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Department of Economics
School of Economics and Management

Lost in Translation

Rethinking the Inequality-Equivalence Criteria for Bounded Health Variables

Gustav Kjellsson
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Lost in translation: Rethinking the inequality-equivalence criteria for bounded health variables

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Abstract

What change in the distribution of a population's health preserves the level of inequality? The answer to the analogous question in the context of income inequality lies somewhere between a uniform and a proportional change. These polar positions represent the absolute and the relative Inequality Equivalence Criterion (IEC), respectively. A bounded health variable may be presented in terms of both health attainments and shortfalls. As a distributional change cannot simultaneously be proportional to attainments and to shortfalls, relative inequality measures may rank populations differently from the two perspectives. In contrast to the literature that stresses the importance of measuring inequality in attainments and shortfalls consistently using an absolute IEC, this paper formalizes a new compromise concept for a bounded variable by explicitly considering the two relative IECs, defined with respect to attainments and shortfalls, to represent the polar cases of defensible positions.

We use a surplus-sharing approach to provide new insights on commonly used inequality indices by evaluating the underpinning IECs in terms of how infinitesimal surpluses of health must be successively distributed to preserve the level of inequality. We derive a one-parameter IEC that, unlike those implicit in commonly used indices, assigns constant weights to the polar cases independent of the health distribution.

Keywords: health inequality, bounded variable, inequality equivalence criteria

JEL: D63, I14

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Introduction

Despite decades of enhancing average health status and egalitarian public policies, inequality in health persists in many countries (e.g., Kunst et al., 2004a,b; Marmot et al., 2012). To evaluate levels of and changes in health inequality over time, it is vital to have a measurement framework which captures the distribution of health in an index value. Health economics research frequently uses the (univariate) Gini coefficient to evaluate total health inequalities and the (bivariate) concentration index to measure health inequalities related to a socioeconomic variable (e.g., income). The recent literature intensively discusses how to adjust these rank-dependent inequality indices for health variables that, unlike income, are bounded from above (Erreygers 2009a,b,c; Erreygers and van Ourti 2011a,b; Kjellsson and Gerdtham 2013b; Wagstaff 2009, 2011a,b, see also Aristondo and Lasso de la Vega, this volume). This discussion boils down to the more general issue of the vertical value judgments inherent in an index’s Inequality Equivalence Criterion (IEC); the distributional change to which an inequality measure should be invariant (c.f. Allanson and Petrie, 2013). Choosing an IEC is controversial in the income inequality literature and becomes even more delicate in relation to inequality in a health variable that is bounded and may arbitrarily be coded in terms of either health attainments or shortfalls.

To provide further understanding of the implicit value judgments the different rank-dependent indices embody, we scrutinize the IECs using a surplus-sharing approach, that is, evaluating how an additional infinitesimal surplus should be distributed to keep inequality constant. In particular, we extend the flexible IEC suggested by Zoli (2003) and Yoshida (2005) to bounded health measures. Beyond providing insights into the IECs underpinning commonly used rank-dependent indices such as Wagstaff’s (2005) and the univariate and bivariate version of Erreygers’ index (Erreygers 2009a,b, respectively), we suggest our own intermediate IEC. In the next section, before formalizing a new compromise concept in the third section and deriving a new nonlinear IEC in the fourth, we draw upon the inequality literature to illustrate why it is necessary to rethink existing IECs for bounded health variables.

Rethinking the IECs for a bounded variable

Income inequality

The income inequality literature has hosted a long-lasting discussion of whether it is appropriate to adopt an absolute or a relative IEC. That is, using an inequality measure that is invariant to either equiproportionate or uniform changes of the variable of interest. In a seminal article, Kolm (1976) introduces the vocabulary of *rightist* and *leftist* to represent the implicit vertical value judgment that underpins the choice between relative and absolute measures, respectively. Kolm further claims that these two IECs represent the natural polar cases of positions that are generally considered to be ethically defensible, although they do not necessarily represent all ethically defensible positions: Referring to Dalton (1920), among others, Kolm (1976, p. 433) further claims that “many people feel that an equal augmentation in everyone’s income decreases inequality, whereas an equiproportional increase in everyone’s income increases it,” which indicates that both absolute and relative perspectives are important. Consequently, he also introduces an intermediate view of inequalities

as a compromise between the rightist (relative) and the leftist (absolute) views.¹ One may find it hard to defend positions outside these boundaries; for example, a vertical value judgment that implies that inequality increases in response to a uniform increment of income or, alternatively, a vertical value judgment that implies that inequality decreases in response to an equiproportionate increase of income. Zheng (2007) refers to IECs representing such positions outside the boundaries as *extreme leftist* and *extreme rightist*, respectively.

Several intermediate IECs that yield the rightist (relative) and the leftist (absolute) positions as polar cases have since been suggested (e.g., Bossert and Pfingsten, 1990; Krtscha, 1994; Zoli, 2003; Yoshida, 2005; Zheng, 2007). Bossert and Pfingsten (1990) suggest a linear compromise between the two polar cases, but Zheng (2004), Zoli (2003), and Yoshida (2005) all point out that this is overly restrictive, i.e., a linear IEC fails to represent all intermediate vertical value judgments individuals may have. This argument also gains support from experiments (e.g., Amiel and Cowell, 1999). As linearity implies that the *level of intermediateness* depends on the initial income distribution, a surplus of \$1 must be distributed in the same way as a surplus of \$1 million. Consequently, a procedure of distributing a surplus of \$1 million by repeatedly distributing smaller surpluses of \$1 would imply that the distribution of each and every surplus would depend on the initial income distribution.

An alternative approach, promoted by Krtscha (1994), Yoshida (2005), and Zoli (2003), suggests that each infinitesimal amount of extra income must be distributed as a convex combination of the relative and the absolute IEC with respect to the presently prevailing income distribution in order to keep inequality constant. The important difference from the linear IECs is that the next infinitesimal amount of extra income should be distributed according to the present, rather than the initial, distribution. Krtscha (1994) suggests a *fair compromise* between the relative and absolute views, implying that the portions of the surplus that must be distributed proportionally and uniformly to the income distribution are of equal size. Yoshida (2005) generalizes this fair compromise so that the size of the portions depends on a parameter. Zoli (2003) further shows how to use this surplus-sharing approach to identify the local vertical value judgment, or the level of intermediateness, for a given income distribution for any well-behaved IEC.

From income to health

As most health inequality measures originate from the income inequality literature, the discussion of IECs is directly relevant also for health inequality researchers. In addition, the boundedness of health variables further complicates matters. Nevertheless, the discussion of IECs underpinning inequality indices tends to get lost in translation when moving from income to a bounded health variable.

For bounded health variables that may be coded in terms of either attainments or shortfalls, an index can be invariant to equiproportionate changes of either attainments or shortfalls of health, but not to both perspectives at the same time (Erreygers and van Ourti, 2011a). Clarke, Gerdtham, Johannesson, Bingefors and Smith (2002) show that a relative inequality index may rank populations differently according to attainments and shortfalls. This did not cause the health inequality literature to acknowledge that these are two different IECs representing two different vertical value judgments. Instead, the finding has rather started a quest for a *consistent* inequality measure (e.g., Lambert and Zheng, 2011; Lasso de la Vega and Aristondo, 2012) and has been used as an argument in favor of an absolute IEC as it ranks populations

¹Although Kolm (1976) refers to this intermediate view as centralist, we will consistently use the term intermediate to avoid using multiple terms for the same concept.

Table 1: Vocabularies of Inequality Equivalence Criteria

Income inequality vocabulary	Erreygers and van Ourti (2011a)	Definition
Extreme rightist	Inverse absolute	Inequality decreases in response to equiproportionate improvements
Rightist	Quasirelative	Inequality is invariant to equiproportional changes
Intermediate	Mixed	Inequality decreases (increases) in response to uniform (equiproportional) improvements
Leftist	Quasiabsolute	Inequality is invariant to uniform changes
Extreme leftist	Inverse relative	Inequality increases in response to uniform improvements
<i>Note:</i> Erreygers and van Ourti (2011a) use the prefix <i>quasi</i> - to acknowledge that, for a bounded variable, equiproportional and uniform changes are not feasible for all distributions.		

consistently (e.g., Erreygers, 2009a,b,c; Erreygers and van Ourti, 2011a; Lambert and Zheng, 2011). The only exception in the literature, as far as we know, is Allanson and Petrie (2012, 2013). Using a two-dimensional inequality map borrowed from the income inequality literature and applied to a bounded variable standardized in the unit interval, Allanson and Petrie (2012) illustrate that the vertical value judgment is fundamentally different if the relative IEC is defined with respect to attainments or shortfalls. The inequality map in Figure 1, adopted from Allanson and Petrie (2012), represents an economy of two individuals, where the attainment and the shortfall of the richer/healthier (poorer/less-healthy) individual are represented on the first and the second horizontal axis (vertical axis). For coherence between the interpretation for total and income-related health inequality, assume that the richer individual also possesses more health. All equal (egalitarian) distributions constitute a 45-degree line departing from the origin; distributions further from the line of equality are generally considered as more unequal.

Take in Figure 1

Any IEC defines a set of health distributions that are equivalent in terms of inequality. These sets constitute iso-inequality contours, which can be represented in the inequality map. Thus, for an arbitrary initial distribution H, all points on a line that passes point H represent a linear iso-inequality contour of distributions that is equivalent to H. All points in the set below the contour represent distributions that are considered more unequal, while all points in the set above the contour and below the 45-degree line represent distributions that are considered less unequal. All distributions obtained by uniform changes of either attainments or shortfalls constitute the absolute IEC as represented by line II. In contrast, lines III and I consist of distributions obtained by proportional changes of attainments and of shortfalls, respectively. Thus, the graph convincingly illustrates that while the absolute IEC of the two perspectives coincide, the relative IECs with respect to attainments and shortfalls represent two distinct vertical value judgments. To explicitly distinguish between the two, we label a relative IEC with respect to attainments as h-relative and a relative IEC with respect to shortfalls as s-relative.

The previous literature has tended to disregard the boundedness by referring to both the h-relative and the s-relative IEC as either rightist or (quasi)relative. That is, using the income inequality vocabulary

Table 2: IECs of a bounded health variable

	Attainments	Shortfalls
	Extreme rightist	Extreme leftist
H-relative	Rightist	Extreme leftist
	Intermediate	Extreme leftist
Absolute	Leftist	Leftist
	Extreme leftist	Intermediate
S-relative	Extreme leftist	Rightist
	Extreme leftist	Rightist

or a related version² without acknowledging that one—implicitly or explicitly—needs to choose either attainments or shortfalls as a reference point. The exception is again Allanson and Petrie (2012, 2013) who explicitly argue for attainments as the natural reference point “as health is generally considered as a *good* like income” and, therefore, define all IECs in terms of attainments. However, labeling the h-relative IEC as rightist implies that the s-relative IEC is only a subset of the extreme leftist IECs, which are represented in the inequality map by any iso-inequality contour that is above (below) the absolute line to the right (left) of H. That is, inequality increases when health increases uniformly and the IEC is outside the range that Kolm (1976) considers as ethically defensible. Choosing attainments as the reference point further implies that an IEC is intermediate if it is a compromise between the h-relative and the absolute IEC (i.e., represented by an iso-inequality contour between lines II and III), while an IEC is extreme leftist if it is a compromise between the s-relative and the absolute (i.e., represented by an iso-inequality contour in the area between lines I and II). However, reversing the perspective implies that an IEC that was intermediate with respect to attainments is now extreme leftist with respect to shortfalls. Table 2 summarizes the correspondence between the IECs defined with respect to attainments and shortfalls. These issues may be considered semantic. We claim they are not. Rather, they are a symptom of the problem of transferring inequality measures from income to a bounded health variable without considering that the natural polar cases of the ethically defensible positions have changed.

For an unbounded variable such as income, it may be difficult to argue in favor of an extreme leftist IEC (i.e., for most people it appears counterintuitive that inequality increases when both absolute and relative differences decrease). For a bounded health variable, such a position excludes any compromise between the s-relative and the absolute IEC. However, that an equiproportional decrease of the shortfall distribution preserves (or at least does not increase) the inequality may appear as an intuitive concept and be compatible, at least in some contexts, with people’s perception of inequality. For example, Allanson and Petrie (2013) stress that this view is consistent with the principle of proportional universalism presented in the Marmot Review: “To reduce the steepness of the social gradient in health, action must be universal, but with a scale and intensity that is proportionate to the level of disadvantage” (Marmot 2010, pg. 15). That is, to

²Erreygers and van Ourti (2011a) label the leftist, rightist, intermediate, extreme leftist and extreme rightist IEC as quasiabsolute, quasirelative, mixed, inverse relative, and inverse absolute, respectively, see Table 1 for the correspondence between the two vocabularies

reduce (income-related) health inequality, interventions must reduce both relative and absolute inequality in attainments, which is consistent with an extreme leftist IEC. Drawing upon Allanson and Petrie's (2013) argument, we suggest that for a bounded health variable we shall not rule out that individuals may have inequality perceptions that are either a) in line with an IEC that intermediates the h-relative and the absolute IEC, b) in line with an extreme leftist IEC that intermediates the absolute and the s-relative IEC, or compatible with a combination of a) and b). Thus, the natural polar cases of the ethically defensible positions are no longer the (h-)relative and absolute, but rather the h-relative and s-relative IECs.

Contribution of the paper

We formalize this new compromise concept for bounded health variables using the s-relative and the h-relative IEC as the more appropriate polar cases. We show that the surplus-sharing rule of any IEC that satisfies this compromise can be interpreted as a weighted sum of the sharing rules of the two polar cases with weights in the unit interval. Thus, for the level of inequality to remain constant, one portion of an infinitesimal extra amount of health should be distributed proportionally to the distribution of attainments and one portion proportionally to the distribution of shortfalls. Analogous to Erreygers and van Ourti's (2011a) measure of a rank-dependent index's sensitivity to relative inequality in relation to absolute inequality, the weights of the surplus-sharing rules may be interpreted as a measure of an inequality index's sensitivity to relative inequality in attainments in relation to relative inequality in shortfalls. Using these weights, we may evaluate the level of intermediateness of any rank-dependent index, including the indices suggested by Erreygers (2009a,b) and Wagstaff (2005), each of which satisfies our suggested compromise concept. We also derive a nonlinear IEC that, in contrast to the IECs underpinning Erreygers' and Wagstaff's indices, weights the relevant polar cases constantly and independently of the health distribution. That is, we translate Yoshida's (2005) generalization of Krttscha's (1994) fair compromise to a bounded health variable.

In another chapter of this volume, Aristondo and Lasso de la Vega (2013) approach the problem of measuring inequality of a bounded health variable from an alternative perspective. Without explicitly considering the underlying IECs, they acknowledge our compromise concept suggesting measuring inequality of the joint distribution of shortfalls and attainments. For a univariate index that is decomposable (c.f. Shorrocks, 1980), analyzing relative inequality of the joint distribution is equivalent to evaluating inequality of the distribution of either attainments or shortfalls using a subset of the indices suggested in a previous paper by Lasso de la Vega and Aristondo (2012). This class of indices is underpinned by the same IEC as Wagstaff's (2005) index (c.f. Kjellsson and Gerdtham, 2013b). Note, however, that the rank-dependent indices considered in our paper are not included in the class of decomposable indices.

Inequality Equivalence Criteria for bounded variables

Preliminaries

Let the vector $\mathbf{h} = (h_1, h_2, \dots, h_n)$ represent the health distribution of a given population of n individuals or groups of individuals, where each h_i ($i = 1, 2, \dots, n$) is a standardized cardinal health variable in the unit interval. The boundedness implies that we can construct a vector $\mathbf{s} = (s_1, s_2, \dots, s_n)$ that represents the ill-health situation of the whole population defined as shortfalls of health $s_i = 1 - h_i$. By defining the IECs in terms of a standardized (cardinal) health variable, we will, in line with Erreygers and van Ourti

(2011a), only consider real differences in health that are not due to changes in the unit of measurement. For technical convenience, we let individual i 's position in the vector \mathbf{h} be decided by the individual rank based on the position in the distribution of health and income, denoted as ρ_i and ϕ_i for total- and income-related inequalities, respectively.³ The average attainment and shortfall of the population is denoted as $\mu_h = \frac{1}{n} \sum_{i=1}^n h_i$ and $\mu_s = \frac{1}{n} \sum_{i=1}^n s_i$.

We denote that distribution \mathbf{h} is considered at least as equal as distribution $\tilde{\mathbf{h}}$ by $\mathbf{h} \succeq \tilde{\mathbf{h}}$. To denote that two distributions are considered to be equivalent in terms of inequality we use $\mathbf{h} \sim \tilde{\mathbf{h}}$. For income-related health inequality, we further assume that, on average, richer individuals have better health. *More equal*, then means that health is less concentrated among the rich. We define an IEC in terms of a normalized distance between the health vector and the mean; two health distributions are considered equal in terms of inequality if the normalized distances are equal.

Definition 1. (General IEC) $\forall \mathbf{h}, \tilde{\mathbf{h}}; \mathbf{h} \sim \tilde{\mathbf{h}}$ if

$$\frac{\mathbf{h} - \mu_h \mathbf{1}}{g(\mu_h)} = \frac{\tilde{\mathbf{h}} - \mu_{\tilde{h}} \mathbf{1}}{g(\mu_{\tilde{h}})} \quad (1)$$

where $\mathbf{1}$ is the unit vector and $g(\mu_h)$ is a positive, continuous, and (piecewise) differentiable function with the derivative denoted as $g'(\mu_h)$.

Rank-dependent indices

Later in the paper, we will relate the IECs to the families of univariate and bivariate rank-dependent indices defined for a standardized health variable. Following Erreygers (2009a,b), we express the two families as normalized sums of weighted health levels.

Definition 2. (Rank-Dependent Index)

(a) A univariate rank-dependent index takes the form

$$G(\mathbf{h}) = f(\mu_h, n) \sum_i^n w_i h_i \quad (2)$$

(b) A bivariate rank-dependent index takes the form

$$I(\mathbf{h}) = f(\mu_h, n) \sum_i^n z_i h_i \quad (3)$$

Here, $w_i = (n+1)/2 - \rho_i$, $z_i = (n+1)/2 - \phi_i$, and the normalization function $f(\mu_h, n) > 0$.

A rank-dependent index represents a General IEC if it is invariant to the distributional change from \mathbf{h} to $\tilde{\mathbf{h}}$ represented in Eq. (1). This relationship is specified in Proposition 1. (*All proofs in the appendix.*)

Proposition 1. A rank-dependent index, $I(\mathbf{h})$ or $G(\mathbf{h})$, represents a General IEC if the normalization function $f(\mu_h, n) = v(n)/g(\mu_h)$, where $v(n)$ is a positive scalar function.

³The individual with the highest value is ranked first. Any tied individuals are assigned the average rank within the tied subgroup, leaving gaps in the ranking both above and below their rank.

The absolute, h-relative and s-relative IECs

We formally define the absolute, h-relative and s-relative IECs in terms of a General IEC by varying $g(\mu_h)$ in Eq. (1). For an absolute IEC, the level of inequality is constant if health changes uniformly across the distribution.

Definition 3. (Absolute IEC) $\forall h, \tilde{h}; h \sim \tilde{h}$ if

$$h - \mu_h \mathbf{1} = \tilde{h} - \mu_{\tilde{h}} \mathbf{1} \quad (4)$$

As we deal with a bounded health variable, we distinguish between an IEC that implies invariance to equiproportionate changes in attainments and an IEC that implies invariance to equiproportionate changes in shortfalls by labeling them as h-relative and s-relative, respectively.

Definition 4. (h-Relative IEC) $\forall h, \tilde{h}; h \sim \tilde{h}$ if

$$\frac{h - \mu_h \mathbf{1}}{\mu_h} = \frac{\tilde{h} - \mu_{\tilde{h}} \mathbf{1}}{\mu_{\tilde{h}}} \quad (5)$$

Definition 5. (s-Relative IEC) $\forall s, \tilde{s}; s \sim \tilde{s}$ if

$$\frac{s - \mu_s \mathbf{1}}{\mu_s} = \frac{\tilde{s} - \mu_{\tilde{s}} \mathbf{1}}{\mu_{\tilde{s}}} \quad (6)$$

To formally illustrate that these two IECs capture two distinct vertical value judgments, it is illuminating to define the s-relative IEC in terms of attainments.

Definition 6. (s-Relative IEC) $\forall h, \tilde{h}; h \sim \tilde{h}$ if

$$\frac{h - \mu_h \mathbf{1}}{1 - \mu_h} = \frac{\tilde{h} - \mu_{\tilde{h}} \mathbf{1}}{1 - \mu_{\tilde{h}}} \quad (7)$$

Note that Eqs. (4), (5), and (7) are equivalent to Eq. (1) using $g(\mu_h) = 1$, $g(\mu_h) = \mu_h$, and $g(\mu_h) = 1 - \mu_h$, respectively.

The new compromise concept

In line with Kolm's (1976) intermediate view of inequality, Bossert and Pfingsten (1990) define a compromise concept that intermediates the two polar cases for income inequality, absolute and relative. For a bounded variable, we may define this concept both as a compromise between the h-relative and absolute IEC and as a compromise between the s-relative and absolute IEC.

Definition 7. (h-Relative–Absolute Compromise) An IEC is a compromise between the h-relative and the absolute IEC if $\forall h, \tilde{h}$ such that $\mu_h \leq \mu_{\tilde{h}}$,

$$h \succeq \tilde{h} \text{ if } \frac{h - \mu_h \mathbf{1}}{\mu_h} = \frac{\tilde{h} - \mu_{\tilde{h}} \mathbf{1}}{\mu_{\tilde{h}}} \quad (8)$$

$$\tilde{h} \succeq h \text{ if } h - \mu_h \mathbf{1} = \tilde{h} - \mu_{\tilde{h}} \mathbf{1} \quad (9)$$

and if $\forall h, \tilde{h}$ such that $\mu_h \geq \mu_{\tilde{h}}$, the opposite applies.

Definition 8. (s-Relative–Absolute Compromise) An IEC is a compromise between the absolute and the s-relative IEC if $\forall \mathbf{h}, \tilde{\mathbf{h}}$ such that $\mu_h \leq \mu_{\tilde{h}}$,

$$\mathbf{h} \succeq \tilde{\mathbf{h}} \text{ if } \mathbf{h} - \mu_h \mathbf{1} = \tilde{\mathbf{h}} - \mu_{\tilde{h}} \mathbf{1} \quad (10)$$

$$\tilde{\mathbf{h}} \succeq \mathbf{h} \text{ if } \frac{\mathbf{h} - \mu_h \mathbf{1}}{1 - \mu_h} = \frac{\tilde{\mathbf{h}} - \mu_{\tilde{h}} \mathbf{1}}{1 - \mu_{\tilde{h}}} \quad (11)$$

and if $\forall \mathbf{h}, \tilde{\mathbf{h}}$ such that $\mu_h \geq \mu_{\tilde{h}}$, the opposite applies.

In short, these compromise concepts require that an equiproportional increase in attainments (shortfalls) does not decrease inequality, and a uniform increase in attainments (shortfalls) does not increase inequality. Relating back to Allanson and Petrie’s (2013) inequality map, a compromise between the h-relative and the absolute IECs is graphically represented by any contour in the area between lines III and II, whereas a compromise between the s-relative and the absolute IECs is represented by any contour in the area between lines I and II. If we consider both these compromise concepts to represent ethically defensible positions, it is natural to define a new compromise concept that is graphically represented by the union of the two areas. That is, a compromise concept adapted to a bounded variable with the h-relative and the s-relative IECs as the polar cases.

Definition 9. (hs-Relative Compromise) An IEC is a compromise between the h-relative and s-relative IECs if $\forall \mathbf{h}, \tilde{\mathbf{h}}$ such that $\mu_h \leq \mu_{\tilde{h}}$,

$$\mathbf{h} \succeq \tilde{\mathbf{h}} \text{ if } \frac{\mathbf{h} - \mu_h \mathbf{1}}{\mu_h} = \frac{\tilde{\mathbf{h}} - \mu_{\tilde{h}} \mathbf{1}}{\mu_{\tilde{h}}} \quad (12)$$

$$\tilde{\mathbf{h}} \succeq \mathbf{h} \text{ if } \frac{\mathbf{h} - \mu_h \mathbf{1}}{(1 - \mu_h)} = \frac{\tilde{\mathbf{h}} - \mu_{\tilde{h}} \mathbf{1}}{(1 - \mu_{\tilde{h}})} \quad (13)$$

and if $\forall \mathbf{h}, \tilde{\mathbf{h}}$ such that $\mu_h \geq \mu_{\tilde{h}}$, the opposite applies.

In words, an equiproportional increase in attainments must not decrease the inequality and an equiproportional decrease in shortfalls must not increase the inequality. All linear contours in the space that represent the compromise concept constitute Bossert and Pfingsten’s (1990) linear intermediate IEC adapted to bounded health variables—i.e., a General IEC with $g(\mu_h) = \mu_h \kappa + (1 - \kappa)(1 - \mu_h)$, where $0 \leq \kappa \leq 1$. The perfect linear compromise, $\kappa = 0.5$, equals the absolute IEC. The compromise concept is however not limited to linear IECs. In the following section, we will use a surplus-sharing approach to derive nonlinear IECs that are represented by iso-inequality contours within this space where the set of inequality equivalent distributions is represented by a curve instead of a line.

A surplus-sharing approach

As the vertical value judgment behind an IEC tells us what kind of distributional change leaves inequality unchanged, any IEC also entails a rule for how an additional surplus of health must be distributed. In this section, we follow Zoli’s (2003) introduction of a vector function that identifies how an additional surplus ε must be distributed to not alter the inequality with respect to distribution \mathbf{h} :

$$\mathbf{h} + \mathbf{d}(\mathbf{h}, \varepsilon) \sim \mathbf{h} \quad (14)$$

We refer to this vector $\mathbf{d}(\mathbf{h}, \varepsilon)$ as an inequality equivalent distributional vector (IEDV). Eq. (14) represents the set of all distributions that compile an inequality contour in an inequality map. That is, $\mathbf{d}(\mathbf{h}, \varepsilon)$ tells us how the surplus is distributed along the path from the initial health distribution \mathbf{h} to the new distribution $\tilde{\mathbf{h}}$. For a General IEC, the corresponding IEDV is⁴

$$\mathbf{d}(\mathbf{h}, \varepsilon) = \frac{\varepsilon}{n} \mathbf{1} + (\mathbf{h} - \mu_h \mathbf{1}) \left(\frac{g(\mu_h + \frac{\varepsilon}{n})}{g(\mu_h)} - 1 \right) \quad (15)$$

As we assume $g(\mu_h)$ to be continuous and (piecewise) differentiable, Eq. (15) is continuous and has a piecewise continuous partial derivative with respect to ε .⁵ The IEDV in Eq. (15) also satisfies what Zoli (2003) refers to as *path independence* (and is represented by a continuous iso-inequality contour). That is, a surplus $\varepsilon + \varepsilon'$ is identically distributed across the population irrespective of being distributed all at once or successively distributed as two surpluses.⁶ These properties restrict an IEDV to not change dramatically due to marginal changes in ε , assuring that it is possible to evaluate how the surplus-sharing rules are affected by marginal changes in the health distribution.

Dividing each element of the IEDV by the total surplus ε yields a vector $\mathbf{d}(\mathbf{h}, \varepsilon)/\varepsilon$ that equals the shares of the surplus distributed to each of the individuals in the population. Following Zoli (2003), we claim that, for a given distribution \mathbf{h} , this vector, $\mathbf{d}(\mathbf{h}, \varepsilon)/\varepsilon$, can be interpreted as representing the vertical value judgment of an IEC for a given change between the initial and the new distribution. However, the vertical value judgment represented by the vector $\mathbf{d}(\mathbf{h}, \varepsilon)/\varepsilon$ generally depends not only on the initial distribution, \mathbf{h} , but also on the surplus size ε . To isolate the effect of the initial distribution, we follow Zoli's (2003) suggestion of using the vector $\boldsymbol{\delta}(\mathbf{h}) = \lim_{\varepsilon \rightarrow 0^+} [\mathbf{d}(\mathbf{h}, \varepsilon)/\varepsilon]$. As this vector, for a given distribution \mathbf{h} , identifies how an *infinitesimal* positive surplus of health must be distributed to leave inequality unchanged, it represents the *local* vertical value judgment of the IEC for a given distribution \mathbf{h} . Thus, by using $\boldsymbol{\delta}(\mathbf{h})$, we may compare the surplus-sharing rules, or the local vertical value judgment, for a given distribution for any General IECs.

Local vertical value judgment

To relate the local vertical value judgment represented by an IEC to the surplus-sharing rules of the absolute, h-relative, and s-relative IECs, we express the IEDV representing the three IECs as, respectively,

$$\mathbf{d}(\mathbf{h}, \varepsilon) = \frac{\varepsilon}{n} \mathbf{1} \quad (16)$$

$$\mathbf{d}(\mathbf{h}, \varepsilon) = \frac{\varepsilon}{n} \frac{\mathbf{h}}{\mu_h} \quad (17)$$

and

$$\mathbf{d}(\mathbf{h}, \varepsilon) = \frac{\varepsilon}{n} \left(\mathbf{1} - \frac{\mathbf{h} - \mu_h \mathbf{1}}{(1 - \mu_h)} \right) \quad (18)$$

⁴See the proof of Proposition 2 in the appendix.

⁵We assume that the whole surplus ε must be distributed.

⁶For a formal definition see proof of Proposition 3 in the appendix.

Calculating the limit of the function that identifies the shares distributed to each individual, i.e., $\delta(\mathbf{h}) = \lim_{\varepsilon \rightarrow 0^+} [\mathbf{d}(\mathbf{h}, \varepsilon)/\varepsilon]$, for each of the three IECs yields

$$\delta(\mathbf{h}) = \frac{1}{n} \mathbf{1} \quad (19)$$

$$\delta(\mathbf{h}) = \frac{\mathbf{h}}{n\mu_h} \quad (20)$$

and

$$\delta(\mathbf{h}) = \left(\frac{1}{n} \mathbf{1} - \frac{\mathbf{h} - \mu_h \mathbf{1}}{n(1 - \mu_h)} \right) \quad (21)$$

For any General IEC, the local sharing rules—the corresponding $\delta(\mathbf{h})$ —may be expressed as a weighted sum of the local sharing rules of both the h-relative and the absolute IEC and, more importantly, the s-relative and the h-relative IEC.

Proposition 2. *For any General IEC, we may express $\delta(\mathbf{h})$ as*

$$\delta(\mathbf{h}) = \frac{1}{n} \mathbf{1} (1 - \omega_{\text{ra}}(\mu_h)) + \frac{1}{n} \frac{\mathbf{h}}{\mu_h} \omega_{\text{ra}}(\mu_h) \quad (22)$$

and

$$\delta(\mathbf{h}) = \omega_{\text{hs}}(\mu_h) \frac{\mathbf{h}}{n\mu_h} + (1 - \omega_{\text{hs}}(\mu_h)) \left(\frac{1}{n} \mathbf{1} - \frac{\mathbf{h} - \mu_h \mathbf{1}}{n(1 - \mu_h)} \right) \quad (23)$$

where the weights are $\omega_{\text{ra}}(\mu_h) = \frac{g'(\mu_h)\mu_h}{g(\mu_h)}$ and $\omega_{\text{hs}}(\mu_h) = \mu_h + \frac{g'(\mu_h)\mu_h}{g(\mu_h)}(1 - \mu_h)$.

Thus, Eqs. (22) and (23) represent the h-relative $\delta(\mathbf{h})$ for $\omega_{\text{ra}} = 1$ and $\omega_{\text{hs}} = 1$ and the absolute and the s-relative for $\omega_{\text{ra}} = 0$ and $\omega_{\text{hs}} = 0$, respectively. For an IEC that satisfies the hs-relative compromise concepts, the corresponding weights, $\omega_{\text{hs}}(\mu_h)$, will be bounded in the unit interval and, thus, the local surplus-sharing rules will be a convex combination of the polar cases. That is, for the level of inequality to remain constant $100 \times \omega_{\text{hs}}(\mu_h)\%$ of the surplus must be distributed proportionally to the attainment distribution \mathbf{h} and $100 \times (1 - \omega_{\text{hs}}(\mu_h))\%$ must be distributed proportionally to the shortfall distribution $\mathbf{s} = \mathbf{1} - \mathbf{h}$. Analogously, for an IEC that satisfies the h-relative–absolute compromise, $\omega_{\text{ra}}(\mu_h)$ is in the unit interval implying that $100 \times \omega_{\text{ra}}(\mu_h)\%$ of the surplus must be distributed proportionally and $100 \times (1 - \omega_{\text{ra}}(\mu_h))\%$ must be distributed uniformly to the attainment distribution.

Proposition 3. *A General IEC satisfies*

- a) *the h-relative–absolute compromise concept if and only if the weights in Eq. (22) are in the unit interval, i.e., $\omega_{\text{ra}}(\mu_h) \in [0, 1]$.*
- b) *the hs-relative compromise concept if and only if the weights in Eq. (23) are in the unit interval, i.e., $\omega_{\text{hs}}(\mu_h) \in [0, 1]$.*

Relation to Erreygers and van Ourti's (2011a) inequality weights

For relevancy and interpretation of the surplus-sharing rules, it is noteworthy that the weights in Eq. (22), $\omega_{\text{ra}}(\mu_h)$ and $1 - \omega_{\text{ra}}(\mu_h)$, coincide with Erreygers and van Ourti's (2011a) measures of how sensitive a rank-dependent index is to (h-)relative and absolute inequalities, or more precisely how sensitive an index is to relative differences in relation to absolute differences and vice versa. Using the elasticity of the normalization

function of a rank-dependent index,

$$\eta(\mu_h) = \frac{\partial f(\mu_h, n)}{\partial \mu_h} \frac{\mu_h}{f(\mu_h, n)} \quad (24)$$

Erreygers and van Ourti (2011a) define the weight that an index gives to (h-)relative inequality as $-\eta(\mu_h)$ and the weight it gives to absolute inequality as $1 + \eta(\mu_h)$.

As we consider the two relative IECs to be the relevant polar cases for bounded variables, we adapt Erreygers and van Ourti's (2011a) measures to our new hs-relative compromise concept. By normalizing the distance in terms of elasticity to one of the polar cases, we obtain analogous inequality weights that coincide with the weights in Eq. (23). Thus,

$$\omega_{hs}(\mu_h) = \frac{\mu_h/(1 - \mu_h) - \eta(\mu_h)}{1 + \mu_h/(1 - \mu_h)} \quad (25)$$

indicates how much of an additional surplus must be distributed according to the sharing rules of the two relative IECs and may be interpreted as a measure of how sensitive the corresponding rank-dependent index is to relative differences in attainments in relation to relative differences in shortfalls. We express this formally in Proposition 4.

Proposition 4. *Let a rank-dependent inequality index, i.e., $I(\mathbf{h})$ or $G(\mathbf{h})$, represent a General IEC, then the h-relative weight in Eq. (22) equals $\omega_{ra}(\mu_h) = -\eta(\mu_h)$ and the h-relative weight in Eq. (23) equals*

$$\omega_{hs}(\mu_h) = \frac{\mu_h/(1 - \mu_h) - \eta(\mu_h)}{1 + \mu_h/(1 - \mu_h)} \quad (26)$$

A new θ -inequality concept

As the inequality weights in Eqs. (22) and (23), $\omega_{ra}(\mu_h)$ and $\omega_{hs}(\mu_h)$, are functions of the average health in the population, the local vertical value judgment—or level of intermediateness defined by the fractions being distributed according to the surplus-sharing rules of the two polar cases—is generally dependent on the health distribution \mathbf{h} . For the (h-)relative–absolute compromise concept, the only IEC that weights the polar cases constantly and independently of the mean is Yoshida's (2005) generalization of Krtsha's (1994) fair compromise.⁷ For the new hs-relative compromise concept, we adapt Yoshida's (2005) inequality concept to a bounded health variable so that each infinitesimal surplus of health, ε , should be distributed as a convex combination of the h-relative and s-relative sharing rules with weights equal to the parameter θ .

Definition 10. (A θ -IEC) $\forall \mathbf{h}, \tilde{\mathbf{h}}; \mathbf{h} \sim \tilde{\mathbf{h}}$ if

$$\frac{\mathbf{h} - \mu_h \mathbf{1}}{\mu_h^\theta (1 - \mu_h)^{1-\theta}} = \frac{\tilde{\mathbf{h}} - \mu_{\tilde{h}} \mathbf{1}}{\mu_{\tilde{h}}^\theta (1 - \mu_{\tilde{h}})^{1-\theta}} \quad (27)$$

with parameter $\theta \in [0, 1]$.

Proposition 5. *For any General IEC, the corresponding $\delta(\mathbf{h})$ equals*

$$\delta(\mathbf{h}) = (1 - \theta) \left(\frac{1}{n} \mathbf{1} - \frac{\mathbf{h} - \mu_h \mathbf{1}}{n(1 - \mu_h)} \right) + \theta \left(\frac{\mathbf{h}}{n\mu_h} \right) \quad (28)$$

⁷That is, a General IEC with $g(\mu_h) = \mu_h^\lambda$, where $0 \leq \lambda \leq 1$.

if and only if $g(\mu_h) = \mu_h^\theta (1 - \mu_h)^{(1-\theta)}$ where $0 \leq \theta \leq 1$.

Thus, analogously to Yoshida's (2005) suggested IEC, the nonlinear θ -IEC implies that for inequality to remain constant, $100 \times \theta\%$ of the infinitesimal surplus must be distributed proportionally to the attainment distribution and $100 \times (1-\theta)\%$ must be distributed proportionally to the shortfall distribution. The inequality map in Figure 2 illustrates the nonlinear iso-inequality contour representing the θ -IEC for different values of θ . The solid line, $\theta = 0.5$, represents the only IEC that is a perfect compromise between the polar cases. That is, it weights the relative inequality in attainments and relative inequality in shortfalls equally for any health distribution.

Take in Figure 2 here

Extending the θ -IEC

It is further illuminating to use these inequality weights to evaluate the level of intermediateness (for a given health distribution) of a rank-dependent index: how the vertical value judgment relates to the relevant polar cases for bounded variables. For that purpose, we extend the θ -IEC to a more general two-parameter IEC that underpins several of the intensively discussed rank-dependent indices.

Definition 11. (Extended θ -IEC) $\forall \mathbf{h}, \tilde{\mathbf{h}}; \mathbf{h} \sim \tilde{\mathbf{h}}$ if

$$\frac{\mathbf{h} - \mu_h \mathbf{1}}{\mu_h^{\theta_1} (1 - \mu_h)^{\theta_2}} = \frac{\tilde{\mathbf{h}} - \mu_{\tilde{h}} \mathbf{1}}{\mu_{\tilde{h}}^{\theta_1} (1 - \mu_{\tilde{h}})^{\theta_2}} \quad (29)$$

with parameters $\theta_1 \in [0, 1]$ and $\theta_2 \in [0, 1]$.

Note that Eq. (29) equals Eq. (27) for all θ_1 and θ_2 such that $\theta_2 = 1 - \theta_1$. The Extended θ -IEC includes Yoshida's (2005) generalization of the fair compromise (with respect to attainments, $\theta_2 = 0$, or shortfalls, $\theta_1 = 0$) and the IECs represented by the rank-dependent indices suggested by Wagstaff (2005), $\theta_1 = \theta_2 = 1$, and Erreygers (2009a,b), $\theta_1 = \theta_2 = 0$ (i.e., the absolute IEC). For any index representing an Extended θ -IEC, the inequality weights $\omega_{hs}(\mu_h)$ —i.e., the measure of sensitivity to relative inequality in attainments in relation to relative inequality in shortfalls— may be expressed as a linear function of μ_h and the two parameters, θ_1 and θ_2 . Formally, we substitute $g'(\mu_h)\mu_h/g(\mu_h) = -\eta(\mu_h) = \theta_1 - \theta_2 (\mu_h/(1 - \mu_h))$ into $\omega_{hs}(\mu_h)$ and rearrange into

$$\omega_{hs}(\mu_h) = \theta_1 + \mu_h(1 - \theta_1 - \theta_2) \quad (30)$$

Thus, for any rank-dependent index corresponding to the Extended θ -IEC, the level of intermediateness for a given level of μ_h is represented by a line from θ_1 to $(1 - \theta_2)$ for $0 < \mu_h < 1$. For Erreygers' index, the line goes from $\omega_{hs} = 0$ to $\omega_{hs} = 1$. That is, being close to an s-relative IEC in the lower limit of μ_h and approaching the h-relative IEC in the upper limit of μ_h by linearly increasing the weight on relative inequality in attainments (in relation to shortfalls). As Wagstaff's index goes from being h-relative to s-relative, these two indices are each other's opposites in terms of the level of intermediateness.

Discussion and conclusions

Implications of the new compromise concept

There has recently been an intense discussion of the problems of using measures of income inequality to measure health inequality when health is represented by a bounded variable, which is often the case. Although Clarke et al. (2002) showed that relative inequality measures may order populations differently for attainments and shortfalls, the idea that the two relative IECs representing different, but potentially ethically defensible, vertical value judgments did not occur in the literature until Allanson and Petrie (2012, 2013). In line with this idea, we have formalized a general compromise concept with the h-relative IEC (relative inequality with respect to attainments) and the s-relative IEC (relative inequality with respect to shortfalls) as endpoints. Such a compromise concept implies—and we believe most people would not oppose—that an equiproportional increase in attainments does not decrease inequality and an equiproportional decrease in shortfalls does not increase inequality. In addition to all IECs that satisfy the absolute–relative compromise in terms of either attainments or shortfalls, this hs-relative compromise also includes IECs that partly intermediate the absolute and the h-relative (i.e., intermediate with respect to attainments) and partly intermediate the absolute and the s-relative (i.e., extreme leftist with respect to attainments). Unlike for an unbounded variable, we do not necessarily rule out an IEC just because it is extreme leftist for some health distributions.

This position is different from what is presented by Erreygers (2009a,b), Erreygers and van Ourti (2011a), and Lambert and Zheng (2011), who all argue in favor of an absolute IEC and dismiss any relative or intermediate IEC as they do not rank populations consistently. For example, Lambert and Zheng (2011) use the relative–absolute–compromise concept, which is relevant for income inequality,⁸ to show that the absolute IEC is the only one for which an inequality ranking is consistent for both shortfalls and attainments. However, the inequality map in Figure 1 illustrates that this result is a direct consequence of implicitly allowing for both intermediate and extreme leftist IECs without acknowledging that they represent different vertical value judgments. Every IEC with respect to attainments that is represented by an iso-inequality contour in the area between the h-relative and absolute line is mirrored by a contour representing the corresponding IEC with respect to shortfalls in the area between the s-relative and absolute line (i.e., extreme leftist IEC with respect to attainments). For any distribution between the two contours, the ranking in comparison to the initial distribution will vary with the chosen perspective. That means implicitly comparing the ranking of an intermediate IEC with an extreme leftist IEC. As the absolute iso-inequality contour defines the intersection of the areas representing the two relative–absolute compromises, the corresponding inequality measure is the only one that ranks distributions consistently for attainments and shortfalls. If we instead explicitly allow an IEC to be extreme leftist for some values of μ_h by acknowledging the new hs-relative compromise concept, there also exist IECs represented by nonlinear iso-inequality contours that rank distributions consistently. Among these are any Extended θ -IEC such that $\theta_1 = \theta_2$, including the IEC underpinning the index suggested by Wagstaff (2005).⁹

However, if one truly considers it important to jointly measure the distributions of attainments and

⁸Formally, any IEC that may be represented by Zoli’s (2003) flexible two-parameter IEC: $g(\mu_h) = (\kappa\mu_h - (1 - \kappa))^\lambda$, where $\lambda \in [0, 1]$ and $\kappa \in [0, 1]$.

⁹See Lasso de la Vega and Aristondo (2012) for other consistent indices.

shortfalls, which is the main rationale behind the quest for a consistent inequality measure (compare Erreygers, 2009a,b; Lasso de la Vega and Aristondo, 2012; Lambert and Zheng, 2011), would one not call for an IEC that in addition to ranking populations consistently also weights relative differences in attainments and shortfalls equally for any health distribution? Even though the absolute IEC that underpins Erreygers' (2009a,c) index is the perfect linear combination between the two polar cases of the hs-relative compromise concept, it does not weight the polar cases equally (or independent of the mean). Neither does the IEC underpinning Wagstaff's index.

Our surplus-sharing approach characterizes an IEC by how it requires successive infinitesimal surpluses of health to be distributed to leave inequality unchanged. An IEC satisfying the hs-compromise concept implies that, for a given health distribution, an infinitesimal surplus must be distributed as a convex combination of the surplus-sharing rules of the two polar cases. The proportion being distributed according to the h-relative sharing rule is a measure of the level of intermediateness and may be interpreted as how sensitive the corresponding rank-dependent index is to relative differences in attainments in relation to shortfalls. The intermediateness of Erreygers' and Wagstaff's indices are linear functions of the average health in the population. Erreygers' goes from being s-relative to h-relative, while Wagstaff's goes in the opposite direction. This relationship explains the ranking pattern often seen in empirical applications (e.g., Erreygers, 2009b; Fleurbaey and Schokkaert, 2011; Kjellsson and Gerdtham, 2013a). For $\mu_h > 0.5$, the absolute and the s-relative indices, on the one hand, and Wagstaff's and the h-relative indices, on the other hand, tend to rank populations similarly. For $\mu_h < 0.5$, the opposite pairs apply. Consequently, choosing one of the two indices, Erreygers' or Wagstaff's, will for some values of μ_h implicitly imply one of the two relative IECs. In contrast, we show that the only IEC that weights the two polar cases equally and independently of the health distribution is our adaptation of Krtscha's (1994) fair compromise to a bounded health variable: the θ -IEC with $\theta = 0.5$.

However, by formalizing the new compromise concept, we acknowledge that IECs defined with respect to attainments and shortfalls may represent different vertical value judgment (just as the relative and absolute) and thereby question the focus on consistent inequality measures. Consequently, the paper broadens rather than narrows the set of ethically defensible IECs.

Where to go from here

Our reasoning suggests a very general IEC without giving any guidance on the choice of the parameter values. We stress the importance of considering the value judgments implicit in the health inequality measure. But we recognize that the question of how to choose an appropriate measure will be asked by applied researchers. One way forward is to run experiments to find the parameters that represent the views held in the general population. However, rather than focusing on finding an optimal IEC, the main implication of the paper, in order to guide policy, is to use a range of inequality measures bounded by the two relative IECs—preferably complemented by the Extended θ -IEC with various parameter values. Note, however, that before applying this new abundance of inequality concepts, one needs to, in line with Erreygers (2009a,b), derive corresponding indices with desirable properties.

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Appendix

Proof of Proposition 1: For a univariate rank-dependent index, $G(\mathbf{h})$, to represent a General IEC,

$$\frac{\mathbf{h} - \mu_h \mathbf{1}}{g(\mu_h)} = \frac{\tilde{\mathbf{h}} - \mu_{\tilde{h}} \mathbf{1}}{g(\mu_{\tilde{h}})} \quad (\text{A.1})$$

must be a sufficient condition for

$$G(\mathbf{h}) = f(\mu_h, n) \sum h_i w_i = f(\mu_{\tilde{h}}, n) \sum \tilde{h}_i w_i = G(\tilde{\mathbf{h}}) \quad (\text{A.2})$$

Let $w_i = (n+1)/2 - \rho_i$ represent the i th element in the vector $\mathbf{w} = (w_1, w_2, \dots, w_n)$ and let $v(n)$ be a positive scalar function of n . Multiplying both sides of Eq. (A.1) by $v(n)$ and the vector \mathbf{w} yields

$$v(n) \sum_i \frac{h_i - \mu_h}{g(\mu_h)} w_i = v(n) \sum_i \frac{\tilde{h}_i - \mu_{\tilde{h}}}{g(\mu_{\tilde{h}})} w_i \quad (\text{A.3})$$

As $\sum_{i=1}^n w_i = 0$, $\sum_{i=1}^n (h_i - \mu_h) w_i = \sum_{i=1}^n h_i w_i$ and Eq. (A.3) becomes

$$\frac{v(n)}{g(\mu_h)} \sum_i h_i w_i = \frac{v(n)}{g(\mu_{\tilde{h}})} \sum_i \tilde{h}_i w_i \quad (\text{A.4})$$

implying that $G(\mathbf{h}) = G(\tilde{\mathbf{h}})$ if $f(\mu_h, n) = v(n)/g(\mu_h)$. For an analogous proof for a bivariate index, such as $I(\mathbf{h}) = I(\tilde{\mathbf{h}})$, substitute z_i for w_i . ■

Proof of Proposition 2: For a General IEC, $\mathbf{h} \sim \tilde{\mathbf{h}}$ and $\mathbf{h} + \mathbf{d}(\mathbf{h}, \varepsilon) \sim \mathbf{h}$ imply $\mathbf{d}(\mathbf{h}, \varepsilon) = \tilde{\mathbf{h}} - \mathbf{h}$ and $\mu_{\tilde{h}} - \mu_h = \frac{\varepsilon}{n}$. Substituting $\tilde{\mathbf{h}} = \mathbf{h} + \mathbf{d}(\mathbf{h}, \varepsilon)$ and $\mu_{\tilde{h}} = \mu_h + \frac{\varepsilon}{n}$ into Eq. (1) and solving for $\mathbf{d}(\mathbf{h}, \varepsilon)$ yields the corresponding IEDV:

$$\mathbf{d}(\mathbf{h}, \varepsilon) = \frac{\varepsilon}{n} \mathbf{1} + (\mathbf{h} - \mu_h \mathbf{1}) \left(\frac{g(\mu_h + \frac{\varepsilon}{n})}{g(\mu_h)} - 1 \right) \quad (\text{A.5})$$

As we assume $g(\mu_h)$ to be continuous and (piecewise) differentiable, Eq. (A.5) is continuous and has a piecewise continuous partial derivative with respect to ε . For simplicity, rearrange Eq. (A.5) into $\mathbf{d}(\mathbf{h}, \varepsilon) = \frac{\varepsilon}{n} \mathbf{1} - \mu_h \mathbf{1} \left(\frac{g(\mu_h + \frac{\varepsilon}{n}) - g(\mu_h)}{g(\mu_h)} \right) + \mathbf{h} \left(\frac{g(\mu_h + \frac{\varepsilon}{n}) - g(\mu_h)}{g(\mu_h)} \right)$. Let the vector $\boldsymbol{\delta}(\mathbf{h}) = \lim_{\varepsilon \rightarrow 0^+} [\mathbf{d}(\mathbf{h}, \varepsilon)/\varepsilon]$ represent the

local vertical value judgment and calculate the limits as

$$\delta(\mathbf{h}) = \lim_{\varepsilon \rightarrow 0^+} \frac{\varepsilon}{n\varepsilon} \mathbf{1} - \lim_{\varepsilon \rightarrow 0^+} \mu_h \mathbf{1} \left(\frac{1}{\varepsilon} \frac{g(\mu_h + \frac{\varepsilon}{n}) - g(\mu_h)}{g(\mu_h)} \right) + \lim_{\varepsilon \rightarrow 0^+} \mathbf{h} \left(\frac{1}{\varepsilon} \frac{g(\mu_h + \frac{\varepsilon}{n}) - g(\mu_h)}{g(\mu_h)} \right) \quad (\text{A.6})$$

As $\lim_{\varepsilon \rightarrow 0^+} \frac{1}{\varepsilon} \frac{g(\mu_h + \frac{\varepsilon}{n}) - g(\mu_h)}{g(\mu_h)} = \frac{1}{n} \frac{g'(\mu_h)}{g(\mu_h)}$, for instance by L'Hôpital's rule, Eq. (A.6) becomes

$$\delta(\mathbf{h}) = \frac{1}{n} \left(1 - \frac{g'(\mu_h)\mu_h}{g(\mu_h)} \right) + \frac{1}{n} \frac{\mathbf{h}}{\mu_h} \left(\frac{g'(\mu_h)\mu_h}{g(\mu_h)} \right) \quad (\text{A.7})$$

Let $\omega_{\text{ra}}(\mu_h) = g'(\mu_h)\mu_h/g(\mu_h)$, then Eq. (A.7) becomes $\delta(\mathbf{h}) = \frac{1}{n} (1 - \omega_{\text{ra}}(\mu_h)) + \frac{1}{n} \frac{\mathbf{h}}{\mu_h} \omega_{\text{ra}}(\mu_h)$. To show the second part of the proposition, i.e., Eq. (23), add

$$\frac{1}{n} \left((\mathbf{h} - \mathbf{h}) + \mathbf{1}(\mu_h - \mu_h) + \left(\frac{g'(\mu_h)\mu_h}{g(\mu_h)} \right) (\mathbf{h} - \mathbf{h} + \mathbf{1}(\mu_h - \mu_h)) \right) = 0 \quad (\text{A.8})$$

to the right-hand side of Eq. (A.7) and rearrange to

$$\begin{aligned} \delta(\mathbf{h}) &= \frac{1}{n} \left(\mathbf{h} + \frac{g'(\mu_h)\mu_h}{g(\mu_h)} \frac{\mathbf{h}}{\mu_h} - \frac{g'(\mu_h)\mu_h}{g(\mu_h)} \mathbf{h} \right) \\ &\quad + \mathbf{1} \frac{1}{n} \left(1 - \mu_h - \frac{g'(\mu_h)\mu_h}{g(\mu_h)} + \frac{g'(\mu_h)\mu_h}{g(\mu_h)} \mu_h \right) - \frac{1}{n} \left(\mathbf{h} - \mu_h \mathbf{1} - \frac{g'(\mu_h)\mu_h}{g(\mu_h)} \mathbf{h} + \frac{g'(\mu_h)\mu_h}{g(\mu_h)} \mu_h \mathbf{1} \right) \end{aligned} \quad (\text{A.9})$$

which simplifies to

$$\delta(\mathbf{h}) = \left(\mu_h + \frac{g'(\mu_h)\mu_h}{g(\mu_h)} (1 - \mu_h) \right) \frac{\mathbf{h}}{n\mu_h} + \left(1 - \left(\mu_h + \frac{g'(\mu_h)\mu_h}{g(\mu_h)} (1 - \mu_h) \right) \right) \left(\frac{1}{n} - \frac{\mathbf{h} - \mu_h \mathbf{1}}{n(1 - \mu_h)} \right) \quad (\text{A.10})$$

Let $\omega_{\text{hs}}(\mu_h) = \left(\mu_h + \frac{g'(\mu_h)\mu_h}{g(\mu_h)} (1 - \mu_h) \right)$ and Eq. (A.10) becomes

$$\delta(\mathbf{h}) = \omega_{\text{hs}}(\mu_h) \frac{\mathbf{h}}{n\mu_h} + (1 - \omega_{\text{hs}}(\mu_h)) \left(\frac{1}{n} \mathbf{1} - \frac{\mathbf{h} - \mu_h \mathbf{1}}{n(1 - \mu_h)} \right) \blacksquare \quad (\text{A.11})$$

Proof of Proposition 3: First, note that we may rearrange $0 \leq \omega_{\text{ra}}(\mu_h) \leq 1$ and $0 \leq \omega_{\text{hs}}(\mu_h) \leq 1$ into

$$0 \leq \frac{g'(\mu_h)\mu_h}{g(\mu_h)} \leq 1 \quad (\text{A.12})$$

and

$$\frac{-\mu_h}{1 - \mu_h} \leq \frac{g'(\mu_h)\mu_h}{g(\mu_h)} \leq 1 \quad (\text{A.13})$$

respectively. Note also that, for any feasible¹⁰ pair of \mathbf{h} and ε , the definition of a compromise between h-relative and absolute is equivalent to

$$\mathbf{h} + \mathbf{1} \frac{\varepsilon}{n} \succeq \mathbf{h} + \mathbf{d}(\mathbf{h}, \varepsilon) \succeq \mathbf{h} + \frac{\varepsilon \mathbf{h}}{n\mu_h} \quad (\text{A.14})$$

¹⁰For some values of ε , some h_i may exceed its upper bound.

and the definition of a compromise between h-relative and s-relative is equivalent to

$$\mathbf{h} + \frac{\varepsilon}{n} \left(\mathbf{1} - \frac{\mathbf{h} - \mu_h \mathbf{1}}{1 - \mu_h} \right) \succeq \mathbf{h} + \mathbf{d}(\mathbf{h}, \varepsilon) \succeq \mathbf{h} + \frac{\varepsilon \mathbf{h}}{n \mu_h} \quad (\text{A.15})$$

As the IEDV corresponding to a General IEC is continuous and path independent (formally defined as $\mathbf{d}(\mathbf{h}, \varepsilon) + \mathbf{d}[\mathbf{h} + \mathbf{d}(\mathbf{h}, \varepsilon), \varepsilon'] = \mathbf{d}(\mathbf{h}, \varepsilon + \varepsilon') \forall \mathbf{h}, \forall \varepsilon, \varepsilon' > 0$), it is sufficient for proving Proposition 3 to only consider an infinitesimal surplus for any \mathbf{h} by showing that Eqs. (A.12) and (A.13) are necessary and sufficient conditions for an IEC to satisfy the relative–absolute compromise and the hs-relative compromise, respectively. The proof is set out for income-related health inequality, but extends to total health inequality.

Sufficiency: Let an IEC be a compromise between a) the h-relative and the s-relative IECs and b) the h-relative and the absolute IECs. Consider any two individuals (i, j) such that $\phi_i < \phi_j$. The assumption of a pro-rich health distribution implies that the expected value of the difference of h_i and h_j is positive: $E(h_i - h_j) > 0$. (For total inequality, consider instead two individuals such that $\rho_i < \rho_j$, then $h_i - h_j > 0$ holds by definition.) Note that the i th element in $\boldsymbol{\delta}(\mathbf{h})$ corresponding to the absolute, h-relative, and s-relative IECs are $\delta_i = 1/n$, $\delta_i = h_i/n\mu_h$, and $\delta_i = 1/n - (h_i - \mu_h)/(n(1 - \mu_h))$, respectively. Path independency implies that Eqs. (A.14) and (A.15) hold if and only if the expected value of the difference between the shares of an infinitesimal surplus distributed to i and j lies in the range between the expected values of the differences between the surplus shares of the polar cases. That is, a) for a hs-relative compromise,

$$E \left(\frac{1}{n} - \frac{h_i - \mu_h}{n(1 - \mu_h)} - \left(\frac{1}{n} - \frac{h_j - \mu_h}{n(1 - \mu_h)} \right) \right) \leq E(\delta_i - \delta_j) \leq E \left(\frac{h_i}{n\mu_h} - \frac{h_j}{n\mu_h} \right) \quad (\text{A.16})$$

and b) for a relative–absolute compromise,

$$E \left(\frac{1}{n} - \frac{1}{n} \right) \leq E(\delta_i - \delta_j) \leq E \left(\frac{h_i}{n\mu_h} - \frac{h_j}{n\mu_h} \right) \quad (\text{A.17})$$

where

$$E(\delta_i - \delta_j) = E \left(\frac{1}{n} + \frac{g'(\mu_h)\mu_h}{g(\mu_h)} \frac{(h_i - \mu_h)}{n\mu_h} - \left(\frac{1}{n} + \frac{g'(\mu_h)\mu_h}{g(\mu_h)} \frac{(h_j - \mu_h)}{n\mu_h} \right) \right) = \frac{g'(\mu_h)\mu_h}{g(\mu_h)} \left(\frac{E(h_i - h_j)}{n\mu_h} \right) \quad (\text{A.18})$$

Multiplying each term in Eqs. (A.17) and (A.16) by $n\mu_h/E(h_i - h_j)$ yields Eqs. (A.12) and (A.13),¹¹ which completes this part of the proof.

Necessity: Let, instead, a) $-\mu_h/(1 - \mu_h) \leq g'(\mu_h)\mu_h/g(\mu_h) \leq 1$ and b) $0 \leq g'(\mu_h)\mu_h/g(\mu_h) \leq 1$ and reverse the exercise above. ■

Proof of Proposition 4: A rank-dependent index represents a General IEC if $f(\mu_h, n) = v(n)/g(\mu_h)$ (Proposition 1). Then the elasticity of the normalization function is

$$\eta(\mu_h) = f'(\mu_h) \frac{\mu_h}{f(\mu_h)} = -v(n) \frac{g'(\mu_h)}{g(\mu_h)^2} \frac{\mu_h}{\frac{v(n)}{g(\mu_h)}} = -g'(\mu_h) \frac{\mu_h}{g(\mu_h)} \quad (\text{A.19})$$

¹¹Observe that the left-hand side of Eq. (A.16) reduces to $-E(h_i - h_j)/(n(1 - \mu_h))$

To complete the proof, using Proposition 2, substitute Eq. (A.19) into Eqs. (22) and (23) to obtain $\omega_{\text{ra}} = -\eta(\mu_h)$ and

$$\omega_{\text{hs}}(\mu_h) = \mu_h - \eta(\mu_h)(1 - \mu_h) = \frac{\frac{\mu_h}{1-\mu_h} - \eta(\mu_h)}{1 + \frac{\mu_h}{1-\mu_h}}$$

respectively. ■

Proof of Proposition 5: Sufficiency: Represent a General IEC as $\forall \mathbf{h}, \tilde{\mathbf{h}}; \mathbf{h} \sim \tilde{\mathbf{h}}$ if

$$\frac{\mathbf{h} - \mu_h \mathbf{1}}{\mu_h^\theta (1 - \mu_h)^{1-\theta}} = \frac{\tilde{\mathbf{h}} - \mu_{\tilde{h}} \mathbf{1}}{\mu_{\tilde{h}}^\theta (1 - \mu_{\tilde{h}})^{1-\theta}} \quad (\text{A.20})$$

i.e., $g(\mu_h) = \mu_h^\theta (1 - \mu_h)^{1-\theta}$, then $g'(\mu_h)\mu_h/g(\mu_h) = \theta - (1 - \theta)\mu_h/(1 - \mu_h)$. Solving for θ yields $\theta = \mu_h + \frac{g'(\mu_h)\mu_h}{g(\mu_h)}(1 - \mu_h)$. Proposition 2 implies

$$\delta(\mathbf{h}) = (1 - \omega_{\text{hs}}(\mu_h)) \left(\frac{1}{n} - \frac{\mathbf{h} - \mu_h \mathbf{1}}{n(1 - \mu_h)} \right) + \omega_{\text{hs}}(\mu_h) \frac{\mathbf{h}}{n\mu_h} \quad (\text{A.21})$$

where $\omega_{\text{hs}}(\mu_h) = \mu_h + \frac{g'(\mu_h)\mu_h}{g(\mu_h)}(1 - \mu_h) = \theta$. Thus, Eq. (28) holds.

Necessity: Let the local surplus-sharing rules of a General IEC be represented by the vector in Eq. (28). Proposition 2 implies $\omega_{\text{hs}}(\mu_h) = \theta = \mu_h + \frac{g'(\mu_h)\mu_h}{g(\mu_h)}(1 - \mu_h)$. Solve for $g'(\mu_h)\mu_h/g(\mu_h)$ to obtain a differential equation,

$$\frac{g'(\mu_h)\mu_h}{g(\mu_h)} = \theta - (1 - \theta) \frac{\mu_h}{1 - \mu_h} \quad (\text{A.22})$$

that has the single solution: $g(\mu_h) = C\mu_h^\theta (1 - \mu_h)^{1-\theta}$, where C is a constant that may be normalized to one as it appears on both sides of Eq. (1). ■

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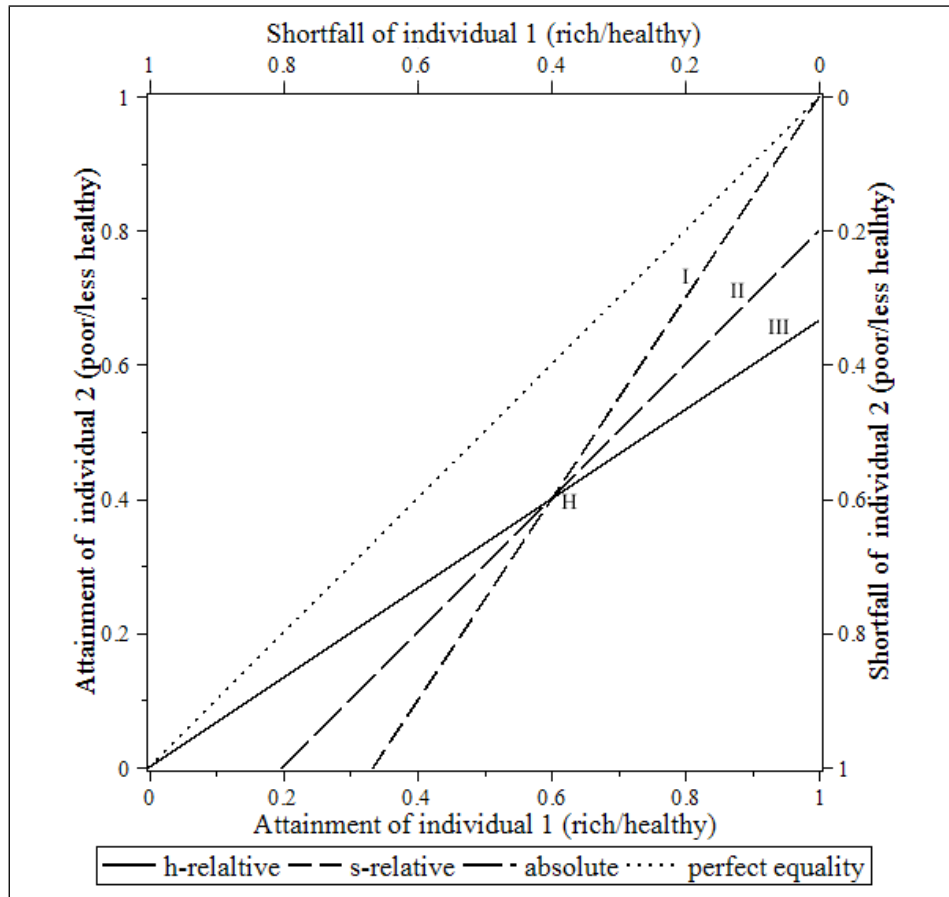


Fig. 1: Inequality map for a bounded variable. The inequality map is adapted from Allanson and Petrie (2012), who present a more comprehensive explanation of the map.

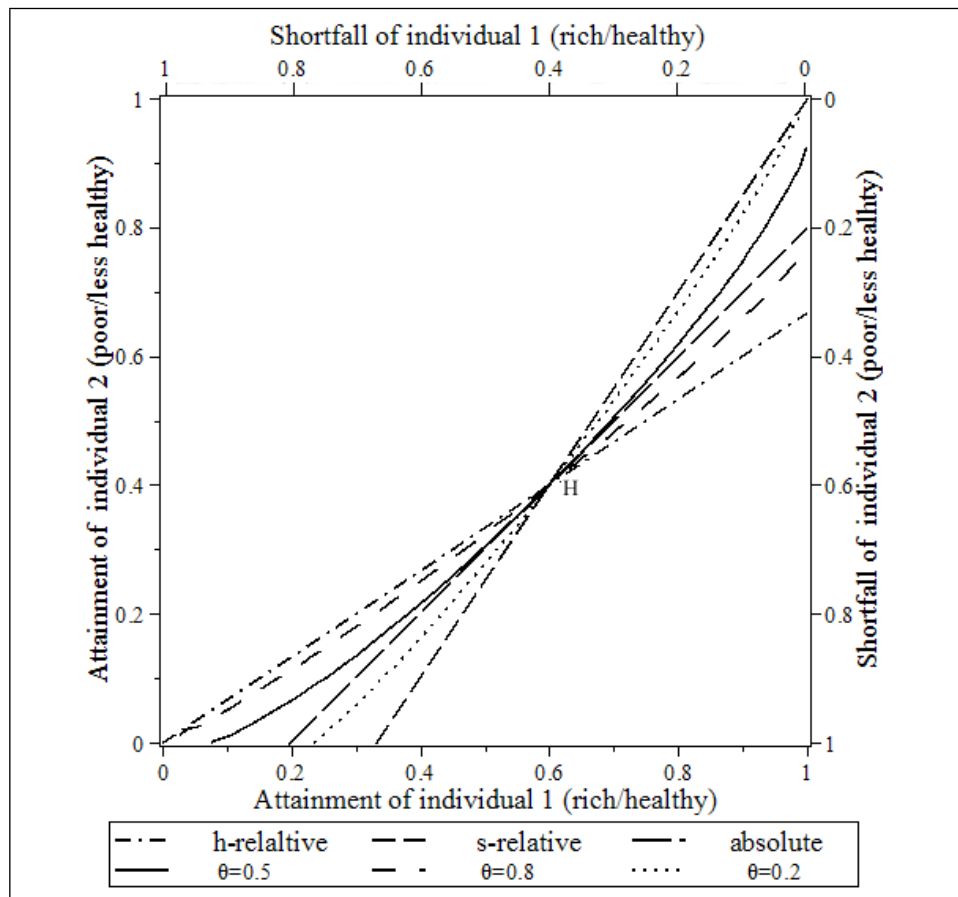


Fig. 2: A θ -IEC. Note that the h-relative (i.e., $\theta = 1$) and the s-relative (i.e., $\theta = 0$) cases are the extrema for the θ -IEC. The absolute line is included in the figure for reference. The inequality map is adapted from Allanson and Petrie (2012)