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The Swing Voter's Curse in Social Networks

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The Swing Voter’s Curse in Social Networks*

Berno Buechel[†] & Lydia Mechtenberg[‡]

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Abstract

We study private communication between jury members who have to decide between two policies in a majority vote. While interests of all agents are perfectly aligned, only some agents (“experts”) receive a private noisy signal about which policy is correct. Each expert can, but need not, recommend a policy to her audience of “non-experts” prior to the vote. We show theoretically and empirically that communication can undermine (informational) efficiency of the vote and hence reduce welfare. Both efficiency and stability of communication hinge on the structure of the communication network. If some experts have distinctly larger audiences than others, non-experts should not follow their voting recommendation. We test the model in a lab experiment and find supporting evidence for this effect and, more generally, for the importance of the network structure.

JEL-Code: *D72, D83, D85, C91.*

Keywords: *Strategic Voting, Social Networks, Majority Rule, Swing Voter’s Curse, Information Aggregation, Information Transmission*

1 Introduction

Consider a number of voters with common interests who, without knowing the true state of the world, have to decide in a majority vote which of two alternative policies

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is the “right” one. For instance, a jury has to decide whether to convict or acquit a defendant; or a parliament aiming at protecting their country from a potential aggressor has to decide whether imposing economic sanctions on the latter is helping them to do so. There are informed and uninformed voters. Informed voters (*experts*) have imperfect private information indicating the right policy; uninformed voters (*non-experts*) only know that both policies are equally likely to be correct. Their lack of information may be due to time constraints or the fact that a specific education is needed to understand the issue at hand. Hence, the decision must be made by experts and non-experts together. Assume now that prior to the vote, some of the experts get the opportunity to privately speak to some of the non-experts. Do the experts have an incentive to reveal to the non-experts which policy their information indicates as the “right” one? And do the non-experts have an incentive to follow their recommendation in the vote? How does such communication affect the information aggregation achieved in the majority vote? We show that, perhaps contrary to intuition, such communication between experts and non-experts reduces the informational efficiency of the voting outcome for certain structures of the communication network, and that this holds true both theoretically and in the lab. Hence, we analyze private communication in a standard context of jury decision making and show that it often undermines the quality of the collective decision.

Jury decision making has been prominently analyzed by the Marquis de Condorcet (De Caritat, 1785), Austen-Smith and Banks (1996), and Feddersen and Pesendorfer (1996, 1997, 1998). In the absence of communication, Feddersen and Pesendorfer (1996) find that it is optimal for rational voters with common interests and imperfect knowledge about the state of the world to abstain if they are uninformed and to vote in line with their independent private signal if they are informed.¹ To understand this result, notice that a vote matters only if it is pivotal. Hence, a strategic voter conditions his considerations about whether – and what – to vote on his own pivotality. Taking into account the strategies of the other voters, the uninformed voter concludes that his vote is more likely to be pivotal if it goes into the wrong direction, i.e. against the majority of the informed votes. Hence, conditional on pivotality, an uninformed vote is more likely to be wrong than correct. It is therefore optimal for the uninformed voter to delegate the collective decision to the informed voters by abstaining from voting himself. By contrast, the informed voter is always more likely to vote in line with the majority of the other informed voters, given that all informed voters cast their ballot in accordance with their private information. Hence, the informed voter concludes that her vote is more likely to be correct than wrong, and that it is optimal for her to “vote her signal.” Since Feddersen’s and Pesendorfer’s ingenious contribution, the finding that uninformed jury members are better off abstaining from the vote has been dubbed the swing voter’s curse. More generally, a voter is “cursed” if his optimal strategy conditional on his pivotality differs from what he would deem optimal if he did not condition his strategy on being pivotal, i.e. what he would choose as a dictator.

Addressing the question initially raised by the Marquis de Condorcet (De Cari-

¹If one deviates from the assumption of common interests by introducing a number of “partisans” who always vote into a pre-specified direction, then abstention does no longer need to be the optimal strategy of the uninformed voters.

tat, 1785), namely how to achieve informational efficiency in a vote, the literature on jury decision making has found that majority votes aggregate private independent information efficiently under standard conditions (i.e. voters have a prior belief of one half and equal precision of signals) if voters have common interests and either behave strategically, i.e., condition on their pivotality, or vote sincerely, i.e. cast their vote for the alternative that yields the highest expected utility given their private information (Austen-Smith and Banks, 1996, Theorem 1). Hence, in a common interest setting with majority voting, the equilibrium in which uninformed voters abstain and informed voters “vote their signal” also exhibits (informational) efficiency. In their experimental study of the model of Feddersen and Pesendorfer (1996), Battaglini, Morton, and Palfrey (2010) find that this equilibrium provides a good prediction for real behavior. Morton and Tyran (2011) have extended the model of Feddersen and Pesendorfer (1996) to include heterogeneity in information quality among the informed voters and found that less well informed voters generally tend to abstain and delegate the collective decision to the better informed voters. Hence, the tendency to “delegate to the expert” seems quite strong in the lab. This suggests that the “let the experts decide” equilibrium might be a good prediction even in more general models of information aggregation by majority votes.

However, this conjecture hinges on the assumption that all participating voters enter the majority vote with *independent private* pieces of information – which is fulfilled in the standard model of jury decision making.² But the picture becomes more complicated when a mechanism is introduced that leads to *correlated* information among voters. To our knowledge, the existing literature on jury decision making has considered two such mechanisms: Public communication (deliberation), and public signals.³ Gerardi and Yariv (2007) show that introducing public communication prior to the vote does not change the degree of information aggregation in the (sequential) equilibria of the model. Intuitively, the information aggregation that the vote would achieve in the standard model is only shifted up the game tree and is now obtained in the communication stage already. Goeree and Yariv (2011) validate this insight experimentally. By contrast, introducing a public signal on the state of the world prior to the vote changes the picture dramatically. Kawamura and Vlaseros (2013) find that the presence of a public signal generates a new class of equilibria in which voters discard their private information in favor of the public signal and information aggregation is inefficient, *even if* voters condition their strategy on their pivotality.

To our knowledge, we are the first to introduce a third way of correlating voters’

²Levy and Razin (2015) provide a model on informed voting which includes heterogeneous preferences among voters, different sources of information for each voter and voters who neglect the correlation between their information sources. They show that correlation neglect may improve the informational efficiency of the vote since it makes voters put more weight on information than on the conflict of interest. As the the standard model of informed voting with common interests, their model assumes that information remains uncorrelated across different voters.

³In a recent theory paper, Battaglini (2015) allows for communication between citizens in separate audiences so that information becomes correlated among the citizens in one audience. However, in his model, citizens cannot vote on policies directly but coordinate on public protest instead, potentially signing a petition against the policy maker’s default policy. Battaglini shows that communication in social media can improve information aggregation and transmission via public protests.

information into the standard model of jury decision making: private communication between informed and uninformed jury members.⁴ We show that the way in which private communication affects information aggregation is closer to the effects of a public signal than to the effects of public communication: Although efficient equilibria always exist, there are also equilibria in which information is inefficiently aggregated. Intuitively, if experts give voting recommendations to non-experts, and if the other non-experts follow their recommendation, then it might be optimal for a given non-expert also to follow the recommendation of “his” expert to offset the “enlarged” voting weights of the other experts. This behavior efficiently aggregates information in some networks, namely if the experts’ audiences are highly similar in size. However, if the network structure is more unbalanced in the sense of some experts having larger audiences than others, then it is possible that experts adhering to the – wrong – minority opinion nevertheless win the election due to their many “followers” who vote in line with their recommendation. In such cases, information is inefficiently aggregated. The general problem in this setting is that private communication between experts and non-experts can correlate the information among voters without already aggregating it prior to the vote. In this respect, the effects of private communication differ from those of public communication but less from the effects of public signals.

The degree of the informational inefficiency caused by private communication depends on the structure of the social network linking experts to non-experts. We study communication networks in which experts give private voting recommendations to non-experts and each non-expert listens to only one expert. Nature draws the binary state of the world and the imperfect but informative signals on it that the experts receive. Both states of the world are equally likely. Each expert receives only one signal, and signals are independent across experts. Some experts have audiences of one or more uninformed voters and can send one out of two possible messages to their audience or keep silent. Each audience receives only one message at most. Then, a vote takes place to decide which of two possible policies shall be implemented. Only the policy matching the true state of the world generates a strictly positive payoff for all individuals (the other policy generates a zero payoff for everyone). Experts and non-experts individually and simultaneously decide between voting for one or the other policy and abstaining. Voting is costless.⁵ The policy that gets a simple majority of votes is implemented. In case the voting outcome is a tie, the policy to be implemented is randomly drawn, where both policies have equal probability.

⁴Bloch, Demange, Kranton, et al. (2014) also allow for private communication between informed and uninformed voters and investigate how the network structure affects information aggregation and voting outcomes. However, in their model there is only one informed but potentially biased voter who sends a truthful or intentionally wrong message, the other voters only decide whether or not to pass this message on to their neighbors. The authors show that high numbers and / or intense clustering of biased agents make it less likely that a biased agent can convince the electorate to vote in his interest.

⁵With costs of voting, the pivot probability which might change across equilibria in different networks would affect the willingness to abstain. Since we want to isolate the effects of communication on voting behavior, we abstract from voting costs. This is also a convention in the literature on jury voting. In the lab, costless voting makes the “willingness to delegate to the expert” harder to find and hence more surprising.

We show that in the networks enabling (some) experts to speak to a non-expert audience, information aggregation through voting becomes inefficient if (i) information transmission occurs prior to the vote and non-experts vote in line with their policy recommendation, and (ii) the degree distribution in the communication network is too unbalanced in the sense that some experts talk to many non-experts while the other experts have only relatively small audiences. Hence, in highly unbalanced networks, e.g., in a network with a star component, the non-experts would want to abstain or vote the opposite rather than follow a voting recommendation of an overly powerful opinion leader. This is due to a new type of the swing voter’s curse: Following the recommendation to vote for a certain alternative is reasonable (because the recommendation is more likely to be true than the opposite), but sub-optimal when the own vote makes the difference. Consider an opinion leader who is so powerful that a large part of the voting population follows her recommendation. Being pivotal with a vote that follows her recommendation implies that many voters from the rest of the population voted for the opposite, which implies, in turn, that they had information contradicting the opinion leader’s recommendation. Hence, conditioning on pivotality, it is more likely that the voting recommendation of the opinion leader is wrong rather than correct. More generally, in highly unbalanced networks following the voting recommendation is neither informationally efficient nor equilibrium behavior. We theoretically find that voting equilibria are characterized by informational efficiency if the communication network is sufficiently balanced or sufficiently unbalanced. For “mildly unbalanced” communication networks there are informationally inefficient voting equilibria with information transmission.

Testing our theoretical predictions in a lab experiment, we find that uninformed voters are indeed more inclined to abstain when they listen to an overly powerful opinion leader, but that abstention still occurs too rarely to prevent a loss in informational efficiency induced by highly unbalanced communication. In the experiment, the loss in informational efficiency is the larger, the more unbalanced the communication network becomes. Intuitively, the more unbalanced the network structure, the less balanced is power such that the final outcome is determined by the message of a few agents, in contrast to Condorcet’s original idea of aggregating information in a large population.

2 The Model

Nature draws one state of the world, ω , which has two possible realizations, A and B , that occur with equal probability and are not directly observable. There is a finite set of agents partitioned into a group of experts M and a group of non-experts N . Experts $j \in M$ receive a private independent signal $s_j \in \{A^*, B^*\}$ about the true state of the world. The signal is imperfectly informative with quality $p = \Pr \{s_j = A^* \mid \omega = A\} = \Pr \{s_j = B^* \mid \omega = B\} \in (\frac{1}{2}, 1)$. Non-experts $i \in N$ do not receive a signal, but can potentially receive a message from an expert. A bipartite graph g , consisting of links $(i, j) \subseteq N \times M$, represents the communication structure between non-experts and experts. The degree d_i is the number of links of agent i . An expert j with $d_j > 1$ is called sender and all non-experts linked to j

are called the “audience of j .” Different audiences do not overlap, i.e. the degree of each non-expert is at most one, and the network structure is common knowledge.⁶

After receiving the signal, each sender may send message “A” or message “B” or an empty message \emptyset to her audience. Then, all agents simultaneously participate in a majority vote the outcome of which determines which of two alternative policies, P_A or P_B , shall be implemented. Voters simultaneously vote for one of the two policies or abstain. If one policy obtains a simple majority of votes, it is implemented; otherwise, the policy to be chosen is randomly drawn with equal probability from the two alternatives.

All agents have the same preferences: They want the policy to match the state of the world. More precisely, their utility is $u(P_A|A) = u(P_B|B) = 1$ and $u(P_B|A) = u(P_A|B) = 0$.

The sequence of actions is as follows. First, nature draws the state of the world and the signals of the experts. Second, each sender decides which message to communicate to her audience, if any. Third, all agents vote or abstain and the outcome is determined by the simple majority rule. Hence, strategies are defined as follows: A communication and voting strategy σ_j of a sender $j \in M$ defines which message to send and whether and how to vote for each signal received, i.e. $\sigma_j : \{A^*, B^*\} \rightarrow \{A, B, \emptyset\} \times \{A, B, \emptyset\}$ if $d_j \geq 1$ and $\sigma_j : \{A^*, B^*\} \rightarrow \{A, B, \emptyset\}$ if $d_j = 0$. A voting strategy of a non-expert $i \in N$ with a link is a mapping from the set of messages into the voting action $\sigma_i : \{A, B, \emptyset\} \rightarrow \{A, B, \emptyset\}$, and a voting strategy of an agent $i \in N$ without a link is simply a voting action $\sigma_i \in \{A, B, \emptyset\}$. A strategy profile σ consists of all experts’ and all non-experts’ strategies.

We analyze this model using the concept of perfect Bayesian equilibrium, i.e., agents use sequentially rational strategies, given their beliefs, and beliefs are updated according to Bayes’ rule whenever possible. Equilibrium analysis is restricted to symmetric pure-strategy equilibria.

Note that if all non-experts in a given audience do not condition their voting action on the message received, then the outcome of the game is as if communication was not possible at all (“babbling equilibrium”). Similarly, if all non-experts in a given audience vote B if the message is A and vote A if the message is B , then the outcome of the game is as if their sender has chosen another communication strategy, where messages A and B are permuted (“mirror equilibria”). We will not differentiate between mirror equilibria, i.e. on the basis of the syntax of information transmission. Instead, we will identify equilibria via the semantics of information transmission, i.e. on the basis of the meanings that messages acquire in equilibrium.⁷

A desirable property of an equilibrium is *informational efficiency* which is defined as follows:

Definition 1. *A strategy profile σ is **efficient** if it maximizes the probability of the implemented policy matching the true state of the world. Equivalently, a strategy*

⁶Under these assumptions on the network structure – being bipartite and admitting each non-expert to have at most one link – no agent can access more than one piece of information. This assures that information aggregation can only take place in the voting stage but not in the communication stage. We currently work on an extension of the model that yields insights into the general case where this assumption is relaxed.

⁷This is standard in the cheap talk literature.

profile σ is efficient if it maximizes the sum of expected utilities of all experts and non-experts.

Let us call the signal that has been received by most experts the *majority signal*. In our model, an efficient strategy profile is characterized by always implementing the policy indicated by the majority signal.⁸ For convenience, we let the number of experts $m := |M|$ be odd such that there is always a unique majority signal indicating the policy that *should* be implemented.⁹ While the definition of informational efficiency above is binary, strategy profiles can also be ranked according to their informational efficiency by comparing their corresponding probabilities of matching the true state.

Hereafter, we will slightly misuse notation by using “*A*” and “*B*” to denote the corresponding state of the world, signal content, message content, and policy, whenever the context prevents confusion.

2.1 Let the Experts Decide

One important feature of the model is that informational efficiency can always be obtained in equilibrium, regardless of the network structure. Consider for instance the strategy profile σ^* in which all experts vote their signal and all non-experts abstain. Under the simple majority rule this “let the experts decide” strategy profile σ^* is efficient since for any draw of nature the signal received by a majority of experts is implemented. Moreover, because preferences are homogeneous, efficient strategy profiles do not only maximize the sum of utilities, but also each individual agent’s utility. Thus, there is no room for improvement, as already argued in McLennan (1998).

Proposition 1. *There exist efficient equilibria for any network structure. For instance, the “let the experts decide” strategy profile σ^* is efficient and an equilibrium for any network structure.*

Importantly, while efficient strategies constitute an equilibrium, the reverse does not hold true: Existence of an equilibrium does not imply that it is efficient. On the contrary, there are (trivial and non-trivial) inefficient equilibria of the game. One non-trivial inefficient equilibrium will be discussed as Example 3 below.

Among the efficient equilibria, we consider the “let the experts decide” equilibrium σ^* focal for two reasons. First, it is symmetric: Agents of one type all use the same strategy. Second, it is intuitive to abstain as a non-expert and to vote one’s signal as an expert, as already argued, e.g., by Feddersen and Pesendorfer (1996). However, since it is also intuitive for experts to send informative messages and for receivers to vote their messages, it may nonetheless be difficult to coordinate on this

⁸Given efficient strategy profiles, the probability of matching the true state is maximized but not equal to one because it might always happen by chance that most experts receive the wrong signal. Letting the number of experts grow, this probability approaches one as in Condorcet’s Jury Theorem.

⁹Admitting an even number of experts would not change the results qualitatively, but it would make the analysis cumbersome because more cases had to be distinguished.

equilibrium. In particular, consider the strategy profile $\hat{\sigma}$ in which experts communicate and vote their signal and non-experts vote their message and abstain if they did not receive any information. This strategy profile $\hat{\sigma}$ is *sincere* in the sense that each agent communicates and votes the alternative that she considers as most likely given her private information.¹⁰ We now proceed by investigating conditions under which the sincere strategy profile is an equilibrium.

2.2 Sincere Voting

The following proposition characterizes under which conditions on the network structure the sincere strategy profile $\hat{\sigma}$ is an equilibrium and how the network structure affects the extent to which $\hat{\sigma}$ aggregates the experts' information efficiently.

Proposition 2. *Let $m = |M| \geq 5$, and let m be odd. Let the number of links $l := \sum_{i=1}^m d_i$ be even and order the degree distribution in M in decreasing order (d_1, d_2, \dots, d_m) such that $d_j \geq d_{j+1}$. The sincere strategy profile $\hat{\sigma}$ is an equilibrium if (a) $d_1 + \sum_{j=2}^{\frac{m-1}{2}} d_j \leq \frac{l}{2}$, and only if either (b) $d_1 + \sum_{k=\frac{m+5}{2}}^m d_k \leq \frac{l}{2}$ or there is an agent who is never pivotal. The sincere strategy profile $\hat{\sigma}$ is efficient if and only if condition (a) is satisfied.*

The two conditions can be interpreted in terms of a sender's "power" that manifests itself in the size of her audience d_i . The sufficient condition (a) says that at most half of the links may be concentrated on the most powerful sender and the $\frac{m-3}{2}$ most powerful *other* senders; while the necessary condition (b) says that at most half of the links may be concentrated on the most powerful sender and the $\frac{m-3}{2}$ least powerful senders. We call networks that satisfy condition (a) "*strongly balanced*" and networks that satisfy condition (b) "*weakly balanced*."

Strong balancedness and hence the sufficient condition (a) in Proposition 2 is illustrated in the following example.

Example 1 (strongly balanced). *Let $n = 4$, $m = 5$, and the degree distribution of experts $(d_1, d_2, d_3, d_4, d_5) = (1, 1, 1, 1, 0)$ as illustrated in the left panel of Figure 1. This network is strongly balanced (since $d_1 + d_2 \leq \frac{l}{2}$ which is $2 \leq 2$). By Proposition 2 the sincere strategy profile $\hat{\sigma}$ is efficient and an equilibrium.*

Observe in the example that under the sincere strategy profile $\hat{\sigma}$ any three experts who vote and communicate the same alternative determine the final outcome. Thus, for any draw of nature the policy indicated by the majority signal is implemented, which means that information is aggregated efficiently.

Strong balancedness is a very strong condition on the equality of the degree distribution requiring that almost all experts have the same degree. In the proof of Proposition 2 we show that strong balancedness is not only equivalent to efficiency of the sincere strategy profile $\hat{\sigma}$, but also equivalent to the following property of $\hat{\sigma}$: The hypothetical outcome of a vote in which the experts M alone participate

¹⁰The "let the experts decide" strategy profile σ^* , in contrast, is not "fully sincere" for the following reason. The aspect that information is not transmitted either means that senders do not communicate their signal or that receivers do not follow their message.

coincides with the outcome of voting in the entire society $M \cup N$ for any realization of the draws of nature (cf. Lemma B.1).

To see the intuition for this equivalence interpret the votes of all non-experts that form an audience of some sender j as copies of his vote. Consider first the extreme case when all audiences are of the exact same size d . Then the voting outcome restricted to the experts must equal the voting outcome in the society since the effect of the additional non-expert votes is as if d copies of each sender's vote would be added. Slightly perturbing the degree distribution does not change this observation as long as the most powerful minority of experts, i.e. those with degrees $d_1, \dots, d_{\frac{m-1}{2}}$, cannot determine the outcome against the votes of the remaining majority, i.e. those with degrees $d_{\frac{m-1}{2}+1}, \dots, d_m$. Strong balancedness of a network simply incorporates this condition such that the outcome of voting (under the sincere strategy profile $\hat{\sigma}$) is as if the votes of all non-experts were ignored. To better understand the equivalence of strong balancedness and efficiency of the sincere strategy profile $\hat{\sigma}$ note that in a strongly balanced network the outcome of the sincere strategy profile $\hat{\sigma}$ coincides with the outcome of the “let the experts decide” strategy profile σ^* , which is efficient. Finally, since any efficient strategy profile is an equilibrium, we get that strong balancedness of the network is sufficient for the sincere strategy profile $\hat{\sigma}$ to be an equilibrium.

Consider now weak balancedness, which is condition (b) in Proposition 2. The proposition states that if all players can be pivotal under the sincere strategy profile $\hat{\sigma}$, then weak balancedness is a necessary condition for $\hat{\sigma}$ to be an equilibrium. Necessity can be illustrated with an example of a network that violates weak balancedness when showing that $\hat{\sigma}$ is not an equilibrium. Networks violating weak balancedness also violate strong balancedness and will be called “unbalanced” hereafter.

Example 2 (star). *Let $n = 4$, $m = 5$, and the degree distribution of experts $(d_1, d_2, d_3, d_4, d_5) = (4, 0, 0, 0, 0)$ as illustrated in the right panel of Figure 1. This network violates weak balancedness (b) (since $d_1 + d_5 \not\leq \frac{1}{2}$ which is $4 \not\leq 2$) and therefore also strong balancedness. Hence, by Proposition 2 the sincere strategy profile $\hat{\sigma}$ is neither efficient, nor an equilibrium (every voter is pivotal for some draw of nature).*

To see why $\hat{\sigma}$ is inefficient in the example, consider a draw of nature by which the most powerful expert, i.e. the expert with the highest degree, receives the minority signal. Assume now, for the sake of argument, that the sincere strategy profile $\hat{\sigma}$ is played. In this case the minority signal determines which policy is implemented; information is hence aggregated inefficiently. To see why $\hat{\sigma}$ is no equilibrium, consider the following two deviation incentives. First, the most powerful expert would want to deviate to not communicating, but still voting, her signal. This would lead to an efficient strategy profile of the “let the experts decide” type. Second, the non-experts, too, can improve by deviating. In particular, consider a non-expert receiving message A . His posterior belief that A is true is $p_i(A|A) = p > \frac{1}{2}$. However, his posterior belief that A is true, given that he is pivotal, is $p_i(A|A, piv) < \frac{1}{2}$ because in this simple example pivotality only occurs when all other experts have received signal B . Thus, abstention or voting the opposite of the message is a strict improvement for any non-expert.

Figure 1

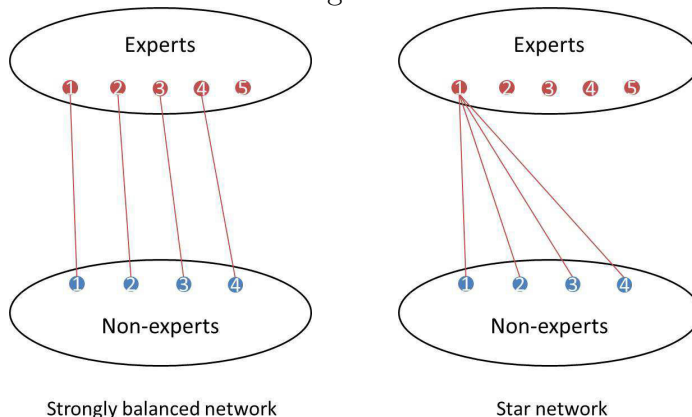


Figure 1: Left: Example 1, which is a network satisfying strong balancedness. Right: Example 2, the star network, which is an unbalanced network.

Example 2 illustrates the swing voter’s curse in an extremely simple setting of our model. The argument, however, is much more general. Assume that all agents play according to the sincere strategy profile $\hat{\sigma}$ and consider the receivers who belong to the largest audience. These receivers know that their sender is very powerful. Hence, if they are pivotal in the vote, this implies that a considerable number among the other experts must have got a signal that contradicts the message they received. Thus, if following the message has any effect on the outcome, it has most likely a detrimental effect. If a receiver in the largest audience realizes that he is “cursed” in this sense, he wants to deviate from the *sincere* strategy and prefers to abstain or to vote the opposite.

In fact, this is the intuition for the proof of the necessary condition in Proposition 2. If condition (b) does not hold, an agent i listening to the most powerful sender infers from her pivotality that the received message must be the minority signal and is thus less likely to match the true state of the world than the opposite choice. Thus, the necessary condition (b) can be interpreted as an upper bound of expert power since it restricts the size of the largest audience.

Note that Proposition 2 provides one sufficient and one necessary condition for the sincere strategy profile $\hat{\sigma}$ to be an equilibrium, but no condition that is both sufficient and necessary. For such a condition, see Proposition B.1 in Appendix B. For networks that satisfy the necessary condition (weak balancedness), but violate the sufficient condition (strong balancedness) the sincere strategy profile $\hat{\sigma}$ is inefficient, but potentially still an equilibrium. More generally, the question arises whether there are equilibria with information transmission prior to the vote that are inefficient.

Proposition 3. *There are networks in which the sincere strategy profile $\hat{\sigma}$ is both an equilibrium and exhibits informational inefficiency.*

One example illustrating the above proposition is given below.

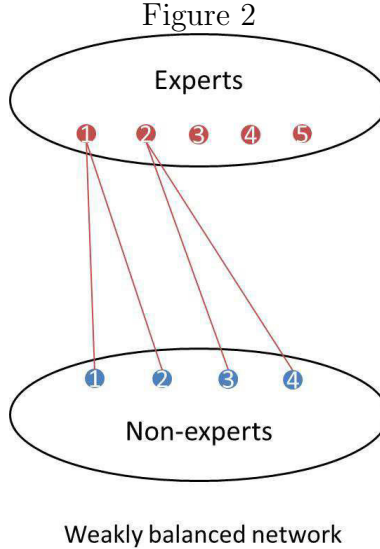


Figure 2: Example 3, which is a network such that the sincere strategy profile $\hat{\sigma}$ is both inefficient and an equilibrium.

Example 3 (weakly balanced). *Let $n = 4$, $m = 5$, and the degree distribution of experts $(d_1, \dots, d_5) = (2, 2, 0, 0, 0)$ as illustrated in Figure 2. In this network the sincere strategy profile $\hat{\sigma}$ is inefficient because the network violates strong balancedness. However, the sincere strategy profile $\hat{\sigma}$ is an equilibrium in this network (see proof of Proposition 3 in Appendix B).*

Overall, we can conclude that communication need not, but can impair information aggregation in equilibrium, depending on the inequality of the network structure. In strongly balanced networks (such as in Example 1), $\hat{\sigma}$ is both efficient and an equilibrium. In weakly balanced networks that are no longer strongly balanced (such as in Example 3), $\hat{\sigma}$ can still be an equilibrium, but is always informationally inefficient. Finally, in unbalanced networks (such as in Example 2) neither property holds. There the swing voter’s curse occurs such that non-experts can profitably deviate from $\hat{\sigma}$ by not following their message.

Whether real people account for the swing voter’s curse in unbalanced networks is an empirical question. Therefore, it may be helpful to bring the theory to the lab and find out how experimental subjects play the game in various networks that differ in the balancedness of their degree distribution. The second main purpose of the laboratory experiment is to do equilibrium selection. Recalling that the “let the experts decide” equilibrium σ^* always exist, we can have multiple equilibria. In particular, in the case of weakly balanced networks that are not strongly balanced the quality of information aggregation depends on whether the agents manage to coordinate on the efficient “let the experts decide” equilibrium or whether they coordinate on the inefficient sincere equilibrium, or on other potential equilibria. This question is hard to answer theoretically, since both the “let the experts decide” strategy profile σ^* and the sincere strategy profile $\hat{\sigma}$ are intuitive and hence focal.

3 The Experiment

In our experiment, we test an extended version of the model analyzed above which includes biased senders. In the extended model, non-experts can no longer be sure that they listen to an expert sharing their interest in matching the true state of the world; instead, they have to account for the possibility of being in the audience of a biased sender (“partisan”) who has a strict preference for one of the two policies, regardless of the state of the world. Partisans preferring policy P_A (P_B) will be called A -partisans (B -partisans). We only consider networks with the same number of A - and B -partisans which has basically the same effect as decreasing the expected reliability that non-experts ascribe to their sender’s message. In Appendix C, we provide a full description of the extended model and show that, unsurprisingly, all theoretical results obtained for the model without partisans also hold true for the model with partisans. This extension serves a twofold purpose: First, it shows that our results are robust in a more realistic setting in which a “sender” of information on the state of the world is not always an honest expert but can also be biased. Second, it captures some aspects of propaganda since in reality, partisans and experts both communicate to audiences of different size, for instance via Internet blogs. Hence, it is of some interest to test whether propaganda can have an unlimited influence on the outcome of votes in which the uninformed and the experts have the common interest of “getting it right,” or whether there is an upper limit to the influence of propaganda due to the deviation incentive that drives non-experts in over-sized audiences to abstain.

Our experimental design implements the four different communication networks depicted in Figure 3. In all networks, seven experimental subjects and four computerized partisans – two A - and two B -partisans – fill the network positions. The four computerized partisans are randomly assigned to four network positions in M , the set of potential senders; and the seven experimental subjects are randomly assigned to the remaining network positions. Each subject knows his own network position and the structure of the network. In all networks, the quality of the signal that the three experts receive is $p = 0.8$.

The four networks differ in the following respects: Network 1 is the *empty network* and serves as a benchmark, since the sincere strategy profile $\hat{\sigma}$ is impossible to play in the empty network because no sender can send any message in the empty set. Hence, the “let the experts decide”-equilibrium is the only focal equilibrium in the empty network. Network 2 is the *weakly balanced network* and is the unique network among the four in which the sincere strategy profile $\hat{\sigma}$ is both an equilibrium and inefficient, as demonstrated in the proof of Proposition 3. Network 3, which we will call the *unbalanced network*, makes sender 1 too powerful compared to the other sender, and the inefficient strategy profile $\hat{\sigma}$ is no longer an equilibrium, though possible to play. The same holds true for network 4, the *star network*, which is even more unbalanced. Apart from the baseline treatment, the empty network, the density of the networks is held constant while the equality of the degree distribution is varied.

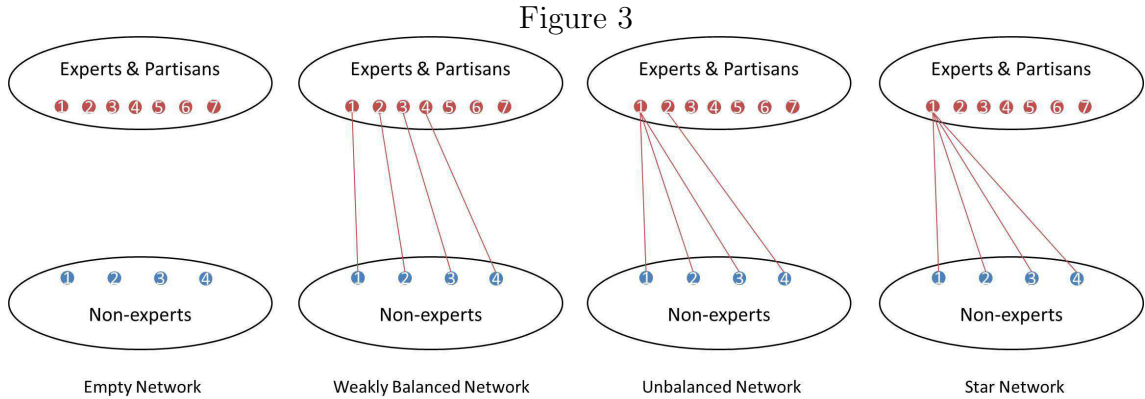


Figure 3: The four treatments of the experiment.

3.1 Experimental Implementation

The experiment addresses two questions: First, do real people forming the audience of a potentially informed sender understand that they should stop voting in line with their sender’s recommendation if the audience becomes too large (compared to the audiences of other senders)? That is, do they act in accordance with the comparative statics of the model? If the answer is yes, this will have the following consequences. There will be a limit to sincere voting in networks that are not strongly balanced, and to the resulting informational inefficiency. Moreover, there will be an upper limit to the influence that partisan propaganda can have on the outcome of votes in which all non-partisans want to implement the “right” policy. The second question addressed by the experiment is whether private communication in networks of varying balancedness can lead to significant inefficiencies in information aggregation. This would be true if all participants played sincerely, but, recall, there is always the option to play “let the experts decide” which is efficient and an equilibrium.

The experiment was conducted in the WISO-lab of the University of Hamburg in fall 2014, using the software z-tree. We ran five sessions. In each session, 28 subjects played the game described above in all four networks over 40 rounds in total. At the beginning of each session, subjects randomly received the role of an expert or the role of a non-experts. These roles stayed throughout the experiment. In each round, subjects were randomly matched into groups of seven consisting of three experts and four non-experts. The four partisans were computerized. In line with the equilibrium behavior in the extended model, *A*-partisans always vote *A* and send the message *A*, and *B*-partisans always vote *B* and send the message *B*. Groups were randomly re-matched each round. Each network game was played in ten rounds in total, but the order of networks across rounds was randomized. Instructions that described the experimental session in detail were handed out at the beginning of each session and were followed by a short quiz that tested the subjects’ understanding of the game. Hence, the experiment started only after each subject understood the rules of the game. Moreover, there were four practicing rounds, one for each treatment, that were not payout relevant. At the end of each session, three rounds were randomly drawn and payed out in cash and in private.

On average, sessions lasted for 1.5 hours and subjects earned EUR 16.7.

Table 1 in Appendix A gives a summary of the number of observations. On the group level we have 800 observations. On the individual level we have 5,600 observations with 40 decisions per subject. 160 subjects participated in the experiment.

3.2 Results on Individual Behavior Across Networks

In the equilibrium analysis of the model, we focused on two pure and symmetric strategy profiles that we consider focal, namely on the sincere profile $\hat{\sigma}$ and the “let the experts decide” profile σ^* . Hence, the question arises how often these two strategy profiles are indeed played in the lab, both in general and depending on the network structure. All tables reporting our experimental results can be found in Appendix A.

Playing sincere versus strategic As it turns out, almost 90% of the time experts vote and communicate in accordance with their signal, which is playing the sincere strategy $\hat{\sigma}_j$ (Table 2). Importantly, when having an audience, only 3% of the time, experts send a message that contradicts their signal and only about 7% of the time, they send no message (which is the residual category in Table 2).

These frequencies are independent of the network structure.¹¹ Thus, most of the time experts play sincere, independent of the communication structure. Note, however, that this does not necessarily imply that experts never target the “let the experts decide” equilibrium; it might also mean that the subjects in the role of the experts intentionally delegate equilibrium selection (or “strategy profile selection”) to the non-experts.¹²

As our experimental data reveal, it is indeed the non-experts who condition their behavior on the network structure; and what is even more interesting, their behavior changes in line with what theory predicts: As displayed in Table 3, a large majority of 70% vote according to their received message in the weakly balanced network, where the sincere strategy profile is an equilibrium, but only 54% and 47% do so in the unbalanced network and the (also unbalanced) star network, respectively. These differences are highly significant as it can be seen from the logistic regressions in Table 4. Moreover, the frequency with which non-experts who received a message abstain from voting increases from 27% in the weakly balanced network to 39% and 47% in the unbalanced network and the star network, respectively (Table 5). Again, these differences are highly significant. However, an abstention rate of 73% of the non-experts in the empty network, compared with the much lower abstention rates of non-experts in the other networks, suggests that, as predicted, coordination on the “let the experts decide” equilibrium works best when communication is not possible.

¹¹Differences in the frequency of sincere behavior arise when comparing experts with an audience with experts without an audience. This is for the trivial reason that satisfying sincerity is more demanding when choosing both a voting action and a message compared with only choosing a voting action.

¹²Another reason might be lying aversion which is common in lab experiments. Not sending a message or sending a message that contradicts the own signal might “feel like” lying.

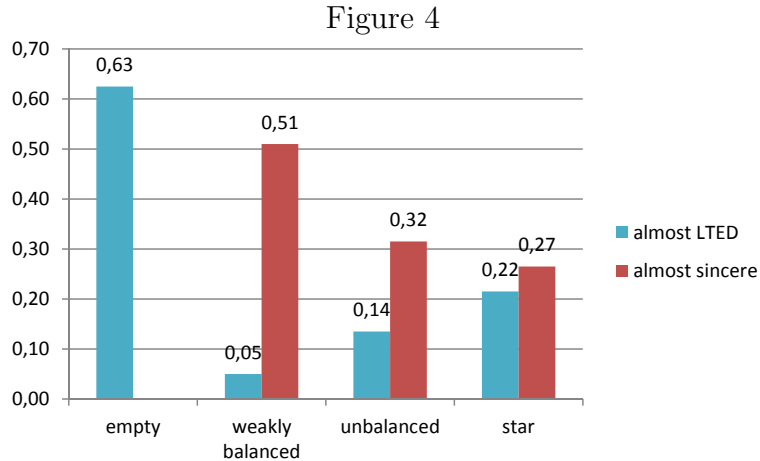


Figure 4: Frequency of strategy profiles σ^* and $\hat{\sigma}$ by treatment. A strategy profile is “almost” played if at most one agent has chosen a different strategy.

To get even more detailed evidence on the question whether, in the non-empty networks, non-experts react to the relative degree of their sender as predicted by the theory, it would be good to test how the network *position* affects behavior of the non-experts on top of the network type. Since sender degree and network type are almost perfectly correlated in our experimental setting, we can do so only by concentrating on the unbalanced network in which the degree varies across senders. Indeed, restricting the analysis to the unbalanced network reveals that in qualitative terms, the sender’s (relative) degree has the predicted influence on behavior of the non-experts: It is indeed the audience of the sender with the highest degree that becomes more reluctant to vote their message and more prone to abstain from voting. As Table 3 reports, the non-experts linked to the less powerful sender vote their sender’s message in 72% of the time, while the non-experts linked to the more powerful sender do so in only 48% of the time on average. This difference, too, is statistically significant.

Proceeding from individual strategies to strategy profiles, Figure 4 shows the frequency with which groups play either σ^* or $\hat{\sigma}$. We consider a group as *playing almost a strategy profile* if at most one of the seven subjects has chosen a different strategy.¹³ Considering the empty network, in which $\hat{\sigma}$ cannot be played, we find the highest level of coordination on σ^* . Considering the networks in which both profiles are possible to play, a decrease in network balancedness leads to a drop in the frequency with which groups coordinate (almost) on the sincere strategy profile $\hat{\sigma}$ and to a sizable increase in the frequency with which groups coordinate (almost) on the “let the experts decide” strategy profile σ^* . Fisher exact tests reveal that, apart from the comparison of unbalanced and star network, these differences are significant (Tables 6 and 7). Hence, we find support for the comparative statics of the theory on the group level, too.

Result 1. *Experts play sincere regardless of the network structure, but non-experts do not. With decreasing balancedness of the network, non-experts linked to the expert*

¹³Recall every group consists of seven real subjects and four computerized partisans. The partisans play according to σ^* and $\hat{\sigma}$ by default.

with the highest degree vote their message significantly less often and abstain significantly more often, and groups coordinate less often on the sincere strategy profile $\hat{\sigma}$ and more often on the “let the experts decide” equilibrium σ^* . Coordination on σ^* is highest in the empty network.

Hence, we find that, although off-equilibrium play occurs, e.g. playing $\hat{\sigma}$ in the unbalanced network, the comparative-static predictions of the theory are well supported by our experimental findings. Before we proceed to our results on the efficiency of information aggregation, a few remarks on uninformed voting are in order. As Table 5 (last column) reveals, non-experts who received no message do not always abstain but participate in the vote in 28% of the time, independent of the network structure. This finding is in line with the literature, since positive rates of uninformed voting are found in all experiments on jury voting. Uninformed voting, i.e., votes that are no better than flips of a coin, have large detrimental effects on informational efficiency, as is known from the literature. In our experiment, it is the empty network in which all non-experts, trivially, receive no message; hence, if they participate in the vote, this necessarily implies uninformed voting. Consequently, the absolute number of uninformed votes is much higher in the empty network than in the other networks. Thus, the possibility to communicate may serve informational efficiency by reducing the extent of uninformed voting. However, there might also be detrimental effects of communication as we will see next.

3.3 Informational Efficiency

Figure 5 below displays the degree of informational efficiency of voting outcomes across networks. Note that all signal distributions are either non-uniform of the form “2:1” (e.g. when two experts have received signal A and one expert signal B) or uniform of the form “3:0” (i.e., when all experts have received the same), and that voting errors are more likely to impair informational efficiency in the first case than in the second. Hence, Figure 5 displays results separately for these two cases: The left panel concentrates on votes with signal distributions of the form “3:0” and the right panel on votes with signal distributions of the form “2:1”. As is easy to see, in both cases the star network performs worst in terms of informational efficiency. Moreover, informational efficiency of the unbalanced network lies between the star network and the other two networks.

To test whether differences in informational efficiency across networks are significant, we create the variable *efficiency* that takes the value -1 if the voting outcome matches the minority signal, the value 0 if a tie occurs, and the value 1 if the voting outcome matches the majority signal. Fisher exact tests reveal that the star network exhibits significantly less informational efficiency than the weakly balanced and the empty network. This holds true for both uniform signals (Table 8) and non-uniform signals (Table 9). Other differences are not significant (except between the empty and the unbalanced network for non-uniform signals on the 10 percent level). Furthermore, using ordered logit and probit models, we regress *efficiency* on the network type, controlling for the signal distribution. Results are displayed in Table 10. We find again that informational efficiency is significantly lower in the

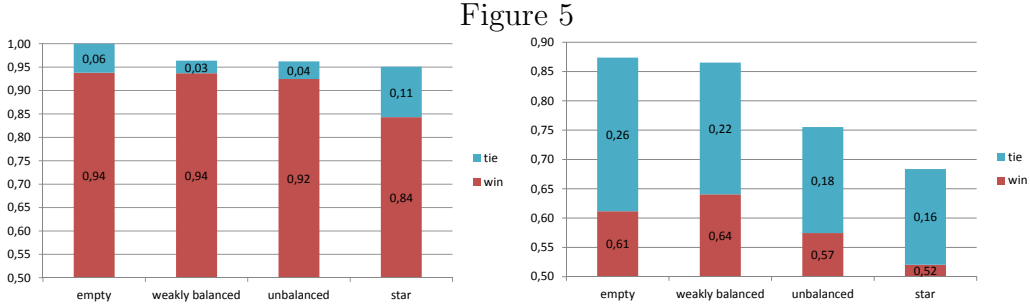


Figure 5: Frequency of informationally efficient group decisions by treatment. Results for uniform signals (of the form “3:0”) are displayed in the left panel; results for non-uniform signals (of the form “2:1”) are displayed in the right panel. ‘Win’ means that the outcome of voting is the majority signal. ‘Tie’ means that there were as many votes for A as for B such that the outcome is correct with probability one half.

star network than in the empty network and the weakly balanced network. There is also weak evidence that the unbalanced network is less efficient than the empty network, but this finding is not as robust.

Result 2. *Informational efficiency is significantly lower in the star network, compared to the empty and the weakly balanced network. There is also weak evidence that the unbalanced network exhibits lower informational efficiency than the empty network.*

To test whether the low informational efficiency in the star network, and probably also in the unbalanced network, affects subjects in an economically meaningful way, we compute the expected payoff EP for each group in each round.¹⁴ If the group decision matches the true state, each member of the group earns 100 points. Hence the variable EP coincides with the likelihood (in percentage points) of a correct decision, given all signals in the group. For instance, if two experts have received signal A and one expert B and the outcome of the majority vote is A , then $EP = \frac{p^2(1-p)}{p^2(1-p)+(1-p)^2p} * 100 = 80$. Computing EP by network type yields on average 80 points in the empty network, 81 points in the weakly balanced network, 77 points in the unbalanced network, and 73 points in the star network. Note that when not controlling for whether the signal was uniform or not, there is additional noise because some treatments might happen to have uniform signals more often than others which makes higher expected payoffs more likely. We test for significant differences using OLS regressions and control for uniformity of the signal (Table 11). The findings are fully analogous to those of Result 2 and thus the inefficiency of the unbalanced networks, in particular the star network, is confirmed. Moreover, t-tests reveal that these values of EP found in the data are significantly below the EP of an efficient strategy profile (not in the appendix).

¹⁴Due to the noise induced by imperfect signals and since only three rounds are paid out, informational efficiency and actual earnings are only loosely coupled.

Result 3. *Expected payoffs are significantly lower in the star network, compared to the empty and weakly balanced network. There is also weak evidence that the unbalanced network exhibits lower expected payoffs than the empty network.*

Result 3 consists of two separate findings. The comparison among the networks in which communication is possible shows that the unbalanced communication structure is detrimental for informational efficiency. The comparison of the unbalanced networks with the empty network, where communication is precluded, shows that communication itself can be detrimental for informational efficiency.

Finally, we consider only the inefficient networks and ask how many deviations would have been necessary in order to induce the efficient outcome. For this purpose Table 12 reports for all inefficient cases, i.e., for all groups for which the minority signal has received weakly more votes than the majority signal, how many more votes the minority signal has received. On average we have a vote difference of 0.68 reflecting that most inefficient outcomes are close calls such as ties (where the vote difference is zero) or wins of the minority signal by one vote (where the vote difference is one). We compare this number to the number of experts and the number of non-experts who voted for the minority signal to see who could have prevented the inefficiency. Apart from the empty network there are on average roughly two non-experts who have voted for the minority signal. If they abstained, the efficient outcome would have been reached in virtually all cases. In the empty network, inefficiency usually means that a tie has been reached. As there is on average roughly one non-expert who, without having any information, voted for the minority signal, we can conclude that also in this network structure inefficiency could have been avoided by more abstention of the non-experts. This observation indicates that there are two sources of inefficiency on the side of the non-experts: uninformed voting when communication is impossible; and following too powerful leaders under unbalanced communication.

To summarize, it appears that a strong decrease in balancedness, i.e., a sizable shift of audience from some senders to one other (or a few others), impairs efficient information aggregation and hence also voters' welfare. Hence, although we find evidence in favor of the comparative statics of our theory and our subjects do switch from sincere voting to the "let the experts decide" equilibrium if network balancedness decreases, this switching behavior is not pronounced enough to prevent detrimental effects of unbalanced communication on informational efficiency.

4 Conclusion

Our experimental evidence supports the comparative-static predictions of the theory but contradicts the theory's predictions on the way in which informational efficiency depends on the network structure. While experts always play the sincere strategy, non-experts are highly sensitive to the network structure. The more unbalanced the network, the more averse are the non-experts linked to the expert with the highest degree to "follow" her by voting her message, and the more they tend to abstain. Put differently, non-experts linked to a powerful expert play the less often sincerely

and the more often strategically, the less balanced the network is. Similarly, groups coordinate more often on the “let the experts decide” equilibrium and less often on the sincere strategy profile if network balancedness decreases. Nonetheless, this tendency is insufficient to compensate the negative effects exerted by overly powerful experts in very unbalanced networks, like the star network. Hence, it is still the empty network in which coordination on the “let the experts decide” equilibrium works best. However, the empty network opens the door much wider for uninformed voting than the other networks and does therefore not exhibit a higher level of informational efficiency than do networks that are no longer strongly balanced but still weakly balanced.

We have analyzed preplay communication in a voting game of common interest. In contrast to public communication, where information aggregation occurs in the communication stage (Gerardi and Yariv, 2007; Goeree and Yariv, 2011), we have studied private communication, which only admits information aggregation in the voting stage. Both scenarios can be considered as extreme cases of more general communication structures. For instance, if experts’ audiences overlap, non-experts can listen to multiple senders. Thus, it seems a natural extension of our model to consider more general networks.

A Appendix: Tables

Table 1. Observations

Treatment	Groups	experts	non-experts	senders	receivers
empty	200	600	800	0	0
w.balanced	200	600	800	347	800
unbalanced	200	600	800	178	800
star	200	600	800	83	800
Total	800	2,400	3,200	608	2,400

Table 1: Number of observations. Senders are experts who are in a network position with an audience. The number of partisan senders is not displayed. Non-experts are receivers if they are linked to a sender (expert or partisan).

Table 2. Behavior of experts by position

	vote signal	vote opposite	send signal	send opposite	sincere
empty ($N = 600$)	560 93.3%	21 3.5 %	- -	- -	560 93.3%
weakly bal.					
<i>position 1-4</i> ($N = 347$)	321 92.5%	11 5.2%	309 90.1%	11 3.2%	248 71.5%
<i>position 5-7</i> ($N = 253$)	229 90.5%	13 5.1%	- -	- -	229 90.5%
unbalanced					
<i>position 1</i> ($N = 86$)	84 97.7%	1 1.2%	76 88.4%	2 2.3%	60 69.8%
<i>position 2</i> ($N = 92$)	90 97.8%	1 1.1%	82 89.1%	2 2.2%	69 75%
<i>position 3-7</i> ($N = 422$)	378 89.6%	20 4.7%	- -	- -	378 89.6%
star					
<i>position 1</i> ($N = 83$)	82 98.8%	1 1.2%	76 91.6%	1 1.2%	62 74.7%
<i>position 2-7</i> ($N = 517$)	474 91.7%	26 5.0%	- -	- -	474 91.7%
Total ($N = 2,400$)	2218 92.4%	101 4.2%	543 89.9%	16 2.7%	2,080 86.7%

Table 2: Behavior of experts by treatment and position. The network positions refer to Figure 3. The action ‘vote (send) opposite’ means vote (send message) A when signal is B and vice versa. In addition to the displayed categories ‘vote signal’ and ‘vote opposite’ experts could abstain. In addition to the displayed categories ‘send signal’ and ‘send opposite’ experts could send an empty message. Experts without an audience are sincere if they vote their signal. Experts with an audience are sincere if they vote their signal and also send it.

Table 3. Behavior of non-experts

	vote message	vote opposite	vote uninformed	sincere
empty ($N = 800$)	- -	- -	220 27.5%	(580) (72.5%)
weakly balanced ($N = 800$)	540 69.5%	37 4.8%	6 26.1%	557 69.6%
unbalanced ($N = 800$)	417 54.3%	61 7.9%	11 34.4%	438 54.8%
<i>position 1-3</i>	278	52	9	293
($N = 600$)	48.3%	9.0%	37.5%	48.8%
<i>position 4</i>	139	9	2	145
($N = 200$)	72.4%	4.7%	25.0%	72.5%
star ($N = 800$)	360 46.4%	59 7.6%	7 29.2%	377 47.1%
Total ($N = 3,200$)	1,317 56.7%	157 4.9%	244 27.8%	1,952 61.0%

Table 3: Behavior of non-experts by treatment (and position). The network positions in the unbalanced network refer to Figure 3. In the empty network all non-experts are uninformed. In the other networks this happens only in 79 cases, where an expert sender chose the empty message. The action ‘vote message’ means that A (B) is voted after message A (B) has been received. In addition to the displayed categories ‘vote message’ and ‘vote opposite’ non-experts who received message A or B could abstain. In addition to the displayed category ‘vote uninformed’ non-experts who received an empty message could abstain.

Table 4. Dependent variable: Following of non-experts

Variable	Logit 1		Logit 2	
	Coefficient	(Std. Err.)	Coefficient	(Std. Err.)
unbalanced	-0.651***	(0.170)	-0.639***	(0.164)
star	-0.968***	(0.193)	-0.945***	(0.186)
Intercept	0.824***	(0.200)	0.830***	(0.194)
N	2,321		2,400	
Log-likelihood	-1543.25		-1595.30	
$\chi^2_{(2)}$	25.61		25.91	

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: Robust standard errors in parentheses adjusted for subjects.

Table 4: Estimation results: Logistic regression with decision to follow message as dependent variable. Baseline category is the weakly balanced network. Model 1 restricts attention to non-experts who received message A or B . Model 2 also considers non-experts who received an empty message, for which following means abstention. Following coincides with sincere behavior of non-experts.

Table 5. Abstention

	experts	non-experts	with message	without message
empty	3,17% N=600	72,50% N=800	- -	72,50% N=800
weakly balanced	3,17% N=600	27,13% N=800	25.7% N=777	73,91% N=23
unbalanced	4,33% N=600	38,88% N=800	37.8% N=768	65,63% N=32
star	2,83% N=600	46,75% N=800	46.0% N=776	70,83% N=24
Total	3,38% N=2.400	46,31% N=3.200	36.5% N=2.321	72,24% N=879

Table 5: Abstention by treatment. The columns ‘experts’ and ‘non-experts’ contain all agents of this type. The column with (without) message contains only those non-experts who received a non-empty (empty) message. If they do not abstain, then this is uninformed voting.

Table 6. Fisher exact test on almost σ^*

	weakly balanced	unbalanced	star
empty	0.000	0.000	0.000
weakly balanced		0.010	0.000
unbalanced			0.048

Table 6: p-values of Fisher exact tests comparing the frequency of the “let the experts decide” strategy profile σ^* between two treatments. A group plays “almost” σ^* if there is at most one player whose strategy differs from the profile.

Table 7. Fisher exact test on almost $\hat{\sigma}$

	unbalanced	star
w.balanced	0.000	0.000
unbalanced		0.321

Table 7: p-values of Fisher exact tests comparing the frequency of the sincere strategy profile $\hat{\sigma}$ between two treatments (in the empty network $\hat{\sigma}$ cannot be played). A group plays “almost” $\hat{\sigma}$ if there is at most one player whose strategy differs from the profile.

Table 8. Efficiency for uniform signals

	weakly balanced	unbalanced	star
empty	0.102	0.160	0.034
weakly balanced		0.921	0.050
unbalanced			0.131

Table 8: p-values of Fisher exact tests comparing efficiency between two treatments for the case of uniform signals. Efficiency is 1 if majority signal wins, 0 in case of a tie, and -1 if majority signal loses.

Table 9. Efficiency for non-uniform signals

	weakly balanced	unbalanced	star
empty	0.869	0.070	0.003
weakly balanced		0.151	0.012
unbalanced			0.546

Table 9: p-values of Fisher exact tests comparing efficiency of two treatments for the case of non-uniform signals. Efficiency is 1 if majority signal wins, 0 in case of a tie, and -1 if majority signal loses.

Table 10. Dependent variable: Efficiency

Variable	ologit 1		oprobit 1		ologit 2	
	Coeff.	(Std. Err.)	Coeff.	(Std. Err.)	Coeff.	(Std. Err.)
uniform signal	2.027***	(0.135)	1.136***	(0.065)	2.027***	(0.135)
empty					-0.059	(0.140)
w.balanced	0.059	(0.140)	-0.001	(0.068)		
unbalanced	-0.276*	(0.164)	-0.196**	(0.088)	-0.335	(0.210)
star	-0.711**	(0.319)	-0.444**	(0.174)	-0.770***	(0.243)
Intercept cut 1	-1.611***	(0.208)	-0.979***	(0.115)	-1.670***	(0.251)
Intercept cut 2	-0.572***	(0.122)	-0.385***	(0.071)	-0.631***	(0.179)
N	800		800		800	
Log-likelihood	-513.262		-512.771		-513.262	

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: Robust standard errors in parentheses adjusted for sessions.

Table 10: Estimation results: Ordered logit and orderd probit. The first and the second model use the empty network as baseline category. The third model uses the weakly balanced network as baseline category.

Table 11. Dependent variable: Expected payoff (EP)

Variable	OLS 1		OLS 2	
	Coefficient	(Std. Err.)	Coefficient	(Std. Err.)
uniform signal	31.21***	(0.84)	31.21***	(0.84)
empty			0.75	(0.80)
weakly balanced	-0.75	(0.80)		
unbalanced	-3.53**	(1.00)	-2.77**	(0.91)
star	-7.70*	(2.85)	-6.95**	(2.22)
Intercept	64.41***	(1.31)	63.66***	(1.62)
N	800		800	
R ²	0.347		0.347	

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: Robust standard errors in parentheses adjusted for sessions.

Table 11: Estimation results: OLS with expected payoff (EP) as dependent variable. The first model uses the empty network as baseline category. The second model uses the weakly balanced network as baseline category.

Table 12. Extent of inefficiency

	vote difference	“wrong” experts	“wrong” non-experts
empty N=46	0.33	1.09	1.13
weakly balanced N=39	0.56	1.13	2.05
unbalanced N=48	0.75	0.98	2.06
star N=63	0.95	0.90	2.30
Total N=196	0.68	1.01	1.92

Table 12: Extent of inefficiency. The variable ‘vote difference’ refers to the absolute difference of the number of votes. A vote difference of, e.g., 2 means that the minority signal has received two more votes than the majority signal; and a vote difference of 0 means that a tie has occurred. The label “wrong” refers to an agent who voted for the minority signal. The table reports the mean of these variables over all inefficient cases, i.e. for all groups where the majority signal did not receive a majority of votes.

B Mathematical Appendix

To prove Prop. 2 we first state a helpful lemma which is based on the following two definitions.

Definition 2. (*Strong balancedness*): Let $m = |M|$ be odd and order the degree distribution in M in decreasing order (d_1, d_2, \dots, d_m) such that $d_j \geq d_{j+1}$. Recall that $l := \sum_{j=1}^m d_j$. We call a network **strongly balanced** if the degree distribution in M satisfies condition (a) of Prop. 2, which is $d_1 + \sum_{j=2}^{\frac{m-1}{2}} d_j \leq \frac{l}{2}$.

Definition 3. (*Representation*): For a social network g and a strategy profile σ we say that the agents in M (i.e. the experts) represent the society if the outcome of voting among the agents in M coincides with the outcome of voting in the society $N \cup M$ for any realization of the draws of nature. We call a network g **representative** under strategy profile σ if the agents in M represent the society.

Lemma B.1. Let $m = |M|$ be odd. The following statements about a network g are equivalent:

1. g is such that sincere voting $\hat{\sigma}$ is efficient.
2. g is representative under sincere voting $\hat{\sigma}$.
3. g is strongly balanced.

Proof. Suppose g is such that $\hat{\sigma}$ is efficient. Then for any draw of nature the majority signal must be implemented. Restricting attention to votes within M , indeed for any draw of nature the majority signal receives a majority of votes under $\hat{\sigma}$ since each expert simply votes her signal. Thus, (1.) implies (2.).

To show that (2.) implies (3.) denote the number of links $l := \sum_{j=1}^m d_j$ and let the degree distribution be ordered in decreasing order as in Definition 2. Consider a draw of nature in which the outcome of voting restricted to M is A with one vote more than B . If all experts with signal B happen to be the most powerful experts, the number of votes for B is: $\sum_{j=1}^{\frac{m-1}{2}} (1 + d_j)$. By representativeness, A must be the outcome among all voters. Thus, the number of votes for B must be smaller than $\frac{m+l}{2}$ (which is half of the total number of votes). Strong balancedness follows from a simple rearrangement of this condition:

$$\sum_{j=1}^{\frac{m-1}{2}} (1 + d_j) < \frac{m+l}{2} \quad (\text{B.1})$$

$$d_1 + \sum_{j=2}^{\frac{m-1}{2}} d_j < \frac{1}{2} + \frac{l}{2} \quad (\text{B.2})$$

$$d_1 + \sum_{j=2}^{\frac{m-1}{2}} d_j \leq \frac{l}{2} \quad (\text{B.3})$$

The last inequality holds true because we have a sum of integers on the left-hand side (LHS) (such that “ $+\frac{l}{2}$ ” can be omitted when changing to a weak inequality).

It remains to show that (3.) implies (1.), i.e. in strongly balanced networks the majority signal is always implemented under $\hat{\sigma}$. Without loss of generality let A be the majority signal of some draw of nature. Since all experts communicate their signal, the number of votes for B is smaller or equal to $\sum_{j=1}^{\frac{m-1}{2}} (1 + d_j)$ (which is the number of votes of the most powerful minority). If g is strongly balanced, this is the minority (see inequality B.1) such that the majority signal A wins. \square

Proof of Proposition 2

Note first that the definition of strong balancedness above is just the condition labeled (a) in Prop. 2. The efficiency statement in Prop. 2 is the equivalence of statements (1) and (3) of Lemma B.1 and has been shown in its proof. The sufficiency of property (a) (strong balancedness) for $\hat{\sigma}$ to be an equilibrium follows from its efficiency. By deviating from this strategy profile, expected utility can only be reduced.

It remains to show that condition (b) is a necessary condition for $\hat{\sigma}$ to be an equilibrium; or there must be an agent that is never pivotal under $\hat{\sigma}$.

Suppose that all agents can be pivotal under $\hat{\sigma}$. Let i be a non-expert listening to a sender with maximal degree d_1 . If i is pivotal under $\hat{\sigma}$, the number of votes for i 's message, say A , is $\frac{m+l+1}{2}$ such that A wins by one vote (because i can only deviate by abstaining or voting B and recall that m is odd and l is even). A number $d_1 + 1$ of these votes are due to i 's sender. All remaining votes for A can also be partitioned such that each vote corresponds to one agent in M . The maximal number of agents in M for this purpose is attained when the least powerful agents in M (besides i 's sender) vote for A . If even this maximal number of agents in M (who vote for A) is

less than or equal to $\frac{m-1}{2}$, then A is the minority signal whenever i is pivotal. This is true if the number of votes under $\hat{\sigma}$ that correspond to i 's sender and the $\frac{m-3}{2}$ “weakest” members of M is already equal or larger than $\frac{m+l+1}{2}$ what is required by pivotality. This is incorporated in the following inequality

$$d_1 + 1 + \sum_{k=\frac{m+5}{2}}^m (d_k + 1) \geq \frac{m+l+1}{2},$$

which simplifies as follows:

$$d_1 + \sum_{k=\frac{m+5}{2}}^m d_k + \frac{m-1}{2} \geq \frac{l}{2} + \frac{m+1}{2} \quad (\text{B.4})$$

$$d_1 + \sum_{k=\frac{m+5}{2}}^m d_k \geq \frac{l}{2} + 1 \quad (\text{B.5})$$

$$d_1 + \sum_{k=\frac{m+5}{2}}^m d_k > \frac{l}{2}. \quad (\text{B.6})$$

Since these conditions are sufficient for a profitable deviation of i , the converse condition is necessary for σ to be an equilibrium:

$$d_1 + \sum_{k=\frac{m+5}{2}}^m d_k \leq \frac{l}{2}. \quad (\text{B.7})$$

□

Proposition B.1. *Let m be odd and $\sum_j d_j =: l$ be even. The sincere strategy profile $\hat{\sigma}$ is an equilibrium if and only if the following conditions hold.*

1. *If $\exists i \in N$ with $d_i = 0$, then*

$\sum_{x=1,3,\dots,m} \left(p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \right) [\nu(x, 1) - \nu(-x, 1)] \geq 0$, where $\nu(x, 1)$ denotes the number of “sub-multisets” of multiset $\{d_1 + 1, \dots, d_m + 1\}$ which are of size $\frac{m+x}{2}$ and whose elements sum up to $\frac{m+l+1}{2}$.¹⁵

2. $\forall d_j \in \{d_1, \dots, d_m\}$ such that $d_j > 0$ and for all $\bar{y} \in \{1, 2, d_j, d_j + 1, d_j + 2, 2d_j, 2d_j + 1, 2d_j + 2\}$ the following holds:

(i) *if \bar{y} even, then $\sum_{x=-m+2, -m+4, \dots, m} \left(p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \right) \cdot \sum_{y=1,3,\dots,\bar{y}-1} \nu(x, y|d_j) \geq 0$, and*

(ii) *if \bar{y} odd, then $\sum_{x=-m+2, -m+4, \dots, m} \left(p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \right) \cdot \left[\sum_{y=1,3,\dots,\bar{y}-2} (\nu(x, y|d_j) + \frac{1}{2}\nu(x, \bar{y}|d_j)) \right] \geq 0$,*

¹⁵In a multiset the same numbers can occur several times. In full analogy to the notion of a subset, we call a multiset that is contained in another multiset a “sub-multiset.”

where $\nu(x, y|d_j)$ denotes the number of “sub-multisets” of multiset $\{d_1+1, \dots, d_m+1\}$ which include element d_j+1 , are of size $\frac{m+x}{2}$, and whose elements sum up to $\frac{m+l+y}{2}$.

Proof. Part I shows necessity; part II shows sufficiency.

Part I. “ONLY IF” Suppose $\hat{\sigma}$ is an equilibrium. We show that the two conditions of Prop. B.1 are satisfied.

1. Since $\hat{\sigma}$ is an equilibrium, no player can beneficially deviate. In particular, if there is a non-expert $i \in N$ without a link, i.e. the qualification of the first condition of Prop. B.1 holds, then for any deviation $\sigma'_i \in \Sigma'_i = \{A, B\}$, we have $EU(\hat{\sigma}_{-i}, \hat{\sigma}_i) \geq EU(\sigma_{-i}, \sigma'_i)$. W.l.o.g. suppose that $\sigma'_i = B$. Letting y denote the outcome under $\hat{\sigma}$ defined as the number of votes for A minus the number of votes for B , we observe that the deviation reduces the outcome y by one vote (because i votes for B instead of abstaining). The deviation σ'_i thus only affects the outcome if $y = +1$ and turns it into $y' = 0$ (i.e. if A wins by one vote under $\hat{\sigma}$, while there is a tie under $\sigma' := (\hat{\sigma}_{-i}, \sigma'_i)$). Restricting attention to these draws of nature, we must still have that the sincere strategy profile leads to higher expected utility since it is an equilibrium by assumption:

$$EU_{|y=1}(\hat{\sigma}_{-i}, \hat{\sigma}_i) \geq EU_{|y=1}(\hat{\sigma}_{-i}, \sigma'_i) = \frac{1}{2}. \quad (\text{B.8})$$

The right-hand side (RHS) is $\frac{1}{2}$ because this is the expected utility of a tie. Some more notation is helpful. Let x denote a distribution of signals defined as the number of A -signals minus the number of B -signals received by all experts. Let $P(x|A)$ denote the likelihood that the signals are x when the true state is A , and likewise for $P(x|B)$. Let $\hat{P}(x, y)$ designate the probability that signals x lead to outcome y under $\hat{\sigma}$. Then we can rewrite inequality B.8 as

$$\frac{\frac{1}{2} \sum_{x=-m, -m+2, \dots, m} P(x|A) \hat{P}(x, 1)}{\frac{1}{2} \sum_{x=-m, -m+2, \dots, m} (P(x|A) \hat{P}(x, 1) + P(x|B) \hat{P}(x, 1))} \geq \frac{1}{2}, \quad (\text{B.9})$$

since the expected utility under $\hat{\sigma}$ when restricting attention to the draws of nature that lead to a win of A by one vote equals the probability that A is true under these conditions.

This simplifies to

$$\sum_{x=-m, -m+2, \dots, m} P(x|A) \hat{P}(x, 1) \geq \sum_{x=-m, -m+2, \dots, m} P(x|B) \hat{P}(x, 1) \quad (\text{B.10})$$

and further to

$$\sum_{x=-m, -m+2, \dots, m} (P(x|A) - P(x|B)) \hat{P}(x, 1) \geq 0. \quad (\text{B.11})$$

Now, we split the sum into positive and negative values of x and finally rejoin them by using $P(x|A) = P(-x|B)$:

$$\begin{aligned}
& \sum_{x=-m, -m+2, \dots, m} (P(x|A) - P(x|B)) \hat{P}(x, 1) \geq 0 \\
& \Leftrightarrow \sum_{x=1, 3, \dots, m} (P(x|A) - P(x|B)) \hat{P}(x, 1) \\
+ & \sum_{x=-m, -m+2, \dots, -1} (P(x|A) - P(x|B)) \hat{P}(x, 1) \geq 0 \\
& \Leftrightarrow \sum_{x=1, 3, \dots, m} (P(x|A) - P(x|B)) \hat{P}(x, 1) \\
+ & \sum_{x=1, 3, \dots, m} (P(-x|A) - P(-x|B)) \hat{P}(-x, 1) \geq 0 \\
& \Leftrightarrow \sum_{x=1, 3, \dots, m} (P(x|A) - P(x|B)) \hat{P}(x, 1) \\
& + \sum_{x=1, 3, \dots, m} (P(x|B) - P(x|A)) \hat{P}(-x, 1) \geq 0 \\
& \Leftrightarrow \sum_{x=1, 3, \dots, m} (P(x|A) - P(x|B)) [\hat{P}(x, 1) - \hat{P}(-x, 1)] \geq 0 \\
& \Leftrightarrow \sum_{x=1, 3, \dots, m} (P(x|A) - P(-x|A)) [\hat{P}(x, 1) - \hat{P}(-x, 1)] \geq 0.
\end{aligned}$$

Independent of the strategy profile, $P(x|A) = \binom{m}{\frac{m+x}{2}} p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}}$. For a draw of signals with difference x (in numbers of A -signals and B -signals), the outcome $y = +1$ is reached under $\hat{\sigma}$ if there are exactly $\frac{m+l+1}{2}$ votes for A . All of the A -votes under $\hat{\sigma}$ can be partitioned such that each element of the partition is referred to an expert j with signal A . Such an expert accounts for $d_j + 1$ votes because there is her vote and the votes of her audience. Hence the probability that draw of nature x leads to outcome $y = +1$ is determined by the frequency with which $\frac{m+x}{2}$ experts who have received signal A account for exactly $\frac{m+l+1}{2}$ votes. This frequency is given by the number of “sub-multisets” of multiset $\{d_1 + 1, \dots, d_m + 1\}$ which have size $\frac{m+x}{2}$ and whose elements sum up to $\frac{m+l+1}{2}$.

Considering all possible allocations of $\frac{m+x}{2}$ A -signals among m experts, there are $\binom{m}{\frac{m+x}{2}}$ possibilities (which is the number of all “sub-multisets” of multiset $\{d_1 + 1, \dots, d_m + 1\}$ of size $\frac{m+x}{2}$). Therefore, the probability that signals x lead to outcome $y = +1$ is

$$\hat{P}(x, +1) = \frac{\nu(x, 1)}{\binom{m}{\frac{m+x}{2}}},$$

where $\nu(x, 1)$ denotes the number of “sub-multisets” of multiset $\{d_1 + 1, \dots, d_m + 1\}$ of size $\frac{m+x}{2}$ and sum $\frac{m+l+1}{2}$.

Plugging the equations for $P(x|A)$ and $\hat{P}(x, 1)$ into the inequality derived

above yields:

$$\sum_{x=1,3,\dots,m} \left(\binom{m}{\frac{m+x}{2}} p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} - \binom{m}{\frac{m-x}{2}} (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \right) \cdot \left[\frac{\nu(x,1)}{\binom{m}{\frac{m+x}{2}}} - \frac{\nu(-x,1)}{\binom{m}{\frac{m-x}{2}}} \right] \geq 0. \quad (\text{B.12})$$

Since $\binom{m}{\frac{m-x}{2}} = \binom{m}{\frac{m+x}{2}}$, these factors cancel out such that we get

$$\sum_{x=1,3,\dots,m} \left(p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \right) [\nu(x,1) - \nu(-x,1)] \geq 0. \quad (\text{B.13})$$

This shows that the first condition of Prop. B.1 is indeed implied by the assumption that $\hat{\sigma}$ is an equilibrium.

2. Let us turn to the second condition of Prop. B.1 by considering some expert $j \in M$ with $d_j > 0$. W.l.o.g. let her signal be A. Under the sincere strategy profile j will vote and communicate her signal, i.e. A. Abstention reduces the outcome y by one vote, voting the opposite reduces the outcome y by two votes. Sending no message reduces the outcome by d_j votes. Sending the opposite message reduces the outcome by $2d_j$ votes. Therefore, there are feasible deviations for j that reduce the outcome by a number of votes \bar{y} which is in the following set $\{1, 2, d_j, d_j + 1, d_j + 2, 2d_j, 2d_j + 1, 2d_j + 2\}$.

By the assumption that $\hat{\sigma}$ is an equilibrium, there is no beneficial deviation for j . That is, for any deviation $\sigma'_j \in \Sigma'_j$, we have $EU^{s_j=A^*}(\hat{\sigma}_{-j}, \hat{\sigma}_j) \geq EU^{s_j=A^*}(\hat{\sigma}_{-j}, \sigma'_j)$. Considering some deviation σ'_j and the corresponding reduction of the outcome by \bar{y} , the implemented alternatives only differ for draws of nature such that $y > 0$ and $y' \leq 0$, i.e. for outcomes y such that $0 < y \leq \bar{y}$ (because only then the reduction of support for the received message has any effect). Therefore, the inequality of expected utility must also hold when focusing on these cases, i.e.

$$EU_{|0 < y \leq \bar{y}}^{s_j=A^*}(\hat{\sigma}_{-j}, \hat{\sigma}_j) \geq EU_{|0 < y \leq \bar{y}}^{s_j=A^*}(\hat{\sigma}_{-j}, \sigma'_j). \quad (\text{B.14})$$

- (i) Suppose first that \bar{y} is even. Then the deviation σ'_j turns all outcomes in which A wins and $0 < y \leq \bar{y} - 1$ into a win of alternative B (outcomes $y = \bar{y}$ are not possible because y is odd). Therefore, the expected utility of strategy profile $\hat{\sigma}$ (respectively, $\sigma' := (\hat{\sigma}_{-j}, \sigma'_j)$), focusing on these cases, is the probability that A (respectively, B) is true in these cases. Let $P_{s_j=A^*}(x|\omega = A) =: P_A(x|A)$ denote the probability that the signal distribution is x and that expert j has received signal A when the true state is A, and similarly for $P_{s_j=A^*}(x|\omega = B) =: P_A(x|B)$. Moreover, let $\hat{P}_{s_j=A^*}(x, y) =: \hat{P}_A(x, y)$ be the probability that the signals x lead to outcome y under $\hat{\sigma}$, given that expert j has received signal A. Note that $\hat{P}_A(x, y)$ is not defined for $x = -m$ because if

all experts have received signal B it is not possible that expert j has received signal A . Then we can rewrite inequality B.14 as

$$\begin{aligned} \sum_{x=-m+2,-m+4,\dots,m} P_A(x|A) \sum_{y=1,3,\dots,\bar{y}-1} \hat{P}_A(x,y) &\geq \\ \sum_{x=-m+2,-m+4,\dots,m} P_A(x|B) \sum_{y=1,3,\dots,\bar{y}-1} \hat{P}_A(x,y). \end{aligned} \quad (\text{B.15})$$

inequality B.15 incorporates that the likelihood of A being true is greater or equal than the likelihood of B being true given that the deviation is effective and that expert j has received signal A .¹⁶ This inequality simplifies to

$$\begin{aligned} \sum_{x=-m+2,-m+4,\dots,m} (P_A(x|A) - P_A(x|B)) \\ \cdot \sum_{y=1,3,\dots,\bar{y}-1} \hat{P}_A(x,y) &\geq 0. \end{aligned} \quad (\text{B.16})$$

Independent of the strategy profile, $P_A(x|A) = \binom{m}{\frac{m+x}{2}} p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} \cdot \frac{m+x}{m}$ and $P_A(x|B) = \binom{m}{\frac{m-x}{2}} p^{\frac{m-x}{2}} (1-p)^{\frac{m+x}{2}} \cdot \frac{m+x}{m}$. The factor before the multiplication sign is the probability that there are exactly $\frac{m+x}{2}$ A -signals. Given such a distribution, the factor after the multiplication sign is the probability that expert j has received signal A .

For a distribution of signals x , the outcome y is reached under $\hat{\sigma}$ if there are exactly $\frac{m+l+y}{2}$ votes for A . All of the A -votes under $\hat{\sigma}$ can be partitioned such that each element is referred to an expert k with signal A . Such an expert accounts for $d_k + 1$ votes (because there is her vote and the votes of her audience). By assumption, expert j has received signal A and thus there are at least $d_j + 1$ votes for A under $\hat{\sigma}$. The probability that draw of nature x leads to outcome y is determined by the frequency that the $\frac{m+x}{2}$ experts who have received signal A account for exactly $\frac{m+l+y}{2}$ votes. Hence, this frequency is given by the number of “sub-multisets” of multiset $\{d_1 + 1, \dots, d_m + 1\}$ which include element $d_j + 1$, are of size $\frac{m+x}{2}$, and whose elements sum up to $\frac{m+l+y}{2}$. Considering all possible allocations of $\frac{m+x}{2}$ A -signals among m experts such that j also receives signal A , there are $\binom{m-1}{\frac{m+x}{2}-1}$ possibilities (which is the number of all “sub-multisets” of multiset $\{d_1 + 1, \dots, d_m + 1\}$ which include element $d_j + 1$ and are of size $\frac{m+x}{2}$). Therefore, the probability that signals x lead to outcome y , given that expert j has received signal A , is

$$\hat{P}_A(x,y) = \frac{\nu(x,y|d_j)}{\binom{m-1}{\frac{m+x}{2}-1}},$$

where $\nu(x,y|d_j)$ denotes the number of “sub-multisets” of multiset $\{d_1 + 1, \dots, d_m + 1\}$ which include element $d_j + 1$, are of size $\frac{m+x}{2}$, and whose elements sum up to $\frac{m+l+y}{2}$.

¹⁶To get the absolute probabilities of A (respectively B) being true, we can divide the LHS (respectively the RHS) of inequality B.15 by the sum of the LHS and the RHS.

Hence, we can rewrite inequality B.16 as follows

$$\begin{aligned}
& \sum_{x=-m+2, -m+4, \dots, m} (P_A(x|A) - P_A(x|B)) \sum_{y=1, 3, \dots, \bar{y}-1} \hat{P}_A(x, y) \geq 0 \\
& \Leftrightarrow \sum_{x=-m+2, -m+4, \dots, m} \left(\left(\frac{m}{\frac{m+x}{2}} \right) p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} \frac{\frac{m+x}{2}}{m} \right. \\
& \quad \left. - \left(\frac{m}{\frac{m-x}{2}} \right) (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \frac{\frac{m+x}{2}}{m} \right) \sum_{y=1, 3, \dots, \bar{y}-1} \frac{\nu(x, y|d_j)}{\binom{\frac{m-1}{2}}{\frac{m+x}{2}-1}} \geq 0 \\
& \Leftrightarrow \sum_{x=-m+2, -m+4, \dots, m} \left(\frac{m}{\frac{m+x}{2}} \right) \frac{\frac{m+x}{2}}{m} \left(p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \right) \\
& \quad \cdot \sum_{y=1, 3, \dots, \bar{y}-1} \frac{\nu(x, y|d_j)}{\binom{\frac{m-1}{2}}{\frac{m+x}{2}-1}} \geq 0.
\end{aligned}$$

We have used that $\binom{m}{\frac{m+x}{2}} = \binom{m}{\frac{m-x}{2}}$. Finally, we observe that the factors $\binom{m}{\frac{m+x}{2}}$, $\frac{\frac{m+x}{2}}{m}$, and $\frac{1}{\binom{\frac{m-1}{2}}{\frac{m+x}{2}-1}}$ simplify to one because $\frac{\binom{m}{\frac{m+x}{2}}}{\binom{\frac{m-1}{2}}{\frac{m+x}{2}-1}} = \frac{m}{\frac{m+x}{2}}$ such that we get

$$\sum_{x=-m+2, -m+4, \dots, m} \left(p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \right) \sum_{y=1, 3, \dots, \bar{y}-1} \nu(x, y|d_j) \geq 0 \quad (\text{B.17})$$

We have shown that inequality B.17, which coincides with condition 2(i) of Prop. B.1, holds for any $\bar{y} \in \{1, 2, d_j, d_j + 1, d_j + 2, 2d_j, 2d_j + 1, 2d_j + 2\}$ even.

(ii) Suppose now that \bar{y} is odd. (Still, we keep the assumption that some expert $j \in M$ with $d_j > 0$ has received signal A and considers a deviation σ'_j that reduces the outcome by \bar{y}). Then the deviation σ'_j turns all outcomes in which A wins and $0 < y \leq \bar{y}$ into a win of alternative B for $y = 1, 3, \dots, \bar{y} - 2$ and into a tie for $y = \bar{y}$. Therefore,

$$\begin{aligned}
& EU_{|0 < y \leq \bar{y}}^{s_j=A^*}(\hat{\sigma}_{-j}, \sigma'_j) = \\
& \frac{\sum_{x=-m+2, -m+4, \dots, m} (P_A(x|B) (\sum_{y=1, 3, \dots, \bar{y}-2} P_A(x, y) + \frac{1}{2} \hat{P}_A(x, \bar{y})) + \frac{1}{2} P_A(x|A) \hat{P}_A(x, \bar{y}))}{\sum_{x=-m+2, -m+4, \dots, m} (P_A(x|A) + P_A(x|B)) \sum_{y=1, 3, \dots, \bar{y}} \hat{P}_A(x, y)}.
\end{aligned}$$

The denominator is the probability that an outcome under $\hat{\sigma}$ is reached such that the deviation has some effect. The numerator consists of the probability that B is true for the cases where the deviation leads to a win of alternative B and of half the probabilities that A or B are true when the deviation leads to a tie.

The expected utility of the sincere strategy profile amounts to

$$EU_{|0 < y \leq \bar{y}}^{s_j=A^*}(\hat{\sigma}_{-j}, \hat{\sigma}_j) = \frac{\sum_{x=-m+2, -m+4, \dots, m} P_A(x|A) (\sum_{y=1, 3, \dots, \bar{y}-2} \hat{P}_A(x, y) + \hat{P}_A(x, \bar{y}))}{\sum_{x=-m+2, -m+4, \dots, m} (P_A(x|A) + P_A(x|B)) \sum_{y=1, 3, \dots, \bar{y}} \hat{P}_A(x, y)}.$$

The numerator is the probability that A is true under the cases where the deviation has some effect. Since the denominator is the same as above, we can rewrite inequality B.14 as

$$\begin{aligned}
& \sum_{x=-m+2, -m+4, \dots, m} \left(P_A(x|A) \left(\sum_{y=1, 3, \dots, \bar{y}-2} \hat{P}_A(x, y) + \hat{P}_A(x, \bar{y}) \right) - P_A(x|B) \right. \\
& \cdot \left. \left(\sum_{y=1, 3, \dots, \bar{y}-2} \hat{P}_A(x, y) + \frac{1}{2} \hat{P}_A(x, \bar{y}) \right) - \frac{1}{2} P_A(x|A) \hat{P}_A(x, \bar{y}) \right) \geq 0 \text{ and further}
\end{aligned}$$

simplify it to

$$\sum_{x=-m+2, -m+4, \dots, m} (P_A(x|A) - P_A(x|B)) \cdot \left(\sum_{y=1, 3, \dots, \bar{y}-2} \hat{P}(x, y|d_j) + \frac{1}{2} \hat{P}_A(x, \bar{y}) \right) \geq 0. \quad (\text{B.18})$$

Now, we plug in $P_A(x|A) = \binom{m}{\frac{m+x}{2}} p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} \frac{m+x}{m}$ and $P_A(x|B) = \binom{m}{\frac{m-x}{2}} p^{\frac{m-x}{2}} (1-p)^{\frac{m+x}{2}} \frac{m-x}{m}$; as well as $\hat{P}_A(x, y) = \frac{\nu(x, y|d_j)}{\binom{m-1}{\frac{m+x}{2}-1}}$. This yields:

$$\sum_{x=-m+2, -m+4, \dots, m} \binom{m}{\frac{m+x}{2}} \frac{m+x}{m} \left(p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \right) \cdot \left(\sum_{y=1, 3, \dots, \bar{y}-2} \frac{\nu(x, y|d_j)}{\binom{m-1}{\frac{m+x}{2}-1}} + \frac{1}{2} \frac{\nu(x, \bar{y}|d_j)}{\binom{m-1}{\frac{m+x}{2}-1}} \right) \geq 0. \quad (\text{B.19})$$

Again, the factors $\binom{m}{\frac{m+x}{2}}$, $\frac{m+x}{m}$, and $\frac{1}{\binom{m-1}{\frac{m+x}{2}-1}}$ cancel out since their product is 1. Hence, inequality B.19 becomes

$$\sum_{x=-m+2, -m+4, \dots, m} \left(p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \right) \cdot \left(\sum_{y=1, 3, \dots, \bar{y}-2} \nu(x, y|d_j) + \frac{1}{2} \nu(x, \bar{y}|d_j) \right) \geq 0. \quad (\text{B.20})$$

inequality B.20 holds for any $\bar{y} \in \{1, 2, d_j, d_j + 1, d_j + 2, 2d_j, 2d_j + 1, 2d_j + 2\}$ odd and coincides with condition 2(ii) of Prop. B.1.

We have derived the implications for an arbitrary expert with degree $d_j > 0$ and for some arbitrary $\bar{y} \in \{1, 2, d_j, d_j + 1, d_j + 2, 2d_j, 2d_j + 1, 2d_j + 2\}$. The derived conditions 2(i) and 2(ii) must hence hold for any $d_j \in \{d_1, \dots, d_m\}$ such that $d_j > 0$. For the case of the empty network, in which no single expert has an audience, the strategy profile $\hat{\sigma}$ is not interesting to study because communication is impossible, but formally still Prop. B.1 applies. In this special case condition 2 is trivially satisfied. Thus, we have shown that if $\hat{\sigma}$ is an equilibrium, then the second condition of Prop. B.1 is also satisfied.

Part II. “IF” Suppose that the two conditions of Prop. B.1 are satisfied. We show that $\hat{\sigma}$ is an equilibrium by deriving the implications of these two conditions for every kind of player.

- Non-experts without a link: Consider any non-expert $i \in N$ with $d_i = 0$. The set of strategies is $\{A, B, \phi\}$ and $\hat{\sigma}_i = \phi$. Suppose condition 1 of Prop. B.1

holds, which is inequality B.13. In part I of the proof we used a sequence of transformations to rewrite inequality B.8 as inequality B.13. Since these were all equivalence transformations, the assumption that inequality B.13 holds implies that inequality B.8 holds. Thus, condition 1 of Prop. B.1 implies that for a non-expert without a link deviating from $\hat{\sigma}$ does not increase expected utility, given that the outcome is $y = +1$, i.e. given that the deviation has any effect on the outcome.

- **Experts with an audience:** Consider any expert $j \in M$ with $d_j > 0$. This expert has $(3 \times 3)^2 = 81$ strategies because she chooses one of three messages and one of three voting actions after receiving one of two signals. To evaluate different strategies we can assume w.l.o.g. that the expert has received signal A because neither the utility function nor the strategy profile depends on the label of the alternatives. This reduces the number of strategies to nine. Consider any deviation σ'_j . This deviation reduces the voting outcome y that is attained under $\hat{\sigma}$ by a number $\bar{y} \in \{1, 2, d_j, d_j + 1, d_j + 2, 2d_j, 2d_j + 1, 2d_j + 2\}$. For each of these numbers conditions 2(i) and 2(ii) of Prop. B.1 are equivalent to inequality B.14 since the conditions 2(i) and 2(ii) were derived by equivalence transformations of inequality B.14. Thus, for any deviation of an expert with an audience, the expected utility is weakly smaller than under $\hat{\sigma}$, when restricting attention to the cases where the deviation has some effect on the outcome and hence in general as well.
- **Experts without an audience:** Consider any expert $j \in M$ with $d_j = 0$. W.l.o.g. assume that j has received signal A . Under $\hat{\sigma}$ expert j would vote A . Alternatively, she can vote B respectively abstain, which reduces the outcome y by two respectively by one vote. (These deviations have already been considered for experts with an audience when letting $\bar{y} = 2$, respectively, $\bar{y} = 1$.) These deviations are not increasing expected utility since condition 2(i) of Prop. B.1 holds in particular for $\bar{y} = 2$ and condition 2(ii) of Prop. B.1 holds in particular for $\bar{y} = 1$ such that inequality B.14 is satisfied.
- **Non-experts with a link:** Consider any non-expert $i \in N$ with $d_i = 1$. W.l.o.g. assume that i has received message A . Under $\hat{\sigma}$ non-expert i votes A . Alternatively, he can vote B respectively abstain, which reduces the outcome y by two respectively by one vote. (The effect of these two deviations is as if an expert with signal A would vote for B respectively abstain.) Again, since condition 2(i) of Prop. B.1 holds in particular for $\bar{y} = 2$ and condition 2(ii) of Prop. B.1 holds in particular for $\bar{y} = 1$, inequality B.14 is satisfied such that these deviations do not increase expected utility.

We have shown in part II of the proof that the conditions 1 and 2 provided in Prop. B.1 imply that no player can beneficially deviate from $\hat{\sigma}$. \square

Proof of Proposition 3

We show existence of inefficient strategy profiles with the network introduced in Example 3. This network violates strong balancedness such that by Prop. 2 the sincere strategy profile $\hat{\sigma}$ is inefficient. To show that $\hat{\sigma}$ is an equilibrium we show that there is no profitable deviation for any player. We distinguish between the non-experts, the experts with degree zero, and the experts with degree 2.

Non-experts Under $\hat{\sigma}$, non-expert $i \in N$ is pivotal if and only if the votes of all others would lead to a tie (4:4) because vote differences of others are even numbers. Moreover, in this example a tie of all others is only reached under $\hat{\sigma}$ if signals of the “other” experts (the ones not linked to i) form a tie as well (2:2). In particular, it must be the case that the expert with an audience has received the same signal as exactly one expert without an audience. Since the “own” expert (the one linked to i) reports truthfully under $\hat{\sigma}$, the message received by i is the majority signal (3:2). Thus, the posterior belief that the received message indicates the correct alternative is above 0.5, even when conditioning on pivotality. Therefore, voting for this alternative leads to higher expected utility than abstaining (which would lead to an expected utility of 0.5) and to a higher expected utility than voting the opposite. Thus, a non-expert cannot improve upon $\hat{\sigma}_i$, which prescribes to vote the message.

Experts without an audience Under $\hat{\sigma}$, expert $j \in M$ with degree $d_j = 0$ is pivotal if and only if the votes of all others would lead to a tie (4:4) because vote differences of others are even numbers. In this example, a tie of all others is only reached under $\hat{\sigma}$ if signals of the “other” experts form a tie as well (2:2). (In particular, it must be the case that each signal has been received exactly by one out of two experts with an audience and by one out of two other experts without an audience.) Thus, the own signal is the majority signal (3:2). Now, for the same reason as for non-experts above, voting for this alternative leads to highest expected utility. Hence, there is no profitable deviation from $\hat{\sigma}_j$ for an expert without an audience.

Experts with an audience Consider an expert $j \in M$ of degree $d_j = 2$. There are 80 ways in which this expert can deviate from the sincere strategy profile $\hat{\sigma}$ since experts with an audience can choose both voting actions and messages as a function of their signal (see proof of Prop. B.1 for why we have 81 strategies). A deviation only affects the outcome if the signal that j has received wins under $\hat{\sigma}$, but not when j deviates. W.l.o.g. assume that expert j has received signal A . Then the outcomes ($\#A : \#B$) that expert j can overturn are 7:2, 6:3, and 5:4 (i.e. $y = 5, 3, 1$). We proceed by showing for each of these outcomes that the probability that A is correct is above 0.5 such that there is no incentive to deviate from $\hat{\sigma}$. The outcome 7:2 with j receiving A is reached under $\hat{\sigma}$ only if signals were 3:2 in favor of A . The same holds true for the outcome 5:4. Thus, overruling these outcomes is decreasing expected utility. Finally, the outcome 6:3 can be based on two situations. Either signals are 4:1 and the other expert with an audience has received signal B (while all experts without an audience have also received signal A); or signals are 2:3 and

both experts with an audience have received signal A (while all others have received signal B).

Using the probabilities of these two events, we observe that A is more likely to be true than B , given that j has received signal A and the outcome is 6:3, if and only if the following inequality holds:

$$5p^4(1-p)\frac{4}{5} * \frac{1}{4} + 10p^2(1-p)^3\frac{2}{5} * \frac{1}{4} \geq 5p(1-p)^4\frac{4}{5} * \frac{1}{4} + 10p^3(1-p)^2\frac{2}{5} * \frac{1}{4}.$$

The inequality compares the probability that A is true (when signals are 4:1 and 2:3) on the LHS with the probability that B is true (when signals are 4:1 and 2:3) on the right-hand side, given that j has received signal A and the outcome is 6:3. The inequality simplifies to

$$p^4(1-p) - p^3(1-p)^2 \geq p(1-p)^4 - p^2(1-p)^3,$$

which is true (since the LHS is positive and the RHS is negative for $p > 0.5$). Hence, any outcome that an expert with an audience can overturn in this example is more likely to match the true state than the alternative.

Therefore, there cannot be any profitable deviation from $\hat{\sigma}$. □

C Partisans

C.1 Model with Partisans

We have so far assumed that all agents have the same preferences, namely they want the policy to match the state of the world. Now, we introduce agents who try to induce a specific policy regardless of the state of the world, e.g., due to the expectation of personal perquisites. We call them A -partisans or B -partisans according to their preferred policy. Throughout we assume that the number of A -partisans equals the number of B -partisans. We introduce partisans as members of the set M who can potentially communicate with non-experts in N . Non-experts cannot directly observe whether “their” sender is an expert or a partisan, but the number of experts m_E and the number of partisans $m_A = m_B$ are known.

Formally, we assume that the network g is given and that nature draws an allocation of the given experts and partisans to the nodes in M . Assuming that each allocation has the same probability, the probability that a given sender is an expert is simply $\frac{m_E}{m}$. We consider the position of each expert or partisan as her private information. Since partisans have no incentive to utilize signals about the true state of the world, we assume that they do not receive a signal.

We extend the definition of the two focal strategy profiles σ^* (“let the experts decide”) and $\hat{\sigma}$ (sincere) to the model with partisans by assuming that the latter communicate and vote their preferred alternative.¹⁷ For each partisan j voting and

¹⁷By allocating experts to positions in M , the strategy space of an expert increases since they can potentially condition their strategy on the position. In the focal strategy profiles σ^* and $\hat{\sigma}$ the experts’ strategies do not rely on their position.

communicating the preferred alternative is a best response to σ_{-j}^* , respectively to $\hat{\sigma}_{-j}$. For the “let the experts decide” strategy profile σ^* we assume in addition that all non-experts abstain independent of their received message such that they follow neither an expert’s nor a partisan’s message.

The notion of informational efficiency of Definition 1 still applies to this extension of the model. Note, however, that an informationally efficient strategy profile only maximizes the expected utility of all experts and non-experts, but generally not of any partisan.

The extension of the baseline model that incorporates partisans does not alter the results we have established so far. In particular, given that the number of A -partisans equals the number of B -partisans, Propositions 1, 2, and 3 carry over. This is formally shown as Propositions C.1, C.2, and C.3 in the next subsection.

C.2 Propositions with Partisans

Proposition C.1. *In the extended model with an equal number of partisans ($m_A = m_B$), there exist efficient equilibria for any network structure. For instance, the “let the experts decide” strategy profile σ^* is efficient and an equilibrium for any network structure.*

Proof. Since the votes of the partisans balance each other out, the “let the experts decide” strategy profile σ^* always implements the majority signal and is hence efficient. Therefore, it maximizes the expected utility for any expert and any non-expert.¹⁸ Thus, we only have to check potential deviations of partisans. Deviations in the communication strategy are ineffective because all members of the audience abstain unconditionally under σ^* . Changing the voting action cannot increase expected utility because an A -partisan cannot increase the likelihood that A is chosen when deviating from voting for A ; and analogously for B -partisans. \square

Proposition C.2. *Consider the extended model with an equal number of partisans ($m_A = m_B$). Let $m = |M|$ be odd and greater than or equal to 5. Let the number of links $l := \sum_{i=1}^m d_i$ be even and order the degree distribution in M in decreasing order (d_1, d_2, \dots, d_m) such that $d_j \geq d_{j+1}$. The sincere strategy profile $\hat{\sigma}$ is an equilibrium if (a) $d_1 + \sum_{j=2}^{\frac{m-1}{2}} d_j \leq \frac{l}{2}$, and only if either (b) $d_1 + \sum_{k=\frac{m+5}{2}}^m d_k \leq \frac{l}{2}$ or there is an agent who is never pivotal. The sincere strategy profile $\hat{\sigma}$ is efficient if and only if condition (a) is satisfied.*

Proof. First, we observe that Lemma B.1 carries over to the model extension with partisans. In particular, representativeness (of g under $\hat{\sigma}$) is unaffected and it is equivalent to both condition (a), i.e. strong balancedness, and to informational efficiency (under $\hat{\sigma}$), given that $m_A = m_B$.

Now, suppose condition (a), i.e. strong balancedness, is satisfied. Then $\hat{\sigma}$ is efficient and, hence, experts and non-experts cannot improve by deviating. When

¹⁸With the presence of partisans efficient strategy profiles are not automatically equilibria anymore, but efficient strategy profiles with partisans who cannot improve are.

an A -partisan effectively deviates from $\hat{\sigma}$ either he or his audience stops voting for A . This does not increase the likelihood that A is implemented. This holds analogously for B -partisans. Thus, there is no profitable deviation for any player.

It remains to show that condition (b) is a necessary condition for $\hat{\sigma}$ to be an equilibrium; or there must be an agent that is never pivotal under $\hat{\sigma}$. Suppose that all agents can be pivotal under $\hat{\sigma}$. Let i be a non-expert listening to a sender with maximal degree d_1 and let w.l.o.g. A be the message received. Following the arguments of the proof of Prop. 2, we get the following: If condition (b) (i.e. inequality B.7) is violated, then pivotality of non-expert i implies that a majority of the members of M (experts and partisans) have voted message B . Since $m_A = m_B$, B is then the majority signal and thus, the probability that A is true is below 0.5. Hence, non-expert i can improve by not voting the message. \square

Proposition C.3. *In the extended model with an equal number of partisans ($m_A = m_B$), there are networks in which the sincere strategy profile $\hat{\sigma}$ is both an equilibrium and exhibits informational inefficiency.*

Proof. We show the proposition by an example. Let $m = 7$, $m_A = m_B = 2$, and $n = 4$. Let the network structure be as in the weakly balanced network of the experimental treatments (i.e. the third panel in Figure 3) such that the degree distribution of the experts and partisans is $(d_1, d_2, d_3, d_4, d_5, d_6, d_7) = (1, 1, 1, 1, 0, 0, 0)$. We first show that $\hat{\sigma}$ exhibits informational inefficiency and then that $\hat{\sigma}$ is an equilibrium.

Inefficiency To see that $\hat{\sigma}$ is inefficient, consider the relation between the signal distribution and the voting outcome. Suppose that two experts have received signal A and one expert has received signal B . Assume that the four non-experts happen to be linked to the two B -partisans, to the expert who received the signal B , and to one of the experts who received signal A . In this case, $\hat{\sigma}$ implies that B wins by one vote. Since this is an instance in which the majority signal is not chosen by the group, $\hat{\sigma}$ is not efficient in the current network.

Equilibrium We show that none of the agents has an incentive to deviate from $\hat{\sigma}$. Consider first any non-expert $i \in N$. He is pivotal if without his vote the outcome of the election is a tie (5:5). This occurs either if there are two messages of each kind and i has received the majority signal as the message; or if there are 3 messages of the minority signal and 1 message of the majority signal and i has received the minority signal as the message. Non-expert i 's belief that his message, say A , is true, conditional on his pivotality, amounts to

$$p_i(A|A, piv) = \frac{3p^2(1-p)^{\frac{4}{7}} * \frac{9}{20} + 3p(1-p)^{\frac{2}{7}} * \frac{4}{20}}{3p^2(1-p)^{\frac{4}{7}} * \frac{9}{20} + 3p(1-p)^{\frac{2}{7}} * \frac{4}{20} + 3p(1-p)^{\frac{2}{7}} * \frac{9}{20} + 3p^2(1-p)^{\frac{3}{7}} * \frac{4}{20}}$$

and simplifies to

$$p_i(A|A, piv) = \frac{p^2(1-p)3 + p(1-p)^2}{[p^2(1-p) + p(1-p)^2](3+1)} > \frac{1}{2}.$$

Hence, non-expert i 's expected utility from following the message as prescribed by $\hat{\sigma}$ is larger than his utility from abstention or voting the opposite.

Now, consider an expert j with $d_j = 0$. Assume w.l.o.g. that j has received signal A . By deviating from $\hat{\sigma}_j$ this expert only changes the outcome if A would win by one vote (it is not possible that A wins with two votes). The draws of nature that lead to this outcome are all such that A is the majority signal. If A were the minority signal and expert j with $d_j = 0$ had received A , alternative B would get at least 6 votes (because there are two B -partisans and two experts with signal "B" and at least two of them have a non-expert who listens to them) and always win under $\hat{\sigma}$. Thus, j can only affect the outcome if A is the majority signal. Since the probability that A is correct is then above 0.5, a deviation from $\hat{\sigma}_j$ cannot increase expert j 's expected utility.

Now, consider an expert j with $d_j = 1$. A deviation only affects the outcome if the signal that j has received wins under $\hat{\sigma}$, but not when j deviates. W.l.o.g. assume that expert j has received signal A . Since j can reduce the number of votes for A by at most two and increase the number of votes for B by at most two (when he communicates and votes the opposite), the outcomes $\#A : \#B$ that expert j can overturn are 7:4 and 6:5. We proceed by showing for each of these outcomes that the probability that A is correct is above 0.5 such that there is no incentive to deviate from $\hat{\sigma}$, which implements A . The outcome 7:4 with j receiving A is reached under $\hat{\sigma}$ only if signals were 3:0 or 2:1 in favor of A . Since in these two cases the probability that A is true is above 0.5, overruling outcome 7:4 decreases expected utility. The outcome 6:5 can be based on two situations (as in the discussion of non-experts above). First, it is possible that A is the majority signal and there were two messages A and two messages B . Second, it is possible that A is the minority signal and two A -partisans plus one expert (the one holding the minority signal) have sent message A . Using the probabilities of these two events, we observe that A is more likely to be true than B , given that j has received signal A and the outcome is 5:4, if and only if the following inequality holds:

$$3p^2(1-p)\frac{2}{3} * \frac{9}{20} + 3p(1-p)^2\frac{1}{3} * \frac{4}{20} \geq 3p(1-p)^2\frac{2}{3} * \frac{9}{20} + 3p^2(1-p)\frac{1}{3} * \frac{4}{20}.$$

The equation compares the probability that A is true when signals are 2:1 and 1:2 on the left-hand side with the probability that B is true when signals are 2:1 and 1:2 on the right-hand side, given that j has received signal A and the outcome is 5:4. The inequality simplifies to

$$(3p^2(1-p) - 3p(1-p)^2) \left[\frac{2}{3} * \frac{9}{20} - \frac{1}{3} * \frac{4}{20} \right] \geq 0$$

which is true (since $p > \frac{1}{2}$). Hence, any outcome that an expert with an audience can overturn in this example is more likely to match the true state than the alternative.

Finally, partisans cannot improve by a deviation because, given the others' strategies under $\hat{\sigma}$, they can only reduce the likelihood of their preferred outcome by a deviation. Hence, $\hat{\sigma}$ is an equilibrium despite its informational inefficiency. \square

D Instructions

The original instructions are written in German and can be found in an online appendix or requested from the authors. On the next pages we provide an English version which is a sentence-by-sentence translation of the original instructions.

Welcome to today's experiment!

Please note that **no communication** is allowed from now on and during the whole experiment. If you have a question please raise your hand from the cabin, one of the experimenters will then come to you. The use of cell phones, smart phones, tablets, or similar devices is prohibited during the entire experiment. Please note that a violation of this rule leads to exclusion from the experiment and from any payments.

All decisions are taken anonymously, i.e. none of the other participants comes to know the identity of the others. The payoff is also conducted anonymously at the end of the experiment.

Instructions

In this experiment you will choose along with your group one out of two alternatives whereupon just one alternative is correct and the other is wrong. Only the correct alternative leads to a positive payoff for each member of the group. Some members of the group will receive information about the correct alternative. This information is accurate in 80 out of 100 cases. The group decides by voting which alternative will be implemented. The group is furthermore arranged in a communication network. Certain members of the group can – depending on the network structure – transmit a message to other members before the group ballots for the alternatives.

The sequence of each individual round consists of the following 4 parts.

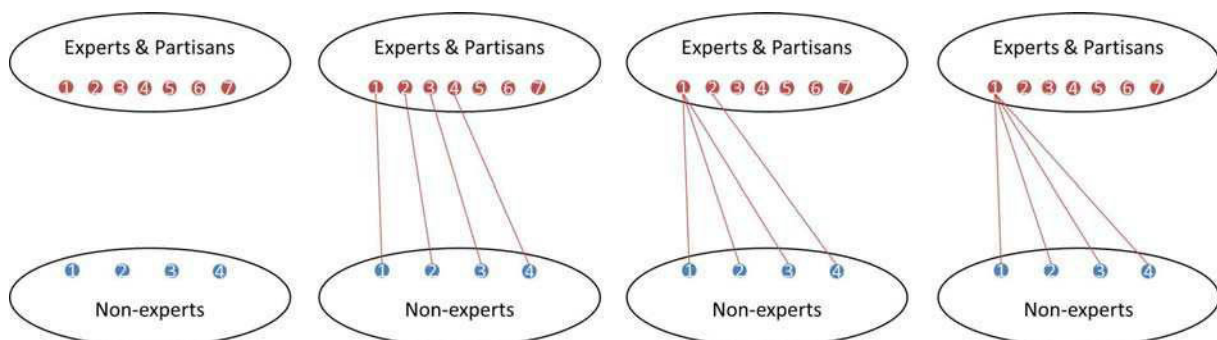
1. Information

You will receive the role of an Informed or an Uninformed at random (and you will keep it during the entire experiment). There are two alternatives: alternative "circle" and alternative "triangle". At the beginning of each round one of the two alternatives will be assigned at random and with equal likelihood as the correct alternative. The "Informed" receive information about the correct alternative which is accurate in 80 out of 100 cases. (The Informed will not necessarily all receive the same information). The "Uninformed" will not receive any information about what the correct alternative is.

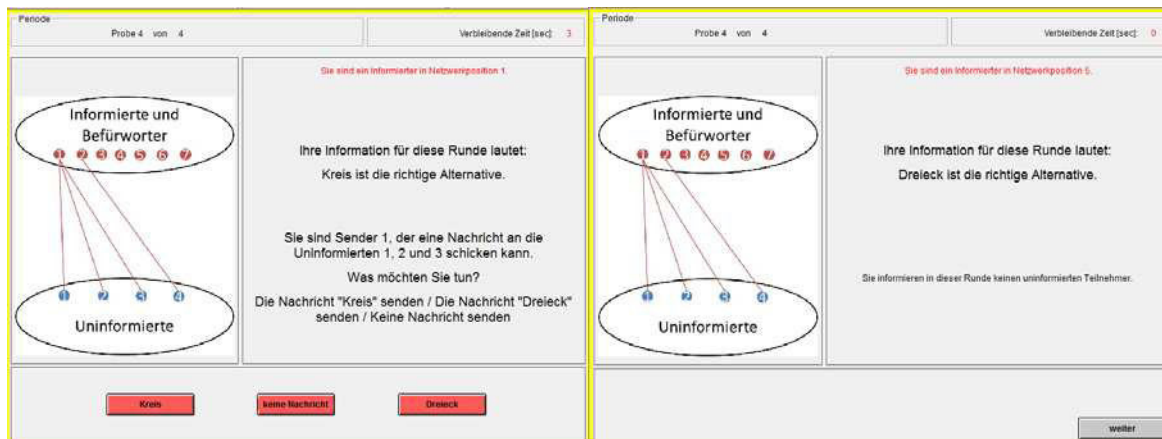


2. Communication

You will randomly be divided into groups of 11 members out of whom 7 are real participants and the remaining 4 being represented by the computer. A group is composed of 3 Informed and 4 Uninformed (a total of 7 real participants of the experiment) as well as 2 Circle-advocates and 2 Triangle-advocates (group members represented by the computer). The Circle-advocates categorically vote for “circle;” and the Triangle-advocates categorically vote for “triangle.” All group members are arranged in a communication network. At the beginning of a round you get to know the network structure and your position in the network. You can see the possible networks pictured in the figure below.

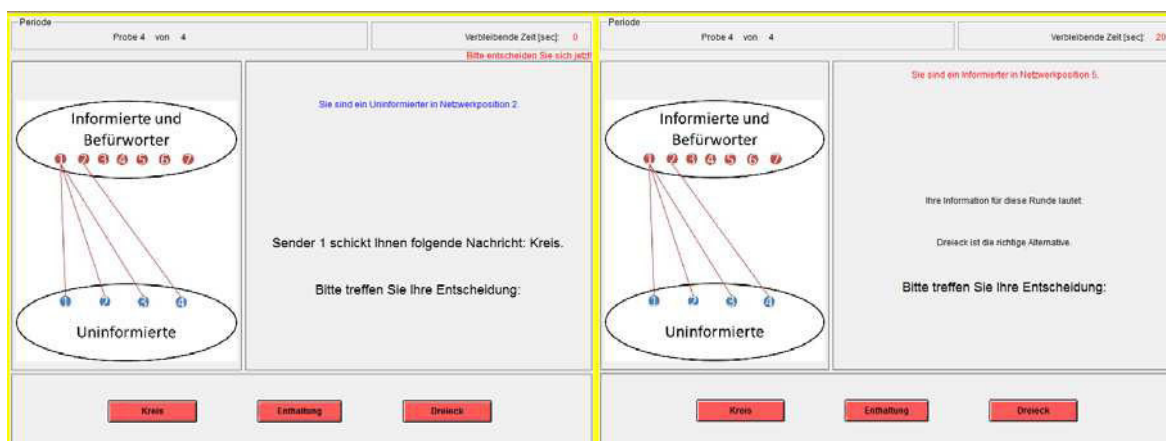


3 Informed and 4 Advocates receive in randomized arrangement the positions *Above 1 to 7* in the network. 4 Uninformed receive in randomized arrangement the positions *Below 1 to 4* in the network. Everyone knows therefore that someone with an upper position is either an Informed or an Advocate and that someone with a lower position is an Uninformed. The network structure reveals who can communicate with whom. The Uninformed can be recipients but not senders of a message. Sender of the message is – depending on the network position – an Informed or an Advocate. The Circle-advocates send the message “circle” to their recipient(s) and the Triangle-advocates send the message “triangle.” The Informed send either the message “circle” or the message “triangle” or they don’t send any message to their recipient(s). Each sender can send exactly one message to all of its (his/her) recipients. Not every Informed or Advocate is necessarily a sender. This depends on the network structure and the network position. The connecting lines between upper and lower positions in the network display who can send a message to whom.



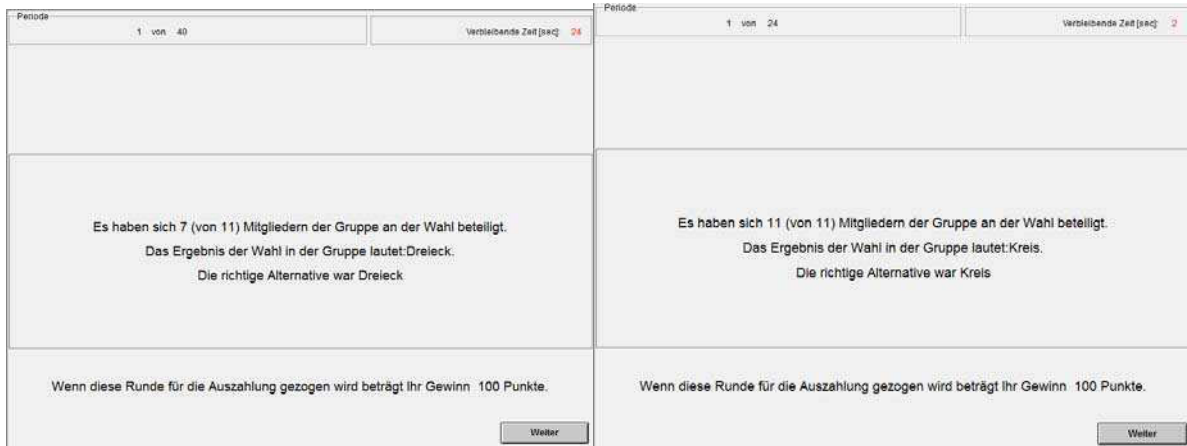
3. Voting

You can decide to vote for “circle,” to abstain from voting, or to vote for “triangle.” The 2 Circle-advocates always vote for “circle” and the 2 Triangle-advocates always for “triangle.” The voting result in the group is the alternative (circle or triangle) with the most votes. In case of a tie the computer will pick one of the two alternatives at random and with the same probability.



4. Outcome

At the end of the round you will get to know the voting outcome as well as the right alternative. If they match, e.g. the voting outcome is triangle and the right alternative is triangle, you will receive 100 points. Otherwise you will not receive any points. At the end of 40 rounds 3 rounds will be drawn randomly, which are then relevant for the payoffs. The rate of exchange between points and Euro is the following: 20 points correspond to 1 Euro. You will receive 5 Euro additionally for your participation in the experiment.



Procedure of the experiment

40 rounds will be played in total. The composition of the group changes from round to round. The network structure changes every 5 rounds. There will be a short questionnaire subsequent to the 40 rounds of the experiment. Prior to the 40 rounds of the experiment 4 sample rounds are played. These are not payoff-relevant. (In each sample round a different network is introduced.)

Summary of the procedure of the experiment:

1. Reading of the instructions
2. Questions of comprehension concerning the instructions
3. 4 sample rounds
4. 40 EXPERIMENTAL ROUNDS
5. Questionnaire
6. Payoffs

If you have a question, please raise your hand from the cabin, we will then come to you.

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