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How Flexible are the Inflation Targets? A Bayesian MCMC Estimator of the Long Memory Parameter in a State Space Model

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Abstract: Several central banks have adopted inflation targets. The implementation of these targets is flexible; the central banks aim to meet the target over the long term but allow inflation to deviate from the target in the short-term in order to avoid unnecessary volatility in the real economy. In this paper, we propose modeling the degree of flexibility using an AFRIMA model. Under the assumption that the central bankers control the long-run inflation rates, the fractional integration order captures the flexibility of the inflation targets. A higher integration order is associated with a more flexible target. Several estimators of the fractional integration order have been proposed in the literature. Grassi and Magistris (2011) show that a state-based maximum likelihood estimator is superior to other estimators, but our simulations show that their finding is over-biased for a nearly non-stationary time series. We resolve this issue by using a Bayesian Monte Carlo Markov Chain (MCMC) estimator. Applying this estimator to inflation from six inflation-targeting countries for the period 1999M1 to 2013M3, we find that inflation is integrated of order 0.8 to 0.9 depending on the country. The inflation targets are thus implemented with a high degree of flexibility.

Keywords: *fractional integration, inflation-targeting, state space model*

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1 Introduction

Several central banks around the world have switched from a fixed exchange rate regime to an inflation-targeting regime during the last 25 years. The inflation target is either a constant or a range and is calculated using the year-on-year change in the consumer price index (CPI). Facing a trade-off between inflation stability and real economic stability in the short run (Svensson, 1997), most central banks have chosen a flexible inflation target to maintain real economic stability. In the words of the previous Governor of the Bank of England, Mervyn King, central bankers are not "inflation nutters". Consequently, inflation contains long swings around its mean. Moreover, there is evidence that inflation is covariance non-stationary yet mean-reverting, i.e., fractionally integrated (Hassler and Wolters, 1995; Caggio and Castelnuovo, 2011).

The flexibility of the inflation targets is commonly modeled using a Taylor rule (Clarida, Gali and Gertler, 1998; Cobion and Goldstein, 2012). According to the Taylor rule, the central bankers set the interest rate based on the deviation of inflation from the target and the size of the output gap. Empirical estimation of the Taylor rule is difficult because it requires both an estimate of the long-run equilibrium real interest rate and an estimate of the output gap. The model also assumes that central bankers do not consider other variables when making interest rate decisions (Svensson, 2003).

An alternative approach is to estimate the degree of flexibility by modeling inflation with an ARFIMA model. Here, AR and MA components capture the short-run dynamics of the inflation target and the fractional integration order of the long-run dynamics. Using the fractional integration order under the assumption that the central bank controls the long-run inflation rate, we can estimate how flexibly the inflation targets are implemented. A higher integration order indicates that the central banker is more willing to allow inflation to deviate from its target - thus

the inflation target is more flexible, and conversely, the central banker is then more willing to allow inflation to deviate from its target. By analyzing the fractional integration order in an ARFIMA model, it is possible to determine whether the central bankers are "inflation nutters", or, flexible in their implementation of the inflation target.

Several estimators of ARFIMA models have been proposed. These estimators include the parametric method, which is based on the maximum likelihood function (Fox and Taqqu, 1986; Sowell, 1992; Giraitis and Taqqu, 1999), and the regression-based approach in spectral domain (Geweke and Porter-Hudak, 1983). Additional estimators include the semi-parametric (Robison 1995a, b; Shimotsu and Phillips, 2005), and the wavelet-based semi-parametric (McCoy and Walden 1996; Jensen 2004).

Chan and Palma (1998) established a theoretical foundation to estimate the ARFIMA model with an approximate maximum likelihood estimation (MLE)-based state space model. The authors truncate the infinite AR or MA representations of the ARFIMA model into finite lags and calculate the approximate maximum likelihood using the Kalman filter. Chan and Palma (1998) show that the approximate MLE-based state space model has desirable asymptotic properties and a rapid converging rate. Recently, Grassi and Magistris (2011) conducted a simulation study to compare the state space model-based long memory estimation with several widely applied parametric and semi-parametric methods. Grassi and Magistris (2011) show that compared with the other estimations, the state space model method is robust to the *t* distribution and is missing value, measurement error and level shift. However, both Chan and Palman (1998) and Grassi and Magistris (2011) concentrate mainly on estimating the fractional difference parameter d in stationary time series when $0 < d < 0.4$, and they ignore the estimation of the nuisance parameter σ_ε, which is the variance of the disturbance term. Simulations in our paper

include a wide range of d and σ_ε, where 0 < d < 1 and σ_ε =1, 3, 5. Our results show that the state space-based estimators perform well in the near integrated case where d is in the neighborhood of 1. However, the estimates for both d and σ_ε are over-biased in the nearly covariance non-stationary case, i.e., when $0.4 < d < 0.5$.

In some cases, we know that the integration order is between certain intervals, e.g., covariance stationary (-0.5 ≤ d < 0.5), or mean-reverting but not covariance-stationary (0.5 ≤ d < 1). We can combine this essential prior information with the state space model in the framework of Bayesian inference; if we know within which interval d is located, we can assign a prior distribution $\pi(\theta)$ and generate parameter samples direct from the posterior distribution $L(\theta|y)$ where $L(\theta|y) \infty L(y|\theta)\pi(\theta)$. The estimator for the parameters is the expected value from the posterior distribution. When the posterior distribution $L(\theta|y)$ is not analytically tractable, the Markov chain Monte Carlo (MCMC) algorithm can be applied to draw samples from $L(\theta|y)$ and to estimate both d and σ_ε. Simulation studies show that using the Bayesian MCMC extension solves the bias problem when $0.4 < d < 0.5$ and that the Bayesian MCMC estimator performs well also for other values of d, including $d \geq 0.5$.

ARFIMA models are estimated for inflation from six inflation-targeting regions: Canada, the Euro Area, Norway, Sweden, the United Kingdom and the United States, for the period of 1999M1 to 2013M3. Our results show that most of the inflation persistence is caused by the long-run dynamics and that the short-run dynamics exhibits low persistence. The fractional integration order falls within the interval of 0.8 and 0.90 for all regions except Norway, where the integration order is 1.05. However, the estimated integration order for Norway is not

significantly greater than one at the 5% significance level. Overall, our results show that none of the central banks are "inflation nutters".

The rest of the paper is organized as follows: section 2 introduces the state space model-based MLE for long memory series, section 3 combines the state space model with the MCMC algorithm to estimate the fractional difference parameters, section 4 applies empirical examples, and the conclusions can be found in the final sections.

2 State Space Maximum Likelihood Estimator

Consider the ARFIMA(*p,d,q*) model $\phi(B)(1-L)^d Y_t = \theta(B)\varepsilon_t$, $0 < d < 1$ and $\varepsilon_t \sim i.i.d.N(0,\sigma_\varepsilon^2)$, $0 < d < 1$ and $\varepsilon_t \sim i.i.d.N(0,\sigma_\varepsilon^2)$. When *p,q* are less than or equal to one, we can obtain a truncated AR or MA representation of the ARFIMA(*p,d,q*) model, and estimate the parameters by approximate *MLE*. However, it is difficult to write out closed form AR or MA representations and carry out the estimation when *p* and *q* exceed one. However, we can use Hosking's (1981) method and estimate the parameters in the ARFIMA model recursively:

Step 1: Estimate d^0 by viewing Y_t as pure fractional difference series and then applying the ARIMA(*p,0,q*) process $u_t^0 = (1-L)^{d^0} Y_t$.

Step 2: Use the Box-Jenkins method to identify and estimate ϕ^0 and θ^0 parameters in the ARIMA(*p,0,q*) model $\phi(B)u_t^0 = \theta(B)\varepsilon_t$.

Step 3: Apply the ARIMA(*0,d,0*) process $x_t^0 = \{\theta^0(B)\}^{-1}\phi^0(B)Y_t$, and estimate d^1 in the fractional difference process $(1-L)^{d^1} x_t = \varepsilon_t$.

Step 4: Check for convergence with the convergence rule $d^i - d^{i-1} < 0.005$, and obtain the estimation results d^i, ϕ^i and θ^i.

For simplicity, consider the pure long memory series $(1-L)^d Y_t = \varepsilon_t$. There are three reasons to consider a long memory model in state space form. First, it can streamline the prediction and interpolations through the recursive expression in Kalman filter. Second, it can utilize the skipping approach (Durbin and Koopman, 2001), which controls for missing data by extrapolating the data that is available rather than relying on assumptions.. Third, the state equation form addresses indirectly observed process.

To obtain the state space form representation for the long memory series $(1-L)^d Y_t = \varepsilon_t$, $0 < d < 1$ and $\varepsilon_t \sim i.i.d.N(0, \sigma_\varepsilon^2)$, Chan and Palman (1998) suggested writing the model in the form of truncated MA and AR expansions: $y_t = \sum_{j=1}^{\infty} \pi_j y_{t-j} + \varepsilon_t$ or $y_t = \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j}$, where π_j and ψ_j can refer to Hosking (1981). This paper use AR representation and $\pi_j = \frac{-(j-d-1)!}{j!(-d-1)!}$. The state space form representation can be expressed as:

$$\begin{cases} y_t = Z\alpha_t & \text{(Measurement equation)} \\ \alpha_{t+1} = T\alpha_t + H\eta_t, \ \ \eta_t \sim NID(0, Q) & \text{(State equation)} \end{cases}.$$

With the truncated lag length setting as m, we have $\alpha_t = \begin{pmatrix} y_t \\ y_{t-1} \\ \dots \\ y_{t-(m-1)} \end{pmatrix}_{m*1}$

$Z_{1*m} = [1, 0, ..., 0]$, $\mathrm{T_{m*m}} = \begin{pmatrix} \pi_1 & \dots & \pi_m \\ 0 & & \mathrm{I_{m-1}} \end{pmatrix}$, $\mathrm{H_{m*1}} = (1, 0, ...0)$, $\mathrm{Q_{m*m}} = \begin{pmatrix} \sigma_\varepsilon^2 & \dots & 0 \\ & 0 & \end{pmatrix}$. Based on the truncated state space form representation, we can obtain the approximate likelihood function with the corresponding estimation algorithm order being $O(n)$. Compared with order $O(n^3)$ in exact

MLE, the reduced computation order will achieve a more efficient estimation and faster computation time (Chan and Palman, 1998).

The Kalman filter is utilized to calculate the likelihood function. Let I_{t-1} denote the information set at time $t-1$. The optimal predictor of the state α_t and its variance matrix are, respectively: $\alpha_{t|t-1} = E[\alpha_t | I_{t-1}] = T_t \alpha_{t-1}$, and $P_{t|t-1} = Var[\alpha_t | I_{t-1}] = T_t P_{t-1} T_{t-1}^{'} + Q_t$. The corresponding optimal predictor for y_t is then $y_{t|t-1} = Z_t \alpha_{t|t-1}$. Once the new observation y_t is available, the optimal predictors $\alpha_{t|t-1}$ and $P_{t|t-1}$ are updated as: $\alpha_t = \alpha_{t|t-1} + P_{t-1} Z_t^{'} F_t^{-1} (y_t - Z_t \alpha_{t|t-1}) = \alpha_{t|t-1} + P_{t-1} Z_t^{'} F_t^{-1} \nu_t$, $P_{t|t-1} = P_{t|t-1} - P_{t|t-1} Z_t^{'} F_t^{-1} Z_t P_{t|t-1}$. When the initial value $\alpha_{1|0}$ and $P_{1|0}$ are specified, the Kalman filter returns prediction errors $\nu_t = y_t - Z_t \alpha_{t|t-1}$ and the variance matrix $E(\nu_t \nu_t^{'}) = F_t = Z_t P_{t|t-1} Z_t^{'} + H_t$. Finally, by maximizing the log likelihood function $\ln L(y|\theta) = -\frac{NT}{2} \ln(2\pi) - \frac{1}{2} \left\{ \sum_{t=1}^{T} \ln |F_t| + \sum_{t=1}^{T} \nu_t^{'} F_t^{-1} \nu_t \right\}$, the parameters $\theta = (d, \sigma_\varepsilon^2)$ can be estimated. Chan and Palman (1998) established the asymptotic properties of the approximate maximum likelihood estimation, and the simulation shows that the approach is efficient. The most current study on the state space model long memory estimation is the one conducted by Grassi and Magistris in 2011. However, Chan and Palman (1998) and Grassi and Magistris (2011) only consider stationary series with $0 < d < 0.4$ where $\sigma_\varepsilon^2 = 1$ and is assumed to be known.

The range of integration orders considered in their simulations is relatively narrow from an economic point of view. Several economic time series such as exchange rates (Andersson, 2013), inflation (Hassler and Wolters, 1995; Caggio and Castelnuovo, 2011) and interest rates (Tkacz,2001; Coleman and Sirichand, 2012) have been found to be covariance non-stationary yet

mean-reverting. We thus expand the simulations (see Tables 1 to 3) to also include nearly non-stationary time series ($d = 0.45, 0.48$), non-stationary though mean-reverting ($d = 0.7, 0.8, 0.9$) and nearly unit root ($d = 0.95, 0.98$). Unlike Chan and Palman (1998) and Grassi and Magistris (2011) we also consider both the case when σ_ε^2 is known (Table 1), and the case when σ_ε^2 is unknown and estimated jointly with *d* (Table 2). In the simulation, the initial value of $\alpha_{1|0}$ is set as 0, and $P_{1|0}$ is the empirical auto-covariance matrix up to lag m, which is set to 10. We concentrate on the case where $T = 170$, which corresponds to the sample size in our empirical analysis. The standard deviation of the shocks is set to $\sigma_\varepsilon \in (1,3,5)$. The simulation is based on 500 repetitions.

The estimates of the integration order are unbiased for all cases except where *d* is close to 0.5 and the estimates contain a positive bias. The bias is relatively large (between 0.10 and 0.12). In an empirical analysis, this large and positive bias increases the risk of concluding that a series is non-stationary when it is actually stationary.

[Table 1]

[Table 2]

As can also be seen in the tables, the bias is independent of whether σ_ε^2 is known or unknown and of the value of σ_ε^2. The estimates of σ_ε^2 are unbiased irrespective of *d* (see Table 3), and only the estimates of *d* are biased for the series with an integration order close to 0.5. Overall, the state space model based estimation gives out a satisfactory result in most cases except when $d = 0.45, 0.48$.

[Table 3]

3 Bayesian MCMC Estimator

In certain situations, we have some prior knowledge of the series' properties, whether they are covariance-stationary, mean-reverting or non-stationary. Such information can potentially be used to improve the accuracy of the estimator, and in the case of the MLE described in Section 2, it can solve the over-bias problem for the nearly non-stationary series.

The estimation in Section 2 is based on the maximization of the log likelihood function $\ln L(y|\theta) = -\frac{NT}{2}\ln(2\pi) - \frac{1}{2}\left\{\sum_{t=1}^{T}\ln|F_t| + \sum_{t=1}^{T} v_t' F_t^{-1} v_t\right\}$, where we assume θ is fixed but unknown. If we know whether the series is stationary or non-stationary, we can set d as a random variable with the definition domain as $0 < d < 0.5$ or $0.5 < d < 1$ respectively. To estimate the parameters in the fractional difference series using the Bayesian methodology, we can refer to Koop *et al.* (1997), Petris (1997) and the recent literature (Jensen 2004; Ko and Vannuchi 2006 a; Holan *et al.* 2009; Ko *et al.* 2009). Rather than estimating the parameters by maximizing the log likelihood function $\ln L(y|\theta)$, we first construct the posterior distribution $L(\theta|y)$ based on the prior distribution $\pi(\theta)$, and we construct the approximate likelihood function $L(y|\theta)$ by $L(\theta|y) \propto L(y|\theta)\pi(\theta)$.

The prior information $\pi(\theta)$ is chosen as the independent priors for d and σ_ε with $\pi(\theta) = \pi(d)\pi(\sigma)$. For d, where we have prior knowledge that the series is stationary with $0 < d < 0.5$ and non-stationary with $0.5 < d < 1$, we choose a respectively uniform distribution $Unif(0, 0.5)$ and $Unif(0.5, 1)$. The prior distribution σ_ε does not depend on d, and this paper

uses $Unif(0,10)$. The posterior distributions for d and σ_ε are:

$$p(d|\sigma, y) \propto (2\pi)^{-NT/2} \exp\left\{-\frac{1}{2}\sum_{t=1}^{T}\ln|F_t| - \frac{1}{2}\sum_{t=1}^{T} v_t' F_t^{-1} v_t\right\} \times I_{(0,0.5)}(d);$$

$$p(\sigma|d, y) \propto (2\pi)^{-NT/2} \exp\left\{-\frac{1}{2}\sum_{t=1}^{T}\ln|F_t| - \frac{1}{2}\sum_{t=1}^{T} v_t' F_t^{-1} v_t\right\} \times I_{(0,10)}(\sigma).$$

The estimators for d and σ_ε are simply the posterior mean and $\hat{\sigma} = \int \sigma \, \mathrm{d}P(\sigma|y)$. Because the marginal posterior $P(d|y)$ and $P(\sigma|y)$ result in the integration being analytically intractable, and the posterior distribution for d, σ_ε are conditionally depend on each other, a two-step iterative Metropolis-Hasting method is applied (Scollnik, 1996; Brooks, 1998; Besag, 2004).

Simulation results using the Bayesian approach are presented in Tables 4 to 6. We use the same simulation set-up as in Section 2. Table 4 contains the results when σ_ε is known. Table 5 contains the results when σ_ε i are unknown and estimated jointly with *d*. The results for the Bayesian approach are similar to the result of the MLE for all *d,* except $d \in (0.4, 0.45, 0.48)$. In this case, the bias issue has disappeared. Therefore, with certain prior information, the Bayesian-based method can improve the estimation for the nearly non-stationary series and generate the same accurate results for the other integration orders.

[Table 4]

[Table 5]

Compared with the MLE estimates of σ_ε, however, the Bayesian estimates are more biased when $d = 0.48$, which is a result of choosing a prior distribution. But, given the reduction in the bias of *d*, the bias of $\hat{\sigma}_\varepsilon$ is acceptable.

[Table 6]

4 Empirical Analysis

The integration order is estimated using an ARFIMA model for six inflation-targeting regions (Canada, the Euro Area, Norway, Sweden, the United Kingdom and the United States) for the period following the introduction of the Euro (i.e., 1999M1 to 2013M3). Inflation is measured as the year-on-year increase in the Harmonized Index of Consumer Prices (HICP) in all regions but Canada and the United States, where the Consumer Price Index (CPI) is used because there is no HICP data. All data are collected from Eurostat[1].

Descriptive statistics are available in Table 7. Table 7 also presents the official inflation targets. Average inflation is fairly close to the targets in all countries, although average inflation has been approximately 0.5 percentage points lower than the target in Norway and Sweden and approximately 0.5 percentage points higher than the 2012-defined United States target.

[TABLE 7]

The estimated parameters in the ARIMA model are presented in Table 8. The variance of the inflation shocks ranges between 0.243 (Euro Area) and 0.578 (the United States), and the shocks are auto-correlated in all countries. The degree of autocorrelation is relatively low. In most countries, the shocks follow an AR(1) model with an AR-parameter between 0.20 and 0.30. The exceptions are Norway and the United States, where the AR-parameters are within the range of 0.65 to 0.8. The variance of the inflation shocks are also the highest in Norway and the United States.

[TABLE 8]

Because of the relatively low degree of autocorrelation in the inflation shocks, most of the long swings in inflation are not caused by the inflation shocks but by the central bankers' response to

[1] http://epp.eurostat.ec.europa.eu/portal/page/portal/eurostat/home/

those shocks. The estimated fractional integration orders ranges between 0.824 (the United States) to 1.049 (Norway). The integration order for Norway is explosive, but the integration order is not significantly greater than one, and we cannot also reject mean-reversion for Norway.

Excluding Norway, the estimated integration orders are similar amongst regions and within the range of 0.824 to 0.887. Central bankers in these countries appear to agree upon how flexibly the inflation target should be implemented. The integration orders are, moreover, relatively high and show that the inflation targets are implemented flexibly and that additional issues play an important role in the central banks' policies. Although relatively high, the integration orders are all considerably smaller than 1 at the 5% significance level.

5 Conclusion

In this paper, we model the degree of inflation flexibility using an AFRIMA model in the framework of state space models. We divide the fractional difference series into four groups: pure stationary, nearly non-stationary, pure non-stationary and nearly unit root. We estimate both the difference parameter d and the variance σ_ε. The simulation result indicates that the method calculates quite precise estimation in most cases, other than when d nears 0.5. We argue that in certain situations, we have prior knowledge of whether the series is stationary or non-stationary. This knowledge can improve the estimation when we set the prior distribution for d and σ_ε in the framework of Bayesian inference. We utilized the Metropolis–Hastings algorithm to show that this methodology can improve the estimation to a large extent when $0.4 < d < 0.5$. Because the state space-based estimator works quite well when $0.7 < d < 1$, we use it to estimate inflation-targeting in the empirical example. The result shows that inflation contains long swings and that these swings are caused by the central bankers' preferences rather than the nature of the

inflation shocks. All central banks in the study pursue an inflation-targeting policy with a high degree of flexibility.

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Table 1 Estimation of *d* based on the state space model when σ_ε is known

d	0.20	0.30	0.40	0.45	0.48	0.70	0.80	0.90	0.95	0.98
$\sigma=1$										
Bias	0.001	0.014	0.044	0.079	0.124	0.033	0.014	0.003	-0.003	-0.011
RMSE	0.076	0.070	0.090	0.124	0.167	0.079	0.069	0.059	0.056	0.060
$\sigma=3$										
Bias	0.001	0.010	0.041	0.081	0.131	0.027	0.018	0.002	-0.004	-0.008
RMSE	0.066	0.067	0.093	0.125	0.172	0.077	0.071	0.062	0.059	0.061
$\sigma=5$										
Bias	-0.003	0.017	0.048	0.077	0.134	0.025	0.018	0.003	-0.004	-0.005
RMSE	0.065	0.075	0.099	0.121	0.171	0.076	0.074	0.062	0.056	0.058

Table 2 Estimation of *d* based on the state space model when σ_ε is unknown

d	0.20	0.30	0.40	0.45	0.48	0.70	0.80	0.90	0.95	0.98
$\sigma=1$										
Bias	-0.001	0.017	0.043	0.079	0.125	0.029	0.014	0.004	-0.005	-0.009
RMSE	0.077	0.069	0.096	0.121	0.166	0.079	0.065	0.064	0.058	0.057
$\sigma=3$										
Bias	0.006	0.020	0.041	0.084	0.112	0.030	0.011	0.004	-0.004	-0.007
RMSE	0.077	0.064	0.088	0.123	0.155	0.081	0.067	0.061	0.058	0.058
$\sigma=5$										
Bias	0.002	0.015	0.041	0.067	0.121	0.036	0.016	0.001	-0.008	-0.009
RMSE	0.056	0.066	0.089	0.100	0.170	0.078	0.070	0.060	0.049	0.050

Table 3 Estimation of σ_ε based on the state space model

d	0.20	0.30	0.04	0.45	0.48	0.70	0.80	0.90	0.95	0.98
$\sigma=1$										
Bias	-0.001	-0.001	0.004	0.015	0.025	-0.001	-0.001	-0.004	-0.005	0.006
RMSE	0.055	0.054	0.054	0.059	0.064	0.053	0.055	0.055	0.052	0.055
$\sigma=3$										
Bias	-0.002	0.002	0.023	0.041	0.062	0.010	-0.006	0.011	-0.003	-0.013
RMSE	0.166	0.155	0.167	0.181	0.180	0.153	0.152	0.132	0.135	0.150
$\sigma=5$										
Bias	-0.010	-0.016	0.019	0.065	0.142	0.002	-0.015	-0.008	-0.037	-0.038
RMSE	0.227	0.234	0.224	0.255	0.314	0.221	0.208	0.225	0.266	0.228

Table 4 Estimation of *d* based on the Bayes model when σ_ε is known

D	0.20	0.30	0.40	0.45	0.48	0.70	0.80	0.90	0.95	0.98
$\sigma=1$										
Bias	0.012	0.020	0.018	0.003	-0.006	0.037	0.018	-0.007	-0.024	-0.037
RMSE	0.061	0.067	0.054	0.042	0.031	0.079	0.067	0.054	0.050	0.051
$\sigma=3$										
Bias	0.014	0.016	0.018	0.005	-0.006	0.035	0.024	-0.006	-0.020	-0.036
RMSE	0.060	0.067	0.055	0.039	0.032	0.077	0.070	0.050	0.046	0.053
$\sigma=5$										
Bias	0.008	0.011	0.018	0.003	-0.004	0.039	0.023	-0.003	-0.024	-0.039
RMSE	0.065	0.064	0.055	0.040	0.026	0.081	0.066	0.052	0.048	0.053

Table 5 Estimation of *d* based on the Bayes model when σ_ε is unknown

D	0.20	0.30	0.40	0.45	0.48	0.70	0.80	0.90	0.95	0.98
$\sigma=1$										
Bias	0.010	0.019	0.019	0.003	-0.007	0.035	0.016	0.001	-0.024	-0.039
RMSE	0.066	0.062	0.057	0.042	0.031	0.80	0.067	0.047	0.046	0.052
$\sigma=3$										
Bias	0.009	0.018	0.016	0.005	-0.004	0.033	0.023	-0.001	-0.020	-0.038
RMSE	0.063	0.067	0.055	0.037	0.030	0.077	0.067	0.051	0.045	0.059
$\sigma=5$										
Bias	0.011	0.016	0.017	0.005	-0.004	0.034	0.020	-0.004	-0.020	-0.037
RMSE	0.050	0.068	0.056	0.038	0.027	0.079	0.069	0.051	0.048	0.050

Table 6 Estimation of σ_ε based on the Bayesian model

D	0.20	0.30	0.04	0.45	0.48	0.70	0.80	0.90	0.95	0.98
$\sigma=1$										
Bias	0.007	0.009	0.025	0.042	0.101	0.018	0.019	0.011	-0.009	0.004
RMSE	0.086	0.069	0.071	0.094	0.183	0.068	0.071	0.064	0.063	0.059
$\sigma=3$										
Bias	0.022	0.035	0.060	0.120	0.350	0.044	0.030	0.018	0.017	0.019
RMSE	0.160	0.162	0.164	0.200	0.500	0.165	0.150	0.170	0.150	0.165
$\sigma=5$										
Bias	0.020	0.061	0.078	0.190	0.557	0.052	0.038	0.037	0.024	0.023
RMSE	0.27	0.230	0.260	0.400	0.092	0.290	0.244	0.269	0.235	0.228

Table 7 Descriptive statistics

	Canada	Euro Area	Norway	Sweden	United Kingdom	United States
Inflation Target	1% - 3%	<2%	2.5%	2.0%	2.0%	2.0%
Average	2.06%	2.08%	1.82%	1.64%	2.18%	2.51%
Std. Dev.	0.95	0.77	1.17	0.87	1.10	1.65

Table 8 Estimation results

	Canada	Euro Area	Norway	Sweden	United Kingdom	United States
d	0.835	0.857	1.048	0.857	0.887	0.824
	(0.041)	(0.036)	(0.025)	(0.040)	(0.038)	(0.044)
σ_ε	0.482	0.243	0.511	0.346	0.291	0.578
	(0.019)	(0.009)	(0.019)	(0.013)	(0.011)	(0.022)
AR(1)	0.220	0.338	0.807	0.201	0.237	0.653
	(0.076)	(0.081)	(0.050)	(0.080)	(0.076)	(0.075)
AR(2)	---	0.081	-0.714	---	0.145	-0.221
		(0.081)	(0.076)		(0.076)	(0.075)
MA(1)	---	---		---	---	---