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# Jointness of Growth Determinants 

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## Jointness of Growth Determinants


#### Abstract

This paper introduces a new measure of dependence or jointness among explanatory variables. Jointness is based on the joint posterior distribution of variables over the model space, thereby taking model uncertainty into account. By looking beyond marginal measures of variable importance, jointness reveals generally unknown forms of dependence. Positive jointness implies that regressors are complements, representing distinct, but mutually reinforcing effects. Negative jointness implies that explanatory variables are substitutes and capture similar underlying effects. In a cross-country dataset we show that jointness among 67 determinants of growth is important, affecting inference and informing economic policy.


## JEL Code: C11, C52, O20, O50.

Keywords: model uncertainty, dependence among regressors, jointness, determinants of economic growth.

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## 1 Introduction

Model uncertainty is encountered in many areas of empirical work in economics. Typically there are a number of dimensions to such uncertainty, including theory, data and specification uncertainty. A policymaker faces considerable uncertainty given a set of multiple, overlapping theories which emphasize different channels in making inference or forecasting. Brock and Durlauf (2001) refer to this as "openendedness" of economic theories, in the sense that one theory being true does not imply that another one is wrong. Within each theoretical channel there may be multiple competing measures representing the same theory. Since it is often not clear a priori which theory is correct and which variables should be included in the "true" regression model, a naive approach that ignores specification and data uncertainty generally results in biased parameter estimates, overconfident (too narrow) standard errors and misleading inference and predictions.

Model averaging addresses issues of model uncertainty explicitly, and there is a recent and growing literature on this topic. ${ }^{1}$ Following Leamer's (1978) seminal contribution, Sala-i-Martin, Doppelhofer and Miller (2004), henceforth SDM (2004), suggest Bayesian Averaging of Classical Estimates or BACE to estimate the importance of explanatory variables. This BACE approach estimates the posterior distribution of parameters of interest and introduces a minimum of prior information by using Classical ordinary least squares (OLS) estimates. The resulting approximate posterior model weights are proportional to the Bayesian Information Criterion (BIC) developed by Schwarz (1978). ${ }^{2}$

The posterior probability of including a given explanatory variable in the regression can be calculated by summing the posterior weights of models that include that regressor. Such an unconditional, scale-free probability measure of relative variable importance can be a useful tool for policy decisions, inference and prediction, over and above parameter estimates such as the posterior mean and standard deviation. The emphasis on marginal measures of variable importance make it difficult (if not impossible) to detect dependence among explanatory variables. Such dependence will affect the posterior probability of any given model and the form of the posterior distribution of variables over the space of models $\mathcal{M}$. Much of the model averaging literature has neglected dependence among explanatory variables, both in the specification of independent prior model probabilities and in posterior inference.

We introduce the concept of jointness which in general terms captures depen-

[^2]dence between explanatory variables in the posterior distribution over the model space $\mathcal{M}$. By emphasizing dependence and conditioning on a set of one or more other variables, jointness moves away from marginal measures of variable importance. However, jointness is calculated unconditionally with respect to the model space and therefore takes model uncertainty into account. Jointness investigates the sensitivity of posterior distributions of parameters of interest to dependence across regressors. For example, if two variables are complementary in their posterior distribution over $\mathcal{M}$, models that either include and exclude both variables together receive relatively more weight than models where only one variable is present.

The jointness statistic introduced in this paper is defined as the logarithm of the cross product ratio associated with the joint posterior distribution of explanatory variables in regressions. Jointness can also be viewed as the log posterior odds ratio of dependence, compared to independent inclusion of explanatory variables (see for example Lehmann, 1966). To further investigate dependence between explanatory variables, we calculate the standardized posterior mean and so-called sign certainty of explanatory variables conditional on joint inclusion with other variables in the regression. We conceive of jointness in two fundamental ways.

Positive jointness among regressors could be due to two or more variables representing distinct, but complementary economic effects. In the context of economic growth for example, geographic characteristics of a country or region may be important determinants of its long term growth rate. However, a policymaker may be interested in the extent to which these growth determinants interact with other variables such as the disease environment a country is facing or institutional variables. In models of financial crises, institutional structures such as the rule of law and property rights may be required to successfully reform corporate governance. We call variables $\mathbf{x}_{i}$ and $\mathbf{x}_{l}$ complements if they exhibit positive jointness, and our assessment of posterior probability depends positively on their joint inclusion and exclusion relative to entering the regression model separately. Jointness between variables can be further analyzed by considering both the inclusion and exclusion margin of the joint posterior distribution.

Negative jointness on the other hand may be manifest when two or more variables are capturing a similar underlying mechanism. For example, in explaining economic growth there are competing measures of geographic and cultural characteristics and "quality of life" measures such as life expectancy or primary school enrolment. In propensity score models of financial loans, banks face a large amount of information designed to represent the risk of default. In a well known dataset, originally described in Lee (2001), there are 23 regressors measuring stability, demographic and financial variables, and there is substantial correlation between these indicators of default. We call variables $\mathbf{x}_{i}$ and $\mathbf{x}_{l}$ substitutes if they exhibit negative jointness and are measuring the same underlying mechanism.

The principal difference between the jointness measure proposed in this paper and other approaches suggested in the literature, is that jointness is revealed in the posterior distribution and not introduced through prior information on the type of interactions. ${ }^{3}$ In this respect our jointness approach allows for more general (unrestricted) forms of dependence among regressors and does not require decisions on an a priori structure on interactions among explanatory variables or different theories (see Chipman, 1996). Hence, the jointness approach differs from the hierarchical structure used by Brock, Durlauf and West (2003), where variables are restricted to fall into certain categories a priori. ${ }^{4}$

We apply jointness to the cross-sectional dataset of SDM (2004), containing observations for 88 countries and 67 candidate variables in explaining economic growth. Fernandez, Ley and Steel (2001a) also apply Bayesian model averaging to assess the effect of model uncertainty in cross-country growth regressions. The main difference between these studies and our approach is that we focus on dependence or jointness among growth determinants and not marginal measures of variable importance. ${ }^{5}$

We find an important role for jointness among growth determinants. In particular, we find evidence of significant positive jointness or complementarity among a broad set of explanatory variables, including some regressors labeled "insignificant" according to marginal measures of variable importance. In contrast to negative jointness, complementarity between variables reinforces size and significance of their effects on economic growth. We find evidence of significant negative jointness for variables with relatively high marginal posterior inclusion probability and other regressors. This finding implies that only variables found to be significant growth determinants are being flagged up as significant substitutes for other regressors. Compared to the potentially very large number of dependencies among growth determinants, we find a moderate number of significant jointness entries that affect inference. This implies that policy decisions are not becoming too complex, even when those dependencies are taken into account.

The remainder of the paper is organized as follows: Section 2 describes the statistical method of model averaging. Section 3 derives the jointness statistic and discusses its importance for prior specification and posterior inference. Section 4 presents the empirical results for jointness of determinants of economic growth and sensitivity analysis with respect to prior assumptions, and section 5 concludes.

[^3]
## 2 Model Averaging

Consider the following general linear regression model

$$
\begin{equation*}
\mathbf{y}=\mathbf{X} \beta+\varepsilon \tag{1}
\end{equation*}
$$

where $\mathbf{y}$ is a $(T \times 1)$ vector, $\mathbf{X}=\left[\mathbf{x}_{1}, \ldots, \mathbf{x}_{k}\right]$ is a $(T \times k)$ matrix of explanatory variables (including an intercept $\alpha$ ), $\beta$ is a $(k \times 1)$ vector of unknown parameters and $\varepsilon$ is a $(T \times 1)$ vector of residuals which are assumed to be normally distributed, $\varepsilon \sim N\left(\mathbf{0}, \sigma^{2} \mathbf{I}\right)$, and conditionally homoscedastic. ${ }^{6}$

Consider the problem of making inference on the determinants of the dependent variable, given data $\mathbf{D} \equiv[\mathbf{y}, \mathbf{X}]$. Suppose that there are $K$ potential regressors, then the model space $\mathcal{M}$ is the set of all $2^{K}$ linear models. Each model $M_{j}$ is described by a $(k \times 1)$ binary vector $\gamma=\left(\gamma_{1}, \ldots, \gamma_{K}\right)^{\prime}$, where a one (zero) indicates the inclusion (exclusion) of a variable $\mathbf{x}_{i}$ in regression (1). Let $\mathbf{X}_{j}$ be the set of regressors included in model $M_{j}$. For a given model $M_{j}$, the unknown parameter vector $\beta$ represents the effects of the variables included in the regression model. We can estimate its density $p\left(\beta \mid M_{j}\right)$ conditional on model $M_{j}$.

Given the (potentially large) space of models $\mathcal{M}$ the unconditional effects of model parameters can be derived by integrating out all aspects of model uncertainty, including the space of models $\mathcal{M}$. A maintained assumption throughout is that the explanatory variables $\mathbf{X}$ are predetermined (weakly exogenous) and independent of parameters $\beta$ and $\sigma$. The posterior density $p(\beta \mid \mathbf{y})$ can then be expressed as function of sample observations of $\mathbf{y}$.

The unconditional posterior distribution of the slope coefficient $\beta$ is given by

$$
\begin{equation*}
p(\beta \mid \mathbf{y})=\sum_{j=1}^{2^{K}} p\left(\beta \mid M_{j}, \mathbf{y}\right) \cdot p\left(M_{j} \mid \mathbf{y}\right) \tag{2}
\end{equation*}
$$

where $p\left(\beta \mid M_{j}, \mathbf{y}\right)$ is the conditional distribution of $\beta$ given model $M_{j}$. The posterior model probability $p\left(M_{j} \mid \mathbf{y}\right)$ propagates model uncertainty into the posterior distribution of model parameters. By Bayes' rule $p\left(M_{j} \mid \mathbf{y}\right)$ can be written as

$$
\begin{align*}
p\left(M_{j} \mid \mathbf{y}\right) & =\frac{l\left(\mathbf{y} \mid M_{j}\right) \cdot p\left(M_{j}\right)}{p(\mathbf{y})}  \tag{3}\\
& \propto l\left(\mathbf{y} \mid M_{j}\right) \cdot p\left(M_{j}\right)
\end{align*}
$$

such that the posterior model probability (weight) is proportional to the product of the model-specific marginal likelihood $l\left(\mathbf{y} \mid M_{j}\right)$ and the prior model probability

[^4]$p\left(M_{j}\right)$. The model weights are converted into probabilities by normalizing relative to the set of all $2^{K}$ models:
\[

$$
\begin{equation*}
p\left(M_{j} \mid \mathbf{y}\right)=\frac{l\left(\mathbf{y} \mid M_{j}\right) \cdot p\left(M_{j}\right)}{\sum_{i=1}^{2^{K}} l\left(\mathbf{y} \mid M_{i}\right) \cdot p\left(M_{i}\right)} \tag{4}
\end{equation*}
$$

\]

We follow the Bayesian model averaging literature by assuming the following prior structure for parameters in each model. The prior slope coefficients $\beta$ are normally distributed with mean zero and variance $\sigma^{2} \mathbf{V}_{0 j}$ :

$$
\begin{equation*}
p\left(\beta \mid \sigma^{2}, M_{j}\right) \sim N\left(\mathbf{0}, \sigma^{2} \mathbf{V}_{0 j}\right) \tag{5}
\end{equation*}
$$

The prior variance matrix $\mathbf{V}_{0 j}$ is assumed to be proportional to the sample covariance

$$
\begin{equation*}
\mathbf{V}_{0}=\left(g_{0} \mathbf{X}_{j}^{\prime} \mathbf{X}_{j}\right)^{-1} \tag{6}
\end{equation*}
$$

with factor of proportionality $g_{0}$. This $g$-prior was first suggested by Zellner (1986), and is a convenient way to specify the prior variance matrix, in particular in the presence of considerable model uncertainty. For the benchmark case, we assume that the prior distribution is dominated by the sample information and the prior variance is sufficiently diffuse which implies that $g_{0}$ approaches zero in (6). We examine the sensitivity of our results to changing this assumption in section 4.3.2.

For the two parameters that are common across all model, the prior error variance $\sigma^{2}$ and the intercept $\alpha$, we assume non-informative (improper) priors that impose a minimum of prior information:

$$
\begin{align*}
p\left(\sigma^{2}\right) & \propto \frac{1}{\sigma^{2}}  \tag{7}\\
p(\alpha) & =1 \tag{8}
\end{align*}
$$

Alternatively, one could assume a proper, inverse-Gamma prior distribution for the error variance $\sigma^{2}$ which is the natural conjugate prior for the normal regression model (see for example Koop, 2003).

The prior probability for model $M_{j}$ is

$$
\begin{equation*}
p\left(M_{j}\right)=\prod_{i=1}^{K} \pi_{i}^{\gamma_{i}}\left(1-\pi_{i}\right)^{1-\gamma_{i}} \tag{9}
\end{equation*}
$$

where $\pi_{i}$ is the (independent) prior inclusion probability of variable $\mathbf{x}_{i}$ in $M_{j}$, with corresponding indicator $\gamma_{i}$. The assumption that each variable has an equal prior probability of inclusion corresponds to the adoption of uniform priors and setting the hyper-parameter $\pi_{i}=1 / 2$ for all $i$ (see George and McCulloch, 1993). However, with a relatively large number of regressors $K$, a uniform prior implies that the great majority of prior probability is allocated to models with a large number of

Figure 1: Prior Probabilities by Model Size: Benchmark Case with Prior Model Size $\bar{k}=7$ and Uniform Prior with $\bar{k}=33$.

variables. As an alternative we follow SDM (2004) and introduce a prior expected model size $\bar{k}$ and corresponding prior inclusion probability $\pi_{i}^{B A C E}=\bar{k} / K$. Figure 1 shows the prior distribution over model sizes for the benchmark case with $\bar{k}=7$ (implied prior inclusion probability $\pi_{i}=7 / 67=0.104$ ) and for the uniform prior case with $\pi_{i}=1 / 2$ (and implied prior model size $\bar{k}=33.5$ ). ${ }^{7}$

The assumed prior structure introduces a minimum of prior information into the estimation. In the limit, when the sample information dominates the prior information, Leamer (1978) shows that the marginal likelihood of model $M_{j}$ may be written as

$$
\begin{equation*}
l\left(\mathbf{y} \mid M_{j}\right) \propto T^{-k_{j} / 2} \cdot S S E_{j}^{-T / 2} \tag{10}
\end{equation*}
$$

where $k_{j}$ is the number of regressors and $S S E_{j}=\left(\mathbf{y}-\mathbf{X}_{j} \beta\right)^{\prime}\left(\mathbf{y}-\mathbf{X}_{j} \beta\right)$ is the sum of squared errors in model $M_{j}$. The posterior model probability of model $M_{j}$ is obtained by pre-multiplying (10) by the prior model probability $p\left(M_{j}\right)$ and dividing

[^5]by the sum over all $2^{K}$ possible models:
\[

$$
\begin{equation*}
p\left(M_{j} \mid \mathbf{y}\right)=\frac{p\left(M_{j}\right) \cdot T^{-k_{j} / 2} \cdot S S E_{j}^{-T / 2}}{\sum_{r=1}^{2^{K}} p\left(M_{r}\right) \cdot T^{-k_{r} / 2} \cdot S S E_{r}^{-T / 2}} \tag{11}
\end{equation*}
$$

\]

The posterior model weights (11) equal the prior model weights times the (exponentiated) Bayesian Information Criterion (BIC) developed by Schwarz (1978). The BIC weights depend on the likelihood, but penalizes relatively large models through the penalty term $T^{-k_{j} / 2}$. The implied preference for smaller models addresses to a certain extent collinearity among regressors. Explanatory variables that are very similar explain relatively less of the variation of the dependent variable which implies less weight on such models.

BIC model weights (11) have been extensively discussed in the literature. Alternative derivations include the so-called "unit information prior" discussed in Kass and Wassermann (1995), approximation to Bayes Factors by Kass and Raftery (1995) and Raftery (1995), benchmark priors by Fernandez, Ley and Steel (2001b), or the limiting case of a non-informative Jeffreys prior for the error variance with a particular choice of normalizing constant (see for example Wasserman, 2000). Klein and Brown (1984) show that by minimizing the so-called Shannon information in the prior distribution, the BIC model weights (11) can be used in small samples. We adopt the BIC posterior model weights since they provide a reasonable approximation to proper Bayesian model weights and are consistent in large samples. In section 4.3 .2 we examine the sensitivity of results by considering the impact of different prior model sizes $\bar{k}$, prior distributions of regression errors, and the addition of multiplicative interaction terms as regressors.

We also experiment with alternative model weights such as Akaike's Information Criterion (AIC) in section 4.3.3, where AIC weights can be derived by minimizing the Kullback-Leibler distance between the true and predictive distribution (see Kass and Raftery, 1995, or Wasserman, 2000).

### 2.1 Marginal Posterior Objects

The unconditional mean and variance of slope parameters $\beta$ can be calculated in a straightforward manner from conditional (model specific) parameter estimates (see Leamer (1978), p. 118). The mean of the posterior distribution of slope parameter $\beta_{i}$ associated with variable $\mathbf{x}_{i}$, unconditional with respect to space of models $\mathcal{M}$, is given by

$$
\begin{equation*}
E\left(\beta_{i} \mid \mathbf{y}\right)=\sum_{j=1}^{2^{K}} p\left(M_{j} \mid \mathbf{y}\right) \cdot \widehat{\beta}_{i j} \tag{12}
\end{equation*}
$$

where $\hat{\beta}_{i j}=E\left(\beta_{i} \mid \mathbf{y}, M_{j}\right)$ is the OLS estimate for slope parameter $\beta_{i}$ given model $M_{j}$. The posterior variance of slope $\beta_{i}$ is given by

$$
\begin{equation*}
V\left(\beta_{i} \mid \mathbf{y}\right)=\sum_{j=1}^{2^{K}} p\left(M_{j} \mid \mathbf{y}\right) \cdot V\left(\beta_{i} \mid \mathbf{y}, M_{j}\right)+\sum_{j=1}^{2^{K}} p\left(M_{j} \mid \mathbf{y}\right) \cdot\left[\widehat{\beta}_{i j}-E\left(\beta_{i} \mid \mathbf{y}\right)\right]^{2} \tag{13}
\end{equation*}
$$

where the conditional variance is estimated by the maximum likelihood ${ }^{8}$ estimator $V\left(\beta_{i} \mid \mathbf{y}, M_{j}\right)=\hat{\sigma}_{j}^{2}\left(\mathbf{X}_{j}^{\prime} \mathbf{X}_{j}\right)_{i i}^{-1}$, with error variance estimate $\hat{\sigma}_{j}^{2} \equiv S S E_{j} /\left(T-k_{j}\right)$. Notice that the posterior variance (13) of coefficient $\beta_{i}$ consists of two terms: the weighted sum of conditional (model-specific) variances and an additional term taking into account the difference between conditional and posterior estimates of mean coefficients.

A policymaker might be interested to know how important variables are in explaining the dependent variable. The posterior inclusion probability of variable $\mathbf{x}_{i}$

$$
\begin{equation*}
p(i \mid \mathbf{y})=\sum_{j=1}^{2^{K}} \mathbf{1}\left(\gamma_{i}=1 \mid \mathbf{y}, M_{j}\right) \cdot p\left(M_{j} \mid \mathbf{y}\right) \tag{14}
\end{equation*}
$$

represents the probability that, conditional on the data, but unconditional with respect to the model space $\mathcal{M}$, variable $\mathbf{x}_{i}$ is relevant in explaining the dependent variable (see Leamer, 1978, and Mitchell and Beauchamp, 1988). This measure is therefore a model-weighted measure of the relative importance of including a variable in the regression.

Alternatively, we might be interested in estimates of the mean and variance of the slope coefficients conditional on a variable's inclusion, but unconditional with respect to $\mathcal{M}$. The conditional posterior mean for $\beta_{i}$ is obtained by dividing the unconditional posterior mean (12) by the posterior inclusion probability (14):

$$
\begin{equation*}
E\left(\beta_{i} \mid \gamma_{i}=1, \mathbf{y}\right)=\frac{E\left(\beta_{i} \mid \mathbf{y}\right)}{p(i \mid \mathbf{y})} \tag{15}
\end{equation*}
$$

Similarly, the variance conditional on including variable $\mathbf{x}_{i}$ is calculated from the unconditional posterior estimates of moments (12), (13), and the posterior inclusion probability (14):

$$
\begin{equation*}
V\left(\beta_{i} \mid \gamma_{i}=1, \mathbf{y}\right)=\frac{V\left(\beta_{i} \mid \mathbf{y}\right)+\left[E\left(\beta_{i} \mid \mathbf{y}\right)\right]^{2}}{p(i \mid \mathbf{y})}-\left[E\left(\beta_{i} \mid \gamma_{i}=1, \mathbf{y}\right)\right]^{2} \tag{16}
\end{equation*}
$$

To facilitate comparison across variables, we divide the posterior mean coefficient (12) by its posterior standard deviation $s_{i}$, defined as the square root of the posterior

[^6]variance (13). We use this posterior standardized coefficient associated with variable $\mathbf{x}_{i}$ conditional on its inclusion
\[

$$
\begin{equation*}
E\left(\beta_{i} / s_{i} \mid \gamma_{i}=1, \mathbf{y}\right) \equiv \frac{E\left(\beta_{i} \mid \gamma_{i}=1, \mathbf{y}\right)}{\sqrt{V\left(\beta_{i} \mid \gamma_{i}=1, \mathbf{y}\right)}} \tag{17}
\end{equation*}
$$

\]

to assess the relative size and significance of the effect of variable $\mathbf{x}_{i}$ in explaining the dependent variable $\mathbf{y} .{ }^{9}$

SDM (2004) also estimate the probability that the coefficient has the same sign as the posterior mean. This posterior sign certainty probability is given by

$$
p\left(\operatorname{sign}_{i} \mid \mathbf{y}\right)=\left\{\begin{array}{cl}
\sum_{j=1}^{2^{K}} p\left(M_{j} \mid \mathbf{y}\right) \cdot C D F\left(t_{i j} \mid M_{j}\right), & \text { if } \operatorname{sign}\left[E\left(\beta_{i} \mid \mathbf{y}\right)\right]>0  \tag{18}\\
1-\sum_{j=1}^{2^{K}} p\left(M_{j} \mid \mathbf{y}\right) \cdot C D F\left(t_{i j} \mid M_{j}\right), & \text { if } \operatorname{sign}\left[E\left(\beta_{i} \mid \mathbf{y}\right)\right]<0
\end{array}\right.
$$

For each model $M_{j}$ a non-centered cumulative $t$-distribution function is evaluated at standardized parameter estimates $t_{i j} \equiv\left(\hat{\beta}_{i j} / \hat{s}_{i j} \mid M_{j}\right)$, where $\hat{s}_{i j}$ is the square root of the conditional (model-specific) variance estimate $V\left(\beta_{i} \mid M_{j}\right)$. The posterior sign certainty conditional on including variable $\mathbf{x}_{i}$ is given by

$$
\begin{equation*}
E\left(\operatorname{sign}_{i} \mid \gamma_{i}=1, \mathbf{y}\right)=\frac{p\left(\operatorname{sign}_{i} \mid \mathbf{y}\right)}{p(i \mid \mathbf{y})} \tag{19}
\end{equation*}
$$

All the posterior statistics presented in this section - standardized coefficient estimate (17), sign certainty (18), and inclusion probability (14) - allow for unconditional inference on the marginal importance of the variable $\mathbf{x}_{i}$. However, such marginal objects have the drawback of not revealing relationships between explanatory variables, unless all regressors are independent. To address this issue we introduce a posterior object, positioned between the posterior measures of variable importance and model uncertainty, which will allow us to capture dependencies or jointness among explanatory variables.

## 3 Jointness

This section introduces our measure of jointness. We start by equating model uncertainty with the best subset variable selection problem, rewriting the posterior probability for model $j$ as

$$
\begin{equation*}
p\left(M_{j} \mid \mathbf{y}\right)=p\left(\gamma_{1}=c_{1}, \gamma_{2}=c_{2}, \ldots, \gamma_{K}=c_{K} \mid \mathbf{y}, M_{j}\right) \tag{20}
\end{equation*}
$$

$c_{i}=1$ or 0 depending on whether variable $\mathbf{x}_{i}$ is included in model $M_{j}$ or not. Although the posterior probability $p\left(M_{j} \mid \mathbf{y}\right)$ measures model uncertainty, it is difficult

[^7]to observe the nature of dependencies among regressors. In the case of bivariate dependence, the posterior joint probability of inclusion of variable $\mathbf{x}_{i}$ and $\mathbf{x}_{l}$ is given by
\[

$$
\begin{equation*}
p(i \cap l \mid \mathbf{y})=\sum_{j=1}^{2^{K}} \mathbf{1}\left(\gamma_{i}=1 \cap \gamma_{l}=1 \mid \mathbf{y}, M_{j}\right) \cdot p\left(M_{j} \mid \mathbf{y}\right) \tag{21}
\end{equation*}
$$

\]

which sums posterior model probabilities for each model in which the two variables appear. ${ }^{10}$ By comparing estimates of the posterior joint probability, $p(i \cap l \mid \mathbf{y})$, with marginal estimates $p(i \mid \mathbf{y})$ and $p(l \mid \mathbf{y})$, we may determine whether $\mathbf{x}_{i}$ and $\mathbf{x}_{l}$ are independent over the model space or if jointness is manifest in terms of a complementary or substitution relationship between $\mathbf{x}_{i}$ and $\mathbf{x}_{l}$.

In motivating our jointness concept we first consider the random vector $\left(\gamma_{i}, \gamma_{l}\right)$ with joint posterior distribution $p\left(\gamma_{i}, \gamma_{l} \mid \mathbf{y}\right)$. Probability mass points defined on $\{0,1\}^{2}$ are given below:

| $p\left(\gamma_{i}, \gamma_{l} \mid \mathbf{y}\right)$ |  | $\gamma_{l}=0$ | $\gamma_{l}=1$ | Total |
| :--- | :--- | :--- | :--- | :--- |
|  | $\gamma_{i}=0$ | $p(\bar{\imath} \cap \bar{l} \mid \mathbf{y})$ | $p(\bar{\imath} \cap l \mid \mathbf{y})$ | $p(\bar{\imath} \mid \mathbf{y})$ |
|  | $\gamma_{i}=1$ | $p(i \cap \bar{l} \mid \mathbf{y})$ | $p(i \cap l \mid \mathbf{y})$ | $p(i \mid \mathbf{y})$ |
|  | Total | $p(\bar{l} \mid \mathbf{y})$ | $p(l \mid \mathbf{y})$ | 1 |

For example, the marginal probability of including variable $\mathbf{x}_{l}, p(l \mid \mathbf{y})=p(\bar{\imath} \cap l \mid \mathbf{y})+$ $p(i \cap l \mid \mathbf{y})$, represents the unconditional probability that $\mathbf{x}_{l}$ is included. Analogously, the complementary marginal probability of excluding variable $\mathbf{x}_{l}$ is equal to $p(\bar{l} \mid \mathbf{y}) \equiv$ $1-p(l \mid \mathbf{y})=p(\bar{\imath} \cap \bar{l} \mid \mathbf{y})+p(i \cap \bar{l} \mid \mathbf{y})$. To the extent that variables $\mathbf{x}_{i}$ and $\mathbf{x}_{l}$ exhibit a form of dependence over the space of models $\mathcal{M}$, we would expect diagonal entries $p(\bar{\imath} \cap \bar{l} \mid \mathbf{y})$ and $p(i \cap l \mid \mathbf{y})$ to exceed off-diagonal entries. Subsequently a natural measure of dependence between the two binary random variables $\gamma_{i}$ and $\gamma_{l}$ is the cross-product ratio, denoted cpr $(i, l \mid \mathbf{y})$, given by ${ }^{11}$

$$
\begin{equation*}
\operatorname{cpr}(i, l \mid \mathbf{y})=\frac{p(i \cap l \mid \mathbf{y})}{p(i \cap \bar{l} \mid \mathbf{y})} \cdot \frac{p(\bar{\imath} \cap \bar{l} \mid \mathbf{y})}{p(\bar{\imath} \cap l \mid \mathbf{y})} \tag{22}
\end{equation*}
$$

To motivate the cross product ratio as a natural measure of dependence over the joint posterior distribution, we write the bivariate probability distribution $p\left(\gamma_{i}, \gamma_{l} \mid \mathbf{y}\right)$ as

$$
\begin{equation*}
p\left(\gamma_{i}, \gamma_{l} \mid \mathbf{y}\right)=p(\bar{\imath} \cap \bar{l} \mid \mathbf{y})^{\left(1-\gamma_{i}\right)\left(1-\gamma_{l}\right)} \cdot p(\bar{\imath} \cap l \mid \mathbf{y})^{\left(1-\gamma_{i}\right) \gamma_{l}} \cdot p(i \cap \bar{l} \mid \mathbf{y})^{\gamma_{i}\left(1-\gamma_{l}\right)} \cdot p(i \cap l \mid \mathbf{y})^{\gamma_{i} \gamma_{l}} \tag{23}
\end{equation*}
$$

[^8]Taking logarithms of the bivariate distribution and collecting terms in $\gamma_{i}$ and $\gamma_{l}$ gives

$$
\begin{align*}
\ln p\left(\gamma_{i}, \gamma_{l} \mid \mathbf{y}\right)= & \ln p(\bar{\imath} \cap \bar{l} \mid \mathbf{y})+\gamma_{i} \ln \left[\frac{p(i \cap \bar{l} \mid \mathbf{y})}{p(\bar{\imath} \cap \bar{l} \mid \mathbf{y})}\right]+\gamma_{l} \ln \left[\frac{p(\bar{\imath} \cap l \mid \mathbf{y})}{p(\bar{\imath} \cap \bar{l} \mid \mathbf{y})}\right]+ \\
& +\gamma_{i} \gamma_{l} \ln \left[\frac{p(i \cap l \mid \mathbf{y})}{p(i \cap \bar{l} \mathbf{y})} \cdot \frac{p(\bar{\imath} \cap \bar{l} \mid \mathbf{y})}{p(\bar{\imath} \cap l \mid \mathbf{y})}\right] \tag{24}
\end{align*}
$$

A necessary and sufficient condition for marginal independence is that $\ln p\left(\gamma_{i}, \gamma_{l} \mid \mathbf{y}\right)$ is additive in the logarithmic marginal distributions, $\ln p\left(\gamma_{i} \mid \mathbf{y}\right)$ and $\ln p\left(\gamma_{l} \mid \mathbf{y}\right)$. This condition is satisfied if and only if the coefficient of the product $\gamma_{i} \cdot \gamma_{l}$ in the $\log$ arithmic bivariate distribution (24) equals zero (see Whittaker, 1990, Proposition 2.4.1). Note that this coefficient equals the natural logarithm of the cross-product ratio, $\ln c p r(i, l)$, in equation (22).

To formalize the degree of dependence between variables $\mathbf{x}_{i}$ and $\mathbf{x}_{l}$ we introduce the following jointness statistic

$$
\begin{align*}
J_{i l} & \equiv \ln \operatorname{cpr}(i, l \mid \mathbf{y})=\ln \left[\frac{p(i \cap l \mid \mathbf{y})}{p(\bar{\imath} \cap l \mid \mathbf{y})} \cdot \frac{p(\bar{\imath} \cap \bar{l} \mid \mathbf{y})}{p(i \cap \bar{l} \mid \mathbf{y})}\right]  \tag{25}\\
& =\ln \left[\frac{p(i \mid l, \mathbf{y})}{p(\bar{\imath} \mid l, \mathbf{y})} \cdot \frac{p(\bar{\imath} \mid \bar{l}, \mathbf{y})}{p(i \mid \bar{l}, \mathbf{y})}\right]=\ln \left[P O_{i \mid l} \cdot P O_{\bar{i} \mid \bar{l}}\right] \tag{26}
\end{align*}
$$

The jointness statistic (25) can be written as the logarithm of the product of two posterior odds ratios: $P O_{i \mid l}$ denotes the posterior odds of including $\mathbf{x}_{i}$ conditional on $\mathbf{x}_{l}$ being included. $P O_{\bar{\imath} \bar{l} \bar{l}}$ represents the posterior odds of excluding $\mathbf{x}_{i}$ conditional on $\mathbf{x}_{l}$ being excluded. Jointness therefore incorporates both the inclusion and exclusion margin of the joint posterior distribution. ${ }^{12}$ By inspection, the jointness statistic is symmetric in variables $\mathbf{x}_{i}$ and $\mathbf{x}_{l}$ and can also be written as $J_{i l}=J_{l i}=\ln \left[P O_{l \mid i} \cdot P O_{\bar{l} \bar{l} \bar{i}}\right]$.

Jointness is zero if and only if each conditional term in (26) reduces to the respective marginal probability, implying independence of variables $\mathbf{x}_{i}$ and $\mathbf{x}_{l}$. We call variables with positive jointness complements, reflecting the increase in posterior probability of jointly including and excluding variables relative to including only one. Similarly, we call variables with negative jointness substitutes, if models that include only one of these variable have higher posterior probability compared to models that include and exclude them jointly.

A useful property of the jointness statistic is that it is readily extended to consider measures of dependence beyond the bivariate case. For example, if we were to take the trivariate random vector $\left(\gamma_{i}, \gamma_{l}, \gamma_{h}\right)$ and perform the equivalent log-linear

[^9]expansion then all pairwise terms represent measures of conditional dependence. Coefficients of products $\gamma_{i} \cdot \gamma_{l}, \gamma_{i} \cdot \gamma_{h}$, and $\gamma_{l} \cdot \gamma_{h}$, represent the jointness statistics, conditional on excluding, respectively, $\mathbf{x}_{h}, \mathbf{x}_{l}$, and $\mathbf{x}_{i}$. Note that the coefficient of $\gamma_{i} \cdot \gamma_{l} \cdot \gamma_{h}$, which can be written as
\[

$$
\begin{equation*}
\ln \left[\frac{c p r(i, l \mid h, \mathbf{y})}{c p r(i, l \mid \bar{h}, \mathbf{y})}\right]=J_{i l}\left|h-J_{i l}\right| \bar{h} \tag{27}
\end{equation*}
$$

\]

represents the difference in the jointness statistic for variables $\mathbf{x}_{l}$ and $\mathbf{x}_{i}$ conditional on $\mathbf{x}_{h}$ being included or excluded. ${ }^{13}$ If jointness for variables $\mathbf{x}_{l}$ and $\mathbf{x}_{i}$ is invariant to the inclusion of $\mathbf{x}_{h}$, then the statistic $J_{i l}$ can be interpreted as a partial measure of dependence.

We can generalize the pairwise measure of jointness of variables $\mathbf{x}_{i}$ and $\mathbf{x}_{l}$ to an extended set of control variables. Let $\gamma$ be the $k \times 1$ vector of Bernoulli indicators and $\gamma^{*}=\gamma_{/\{i, j\}}$ be the $(k-2) \times 1$ vector which excludes variables $\mathbf{x}_{i}$ and $\mathbf{x}_{l}$. Suppose $\mathbf{c}^{*}$ denotes a specific set of values on $\gamma^{*}$. For each model in $\mathcal{M}$, conditional on the inclusion of variables $\mathbf{x}_{i}$ and $\mathbf{x}_{l}$, it is possible to calculate a jointness statistic for each of the $2^{k-2}$ combinations on $\gamma^{*}$. In the trivariate case $\gamma$ is a $3 \times 1$ vector, $\gamma^{*}=\gamma_{h}$, and the jointness statistic is then given by

$$
\begin{equation*}
\left.J_{i l}\right|_{c^{*}}=\ln \left[\frac{p\left(i \cap l \mid c^{*}, \mathbf{y}\right)}{p\left(\bar{\imath} \cap l \mid c^{*}, \mathbf{y}\right)} \cdot \frac{p\left(\bar{\imath} \cap \bar{l} \mid c^{*}, \mathbf{y}\right)}{p\left(i \cap \bar{l} \mid c^{*}, \mathbf{y}\right)}\right] \tag{28}
\end{equation*}
$$

If the jointness statistic $\left.J_{i l}\right|_{c^{*}}$ varies significantly according to $\gamma_{h}=1$ or $\gamma_{h}=0$, then higher order interactions may be of interest. A corollary of this statement follows from the well known "Simpson's paradox", namely that the marginalization of higher order interactions may be misleading.

Given the definition of joint inclusion probabilities we now redefine the posterior, marginal objects associated with variable $\mathbf{x}_{i}$ introduced in section 2 . The conditional mean of the posterior distribution of model parameter $\beta_{i}$ unconditional with respect to space of models $\mathcal{M}$, but conditional on the inclusion of another variable $\mathbf{x}_{l}$, is given by

$$
\begin{equation*}
E\left(\beta_{i} \mid \gamma_{i}=1 \cap \gamma_{l}=1, \mathbf{y}\right)=\frac{E\left(\beta_{i} \mid \gamma_{l}=1, \mathbf{y}\right)}{p(i \cap l \mid \mathbf{y})} \tag{29}
\end{equation*}
$$

In comparing the mean of $\beta_{i}$ conditional on the joint inclusion of variables $\mathbf{x}_{i}$ and $\mathbf{x}_{l}$ with the conditional marginal estimate of the mean (15), we note the following. First, the mean of $\beta_{i}$ is estimated conditionally on including variable $\mathbf{x}_{l}$ which affects its value unless $\mathbf{x}_{i}$ and $\mathbf{x}_{l}$ are orthogonal. Second, unless $p(i \cap l \mid \mathbf{y})=p(i \mid \mathbf{y})$, the conditional mean is defined on a subset of models. The difference between the two means (29) and (15) can be used for inference on the degree of dependence in jointly including variable $\mathbf{x}_{i}$ and $\mathbf{x}_{l}$.

[^10]Analogous to the derivation of the conditional mean, we can calculate the posterior variance for the slope coefficient $\beta_{i}$, conditional on inclusion of variable $\mathbf{x}_{l}$ :

$$
\begin{align*}
V\left(\beta_{i} \mid \gamma_{i}=1 \cap \gamma_{l}=1, \mathbf{y}\right)= & \frac{V\left(\beta_{i} \mid \gamma_{l}=1, \mathbf{y}\right)+\left[E\left(\beta_{i} \mid \gamma_{l}=1, \mathbf{y}\right)\right]^{2}}{p(i \cap l \mid \mathbf{y})}- \\
& -\left[E\left(\beta_{i} \mid \gamma_{i}=1 \cap \gamma_{l}=1, \mathbf{y}\right)\right]^{2} \tag{30}
\end{align*}
$$

The conditional standardized coefficient is obtained by dividing the conditional mean (29) by the corresponding conditional standard deviations

$$
\begin{equation*}
E\left(\beta_{i} / s_{i} \mid \gamma_{i}=1 \cap \gamma_{l}=1, \mathbf{y}\right) \equiv \frac{E\left(\beta_{i} \mid \gamma_{i}=1 \cap \gamma_{l}=1, \mathbf{y}\right)}{\sqrt{V\left(\beta_{i} \mid \gamma_{i}=1 \cap \gamma_{l}=1, \mathbf{y}\right)}} \tag{31}
\end{equation*}
$$

The same conditioning is used to calculate the conditional sign certainty probability for coefficient $\beta_{i}$, conditional on inclusion of variable $\mathbf{x}_{l}$

$$
\begin{equation*}
p\left(\operatorname{sign}_{i} \mid \gamma_{i}=1 \cap \gamma_{l}=1, \mathbf{y}\right)=\frac{p\left(\operatorname{sign}_{i} \mid \gamma_{l}=1, \mathbf{y}\right)}{p(i \cap l \mid \mathbf{y})} \tag{32}
\end{equation*}
$$

which may be compared to the marginal conditional expression (19). Note that in contrast to jointness which is symmetric $J_{i l}=J_{l i}$, the conditional standardized coefficients and sign certainty statistics are not symmetric, since conditional moments are not. For example, $E\left(\beta_{i} \mid \gamma_{i}=1 \cap \gamma_{l}=1, \mathbf{y}\right) \neq E\left(\beta_{l} \mid \gamma_{i}=1 \cap \gamma_{l}=1, \mathbf{y}\right)$.

### 3.1 Jointness and Posterior Inference

A policymaker can be confronted with several dimensions of model uncertainty, including partially overlapping theories and a potentially very large set of competing explanatory variables. These types of uncertainty have been used to motivate a model averaging approach to conducting policy inference. For example, Brock et al (2003, p.281f) argue that a policymaker, following a particular " $t$-statistic" decision rule based on a quadratic loss function, evaluates policies utilizing marginal standardized coefficients (17). However, there are limits to inference about the posterior distribution of a policy effect in a purely unconditional setting.

Although model uncertainty with respect to individual variables is taken into account, dependence among variables is not revealed. For example, when evaluating a given policy, it matters if the effect is constant over a population, or whether it varies according to one or more characteristics of the observations (e.g. countries or firms). Brock and Durlauf (2001) argue that this type of heterogeneity can be readily integrated within a model averaging context by including interactions with dummy variables. However, as the number of competing theories increases, accounting for dependencies in this manner requires increasingly specific and complex prior information.

Dependence among variables could also occur in situations where policies are not considered independently, but administered as a "package". ${ }^{14}$ If two or more policy variables or economic theories are highly complementary, then the use of simple marginal posterior objects, such as variable inclusion probability or standardized coefficients, may obscure such relationships. Alternatively, different theories might operate independently of one another in many economic applications, and Brock and Durlauf (2001) call this "open-endedness" of theories.

The jointness statistic presented in (25) can be viewed as a posterior odds ratio of jointly including or excluding variables relative to mutually exclusive inclusion. The relative posterior probabilities can be decomposed into relative prior probabilities and relative marginal likelihoods, the so-called Bayes factor. By assuming independent prior inclusion probabilities (9), we do not impose different prior probabilities on the joint or separate inclusion of variables. We can therefore use a classification of Bayes factors, similar to that suggested in the Bayesian literature, to assess the significance of jointness among variables (see Jeffreys, 1961, or Kass and Raftery, 1995).

| $J_{i l}<-2$ | strong substitutes |
| :---: | :--- |
| $-2<J_{i l}<-1$ | significant substitutes |
| $-1<J_{i l}<1 \quad$ if variables are | not significantly related |
| $+1<J_{i l}<+2$ | significant complements |
| $J_{i l}>+2$ | strong complements |

To consider in more detail the additional information revealed by jointness and related conditional statistics (standardized coefficients and sign certainty), we consider a number of hypothetical scenarios.

Suppose we find very little significant positive jointness among variables acting as policy instruments and controls, implying little complementarity. One might then conclude that little positive jointness is good for a policymaker, since the distribution of policy instruments and conditioning variables is less complex (and unknown). Alternatively, suppose we find significant negative jointness for a set of regressors. In this case policies act as substitutes in explaining the dependent variable, implying that collinearity and issues of prior distributions are important.

If policies are implemented based upon a "package" of measures, an analysis of marginal measures, such as $p(i \mid \mathbf{y})$, may be misleading. An example is a growth determinant that requires appropriate conditioning on a set of complementary explanatory variables (e.g. controls for initial conditions) to exert a significant effect. Suppose the marginal posterior inclusion probability $p(i \mid \mathbf{y})$ for a particular regressor $\mathbf{x}_{i}$ is relatively low, but the variable has significant positive jointness with other regressors $\mathbf{x}_{l}$. This suggests that $\mathbf{x}_{i}$ requires complementary variables $\mathbf{x}_{l}$ to explain the

[^11]dependent variable. In this case we would expect the sign certainty of the coefficient $\beta_{i}$ to increase when complementary variables are included in the regression. We may also wish to examine whether the conditional posterior standardized coefficient and sign certainty indicate a statistically and economically important effect of variable $\mathbf{x}_{i}$ given that we condition on the inclusion of variable $\mathbf{x}_{l}$.

Suppose the marginal posterior probability $p(i \mid \mathbf{y})$ of including variable $\mathbf{x}_{i}$ is high, but the variable has significant negative jointness with other regressors $\mathbf{x}_{l}$. This implies that variable $\mathbf{x}_{i}$ with high marginal inclusion probability is a close substitute for $\mathbf{x}_{l}$. An example, could be a "catch-all" variable that measures similar underlying characteristics of the dependent variable as other regressors. We would expect conditional normalized coefficients to be shrinking towards zero when variables measuring "similar" underlying concepts are included in the regression.

In the following empirical application to the determinants of economic growth in a cross-section of countries, we illustrate implications of jointness and related conditional statistics for statistical inference and economic policy decisions.

## 4 Jointness of Growth Determinants

This section presents results of applying jointness to the determinants of economic growth dataset used by SDM (2004). The explanatory variables are chosen from regressors found to be related to economic growth in earlier studies (see for example the list in Durlauf and Quah, 1999). SDM (2004) select variables that represent "state variables" in economic growth models and measure them as close as possible to the start of the sample period in 1960. Furthermore, the dataset is restricted to be balanced, i.e. without missing observations. Under these criteria the total number of explanatory variables is $K=67$ with observations for $T=88$ countries. ${ }^{15}$ The results are based on approximately 60 million randomly drawn regressions (see Computation Appendix A for details).

The dependent variable, average growth rate of GDP per capita between 1960-96, and the 67 explanatory variables are listed in the Data Appendix B. The appendix also present abbreviated variable names, variable descriptions, and sample mean and standard deviations. In the Data Appendix and Tables explanatory variables are ranked by marginal posterior inclusion probability $p(i \mid \mathbf{y})$ defined in equation (14), which is shown in the fourth column of the Data Appendix B. The posterior can be compared to the prior probability of inclusion, which in the benchmark case with prior model size $\bar{k}=7$ equals $\pi_{i}^{B A C E}=\bar{k} / K=7 / 67=0.104$. SDM (2004) call the 18 highest ranked explanatory variables with posterior inclusion probability

[^12]Figure 2: Jointness Among Selected Regressors

greater than the prior, "significantly" related to economic growth.
Table 1 shows the results of bivariate jointness, defined in equation (25), between the explanatory variables. As described in section 3, a negative jointness value indicates that two explanatory variables are substitutes in explaining economic growth, whereas a positive value indicates that they are complements. We call absolute values of jointness in excess of unity "significant", reflecting evidence from the posterior odds ratio. ${ }^{16}$ Figure 2 shows the relative frequency of (significant) negative or positive values of jointness among (i) all 67 explanatory variables and also selected subsets of regressors: (ii) the 21 top ranked, (iii) the top and (iv) the lowest ranked regressor, all ordered by marginal posterior inclusion probability $p(i \mid \mathbf{y})$.

We also report the conditional standardized coefficients (31) and the conditional sign certainty statistics (32) in Tables 2 and 3, respectively. Notice that these Tables contain conditional statistics associated with variable $\mathbf{x}_{i}$ (in rows numbered 1 to 67 ), conditional on inclusion of variable $\mathbf{x}_{l}$ (in columns numbered in the first row of each table). ${ }^{17}$

[^13]The main empirical findings are as follows. We find evidence of both positive and negative jointness among growth determinants. Jointness detects dependencies among regressors in an unconditional sense (across many possible growth models) and therefore differs from simple correlation measures which are independent of the models under consideration. We find no simple relationship between jointness and bivariate correlations. ${ }^{18}$

Instances of Significant positive jointness (with $J_{i l}>1$ ) are not restricted to variables considered "significant" by the marginal posterior inclusion probability exceeding the prior inclusion probability. In this respect a number of growth determinants benefit from inclusion of complementary variables in explaining economic growth and become significant determinants of economic growth, conditional on the joint inclusion of the complimentary variables in the regression.

Significant negative jointness (with $J_{i l}<-1$ ) is concentrated among "significant" variables (again measured by marginal posterior inclusion probability) and other explanatory variables, including significant variables themselves. This indicates that only variables with relatively high inclusion probability in explaining growth are flagged as significant substitutes for other variables. Variables with posterior inclusion probability smaller than the prior inclusion probability do not exhibit significant negative jointness with variables outside the top 18 regressors.

Conditional standardized coefficients behave qualitatively similar to the jointness results. Negative jointness is generally associated with smaller conditional standardized coefficients (in absolute value terms) given that the inclusion of substitutes tends to shrink standardized coefficients towards zero. Positive jointness gives the opposite results: the inclusion of complementary variables strengthens standardized coefficients and increases their importance in a conditional sense. These findings are potentially very important for a policymaker interested in the size and significance of variables, conditional on a set of controls or other policies.

The conditional sign certainty statistic follows the same qualitative pattern as standardized coefficients: positive (negative) jointness is associated with higher (lower) conditional sign certainty of effects. This is not surprising, given that the sign certainty averages $t$-statistics across models, whereas standardized coefficients are calculated from averaged coefficient estimates and their standard deviations.

The following two subsections discuss these findings in greater detail. We conclude the section by examining the sensitivity of jointness with respect to prior model size $\bar{k}$, prior distribution of regression errors and heteroscedasticity, alterna-
other entries, that are conditional on inclusion of variables $\mathbf{x}_{i}$ and $\mathbf{x}_{l}$, the diagonal entries are also set in boldface in Tables 2 and 3. Notice that the last column for Interior Density ranked lowest (67th) by marginal posterior inclusion probability is not shown in Tables 2 and 3, but is available on request. Conditioning on this variable does not affect inference about the other regressors.
${ }^{18}$ Bivariate correlations are not reported due to space constraints, but are available at http://www.econ.cam.ac.uk/faculty/doppelhofer/research/Jointness.html.
tive AIC model weights and the inclusion of multiplicative interaction among some of the original regressors.

### 4.1 Positive Jointness and Complements

We find evidence of significant positive jointness among a broad set of variables, including some with unconditional posterior inclusion probability lower than the prior. In the following discussion we focus on variables whose standardized coefficient (or sign certainty) indicates a significant effect on economic growth once we condition on a complementary variable. In such cases a policymaker would consider the variable statistically and economically important in a conditional sense. We find evidence of significant positive jointness among the following groups of variables:

1. Initial Conditions: Primary School Enrolment and the Price of Investment Goods are significant complements with each other and also with Coastal Density. For example, Primary School Enrolment and the Price of Investment Goods have a value of jointness of 1.53 in Table 1. Since this value exceeds one we conclude that these two variables exhibit significant complementarity. The jointness values in Table 1 are calculated from joint and marginal inclusion probabilities as we illustrate with the following example.
The joint posterior inclusion probability for Primary School Enrolment (P60) and Price of Investment Goods (IPRICE1) equals $p($ P60,IPRICE1 $\mid \mathbf{y})=0.67$. The marginal inclusion probabilities are $p(\mathrm{P} 60 \mid \mathbf{y})=0.80$ and $p($ IPRICE1 $\mid \mathbf{y})=$ 0.77 , respectively. The probabilities of mutually exclusive inclusion are given by $p(\mathrm{P} 60 \cap n o t \operatorname{IPRICE} 1 \mid \mathbf{y})=p(\mathrm{P} 60 \mid \mathbf{y})-p(\mathrm{P} 60 \cap \operatorname{IPRICE1} \mid \mathbf{y})=0.80-0.67=$ 0.13 , and $p$ (IPRICE1 $\cap$ not P60|y) $=0.77-0.67=0.10$; by the addingup constraint, the probability of joint exclusion equals $p$ (not IPRICE1 n not $\mathrm{P} 60 \mid \mathbf{y})=0.10$. We can therefore calculate jointness by substituting for the probabilities in equation (25):

$$
\begin{equation*}
J_{\text {P60,IPRICE1 }}=\ln \left(\frac{0.67 \cdot 0.10}{0.10 \cdot 0.13}\right)=1.53 \tag{33}
\end{equation*}
$$

Note that the jointness statistic can also be written as the logarithm of the product of two posterior odds ratio (26): the posterior odds ratio of including P60 conditional on inclusion of the IPRICE1, $P O_{\text {P60|IPRICE1 }}=6.22$, times the posterior odds of excluding P60 conditional on exclusion of the IPRICE1, $P O_{\text {not P60|not IPRICE1 }}=0.75$. Jointness therefore captures dependence over the entire posterior distribution, and the decomposition of the joint distribution into inclusion and exclusion margin can shed further light on the source of dependence.
Primary school enrolment is also a significant complement with the Fraction
of Tropical Area and Coastal Density, indicating that conditional on these geographic characteristics the effect of primary education is more important in explaining economic growth (and vice versa). Initial Income is complementary with the Population Density, implying a significant conditional effect of the latter on growth. Life Expectancy has a complementary relationship with Initial Income, the Fraction of GDP in Mining and Fraction Speaking Foreign Language, implying that the sizes and significance of the associated coefficients are strengthened conditional on controlling for Life Expectancy.
The posterior coefficient of Initial Income conditional on being included with other variables ranges from -0.0104 to -0.0073 , similar the marginal effect -0.0085 shown in Table 2 of SDM (2004). ${ }^{19}$ The implied "speed of convergence" ranges from $0.8 \%$ to $1.3 \%$ which is lower than the two percent figure usually found in cross-sectional growth regressions. This can be explained by the systematic control for alternative growth determinants by the model averaging procedure.
2. Geography: The East Asian dummy is a significant complement with the Fraction of Tropical Area and Malaria Prevalence. Conditional on joint inclusion with the East Asian dummy, the size of the standardized coefficients of the complementary variables (Tropical Area and Malaria Prevalence) is larger in absolute value. Tropical Area in turn is complementary with Coastal Density. Their joint inclusion reinforces the negative coefficient of Tropical Area and the positive effect of Coastal Density, which is typical for complementary variables. ${ }^{20}$
3. Disease Environment and Colonial History: Malaria Prevalence and the former Spanish Colony dummy have a highly complementary relationship $\left(J_{\text {MALFAL66,SPAIN }}=2.85\right)$, strengthening the negative sign of the associated standardized coefficients from the marginal size of -2.56 to a conditional size of -3.68 in the case of Malaria Prevalence, and from -2.13 to -2.80 for the Spanish colony in Table 2. Based on marginal results, we might conclude that former Spanish colonies with high Malaria prevalence would have significantly lower growth rates. However, the respective negative effects are reinforced when these two variables enter the model jointly. ${ }^{21}$

[^14]4. Cultural Variables: The Fraction Confucian, the Sub Saharan Africa as well as the Latin America dummies, are significant complements with one another. The Fraction Confucian is also complementary to the population Fraction Buddhist and Muslim, and the latter is also complementary with the share of Primary Exports and the Fraction Buddhist and Hindu. This pattern of complementarity indicates that these regional and cultural variables should be jointly included in the model and form a "conditioning set" in explaining growth. As pointed out earlier, the East Asian dummy is a substitute for all variables in this conditioning set of variables. If not included in the regression model, a larger set of the conditioning dummy variables is suggested. The Fraction Protestant is complementary with the Spanish Colony dummy and Fraction Catholic. In all cases, positive (complementary) jointness strengthens the standardized coefficients in Table 2 and increases the sign certainty in Table 3, conditional on including the significant complementary variables.
5. Sectoral and Macroeconomic Variables: The Mining Share of GDP has significant positive jointness with Real Exchange Rate Distortions and the Public Investment Share in GDP. The coefficients of Public Investment Share and the Government Share are significantly negative conditional on joint inclusion with geographic dummies for Sub Saharan Africa and Latin America. Also, the share of Primary Exports has a significant negative sign conditional on the Fraction Muslim. The signs of the effects on economic growth are reinforced when conditioning on complementary variables, implying a stronger positive effect of the Mining share and a more negative effect of the Real Exchange Rate Distortions, the Government Share, Primary Exports and Public Investment Share on economic growth. These conditional estimates could be of interest to policymakers, in particular if a country has a relatively important Mining industry.
6. Geography and Population Density: Air Distance to Big Cities has a significant complementary relationship with the Population Density, implying a more negative and significant effect of longer distance conditional on controlling for population density and a positive effect of density controlling for air distance. The Fraction of Population in Tropics and Fraction of Land Area Near Navigable Water are significant complements in explaining growth. Table 2 shows that the standardized negative size of the coefficient for proximity to Navigable Water is strengthened from -0.46 unconditionally to -1.69 conditional on controlling for the Tropical Population (the corresponding conditional sign certainty in Table 3 equals 0.95 , compared to 0.66 unconditionally).
7. Political Institutions: Variables measuring Political Rights and Civil Lib-
erties are significant complements and also have a high correlation coefficient -0.83 . Neither variable on its own is significantly related to economic growth, but conditional on joint inclusion the positive effect ${ }^{22}$ of Political Rights and negative effect of Civil Liberties are reinforced. It appears that conditional on joint inclusion, the two variables measure different aspects of how political institutions effect economic growth in a cross-section of countries.

So what can an analyst or policymaker learn from positive jointness? First, we find that some growth determinants require a set of conditioning variables to jointly explain growth across countries. This appears to be true in particular for some variables measuring geographic and cultural differences across countries, capturing heterogeneity of cross-country growth rates. Second, some positively related variables capture different aspects of the cross-country growth mechanism, and provide a differentiated explanation. Examples include the Mining Share and Public Investment Share or Real Exchange Rate Distortions and Terms of Trade Growth. When implementing a particular policy, a policymaker might want to take these dependencies into account.

To inform policy makers we recommend to look at instances where the size of the standardized coefficients conditional on inclusion of a complementary variable becomes significant or alternatively the conditional sign certainty indicates robustness. Given the number of growth determinants under consideration and the large number of potential dependencies, the number of cases of significant positive jointness and changes of size and significance of coefficients is relatively small.

### 4.2 Negative Jointness and Substitutes

Negative jointness implies that growth determinants act as substitutes for each other and as a consequence have lower posterior probability of appearing jointly relative to the respective marginal inclusion probabilities. Interestingly, significant negative jointness among growth determinants is limited to variables with high marginal posterior inclusion probability. Only variables with high marginal posterior weight (relative to prior inclusion probability) are significantly affected by joint inclusion with other variables capturing "similar" aspects of the cross-country variation in economic growth. The standardized coefficients shown in Table 2 tend to be smaller in absolute value conditional on the joint inclusion of other variables exhibiting negative jointness, compared to marginal effects on the diagonal. Similarly, the sign certainties in Table 3 tend to fall conditional on the joint presence of substitutable variables. There are several groups of variables exhibiting significant negative jointness:

[^15]1. Geography and Cultural Variables: The East Asian dummy has significant negative jointness with several other regional and cultural variables. The East Asian Dummy is also negatively associated with other important growth determinants, indicating that the East Asian dummy acts as a "catch-all" variable that looses significance once other regional and cultural variables are included as regressors. ${ }^{23}$
2. Initial Conditions: Primary School Enrollment and Investment Price exhibit significant negative jointness with Malaria Prevalence. Primary School Enrollment is a significant substitute for Life Expectancy, the Government Consumption Share, and Real Exchange Rate Distortions. The latter variable is also a significant substitute for the Investment Price. Initial income has negative jointness with Higher Education Enrollment, the Fraction Protestants and the Fraction of the Population Less than 15. Conditional on joint inclusion of initial income, these variables become even less important in explaining growth.
3. Geography and Disease Environment: The Fraction of Tropical Area and the Sub Saharan Africa dummy show significant negative jointness with Malaria Prevalence, consistent with unfavorable disease environments present in these regions ${ }^{24}$. Malaria Prevalence has also significant negative jointness with Coastal Population Density and overall Population Density, showing interesting dependence between adverse disease environment captured by Malaria Prevalence and population density. The Fraction of Tropical Area exhibits significant negative jointness with the Latin America and Spanish Colony dummy, the latter variable exhibiting negative jointness with Coastal Population Density, reflecting geographic characteristics of former Spanish colonies in Latin America. Perhaps more interesting is the finding that the Fraction of Tropical Area is also a substitute for Primary Exports.
4. Sectoral Variables and Geography: The Fraction of GDP in Mining exhibits strong negative jointness with Population Density. Coastal Population Density and the share of Exports and Imports in GDP act as significant substitutes. Countries with high coastal density are likely to have more opportunities to trade leading to higher trade shares of GDP. This finding can be interpreted as a version of the "gravity equation" in the empirical trade literature.
5. Government Spending: Government Consumption Share and Government
[^16]Share of GDP are significant substitutes for one another, reflected also in the very high correlation 0.93 . An analyst might conclude that these two variables measure very similar aspects of government spending, adding little information when being jointly included in the regression. Table 2 shows that the standardized posterior coefficient for the Government Share of GDP switches sign (to positive) conditional on joint inclusion with Government Consumption Share. The sign certainty reported in Table 3 falls for both variables conditional on joint inclusion: from 0.97 to 0.87 for the Government Consumption Share, and from 0.93 to 0.71 for the Government Share of GDP.
6. Political Institutions, Geography and Disease Environment: Political Rights exhibits negative jointness with Malaria Prevalence, Life Expectancy, and dummy variables for Latin America, former Spanish Colony and (marginally) Sub-Saharan Africa. We conclude that Political Rights as a measure of political institutions seem to be closely related to other important determinants of growth measuring disease environment or geographic characteristics (see Acemoglu, Johnson and Robinson, 2001).

From observing the posterior distribution of coefficients and the negative jointness described above, a policymaker can learn about groups of variables acting as substitutes for one another in explaining economic growth across countries. In some cases, negative jointness can be spotted upfront looking at simple correlations. However, jointness is more general and allows to uncover dependencies in an unconditional sense after averaging across models. The estimated jointness statistic could be used to "dilute" the weight on variables capturing similar economic phenomena. As advocated by Leamer (1973), one can also use additional prior information (based on economic theory, for example) to interpret the jointness results further.

### 4.3 Sensitivity Analysis of Jointness

This section investigates the robustness of results of jointness obtained using the benchmark case presented in section $2 .{ }^{25}$ We analyze the sensitivity of results by considering the impact of different prior model sizes $\bar{k}$, prior distributions of errors, alternative (AIC) model weights, and the addition of multiplicative interaction terms as regressors.

### 4.3.1 Sensitivity to Prior Model Size

We investigate the sensitivity of jointness with respect to the prior model size $\bar{k}$. We calculate jointness for different prior model sizes $(\bar{k}=4,14,21,28)$ and contrast

[^17]Figure 3: Robustness to Prior Model Size: Top 21 Regressors


Figure 4: Robustness to Prior Model Size: Regressors Ranked 22-67

the results with the benchmark case with $\bar{k}=7$. Figure 3 plots the densities of jointness for the top 21 regressors for these different prior model sizes. Figure 4 shows the densities for the remaining regressors, ranked 22 to 67 by marginal posterior inclusion probabilities. In both cases we observe that the degree of dependence among explanatory variables weakens with increasing prior model size. Larger prior model size $\bar{k}$ implies a smaller penalty of adding regressors in the posterior model weights (11), which implies a richer set of other conditioning variables included in each model $M_{j}$. This reduces both negative and positive jointness for any particular combination of variables, but comes at the price of allowing for a richer set of control variables and interdependence within each regression model.

### 4.3.2 Sensitivity to Prior Distribution of Errors

The results presented in sections 4.1 and 4.2 above are based on the benchmark case which assumes that the residuals in the linear regression model (1) are normally distributed and conditionally homoscedastic. To accommodate possible outliers and unequal variances, we allow for independently distributed, but heteroscedastic errors, $\varepsilon \sim N\left(\mathbf{0}, \sigma^{2} \boldsymbol{\Omega}\right)$. The covariance matrix is assumed diagonal, $\boldsymbol{\Omega} \equiv$ $\operatorname{diag}\left(w_{1}, \ldots, w_{T}\right)$ with independent variances $w_{t}, t=1, \ldots, T$. Geweke (1993) proposes a Chi-squared distribution with degrees of freedom $v$ for $\left(w_{1}, \ldots, w_{T}\right)$ :

$$
\begin{equation*}
v / w_{t} \sim \chi^{2}(v), \quad t=1, \ldots, T \tag{34}
\end{equation*}
$$

Lower values of $v$ imply a more skewed distribution with a higher probability of outliers and relatively larger variances. High values of $v$ on the other hand imply errors drawn from a distribution close to the homoscedastic normal benchmark case described in section 2. Geweke (1993) shows that the normal mixture model with independence prior (34) is equivalent to a model with independent Student$t$ errors with $v$ degrees of freedom. Intuitively, the degrees of freedom parameter $v$ determines the fatness of the tails of the Student- $t$ distribution and the prior weight on outliers. The parameters of interest conditional on each model $M_{j}$ are estimated by generalized least squares (GLS) which downweights observations by their variances.

In examining the robustness of the benchmark results, this section introduces a proper Normal prior for the slope coefficient $\beta$. The prior for slope coefficients is still given by (5). The prior variance $\mathbf{V}_{0 j}$ takes the form of the $g$-prior (6). However, following Fernandez et al. (2001b), we set the hyper-parameter $g_{0}$ equal to $1 / T$ where $T$ is the number of observations. ${ }^{26}$ The non-informative priors for the common scale component of the error variance $\sigma^{2}(7)$ and the intercept $\alpha$ (8) are maintained.

[^18]These prior assumptions imply the following convenient hierarchical structure: ${ }^{27}$

1. The posterior distribution of the slope parameters $\beta$ in model $M_{j}$ conditional on other parameters is given by

$$
\begin{equation*}
p\left(\beta \mid \sigma^{2}, \boldsymbol{\Omega}, M_{j}\right) \sim N\left(\tilde{\beta}_{j}, \sigma^{2} \tilde{\mathbf{V}}_{j}\right) \tag{35}
\end{equation*}
$$

The conditional mean of $\beta$ is estimated using the generalized least squares (GLS) estimator (denoted by a " $\sim$ ")

$$
\begin{equation*}
\tilde{\beta}_{j}=\tilde{\mathbf{V}}_{j}\left(\mathbf{X}_{j}^{\prime} \mathbf{\Omega}^{-1} \mathbf{y}\right) \tag{36}
\end{equation*}
$$

The posterior variance of $\beta$ is given by $\sigma^{2} \tilde{\mathbf{V}}_{j}$, where

$$
\begin{equation*}
\tilde{\mathbf{V}}_{j}=\left(\mathbf{X}_{j}^{\prime} \boldsymbol{\Omega}^{-1} \mathbf{X}_{j}+\sigma^{2} \mathbf{V}_{0 j}^{-1}\right)^{-1} \tag{37}
\end{equation*}
$$

2. The posterior distribution of the error variance parameter $\sigma^{2}$ conditional on the other parameters is given by

$$
\begin{equation*}
\left[\sum_{t=1}^{T}\left(e_{j, t}^{2} / w_{t}\right) / \sigma^{2}\right] \mid(\beta, \boldsymbol{\Omega}) \sim \chi^{2}(T) \tag{38}
\end{equation*}
$$

where $e_{j, t}=y_{t}-\mathbf{x}_{j, t}^{\prime} \beta_{j}$. Notice that the degrees of freedom equal $T$ and not $T-k_{j}$, since we condition on $\beta_{j}$.
3. The posterior distribution of the elements of the error variance matrix $\Omega$ conditional on the other parameters is proportional to

$$
\begin{equation*}
\left[\left(\sigma^{-2} e_{j, t}^{2}+v\right) / w_{t}\right] \mid\left(\beta_{j}, \sigma^{2}\right) \sim \chi^{2}(v+1), \quad t=1, \ldots, T \tag{39}
\end{equation*}
$$

The $v+1$ degrees of freedom follow from combining the prior distribution of $w_{t}$ (34) with $v$ degrees of freedom with terms $\sigma^{-2} e_{j, t}^{2} / w_{t}$ from the likelihood function with a $\chi^{2}(1)$ kernel.
4. Values of the degrees of freedom parameter $v$ are drawn from a Gamma prior distribution

$$
\begin{equation*}
v \sim \Gamma(a, b) \tag{40}
\end{equation*}
$$

where $a$ and $b$ are hyper-parameters. We investigate the sensitivity of the jointness results from the benchmark case with diffuse (sample-dominated) priors by comparing them with results using proper Bayesian priors. The values of the hyper-parameters $a$ and $b$, with mean $E(v)=a b$ and variance $\operatorname{Var}(v)=a b^{2}$ implied by the prior Gamma distribution (40), are as follows:

[^19]| $a$ | 1000 | 500 | 100 | 20 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $b$ | $1 / 10$ | $1 / 10$ | $1 / 4$ | $1 / 2$ | $1 / 2$ |
| $E(v)$ | 100 | 50 | 25 | 10 | 4 |
| $\operatorname{Var}(v)$ | 10 | 10 | 6 | 5 | 2 |

The hierarchical structure of steps 1 to 4 leads naturally to estimation by the Gibbs sampler. Starting from initial values, the Gibbs sampler estimates parameters iteratively by drawing from the conditional posterior distributions of parameters, given by (35)-(37) for mean and variance of the slope coefficient $\beta$, (38) and (39) for the error variance $\sigma^{2} \Omega$, and (40) for the degrees of freedom $v .^{28}$

Conditional on $\boldsymbol{\Omega}$ and the proper normal-inverse Gamma prior, the marginal likelihood of model $M_{j}$ is proportional to

$$
\begin{equation*}
l\left(\mathbf{y} \mid M_{j}\right) \propto\left(\frac{\left|\overline{\mathbf{V}}_{j}\right|}{\left|\mathbf{V}_{0 j}\right|}\right)^{1 / 2}\left(T \cdot \bar{\sigma}^{2}\right)^{-T / 2} \tag{41}
\end{equation*}
$$

where $\overline{\mathbf{V}}_{j}$ is the posterior variance of the slope coefficient $\beta_{j}$, and $\bar{\sigma}^{2}$ is the common term of the posterior error variance. In the calculation of the marginal likelihood (41), we use the analytical expression of the posterior variance $\overline{\mathbf{V}}_{j}^{a}$ defined in equation (44) of the Computational Appendix A. The marginal likelihood (41) combined with the independence prior model probabilities $p\left(M_{j}\right)$ give the posterior model weights (4) that we use for model averaging.

We examine the robustness of jointness with respect to the following deviations from the benchmark prior structure. First, we compare the results and inference for bivariate jointness under the benchmark case with sample-dominated priors and proper Bayesian priors with "nearly" normally distributed priors. ${ }^{29}$ We find a strong qualitative agreement between the two cases such that inference about jointness is not much affected. In terms of positive and negative significant jointness (defined as $|J|>1$ ) the two cases disagree in 2.1 and 1.3 percent of all cases, respectively (see following Table). In terms of even stronger positive or negative jointness (defined as $|J|>2)$ they disagree in only 0.3 and 0.1 percent of cases. We therefore conclude that our benchmark results are not much affected by using proper Bayesian priors.

[^20]| $\begin{gathered} \text { Jointness } \\ J_{i l} \end{gathered}$ | Mean degrees of freedom $E(v)$ $(\operatorname{Var}(v)$ in parenthesis below) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & 100 \\ & (10) \end{aligned}$ | $\begin{gathered} 50 \\ (10) \end{gathered}$ | $\begin{aligned} & 25 \\ & (6) \end{aligned}$ | $\begin{aligned} & 10 \\ & (5) \end{aligned}$ | 4 <br> $(2)$ |
| $>+2$ | 0.3 | 0.8 | 1.3 | 1.1 | 2.1 |
| $>+1$ | 2.1 | 4.5 | 5.0 | 5.7 | 7.6 |
| $<-1$ | 1.3 | 6.6 | 8.4 | 4.8 | 10.6 |
| $<-2$ | 0.1 | 0.5 | 1.4 | 0.6 | 3.7 |

Second, we also contrast the estimates of bivariate Jointness under the (normal) benchmark case with models that allow for heteroscedastic errors in the regression models. The errors are drawn from $t$-distributions with degrees of freedom $v=$ $4,10,25,50$ and 100 . Lower values of $v$ imply fatter tails of the $t$-distribution allowing for more extreme outlying observations of the regression errors. We observe that the simple correlation of instances of significant jointness (defined as pairs with $\left|J_{i l}\right|>1$ ) falls as $v$ decreases and we assume more heteroscedastic errors a priori. Also a more heteroscedastic prior distribution of errors (smaller value of $v$ ) implies more cases of significant jointness. However, there are few qualitative differences, measured in terms of percentage deviations of significant instances of jointness as shown in the Table above. This indicates that the effect on the nature of inference when we allow for considerable heteroscedasticity of the regression errors is relatively small.

### 4.3.3 Sensitivity to AIC Posterior Model Weights

We consider the effect on jointness of allowing for alternative posterior model weights. In particular, the AIC model weights developed by Akaike (1973) have been suggested as alternatives in the model averaging literature. ${ }^{30}$ The AIC model selection criterion is given by $A I C \equiv l\left(\mathbf{y} \mid M_{j}\right)-k_{j}$, where $l\left(\mathbf{y} \mid M_{j}\right)$ is model-specific marginal likelihood and $k_{j}$ is the number of regressors. Exponentiating, normalizing by the sum over all $2^{K}$ models and ignoring constants gives the AIC model weights:

$$
\begin{equation*}
p^{A I C}\left(M_{j} \mid \mathbf{y}\right)=\frac{p\left(M_{j}\right) \cdot e^{-k_{j}} \cdot S S E_{j}^{-T / 2}}{\sum_{r=1}^{2^{K}} p\left(M_{r}\right) \cdot e^{-k_{r}} \cdot S S E_{r}^{-T / 2}} \tag{42}
\end{equation*}
$$

The difference between the AIC model weights (42) and the posterior BIC model weights (11) used in the benchmark case is the degrees of freedom penalty term, $e^{-k_{j}}$ instead of $T^{-k_{j} / 2}$. Given $T=88$ observations in this application, the AIC weights penalize larger models less severely than the benchmark model weights. The effect of using the AIC model weights is therefore similar to accommodating a relatively

[^21]larger prior model size with $\bar{k}>7$ - jointness among regressors is generally less pronounced.

In section 4.3.1 we found that jointness is sensitive to changes in the prior model size and that dependence among explanatory variables is reduced in larger models. This result is not surprising given the richer set of conditioning variables permitted in larger models. This result is further underlined when using a version of AIC model weights that penalizes the inclusion of additional regressors less severely than the benchmark posterior BIC model weights.

### 4.3.4 Sensitivity to Multiplicative Interaction Terms

Finally, we analyze the effect of adding additional multiplicative interaction terms as regressors for variables that exhibit significant positive jointness (complementarity) in the benchmark case shown in Table 1. In particular, we add two interaction terms, one for (Malaria Prevalence $\times$ Spanish Colony) with $J_{\text {MALFAL66,SPAIN }}=2.93$, and the other for (Fraction Protestant $\times$ Fraction Catholic) with $J_{\text {Protoo, } \text { Сathoo }}=2.66$. The number of candidate regressors increases therefore to 69, and we leave the prior model size unchanged at the benchmark value $\bar{k}=7$.

The two new interaction terms do not exhibit significant jointness with others variable in the original set, except for positive jointness between the (Malaria Prevalence $\times$ Spanish Colony) and the East Asian dummy, the Spanish Colony dummy and negative jointness with the Sub-Saharan Africa dummy. We observe that the jointness between the originally significant complements Malaria Prevalence and Spanish Colony exhibits a small change, from 2.93 in the benchmark case to 3.02 when the interaction terms are added to the regressors. Similarly, when the multiplicative interaction terms are included the significant positive jointness between the population Fractions Protestant and Catholic is marginally increased from 2.66 to 2.72 .

For the remaining variables adding the two interaction terms as candidate regressors acts much like a reduction in prior model size, i.e. we observe a slightly higher incidence of negative jointness. For some variables, e.g. some geographic variables, positive dependence is also strengthened with the addition of the interaction terms as regressors.

## 5 Conclusion

This paper proposes a new measure of dependence or jointness among explanatory variables in regressions. Jointness differs from existing approaches in two respects. First, jointness among regressors is calculated unconditionally by averaging across models, thereby fully addressing model uncertainty. Second, jointness emphasizes dependence among explanatory variables in the posterior distribution without introducing any prior information. Positive values of jointness imply that variables are complements, representing distinct, but mutually reinforcing economic effects. Negative values of jointness imply that variables are substitutes and measure similar underlying mechanisms.

We estimate jointness among 67 determinants of economic growth, using crosscountry data from SDM (2004). We find evidence of complementary relationships in the form of significant positive jointness among a wider set of growth determinants. In contrast to negative jointness, significant positive jointness is also present among some variables which would be labeled "insignificant" by marginal measures of variable importance. Such variables require a richer conditioning set of complementary regressors in explaining growth.

For example, Air Distance to Big Cities and Population Density have significant positive jointness and act as complements in explaining economic growth. In contrast to negative jointness, complementary variables showing positive jointness reinforce the size and significance of their mutual effect on economic growth. We find evidence of significant negative jointness among some relatively important growth determinants, indicating that these variables are substitutes in explaining cross-country economic growth. For example, Initial Income has significant negative jointness and is a substitute for Higher Education Enrolment, the Fraction Protestants among the population and the Fraction of the Population Less than 15.

Compared to the possibly very large number of dependencies among growth determinants, we find a relatively small number of significant positive jointness or complementarity. This implies that policy decisions and inference are not becoming too complex, even when taking jointness into account. Our jointness results can inform a policymaker about potentially important heterogeneity of effects and interdependence between policy instruments and control variables. The concept and analysis of jointness is likely to be an important tool of use for inference and economic decision making in applications where dependence across explanatory variables can play an important role.

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## A Computational Appendix

This appendix discusses some of the details of the sampling and model averaging procedure. First, we discuss the random and stratified sampling procedures for the benchmark case presented in section 2. Second, we discuss details of the Gibbs sampler used in section 4.3.2.

## A. 1 Random and Stratified Sampling

For the benchmark normal case of sections $2,4.1$ and 4.2 , we can analytically calculate the marginal likelihood (10) and OLS estimates of mean and variance of $\beta_{j}$ conditional on each model $M_{j}$. The random sampler therefore draws directly from the posterior distribution of $p(\beta \mid \mathbf{y})$. With $K=67$ regressors the number of possible regression models equals $2^{67} \approx 1.48 \times 10^{20}$. Each regression takes approximately 0.0005 seconds using GAUSS on a recent PC. An exhaustive search over all models is therefore not feasible. Instead we draw from the posterior distribution and related parameters of interest until the parameters have converged numerically (see convergence criteria below).

The stratified sampler introduced by SDM (2004, Technical Appendix) allows for sampling inclusion probabilities to differ from prior inclusion probabilities $\pi=$ $\bar{k} / K$. After every 100,000 regressions, the sampling inclusion probability for each variable $\pi_{i}^{S}$ are set equal to a weighted average (with weight 0.5 in the benchmark case) of the initial inclusion probability and the posterior inclusion probability (4) estimated on those runs. To avoid sampling only a very small set of variables, the sampling inclusion probabilities are restricted to lie in the interval $[0.1,0.85]$. The stratified sampler over-samples models that include variables with high inclusion probability which greatly speeds up numerical convergence. To correct for differences in sampling probabilities of different regressors, we scale the posterior weights by the ratio of prior to sampling probabilities $\pi / \pi_{i}^{S}$ for $i=1, \ldots, K$.

To check the numerical convergence in the benchmark case, we monitor changes in the posterior mean of the slope coefficient $\beta$, normalized by the ratio of the standard deviation of $\mathbf{y}$ to the standard deviation of $\mathbf{X}$. Numerical convergence is achieved if the maximum change of the normalized coefficients falls below $10^{-6}$ for ten consecutive sets of 10,000 runs.

## A. 2 Gibbs Sampling

For the mixture-normal case of section 4.3.2, analytic expressions of the marginal likelihood are not available. However, we can make a sufficiently large number of draws from the conditional posterior distributions (35)-(37) for mean and variance of the slope coefficient $\beta$, (38) and (39) for the error variance $\sigma^{2} \Omega$, and (40) for the degrees of freedom $v$. Under certain regularity conditions the chain of draws from
the Gibbs sampler converges to the posterior distribution as the number of draws becomes large (see for example Chib, 2001).

Let $s=1, \ldots, S$ be the number of draws from the posterior distributions. We discard $S_{0}$ burn-in draws and estimate the parameters using the remaining $S_{1}=$ $S-S_{0}$ draws.

- The posterior mean $\bar{\beta}_{j} \equiv E\left(\beta_{j} \mid M_{j}, \mathbf{y}\right)$ is estimated by $\frac{1}{S_{1}} \sum_{s=S_{0}+1}^{S} \tilde{\beta}_{j, s}$ using draws from (35).
- The posterior variance matrix $\overline{\mathbf{V}}_{j} \equiv \operatorname{Var}\left(\beta_{j} \mid M_{j}, \mathbf{y}\right)$ can be calculated either numerically ( n ) or analytically (a) using

$$
\begin{align*}
& \overline{\mathbf{V}}_{j}^{\mathrm{n}}=\frac{1}{S_{1}} \sum_{s=S_{0}+1}^{S} \tilde{\beta}_{j, s}^{2}-\left[E\left(\beta_{j} \mid M_{j}, \mathbf{y}\right)\right]^{2}  \tag{43}\\
& \overline{\mathbf{V}}_{j}^{\mathrm{a}}=\left(\mathbf{X}_{j}^{\prime} \bar{\Omega}^{-1} \mathbf{X}_{j}+\bar{\sigma}^{2} \mathbf{V}_{0 j}^{-1}\right)^{-1} \tag{44}
\end{align*}
$$

$\bar{\sigma}^{2} \equiv E\left(\sigma^{2} \mid M_{j}, \mathbf{y}\right)$ is estimated by $\frac{1}{S_{1}} \sum_{s=S_{0}+1}^{S} \sigma_{s}^{2}$ using draws from (38).

- The posterior error variance $\overline{\boldsymbol{\Omega}} \equiv E\left(\boldsymbol{\Omega} \mid M_{j}, \mathbf{y}\right)$ is estimated using draws from (39)

$$
\begin{equation*}
\frac{1}{S_{1}} \sum_{s=S_{0}+1}^{S} \Omega_{s}=\frac{1}{S_{1}} \sum_{s=S_{0}+1}^{S} \operatorname{diag}\left(w_{1, s}, \ldots, w_{T, s}\right) \tag{45}
\end{equation*}
$$

We combine the estimates from the Gibbs sampler for the conditional mean, $\bar{\beta}_{j}$ and conditional variance, $\bar{\sigma}^{2} \overline{\mathbf{V}}_{j}$ with posterior model weights (41) to estimate the posterior mean and variance unconditionally over the model space $\mathcal{M}$.

To check for numerical convergence of model averaging with Gibbs sampling, we observe that for a large enough number of draws $S$ from the Gibbs sampler, the numerical (43) and analytic (44) estimates of the posterior variance should be similar. By default, we use 260 draws from the Gibbs sampler (excluding 10 draws for burn-in) and 200,000 draws from the model averaging loops. ${ }^{31}$ We use the difference between posterior standardized coefficients using the numerical and analytic posterior variance as numerical convergence criterion in this case. In particular, we choose the number of Gibbs draws $S^{*}$, so that

$$
\begin{equation*}
\left|E\left(\beta_{i}\right) / \sqrt{V^{\mathrm{n}}\left(\beta_{i}\right)}-E\left(\beta_{i}\right) / \sqrt{V^{\mathrm{a}}\left(\beta_{i}\right)}\right|_{S=S^{*}}<0.1, \quad i=1, \ldots, K \tag{46}
\end{equation*}
$$

[^22]
## B Data Appendix

| Rank | Short Name | Variable Description | $p(i \mid \mathbf{y})$ | Mean | S.D. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Depend. Variable | GROWTH | Average Growth Rate of PPP-adjusted GDP per Capita between 1960-1996 | - | 0.0182 | 0.019 |
| 1 | EAST | East Asian Dummy | 0.828 | 0.1136 | 0.3192 |
| 2 | $\mathrm{P} 60$ | Primary Schooling Enrollment | 0.796 | 0.7261 | 0.2932 |
| 3 | IPRICE1 | Investment Price | 0.774 | 92.47 | 53.68 |
| 4 | GDPCH60L | Initial Income (Log GDP in 1960) | 0.685 | 7.3549 | 0.9011 |
| 5 | TROPICAR | Fraction of Tropical Area | 0.570 | 0.5702 | 0.4716 |
| 6 | DENS65C | Population Coastal Density | 0.433 | 146.87 | 509.83 |
| 7 | MALFAL66 | Malaria Prevalence | 0.251 | 0.3394 | 0.4309 |
| 8 | LIFE060 | Life Expectancy | 0.210 | 53.72 | 12.06 |
| 9 | CONFUC | Fraction Confucian | 0.203 | 0.0156 | 0.0793 |
| 10 | SAFRICA | Sub-Saharan Africa Dummy | 0.153 | 0.3068 | 0.4638 |
| 11 | LAAM | Latin American Dummy | 0.147 | 0.2273 | 0.4215 |
| 12 | SPAIN | Spanish Colony Dummy | 0.124 | 0.1705 | 0.3782 |
| 13 | MINING | Fraction GDP in Mining | 0.123 | 0.0507 | 0.0769 |
| 14 | GVR61 | Government Consumption Share | 0.114 | 0.1161 | 0.0745 |
| 15 | MUSLIM00 | Fraction Muslim | 0.110 | 0.1494 | 0.2962 |
| 16 | YRSOPEN | Years Open 1950-94 | 0.109 | 0.3555 | 0.3444 |
| 17 | AVELF | Ethnolinguistic Fractionalization | 0.105 | 0.3476 | 0.3016 |
| 18 | BUDDHA | Fraction Buddhist | 0.104 | 0.0466 | 0.1676 |
| 19 | DENS60 | Population Density | 0.085 | 108.07 | 201.44 |
| 20 | RERD | Real Exchange Rate Distortions | 0.081 | 125.03 | 41.71 |
| 21 | OTHFRAC | Fraction Speaking Foreign Language | 0.080 | 0.3209 | 0.4136 |
| 22 | OPENDEC1 | Openness 1965-74 | 0.076 | 0.5231 | 0.3359 |
| 23 | PRIGHTS | Political Rights | 0.065 | 3.8225 | 1.9966 |
| 24 | GOVSH61 | Government Share of GDP | 0.064 | 0.1664 | 0.0712 |
| 25 | GOV60 | Higher Education Enrollment | 0.060 | 0.0376 | 0.0501 |
| 26 | PRIEXP70 | Primary Exports. | 0.055 | 0.7199 | 0.2827 |
| 27 | TROPPOP | Fraction Population In Tropics | 0.052 | 0.3000 | 0.3731 |
| 28 | GGCFD3 | Public Investment Share | 0.051 | 0.0522 | 0.0388 |
| 29 | PROT00 | Fraction Protestant | 0.044 | 0.1354 | 0.2851 |
| 30 | HINDU00 | Fraction Hindu | 0.043 | 0.0279 | 0.1246 |
| 31 | POP1560 | Fraction Population Less than 15 | 0.041 | 0.3925 | 0.0749 |
| 32 | AIRDIST | Air Distance to Big Cities | 0.039 | 4324 | 2614 |
| 33 | GOVNOM1 | Nominal Government Share | 0.037 | 0.1490 | 0.0584 |
| 34 | ABSLATIT | Absolute Latitude | 0.032 | 23.21 | 16.84 |
| 35 | CATH00 | Fraction Catholic | 0.032 | 0.3283 | 0.4146 |
| 36 | COLONY | Colony Dummy | 0.031 | 0.7500 | 0.4355 |
| 37 | FERTLDC1 | Fertility | 0.031 | 1.5620 | 0.4193 |
| 38 | CIV72 | Civil Liberties | 0.029 | 0.5095 | 0.3259 |
| 39 | REVCOUP | Revolutions and Coups | 0.029 | 0.1849 | 0.2322 |
| 40 | EUROPE | European Dummy | 0.029 | 0.2159 | 0.4138 |
| 41 | SCOUT | Outward Orientation | 0.028 | 0.3977 | 0.4922 |
| 42 | BRIT | British Colony Dummy | 0.027 | 0.3182 | 0.4684 |
| 43 | LHCPC | Hydrocarbon Deposits | 0.024 | 0.4212 | 4.3512 |
| 44 | TOT1DEC1 | Terms of Trade Growth in 1960s | 0.024 | -0.0021 | 0.0345 |
| 45 | POP60 | Population in 1960 | 0.023 | 20308 | 52538 |
| 46 | GDE1 | Defense Spending Share | 0.021 | 0.0259 | 0.0246 |
| 47 | POP6560 | Fraction Population Over 65 | 0.021 | 0.0488 | 0.0290 |
| 48 | NEWSTATE | Timing of Independence | 0.021 | 1.0114 | 0.9767 |
| 49 | SIZE60 | Size of Economy | 0.021 | 16.15 | 1.82 |
| 50 | ENGFRAC | English Speaking Population | 0.020 | 0.0840 | 0.2522 |
| 51 | P16090 | Average Inflation 1960-90 | 0.019 | 13.13 | 14.99 |
| 52 | LANDLOCK | Landlocked Country Dummy | 0.019 | 0.1705 | 0.3782 |
| 53 | LT100CR | Land Area Near Navigable Water | 0.019 | 0.4722 | 0.3802 |
| 54 | HERF00 | Religion Measure | 0.019 | 0.7803 | 0.1932 |
| 55 | DPOP6090 | Population Growth Rate 1960-90 | 0.019 | 0.0215 | 0.0095 |
| 56 | OIL | Oil Producing Country Dummy | 0.019 | 0.0568 | 0.2328 |
| 57 | GEEREC1 | Public Education Spending Share | 0.019 | 0.0244 | 0.0096 |
| 58 | SQPI6090 | Square of Inflation 1960-90 | 0.019 | 394.54 | 1119.70 |
| 59 | SOCIALIST | Socialist Dummy | 0.018 | 0.0682 | 0.2535 |
| 60 | ZTROPICS | Tropical Climate Zone | 0.016 | 0.1900 | 0.2687 |
| 61 | WARTIME | Fraction Spent in War 1960-90 | 0.016 | 0.0695 | 0.1524 |
| 62 | TOTIND | Terms of Trade Ranking | 0.016 | 0.2813 | 0.1904 |
| 63 | WARTORN | War Participation 1960-90 | 0.015 | 0.3977 | 0.4922 |
| 64 | ORTH00 | Fraction Orthodox | 0.015 | 0.0187 | 0.0983 |
| 65 | ECORG | Capitalism | 0.015 | 3.4659 | 1.3809 |
| 66 | LANDAREA | Land Area | 0.015 | 867189 | 1814688 |
| 67 | DENS65I | Interior Density | 0.014 | 43.37 | 88.06 |

Variables ranked by Posterior Inclusion Probability $p(i \mid \mathbf{y})$. Prior inclusion probability for bench-
mark case equals $\bar{k} / 67=7 / 67=0.10$.






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[^2]:    ${ }^{1}$ Hoeting, Madigan, Raftery and Volinsky (1999) surveys the literature on Bayesian model averaging (BMA). Hjort and Claeskens (2003) discuss frequentist model averaging, and Brock, Durlauf and West (2003) apply model averaging to economic policy decisions.
    ${ }^{2}$ Raftery (1995) also suggests the use of BIC model weights to average maximum likelihood estimates. Raftery's approach differs in the specification of prior probabilities over the model space and sampling method.

[^3]:    ${ }^{3}$ George (1999) introduces so-called "dilution priors" to redistribute prior probability away from models that include explanatory variables exhibiting significant collinearity.
    ${ }^{4}$ A possible way to uncover interesting hierarchies is the Bayesian classification and regression tree (CART) method. See Chipman, George and McCulloch (2001) for further discussion.
    ${ }^{5}$ Following our own recent work (Doppelhofer and Weeks, 2005), Ley and Steel (forthcoming) discuss alternative measures of dependence and apply them to cross-sectional growth regressions.

[^4]:    ${ }^{6}$ Following the suggestion by an anonymous referee, we modify the model averaging procedure to allow for heteroscedastic errors in section 4.3.2.

[^5]:    ${ }^{7} \mathrm{~A}$ disadvantage of this approach is that the notion of what constitutes a reasonable prior model size may vary across analysts. In response to this criticism Godsill, Stone and Weeks (2004) introduce another layer of prior information by combining independent Bernoulli sampling for each variable with a conjugate Beta prior for the binomial proportion parameter $\pi_{i}$.

[^6]:    ${ }^{8}$ Conditional on each model $M_{j}$, the maximum likelihood estimator is optimal, given that we consider a prior structure that is dominated by sample information. Alternatively, the variance estimator would result from diffuse priors starting with a Student prior for $\beta$ (see Leamer, 1978, p.79-80). We check the sensitivity of our estimates to using informative priors in section 4.3.2.

[^7]:    ${ }^{9}$ Brock and Durlauf (2001) provide a decision-theoretic foundation to using such standardized coefficients for policy analysis.

[^8]:    ${ }^{10}$ Analogous expressions of joint probabilities can be written for groups of more than two variables.
    ${ }^{11}$ This discussion follows Whittaker (1990), chapter 2. The difference in our paper is that the joint distributions and related statistics are estimated unconditionally over the model space $\mathcal{M}$.

[^9]:    ${ }^{12}$ The jointness statistic (26) is well-defined as long as a variable is not included or excluded in all models. Suppose we wish to condition on the inclusion or exclusion of a variable from all models, say $p(l \mid \mathbf{y})=\gamma_{l}=c$ (with $c=0$ or 1 ). In this case we suggest to use the (still well-defined) posterior odds of including variables $\mathbf{x}_{i}$ as modified jointness measure, $\ln P O_{i \mid \gamma_{l}=c}=$ $\ln \left[p\left(i \mid \gamma_{l}=c, \mathbf{y}\right) / p\left(\bar{\imath} \mid \gamma_{l}=c, \mathbf{y}\right)\right]$.

[^10]:    ${ }^{13}$ By symmetry, this coefficient can also be written as $\ln [\operatorname{cpr}(i, h \mid l, \mathbf{y}) / \operatorname{cpr}(i, h \mid \bar{l}, \mathbf{y})]$ or as $\ln [\operatorname{cpr}(l, h \mid i, \mathbf{y}) / \operatorname{cpr}(l, h \mid \bar{\imath}, \mathbf{y})]$.

[^11]:    ${ }^{14}$ In laying out a framework for policy evaluation in the face of various dimensions of uncertainty, Brock et al (2003) treat the probabilities of each theory as approximately independent.

[^12]:    ${ }^{15}$ The dataset, including a list of data sources and countries with complete observations, is available at http://www.econ.cam.ac.uk/faculty/doppelhofer/research/Jointness.html. We are addressing data issues, such as missing observations, in ongoing research.

[^13]:    ${ }^{16}$ Critical values are shown in section 3.1. For convenience, entries for variables exhibiting significant positive jointness that are complements with $J_{i l}>1$ are set in boldface in Tables 1 to 3. Entries for variables exhibiting significant negative jointness that are substitutes with $J_{i l}<-1$ are set in italics.
    ${ }^{17}$ The diagonal entries contain the marginal standardized coefficients (17) in Table 2 and the sign certainty statistics (18) in Table 3 associated with variable $\mathbf{x}_{i}$. To ease comparison with the

[^14]:    ${ }^{19}$ The posterior mean (29) and variance (30) conditional on joint inclusion of variables are not shown due to space constraints, but are available at http://www.econ.cam.ac.uk/faculty/doppelhofer/research/Jointness.html.
    ${ }^{20}$ Notice that in contrast to jointness, the conditional statistics (standardized means and sign certainties) reported in Tables 2 and 3 are conditional on joint inclusion $P O_{i \mid j}$ of variables, that is, they do not take the joint exclusion margin $P O_{\bar{\imath} \mid \bar{j}}$ into account. To evaluate dependence and inform policy, we suggest to look at both jointness, as well standardized coefficients or sign certainties that are conditional on the joint inclusion of variables.
    ${ }^{21}$ It is unlikely that the identity of former colonial power and disease environment are indepen-

[^15]:    ${ }^{22}$ Note that an increase in the Political Rights index implies less political freedom. The negative standardized coefficient associated with Political Rights in Table 2 corresponds implies a positive relation with economic growth.

[^16]:    ${ }^{23}$ This jointness pattern is consistent with the finding that the unconditional inclusion probability for the East Asian dummy falls with larger prior model sizes as more explanatory variables are included. See Table 3 of SDM (2004).
    ${ }^{24}$ This finding agrees with Gallup and Sachs (2001) on the interaction of geographic factors and the economic burden of Malaria.

[^17]:    ${ }^{25}$ Detailed results of the sensitivity analysis in this section are not shown in the paper, but are available at http://www.econ.cam.ac.uk/faculty/doppelhofer/research/Jointness.html.

[^18]:    ${ }^{26}$ If the number of candidate variables $K$ is sufficiently large, so that $K^{2}>T$, Fernandez et al. (2001b) recommend to set $g_{0}=1 / K^{2}$. This implies a more diffuse prior distribution of $\beta$, similar to our benchmark case of sample-dominated prior information.

[^19]:    ${ }^{27}$ See Geweke (1993) for further details. Koop (2003) contains an excellent discussion of the case of $t$-distributed errors used here and of other error distributions resulting from mixed normal distributions.

[^20]:    ${ }^{28}$ Details of the Gibbs sampling procedure, including numerical convergence criteria, are discussed in the Computational Appendix A.
    ${ }^{29}$ The values of hyper-parameters $a=1000$ and $b=1 / 10$ are chosen to imply $E(v)=100$ and $\operatorname{Var}(v)=10$.

[^21]:    ${ }^{30}$ Burnham and Anderson (2002) provide an extensive discussion of both the theoretical and empirical issues related to the use of Akaike weights.

[^22]:    ${ }^{31}$ For the case assuming most heteroscedasticity a priori ( $a=8, b=1 / 2$ and implied values $E(v)=4, \operatorname{Var}(v)=2$ ), we use 1050 Gibbs draws (minus 50 burn-in) and 1 million model averaging draws. Despite the large number of draws the estimates have not fully converged according to numerical convergence criterion (46). We are exploring this issue in ongoing work.

