumulative distribution function of the standard normal distribution at the steady state value of  $\rho^n$ .

Given  $\underline{a} = e^{\sigma_a z} = 0.7892$  in steady state, we compute

$$H(\underline{a}) := \int_{\underline{a}}^{\infty} a^{1/(1-\alpha)} f(a) da$$

using Simpson's method.

According to our specification of the functions  $\Phi$  and  $\iota$  the dynamics of the model only depends on the elasticities  $\sigma_\Phi$  and  $\sigma_\iota$  of the functions  $\Phi'$  and  $\iota'$  with respect to their arguments, respectively.

## 3.2 Parameterization

We analyze the sensitivity of our model with respect to the introduction of capital adjustment costs and variable capital utilization. Therefore, our main interest is the sensitivity of the model with regard to the choice of the parameters  $\sigma_\Phi$  and  $\sigma_\iota$ , respectively. In addition, we study the model's behavior depending on the parameter values for the price rigidity  $\omega$  and the habit parameter  $h$ . Periods correspond to quarters.

**Preferences.** Following Walsh (2005), we set the discount factor  $\beta = 0.989$ , the intertemporal elasticity of substitution  $1/\sigma = 0.5$ , and the habit parameter  $h = 0.78$ . The parameter values are summarized in Table 1.

**Table 1:**  
Parameter choice in the steady state of the benchmark model

Preferences	$\beta=0.989$	$\sigma=2$	$h=0.78$	
Labor Market	$\rho^x=0.068$	$\rho^n=0.0343$	$\kappa^f=0.7$	$\kappa^w=0.6$
	$\eta=0.5$	$\chi=0.4$		
Production	$\alpha=0.36$	$\delta=0.025$	$\sigma_a=0.13$	$\sigma_\Phi = 0.5$
	$\sigma_\iota = 0.01$	$\theta = 11.0$	$\rho_z = 0.95$	$\sigma_\epsilon = 0.007$
Price adjustment	$\omega = 0.85$			
Monetary policy	$\rho_R = 0.9$	$\phi_\pi = 1.1$	$\sigma_\pi = 0.002$	

**Matching and the labor market.** Walsh (2005) and den Haan et al. (2000) assume a total separation rate  $\rho^s = 1 - (1 - \rho^x)(1 - \rho^n)$  equal to 0.1 and an exogenous separation rate  $\rho^x = 0.068$ . In steady state, the endogenous separation rate therefore amounts to  $\rho^n = 0.0343$ . In the matching function,  $\chi$  is set equal to 0.4 in accordance with empirical estimates by Blanchard and Diamond (1989). Furthermore, the steady state values of the matching probabilities are chosen as  $\kappa^f = 0.7$  and  $\kappa^w=0.6$ . The workers and the firms split the surplus evenly implying  $\eta = 0.5$ .

**Production and capital adjustment.** In addition to Walsh (2005), we introduce capital into production. The capital elasticity of output is set equal to  $\alpha = 0.36$ . Capital depreciates at the rate  $\delta = 0.025$ . Following Christiano et al. (2005), we set  $\sigma_\iota = 0.01$ , but we will also consider the case of a constant utilization rate with  $u_t \equiv 1.0$ . As empirical estimates of the adjustment-cost elasticity vary considerably, we consider a wide range of values for  $\sigma_\Phi \in \{1/15, 1/2\}$ . In our benchmark case, we choose  $\sigma = 1/2$ . In our sensitivity analysis, we apply  $\sigma_\Phi = 1/15$  in accordance with Baxter and Crucini (1993).

The log of the aggregate technology shock follows an AR(1) process,  $\log Z_t = \rho_z \log Z_{t-1} + \epsilon_t$ , with autoregressive parameter  $\rho_z = 0.95$  and standard deviation  $\sigma_\epsilon = 0.007$ . In the wholesale sector, the demand elasticity is equal to  $\theta = 11$  implying an average mark-up equal to 10%.

**Price rigidity.** We set the probability  $\omega$  that a firm is not allowed to change its price optimally in a given period equal to 0.85. Walsh (2005) uses the same value that implies the average time between price adjustment of 6.5 quarters. Alternatively, we will also consider a more frequent price adjustment  $\omega = 0.5$  in our sensitivity analysis.

**Monetary policy** The parameters of the monetary policy rules applied by Walsh (2005) reflect a high degree of inertia in the interest rate,  $\rho_R = 0.9$ , and a long-run response of the interest rate to the inflation rate by 1.10 implying  $\phi_\pi = 1.10$ . Trigari (2004) chooses  $\phi_\pi = 1.5$  which we will also use in our computation in order to study the effects of the policy rule. The monetary policy shock displays a standard deviation  $\sigma_\phi = 0.002$ .

### 3.3 Computation

We use a log-linear approximation of the model around the steady state in order to compute the dynamics. The log-linearized version of the model is provided in the Appendix. For the numerical solution, we use the techniques proposed by King and Watson (2002). It relies upon the Schur factorization of the matrix that is describing the autoregressive part of the dynamic system.<sup>5</sup>

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<sup>5</sup>See Section 2.3 in Heer and Maussner (2005) for a detailed description.



## 4 Results

In this section, we present our results on the dynamics of output and inflation in the labor market search model with capital. First, we study the effects of a shock to the interest rate; subsequently, we look at the impact of a productivity shock.

### 4.1 Interest rate shock

Figure 2 plots the impulse responses of the model variables following an unexpected rise of the interest rate by one standard deviation (equal to 0.2 percentage points). In this benchmark case, we replicate the findings for the model of Walsh (2005) without capital (compare his Figure 1). In particular, we choose his calibration with  $\{\omega, \rho_R, h\} = \{0.85, 0.9, 0.78\}$ .<sup>6</sup> In addition, capacity utilization is fixed ( $u_t \equiv 1.0$ ) and adjustment costs of capital are infinite.<sup>7</sup> Therefore, the capital stock remains constant. Following an increase of the nominal interest rate by 0.2 percentage points, output falls by 0.2% and displays a hump-shaped response, while inflation inertia is pronounced and inflation attains its minimum value at 0.12 percentage points below its steady state value after six quarters.

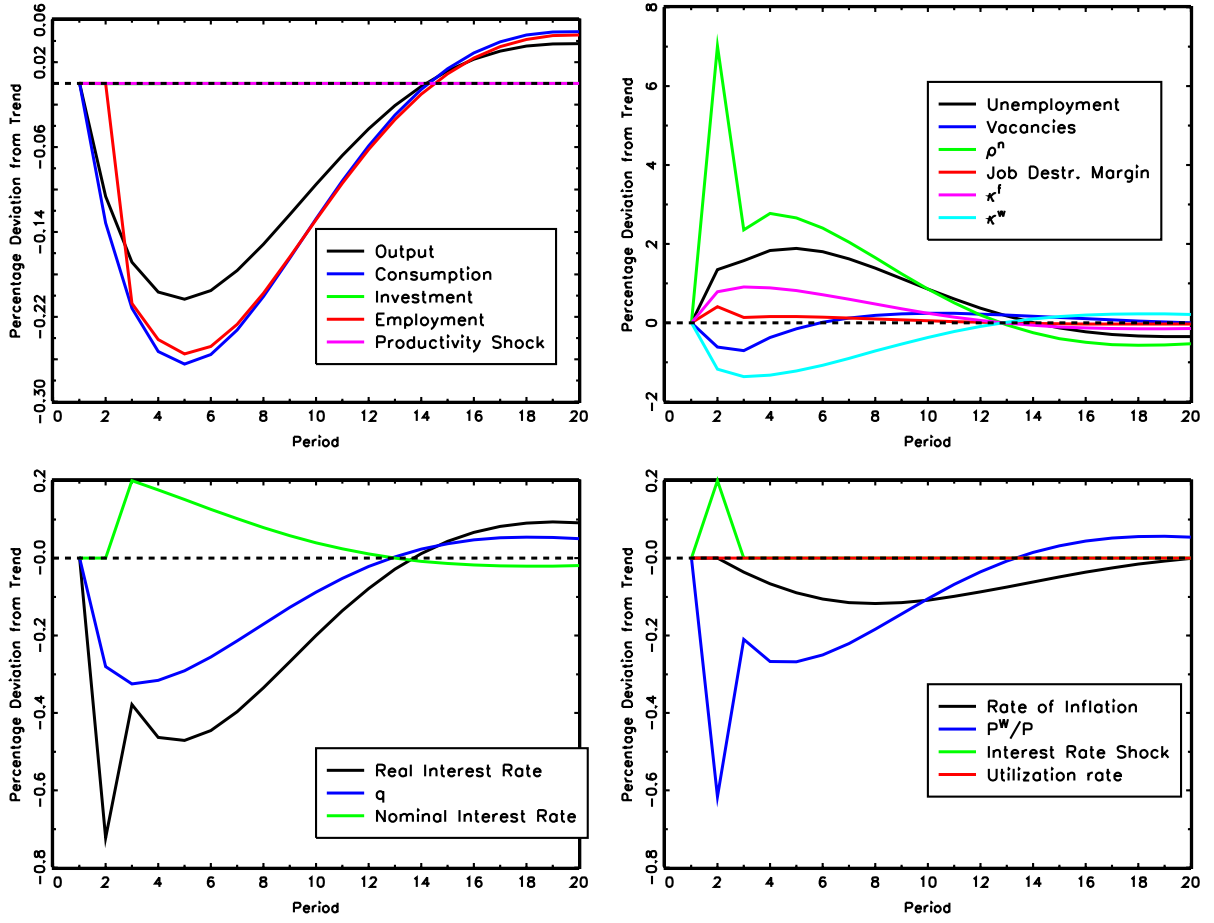
In the presence of sticky prices, a rise in the nominal interest rate  $R$  on bonds results in a rise of the real interest rate on bonds,  $R/\pi$ . As a consequence, output demand declines. Since intermediate sector prices are flexible while wholesale prices are sticky the relative price of the intermediate sector output  $g_t$  deteriorates (see the line labeled  $P^W/P$  in Figure 2). Therefore, the demand for capital services declines (see equation (10)), and subsequently the real interest rate falls. Since households are characterized by habit persistence, consumption adjusts only gradually. Furthermore, as demand declines, the surplus of an employment relationship for given individual productivity  $a$  declines and firms post less vacancies. For this reason, job matches  $m(U_t, V_t)$  and employment  $N_t$  decline. Also, the job finding probability  $\kappa^w$  of the workers decreases, while the job destruction margin  $\underline{a}$  increases (see upper right picture in Figure 2). In addition, the central bank policy displays a high degree of inertia as nominal interest rates are highly

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<sup>6</sup>Our impulse response functions are smaller by the factor 5 as we consider a shock of one standard deviation rather than one percentage point.

<sup>7</sup>In our computation, we use  $\sigma_\Phi = 10,000$  and  $\sigma_\zeta = 10000,00$  so that both  $\bar{K}_t$  and  $u_t$  do not change.

**Figure 2:** Effects of a negative interest rate shock, preset prices, constant capital



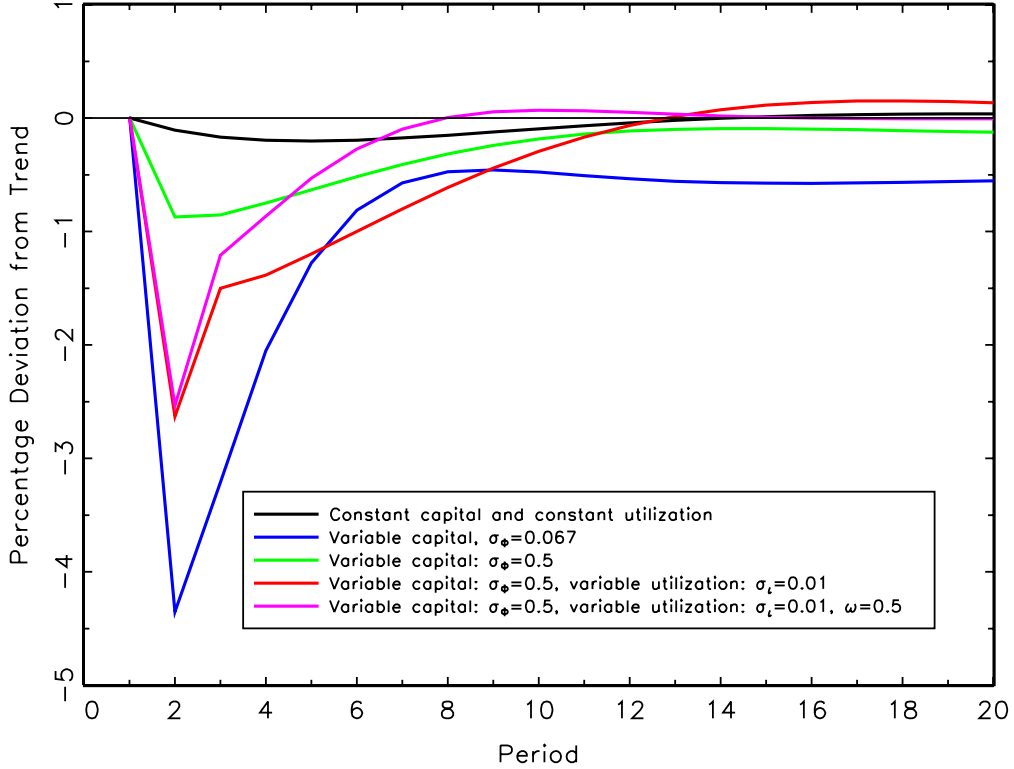
auto-correlated ( $\rho_R = 0.9$ ). As a consequence, the response of output is hump-shaped.<sup>8</sup>

In the following, we depart from this benchmark case and study the sensitivity of these results with regard to the introduction of capital. In addition, we will consider the role of sticky prices, preset prices, and the inertia of the central bank policy.

**Variable investment and capacity utilization.** In Figure 3, we graph the effects of variable capital on the dynamics of output. Notice that if we introduce very elastic capital adjustment costs,  $\sigma_\Phi = 0.067$ , investment demand falls significantly in response to a rise

<sup>8</sup>If we set  $\rho_R$  equal to zero, the maximum absolute response of output already takes place in the first period of the shock.

**Figure 3:** Effects of a negative interest rate shock on output I

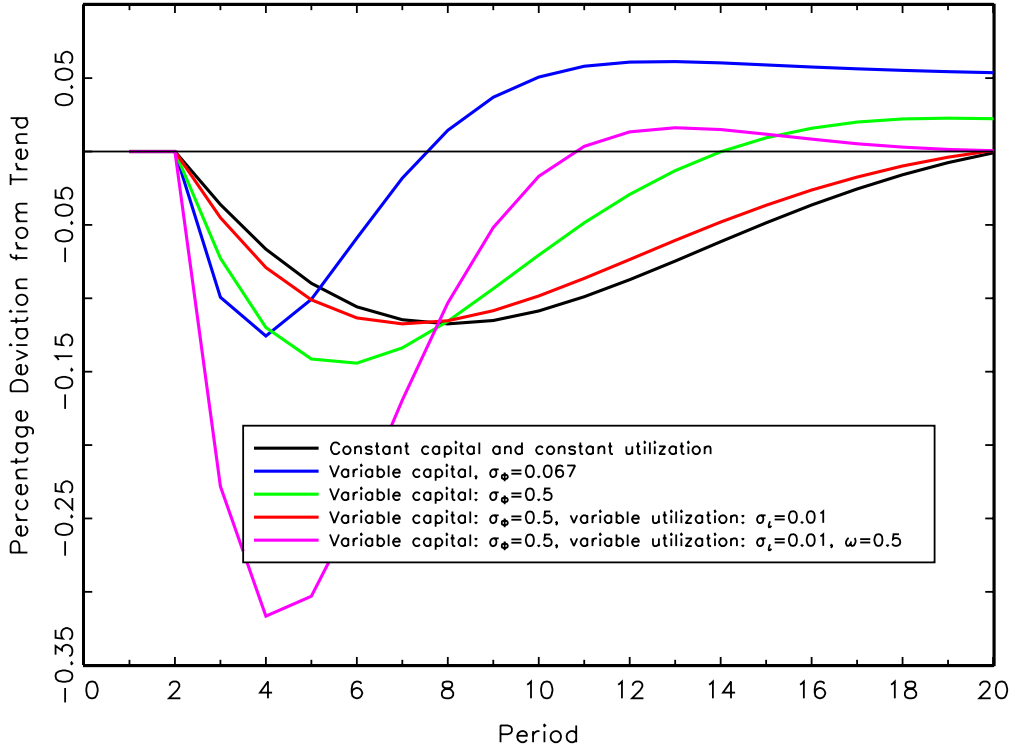


in interest rate of only 0.2 percentage points. In this case, we observe an output response that is much larger than observed empirically. Therefore, a value of  $\sigma_\Phi = 0.5$  that is in the upper range of the empirical estimates for the capital adjustment elasticity is in better accordance with recent VAR estimates on the US output response to a rise in the Federal Funds Rate. If capacity utilization is also variable, capital services are reduced rather by a fall in utilization rate  $u_t$  than by a decrease of the capital stock  $K_t$ . The effect on output, however, is negligible. Notice, however, that the response of output is not hump-shaped. The maximum impact of the interest rate shock on output occurs in the first period.

Figure 4 plots the dynamics of inflation in response to a rise of the nominal interest rate by one standard deviation. Obviously, inflation displays too little inertia in the case of variable capital, but constant capacity utilization (blue and green lines). In this case, the interest rate costs decrease too much and, therefore, marginal costs fall. As a consequence, the drop in inflation is pronounced and almost immediate.<sup>9</sup> In the case of variable capacity

<sup>9</sup>If prices were not preset, the biggest impact would be in the very first period following the shock.

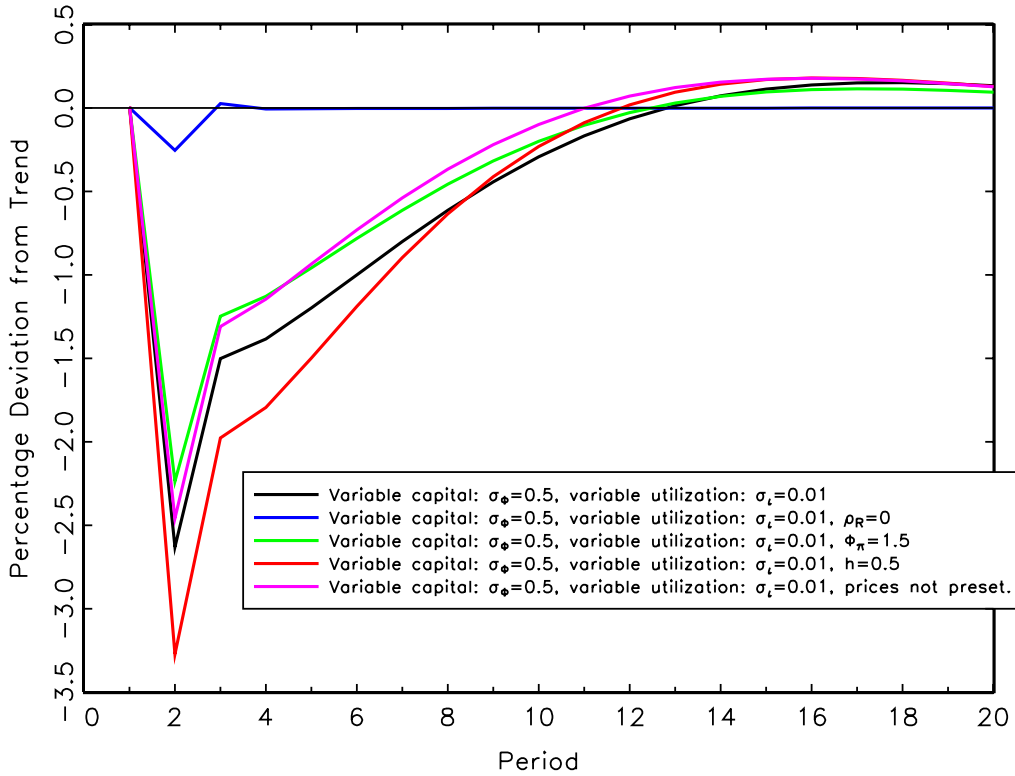
**Figure 4:** Effects of a negative interest rate shock on inflation I



utilization, however, investment demand changes little and all the adjustment takes place by using the existing capital stock less intensively. As a consequence, the change in marginal costs is much smaller and smoother. The dynamics mainly reflect the sluggish response of consumption that is driven by the households’ preferences with regard to habit formation. Therefore, only variable capacity utilization is in accordance with the observed inflation inertia.

**Price stickiness and preset prices.** In accordance with Walsh (2005), a higher degree of price stickiness generates a more persistent inflation response. In Figure 4, the pink line plots the inflation response if firms can set their prices optimally every second quarter on average corresponding to  $\omega = 0.5$ . In this case, the inflation response is more immediate, while output returns more rapidly to its steady-state value (see also the pink line in Figure 3). Notice that we assume variable capital utilization and a capital adjustment cost elasticity  $\sigma_\Phi = 0.5$  in this and the following cases when we consider the effects of an interest

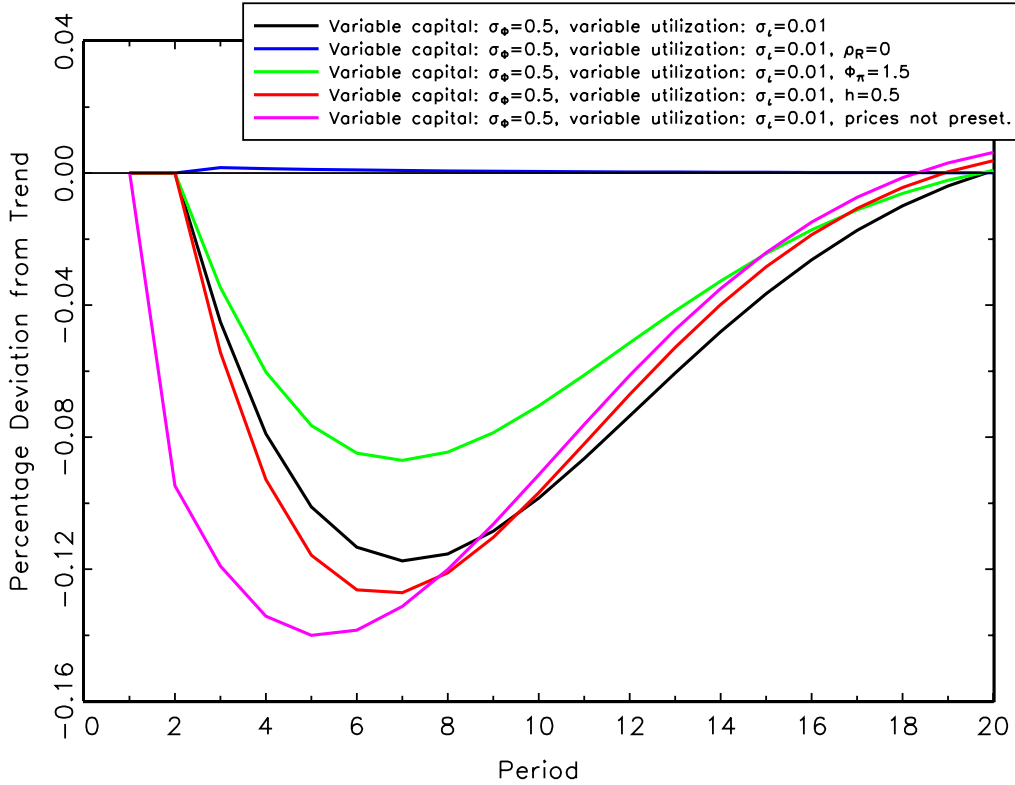
**Figure 5:** Effects of a negative interest rate shock on output II



rate shock. In particular, we study the sensitivity of our results as we change only one of the parameters from our benchmark calibration as presented in Table 1. Furthermore, prices are set prior to the observation of the interest rate shock. This assumption, however, is rather innocent. If we assume that prices can also be set after the observation of the shock so that the New-Keynesian Phillips curve is presented by (8) rather than (7), there is not any noteworthy effect on the dynamics of output, whereas the effect on inflation is more immediate and more pronounced in the first six quarters after the impact of the shock (occurring in period two). In order to notice this compare the pink line (prices are not preset) with the black line (prices are preset) in Figures 5 and 6, respectively.

**Monetary policy.** As his main result, Walsh (2005) shows that policy inertia is the most important factor in accounting for the hump-shaped response of output and the persistent response of inflation. As we already showed above, in the presence of capital, output does not display a hump-shaped response any more. However, we are able to confirm his second

**Figure 6:** Effects of a negative interest rate shock on inflation II



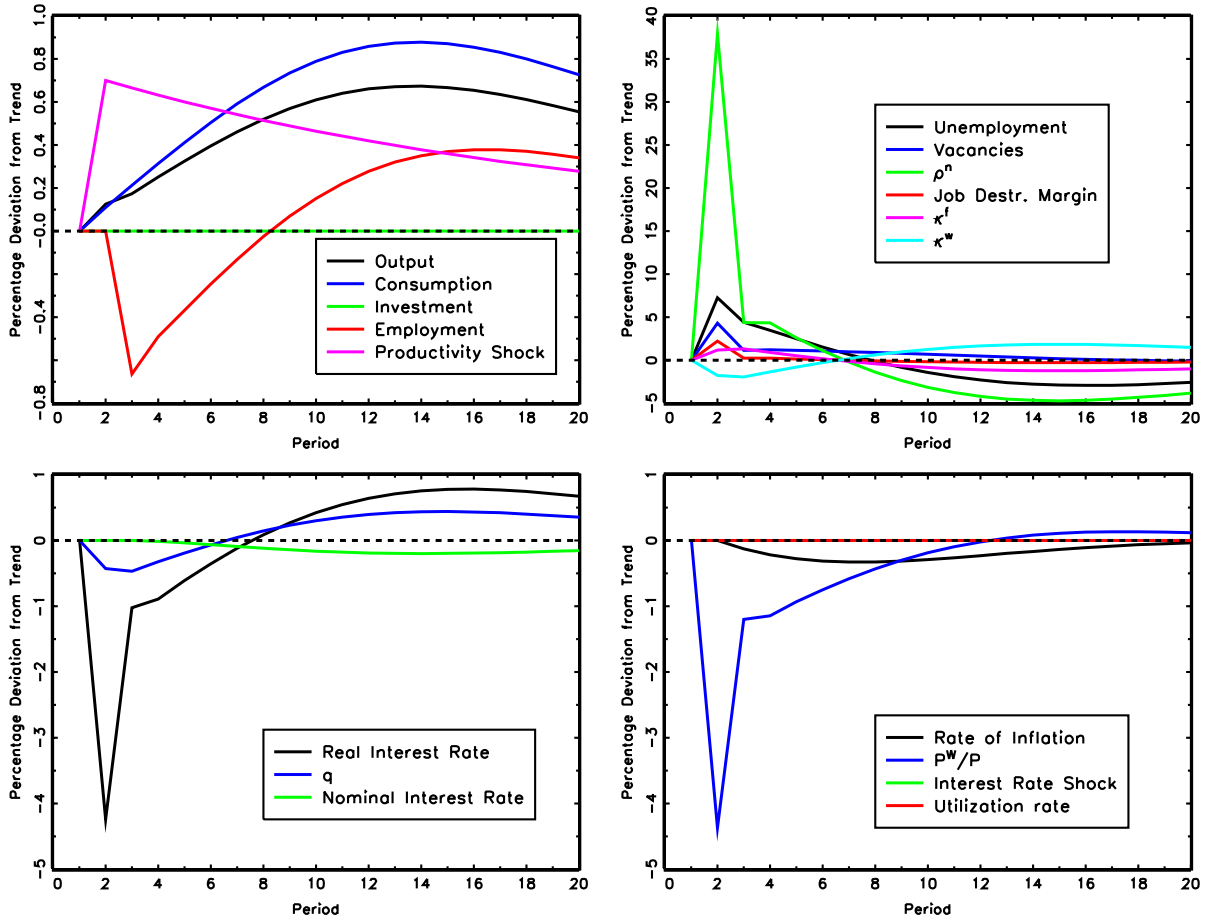
result for the economy with capital as soon as we assume capacity utilization to be variable. In Figures 6, we show that the persistent response of inflation depends crucially on the inertia of the policy rule. If the autoregressive parameter of the Taylor rule with respect to the interest rate is reduced from  $\rho_R = 0.9$  to  $\rho_R = 0$ , the impulse response of inflation is flat (compare the blue line with the black line in Figure 6). The response of inflation is less sensitive to the other parameters of the model like, for example, the inflation parameter  $\phi_\pi$  of the policy rule. In Figure 6, the green line represents the case when we increase  $\phi_\pi$  from 1.2 to 1.5. If the interest rate  $R$  is more sensitive with regard to the inflation rate, the response of inflation is smaller, but still persistent.<sup>10</sup>

<sup>10</sup>The persistence of the inflation response does also not depend on the degree of habit persistence  $h$ . In Figure 6, we present the case  $h = 0.5$ . Even without habit persistence,  $h = 0$ , (not illustrated), inflation is still persistent, while the output response is increased.

## 4.2 Technology shock

In the previous Section, we found that the model with variable capacity utilization helps to explain the persistent response of inflation, even though it cannot account for the hump-shaped response of output. In this section, we analyze if this model is also able to explain the output-inflation dynamics in response to a productivity shock. Figure 7 shows the impulse response of key variables to a one-time productivity shock in period  $t = 2$  of size  $\sigma_Z = 0.007$  with fixed capital services (i.e.  $\sigma_\Phi = 10,000$  and  $\sigma_\iota = 10,000$ ). All other parameters are calibrated as in Table 1, and prices are preset.

Figure 7: Effects of a technology shock



The response of output and employment is consistent with the evidence provided by Galí (1999) and Francis and Ramey (2002) who show that a supply shock raises output but

depresses employment in the first few quarters.<sup>11</sup> To understand the mechanism behind this result in our model consider again the relative price of intermediate goods  $g_t$  (the line  $P^W/P$  in the lower right panel of Figure 7). On impact, the increased productivity entails a lower nominal price of intermediate goods. Since wholesale prices are fixed in the impact period, the relative price of intermediate products falls and counteracts the outward shift of the production function. Thus, the job destruction margin increases and more employment relationships separate endogenously. As soon as prices adjust (see the spikes in the separation rate  $\rho_t^n$ , the relative price of intermediate products,  $P^W/P$ , and the real interest rate in Figure 7) the positive effect of the technology shock begins to predominate. Note also that there is a protracted hump-shaped decline of the inflation rate, which is in accordance with the persistent negative impact on inflation found empirically by Galí (1999).

As in the case of an interest rate shock the dynamics of output and employment is sensitive with regard to the assumption of fixed capital services. Figures 8 and 9 display the impulse responses of output and employment for different values of key parameters.

If the utilization of capital services is endogenously determined ( $\sigma_l = 0.01$ ) the effect of predetermined prices on employment is so large that it outweighs the outward shift of the production function and output declines in the first quarter. This effect is somewhat smaller if the degree of nominal rigidity as measured by the parameter  $\omega$  is considerably decreased (compare the red and the blue line in Figure 8) and even disappears if prices are moderately rigid and are allowed to change in the same quarter where the shock hits the economy (see the pink line in Figure 8).

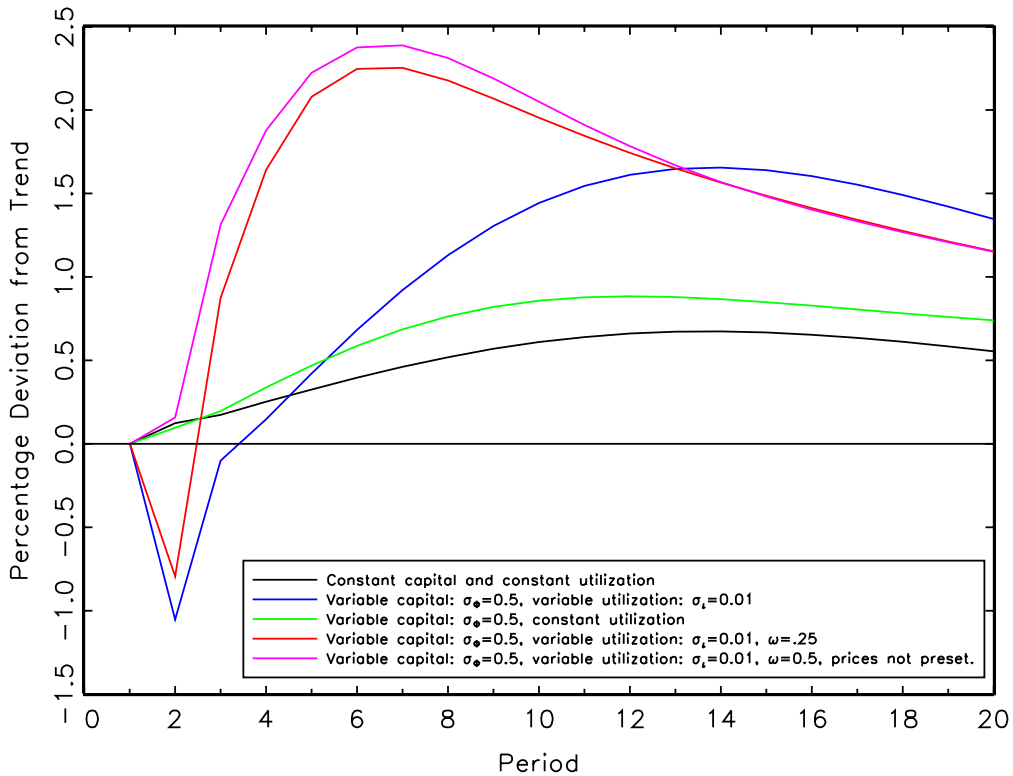
Figure 9 corroborates the finding that the negative effects on output originate in the flexible use of capital services. With fixed capital, employment alone bears the burden of adjustment. The more flexible capital services are, the smaller is the fall of employment (compare the black, blue, and green lines in Figure 9). However, it requires a substantial amount of price flexibility for employment to increase immediately after a technology shock (see the pink line in Figure 9). Using the parameter values from Table 1 together with  $\omega = 0.5$  – so that firms can adjust their prices on average every second quarter – we also

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<sup>11</sup>The empirical evidence on the effects of technology shocks on employment depend crucially on the question whether or not hours per worker are stationary. In the latter case Christiano et al. (2003, 2004) demonstrate that hours increase after a technology shock.



**Figure 8:** Effects of a technology shock on output



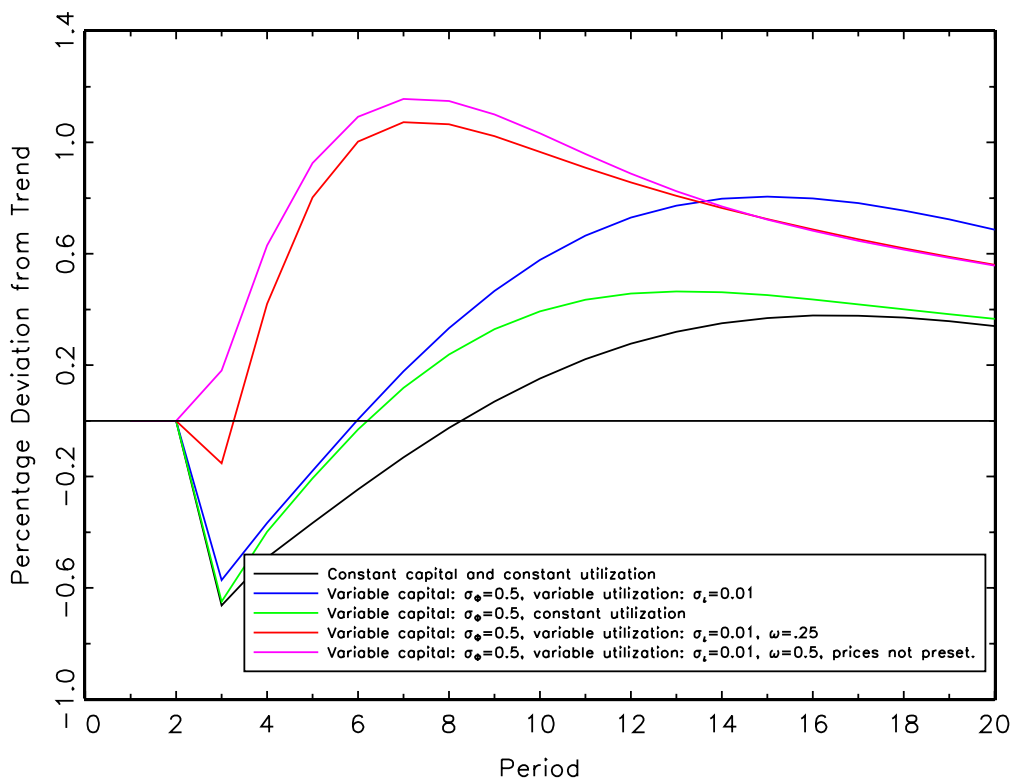
need to assume that the firms which are receiving the signal to change their price can do so immediately after the realization of the technology shock.

In summary, a sharp decrease of output in response to a technology shock is at odds with the empirical findings provided by Galí (1999) and Francis and Ramey (2002). In the present model this puzzle can only be resolved if either the marginal cost function  $l'(u_t)$  is very elastic or the degree of price rigidity is only modest.

## 5 Conclusion

In this paper we have studied the inflation and output dynamics in the labor market search model with capital. In the presence of capital adjustment costs, variable capacity utilization helps to reconcile the model's inflation response to a rise in the nominal interest rate with the one that is observed empirically. However, in this case, the magnitude of

**Figure 9:** Effects of a technology shock on employment



the output response is much stronger than observed in the US economy and, in particular, the output response is no longer hump-shaped. Therefore, we conclude that the output-inflation dynamics of the labor market search model in response to an interest-rate shock is sensitive with regard to the introduction of capital. This conclusion also applies to the case when we consider the consequences of a technology shock. Contrary to empirical findings, an unexpected productivity increase causes a decline in output if capital services are sufficiently flexible and prices are rigid.

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## 6 Appendix: The log-linear model

The equilibrium conditions of the model are presented by the 11 contemporaneous equations (14), (12), (31), (32), (24), (18), (19), (15), (33), (28e), and (28b) which we restate for the readers' convenience:

$$F(\underline{a}_t) = \rho_t^n = \int_0^{\underline{a}_t} f(a) da, \quad (\text{A.1a})$$

$$l + b = q_t + (1 - \alpha) (g_t \underline{a}_t Z_t)^{1/(1-\alpha)} \left( \frac{\alpha}{r_t} \right)^{\alpha/(1-\alpha)}, \quad (\text{A.1b})$$

$$u_t \bar{K}_t = (1 - \rho^x) N_t H(\underline{a}_t) \left( \frac{\alpha g_t Z_t}{r_t} \right)^{1/(1-\alpha)}, \quad (\text{A.1c})$$

$$Y_t = Z_t \left[ (1 - \rho^x) N_t H(\underline{a}_t) \right]^{1-\alpha} (u_t \bar{K}_t)^\alpha, \quad (\text{A.1d})$$

$$q_t = \frac{c(1 - \eta \kappa_t^w)}{(1 - \eta) \kappa_t^f}, \quad (\text{A.1e})$$

$$\kappa_t^f = \frac{M_t}{V_t} = \psi (V_t/U_t)^{-\chi}, \quad (\text{A.1f})$$

$$\kappa_t^w = \frac{M_t}{U_t} = \psi (V_t/U_t)^{1-\chi}, \quad (\text{A.1g})$$

$$U_t = 1 - (1 - \rho^x)(1 - \rho_t^n) N_t, \quad (\text{A.1h})$$

$$I_t = Y_t - cV_t - C_t - \iota(u_t) \bar{K}_t, \quad (\text{A.1i})$$

$$\xi_t = \frac{1}{\Phi'(I_t/K_t)}, \quad (\text{A.1j})$$

$$\iota'(u_t) = r_t. \quad (\text{A.1k})$$

and the 7 dynamic equations (28a), (28c), (28f), (27), (16), (23), and (29)

$$\lambda_t = (C_t - hC_{t-1})^{-\sigma} - \beta h E_t (C_{t+1} - hC_t)^{-\sigma} \quad (\text{A.2a})$$

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{R_{t+1}}{\pi_{t+1}}, \quad (\text{A.2b})$$

$$\xi_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[ r_{t+1} u_{t+1} - \iota(u_{t+1}) - \frac{I_{t+1}}{\bar{K}_{t+1}} + \xi_{t+1} \left( 1 - \delta + \Phi(I_{t+1}/\bar{K}_{t+1}) \right) \right], \quad (\text{A.2c})$$

$$\bar{K}_{t+1} = \Phi \left( \frac{I_t}{\bar{K}_t} \right) \bar{K}_t + (1 - \delta) \bar{K}_t, \quad (\text{A.2d})$$

$$N_{t+1} = (1 - \rho^x)(1 - \rho_t^n) N_t + \psi U_t^\chi V_t^{1-\chi}, \quad (\text{A.2e})$$

$$q_t \lambda_t = \beta E_t \lambda_{t+1} (1 - \rho^x)(1 - \eta \kappa_t^w)(1 - \alpha) \left( \frac{\alpha}{r_{t+1}} \right)^{\alpha/(1-\alpha)} (g_{t+1} Z_{t+1})^{1/(1-\alpha)}, \quad (\text{A.2f})$$

$$\times \left( H(\underline{a}_{t+1}) - (1 - \rho_{t+1}^n) \underline{a}_{t+1}^{1/(1-\alpha)} \right),$$

$$R_{t+1} = \bar{\pi}^{(1-\rho_R)(1-\phi_\pi)} \beta^{-(1-\rho_R)} R_t^{\rho_R} \pi_t^{\phi_\pi(1-\rho_R)} e^{\phi_t}, \quad \phi_t \sim N(0, \sigma_\phi). \quad (\text{A.2g})$$

The log-linearized system of equations (A.1) is:

$$\hat{\rho}_t^n - \varepsilon_{F,a} \hat{a}_t = 0, \quad (\text{A.3a})$$

$$\Gamma_1 \hat{r}_t - \Gamma_2 \hat{a}_t = -\hat{q}_t + \Gamma_2 \hat{g}_t + \Gamma_2 \hat{Z}_t, \quad (\text{A.3b})$$

$$\varepsilon_{H,a} \hat{a}_t - \frac{1}{1-\alpha} \hat{r}_t - \hat{u}_t = \hat{K}_t - \hat{N}_t - \frac{1}{1-\alpha} \hat{g}_t - \frac{1}{1-\alpha} \hat{Z}_t, \quad (\text{A.3c})$$

$$(\alpha - 1) \varepsilon_{H,a} \hat{a}_t + \hat{Y}_t - \alpha \hat{u}_t = \alpha \hat{K}_t + (1 - \alpha) \hat{N}_t + \hat{Z}_t, \quad (\text{A.3d})$$

$$\hat{\kappa}_t^f + \frac{\eta \kappa^w}{1 - \eta \kappa^w} \hat{\kappa}_t^w = -\hat{q}_t, \quad (\text{A.3e})$$

$$\hat{\kappa}_t^f + \chi \hat{V}_t - \chi \hat{U}_t = 0, \quad (\text{A.3f})$$

$$\hat{\kappa}_t^w + (\chi - 1) \hat{V}_t + (1 - \chi) \hat{U}_t = 0, \quad (\text{A.3g})$$

$$(U/N) \hat{U}_t - (1 - \rho^x) \rho^n \hat{\rho}_t^n = -(1 - \rho^x) (1 - \rho^n) \hat{N}_t, \quad (\text{A.3h})$$

$$\hat{I}_t - (Y/I) \hat{Y}_t + (cV/I) \hat{V}_t + (r/\delta) \hat{u}_t = -(C/I) \hat{C}_t, \quad (\text{A.3i})$$

$$\sigma_\Phi \hat{I}_t - \hat{\xi}_t = \sigma_\Phi \hat{K}_t, \quad (\text{A.3j})$$

$$\sigma_\iota \hat{u}_t - \hat{r}_t = 0, \quad (\text{A.3k})$$

$$\begin{aligned} \Gamma_1 &= \frac{\alpha}{1-\alpha} \frac{q-l-b}{q}, \quad \Gamma_2 = \frac{\Gamma_1}{\alpha}, \\ \varepsilon_{F,a} &= \frac{F'(\underline{a})\underline{a}}{F(\underline{a})}, \quad \varepsilon_{H,a} = \frac{H'(\underline{a})\underline{a}}{H(\underline{a})}, \\ \sigma_\Phi &:= -\frac{\phi''(\delta)\delta}{\phi'(\delta)}, \quad \sigma_\iota := \frac{\iota''(\bar{u})\bar{u}}{\iota'(\bar{u})}, \end{aligned}$$

In (A.3i), we used the steady-state conditions  $\iota'(\bar{u}) = r$  and  $\bar{K}/I = 1/\delta$ .

Log-linearizing equations (A.2) yields

$$\beta h \Gamma_3 E_t \hat{C}_{t+1} - (1 + \beta h^2) \Gamma_3 \hat{C}_t \quad (\text{A.4a})$$

$$+ h \Gamma_3 \hat{C}_{t-1} - \hat{\lambda}_t = 0,$$

$$E_t \hat{\lambda}_{t+1} - \hat{\lambda}_t = -\beta r E_t \hat{r}_{t+1} - \beta E_t \hat{\xi}_{t+1} + \hat{\xi}_t \quad (\text{A.4b})$$

$$\hat{K}_{t+1} + (\delta - 1) \hat{K}_t = \delta \hat{I}_t, \quad (\text{A.4c})$$

$$\begin{aligned} \hat{N}_{t+1} - (1 - \rho^x) (1 - \rho^n) \hat{N}_t &= \kappa^w (U/N) \hat{U}_t + \kappa^w (U/N) \hat{\kappa}_t^w \\ &\quad - (1 - \rho^x) \rho^n \hat{\rho}_t^n, \end{aligned} \quad (\text{A.4d})$$

$$\begin{aligned} E_t \hat{\lambda}_{t+1} + \frac{1}{1-\alpha} E_t \hat{g}_{t+1} - \hat{\lambda}_t - \hat{q}_t &= \frac{\alpha}{1-\alpha} E_t \hat{r}_{t+1} - \Gamma_4 E_t \hat{a}_{t+1} - \Gamma_5 E_t \hat{\rho}_{t+1}^n \\ &\quad + \frac{\eta \kappa^w}{1 - \eta \kappa^w} \hat{\kappa}_t^w - \frac{1}{1-\alpha} E_t \hat{Z}_{t+1}, \end{aligned} \quad (\text{A.4e})$$

$$E_t \hat{\lambda}_{t+1} + \hat{R}_{t+1} - E_t \hat{\pi}_{t+1} - \hat{\lambda}_t = 0, \quad (\text{A.4f})$$

$$\hat{R}_{t+1} - \rho_R \hat{R}_t - \phi_\pi (1 - \rho_R) \hat{\pi}_t = \phi_t, \quad (\text{A.4g})$$

$$\hat{\pi}_t - \frac{1}{1 + \beta} \hat{\pi}_{t-1} - E_t \hat{\pi}_{t+1} - \frac{1}{1 + \beta} \hat{\pi}_t \quad (\text{A.4h})$$

$$- \frac{\beta}{1 + \beta} E_t \hat{\pi}_{t+2} - \Gamma E_t \hat{g}_{t+1} = 0,$$

$$\Gamma_3 := \frac{\sigma}{(1 - \beta h)(1 - h)},$$

$$\Gamma_4 := \Delta \left[ H(\underline{a}) \varepsilon_{H,a} - \frac{1 - \rho^n}{1 - \alpha} \underline{a}^{1/(1-\alpha)} \right],$$

$$\Gamma_5 := \Delta \rho^n \underline{a}^{1/(1-\alpha)},$$

$$\Delta := \left[ H(\underline{a}) - (1 - \rho^n) \underline{a}^{1/(1-\alpha)} \right]^{-1},$$

$$\Gamma = \frac{(1 - \omega)(1 - \beta\omega)}{(1 + \beta)\omega},$$

where we have also used the New Keynesian Phillips curve (7) in (A.4h) with the time index shifted one period forward.

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