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# Correcting the Concentration Index for Binary Variables

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## Abstract:

This paper discusses measurement of socioeconomic inequalities in prevalence of a health condition. As its point of departure, it uses the recent exchange between Guido Erreygers and Adam Wagstaff in this journal, where they discuss merits of their own corrections of the frequently used concentration index. The paper first reconciles the debate between Erreygers and Wagstaff and discusses the value judgments that are hidden behind their indices. Secondly, it discusses when the property *level independence* is desirable. Thirdly, it empirically illustrates whether and when the choice of index affects comparisons between contexts using binary health indicators from the European Survey of Health, Ageing and Retirement (SHARE). The results show that choice of index matters and that it not only affects the magnitude of measured inequalities but also internal ranking between countries.

**JEL Classification:** I10 D63

**Keywords:** health inequalities, socioeconomic inequality, concentration index, binary variables

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## 1. Introduction

In Health Economics, the concentration index approach has since the 1990's become the standard tool to evaluate socioeconomic inequalities in health. As the concentration index ( $C$ , hereafter) is derived from the Gini coefficient of income inequalities, it requires that the health variable is on the same scale as income, i.e. a ratio-scaled measure without an upper bound (Erreygers, 2009a). As health differs from income in numerous aspects such a variable is rarely at hand. Instead, health measures tend to be bounded and either ordinal or cardinal.

For bounded variables, i)  $C$  may rank countries by inequalities in health and ill-health differently (Clarke et al., 2002), ii) the maximum and minimum value of  $C$  depend on the mean health in the society (Wagstaff, 2005), and iii) the value of  $C$  depends on the scale of the health variable (Erreygers, 2009a). To account for these issues, Erreygers (2009a) and Wagstaff (2005) develop their respective corrections of  $C$  for bounded variables. In a recent exchange in this journal, Wagstaff (2009) and Erreygers (2009a; 2009b) debate the merits of these corrections.

Although the debate mainly concerns cardinal variables, Erreygers (2009a) uses a binary variable, which are bounded but rather ordinal than cardinal, to illustrate the importance of correcting  $C$ . In practice, many health variables are binary. The empirical literature is full of examples analyzing inequalities in the prevalence of health conditions such as malnutrition, obesity, and a variety of self assessed measures (see. Hernández-Quevedo et al., 2006; Harper and Lynch, 2007; van de Poel et al., 2008; Mackenbach et al., 2008). Moreover, the methodological literature suggests dichotomizing other categorical or ordinal variables (O'Donnell et al., 2008). Without an in depth discussion, researchers and practitioners may use an inequality measure without considering the value judgment behind it (e.g. Mishra and Joe, 2010; de Poel et al., 2008, Hernández-Quevedo et al., 2010). Focusing on binary

variables, our paper reconciles the debate of Erreygers and Wagstaff and examines the methodological and empirical differences between their suggested indices.

As Erreygers (2009a, p. 515) highlights, the concentration index approach does not meaningfully measure inequalities for ordinal health variables. Therefore, Section 2, which presents the methodological background of measuring health inequalities of a bounded variable, provides a justification of using the concentration index approach with binary health measures.

Erreygers (2009a; 2009b; and van Ourti, 2010) claims his index to be superior as it satisfies *level independence* (i.e. that an equal increment of health for all individuals does not affect the value of the index), while Wagstaff (2009b) questions the desirability of the same property. Although we applaud Erreygers quest to extend the concentration index to cardinal variables, we argue that Erreygers's index ( $E$ , hereafter) is not necessarily superior to Wagstaff's ( $W$ , hereafter). Relating to the discussion of relative and absolute inequalities between Erreygers (2001a; 2009b) and Wagstaff (2005),<sup>1</sup> Section 3 reveals the underlying question and value judgment of the indices. As the two indices condition the absolute inequalities on different definitions of *the most unequal society*, they capture different perspectives of socioeconomic inequalities and the choice of index therefore depends on the preferred value judgment. The section further illustrates that all the technical advantages of  $E$  that Erreygers (2009a; 2009b) highlights are a consequence of this definition.

Following Erreygers (2009a; 2009b) and Erreygers and van Ourti (2010), we acknowledge that the notion of relative and absolute value judgments changes when the health measure no longer is an unbounded ratio-scaled variable but a binary one. While Erreygers and van Ourti (2010) therefore exclude any relative value judgment for bounded variables (unless one

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<sup>1</sup> See also Erreygers and van Ourti (2010)

relaxes other conditions), we claim that, as no relative inequality preserving changes in health are feasible, the value judgment of the indices ought to be evaluated by how they weight absolute inequalities. As  $E$  weights absolute inequalities constantly and independently of the prevalence in the society, it captures an absolute value judgment.  $C$  of health suggests that the same level of absolute inequalities is more severe when the prevalence is low while  $C$  of ill-health suggests that the inequalities is more severe when the prevalence is high. As a consequence of the definition of the most unequal society,  $W$  combines these two perspectives suggesting that the same level of absolute inequalities are more severe for both high and low values of the prevalence. This value judgment reflects that being of ill-health is, in relative terms, more disadvantageous in a low prevalence society, while being of good health is, at the same time, more privileged in a high prevalence society.

We argue in Section 4 that for a specific class of binary health measures, *level independence* is desirable and  $E$  is the preferred index due to reasons not previously discussed in the literature. The nature of the latent variable that underlies the binary representation of the health measure is crucial. Stipulating that the health measure is open for subjectivity or cultural differences, i.e. a health measure which is exposed to a high risk of reporting heterogeneity, then  $E$  should be the preferred index, irrespective of the preferred value judgment. For less subjective variables, the choice should depend on the preferred value judgment.

Using binary health indicators from the European Survey of Health, Ageing and Retirement (SHARE), Section 5 empirically confirms the conclusions from methodological discussion illustrating whether, and when, the choice of index affects comparisons between countries. The empirical results show that the methodological discussion is important rather than just a matter of semantics. Finally, Section 6 concludes.

## 2. Rank dependent inequality indices

$C$  quantifies relative socioeconomic inequalities in a health variable,  $h_i$ , by calculating the cumulative percentage of  $h_i$  concentrated to a cumulative percentage of the population ranked by a socioeconomic variable (cf. Kakwani et al., 1997).  $C$  is equal to twice the area between the concentration curve and the line of equality (the 45 degree line). The related generalized concentration index ( $V$ , hereafter) quantifies absolute inequalities and equals  $C$  multiplied by the prevalence, or mean, of the health variable  $\mu_h$  (Clarke et al., 2002).

Erreygers (2009a) shows that  $C$ ,  $V$ ,  $W$ , and  $E$  all belong to the family of rank dependent indicators of health inequalities. Following Erreygers, we express the general form of this family of indicators as a normalized sum of weighted health levels:

$$I(h) = f(H) \sum_{i=1}^n z_i h_i \quad Eq(1)$$

where  $f(H) > 0$ ,  $n$  is the number of individuals in a given population and  $z_i = \frac{(n+1)}{2} - \lambda_i$  where  $\lambda_i$  is the socioeconomic rank of the individual ranging from the richest ( $\lambda_i = 1$ ) to the poorest ( $\lambda_i = n$ ).  $z_i$  takes on a positive value if individual  $i$  is rich (e.g. from the upper half of the income distribution) and a negative value if individual  $i$  is poor (e.g. from the lower half of the income distribution). Consequently, if  $h_i$  is a good, a positive (negative) value of  $I(h)$  suggests a pro-rich (pro-poor) concentration of  $h_i$ . Conversely, if  $h_i$  is a bad, the reverse pattern applies.

As the function normalizing the weighted sum  $f(H)$ , where  $H=(\mu_h, n)$  is a vector,<sup>2</sup> is the only variation between the indices, it determines the specific form and properties of indices. We express  $C$ ,  $V$ ,  $W$ , and  $E$  for binary variables as:

$$C = \frac{2}{n^2 \mu_h} \sum_{i=1}^n z_i h_i \quad Eq(2)$$

$$V = \frac{2}{n^2} \sum_{i=1}^n z_i h_i \quad Eq(3)$$

$$W = \frac{2}{n^2 (1-\mu_h) \mu_h} \sum_{i=1}^n z_i h_i \quad Eq(4)$$

$$E = \frac{8}{n^2} \sum_{i=1}^n z_i h_i \quad Eq(5)$$

Moreover, Erreygers (2009a) shows that  $E$  is the only indicator within this family that satisfies four, according to Erreygers desirable, properties:

- 1) **transfer**: a small transfer of health from a richer (poorer) to a poorer (richer) individual translates into a pro-poor (pro-rich) change of the index.
- 2) **mirror**: the inequality indices of health and ill-health should be *mirror* images of each other, i.e.  $I(\text{health})$  are equal to the absolute value of  $I(\text{ill-health})$ , but with opposite sign.
- 3) **level independence**: an equal increment of health for all individuals does not affect the index, i.e. the index is invariant to scalar addition even though the upper and lower limits are unchanged.
- 4) **cardinal invariance**: no positive linear transformation of the health variable,  $h_i$ , affects the value of the index, i.e. cardinal invariance implies that the measured degree of inequalities is the same irrespective of the cardinal scale of the health variable (e.g.

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<sup>2</sup>In comparison to Erreygers (2009a),  $f(H)$  is simplified for binary variables as the lower and upper bound of the health variable equals 0 and 1, respectively.

$I(h)$  of body temperature would be the same whether measured in Celsius or Fahrenheit).

$C$  satisfies only the transfer property, while  $V$  satisfies all but *cardinal invariance* and  $W$  satisfies all but *level independence*. Unless there is a strong a priori reason to why health inequalities should be based on either the prevalence of health or ill-health, it is sensible to impose the mirror condition when armed with a binary variable. Further, cardinal invariance means that the measured degree of inequalities is the same irrespective of which two numeric values that represent good and bad health (i.e. any dichotomous variable could be used). Thus, the set of possible rank dependent indices is reduced to  $W$  and  $E$ . The further discussion in this paper focuses on the choice between the two indices and the *level independence* property which constitutes the main difference between them.

## **Table 1**

### **2.1. A justification for binary variables**

A binary variable is a specific representation of a dichotomous variable, which is of ordinal or qualitative nature. As such a scale is not applicable with the rank dependent indices (Erreygers, 2006; 2009a; Erreygers and van Ourti, 2010), another interpretation of the variable is needed.

Although an arbitrary dichotomous variable is at most ordinal, the binary representation  $\{0,1\}$  has an absolute interpretation. The zero situation of a binary variable, such as prevalence of any health condition, corresponds to a situation of complete absence, i.e. zero indicates that the individual is not of the examined health condition (cf. Roberts, 1979, p. 64-65). Thus, the binary representation is compatible with the cumulative nature of  $C$ , as adding up the “zeroes and ones” intuitively can be interpreted as the cumulative number of healthy



or unhealthy individuals concentrated within a cumulative proportion of the population. Relating to this understanding, one could consider the binary variable being projected onto a cardinal or ratio-scaled measure of health bounded at 0 and 1 ( $h_i^*$ , hereafter) by using the prevalence at an aggregated level of defined subgroups (e.g. deciles, percentiles etc) (cf. Erreygers and van Ourti, 2010). In fact, any rank dependent index of  $h_i^*$  is asymptotically equal to an index of  $h_i$ .<sup>3</sup>

Interpreting the binary variable as a ratio-scaled proxy does i) facilitate the discussion of relative and absolute inequalities in Section 3 and ii) provide a more intuitive understanding of *level independence* for binary variables. Instead of corresponding to being invariant to a shift of all individuals from 0 to 1 – the only possible equal increment of a binary variable– *level independence* now corresponds to being invariant to equal increments in prevalence across the deciles or percentiles.

### 3. Relative and absolute value judgments

For unbounded ratio-scaled variables the notion of relative and absolute value judgments is clear-cut. Being invariant to an equal health increment but not to an equiproportionate change in the health variable is equivalent to being a measure of absolute inequalities (e.g.  $V$ ). The opposite applies for measures of relative inequalities (e.g.  $C$ ). It is clear from the discussion in Erreygers (2009a; 2009b) and Wagstaff (2009) that it is not as straightforward for bounded health variables.

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<sup>3</sup> Let  $I(h^*) = f(H) \sum_{i=1}^n z_i h_i^*$ , where  $h_i^* = \sum_{j=1+k(l-1)}^{kl} \frac{h_j}{k}$ ,  $k$  is equal to the number of observation in each subgroup and  $l$  is the index of the subgroup, then  $I(h^*)$  will converge to  $I(h)$  when either the number of observations or the number of groups increases. Increasing the number of groups is analogous to decreasing the number of observations within the group towards one. We are thankful to Guido Erreygers and Tom van Ourti for comments on this specific issue.

Although using  $h_i^*$  facilitates relating to the concepts of relative and absolute inequalities, it does not address the issues induced by the bounds of the variable. First, an increase of health is mirrored by a decrease in ill-health. Second, a proportional change of health is not a proportional change of ill-health. Third, the bounds act as constraints to (proportionally) equal transformations of the health variable. However, asymptotically it is always possible to define subgroups in a way that marginal changes are feasible for almost all distributions of  $h_i^*$ .<sup>4</sup> Nevertheless, one cannot directly translate the relative and absolute value judgments from unbounded variables to bounded ones.

As a result of the first two points, unless the mirror condition is relaxed, an index of a bounded variable is always sensitive to equiproportionate changes of health and cannot be decreasing as a response to equal increments for all values of  $\mu_h$ .<sup>5</sup> In other words, as Wagstaff points out,  $W$  cannot be a pure measure of relative inequalities. Erreygers and van Ourti (2010) therefore exclude any relative (or non-absolute) value judgment.

By contrast, we argue that even though discussing changes that preserve relative inequalities is futile for binary health variables, the indices weight absolute inequalities differently.  $E$  suggests that the severeness of absolute inequalities is independent of the prevalence (*level independence*), while  $W$  (as well as  $C$ ) suggests that the severeness of the absolute inequalities depends on the prevalence. As the normalization function  $f(H)$  is the only variation between the indices, the weight of the absolute inequalities and the value judgment of the index are implicitly embedded within it. The following sections reveal the underlying value judgments of the indices and explore how, and why, equiproportionate changes and

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<sup>4</sup> Erreygers and van Ourti (2010) address this issue, by redefining the concepts of relative and absolute to quasi-relative and quasi-absolute.

<sup>5</sup> In fact, an equiproportionate increase of health is translated into an equiproportionate decrease **and** an equal increment of ill-health. E.g let  $s_i$  be the level of ill-health and  $h_i\beta = h_i(1+\delta) = h_i + h_i\delta$ , then  $h_i\delta = (1-s_i)\delta = \delta-s_i\delta$ . Thus, to satisfy both the mirror condition and being invariant to equiproportionate changes, an index also has to be insensitive to equal increments (*level independent*).

equal increments affect the indices. However, to understand these value judgments and the response of the indices we first need to discuss how the indices define – and differ in their definition of – the most unequal society.

### 3.1. The most unequal society

$C$ ,  $W$ , and  $E$  all answer the question of how far the society is from the most unequal society by conditioning the actual inequalities on the inequalities in such a state. However, the definition of this state varies between the three indices.

To visualize the differences between  $W$  and  $E$  (and  $C$ ), it is useful to express the indices in terms of a ratio between  $V$  of the observed state (i.e. actual absolute inequalities) and  $V$  of the most unequal society according to the definition of the respective index ( $V^{max-I(h)}$ , hereafter). As  $V$  is equal to twice the area between the line of equality and the *generalized* concentration curve – the cumulative population graphed against the cumulative amount of mean health  $\mu_h$  – the numerator is always equal to area **I+II** in Figure 1. The denominators mirror the indices' respective definition of the most unequal society (see Appendix). According to  $C$ , the most unequal society is a state where all health is concentrated to the richest individual (i.e.  $V^{max-C}$  equals area **I+II+III+IV+VI**).

#### Figure 1

For a binary variable such a state is not feasible unless there is only one unit of health to distribute within the population (i.e. only one individual is healthy). Wagstaff (2005) addresses exactly this issue when he normalizes  $C$  by  $1/(1-\mu_h)$  (compare Eq(2) and Eq(3)). Thus, in the most unequal society according to  $W$ , only the richest share of the individuals is of good health where this share always equals  $\mu_h$  (i.e.  $V^{max-W}$  equals area **I+II+III+IV**). As both the lower left and the upper left corners of the area change with  $\mu_h$ , the size of the denominator will depend on  $\mu_h$ . On the contrary, the society that  $E$  conditions on is

independent of the prevalence in the society and do always corresponds to the richest 50% of the individuals being of good health (i.e.  $V^{max\_E}$  equals area **I+III+V**). The constant denominator of  $E$  corresponds to *level independence*. The denominators, and thus the values, of  $E$  and  $W$  coincide only when  $\mu_h=0.5$ . As the definition of the most unequal society drives the differences of the two indices, it is crucial to keep the definitions in mind when evaluating the properties and the underlying value judgments of the indices.

### 3.2. Linearity and the effect on indices of equiproportionate changes

In line with what is expected from an absolute index,  $E$  increases as a response to an equiproportionate change in health. Erreygers (2009b) further highlights the *linearity* of  $E$ .

**linearity:** A reduction of every individual's health from  $h_i^*$  to  $\beta h_i^*$ , ( $0 \leq \beta < 1$ ), implies that  $\beta I(h^*) = I(\beta h^*)$ .

A rank dependent index satisfies *linearity*, only if it is level independent.<sup>6</sup> Provided that the transformation is feasible, *linearity* also means that if the prevalence is doubled in every decile, the measured degree of inequality is doubled as well. As the numerator of the indices in Figure 1 – the absolute inequalities – satisfies *linearity*,  $E$  satisfies this property due to the constant denominator.

In contrast to the easily interpreted *linearity* of  $E$  or the invariance to equiproportionate changes of  $C$ ,  $W$ 's response is an increasing function of  $\mu_h$ . The ratio of  $W(\beta h^*)$  and  $W(h^*)$  demonstrates this relationship:

$$\frac{W(\beta h^*)}{W(h^*)} = \frac{C/(1-\beta\mu_h)}{C/(1-\mu_h)} = \frac{1-\mu_h}{1-\beta\mu_h} > 1 \text{ if } \beta > 1 \quad Eq(10)$$

<sup>6</sup>  $I(\beta h) = f(\mu_h, \beta, n) \sum_{i=1}^n z_i h_i \beta = f(\mu_h, \beta, n) \beta \sum_{i=1}^n z_i h_i = \beta f(\mu_h, n) \sum_{i=1}^n z_i h_i = \beta I(h)$  if and only if  $f(\mu_h, \beta, n) = f(\mu_h, n)$ , which is equivalent to level independence or  $\partial f(H)/\partial \mu_h = 0$ .

The underlying question of  $W$  is to blame for the seemingly counterintuitive response; specifically for larger values of  $\mu_h$  the society is further away from the most unequal state after the health change. To facilitate the interpretation, the graph in Figure 4 illustrates the change in  $W$  using a ratio of  $C$  instead of  $V$ .<sup>7</sup> As  $W$  satisfies the mirror condition, the graph can simultaneously show the ratio in the perspective of both health and ill-health and present two ways to understand  $W$ 's response. First, the higher  $\mu_h$ , the smaller the difference between the triangle representing  $C^{max-W}$  and the area representing  $C$ . While  $C$  is invariant to the equiproportionate change,  $C^{max-W}$  – the denominator of  $W$  – approaches the most *equal* society (the line of equality) as  $\mu_h$  increases (i.e. decreases with area  $c$ ). Second, the higher  $\mu_h$ , the more disproportional change in ill-health. Thus, the increase in  $C$  of ill-health (i.e. area  $g$ ) is disproportionately larger than the increase in  $C^{max-W}$  of ill-health (i.e. area  $e$ ). That  $W$  weights the inequalities in ill-health higher for higher values of  $\mu_h$  (or equivalently lower values of the prevalence of ill-health) is in line with a relative value judgment and relates to the discussion in the following section.

## Figure 2

### 3.3. The effect on indices of equal increments

As  $V$  – the absolute inequalities and the numerator of  $E$  and  $W$  – is insensitive to equal increments of health, the response of the indices to equal increments depends only on the response of the denominator. Evaluating the response of the denominator is equivalent to evaluating the derivative of the normalization function in Eq(1)-Eq(5) with respect to  $\mu_h$  (i.e.  $\partial f(H)^I / \partial \mu_h$ ). The constant denominator (i.e.  $\partial f(H)^E / \partial \mu_h = 0$ ) confirms that  $E$  satisfies *level*

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<sup>7</sup> By normalizing both the numerator and denominator by  $1/\mu_h$ , we standardize the upper right corner to one. Thus, changes in  $\mu_h$  only vary the size of the denominator.

*independence* and is insensitive to equal increments of prevalence across the deciles – exactly as expected for an absolute index.<sup>8</sup>

As  $W$  satisfies the mirror condition, it cannot be a pure measure of relative inequalities. In contrast to  $C$ 's response to equal increments ( $\partial f(H)^C/\partial\mu_h < 0$ ),  $W$ 's response depends on the value of  $\mu_h$  and increases when  $\mu_h > 1/2$  (i.e.  $\partial f(H)^W/\partial\mu_h > 0$  if  $\mu_h > 1/2$  and  $\partial f(H)^W/\partial\mu_h < 0$  if  $\mu_h < 1/2$ ).<sup>9</sup> An equal increment in health being translated into an increase of the index is the opposite of what is expected from a relative inequality index. This surprising behavior is due to that prevalence is a bounded variable and when health increases from 0.7 to 0.8 there is a simultaneous decrease of ill-health in the same variable from 0.3 to 0.2. Imposing the mirror condition implies that the response to changes in health is required to equal the absolute value of the response to changes in ill-health, but with opposite sign. Consequently, as Figure 3 illustrates, the marginal response of an equal increment  $\partial f(H)^W/\partial\mu_h$  for  $\mu_h < 1/2$  is the negative mirror of the same function for  $\mu_h > 1/2$ , i.e. it is the derivative of a U-shaped function (see Figure 4)

**Figure 3**

**Figure 4:**

Figure 3 also demonstrates that the marginal response of  $W$  ( $\partial f(H)^W/\partial\mu_h$ ) always has the same sign and a similar shape as the marginal response of  $C$  ( $\partial f(H)^C/\partial\mu_h$ ) representing the variable, health or ill-health, with the lowest mean. That is if  $\mu_{health} < \mu_{ill-health}$ , then  $\partial f(H)^W/\partial\mu_h$  has the same sign as  $\partial f(H)^{C(health)}/\partial\mu_h$  and vice versa. In this sense, the result is not as counterintuitive as at first glance (compare Erreygers and van Ourti, 2010). The response of  $W$  reflects that, in relative terms, it is more disadvantageous to be of ill-health (i.e. to have a certain medical condition) in a low prevalence society. Analogously, it is more privileged to be healthy (i.e.

<sup>8</sup> *Level Independence* corresponds to Erreygers and van Ourti's (2010) definition of a quasi-absolute measure for bounded variables.

<sup>9</sup>  $\partial f(H)^W/\partial\mu_h = (4\mu_h - 2) / (n^2(\mu_h - \mu_h^2)^2)$ . Thus,  $\partial f(H)^W/\partial\mu_h > 0$  if  $\mu_h < 0$  and  $\partial f(H)^W/\partial\mu_h < 0$  if  $\mu_h > 0$ .

not to have a certain medical condition) in a high prevalence society.  $W$  simultaneously captures these two perspectives, weighting the absolute inequalities with one divided by the product of the prevalence of health and ill-health (see Appendix). This normative attribute, which we will refer to as *mirror relativity*, suggests that the weight of absolute inequalities always reflects the perspective – health or ill-health - with the lowest prevalence.

Graphing the relationship between  $\mu_h$  and the weight of absolute inequalities - the inverse of the most unequal society  $1/V^{max\_I(h)}$  or equivalently the normalization function  $f(H)$  (see Appendix) –for  $W$ ,  $C(health)$  and  $C(ill-health)$ , Figure 3 illustrates the underlying value judgments and the notion of *mirror relativity*. The weight of  $C(health)$  suggests that the same level of absolute inequalities is more severe when the total amount of health in the population is low, while the weight of  $C(ill-health)$  suggests that the same level is more severe when the total amount of health is high. The weight of  $W$  combines the two perspectives suggesting that the same level of absolute inequalities is more severe when the prevalence is either high or low. As discussing relative inequality preserving changes for a bounded variable is futile, a relative value judgment ought to instead reflex how the index weights a given level of absolute inequalities. Drawing on this argument we claim that, if the health variable is binary and the rank dependent index aim to capture a relative value judgment when the mirror condition is imposed, it is reasonable that  $f(H)$  is a U-shaped function of the prevalence.

Given that the mirror condition is imposed, it is not obvious how a rank dependent index can take relative inequalities into account in another way than this notion of *mirror relativity*. If such a notion of a relative value judgment is seen as either unsatisfying or normatively unreasonable, one has to either rule out a relative value judgment for binary variables (as well

as for any bounded variable) or relax the mirror condition<sup>10</sup> (cf. Erreygers and van Ourti, 2010; Wagstaff, 2009).

To conclude, we cannot impose the value judgments directly from ratio-scaled unbounded variables to binary ones. Furthermore, there is a crucial difference in how the indices respond to equal increments of health and, therefore, how they relate to relative and absolute inequalities. As  $E$  satisfies *level independence*, it captures an absolute value judgment. Drawing on the previous discussion, we claim that  $W$ , even though it is affected by equiproportionate changes and not necessarily decreases for equal increments, captures a value judgment that puts larger weight on relative than absolute inequalities.

### 3.4. Masked critique against the value judgment

Erreygers (2009a; 2009b) puts forward two additional properties – *monotonicity* and *convergence* – as arguments for preferring  $E$  above  $W$ .  $E$  satisfies both these properties while  $W$  does not. However, the following section illustrates that these properties are a result of the definition of the most unequal society, i.e. the constant  $V^{\max\_E}$  in Figure 1.

#### 3.4.1. Individual changes and monotonicity

When considering individual changes of health, *level independence* again constitutes the crucial difference between the indices. Erreygers (2009a) shows that *level independence* implies *monotonicity*.

***monotonicity***: If an individual from the upper half of the income distribution become of good health (a pro-rich health improvement) then  $I(h)$  increases (a pro-rich change).

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<sup>10</sup> There may for example be arguments for using  $C$  of health, which imply a value judgment suggesting that the same level of absolute inequalities is more severe when the total amount of health in the population is low.



As the denominator, the most unequal society, of  $E$  is constant, an individual health change modifies only the numerator of the index and the change in  $E$  depends only on the socioeconomic rank of the individual changing health. As a result a pro-rich health improvement, translates into a pro-rich change of  $E$ .

As  $W$  is not *level independent*, its denominator is not constant and it does not satisfy *monotonicity*; in addition to the socioeconomic rank, the sign (and size) of the change in  $W$  also depends on the initial prevalence  $\mu_h$  and the initial level of absolute inequalities (see Appendix). Therefore, a pro-rich health improvement does not necessarily translates into a pro-rich change of  $W$ .

As a consequence, Erreygers (2009a, p. 508) criticizes  $W$  for producing artificial and counterintuitive results. For example, if the richest 10% of a population is of good health, then an additional rich individual (not ranked in the 11<sup>th</sup> percentile) becoming of good health implies that  $E$  increases but  $W$  decreases. This health improvement is, according to Erreygers, obviously pro-rich but translates into a pro-poor change of  $W$ .

However, the *non-monotonicity* arises from the relative value judgments underlying  $W$  (and  $C$ ). Above all, the change of the health distribution in the example is a movement away from the most unequal society as defined by  $W$  – a society where only the individuals in the top of the income distribution are of good health. Likewise, the *monotonicity* of  $E$  arises from the definition of the most unequal society – a society where the richest 50% of the population is of good health – because then an additional individual from the upper half of the income distribution becoming of good health is always a movement towards such a state. Thus, as Wagstaff (2009) points out, the desirability of *monotonicity* depends on the question behind the index and the value judgment one wants to impose.

### 3.4.2. The lack of convergence

Erreygers (2009b) states that if the health of every individual is gradually reduced to zero, i.e. the society approaches a state of perfect equality, so should also the measured degree of inequalities.

**convergence:** Consider a reduction of every individuals health from  $h_i^*$  to  $\beta h_i^*$  where  $\beta < 1$ , then  $I(h^*)$  converges to zero (i.e.  $\lim_{\beta \rightarrow 0} I(\beta h^*)$ ).

For such a health change, the absolute inequalities or the numerator of the indices (i.e.  $V$  or **I+II** in Figure 1) gradually decreases as  $\mu_h$  decreases. As the denominator of  $E$  ( $V^{max-E} = \mathbf{I+II+V}$ ) is constant, the ratio will converge to zero. On the contrary, the denominator of  $W$  ( $\mathbf{I+II+III+IV}$ ) converges to the one of  $C$  ( $\mathbf{I+II+III+IV+VI}$ ) as area **VI** approaches zero.<sup>11</sup> Neither  $C$  nor  $W$  converges to zero as the relative value judgment of  $C$  means that the measured degree of relative inequalities are the same although the absolute inequalities approach zero (i.e. the ratio between **(I+II)** and **(I+II+III+IV+VI)** is constant).<sup>12</sup> Due to the mirror relativity,  $W$  behaves correspondingly when  $\mu_h$  is approaching one. Erreygers (2009b, p.523) points out this lack of convergence as the major shortcoming of  $W$  because it means that  $W$  may blow up the measured inequalities when  $\mu_h$  is approaching its limits. However, that the measured degree of inequalities is high when the prevalence is low is only a result of a relative value judgment and the underlying question of the index. Thus, the critique against the convergence problem is a hidden critique against the value judgment.

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<sup>11</sup> area **VI** is approaching zero as  $(1-\mu_h)$  approaches one.

<sup>12</sup> Erreygers and van Ourti (2010) shows that  $C$  and  $W$  converge to  $2/n^2$

## 4. A latent variable approach

Besides an intention to measure absolute inequalities or the pure technical arguments discussed in the previous section, there are other reasons to impose *level independence*. The following sections consider the latent variable that often underlies the binary representation of the health measure and derive, from the risk of reporting heterogeneity, a new decision rule.

### 4.1. The threshold of the latent variable

The health economic literature suggests dichotomizing variables as one possible solution to the problem of being armed with only an ordinal (or even cardinal) health measure (O'Donnell et al., 2008). Coding a binary variable from an underlying ordinal variable also appears to be common practice in the empirical literature. Even though the variable is not coded from a latent variable, the respondents themselves may have reported an ordinal or a qualitative health measure derived from a latent variable. Thus, in most cases there is a latent variable,  $y_i$ , such that  $h_i=0$  if  $y_i < t$  and  $h_i=1$  if  $y_i > t$ , where  $t$  is a threshold.

Incremental or equiproportionate changes of the latent variable may or may not affect the binary representation, as it would potentially push additional individuals above the threshold (see Figure 5). If a transformation of the latent variable neither equivalently affects the threshold nor corresponds to true health changes, the operation would be equivalent to shifting the threshold. In turn, shifting the threshold would be analogous to comparisons between contexts – countries, cultures, or time – with different thresholds of the latent variable. For example, given the level of health, Danes tend to report better health than Germans (Jürges, 2007).

**Figure 5**

## 4.2. A new decision rule

If the localization of the threshold within the distribution of the latent variable (compare Figure 5) is due to either arbitrariness or cultural differences, we argue that *level independence* is a desirable property; given that the level of prevalence is due to an arbitrary threshold, it is sensible if the measured degree of inequalities is invariant to the level of prevalence. Consequently, the researcher has to retreat to an absolute index independently of the preferred value judgment. For example, the threshold of an ordinal measure of Self Assessed Health (SAH) is open for a high level of subjectivity and arbitrariness, both for the researcher when coding the binary variable and for the respondent when answering the questionnaire.<sup>13</sup> As the threshold may vary between contexts, e.g. as pointed out Germans may be more prone to report worse health status than Danes,  $E$  would be more reliable for comparisons between countries for such health measures.

By contrast, the threshold for a diagnosed medical condition, e.g. diabetes, is less subjective and there is less variation between contexts. The prevalence of diabetes can therefore be considered to be both accurate and interesting information, and thus an analysis of relative inequalities may be appropriate. In such a case, *level independence* is not necessarily desirable and, given the normative acknowledgment of the mirror relativity of  $W$ , the choice of index should depend on the preferred value judgment.

Accordingly, we can sum up the discussion in a two by two matrix (see Figure 6). If the threshold of the latent variable is arbitrary or subjective, i.e. if there is a risk of reporting

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<sup>13</sup> This problem has been termed state-dependent reporting bias (e.g. Kerkhofs and Lindeboom, 1995), scale of reference bias (e.g. Groot, 2000), response category cut-point shift (e.g. Sadana et al., 2000; Murray et al., 2001), or reporting error (e.g. Van Doorslaer and Gerdtham, 2003).

heterogeneity, one should consider using  $E$ . In turn, if the threshold is objective, the choice ought to depend on the imposed value judgment.

### Figure 6

## 5. Empirical Analysis

### 5.1. Data

Empirically, we examine how the choice of index affects comparisons between countries using nine binary indicators of bad health from the second wave of the European Survey of Health, Ageing and Retirement (SHARE).<sup>14</sup> The nine indicators are having diabetes, having cancer, having a long-term sickness, having more than two limitations in the daily life, having more than two chronic diseases, and three measures of bad SAH. The three SAH-measures are from the same reported ordinal scale, but are coded at different cut-off points.<sup>15</sup> As the socioeconomic ranking variable, we use equivalent income on a household level.<sup>16</sup>

To analyze if the choice of index affects comparison between countries we compute correlation coefficients of the indices. However, what really matters is if the index varies the rank of the countries, e.g. if the health in country A is more equally distributed than in the health in country B according to  $W$ , but the reverse pattern emerges according to  $E$ . Therefore, we compute Spearman's and Kendall's rank correlation coefficients.

### 5.2 Results

The indices in Table 2 and the rank correlation coefficients in Table 3 show that the choice of index may vary the ranking and, thus, may affect the outcome of a comparison. Although the

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<sup>14</sup> The data includes Austria, Germany, Sweden, Netherlands, Spain, Italy, France, Denmark, Greece, Switzerland, Belgium, Czech Republic, Poland, and Ireland.

<sup>15</sup> The ordinal scale is 1) Excellent, 2) Very good, 3) Good, 4) Fair, and 5) Poor. SAH 1 is equal to one if the respondent has stated to have poor health (5). SAH 2 corresponds to less than good (4 or 5) and SAH 3 corresponds to having less than very good health (3,4, or 5)

<sup>16</sup> Reported household income last month divided by the square root of the household size.

extent differs, the ranking of the 14 countries are different for  $E$  and  $W$  in all nine health indicators

**Table 2**

**Table 3**

$W$  and  $E$ , as well as their definitions of the most unequal society, coincide when  $\mu_h = \frac{1}{2}$  and diverges for large and small values of  $\mu_h$  (see Eq(4) and Eq(5)). Because  $E$  is *level independent* while  $W$  is not, the ranking based on the two indices will be different when  $\mu_h$  varies between contexts. The empirical findings presented in Table 2 and 3 confirm these claims. For health variables where  $\mu_h$  is close to  $\frac{1}{2}$  for all countries (e.g. Long sickness) the ranking based of the two indices practically coincide. By contrast, if  $\mu_h$  varies substantially across the countries (e.g. SAH 3) the rank correlation is low. Although the number of observations is small, we can also easily verify this twofold conclusion by running a regression of the rank correlation coefficients on the mean of prevalence, the squared mean of prevalence and the standard deviations of the prevalence. The results in Table 4 first shows that there is a U-shaped association between the mean of the prevalence  $\bar{\mu}_h$  and the rank correlation coefficient, i.e. the ranking according to  $W$  and that according to  $E$  diverge for low and high values of  $\bar{\mu}_h$ . Second, the results also confirm that the larger variation in prevalence, the smaller rank correlation coefficient.

**Table 4**

**6. Conclusion**

Even though  $C$ ,  $V$ ,  $W$ , and  $E$  all belong to the family of rank dependent indicators, the indices still have a different perspective of socioeconomic inequalities in health as they weight absolute inequalities differently.  $C$ ,  $W$ , and  $E$  all condition the absolute inequalities on the

most unequal society, but differ in their definition of that state.  $W$  answers the question of how far the society, given the level of health, is from a state where only the individuals in the top of the income distribution are healthy, while  $E$  answers the question of how far the society is from a state where the upper 50% of the distribution are healthy independently on the prevalence.

As no relative inequality preserving changes are feasible unless we relax the mirror condition, discussing a relative value judgment in the same way as for unbounded health variables is futile. Focusing on the definition of the most unequal society and the weight of absolute inequalities, we illustrate how  $W$  suggests that the same level of absolute inequalities is more severe for high and low values of the prevalence. If this *mirror relativity* is acknowledged as normatively satisfying and researchers are aware of the underlying value judgment,  $W$  provides important information of relative inequalities.

Moreover, we acknowledge the compelling technical simplicity of  $E$ ; i) the *linearity* of equiproportionate changes, ii) unlike  $W$ ,  $E$  approaches zero when  $\mu_h$  approaches zero (or one), and iii) due to the *monotonicity* it is easier to interpret and anticipate the effect of individual changes in health. Still, all these positive features are a result of *level independence* and the definition of the most unequal society. Thus, they are only desirable if one wants to capture an absolute value judgment.

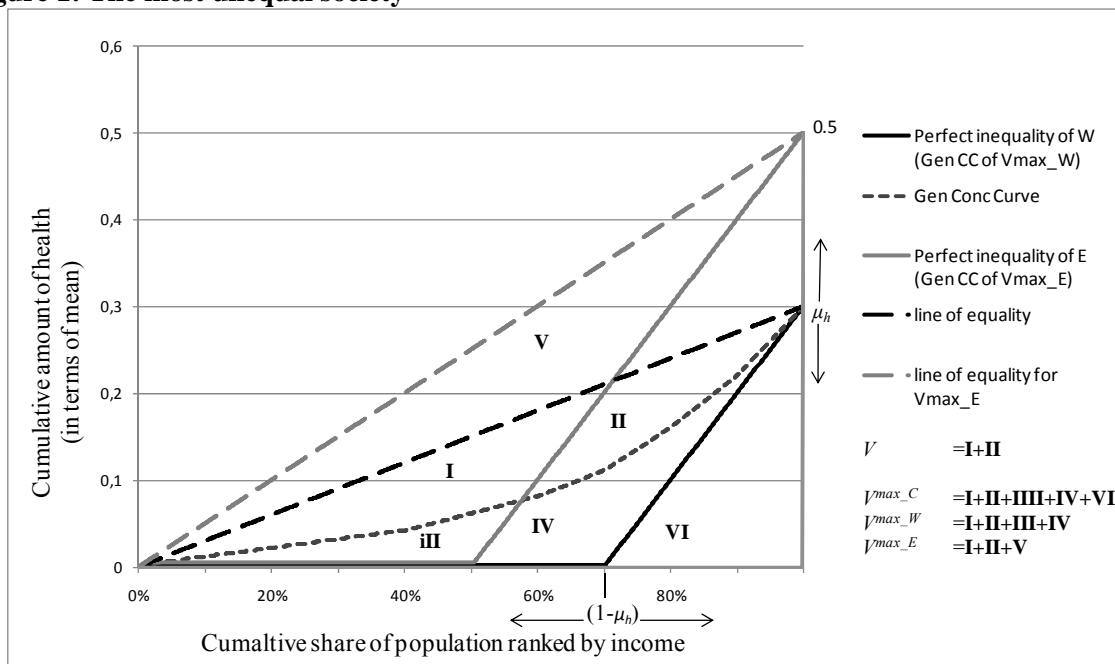
Nevertheless, we claim that *level independence* is desirable independently of the preferred value judgment if there is a high risk of reporting heterogeneity. For such variables it is not sensible if the prevalence affects the index and, thus, we believe that it is more reliable to use  $E$  for comparisons between countries.

Following our empirical results, we conclude that, as comparisons between contexts are affected, the discussion of which index to use is not only a matter of semantics. Therefore, we call for researchers and practitioners to seriously consider their choice of index, and critically reflect on which value judgment to impose, when evaluating health inequalities. For policy makers, these findings are important because using different indices may call for different policy strategies.



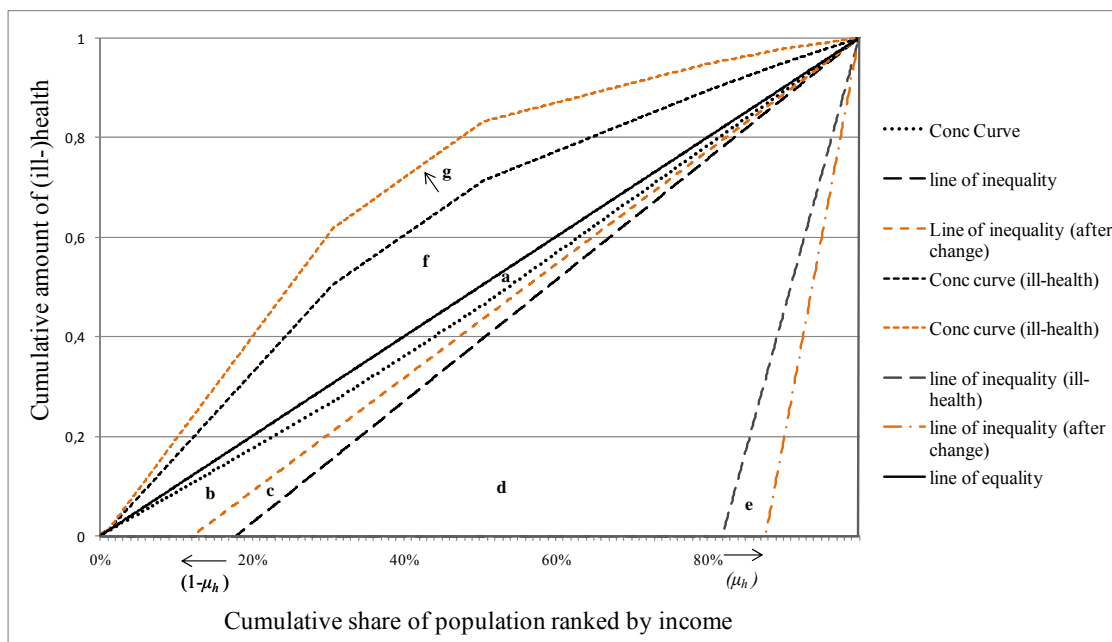
## Figures and Tables

**Figure 1: The most unequal society**



**Note:** If the health variable is a bad (ill-health), then the most unequal society is defined in a reversed perspective, i.e. the ill-health is concentrated to the richest individual ( $C$ ), the richest share of the individuals are of ill-health ( $W$ ), and the richest 50% of the individuals are of ill-health ( $E$ ). However, the area representing these reversed inequalities is equally large as the area between the line of equality and an imaginary line of perfect inequality representing a state where the poorest individuals being of bad health. As these areas would be above the line of equality, using such a definition would require conditioning the absolute inequalities on the absolute value of  $V^{max}_I(h)$

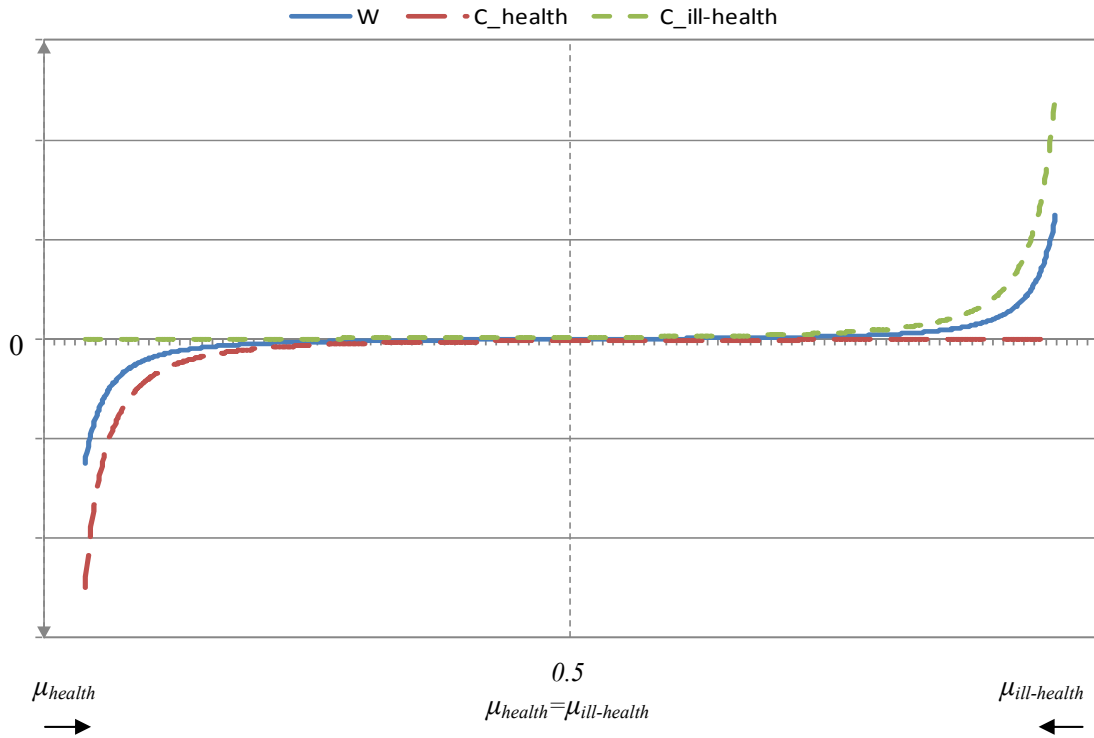
**Figure 2: The response of  $W$  to equiproportional changes**



**Note:** The figure simultaneously graphs the concentration curve of health and ill-health as well as the line of inequalities of the two perspectives.  $W$  of health equals the ratio between  $a$  and  $(a+b+c)$ , while  $W$  of ill-health equals the ratio between  $(-f)$  and  $(a+b+c+d)$ . As  $W$  satisfies the mirror condition, the absolute values of the two

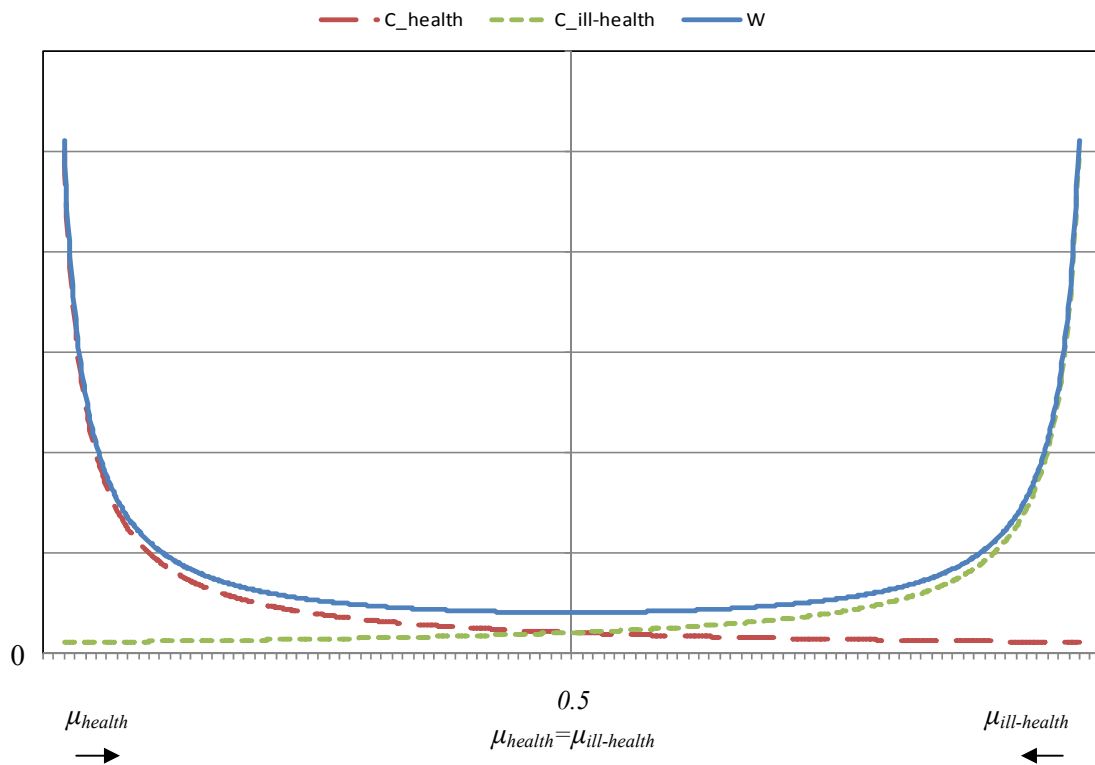
ratios are identical. Note again that although the area representing the most unequal society from the perspective of ill-health is a society where only the richest individuals are of bad health, this area is equally large as the area between the line of equality and an imaginary line of inequality representing a society where only the poorest individuals were of bad health.

**Figure 3: Marginal response ( $\partial f(H)/\partial \mu_h$ ) of  $W$ ,  $C(\text{health})$  and  $C(\text{ill-health})$**

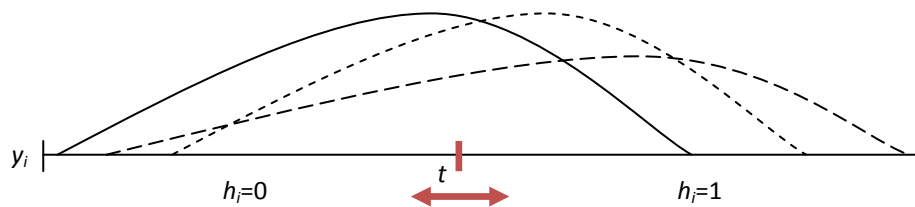


**Note:** The functions are shown for any arbitrary value of  $n$ .  $\partial f(H)/\partial \mu_h$  of  $W$  (line) has the same sign (negative) as  $\partial f(H)^C/\partial \mu_h$  of  $C(\text{health})$  (long dash) as long as  $\mu_{\text{health}} < \mu_{\text{ill-health}}$ , whereas for  $\mu_{\text{health}} > \mu_{\text{ill-health}}$   $\partial f(H)/\partial \mu_h$  of  $W$  has the same sign (positive) as  $\partial f(H)^C/\partial \mu_h$  of  $C(\text{ill-health})$  (short dash).

**Figure 4:  $f(H)$  or  $1/V^{\max-I}$  of  $W$ ,  $C(\text{health})$  and  $C(\text{ill-health})$**



**Figure 5: Thresholds of latent variables**



For a given health distribution (solid line), the threshold decides the value of the binary health measure. The impact of scalar addition (short dash) and scalar multiplication (long dash) are analogous, pushing additional individuals pass the threshold which shifts the value of  $h_i$ . A shift of the threshold within the original distribution would have an equivalent impact.

**Figure 6: The new decision rule**

	<i>Threshold of the underlying variable</i>	
<i>Value judgment</i>	<b>Subjective/Arbitrary</b>	<b>Objective</b>
<b>Relative</b>	<i>Not relevant</i>	<i>W</i>
<b>Absolute</b>	<i>E</i>	<i>E</i>

**Table 1: Properties of the rank-dependent indices**

	<i>Mirror</i>	<i>Transfer</i>	<i>Cardinal Invariance</i>	<i>Level independence</i>
<i>E</i>	✓	✓	✓	✓
<i>W</i>	✓	✓	✓	
<i>C</i>		✓		
<i>V</i>	✓	✓		✓

Table 2: Results

<i>Cancer</i>	$\mu_h$	$W$	$E$	<i>Rank W</i>	<i>Rank E</i>
Austria	0.02	0.30	0.02	1	3
Germany	0.04	-0.18	-0.03	13	14
Sweden	0.07	-0.09	-0.02	10	11
Netherlands	0.03	-0.01	0.00	9	9
Spain	0.02	0.13	0.01	5	4
Italy	0.03	0.24	0.03	2	2
France	0.04	0.20	0.03	3	1
Denmark	0.07	-0.11	-0.03	11	13
Greece	0.01	0.17	0.01	4	5
Switzerland	0.04	0.00	0.00	7	7
Belgium	0.03	-0.19	-0.03	14	12
Czechia	0.05	0.03	0.01	6	6
Poland	0.03	-0.01	0.00	8	8
Ireland	0.05	-0.11	-0.02	12	10
<i>Diabetes</i>	$\mu_h$	$W$	$E$	<i>Rank W</i>	<i>Rank E</i>
Austria	0.11	0.02	0.01	3	3
Germany	0.15	-0.14	-0.07	13	14
Sweden	0.09	-0.13	-0.04	11	7
Netherlands	0.09	-0.07	-0.02	5	5
Spain	0.15	-0.12	-0.06	8	12
Italy	0.12	-0.04	-0.02	4	4
France	0.10	-0.15	-0.05	14	11
Denmark	0.08	-0.12	-0.04	7	6
Greece	0.13	-0.12	-0.05	6	10
Switzerland	0.06	0.07	0.02	1	2
Belgium	0.10	-0.12	-0.04	9	8
Czechia	0.14	-0.13	-0.06	10	13
Poland	0.11	0.06	0.02	2	1
Ireland	0.10	-0.14	-0.05	12	9
<i>Long Sickness</i>	$\mu_h$	$W$	$E$	<i>Rank W</i>	<i>Rank E</i>
Austria	0.45	-0.07	-0.06	3	3
Germany	0.59	-0.11	-0.10	7	7
Sweden	0.54	-0.20	-0.20	13	14
Netherlands	0.44	-0.07	-0.07	4	5
Spain	0.56	-0.15	-0.15	10	10
Italy	0.42	-0.07	-0.07	5	4
France	0.50	-0.12	-0.12	8	8
Denmark	0.48	-0.16	-0.16	11	11
Greece	0.37	-0.21	-0.20	14	13
Switzerland	0.36	-0.08	-0.07	6	6
Belgium	0.44	-0.12	-0.12	9	9
Czechia	0.55	-0.20	-0.19	12	12
Poland	0.66	-0.06	-0.05	2	2
Ireland	0.42	-0.02	-0.02	1	1

<i>SAH 1(poor)</i>	$\mu_h$	$W$	$E$	<i>Rank W</i>	<i>Rank E</i>
Austria	0.07	-0.06	-0.02	2	2
Germany	0.11	-0.27	-0.10	11	12
Sweden	0.07	-0.33	-0.09	14	11
Netherlands	0.05	-0.30	-0.05	12	7
Spain	0.14	-0.16	-0.07	7	9
Italy	0.13	-0.10	-0.05	5	5
France	0.09	-0.19	-0.07	9	8
Denmark	0.06	-0.33	-0.08	13	10
Greece	0.06	-0.09	-0.02	3	3
Switzerland	0.03	0.02	0.00	1	1
Belgium	0.07	-0.10	-0.02	4	4
Czechia	0.14	-0.25	-0.12	10	14
Poland	0.34	-0.12	-0.10	6	13
Ireland	0.07	-0.19	-0.05	8	6
<i>SAH 2(less than good)</i>	$\mu_h$	$W$	$E$	<i>Rank W</i>	<i>Rank E</i>
Austria	0.31	-0.22	-0.19	8	7
Germany	0.41	-0.24	-0.23	10	13
Sweden	0.30	-0.27	-0.22	13	10
Netherlands	0.29	-0.15	-0.13	4	4
Spain	0.46	-0.20	-0.20	6	8
Italy	0.44	-0.11	-0.11	2	3
France	0.36	-0.23	-0.21	9	9
Denmark	0.25	-0.31	-0.23	14	12
Greece	0.30	-0.27	-0.23	12	11
Switzerland	0.18	-0.12	-0.07	3	1
Belgium	0.30	-0.22	-0.18	7	6
Czechia	0.46	-0.26	-0.26	11	14
Poland	0.63	-0.10	-0.09	1	2
Ireland	0.25	-0.20	-0.15	5	5
<i>SAH 3(less than very good)</i>	$\mu_h$	$W$	$E$	<i>Rank W</i>	<i>Rank E</i>
Austria	0.73	-0.19	-0.15	6	6
Germany	0.81	-0.24	-0.15	11	4
Sweden	0.59	-0.27	-0.26	12	13
Netherlands	0.72	-0.20	-0.16	7	8
Spain	0.87	-0.19	-0.09	4	3
Italy	0.81	-0.13	-0.08	1	2
France	0.79	-0.22	-0.15	9	5
Denmark	0.49	-0.28	-0.28	13	14
Greece	0.63	-0.24	-0.22	10	12
Switzerland	0.54	-0.18	-0.18	3	10
Belgium	0.71	-0.19	-0.16	5	7
Czechia	0.82	-0.32	-0.18	14	11
Poland	0.93	-0.21	-0.06	8	1
Ireland	0.54	-0.16	-0.16	2	9

<i>Limitations</i>	$\mu_h$	<i>W</i>	<i>E</i>	<i>Rank W</i>	<i>Rank E</i>
Austria	0.08	-0.04	-0.01	2	2
Germany	0.06	-0.18	-0.04	6	6
Sweden	0.04	-0.38	-0.06	13	10
Netherlands	0.03	-0.28	-0.04	10	5
Spain	0.07	-0.28	-0.08	11	12
Italy	0.08	0.01	0.00	1	1
France	0.07	-0.21	-0.06	8	9
Denmark	0.06	-0.41	-0.09	14	14
Greece	0.06	-0.15	-0.04	4	4
Switzerland	0.02	-0.27	-0.02	9	3
Belgium	0.09	-0.17	-0.05	5	7
Czechia	0.07	-0.30	-0.08	12	13
Poland	0.17	-0.10	-0.05	3	8
Ireland	0.09	-0.21	-0.07	7	11
<i>Chronic disease</i>	$\mu_h$	<i>W</i>	<i>E</i>	<i>Rank W</i>	<i>Rank E</i>
Austria	0.20	-0.05	-0.03	2	3
Germany	0.21	-0.15	-0.10	7	8
Sweden	0.21	-0.29	-0.20	13	13
Netherlands	0.14	-0.15	-0.07	9	5
Spain	0.23	-0.15	-0.11	8	9
Italy	0.27	-0.10	-0.08	5	6
France	0.18	-0.24	-0.14	11	10
Denmark	0.23	-0.32	-0.23	14	14
Greece	0.21	-0.28	-0.18	12	12
Switzerland	0.09	-0.09	-0.03	4	2
Belgium	0.22	-0.13	-0.09	6	7
Czechia	0.25	-0.21	-0.16	10	11
Poland	0.31	-0.03	-0.03	1	1
Ireland	0.22	-0.09	-0.06	3	4
<i>Smoke</i>	$\mu_h$	<i>W</i>	<i>E</i>	<i>Rank W</i>	<i>Rank E</i>
Austria	0.14	0.06	0.03	5	5
Germany	0.18	0.00	0.00	8	8
Sweden	0.17	-0.04	-0.02	13	13
Netherlands	0.25	-0.12	-0.09	14	14
Spain	0.16	0.22	0.12	1	2
Italy	0.17	0.09	0.05	4	4
France	0.15	0.03	0.02	7	7
Denmark	0.28	-0.01	-0.01	10	10
Greece	0.29	0.16	0.13	2	1
Switzerland	0.18	-0.03	-0.02	12	12
Belgium	0.19	0.05	0.03	6	6
Czechia	0.21	0.14	0.09	3	3
Poland	0.27	-0.02	-0.02	11	11
Ireland	0.19	0.00	0.00	9	9

**Table 3: Rank correlations coefficients**

<i>Health Indicator</i>	<i>Spearman</i>	<i>Kendall</i>	<i>Pearson</i>	<i>std of <math>\mu</math></i>	$\bar{\mu}$
<b>Cancer</b>	0.947	0.824	0.940	0.017	0.039
<b>Diabetes</b>	0.824	0.692	0.959	0.025	0.110
<b>Longsick</b>	0.991	0.956	0.998	0.085	0.484
<b>SAH 1</b>	0.741	0.604	0.705	0.075	0.101
<b>SAH 2</b>	0.903	0.758	0.931	0.117	0.352
<b>SAH 3</b>	0.486	0.319	0.617	0.135	0.713
<b>Limitation</b>	0.741	0.582	0.742	0.034	0.070
<b>chronic disease</b>	0.938	0.846	0.975	0.053	0.213
<b>Smoke</b>	0.996	0.978	0.982	0.049	0.202

**Table 4: Regression results**

	<i>St.dev. of <math>\mu</math></i>	$\mu$	$\bar{\mu}^2$
<b>Spearman</b>	-3.06	2.40	-3.07
<b>Kendall</b>	-4.50	3.28	-3.96



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## Appendix

### A1. Most unequal society and the weight of absolute inequalities

We can express  $W$ ,  $E$ , and  $C$  as a ratio between  $V$  of the observed state and  $V$  of the state where the inequalities are maximized according to the definition of the respective index ( $V^{max-I(h)}$ ). Using the definitions from Section 2, we let  $h_i$  be a binary indicator of good health and expresses  $V$  as:

$$V = \frac{2}{n^2} \sum_{i=1}^n z_i h_i = \frac{2}{n^2} \sum_{i=1}^n \left( \frac{n+1}{2} - \lambda_i \right) h_i = \sum_{i=1}^n \frac{h_i}{n} + \sum_{i=1}^n \frac{h_i}{n^2} + \sum_{i=1}^n \frac{2\lambda_i h_i}{n^2}.$$

If we, according to the definition of the most unequal society of  $W$ , let only the richest  $K$  individuals be of good health, where  $K = \sum_{i=1}^n h_i$ , then  $\sum_{i=1}^n \lambda_i h_i = K(K+1)/2$  and  $V^{max-W}$  equals:

$$V^{max-W} = \frac{K}{n} + \frac{K}{n^2} - \frac{2K(K+1)}{2n^2} = \frac{K}{n} \left( 1 - \frac{K}{n} \right) = \mu_h (1 - \mu_h).$$

Thus, we can express  $W$  as:

$$W = \frac{2}{n^2(1-\mu_h)\mu_h} \sum_{i=1}^n z_i h_i = \frac{1}{(1-\mu_h)\mu_h} V = \frac{V}{V^{max-W}}.$$

If we, in accordance with the most unequal society of  $E$ , let  $K=0.5n$  (i.e. only the richest 50% of the individuals are of good health) independent of the actual prevalence, then  $V^{max-E}$  equals:

$$V^{max-E} = \frac{K}{n} \left( 1 + \frac{K}{n} \right) = 0.25.$$

Thus, we can express  $E$  as:

$$E = \frac{8}{n^2\mu_h} \sum_{i=1}^n z_i h_i = \frac{1}{0.25} V = \frac{V}{V^{max-E}}$$

For  $C$ , the most unequal society is defined as a state where the richest individual possesses all health units in the society, i.e.  $h_i = n\mu_h$  for  $i$  such that  $\lambda_i = 1$  and  $h_i = 0$  for everyone else. Thus,  $V^{max-C}$  equals:

$$V^{max-C} = \frac{2}{n^2} \sum_{i=1}^n \left( \frac{n+1}{2} - \lambda_i \right) h_i = \frac{2}{n^2} \left( \frac{n+1}{2} - 1 \right) n \mu_h = \mu_h \left( 1 - \frac{1}{n} \right).$$

For a large enough  $n$ ,  $V^{max-C}$  goes to  $\mu_h$  and we can express  $C$  as:

$$C = \frac{2}{n^2 \mu_h} \sum_{i=1}^n z_i h_i = \frac{1}{\mu_h} V = \frac{V}{V^{max-C}}.$$

## A2. Individual health changes

Let  $N$  represent a given population. Consider a health change of  $m$  individuals represented by the set  $M \subseteq N$ . If these individuals become of good health, then  $E$  changes as

$$\Delta E = \frac{1}{V^{max-E}} \Delta V = \frac{8}{n^2} \sum_{j \in M} z_j h_j$$

Thus,  $\Delta E$  depends only on the socioeconomic rank of the additional individual changing health and  $E$  satisfies *monotonicity*. As  $W$  does not satisfy level independence the increased prevalence affects the most unequal society and the change in  $W$  equals

$$\begin{aligned} \Delta W &= \frac{1}{V_1^{max-W}} \Delta V + \Delta \left( \frac{1}{V^{max-W}} \right) V \\ &= \frac{1}{\left[ 1 - \left( \mu_h + \frac{m}{n} \right) \right] \left( \mu_h + \frac{m}{n} \right) n^2} \sum_{j \in M} z_j h_j + \left[ \frac{1}{\mu_h - \mu_h^2 + \frac{m}{n} \left( 1 - 2\mu_h - \frac{m}{n} \right)} - \frac{1}{\mu_h - \mu_h^2} \right] \frac{2}{n^2} \sum_{i \in N-M} z_i h_i \end{aligned}$$

The first part equals the change in actual inequalities induced by the additional  $m$  individuals becoming of good health (i.e.  $\Delta V$ ) weighted by the most unequal society with the new prevalence  $\left( \mu_h + \frac{m}{n} \right)$  (i.e.  $V_1^{max-W}$ ). This part is, like  $E$ , always *monotonic*. The sign and size of the second part depends on both the initial absolute inequalities (i.e.  $V$ ) and the change in the weight  $(\Delta 1/V^{max-W})$  induced by the increased prevalence. The sign of  $\Delta \left( \frac{1}{V^{max-W}} \right)$  is negative if  $\mu_h < \frac{n-m}{2n}$  and positive if  $\mu_h > \frac{n-m}{2n}$ . As this second part may be of opposite sign and exceed the first,  $W$  is not monotonic.