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Department of Economics
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Banks' Credit-Portfolio Choices and Risk-Based Capital Regulation

Caren Yinxia Nielsen

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Banks' credit-portfolio choices and risk-based capital regulation[☆]

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Abstract

To address banks' risk taking during the recent financial crisis, we develop a model of credit-portfolio optimization and study the impact of risk-based capital regulation (Basel Accords) on banks' asset allocations. The model shows that, when a bank's capital is constrained by regulation, regulatory cost (risk weightings in the Basel Accords) alters the risk and value calculations for the bank's assets. The model predicts that the effect of a tightening of the capital requirements – for banks for which these requirements are (will become) binding – will be to skew the risky portfolio towards high-risk, high-earning assets (low-risk, low-earning assets), provided that the asset valuation – i.e., reward-to-regulatory-cost ratio – of the high-risk asset is higher than that of the low-risk asset. Empirical examination of U.S. banks supports the predictions applicable to the dataset. In addition, our tests show the characteristics of banks with different levels of risk taking. In particular, the core banks that use the internal ratings-based approach under Basel II invest more in high-risk assets.

Keywords: Banks; asset risk; credit risk; portfolio choice; risk-based capital regulation

JEL classification: G11; G18; G21; G28

1. Introduction

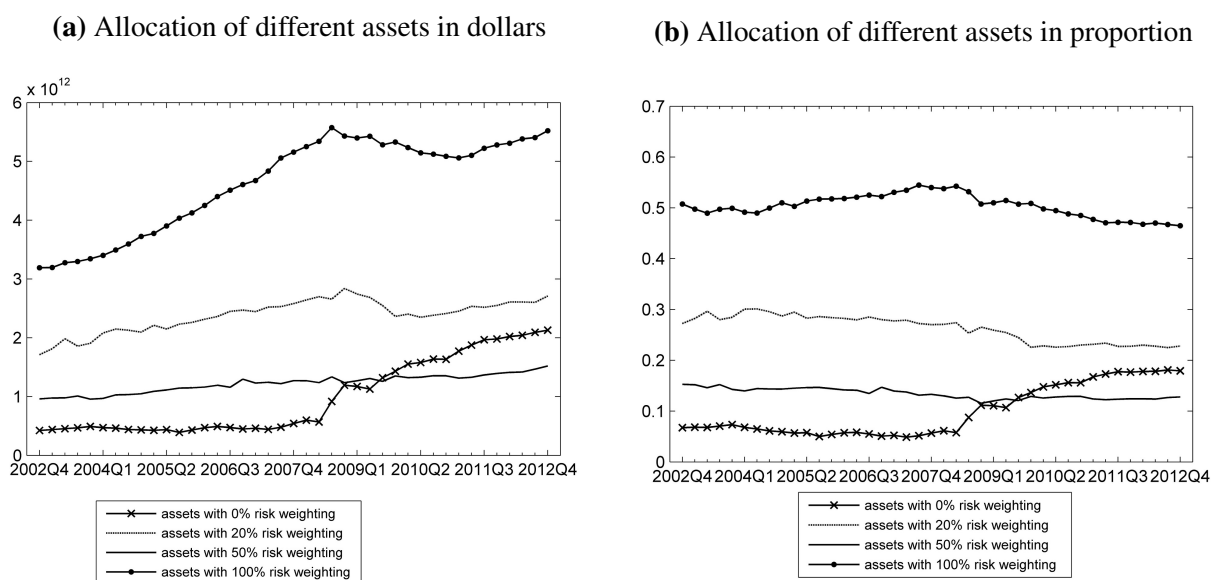
The recent financial crisis has placed a sharp spotlight on banks' risk taking. Banks are blamed for shrugging off risk concerns while pursuing higher earnings, such as on loans with high credit risk.

To investigate banks' risk taking in relation to credit risk, we look into banks' total assets with

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Figure 1: Banks' allocation of assets with different risk weightings



different levels of credit risk, defined by the risk weightings under the Basel Accords.¹ To value the overall risk of a bank's assets, the Basel Accords adopt the total risk-weighted assets, where a higher weight is assigned to assets with higher credit risk.²

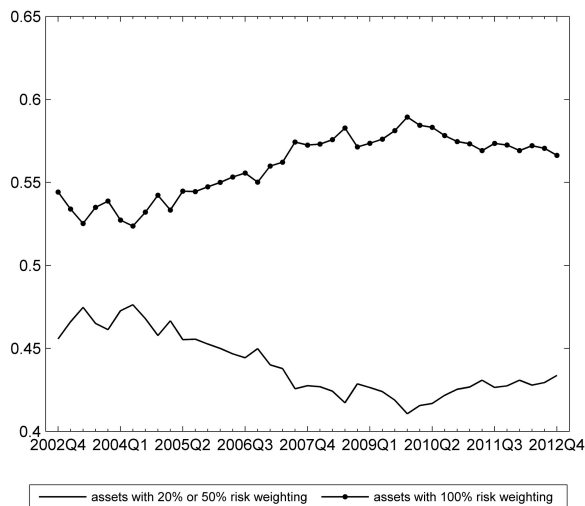
Figures 1 display the sums of the assets with certain risk weightings for all U.S. banks that were insured by the Federal Deposit Insurance Corporation (FDIC) from 2002 to 2012. According to Figure 1, there is a distinct increase in the amount of assets with the highest credit risk (100% risk weighting), although the trend for its proportion of the total is not obvious. Notice that, from the second quarter of 2008, there is an apparent increase in the assets with the lowest credit risk (0% risk weighting), measured in dollars or as a proportion, which verifies the flight-to-safety phenomenon since the crisis. Figure 2 shows banks' total allocation among risky assets whose risk weightings are nonzero. The proportion of assets with 100% risk weighting – i.e., high-risk assets – increases through the years, compared to the sum of assets with 20% and 50% risk weightings – i.e., low-risk assets. The maximum of the difference in their allocations is 17.9% of risky assets: that is, 1.62 trillion dollars. This could be due to a flight to earnings targeting the assets with the 100% risk weighting.

One question that arises from the above observations is how banks allocate resources (deposits

¹The Basel Accords are the supervision accords for banks promulgated by the Basel Committee on Banking Supervision. This study limits its focus to the aspects of the Accords that address capital adequacy, which is the center of the Accords.

²Under the standard approach of Basel I and II, there are four broad categories of risk: 0%, 20%, 50%, and 100% risk weightings. To determine capital adequacy, the Basel Accords use a risk-based capital ratio: the ratio of total regulatory capital to total risk-weighted assets.

Figure 2: Banks' allocation of risky assets with nonzero risk weightings



and capital) for assets with different credit risks conditional on the information on assets' payoffs and default probabilities. Our paper addresses this question. Moreover, the risk-based capital-adequacy requirements under the Basel Accords³ pose additional costs for riskier assets, since banks have to reserve more capital for assets with a higher credit risk. However, how banks react to this regulation is ambiguous. Banks also have incentives to take more risk in order to gain higher earnings and compensate for the higher costs of their capital reserves. Thus, we are not certain that a tightening capital requirement would have the desired effect.

We regard a bank as its assets' manager and develop a model of portfolio allocation with a minimum regulatory capital requirement as a possible binding condition. Then we examine how the bank reshuffles the portfolio basket when the conditional information (i.e., assets' payoffs, default probabilities, default correlation, or regulation) changes. This allows us to explicitly examine the bank's credit risk from the point of view of asset portfolio management and to study the specific effects of the risk-based capital requirements on the bank's asset choice.

Our model shows that, when a bank's capital is not constrained by regulation, its asset allocation decision depends on the risk measure of assets – namely, the cash-flow volatility around the expected loss due to default risk – and on the key measure of an asset's valuation, the Sharpe ratio (Sharpe, 1966), modified according to our settings. However, when the bank's capital is constrained by regulation, regulatory cost (risk weighting in risk-based capital regulation) steps in and weights the cash-flow volatility, and even replaces the volatility in the measure of the assets' valuation

³Under Basel I (1998) and II (2004), a bank has to reserve total capital equal to at least 8% of the value of the bank's total risk-weighted assets; under Basel III (2010), a bank has to hold additional conservation and countercyclical buffers.

(reward-to-regulatory-cost ratio instead of Sharpe's reward-to-variability ratio). If the regulator imposes a new and more stringent regulation, a higher risk-based capital requirement, the bank whose capital is already constrained will skew the risky portfolio to high-risk, high-earning assets, while the bank whose capital will become constrained by the new regulation will do the opposite, investing less in high-risk assets, provided that the valuation (reward-to-regulatory-cost ratio) of a high-risk asset is higher than that of a low-risk asset. The latter effect is similar to the immediate risk-reducing effect of capital regulation demonstrated in the literature.

Moreover, we test the model with bank-level data on assets with different credit-risk categories for all commercial banks insured by the FDIC. The empirical examination verifies the model's predictions of how banks' choices between high-risk, high-earning assets and low-risk, low-earning assets react to the updated information on assets' earnings and default probabilities, and of the impact of a higher capital requirement. In addition, our tests show the characteristics of banks with different levels of risk taking. In particular, the core banks that use the internal ratings-based approach under Basel II invest more in high-risk assets.

Although there are models evaluating portfolio credit risk, only a few articles concern credit-portfolio optimization (see, e.g., Altman and Saunders, 1998; Kealhofer and Bohn, 2001; Mencía, 2012). Regarding the impact of capital regulation on banks' asset risk, the theoretical literature yields mixed predictions, with a few studies from the point of view of portfolio management (see, e.g., Koehn and Santomero, 1980; Kim and Santomero, 1988; Rochet, 1992; Furfine, 2001; Milne, 2002). To understand banks' risk taking in relation to credit risk and the impact of a tightening capital requirement on banks' asset risk, we derive an analytical and tractable solution for credit-portfolio optimization, similar to Koehn and Santomero (1980) and Kim and Santomero (1988). Our paper contributes to studies of banks' risk taking by investigating banks' asset allocation explicitly with respect to credit risk and by disentangling the effects of risk-based capital regulation on banks' asset risk.

The remainder of the paper is organized as follows. Section 2 reviews the literature. Section 3 presents the model. In Section 4, we examine the model empirically using a panel data set, including details of the estimation of conditional default probabilities, default correlation, and payoffs. Section 5 concludes.

2. Literature review

Over the past two decades, we have seen important advances in the modeling of correlated defaults and the evaluation of portfolio credit risk, including Moody's *KMV Portfolio Manager*, JPMorgan's *CreditMetrics*, Credit Suisse's *CreditRisk⁺*, McKinsey's *CreditPortfolioView*, correlated

default-intensity models, and copula-based modeling.⁴

Yet, there are only a few articles on credit-portfolio optimization. Altman and Saunders (1998) measure a portfolio's risk by its unexpected loss, determined by the standard deviation around the expected loss, which is estimated historically over time using bond rating equivalence: the Z'' -score (Altman, 1993). Similarly, Kealhofer and Bohn (2001) measure unexpected loss by the standard deviation of loss only due to default in a default-only model, where there are two states, default and no default. Mencía (2012) models homogeneous loan classes, each comprising conditional independent loans whose conditional default probability is a probit function of a Gaussian state variable. Thus, the return distribution of the loan portfolio is presented by the mean and variance of the state variable. Then, all three articles adapt the mean-variance framework, introduced by Markowitz (1952), to analyze the risk and returns on credit portfolios.

Due to the difficulty of presenting the distribution of portfolio credit losses with sufficient accuracy, various simulation techniques have been developed to approximate the distribution (e.g., Iscoe et al., 1999; Mausser and Rosen, 2001; Zagst et al., 2003). Iscoe et al. (2012) adapt factor models for conditional default probabilities and compare different techniques to minimize the variance, value at risk, and expected shortfall of portfolio credit losses. Yet, they find that a normal approximation to the conditional loss distribution performs best. Normal distribution is also widely used by practitioners to calculate value at risk and expected shortfall.

To understand banks' risk taking on credit risk, we need an analytical and tractable solution for credit-portfolio optimization, which cannot be provided by simulation techniques. Hence, we develop a default-only model of one-period conditional credit-portfolio management for a bank manager; the risk of a type of asset is measured by the volatility around the expected loss of the assets' cash profits. This risk measure is similar to those from Altman and Saunders (1998) and Kealhofer and Bohn (2001). Different from Mencía (2012), capital regulation – i.e., the minimum capital requirement – is explicitly presented in our analytical solution of portfolio allocation. Although we do not model the drivers for the conditional default probabilities, default correlation, and asset payoffs, which are given before the bank manager makes a decision, they are determined by the market data in the empirical examination in Section 4. Moreover, the losses of risky assets in our study are Bernoulli distributed with a positive skewness, and the skewness and kurtosis of the distribution are functions of the first two moments.

Regarding the impact of capital regulation on banks' asset risk, theoretical literature yields mixed predictions, although there is general agreement about the immediate effects of capital requirements on bank lending and the longer-term impact on capital ratio (See VanHoose, 2007,

⁴See detailed descriptions of the models by Gordy (2000), Crouhy et al. (2000), and Duffie and Singleton (2003), among others.

among others). As yet, there are just a few studies of banks' asset risk from the point of view of portfolio management. Koehn and Santomero (1980) and Kim and Santomero (1988) consider a mean-variance portfolio-selection model, showing that a higher uniform regulatory capital ratio constrains the efficient asset investment frontier and might actually result in a higher asset risk and increase banks' insolvency risk, yielding the opposite of the intended effect. Nevertheless, Kim and Santomero (1988) model the optimal weights for the risk-based capital requirement, and predict that, with higher weights for riskier assets, banks would hold more liquid safe assets and fewer risky assets. Rochet (1992) argues that, if banks behave as portfolio managers – maximizing utility instead of the market value of their future profits as in Furlong and Keeley (1989), among others – capital regulation can be effective, but only if the risk weights are proportional to the systematic risks of the assets (their betas). Furfine (2001), developing a dynamic value-maximizing model and calibrating it to U.S. data, finds that Basel I was involved in the credit crunch experienced in the 1990s and predicts that, under Basel II, banks would increase loans relative to securities and safer loans relative to risky ones. Milne (2002) interprets capital regulation as a system of sanctions for ex post violation instead of ex ante enforcement, and his value-maximizing model suggests that there is relatively less need to match risk weightings accurately to portfolio risk.

To study the effects of risk-based capital regulation explicitly, we set up a portfolio-selection model similar to those of Koehn and Santomero (1980) and Kim and Santomero (1988). However, in our model, the bank manager maximizes the utility of one-period net value of assets, instead of the utility of equity returns, as in their model, which has been questioned for modeling returns of a bank's capital regardless of its actual asset risk. The advantage of using the utility of assets' net value is its focus on the bank's asset risk. Further, the model is close to practice and could be enriched with additional features, such as capital structure.

The most marked difference from the aforementioned banking literature is that this paper focuses on credit risk, where there are only two conditional states: default and no default. Thus, we can adapt similar approaches from the credit-portfolio-optimization literature. Moreover, we study whether banks restructure their portfolios from low-risk, low-earning assets to high-risk, high-earning assets to compensate for additional costs imposed by capital requirements. Therefore, our model contributes not only to rationalizing banks' asset allocation with respect to credit risk, but also to addressing the gap in the literature studying the impact of capital regulation on the credit risk of banks' assets.

3. Model

We model a commercial bank as its asset manager makes one-period decisions on allocating resources (deposits and capital) for the assets with different levels of credit risk; its capital might be

constrained by risk-based capital regulation. The model predicts how the bank manager restructures the portfolio of different assets when their conditional default probabilities, default correlation, or payoffs change, or when the regulator tightens risk-based capital requirements.

3.1. Model set-up

The bank aims to maximize a single-period expected quadratic utility of its assets' random cash profit, and the expected utility is an increasing function of the expected cash flow and a decreasing function of the cash-flow variance.⁵ The bank chooses among three types of assets: a high-risk, high-earning asset; a low-risk, low-earning asset; and a risk-free asset.⁶ For simplicity, only the relative sizes of assets are assumed to be under the control of bank management.⁷ In addition, the regulator decides that the bank has to hold capital as minimum k times the total risk-weighted assets, where risky assets are assigned higher weights. It is also assumed that the holding period perfectly matches the maturity of the assets.

The random cash flow, \widetilde{CF} , of a risky asset is $(1 + C)(1 - \widetilde{Z})$, where C is the payoff of the asset if not defaulted and \widetilde{Z} is a variable for a default event with Bernoulli distribution, which takes value 1 with probability p if default happens⁸ and 0 otherwise. That is, there are two credit states, default and no default, and

$$\widetilde{Z} = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability } 1 - p. \end{cases} \quad (1)$$

The third and fourth moments of the Bernoulli distribution are functions of its mean and variance. Therefore, we can use the first two moments to represent the distribution. Naturally, the measure of risk is the volatility around the expected loss (mean) and an approximation of unexpected loss, which is a usual term in the credit-portfolio literature.

⁵Bawa and Lindenberg (1977) show that, as long as the expected utility can be written as an increasing function of the expected return and a decreasing function of the variance of the portfolio only, without any assumption of probability distributions of assets' returns, the optimal portfolio lies on the efficient frontier in the mean-variance framework of Markowitz (1952). Also, when there is a risk-free asset, the two-fund theorem is valid. Here, by applying a quadratic utility function, we could sidestep most of the problems associated with solving a general utility-based portfolio choice and obtain an analytical solution.

⁶The choice of three types of assets is also consistent with the empirical examination in Section 4. In addition, the model with four types of assets, which corresponds to the four risk categories of assets in the Basel Accords, is qualitatively identical.

⁷This simplification serves the purpose of this study. While we could enrich the model with additional features, such as variations in the bank's liabilities and capital, how the bank allocates among assets with different credit risks in a *ceteris paribus* environment is not altered.

⁸For simplification, the recovery rate is assumed to be zero.

The expected value and variance of its cash flow are

$$\begin{aligned} E[\widetilde{CF}] &= (1+C) - E[\widetilde{Z}](1+C) = (1-p)(1+C) \text{ and} \\ \text{Var}[\widetilde{CF}] &= \text{Var}[\widetilde{Z}](1+C)^2 = p(1-p)(1+C)^2, \text{ respectively,} \end{aligned} \quad (2)$$

where E stands for *Expectation* and Var for *Variance*. To ensure that the expected utility is decreasing as default probability increases, we only consider $p < 0.5$, which is consistent with the estimated probabilities of default (far less than 0.5) in the empirical examination (Section 4). Then, the loss due to default risk is captured by a positively skewed Bernoulli distribution.

The cash-flow covariance of the two types of risky assets in the model – high-risk, high-earning assets with type h (*high*) and low-risk, low-earning assets with type l (*low*) – is

$$\text{Cov}[\widetilde{Z}_h, \widetilde{Z}_l] = p_B - p_h p_l = \rho \sqrt{p_h(1-p_h)p_l(1-p_l)}, \quad (3)$$

where p_B and ρ are the pair-wise probability of default and the default correlation between the two types, respectively.

The bank utility function is

$$u(\widetilde{\pi}) = 2a\widetilde{\pi} - \widetilde{\pi}^2, \quad (4)$$

where $\widetilde{\pi}$ is the random cash profit at the end of the decision period:

$$\widetilde{\pi} = A_h \widetilde{CF}_h + A_l \widetilde{CF}_l + G(1+r_f) - Dr_D, \quad (5)$$

where \widetilde{CF}_h and \widetilde{CF}_l are the random cash flows of high-risk, high-earning and low-risk, low-earning assets, respectively; A_h and A_l are their respective amounts in dollars; G is the amount of the risk-free asset with return r_f ; and D is for deposits with rate r_D .

Consequently, the decision problem for the bank manager is

$$\arg \max_{A_h, A_l, G} \{E[u(\widetilde{\pi})]\} = \arg \max_{A_h, A_l, G} \{2aE[\widetilde{\pi}] - (E[\widetilde{\pi}]^2 + \text{Var}[\widetilde{\pi}])\}, \quad (6)$$

subject to

$$A_h + A_l + G = D + K \quad (7a)$$

$$A_h \geq 0, A_l \geq 0, G \geq 0 \quad (7b)$$

$$\frac{K}{W_h A_h + W_l A_l} \geq k \text{ and } 1 \geq W_h > W_l > 0. \quad (7c)$$

The objective function is a function of $a, A_h, A_l, G, C_h, C_l, p_h, p_l, \rho, r_f, D$, and r_D . a is positive, and $a \geq (1+C_h)(D+K) - Dr_D > 0$, which ensures a positive marginal utility of cash profit. C_h and

C_1 are payoffs of asset types h and l, respectively $-1 \geq C_h > C_l > r_f > 0$ – and p_h and p_l are their respective probabilities of default $-0.5 > p_h > p_l > 0$. K stands for capital and the first restriction (Equation (7a)) states the balance-sheet constraint. W_h and W_l are the risk weightings for risky assets used in the calculation of the total risk-weighted assets: They are constant and determined by the regulator, and, by definition, $1 \geq W_h > W_l > 0$. k denotes the minimum risk-based capital ratio determined by the regulator, which is 8% under Basel I and II, and the third restriction (Equation (7c)) expresses the regulatory capital constraint. In practice, actual defaults are positively, but not perfectly positively, correlated (Kealhofer and Bohn, 2001). Hence, default correlation, ρ here, belongs to interval $(0, 1)$.

In addition, the following subsections are based on solutions to the above maximization problem when the risky assets generate positive excess cash flows over risk-free assets: That is, $X_h \equiv (1 - p_h)(1 + C_h) - (1 + r_f) > X_l \equiv (1 - p_l)(1 + C_l) - (1 + r_f) > 0$.

3.2. Optimal portfolio allocation when the capital requirement is not binding

When the capital requirement is not binding, we get the following inner solution to the maximization problem (Equation (6)).

$$A_h^* = \frac{(a - B)(SR_h - \rho SR_l)}{(SR_h^2 + SR_l^2 - 2\rho SR_h SR_l + 1 - \rho^2)\sqrt{V_h}} \quad (8)$$

$$A_l^* = \frac{(a - B)(SR_l - \rho SR_h)}{(SR_h^2 + SR_l^2 - 2\rho SR_h SR_l + 1 - \rho^2)\sqrt{V_l}} \quad (9)$$

$$G^* = D + K - \frac{(a - B)[SR_h(\sqrt{V_l} - \rho\sqrt{V_h}) + SR_l(\sqrt{V_h} - \rho\sqrt{V_l})]}{(SR_h^2 + SR_l^2 - 2\rho SR_h SR_l + 1 - \rho^2)\sqrt{V_h V_l}}, \quad (10)$$

where $B \equiv (D + K)(1 + r_f) - Dr_D < a$,⁹ V stands for cash-flow variance, and SR for Sharpe ratio,¹⁰ which is the ratio of excess cash flow over the risk-free asset (X) to cash-flow volatility (\sqrt{V}).

The optimal allocations to the risky assets are determined by their Sharpe ratios, cash-flow variances, default correlation, and the bank's risk aversion. Finally, the balance-sheet constraint controls the investment in risk-free assets.

From now on, we consider a change in the risk or payoff of one type of the risky assets or in default correlation, and derive how the optimal allocation adjusts. This is to reveal a more dynamic picture of how the bank restructures the portfolio of different assets when the conditional information

⁹Note that, because we assume $a \geq (1 + C_h)(D + K) - Dr_D > 0$, $B < a$.

¹⁰This reward-to-variability ratio (Sharpe, 1966) is modified according to the settings in our model.

on assets changes. Here, we use the two-fund separation theorem (Markowitz, 1952). The two funds refer to the risky fund, which is comprised of high-risk, high-earning and low-risk, low-earning assets, and the risk-free fund, comprised of essentially risk-free assets. Then, the risky fund is the tangential portfolio on the capital market line, which is the ray from the risk-free cash flow with a tangency to the mean-variance efficient frontier of risky assets. Within the risky fund, the portfolio weights of type h and type l assets are defined as $\omega_h^* = \frac{A_h^*}{A_h^* + A_l^*}$ and $\omega_l^* = \frac{A_l^*}{A_h^* + A_l^*}$, respectively. For the portfolio composed of the risky and risk-free fund, the amounts of the allocations $A_h^* + A_l^*$ and G^* represent their relative portfolio weights, since the bank's size does not change.

3.2.1. The risky fund

The following proposition is derived from the first derivative of the optimal weight of the high-risk, high-earning asset (ω_h^*) with respect to the payoff or default probability of any risky asset, or default correlation. Obviously, the weight of the low-risk, low-earning asset (ω_l^*) would consequently change in the opposite direction.

Proposition 1. *Within the risky fund, the bank invests proportionally more (less) in high-risk assets, ceteris paribus, if*

- (a) *its payoff, C_h , increases (decreases); or*
- (b) *its probability of default, p_h , decreases (increases); or*
- (c) *the payoff of the low-risk asset, C_l , decreases (increases); or*
- (d) *the default probability of the low-risk asset, p_l , increases (decreases); or*
- (e) *the default correlation, ρ , increases (decreases), given that $SR_h > SR_l$.*

The proof is provided in Appendix A.1. As expected, the bank invests more in high-risk, high-earning assets when the asset generates higher payoff or its obligor has a lower probability of defaulting, or the other risky asset generates a lower payoff or its obligor has a higher probability of defaulting, *ceteris paribus*. In short, a high-risk, high-earning asset acts as a substitute for a low-risk, low-yield asset, and provides a natural hedge against losses stemming from low-risk assets.

When default correlation increases, the bank allocates more to high-risk, high-earning assets if their Sharpe ratio is larger than that of low-risk, low-yield assets. That is, when the two types of assets are more likely to default at the same time, the best strategy is to compare their Sharpe ratios and choose the asset type with a higher Sharpe ratio.

3.2.2. The risk-free fund

For the risky fund as a single asset, there is no measure of its overall payoff or probability of default. Therefore, as in Section 3.2.1, we consider any change in the payoff or default probability of any

risky asset or in the default correlation, which measures how the earning or risk of the whole fund varies. The following proposition is derived from the first derivative of the optimal investment in a risk-free asset (G^* (Equation (10)) with respect to each of these measures. Obviously, the allocation to the risky fund ($A_h^* + A_l^*$) would consequently change in the opposite direction. Recall that the amounts of the allocations in different funds represent their relative portfolio weights since the bank's size does not change.

Proposition 2. *The bank invests more (less) in the risk-free fund, ceteris paribus, if*

- (a) *the risk-free rate r_f increases (decreases); or*
- (b) *the payoff of the high-risk asset, C_h , increases (decreases), given that $\rho\sqrt{V_h} \geq \sqrt{V_l}$; or*
- (c) *the default probability of the high-risk asset, p_h , decreases (increases), given that $\frac{\rho\sqrt{V_h}}{\sqrt{V_l}} \geq \frac{2X_h}{X_h + X_l} > 1$; or*
- (d) *the payoff of the low-risk asset, C_l , decreases (increases), given that $\frac{\rho\sqrt{V_h}}{\sqrt{V_l}} \geq \frac{X_h + 1 + r_f}{X_l + 1 + r_f}$, $\rho^2 X_h \geq X_l$, $\rho(1 + r_f) > X_l$ and $\frac{X_h^2}{X_l^2} \geq \frac{V_h - \rho\sqrt{V_h V_l}}{V_l - \rho^2 V_l}$; or*
- (e) *the default probability of the low-risk asset, p_l , increases (decreases), given that $\rho\sqrt{V_h} \geq \sqrt{V_l}$ and $SR_l^2 \leq 1 - \rho^2$; or*
- (f) *the default correlation, ρ , increases (decreases), assuming $\rho\sqrt{V_h} \leq \sqrt{V_l}$ and $SR_h \geq SR_l$.*

The proof is in Appendix A.2. As expected, the bank buys more risk-free securities when the risk-free rate is higher.

This proposition shows that when a high-risk asset is far riskier than a low-risk asset in terms of cash-flow variance – i.e., $\rho\sqrt{V_h} > \sqrt{V_l}$ – the risk-free fund is a complement to the high-risk asset (in statements (b) and (c)) and a substitute for the low-risk asset (in statements (d) and (e)), given possible additional conditions: For instance, where the relative difference in variance is greater than that in excess cash flow, and some measures of excess cash flows are limited.

When a high-risk asset generates a lower payoff or is more likely to default, the bank invests less in high-risk assets and more in low-risk assets, as Proposition 1 tells us. However, when a high-risk asset is far riskier than a low-risk asset in terms of cash-flow variance, the increase of the investment in the low-risk asset is much more than the decrease of that in the high-risk asset, so that the investment in the risk-free asset actually decreases. Yet, a risk-free asset is always a substitute for a low-risk asset, since the low-risk asset is always less risky than a high-risk asset.

When default correlation increases, the bank invests more in the risk-free fund, given that the cash-flow variances of the risky assets are relatively close and a high-risk asset earns a higher Sharpe ratio than a low-risk asset. Here, the risk-free fund mitigates the risk that arises when both types of risky assets default at the same time.

3.3. Impact of risk-based capital regulation

This section disentangles the effects of risk-based capital regulation on the bank's asset risk by analyzing how the bank changes its asset allocation when the regulator imposes a new and more stringent capital requirement in the situation where the bank's capital is already constrained by the current regulation or will become constrained by the new regulation.

3.3.1. Optimal portfolio allocation when the capital constraint is currently binding

For the bank that achieves the minimum capital requirement and whose capital is constrained, allocations to risky assets are restricted by the risk-based capital ratio, which is the ratio of capital to the total risk-weighted assets. Under this condition, we derive the following inner solution to the maximization problem (Equation (6)).

$$A_{bh}^* = \frac{(a - B)\varphi_l(\varphi_l SR_h - \varphi_h SR_l) - \frac{K}{k}\{SR_l(\varphi_l SR_h - \varphi_h SR_l) + (\rho\varphi_l - \varphi_h)\}}{(\varphi_h^2 - 2\rho\varphi_h\varphi_l + \varphi_l^2)\sqrt{V_h} + (\varphi_l SR_h - \varphi_h SR_l)^2\sqrt{V_h}} \quad (11)$$

$$A_{bl}^* = \frac{(a - B)\varphi_h(\varphi_h SR_l - \varphi_l SR_h) - \frac{K}{k}\{SR_h(\varphi_h SR_l - \varphi_l SR_h) + (\rho\varphi_h - \varphi_l)\}}{(\varphi_h^2 - 2\rho\varphi_h\varphi_l + \varphi_l^2)\sqrt{V_l} + (\varphi_l SR_h - \varphi_h SR_l)^2\sqrt{V_l}} \quad (12)$$

$$G_b^* = D + K - \frac{K}{kW_1} + \frac{W_h - W_l}{W_1} \times \frac{(a - B)\varphi_l(\varphi_l SR_h - \varphi_h SR_l) - \frac{K}{k}\{SR_l(\varphi_l SR_h - \varphi_h SR_l) + (\rho\varphi_l - \varphi_h)\}}{(\varphi_h^2 - 2\rho\varphi_h\varphi_l + \varphi_l^2)\sqrt{V_h} + (\varphi_l SR_h - \varphi_h SR_l)^2\sqrt{V_h}}, \quad (13)$$

where b stands for *binding*, W_h and W_l are the risk weightings for risky assets used in the calculation of the total risk-weighted assets, and $\varphi_h \equiv \frac{W_h}{\sqrt{V_h}}$ and $\varphi_l \equiv \frac{W_l}{\sqrt{V_l}}$.

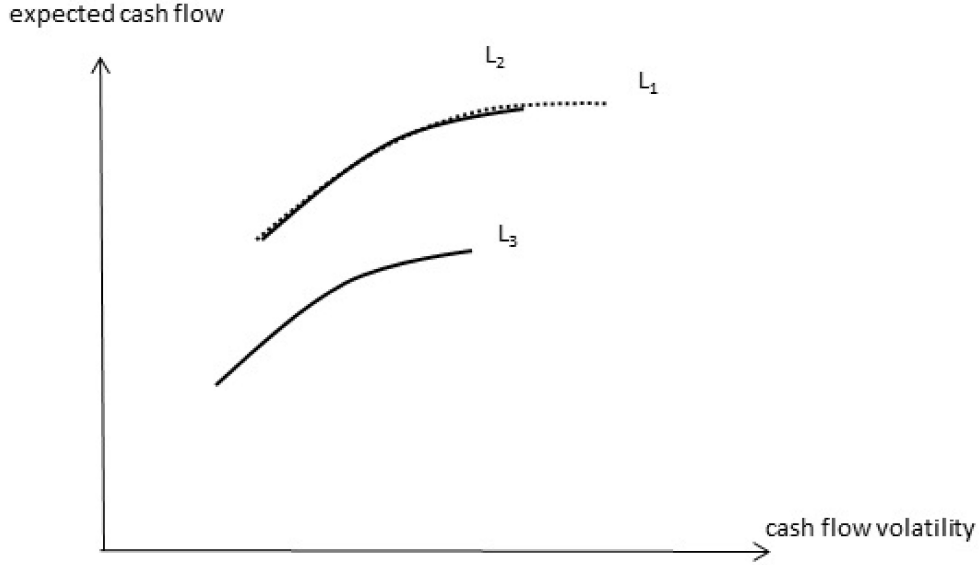
We interpret φ_h and φ_l as the regulatory cost per asset risk for high- and low-risk assets, respectively. Clearly these costs are important in determining the optimal allocation.

3.3.2. Impact of a tightening capital requirement

We then study how the bank reshuffles the portfolio due to new and more stringent capital regulation, such as an increase in the risk-based capital requirement, k , as in Basel Accord III. This would provide a prediction for the impact of Basel III on banks' behavior.

One important effect of a tightening capital requirement on the bank's portfolio is to change its efficient asset investment frontier. For a bank whose capital is not constrained by the current regulation, since it still faces risk-based capital regulation, $W_h A_h + W_l A_l \leq \frac{K}{k}$ (Equation (7c)), there

Figure 3: Banks' efficient asset investment frontier



This figure shows banks' efficient asset investment frontier for risky assets. Each curve represents the best possible expected cash flow of a bank's portfolio of assets for its level of risk (cash-flow volatility) under a certain capital regulation rule.

are upper limits for the portfolio weights in the risky funds, and hence also for their expected values and variances in terms of cash flows. When the regulator imposes a new regulation and requires the bank to have a higher capital ratio, these upper limits for the risky funds are smaller. Subsequently, those risky funds, whose expected values and variances of cash flows are too high, and that locate far upward and right on the efficient frontier, are now out of reach for the bank. Therefore, the available efficient frontier shrinks from the top and right (illustrated by a move from line L_1 to line L_2 in Figure 3), and the bank's capital might become constrained by the new regulation. See proof in Appendix B.1.

For a bank whose capital is already constrained by regulation, all the risky funds on the efficient frontier reach the upper limits of their expected values and variances of cash flows. Thus, when the regulator imposes a new and more stringent regulation – i.e., a higher k – the bank's efficient frontier falls downward and to the left, since any risky fund's expected value and variance of cash flows decrease: This is illustrated by a move from line L_2 to line L_3 in Figure 3. See proof in Appendix B.1.

How will the bank whose capital is already constrained by the current regulation reshuffle its optimal portfolio due to the new regulation? The following proposition is derived from the first derivatives of the optimal portfolio weight of high-risk assets in the risky fund, $\omega_{bh}^* \equiv \frac{A_{bh}^*}{A_{bh}^* + A_{bl}^*}$ and of the optimal investment in the risk-free fund, G_b^* , with respect to k .

It transpires that there are two key parameters determining the bank's choice: $\vartheta_h \equiv [(1 - p_h)(1 + C_h) - (1 + r_f)]/W_h$ and $\vartheta_l \equiv [(1 - p_l)(1 + C_l) - (1 + r_f)]/W_l$. ϑ_h and ϑ_l measure expected excess cash flows per capital cost due to the regulation for high- and low-risk assets, respectively. We call ϑ_h and ϑ_l the reward-to-regulatory-cost ratios.

Proposition 3. *When the bank's capital is constrained by regulation and the regulator imposes a new and more stringent regulation with a higher capital requirement k ,*

- (a) *within the risky fund, the bank invests proportionally more in high-risk assets, ceteris paribus, given that $\vartheta_h > \vartheta_l$, and*
- (b) *the bank invests more in the risk-free fund, ceteris paribus, given that $\vartheta_h \geq \vartheta_l$ and $\rho \varphi_l \geq \varphi_h$.*

The proof is in Appendix B.2. Statement (a) tells us that, if the bank whose capital is already constrained by the regulation is required to have a higher capital ratio, it reshuffles the risky fund and invests proportionally more in the asset with the higher reward-to-regulatory-cost ratio. In contrast with the Sharpe ratio, which is a reward-to-variability ratio, the denominator of the reward-to-regulatory-cost ratio is the risk weighting assigned to that type of risky asset by the regulator. Therefore, if the risk weightings are not consistent with the assets' cash-flow variances, which are measures of risk in this model, we could predict that there would be opportunities for regulatory arbitrage.

Statement (b) shows that when the bank is required to have a higher capital ratio, it invests more in the risk-free fund, if the regulatory cost per asset risk for a low-risk asset is much higher than that for a high-risk asset whose valuation (reward-to-regulatory-cost ratio) is not lower. That is, the risk-free fund mitigates the risk of a higher regulatory cost, when low-risk assets are too costly to provide such mitigation.

How will the portfolio change for the bank whose capital is not constrained by the current regulation but will become constrained by the new regulation? We derive the following proposition by comparing the optimal portfolio weight of a high-risk asset in the risky fund for a bank whose capital is not constrained by regulation (ω_h^*), and that for a bank whose capital is constrained (ω_{bh}^*).

Proposition 4. *When the regulator imposes a higher risk-based capital requirement k and the bank's capital will become constrained by the new regulation, within the risky fund, the bank invests proportionally less in high-risk assets, if $\vartheta_h > \vartheta_l$.*

The proof is in Appendix B.3. Proposition 4 states that, within the risky fund, the bank invests proportionally more in the asset with a lower reward-to-regulatory-cost ratio, when its capital becomes constrained due to a higher capital requirement. That is, a more stringent regulation poses an additional cost, so that the bank cannot invest in the asset with a higher reward-to-regulatory-cost ratio as it could otherwise. Thus, the regulation does restrict the bank's taking on more risk. However, this effect does not exist for the bank whose capital is already constrained by regulation before a

more stringent capital requirement is imposed, as stated in Proposition 3.

4. Empirical examination

This section tests the model using bank-level data for all U.S. commercial banks insured by the FDIC from 2002 to 2012. Due to concerns regarding domestic and international competitiveness, the implementation of capital regulation in the U.S. closely follows the Basel Accords.¹¹

Ideally, an empirical test of the model should be conducted using detailed micro data reflecting each bank's individual assets, and therefore the credit risk and payoff of each asset. Unfortunately, due to business confidentiality, it is not possible to obtain such detailed data. Yet, we have data on the total holding of assets within each risk category characterized by a risk weighting under the Basel Accords – i.e., the total value of assets with 0%, 20%, 50%, or 100% risk. Thus, we presume each bank does business with all the corporations with senior unsecured debentures issued in the market. Thereafter, the credit risk and payoff of each bank's assets within one risk category are valued by those bonds' issuers with external ratings matching this risk category according to Basel II. Therefore, the empirical tests aim to investigate whether banks absorb the market-wide macro information on credit risk in their decision making on asset allocations as predicted by the model. We also test the model's prediction on the impact of a tightening capital regulation on banks' asset allocations.

4.1. Data

We obtain the bank financial data in the Uniform Bank Performance Report (UBPR), provided by the Federal Financial Institutions Examination Council. The source of the data in the UBPR is the Report of Condition and Report of Income (Call Reports), filed quarterly by each bank. To capture the variations in banks' appetites for risk and their decision making in relation to asset allocations, we use yearly data for the empirical tests. We exclude banks with assets of less than one billion dollars and with capital ratios or risk-based capital ratios equal to or larger than 25%, to ensure the relevance of the tests.

We use Standard & Poor's long-term credit ratings of the issuers of all senior unsecured corporate debentures in the market, and actual defaults among these issuers, to estimate the default probability of corporations with certain ratings, and the default correlation between these corporations and

¹¹General risk-based capital rules based on Basel I have been implemented since 1989; the standardized approach for general banking organizations and the advanced internal ratings-based approach for core banks, based on Basel II, have been implemented since 2008. Core banks are those with consolidated total assets of \$250 billion or more or with a consolidated total on-balance-sheet foreign exposure of \$10 billion or more (Treasury, the Federal Reserve System, and Federal Deposit Insurance Corporation, 2007, 2008).

other corporations holding other ratings. To value the payoff of assets within one risk category, we use average yield¹² of the bonds issued by the corporations holding the ratings corresponding to the risk category according to Basel II. These data are obtained from Standard & Poor's Capital IQ.

4.2. Asset categories

Under the standardized approach in Basel Accord II, assets are classified into different risk categories according to their external ratings when the ratings are applicable. Based on Basel Accord II and the requirements for Call Reports, the ratings corresponding to assets with 20%, 50%, and 100% risk are AAA to AA, A, and BBB to BB,¹³ respectively.

Yet, there are so few observations of defaults for corporations holding ratings AAA to AA, or A, that the result is a zero default rate in much of the sample period. Therefore, we combine assets with 20% and 50% risk, and assign them an average risk level of 35%. Consequently, there are three types of asset in the sample: risk-free assets, low-risk, low-earning assets, and high-risk, high-earning assets, with 0%, 35%, and 100% risk, respectively, consistent with the model in Section 3. Hence, the credit qualities of low- and high-risk assets are estimated by the market-wide bond issuers holding ratings AAA to A and BBB to BB, respectively.

4.3. Estimating default probability, default correlation, and payoff

We estimate the probabilities of default by empirical average cumulative default rates for a historical time period, as is commonly done by the major rating agencies. These historical default rates, based on issuer, give equal weights to all issuers in the calculation, regardless of differences in the nominal size of the bonds issued by each issuer.¹⁴ This approach is also cohort based, which tracks the default rates of firms with a certain rating on a given calendar date, and this pool of issuers is a *cohort*. We adopt the method of calculating average cumulative default rates with adjustment for rating withdrawals used by Moody's, as demonstrated by Cantor and Hamilton (2007). We then modify their methodology to estimate the default correlations. The methods are illustrated in Appendix C.

¹²Since the yield on a bond already accounts for its associated risk, using yield would underestimate the payoff of a type of asset. Nevertheless, the average yield in the market provides a macro level (actually a macro low bound) of the average payoff of the type of asset, which is comparable across time.

¹³According to the instructions for Call Reports, only the ratings above B are eligible for the ratings-based approach. Although, in accordance with the Dodd–Frank Wall Street Reform and Consumer Protection Act, the U.S. rules do not reference external credit ratings from 2010, in practice U.S. banks often use external ratings: see *Regulatory Consistency Assessment Programme (RCAP), Assessment of Basel III regulations – United States of America*, available at www.bis.org/bcbs/publ/d301.pdf.

¹⁴We use average default information on issuers instead of issues to obtain the low bound of the macro level of default, since the ratings of issues are generally not higher than that of their issuer.

Since there are relatively more default observations on a quarterly basis than on a monthly basis, especially for investment-grade corporations, we employ quarterly cohort spacing, which also produces more accurate estimates of default correlations. For the same reason, we also choose a longer investment horizon of four years.¹⁵ Then, it is assumed that, when banks' managers make portfolio asset choices, they hold expectations on default probabilities and default correlations based on the historical information during the previous four years.

To value the credit risk and payoffs of the assets with a certain risk type, and to preserve the creditworthiness of issuers, we only employ senior unsecured straight bonds: fixed-rate, U.S.-dollar bonds without any asset-backing or credit-enhancement – e.g., callability, puttability, sinking, or convertibility. We then estimate the payoff of each asset type at a date by an average of four-year-to-maturity yields¹⁶ at that date on all available straight bonds whose issuers hold certain ratings.

4.4. Results

Table 1 summarizes statistics of the data on the variables used in the empirical tests. *Proportion of high-risk assets* and *Proportion of low-risk assets* are the banks' actual shares of high-risk (100% risk) and low-risk (20% and 50% risk) assets among the risky (20%, 50%, and 100% risk) assets, respectively. Thus, they sum to one for each bank. Consistent with Figure 2, on average, banks allocate resources more to high-risk assets. The *Payoff* and *Default probability* of each type of risky asset and their *Default correlation* are the average macro credit information from our estimation. Consistent with the assumptions in the model, high-risk assets have higher probability of default and payoff compared to low-risk assets. These variables are used to test whether banks do absorb macro credit information in the direction that the model predicts.

Basel II is a dummy variable for the years since Basel II was implemented. We use *Basel II* as a proxy for a tightened capital requirement. This approximation is applicable since an asset's risk is valued by the type of its obligors under Basel I instead of by the actual risk of the obligors. For example, assets involving banks in OECD countries are classified as 20% risk category under Basel I; however, among these, those whose obligors have high credit risk will fall to 50% or 100% risk category under Basel II. *Capital becomes constrained*, a dummy, takes one for a bank that fulfills the capital requirement at $t - 1$ but not at t . We use these variables to test the model's predictions on the impact of a tightening capital requirement on banks' asset risk. The remaining variables are

¹⁵We perform robustness checks on two-year and three-year default rates and correlations: the results are qualitatively similar. The maximum possible length of the estimation window is four years because the data on actual defaults date back to 1998.

¹⁶The yield that represents one issuer is an average of yields on all available straight bonds issued by that corporation.

Table 1: Summary statistics of the data

Variable	Mean	Std. Dev.	Min	Max
Proportion of high-risk assets	61.83	15.10	0.08	99.31
Proportion of low-risk assets	38.17	15.10	0.69	99.92
Payoff of high-risk assets	5.80	1.69	2.49	8.46
Payoff of low-risk assets	3.62	1.41	1.19	5.58
Default prob. of high-risk assets	159.9	126.6	20.1	432.0
Default prob. of low-risk assets	14.75	12.97	0.00	43.52
Default correlation	1.55	1.77	0.00	5.83
Basel II	0.48	0.50	0	1
Capital becomes constrained	0.01	0.10	0	1
Risk-based capital ratio	12.98	2.74	1.97	24.96
Capital ratio	10.33	3.03	1.00	24.95
Nonearnings assets	9.45	3.72	0.34	35.66
Loan and lease allowance	1.70	1.71	0.00	100
Size	0.02	0.11	0.00	1.90
Core banks	0.01	0.08	0	1

Proportion of high-risk assets and *Proportion of low-risk assets* are the proportions of banks' allocations within risky assets. Therefore, for each bank, they sum to one. *Payoff*, *Default probability*, and *Default correlation* are the average macro credit information on the risky assets from our estimation. *Basel II*, a time dummy, takes one from 2008 when Basel II was implemented. *Capital becomes constrained*, a dummy variable, takes one for a bank whose ratio of capital to the total risk-weighted assets is more than 8% at time $t - 1$, but not at time t . The bank-level controls are *Risk-based capital ratio* (the ratio of total capital to total risk-weighted assets), *Capital ratio* (the ratio of total capital to total assets), *Nonearnings assets* (average nonearnings assets divided by average total assets), *Loan and lease allowance* (loan and lease allowance to total loans and leases), *Size* (total assets), and *Core banks* (a dummy variable for banks using the advanced internal ratings-based approach under Basel II). The estimated payoffs, default probabilities, and default correlation, used to calibrate the model, are valued at the beginning of the last quarter. The variables are valued in percentages, except for *Default probability* and *Default correlation* (valued in basis points), *Size* (valued in trillions of dollars), and dummy variables.

bank-level controls.

For the panel data, since estimated default probabilities, default correlation, and payoffs are macro information, we use bank fixed-effect regressions to capture the evolution of banks' choices of assets across time. Moreover, we adopt Hoechle's (2007) approach with Driscoll and Kraay (1998) standard errors to produce standard-error estimates consistent cross-sectionally and in terms of heteroscedasticity and autocorrelation.

Table 2 presents the results. The dependent variable is each bank's actual share of high-risk assets within the risky assets. The tests are conducted for the years when the excess cash flows over risk-free assets of high-risk assets and those of low-risk assets are positive (the condition for the propositions in the model¹⁷). We test only the majority of Proposition 1 and a proxy for Proposition 4 (denoted Proposition 4') in the model, since the conditions in Propositions 1(e), 2, and 3¹⁸ are

¹⁷The condition is not fulfilled in years 2003 and 2004.

¹⁸The condition $\vartheta_h > \vartheta_l$ in Proposition 3 is only met in 2011 and there is no bank whose capital is constrained in both 2007 and 2008 when there was a shift of regulation.

Table 2: Bank fixed-effect panel regressions on banks' allocations in high-risk assets

Variable	(1) Prop. 1	(2) Prop. 1	(3) Prop. 4'	(4) Prop. 4'
Payoff of high-risk assets	0.18* (0.08)	0.18 (0.17)		
Payoff of low-risk assets	-0.44*** (0.10)	-1.48*** (0.16)		
Default probability of high-risk assets	-0.03*** (0.00)	-0.03*** (0.00)		
Default probability of low-risk assets	0.49*** (0.01)	0.46*** (0.03)		
Default correlation	-3.15*** (0.11)	-2.82*** (0.27)		
Basel II			1.03 (1.25)	4.27*** (0.65)
D2008			2.50** (0.82)	-0.42 (0.28)
Capital becomes constrained			4.91*** (0.40)	-0.10 (0.77)
(D2008)×(Capital becomes constrained)			-5.31*** (0.61)	-1.68* (0.84)
Capital ratio		1.58*** (0.24)		1.72*** (0.18)
Loan and lease allowance		0.14* (0.06)		0.16** (0.07)
Risk-based capital ratio		-2.01*** (0.18)		-2.17*** (0.16)
Nonearnings assets		-0.48*** (0.09)		-0.50*** (0.08)
Size		-11.14*** (1.50)		-9.25*** (1.56)
Core banks		4.25*** (0.87)		3.82*** (1.11)
Constant	65.54*** (0.45)	82.92*** (1.17)	61.70*** (1.01)	75.27*** (1.35)
Observations	4,068	4,064	4,121	4,117
Number of groups	803	801	804	802
<i>F</i>	3,445	4,129	93.48	168.6
Prob > <i>F</i> within <i>R</i> ²	0 0.0964	0 0.292	9.46e - 07 0.0331	3.43e - 08 0.278

Payoff, *Default probability*, and *Default correlation* are the average macro credit information on the risky assets from our estimation. *Basel II*, a time dummy, takes one from 2008 when Basel II was implemented. *D2008* is a dummy for year 2008. *Capital becomes constrained*, a dummy variable, takes one for a bank whose risk-based capital ratio is more than 8% at time $t - 1$ but not at time t . The bank-level controls are *Risk-based capital ratio* (the ratio of total capital to total risk-weighted assets), *Capital ratio* (the ratio of total capital to total assets), *Nonearnings assets* (average nonearnings assets divided by average total assets), *Loan and lease allowance* (loan and lease allowance to total loans and lease), *Size* (total assets), and *Core banks* (a dummy for banks using the advanced internal ratings-based approach under Basel II). The variables are valued in percentages, except for *Default probability* and *Default correlation* (valued in basis points), *Size* (valued in trillions of dollars), and dummy variables. In parentheses are Driscoll and Kraay (1998) standard errors consistent cross-sectionally and in terms of heteroscedasticity and autocorrelation. *, **, and *** denote significance at the 10%, 5%, and 1% confidence levels, respectively.

not met due to the limited information on assets' actual payoffs and credit risk. Specifications (1) and (2) test Statements (a) to (d) of Proposition 1 for banks whose capital is not constrained by regulation. It is verified that banks do increase allocation of resources in high-risk assets when, in general, they have a higher average payoff or are on average less likely to default, and low-risk assets are substitutes for high-risk assets. We cannot test the exact version of Proposition 4, since the condition for assets' valuations ($\vartheta_h > \vartheta_l$) based on the macro information is not met in 2008 when Basel II was implemented. However, there are banks whose capital became constrained in 2008. Thus, we test the concept of an immediate risk reduction due to a tightening capital requirement, a proxy for Proposition 4. Specification (3) tells us that the banks whose capital became constrained due to the tightening capital requirement do invest less in high-risk assets, although the explanatory power of this is weaker when we add bank-level controls (Specification (4)). After accounting for banks' actual capital levels (*Risk-based capital ratio* and *Capital ratio*) and their valuations of asset risk (*Nonearnings assets* and *Loan and lease allowance*), on average, banks have still skewed the risky portfolio to high-risk assets since 2008 (*Basel II*).

In addition, bank-level controls reveal that the banks rich in capital in terms of capital ratio or those that reserve more loan and lease allowances (which cover expected losses of assets) invest more in high-risk assets to pursue earnings. Naturally, a higher risk-based capital ratio is associated with a higher proportion of high-risk assets. Yet, the larger banks or those with more nonearnings assets are more risk averse in that they invest proportionally less in high-risk assets. However, the core banks¹⁹ invest proportionally more in high-risk assets compared to other banks. This suggests an inconsistency between the internal ratings-based approach used by the core banks and the standardized approach used by other banks.

5. Conclusion

This paper explicitly investigates the credit risk of banks' assets and addresses banks' portfolio allocations under risk-based capital regulation. Adopting a methodology from the credit-portfolio-optimization literature allows us to disentangle the effects of risk-based capital regulation on the credit risk of banks' assets, which have hitherto not been explored in the banking literature.

Our model of portfolio allocation shows that, when risk-based capital regulation is binding, the risk weightings assigned by the regulator affect the original measures of risk and valuation of assets: namely, volatility around expected loss due to default risk and Sharpe ratio, respectively. This raises concerns that, if the risk weightings are not consistent with the assets' true risk measures, there

¹⁹By core banks, we mean banks with total assets of at least \$250 billion at the end of a year since 2008, identified as those using the advanced internal ratings-based approach under Basel II.

could be opportunities for regulatory arbitrage so that banks invest more in the assets with a high level of true risk but a low regulatory risk weighting. If the regulator imposes a new, and more stringent, regulation – i.e., a higher risk-based capital requirement – the bank whose capital is already constrained will skew the risky portfolio to high-risk, high-earning assets, while the bank whose capital will become constrained by the new regulation will do the opposite, investing less in high-risk assets – provided that the valuation (reward-to-regulatory-cost ratio) of high-risk assets is higher than that of low-risk assets. The immediate risk reduction of the latter is consistent with the literature.

The empirical tests support the model’s predictions applicable in the dataset. Due to business confidentiality, detailed data on each asset of each bank are not available. Yet, the average macro information on payoffs and credit risk of assets in each risk category that we estimate is very helpful in explaining banks’ actual asset choices. The tests support the prediction of a “flight to earnings” and avoiding default risk (Proposition 1) and the immediate risk reduction for banks whose capital becomes constrained due to a tightening capital regulation (Proposition 4’). In addition, our results also reveal the characteristics of banks with different levels of risk taking. Particularly, the core banks, which use the internal ratings-based approach under Basel II, take more asset risks compared to other banks, which suggests an inconsistency between the internal ratings-based approach and the standard approach: That is, the regulation is softer for the core banks using the approach of applying lower capital requirements for the same level of asset risk.

Our study contributes to the literature and to ongoing debates on banks’ risk taking and capital regulation from the perspective of credit risk, which, it is hoped, paves a way for future research on banks’ asset risk. For example, our analysis could be extended by using detailed data on assets at the individual-bank level.

Appendix

A. Optimal portfolio allocation when the capital requirement is not binding

A.1. Proof of Proposition 1

(a) Within the risky fund, the bank invests proportionally more (less) in high-risk assets, ceteris paribus, if its payoff, C_h , increases (decreases).

Proof:

$$\begin{aligned}
\frac{\partial \omega_h^*}{\partial C_h} &= \frac{\partial \frac{A_h^*}{A_h^* + A_l^*}}{\partial C_h} \\
&= \frac{(A_h^*)^2 \sqrt{p_h(1-p_h)}}{(A_h^* + A_l^*)^2 (E_h \sqrt{V_l} - \rho E_l \sqrt{V_h})^2 \sqrt{V_l}} \{ \rho (E_l \sqrt{V_h} - E_h \sqrt{V_l})^2 \\
&\quad + (1-\rho) E_l \sqrt{V_h V_l} [\rho(1-p_h)(1+C_h) + 2(1+r_f) - (1-p_h)(1+C_h)] \} \\
&> 0
\end{aligned} \tag{A.1}$$

since $1 > \rho > 0$ and $2(1+r_f) \geq 2 > (1-p_h)(1+C_h)$. Then $\frac{\partial \omega_h^*}{\partial C_h}$ is positive. That is, the bank invests proportionally more (less) in high-risk assets, *ceteris paribus*, if its yield C_h increases (decreases).
(b) *Within the risky fund, the bank invests proportionally more (less) in high-risk assets, ceteris paribus, if its probability of default, p_h , decreases (increases).*

Proof:

$$\begin{aligned}
\frac{\partial \omega_h^*}{\partial p_h} &= \frac{\partial \frac{A_h^*}{A_h^* + A_l^*}}{\partial p_h} \\
&= - \frac{(A_h^*)^2}{(A_h^* + A_l^*)^2 (E_h V_l - \rho E_l \sqrt{V_h V_l})^2} \{ (1-\rho^2) E_l V_h V_l (1+C_h) \\
&\quad + \frac{V_l}{2} (1-2p_h)(1+C_h)^2 [E_l \sqrt{V_h} (SR_h - \rho SR_l) + E_h \sqrt{V_l} (SR_l - \rho SR_h)] \} \\
&< 0
\end{aligned} \tag{A.2}$$

since $p_h < 0.5$, $SR_h - \rho SR_l > 0$ and $SR_l - \rho SR_h > 0$ (as $A_h^* > 0$ and $A_l^* > 0$). Hence, $\frac{\partial \omega_h^*}{\partial p_h} < 0$. That is, the bank invests proportionally more (less) in high-risk assets, *ceteris paribus*, if its probability of default, p_h , decreases (increases).

(c) *Within the risky fund, the bank invests proportionally more (less) in high-risk assets, ceteris paribus, if the payoff of low-risk asset, C_l , decreases (increases).*

Proof: It can be shown as in the proof for statement (a).

(d) *Within the risky fund, the bank invests proportionally more (less) in high-risk asset, ceteris paribus, if the default probability of low-risk asset, p_l , increases (decreases).*

Proof: It can be shown as in the proof for statement (b).

(e) *Within the risky fund, the bank invests proportionally more (less) in high-risk assets, ceteris paribus, if the default correlation, ρ , increases (decreases), given that $SR_h > SR_l$.*

Proof:

$$\frac{\partial \omega_h^*}{\partial \rho} = \frac{\partial \frac{A_h^*}{A_h^* + A_l^*}}{\partial \rho} = \frac{(A_h^*)^2 V_h \sqrt{V_h V_l}}{(A_h^* + A_l^*)^2 (E_h \sqrt{V_l} - \rho E_l \sqrt{V_h})^2} (SR_h^2 - SR_l^2) > 0 \quad (\text{A.3})$$

if $SR_h > SR_l$. Then the bank invests proportionally more (less) in high-risk assets, *ceteris paribus*, if the default correlation, ρ , increases (decreases), given that $SR_h > SR_l$.

A.2. Proof of Proposition 2

(a) The bank invests more (less) in the risk-free fund, *ceteris paribus*, if the risk-free rate, r_f , increases (decreases).

Proof:

$$\frac{\partial G^*}{\partial r_f} = (D + K) \frac{\sqrt{V_l}(X_h \sqrt{V_l} - \rho X_l \sqrt{V_h}) + \sqrt{V_h}(X_l \sqrt{V_h} - \rho X_h \sqrt{V_l})}{M} > 0 \quad (\text{A.4})$$

since $X_h \sqrt{V_l} > \rho X_l \sqrt{V_h}$ and $X_l \sqrt{V_h} > \rho X_h \sqrt{V_l}$ (as $A_h^* > 0$ and $A_l^* > 0$), where $M \equiv X_h^2 V_l + X_l^2 V_h - 2\rho X_h X_l \sqrt{V_h V_l} + (1 - \rho^2) V_h V_l = X_h \sqrt{V_l}(X_h \sqrt{V_l} - \rho X_l \sqrt{V_h}) + X_l \sqrt{V_h}(X_l \sqrt{V_h} - \rho X_h \sqrt{V_l}) + (1 - \rho^2) V_h V_l > 0$.

(b) The bank invests more (less) in the risk-free fund, *ceteris paribus*, if the payoff of the high-risk asset, C_h , increases (decreases), given that $\rho \sqrt{V_h} \geq \sqrt{V_l}$.

Proof:

$$\begin{aligned} \frac{\partial G^*}{\partial C_h} &= \frac{(a - B) \sqrt{V_l}(X_h \sqrt{V_l} - \rho X_l \sqrt{V_h})}{M(1 + C_h)} \\ &+ \frac{(a - B) \sqrt{V_l}(1 + r_f)(X_h \sqrt{V_l} - \rho X_l \sqrt{V_h}) \{X_l \sqrt{V_h}(\sqrt{V_h} - \rho \sqrt{V_l}) + \sqrt{V_l}(X_h \sqrt{V_l} - \rho X_l \sqrt{V_h})\}}{M^2(1 + C_h)} \\ &+ \frac{(a - B) \sqrt{V_l}(1 + r_f) \{ \sqrt{V_h V_l}(X_l \sqrt{V_h} - \rho X_h \sqrt{V_l})(X_h - X_l) + (1 - \rho^2) V_h V_l (\rho \sqrt{V_h} - \sqrt{V_l}) \}}{M^2(1 + C_h)} \end{aligned} \quad (\text{A.5})$$

Since $a > B$, $X_h \sqrt{V_l} > \rho X_l \sqrt{V_h}$, $\sqrt{V_h} > \rho \sqrt{V_l}$ (as $V_h > V_l$ and $\rho < 1$), $X_l \sqrt{V_h} > \rho X_h \sqrt{V_l}$, and $X_h > X_l$, $\frac{\partial G^*}{\partial C_h} > 0$, given that $\rho \sqrt{V_h} \geq \sqrt{V_l}$.

(c) The bank invests more (less) in the risk-free fund, *ceteris paribus*, if the default probability of high-risk asset, p_h , decreases (increases), given that $\frac{\rho \sqrt{V_h}}{\sqrt{V_l}} \geq \frac{2X_h}{X_h + X_l}$.

Proof:

$$\begin{aligned}
\frac{\partial G^*}{\partial p_h} &= \frac{-(a-B)(1+C_h)\sqrt{V_1}}{M^2} \\
&\times \left\{ \sqrt{V_h V_1} (X_1 \sqrt{V_h} - \rho X_h \sqrt{V_1}) (X_h - X_1) + (X_h \sqrt{V_1} - \rho X_1 \sqrt{V_h})^2 \sqrt{V_1} \right. \\
&+ X_1 \sqrt{V_h} (X_h \sqrt{V_1} - \rho X_1 \sqrt{V_h}) (\sqrt{V_h} - \rho \sqrt{V_1}) + (1 - \rho^2) V_h V_1 (\rho \sqrt{V_h} - \sqrt{V_1}) \left. \right\} \\
&- \frac{(a-B)(1-2p_h)\sqrt{V_1}}{2\sqrt{V_h} M^2} \\
&\times \left\{ (X_h - X_1) \left[X_1 \sqrt{V_h} (X_h \sqrt{V_1} - \rho X_1 \sqrt{V_h}) + X_h \sqrt{V_1} (X_1 \sqrt{V_h} - \rho X_h \sqrt{V_1}) \right] \right. \\
&+ \left. \sqrt{V_h} V_1 (1 - \rho^2) \left[(X_h + X_1) \rho \sqrt{V_h} - 2X_h \sqrt{V_1} \right] \right\}.
\end{aligned} \tag{A.6}$$

The term within the first pair of large braces is negative given $\rho \sqrt{V_h} \geq \sqrt{V_1}$ since $a > B$, $X_1 \sqrt{V_h} > \rho X_h \sqrt{V_1}$, $X_h > X_1$, $X_h \sqrt{V_1} > \rho X_1 \sqrt{V_h}$, and $\sqrt{V_h} > \rho \sqrt{V_1}$. The term within the second pair of large braces is negative given that $\frac{\rho \sqrt{V_h}}{\sqrt{V_1}} \geq \frac{2X_h}{X_h + X_1}$, since $p_h < 0.5$. Therefore, $\frac{\partial G^*}{\partial p_h} < 0$ given that

$$\frac{\rho \sqrt{V_h}}{\sqrt{V_1}} \geq \frac{2X_h}{X_h + X_1}.$$

(d) The bank invests more (less) in the risk-free fund, *ceteris paribus*, if the payoff of the low-risk asset, C_l , decreases (increases), given that $\frac{\rho \sqrt{V_h}}{\sqrt{V_1}} \geq \frac{X_h + 1 + r_f}{X_1 + 1 + r_f}$, $\rho^2 X_h \geq X_1$, $\rho(1 + r_f) > X_1$ and

$$\frac{X_h^2}{X_1^2} \geq \frac{V_h - \rho \sqrt{V_h V_1}}{V_1 - \rho^2 V_1}.$$

Proof:

$$\begin{aligned}
\frac{\partial G^*}{\partial C_l} &= -\frac{a-B}{M^2(1+C_l)} \left\{ \rho X_h^2 \sqrt{V_h} V_1 (X_h \sqrt{V_1} - \rho X_1 \sqrt{V_h}) \right. \\
&+ (1 - \rho^2) V_h V_1 \sqrt{V_h} \left[\sqrt{V_h} (1 + r_f - X_1) - \rho \sqrt{V_1} (1 + r_f - X_h) \right] \\
&+ X_h X_1 V_h \sqrt{V_1} \left[(1 + r_f) (\rho \sqrt{V_h} - \sqrt{V_1}) - (X_h \sqrt{V_1} - \rho X_1 \sqrt{V_h}) \right] \\
&+ (1 + r_f) V_h \left[(1 - \rho^2) X_h^2 V_1 - X_1^2 \sqrt{V_h} (\sqrt{V_h} - \rho \sqrt{V_1}) \right] \\
&+ \sqrt{V_h} (X_1 \sqrt{V_h} - \rho X_h \sqrt{V_1}) \\
&\times \left. \left[(1 + r_f) X_h \sqrt{V_1} (\rho \sqrt{V_h} - \sqrt{V_1}) - X_1 \sqrt{V_h} (X_1 \sqrt{V_h} - \rho X_h \sqrt{V_1}) \right] \right\}.
\end{aligned} \tag{A.7}$$

Within the large braces, the term in the first row is positive since $X_h \sqrt{V_1} > \rho X_1 \sqrt{V_h}$; the term in the second row is positive since $\sqrt{V_h} > \rho \sqrt{V_1}$ and $1 + r_f > X_h > X_1$; the term in the third row

is nonnegative given that $\frac{\rho\sqrt{V_h}}{\sqrt{V_l}} \geq \frac{X_h + 1 + r_f}{X_l + 1 + r_f}$; the term in the fourth row is nonnegative given that $\frac{X_h^2}{X_l^2} \geq \frac{V_h - \rho\sqrt{V_h V_l}}{V_l - \rho^2 V_l}$ since $\sqrt{V_h} > \rho\sqrt{V_l}$. In addition, $\frac{X_h^2}{X_l^2} \geq \frac{V_h - \rho\sqrt{V_h V_l}}{V_l - \rho^2 V_l}$ implies that $X_h\sqrt{V_l} > X_l\sqrt{V_h}$ if $\rho\sqrt{V_h} > \sqrt{V_l}$. Then the term in the last row is positive if $(1 + r_f)(\rho\sqrt{V_h} - \sqrt{V_l}) \geq X_l\sqrt{V_h} - \rho X_h\sqrt{V_l}$, which is true given that $\rho\sqrt{V_h} > \sqrt{V_l}$, $\rho(1 + r_f) > X_l$ and $\rho^2 X_h \geq X_l$.

Therefore, $\frac{\partial G^*}{\partial C_1} < 0$ given that $\frac{\rho\sqrt{V_h}}{\sqrt{V_l}} \geq \frac{X_h + 1 + r_f}{X_l + 1 + r_f}$, $\rho^2 X_h \geq X_l$, $\rho(1 + r_f) > X_l$, and $\frac{X_h^2}{X_l^2} \geq \frac{V_h - \rho\sqrt{V_h V_l}}{V_l - \rho^2 V_l}$, as $a > B$.

(e) The bank invests more (less) in the risk-free fund, *ceteris paribus*, if the default probability of the low-risk asset, p_l , increases(decreases), given that $\rho\sqrt{V_h} \geq \sqrt{V_l}$ and $SR_l^2 \leq 1 - \rho^2$.

Proof:

$$\begin{aligned} \frac{\partial G^*}{\partial p_l} &= \frac{(a - B)(1 + C_1)\sqrt{V_h}}{M^2} \\ &\times \left\{ X_h\sqrt{V_l}(\rho\sqrt{V_h} - \sqrt{V_l})(2X_l\sqrt{V_h} - \rho X_h\sqrt{V_l}) + (1 - \rho^2)X_h^2 V_l\sqrt{V_h} \right. \\ &+ \left. V_h(\sqrt{V_h} - \rho\sqrt{V_l})[(1 - \rho^2)V_l - X_l^2] \right\} \\ &+ \frac{(a - B)(1 - 2p_l)\sqrt{V_h}}{2\sqrt{V_l}M^2} \\ &\times \left\{ (X_h - X_l) \left[X_l\sqrt{V_h}(X_h\sqrt{V_l} - \rho X_l\sqrt{V_h}) + X_h\sqrt{V_l}(X_l\sqrt{V_h} - \rho X_h\sqrt{V_l}) \right] \right. \\ &+ \left. \sqrt{V_l}V_h(1 - \rho^2) \left[\rho\sqrt{V_l}(X_h - X_l) + 2(X_l\sqrt{V_h} - \rho X_h\sqrt{V_l}) \right] \right\}. \end{aligned} \tag{A.8}$$

Within the first pair of large braces, the term in the first row is positive given $\rho\sqrt{V_h} \geq \sqrt{V_l}$, since $X_l\sqrt{V_h} > \rho X_h\sqrt{V_l}$; the term in the second row is positive given that $X_l^2 \leq (1 - \rho^2)V_l$ - i.e., $SR_l^2 \leq 1 - \rho^2$ - since $\sqrt{V_h} > \rho\sqrt{V_l}$. Within the second pair of large braces, the term in the first row is positive since $X_h > X_l$, $X_h\sqrt{V_l} > \rho X_l\sqrt{V_h}$, and $X_l\sqrt{V_h} > \rho X_h\sqrt{V_l}$, which also implies that the term in the second row is positive.

Therefore, $\frac{\partial G^*}{\partial p_l} > 0$ given that $\rho\sqrt{V_h} \geq \sqrt{V_l}$ and $SR_l^2 \leq 1 - \rho^2$, since $a > B$ and $p_l < 0.5$.

(f) The bank invests more (less) in the risk-free fund, *ceteris paribus*, if the default correlation, ρ , increases (decreases), given that $\rho\sqrt{V_h} \leq \sqrt{V_l}$ and $SR_h \geq SR_l$.

Proof:

$$\begin{aligned} \frac{\partial G^*}{\partial \rho} &= \frac{(a-B)\sqrt{V_h V_l}}{M^2} \\ &\times \left\{ \sqrt{V_h V_l} \left[(\sqrt{V_h} - \rho\sqrt{V_l})(X_h\sqrt{V_l} - \rho X_l\sqrt{V_h}) + (\sqrt{V_l} - \rho\sqrt{V_h})(X_l\sqrt{V_h} - \rho X_h\sqrt{V_l}) \right] \right. \\ &\left. + (X_h - X_l)(X_h^2 V_l - X_l^2 V_h) \right\} \end{aligned} \quad (\text{A.9})$$

Within the large braces, the term in the first row is positive given that $\rho\sqrt{V_h} \leq \sqrt{V_l}$, since $\sqrt{V_h} > \rho\sqrt{V_l}$, $X_h\sqrt{V_l} > \rho X_l\sqrt{V_h}$, and $X_l\sqrt{V_h} > \rho X_h\sqrt{V_l}$; the term in the second row is nonnegative given that $X_h^2 V_l \geq X_l^2 V_h$: i.e., $SR_h \geq SR_l$, since $X_h > X_l$. Therefore, $\frac{\partial G^*}{\partial \rho} > 0$ given that $\rho\sqrt{V_h} \leq \sqrt{V_l}$ and $SR_h \geq SR_l$.

B. Impact of risk-based capital regulation

B.1. How the efficient frontier changes

For a bank whose capital is not constrained by regulation, since it still faces the regulatory capital constraint ($W_h A_h + W_l A_l \leq \frac{K}{k}$, Equation (7c)), for each risky fund P , there is a constraint for the portfolio weight of the low-risk asset, $\omega_l \leq \frac{1}{A_h + A_l} \left(\frac{K}{W_l k} - \frac{W_h}{W_l} A_h \right)$.

Then the expected value and variance of random cash flow for each risky fund P are:

$$\begin{aligned} E[\widetilde{CF}_P] &= \omega_h(1 - p_h)(1 + C_h) + \omega_l(1 - p_l)(1 + C_l) \\ &\leq \frac{1}{A_h + A_l} [A_h(1 - p_h)(1 + C_h) + \left(\frac{K}{kW_l} - \frac{W_h}{W_l} A_h \right) (1 - p_l)(1 + C_l)] \\ &= \omega_h \left(E[\widetilde{CF}_h] - \frac{W_h}{W_l} E[\widetilde{CF}_l] \right) + \frac{K}{(A_h + A_l)kW_l} E[\widetilde{CF}_l] \equiv E[\widetilde{CF}_P]_{\text{bound}} \end{aligned} \quad (\text{B.1})$$

and

$$\begin{aligned} \text{Var}[\widetilde{CF}_P] &= \omega_h^2 V_h + \omega_l^2 V_l + 2\omega_h \omega_l \rho \sqrt{V_h V_l} \\ &\leq \frac{1}{(A_h + A_l)^2} [A_h^2 V_h + \left(\frac{K}{kW_l} - \frac{W_h}{W_l} A_h \right)^2 V_l + 2A_h \left(\frac{K}{kW_l} - \frac{W_h}{W_l} A_h \right) \rho \sqrt{V_h V_l}] \\ &= \omega_h^2 \left(V_h + \frac{W_h^2}{W_l^2} V_l - 2\frac{W_h}{W_l} \rho \sqrt{V_h V_l} \right) \\ &\quad + 2\frac{\omega_h K \sqrt{V_l}}{(A_h + A_l)kW_l} \left(\rho \sqrt{V_h} - \frac{W_h}{W_l} \sqrt{V_l} \right) + \frac{K^2 V_l}{k^2 W_l^2 (V_h + V_l)^2} \\ &\equiv \text{Var}[\widetilde{CF}_P]_{\text{bound}}, \end{aligned} \quad (\text{B.2})$$

where $E[\widetilde{CF}_P]_{\text{bound}}$ and $\text{Var}[\widetilde{CF}_P]_{\text{bound}}$ are upper limits for $E[\widetilde{CF}_P]$ and $\text{Var}[\widetilde{CF}_P]$, respectively.

Furthermore, as k increases, $E[\widetilde{CF}_P]_{\text{bound}}$ decreases, and $\text{Var}[\widetilde{CF}_P]_{\text{bound}}$ decreases since $\frac{\partial \text{Var}[\widetilde{CF}_P]_{\text{bound}}}{\partial k} = \frac{2K\sqrt{V_1}}{(A_h + A_l)^2 k^2 W_1^2} \left[\left(A_h W_h - \frac{K}{k} \right) \sqrt{V_1} - A_h \rho W_1 \sqrt{V_h} \right] < 0$ (as $A_h W_h \leq \frac{K}{k}$ and $\rho > 0$).

When the regulator imposes a new and more stringent capital requirement – i.e., k increases – for some risky funds, the expected value and variance of their random cash flows are higher than the respective upper limits. Hence, these risky funds are out of reach and the efficient frontier for the bank shrinks from the top and right.

For a bank whose capital is already constrained by regulation, the expected values and variances of cash flows of all the risky funds on its efficient frontier reach their respective upper limits, which decrease as k increases. Therefore, the new regulation forces the efficient frontier to move downward and to the left.

B.2. Proof of Proposition 3

(a) When the bank's capital is constrained by regulation and the regulator imposes a new and more stringent regulation with a higher capital requirement, k , within the risky fund, the bank invests proportionally more in high-risk assets, *ceteris paribus*, given that $\vartheta_h > \vartheta_l$.

Proof:

$$\begin{aligned} \frac{\partial \omega_{bh}^*}{\partial k} &= \frac{\partial \frac{A_{bh}^*}{A_{bh}^* + A_{bl}^*}}{\partial k} = \\ &= \frac{A_{bh}^{*2} K \sqrt{V_h} (a - B) (\varphi_l SR_h - \varphi_h SR_l) [(\varphi_l SR_h - \varphi_h SR_l)^2 + (\varphi_h^2 - 2\rho\varphi_h\varphi_l + \varphi_l^2)]}{(A_{bh}^* + A_{bl}^*)^2 \sqrt{V_1} \{k(a - B)\varphi_l(\varphi_l SR_h - \varphi_h SR_l) - K[SR_l(\varphi_l SR_h - \varphi_h SR_l) + (\rho\varphi_l - \varphi_h)]\}^2} \\ &> 0, \end{aligned} \quad (\text{B.3})$$

given that $\varphi_l SR_h > \varphi_h SR_l$ – i.e., $\vartheta_h > \vartheta_l$ – since $a > B$ and $\varphi_h^2 - 2\rho\varphi_h\varphi_l + \varphi_l^2 = (\varphi_h - \varphi_l)^2 + 2(1 - \rho)\varphi_h\varphi_l > 0$.

(b) When the bank's capital is constrained by regulation and the regulator imposes a new and more stringent regulation with a higher capital requirement, k , the bank invests more in the risk-free fund, *ceteris paribus*, given that $\vartheta_h \geq \vartheta_l$ and $\rho\varphi_l \geq \varphi_h$.

Proof:

$$\begin{aligned} \frac{\partial G_b^*}{\partial k} &= \frac{K}{k^2 W_1} + \frac{W_h - W_l}{W_1} \frac{\partial A_{bh}^*}{\partial k} \\ &= \frac{K}{k^2 W_1} + \frac{W_h - W_l}{W_1} \frac{K \{SR_l(\varphi_l SR_h - \varphi_h SR_l) + (\rho\varphi_l - \varphi_h)\}}{k^2 (\varphi_h^2 - 2\rho\varphi_h\varphi_l + \varphi_l^2) \sqrt{V_h} + k^2 (\varphi_l SR_h - \varphi_h SR_l)^2 \sqrt{V_h}} > 0, \end{aligned} \quad (\text{B.4})$$

given that $\varphi_1 SR_h \geq \varphi_h SR_1$ – i.e., $\vartheta_h \geq \vartheta_1$ – and $\rho \varphi_1 \geq \varphi_h$, since $W_h > W_1$.

B.3. Proof of Proposition 4

Proposition 4. *When the regulator imposes a higher risk-based capital requirement k and the bank's capital will become constrained by the new regulation, within the risky fund, the bank invests proportionally less in high-risk assets, if $\vartheta_h > \vartheta_1$.*

Proof:

$$\begin{aligned} \frac{A_1^*/A_h^*}{A_{bl}^*/A_{bh}^*} &= \\ \frac{SR_1 - \rho SR_h}{SR_h - \rho SR_1} \frac{(a-B)\varphi_1(\varphi_1 SR_h - \varphi_h SR_1) - \frac{K}{k}[SR_1(\varphi_1 SR_h - \varphi_h SR_1) + (\rho \varphi_1 - \varphi_h)]}{(a-B)\varphi_h(\varphi_h SR_1 - \varphi_1 SR_h) - \frac{K}{k}[SR_h(\varphi_h SR_1 - \varphi_1 SR_h) + (\rho \varphi_h - \varphi_1)]} & \quad (B.5) \\ < 1 - \text{i.e., } \frac{A_1^*}{A_h^*} < \frac{A_{bl}^*}{A_{bh}^*} - \text{given that } \vartheta_h > \vartheta_1, \end{aligned}$$

since

$$\begin{aligned} & (SR_1 - \rho SR_h) \left\{ (a-B)\varphi_1(\varphi_1 SR_h - \varphi_h SR_1) - \frac{K}{k}[SR_1(\varphi_1 SR_h - \varphi_h SR_1) + (\rho \varphi_1 - \varphi_h)] \right\} \\ & - (SR_h - \rho SR_1) \left\{ (a-B)\varphi_h(\varphi_h SR_1 - \varphi_1 SR_h) - \frac{K}{k}[SR_h(\varphi_h SR_1 - \varphi_1 SR_h) + (\rho \varphi_h - \varphi_1)] \right\} \\ & = (\varphi_1 SR_h - \varphi_h SR_1) \left(A_h^* W_h + A_1^* W_1 - \frac{K}{k} \right) (SR_h^2 - 2\rho SR_h SR_1 + SR_1^2 + 1 - \rho^2) < 0, \end{aligned}$$

given that $\varphi_1 SR_h > \varphi_h SR_1$ – i.e., $\vartheta_h > \vartheta_1$ – because $A_h^* W_h + A_1^* W_1 < \frac{K}{k}$, $SR_h^2 - 2\rho SR_h SR_1 + SR_1^2 > 0$ and $\rho^2 < 1$.

Therefore, $\omega_h^* = \frac{A_h^*}{A_h^* + A_1^*} = \frac{1}{1 + A_1^*/A_h^*} > \omega_{bh}^* = \frac{A_{bh}^*}{A_{bh}^* + A_{bl}^*} = \frac{1}{1 + A_{bl}^*/A_{bh}^*}$ if $\vartheta_h > \vartheta_1$. That is, when its capital becomes constrained by the capital regulation, the bank invests proportionally more in the asset with a lower reward-to-regulatory-cost ratio.

C. Estimating default probability and default correlation

We adopt the method of calculating average cumulative default rates with adjustment for rating withdrawals used by Moody's, as demonstrated by Cantor and Hamilton (2007).

A cumulative default rate for an investment horizon of length T , denoted as $D(T)$ is formulated

as

$$D_y(T) = d_y(1) + d_y(2)[1 - d_y(1)] + d_y(3)[(1 - d_y(1))(1 - d_y(2))] + \dots \\ + d_y(T)\left(\prod_{t=1}^{T-1} [1 - d_y(t)]\right) = 1 - \prod_{t=1}^T [1 - d_y(t)], \quad (\text{C.1})$$

where $d_y(t)$ is the marginal default rate in the time interval t^{20} for a cohort of issuers formed on date y holding a certain rating and calculated as $d_y(t) = \frac{x_y(t)}{n_y(t)}$, where x is the number of defaults and n is the effective size of the cohort adjusted for rating withdrawals. As displayed, the cumulative default rate is essentially a discrete-time approximation of the nonparametric continuous-time-hazard-rate approach and a conditional probability.

We adopt average cumulative default rates, where the average is taken over many cohort periods, to estimate default probabilities in our study. The average cumulative default rate for an investment horizon of length T , denoted as $\bar{D}(T)$, is derived from the weighted average marginal default rates, $\bar{d}(t)$, where the average is taken over all the available cohort marginal default rates in the historical data set Y .

$$\text{Then } \bar{D}(T) = 1 - \prod_{t=1}^T [1 - \bar{d}(t)], \text{ where } \bar{d}(t) = \frac{\sum_{y \in Y} x_y(t)}{\sum_{y \in Y} n_y(t)}.$$

As we estimate the default correlations, we modify the above methodology accordingly.

The pair-wise default probability, for one corporation with rating 1 and another with rating 2 in the time interval t , is $\frac{x_y^1(t)x_y^2(t)}{n_y^1(t)n_y^2(t)}$, where x_y^1 and x_y^2 are the numbers of defaults for cohorts of issuers holding rating 1 and 2 formed on date y respectively, and n_y^1 and n_y^2 are the corresponding effective sizes of the cohorts. Then, the average pair-wise default rate in the time interval t over all available cohorts is $\bar{d}_{12}(t) = \frac{\sum_{y \in Y} x_y^1(t)x_y^2(t)}{\sum_{y \in Y} n_y^1(t)n_y^2(t)}$.

Hence, we could estimate the default correlation for the investment horizon of length T by an average over all available marginal default correlations in the data set Y :

$$\bar{\rho}_{12}(T) = \frac{1}{T-1} \sum_{t=1}^{T-1} \rho_{12}(t) = \frac{1}{T-1} \sum_{t=1}^{T-1} \frac{\bar{d}_{12}(t) - \bar{d}_1(t)\bar{d}_2(t)}{\sqrt{\bar{d}_1(t)[1 - \bar{d}_1(t)]\bar{d}_2(t)[1 - \bar{d}_2(t)]}} \quad (\text{C.2})$$

where $\rho_{12}(t)$ is the marginal default correlation in the time interval t .²¹

²⁰For example, in the first period after the formation of a cohort, $t = 1$; in the second period after the formation of a cohort, $t = 2$; etc..

²¹ $\rho_{12}(T)$ is excluded in the calculation of $\bar{\rho}_{12}(T)$ because there is only one observation at T resulting from one cohort left with a certain rating, which results in zero correlation.

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