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# A trickle-down theory of incentives with applications to privatization and outsourcing

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## Abstract

The make-or-buy decision is analyzed in a three-layer principal-management-agent model. There is a cost-saving/quality tradeoff in effort provision. The principal faces the choice between employing an in-house management and contracting with an independent management; the cost-saving incentives facing the management are weaker in the former case. Cost-saving incentives trickle-down to the agent, affecting the cost-saving/quality tradeoff. It is shown that: weak cost-saving incentives to the management promotes quality if it is hard enough to measure; a more severe quality-control problem between the principal and the management, as well as a higher valuation of quality, makes an in-house management more attractive.

*JEL Classification:* D23, L22, L24

*Keywords:* make-or-buy decision, multitask principal-agent problem, contracting out

## 1 Introduction

The make-or-buy decision has intrigued economists for generations. In somewhat different disguises – and in terms of somewhat different terminology, such as in-house versus outsourced/independent production – it has been scrutinized by a large number of scholars.<sup>1</sup> The different disguises stem from different questions being asked. Some work focuses on the fundamental – but probably

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<sup>1</sup> Important examples include: Coase (1937), Williamson (1985), Grossman and Hart (1986), Hart and Moore (1990), Holmström and Milgrom (1994), Holmström (1999) and Baker, Gibbons and Murphy (2002).

somewhat quaint to non-economists – question about the nature of the firm and the forces determining its boundaries; other work is more focused on hands-on tradeoffs concerning vertical integration. While this paper falls in the second category by focusing on the choice between in-house production and outsourcing of activities, the most prominent distinguishing feature is the focus on measurement-related determinants of this choice in a comprehensive-contracting context. In addition, the choice between public and privatized management of “public-sector activities” is an important source of motivation and application of the model presented.

In much work on the make-or-buy decision, the assets involved take center stage. The main insight is that activities for which *specific assets* are important when two parties are involved are more likely to be integrated by one party because the owner of the specific asset may be subject to “hold-up” by the other party.<sup>2</sup> The hold-up problem, in turn, undermines incentives to invest in specific assets. Complementary insights are that hold-up possibilities also *may help* creating appropriate investment incentives under certain circumstances, and that the degree of contractual incompleteness is the key determinant of the severity of the problem.<sup>3</sup> Thus, the combination of specific assets and contractual incompleteness is typically taken to predict that “make” is the preferred choice.

While the asset-based approach is conceptually and empirically (though to a lesser extent) convincing, it is clearly only part of the story. An additional set of properties that are relevant for the make-or-buy decision is the measurement and contractibility characteristics of the activity subject to the make-or-buy decision. There is, moreover, empirical work indicating that measurement aspects have more power in explaining the make-or-buy decision: In their work on in-house versus independent sales forces, Anderson and Schmittlein (1984) and Anderson (1985) found measurement-related explanatory variables to stand out most strongly. Holmström and Milgrom (1994) and Holmström (1999) have brought these observations to bear in theoretical analyses of the make-or-buy decision. In Holmström and Milgrom (1994), the *complementarities* among a set of instruments for affecting performance in a given task are explored; it is argued in particular that strong incentives on the one hand, and freedom in choosing how to accomplish a task on the other, are complementary – as are the two opposites, weak incentives and stringent regulation of procedures. Holmström (1999) explores how the power to structure incentives – argued to be a key trait of the firm – may or may not be determined by asset ownership.

In this paper, we employ the measurement approach in trying to answer a number of specific

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<sup>2</sup>A party is subject to hold-up if another party threatens to withdraw from trade – in which case the specific asset would be inefficiently utilized – in order to appropriate all, or a large portion, of the surplus.

<sup>3</sup>See Hart (1995) and Holmström (1999) for clear and simple accounts for the basic logic.

questions relevant when the make-or-buy choice is encountered in practice. More specifically, we consider a three-layer hierarchy with a principal, a management, and an agent. The principal – which may be, for example, a firm or an elected body – delegates a task to a management; the management, in turn, delegates the actual execution of the task to an agent. The success in the undertaking of the task has a cost-saving and a quality dimension, and the agent can allocate his effort between these dimensions; formally, the effort is two-dimensional as well. The distinction between “make” and “buy” is interpreted as the distinction between a management that is employed by the principal, and a management of an independent contractor. The substantive difference is that the principal is constrained to provide weaker direct cost-saving incentives when the manager is employed. While this assumed difference is easy to defend on empirical grounds – and the intended contribution is the analysis of the make-or-buy choice *given this assumed difference* – we sketch a formal justification for it.<sup>4</sup> The core of the analysis deals with how equilibrium contracts depend on the incentive problem faced by the management in rewarding the agent, and by the principal rewarding the management; in particular, the possibilities for rewarding quality. The main results are that:

- the strength of incentives is subject to trickling-down: when the management faces weak incentives, the incentives provided to the agent by the management will be weak as well;
- there is trickling-down in effort allocation too: weakening cost-saving incentives for the management will, under plausible circumstances, lead to more care being devoted to quality “on the ground”;
- the more severe the incentive problem between the principal and the management as regards quality measurement, the more likely is the principal to opt for an employed management;
- the higher the value attributed to quality by the principal, the more likely is the principal to opt for an employed management.

While several of these results are quite intuitive, they are, arguably, generated in an empirically plausible vertical structure which, importantly, accounts for the “make” and “buy” cases

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<sup>4</sup>As we will come back to, Acemoglu, Kremer and Mian (2003) provide a more rigorous justification for the prevalence of weak incentives in firms and governments. See also e.g. Williamson (1998) for a discussion. Muted incentives within organization may be due to forces similar to those causing “soft budget constraints” (Kornai, Maskin and Roland 2004) although this connection seems not to have been articulated.

in a symmetric way.<sup>5</sup>

An important part of the motivation for this paper is the relevance of the analysis of the make-or-buy decision for the organization of public-sector activities – where public-sector activities refer to activities that are publicly financed or subject to extensive regulation and often provided directly by the public sector. We will argue that the results of our analysis illuminate both the popular discussion about and the real tradeoffs involved in the choice between public and private provision. In its focus on this application, the paper relates to a small literature in economics exploring privatization in this sense – often referred to as contracting out – from a contract-oriented perspective.<sup>6</sup>

The most important contribution in this literature is Hart, Shleifer and Vishny (1997) who approach the question of privatization in general – and the issue of privatizing prisons in particular – with a model focusing on incomplete contracts and asset ownership. In their model, the agent in charge of the operation makes two investments, one geared towards cost savings, having adverse consequences for quality, and one geared towards quality-enhancing innovations. The key distinction between an in-house and an independent head of operations is that while the in-house head of operations needs the consent of the principal to implement any investment, the independent head of operations needs consent only for quality-enhancing innovations (in both cases, consent is followed by renegotiation of the incomplete contract). Under these assumptions, it follows that the independent head of operations has excessive incentives for cost savings, and too weak incentives for quality innovations; an in-house head of operations, on the other hand, has too weak incentives for cost savings as well as quality innovations (the latter being even weaker than those facing the independent head of operations). The Hart-Shleifer-Vishny model is clearly rife with insights concerning the significance of asset ownership for privatization in contexts plagued by contractual incompleteness; it also endogenously obtains two distinct regimes – in-house versus independent operations. The drawback of their approach is that the incentives generated by contracts governing privatization cannot be analyzed since contracts are essentially assumed away in their framework. By focusing on the contracts, we consider our work complementary to theirs.

In addition to the bodies of work cited, there are a number of other relevant papers. Most closely related to this work is Acemoglu, Kremer and Mian (2003). The core idea of their

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<sup>5</sup>A common (and valid) criticism of the Williamsonian specific-asset story is the lack of a clear account for how and why the hold-up problem is attenuated under vertical integration.

<sup>6</sup>See Domberger and Jensen (1997) for a sensible overview of the issues – although under-appreciative of the measurement issues – and a review of some empirical evidence.

paper is that market incentives sometimes induce too much “signaling effort,” i.e. effort to inflate others’ assessment of performance without promoting performance *per se*; they mention schooling and delegated asset management as examples where this may be a significant problem. Their analysis is devoted to analyzing why incentives are, in general, weaker in firms and, even more so, in governments, than in markets. They consider a career-concerns model with a “good” and a “bad” component of effort; after showing that market incentives may be excessively strong, they argue that firms can remedy this by creating, by design, a moral-hazard-in-teams free-rider problem that blunts incentives; they also argue that competition between firms will allow remnants of market incentives to trickle down to employees, and that this effect can be avoided by governments. The paper is thus very close to this one in terms of the distinguishing characteristic of firms and governments compared to markets; while they focus on the foundation for this difference, however, our focus is on the implications for associating activities with modes of organization. Acemoglu, Kremer and Mian also identify a trickle-down property of incentives. Importantly, however, they work in a “contract-free” environment, and hence do not address questions about the properties of actual incentive contracts.

There are a number of papers that approach incentive problems in the public sector from somewhat different perspectives. One distinguishing feature of the public sector is, arguably, the feature that agencies and agents in one way or another serve multiple principals or multiple goals – schools, for example, serve students, parents, prospective employers and perhaps also teachers’ unions and other interest groups.<sup>7</sup> Following Wilson (1989) there has been some work on the desirability of creating clear “missions” – essentially undoing multiple-principal problems – for public-sector bodies; the thrust of the idea is that clear missions may be a substitute for incentives in inducing effort. Dewatripont, Jewitt and Tirole (1999) provide formal support for this idea in a multi-task career-concerns model. Somewhat relatedly, it may be argued that the intrinsic motivation of agents is more important in the presence of weaker monetary incentives; this idea is explored by Besley and Ghatak (2003).

The paper proceeds as follows. In the next section, a simple example is analyzed in order to provide some groundwork for the rest of the analysis; this example also highlights the trickling-down effect in the simplest way possible. Next, in Section 3 the two-task agency problem faced by the management is presented; in Section 4 the main results are derived in the full three-layer model, and in Section 5 we discuss applications and elaborations. In Section 6 we conclude the paper. Most of the analysis of the full model is provided in the Appendix.

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<sup>7</sup>See Dixit (2002) for an overview emphasizing this aspect.

## 2 A basic framework and a simple example

In this section we will present the basic framework, and then go on to analyzing a simple – and, indeed, in many respects simplistic – example intended to illuminate the basic idea.

**Basic framework.** We will consider a three-layer agency model with a *principal* at one end. The principal has an exogenously given task that she cannot solve by herself; the principal may be thought of as a political body or the top management of a firm or corporation. At the other end is an *agent*, who in the end solves the task. The agent may be thought of as a worker; many relevant applications of our work will involve multiple agents, but we will abstract from potential moral-hazard-in-teams problems in the interest of analytic tractability.

The principal cannot delegate the task directly to the agent. There could be a number of plausible reasons for this – such as economies of scope in managing several agents solving different tasks – but fundamentally it is an assumption reflecting our focus on the intermediate tier in real-world hierarchies whose role is most clearly different when a task is outsourced rather than solved in-house. We refer to the intermediate tier as the *management*; as indicated, the management may be thought of as the middle management within a firm if the task in question is solved in-house, and as the manager-owner of a subcontractor if the task is outsourced.

All side-contracting between the principal and the agent is ruled out. This too reflects pragmatic notions that in most examples seem reasonable indeed; it does not, for instance, seem relevant to consider possibilities for an elected body to by-pass administrative layers and make side-contracts with individual workers. The assumption is substantive, however, in so far that there would be scope for mutually beneficial side-contracts along these lines as long as the intermediate layer carries a real cost.

The preferences of the parties are simple; they will be somewhat enriched in the full model below.

- The principal,  $P$ , is a risk neutral profit maximizer, valuing the successful completion of the task in question at some  $B > 0$ . Assuming that the task is worthwhile solving, the principal's key objective is to minimize cost, and, in the following sections, ascertain quality.
- The management,  $M$ , is risk neutral and profit maximizing. Since, formally,  $M$  is merely an intermediary, her objective is simply revenues minus cost. The management has a

reservation payoff of zero.<sup>8</sup>

- The agent,  $A$ , cares about income,  $y$ , and the effort he exerts,  $a$ . He is risk averse, and his utility from income  $y$  and effort  $a$  is

$$-\exp\{-r[y - a^2/2]\};$$

the specific utility function is assumed in order for the full model to be reasonably tractable. The agent has reservation payoff  $u_0$ .

The nature of the task will be quite general. In this section we will consider an example where the task is perfectly contractible in all respects other than one, which may be thought of as realized cost; in the sequel there will, in addition, be a quality dimension. Contracts are assumed to be linear in the relevant performance measures; this is not important in the example below, but necessary to have a workable multitask model below.<sup>9</sup>

**In-house versus independent management.** While this is a paper about how incentives trickle down in an organization and not a paper about foundations for the existence of organizations, the presumption that incentives “originating in” an organization are, in general, weaker than incentives generated in contractual relations between organizations is clearly crucial for the rest of the paper.<sup>10</sup> We will refer to the two cases as two *regimes*.<sup>11</sup>

The distinction between in-house and independent management that we will assume is that with an in-house management, the results of cost-saving efforts accrue directly to the principal; i.e., in the model in this section, the principal’s payoff is

$$B + s - R^{\text{in-house}}(z),$$

where  $s$  measures cost-savings and where  $R^{\text{in-house}}(z)$  is the remuneration to the management, based on some performance measure  $z$ ; the management’s revenues are  $R^{\text{in-house}}(z)$ . With an

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<sup>8</sup>This is inconsequential for our results, but does affect the incentives for side-contracting.

<sup>9</sup>The most convincing rationale for linear contracts is provided by Holmström and Milgrom (1987); their rather technical argument reflects a very fundamental notion, *viz.* that non-linear incentive schemes open up for gaming in ways that are likely to be undesirable unless the party facing non-linear incentives has very narrow action possibilities.

<sup>10</sup>As we have noted, this is a widely shared presumption, articulated e.g. by Williamson (1998).

<sup>11</sup>There is also evidence that incentives in non-profit organizations are weaker than incentives in for-profit organizations; see e.g. Roomkin and Weisbrod (1999).



independent management, on the other hand, the results of cost-saving efforts accrue to the management; in this case thus, the principal's payoff is

$$B - R^{\text{indep}}(z)$$

where  $R^{\text{indep}}(z)$  is the remuneration to the management; the management's revenues in this case is  $R^{\text{indep}}(z) + s$ .

This setting is assumed, but it is, arguably, a natural point of departure. If the setting were integrated in a perfect-contracting environment the differences between the two regimes would vanish in the end – i.e., when we arrive at actual incentives and allocations. As we will see below, however, with modest departures from perfect-contracting assumptions, the setting generates substantive differences in end results between the two regimes.

**Example.** Consider now a case where the agent, in the end, exerts effort,  $a$ , on a task whose result – an inverse measure of realized cost – is

$$x = a + \varepsilon$$

where  $\varepsilon$  is a random variable reflecting the fact that the results is affected but not determined by the agent's effort, and where  $\varepsilon$  is normally distributed with mean zero and variance  $v$ .

The principal delegates the task to the management, offering a linear contract

$$R = \alpha + \beta x,$$

for constants  $\alpha$  and  $\beta$ . It would seem natural to impose that  $\beta = 1$ ; as we will discuss shortly, however, other cases are interesting too.

The management, in turn, delegates the task to the agent, and the agent's monetary reward is

$$y = F + mx$$

for constants  $F$  and  $m$ .

**Optimal contracts.** Given the assumption that the contracts are linear, the contract that the management optimally offers to the agent is simple. Since the analysis is a simplified roadmap to the analysis of the multitask model below, which is deferred to an Appendix, we provide the details.

The management solves (where expectations are w.r.t. the distribution of  $\varepsilon$ )

$$\begin{aligned} \max_{m,F} \quad & \alpha + E[\beta x - (F + mx)] = \alpha + [E(\beta - m)(a + \varepsilon) - F] \\ \text{s.t.} \quad & -E \exp \left\{ -r \left[ F + m(a + \varepsilon) - a^2/2 \right] \right\} \geq u_0, \\ & \text{and } a \text{ maximizes } -E \exp \left\{ -r \left[ F + m(a + \varepsilon) - a^2/2 \right] \right\}. \end{aligned}$$

The problem is simplified by the fact that

$$E \exp \left\{ -r \left[ F + m(a + \varepsilon) - a^2/2 \right] \right\} = \exp \left\{ -r(F + ma - a^2/2) + r^2 m^2 \frac{v}{2} \right\},$$

and can thus be written

$$\begin{aligned} \max \quad & \alpha + (\beta - m)a - F \\ \text{s.t.} \quad & -\exp \left\{ -r \left[ (F + ma) - rm^2 v/2 - a^2/2 \right] \right\} \geq u_0 \\ & \text{and } a \in \arg \max \left( -\exp \left\{ -r \left[ (F + ma) - rm^2 v/2 - a^2/2 \right] \right\} \right). \end{aligned}$$

Maximization by the agent yields<sup>12</sup>  $a^* = m$ ; inserting this and taking logarithms in the first constraint we get

$$\begin{aligned} \max \quad & \alpha + (\beta - m)m - F \\ \text{s.t.} \quad & F + m^2 - rm^2 v/2 - m^2/2 \geq -\frac{\ln(-u_0)}{r}. \end{aligned} \tag{1}$$

Solving the constraint – which obviously must bind – for  $F$ , we get, denoting the reduced-form objective function by  $\phi$ ,

$$\phi(m) = \alpha + (\beta - m)m + m^2 - rm^2 v/2 - m^2/2 + \frac{\ln(-u_0)}{r}. \tag{2}$$

The first-order condition is:

$$\frac{d\phi}{dm} = [(\beta - 2m) + (2m - rvm - m)] = 0, \tag{3}$$

implying

$$m = \frac{\beta}{1 + rv}. \tag{4}$$

Thus, the incentives faced by the management provide an upper bound on the incentives provided by the management for the agent; in the model this is seen by noting that  $r$  and  $v$  are non-negative, but the property is clearly true quite generally in principal-agent models with a risk averse agent.

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<sup>12</sup>Since we impose linear contracts, issues about the validity of the “first-order approach” do not arise; see Jewitt (1988).

This extremely simple example highlights a straightforward and natural property that is rarely noted, *viz.* that incentives *trickle down*. In particular, it provides a simple and, arguably, quite plausible explanation of the frequently made observation that incentives are weaker in non-profit firms than in for-profits; indeed, this observation is sometimes considered puzzling. We will come back to this briefly when we discuss applications.

**Origins of muted incentives.** While the empirical underpinnings for the fact that incentives are weaker within organizations than are incentives generated in relations between organizations seem quite solid, as we argued, it is useful to have a theoretical foundation as well.

**Manipulation by the management.** Consider an environment as in the example where the management observes  $x = a + \varepsilon$  as described, but where the management can distort the signal observed by the principal by means of manipulation.<sup>13</sup> To keep things straightforward and simple, let the principal's signal be given by

$$z = x + \gamma d = a + \varepsilon + \gamma d,$$

where  $d$  is the distortion resulting from the management's manipulation, and  $\gamma > 0$  a constant; moreover, we assume that the management suffers disutility  $d^2/2$  from a manipulation  $d$ . Since the distortion does not enter the management's constraints, and since it is separable from the management's other choice variables ( $m$  and  $F$ ), its effect on the management's problem is simply to add a benefit,  $\beta\gamma d$ , and a cost,  $-d^2/2$ , to the objective function which adds up to a benefit of  $\gamma^2\beta^2/2$  since  $d^* = \beta\gamma$  for optimally chosen  $d$ .

The principal's problem, however, is affected more substantially. Again, we present some in-text analysis at this point since it illuminates the ensuing more cumbersome analysis in the general case. The principal maximizes her payoff subject to the standard constraints, *viz.* that the management attains its reservation payoff, and that the management behaves optimally. Formally, the principal solves (using reduced forms in the constraint, and suppressing the constraints of the management's maximization problem):

$$\begin{aligned} \max_{\alpha, \beta} E(x - \beta z - \alpha) &= (1 - \beta)a - \gamma^2\beta^2 - \alpha \\ \text{s.t. } \alpha + (\beta - m)m - F + \gamma^2\beta^2/2 &\geq 0, \\ \text{and } (m, F) &\text{ maximizes } \alpha + (\beta - m)m - F + \gamma^2\beta^2/2. \end{aligned}$$

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<sup>13</sup>This approach is in line with e.g. Holmström (1999).

Noting that  $m = \beta / (1 + rvc)$  and solving the agent's participation constraint for  $F$  as in (1) we have

$$\begin{aligned} & \max_{\alpha, \beta} (1 - \beta) m - \gamma^2 \beta^2 - \alpha \\ \text{s.t. } & \alpha + (\beta - m)a + \gamma^2 \beta^2 / 2 - [-\ln(-u_0) / r + rm^2 v / 2 + a^2 / 2 - ma] \geq 0; \end{aligned}$$

the objective function is, substituting the constraint,

$$\phi(\beta) = (1 - \beta) m - \gamma^2 \beta^2 + (\beta - m)m + \gamma^2 \beta^2 / 2 - [-\ln(-u_0) / r + rm^2 v / 2 - m^2 / 2],$$

and, simplifying, we have

$$\phi(\beta) = m - m^2 / 2 - \gamma^2 \beta^2 / 2 - rm^2 v / 2 + \ln(-u_0) / r.$$

The first-order condition w.r.t.  $\beta$  is, using  $dm/d\beta = 1 / (1 + rvc)$ ,

$$\frac{d\phi}{d\beta} = -\gamma^2 \beta + [1 - m(1 + rv)] \frac{1}{1 + rv} = 0,$$

or, expressing  $m$  in terms of  $\beta$ ,

$$\beta = \frac{1}{1 + \gamma^2 (1 + rv)},$$

showing that  $\beta = 1$  if  $\gamma = 0$ , while  $\beta < 1$  if  $\gamma > 0$ .

### **Interpretations.**

In the case presented, the principal provides incentives to the management to control costs;  $x$  is realized cost-savings – that in the end accrue to the principal – and  $z$  is a distorted measure of cost savings. This is a natural specification if, for example,  $x$  measures long-term cost savings, while  $z$  is an accounting measure of cost savings that to some extent is controlled by the management. The interpretation is simple and clear: the greater the management's manipulation possibilities, the weaker are optimal incentives.

If revenues accrue to the management, on the other hand, revenue sharing corresponds, in effect, to the management giving up a share  $1 - \beta$  of revenues to the principal. Again letting  $x$  denote real cost savings while  $z$  denotes the measure of cost saving on which revenue sharing is based – the management may be able to influence the measure  $z$  along the same lines as above. In this case, manipulability simply reinforces property that  $\beta = 1$  at the optimum. For interpretations, consider a subcontractor to a firm with a contract sharing cost savings; clearly, the subcontractor has incentives to inflate cost estimates whenever the firm's share of additional costs,  $1 - \beta$ , is positive.

### 3 A two-task management-agent model

In this section we will develop and briefly outline a two-task “management-agent model” that will serve as our basic framework for the remainder of the paper. Obviously, it would normally be called a principal-agent model, but for the sake of consistency with the development of the full model below we call the two layers management and agent. The thrust of the model – as well as of other multitask principal-agent models – is that the incentive problem has an *effort allocation* dimension in addition to the *effort extraction* dimension that is the defining element of principal-agent models. From a pragmatic point of view this is extremely easy to rationalize; a large range of real-world delegation problems involve several distinctive dimensions, quality and quantity dimensions of output being a salient example. In our application below, we will distinguish between a cost-saving dimension,  $x_1$ , and a quality dimension,  $x_2$ .

#### 3.1 Laying out the model

In formal terms, the model produces two output measures,  $x_1$  and  $x_2$ , that depend stochastically on two effort (input) dimensions,  $a_1$  and  $a_2$ , controlled by the agent.<sup>14</sup> More precisely we assume that

$$x_i = a_i + \varepsilon_i, \quad i = 1, 2,$$

where  $\varepsilon_i$  is noise, assumed to be normally distributed with mean zero and variance  $v_i$ , and assumed independent across  $i$ . Note that this formulation – combined with the cost-saving and quality interpretations of the two dimensions that we adopt – gives us a cost-saving dimension and a quality dimension of effort as well as output.

The rest of the setting follows the two lower tiers of the example in the previous section closely. The management, being a risk neutral profit maximizer, offers the agent a contract that specifies monetary compensation that is constrained to be linear in the performance measures:

$$y = F + m_1x_1 + m_2x_2.$$

The agent has preferences over monetary compensation and effort,  $(a_1, a_2)$ , according to the von Neumann-Morgenstern utility function

$$u(y; a) = -\exp\{-r[y - c(a)]\}, \quad \text{where } c(a) = a_1^2 + 2\kappa a_1 a_2 + a_2^2;$$

the parameter  $\kappa$  measures the degree of substitutability between  $a_1$  and  $a_2$  in the agent’s disutility-of-effort function. The agent has reservation payoff  $u_0$ .

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<sup>14</sup>The seminal contribution to the development of this framework is Holmström and Milgrom (1991).

The management values the two dimensions of realized output at  $\beta_1$  and  $\beta_2$  per unit, and the problem faced by the management is thus

$$\begin{aligned} & \max E [\beta_1 x_1 + \beta_2 x_2 - (F + m_1 x_1 + m_2 x_2)] \\ & \text{s.t. } -\exp \left\{ -r \left[ F + m_1 (a_1 + \varepsilon_1) + m_2 (a_2 + \varepsilon_2) - (a_1^2 + 2\kappa a_1 a_2 + a_2^2) \right] \right\} \geq u_0, \\ & \text{and } a \text{ maximizes } -\exp \left\{ -r \left[ F + m_1 (a_1 + \varepsilon_1) + m_2 (a_2 + \varepsilon_2) - (a_1^2 + 2\kappa a_1 a_2 + a_2^2) \right] \right\}. \end{aligned}$$

The solution, which is derived in the Appendix, is

$$m_1 = \frac{2rv_2(\beta_1 - \beta_2\kappa) + \beta_1}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1}, \quad (5)$$

and

$$m_2 = \frac{2rv_1(\beta_2 - \beta_1\kappa) + \beta_2}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1}; \quad (6)$$

$F$  is determined residually. The key insight added by the effort-allocation dimension is – unsurprisingly but importantly – that there is, in general, an interdependence between the two output dimensions,  $(x_1, x_2)$ , in the sense that incentives provided for one component of the result affects inputs and results in both dimensions. This interdependence is a bit unwieldy even as we rule out stochastic dependence between the noise terms and assume that each output dimension depends only on one input. Nevertheless, some general – and for our purposes important – properties can be demonstrated by considering some special cases.<sup>15</sup> We will take the case when  $a_1$  and  $a_2$  are substitutes in the agent’s utility function – i.e. when  $\kappa > 0$  – as the main case and only occasionally note results for the other case; the complements case ( $\kappa < 0$ ) gives the effort-extraction problem a “free-lunch flavor” that seems unnatural for most applications.

- First, it may be worth noting that if noise (measured by  $v_i$ ) or risk aversion vanishes, the incentive problem vanishes too, and the solution is  $m_1 = \beta_1$  and  $m_2 = \beta_2$ .
- Second, consider the case where  $x_2$  is a performance measure that has no intrinsic value to the management so that  $\beta_2 = 0$  (note that this case departs somewhat from our assumptions below). This gives

$$m_1 = \frac{\beta_1(2rv_2 + 1)}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1}; \quad m_2 = \frac{-2rv_1\kappa\beta_1}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1},$$

and we see that as long as the two inputs,  $(a_1, a_2)$ , are substitutes, the agent is punished for high output in the  $x_2$ -dimension (granted, substitutability in the utility function is less obvious when  $x_2$  is a pure performance measure).

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<sup>15</sup>The starkness of the cases depends, of course, on the assumptions that rule out some channels of cross-dependence. For some analysis of the general case, see Holmström and Milgrom (1991).

- Finally, consider the case where the informativeness about effort of one dimension of output, say 2, grows small, i.e. when  $v_2 \rightarrow \infty$ . In this case

$$m_1 = \frac{2r(\beta_1 - \beta_2\kappa)}{4r^2(1 - \kappa^2)v_1 + 2r}; \quad m_2 = 0,$$

and we see that the incentives provided for  $x_1$  must be used to control both dimensions of effort; from the expression one sees e.g. that if the uninformative dimension is important enough – more precisely if  $\beta_1 < \beta_2\kappa$  – output in the other dimension is punished.

The last case is important because the main case below will be relatively closely related to it. It also highlights the general point that there are important circumstances under which weak incentives are desirable for “second-best reasons.”

### 3.2 A shortcut to applications

Since our focus in this paper is on the fact that incentives trickle down, elaborations of the two-layer model are of somewhat limited interest. Nevertheless we will make a simple point that will re-emerge in the general analysis below.

Suppose that the principal is constrained to provide cost-saving incentives – that is incentives for  $x_1$  – that are either quite weak or quite strong for reasons described in the previous section. In formal terms,  $m_1 \in \{\mu_0, 1\}$ . In this case, it is straightforward to derive the optimal  $m_2$  conditional on  $m_1$  being fixed, which is

$$m_2 = \frac{\beta_2 - \beta_1\kappa + \kappa m_1}{2r(1 - \kappa^2)v_2 + 1},$$

for the main case with  $\kappa > 0$ ,  $m_2$  depends positively on  $m_1$ . This positive dependence notwithstanding, the full effect of an increase in  $m_1$  is to increase equilibrium  $a_1$  and decrease equilibrium  $a_2$  (this is true for all  $\kappa$ ; it is easily demonstrated but omitted).

While these simple properties are natural, they show that endogenous optimal adjustment of incentives for  $a_2$  does not cancel the direct effect of a given incentive intensity for  $a_1$ . More substantively, they show that imposing weak cost-saving incentives leads – in equilibrium – to less effort being devoted to cost savings and more effort being devoted to the quality dimension, and vice versa for imposing strong cost-saving incentives. This is to say that if we consider this framework relevant, popular arguments about the costs and benefits of privatization are corroborated by the model – making cost-saving incentives stronger goes with the risk that quality is, in the end, compromised. It is important to note that the comparative statics and their relation to real-world conceptions are relevant independently of what is socially optimal cost-saving incentives in the model.

## 4 Incentives in the three-layer two-task model

We will now consider the general principal-management-agent model where the technology of the project delegated to the agent is that specified by the two-task model. The preferences of the principal,  $P$ , and the management,  $M$ , are the same as in the example –  $P$  and  $M$  are both risk neutral profit maximizers,  $M$  having reservation payoff zero – and  $A$ 's preferences and action possibilities were specified in the previous section.  $P$  and  $M$  both observe the same performance measures,  $(x_1, x_2)$ , while the principal cannot observe the contract between  $M$  and  $A$ . The principal ultimately values the performance in the two dimensions according to

$$V = x_1 + px_2; \tag{7}$$

$P$ 's payoff is measured in dollars and cost-saving performance is valued according to this, while the quality-related performance may weigh more or less heavily in the principal's payoff according to the parameter  $p \geq 0$ .  $P$  offers a contract with remuneration

$$R = \beta_0 + \beta_1x_1 + \beta_2x_2 \tag{8}$$

to  $M$ .

Throughout, we will – as in the example and for the reasons articulated there – rule out side-contracting between  $P$  and  $A$ . The important consequence of this is that we are, essentially, dealing with two separate incentive problems, that where  $M$  provides incentives to  $A$ , and that where  $P$  – internalizing the solution of  $M$ 's problem – provides incentives to  $M$ .

We will start by presenting the trivial solution to the full model in the absence of further complications. Turning to distortions, we start by imposing a constraint on the cost-saving incentives that the principal can provide to the management; as we saw in Section 2, such a constraint on the principal's instruments results if cost-saving performance is manipulable by the management. In the interest of tractability we will consider the constraint on  $\beta_1$  directly. This is also in line with our focusing on the case when  $P$  faces a dichotomous choice between weak and strong incentives; as we have argued in Section 2, such a dichotomy follows with a high degree of manipulability. Next, we will consider the case when  $M$  can manipulate the quality-related performance measure observed by  $P$ ; this serves as a reasonable and tractable incentive problem hitting the principal's contract with the management. After having considered the combined effect, we go on to interpretations.



## 4.1 Optimal incentives for the management

In the absence of further complications, the problem is simple. The management delegates the project to the agent with equilibrium incentives according to (5) and (6). The principal's problem is one where a risk-neutral party delegates something to another risk-neutral party with the same relevant information, and it is intuitive and well known that the optimal incentives – directly following (7) – satisfy

$$\beta_1 = 1 \text{ and } \beta_2 = p. \quad (9)$$

Thus, in the absence of complications or distortions, nothing – beyond possibly a conceptually more satisfactory framework – is gained by the third layer.

## 4.2 Two modes for cost-saving incentives

Next, we consider a simple but important constraint on the principal's problem, *viz.* the constraint that the cost-saving incentives, given by  $\beta_1$ , not be higher than some bound  $b < 1$ . This, of course, is the constraint which we envision that principals dealing with an in-house management face.

The results following our imposing this one restriction are straightforward. First, the constraint must bind since the problem – as demonstrated in the Appendix – is appropriately concave, and since the only solution with the constraint inactive is  $\beta_1 = 1$ . Second, the comparative statics of  $\beta_2$  with respect to  $b$  are straightforward and unsurprising:  $\partial\beta_2/\partial b$  has the sign of  $\kappa$ . For the main case where  $a_1$  and  $a_2$  are substitutes in the disutility-of-effort function thus, the exogenously imposed attenuation of cost-saving incentives makes the principal attenuate quality incentives too.

Let us finally consider how equilibrium effort,  $(a_1, a_2)$ , depends on  $b$ . Equations (A.6) and (A.7) in the Appendix show that while the dependence of equilibrium effort on  $b$  is, in general, ambiguous, we have an unambiguous result when  $v_2$  is large – i.e. when the observation of the quality dimension is indeed a poor indicator of the effort exerted on quality by the agent. In that case,  $a_1$  is increasing in  $b$  while  $a_2$  is decreasing. In words, the attenuation of cost-saving incentives provided by the principal to the management leads, in the end, the agent to exert more effort on quality and less effort on obtaining cost savings. This is important enough to state formally.

**PROPOSITION 1** *Let cost-saving incentives to the management be constrained not to exceed  $b < 1$  and consider optimal contracts given  $b$ . When quality measurement is sufficiently imprecise*

( $v_2$  is sufficiently large), equilibrium cost-saving effort ( $a_1$ ) is increasing and equilibrium effort exerted on quality ( $a_2$ ) decreasing in  $b$ .

A few things are worth stressing:

- While this may not be entirely surprising, it is important to remember that the management is free to choose incentives as it pleases – it is only the principal’s incentives to the management that are constrained. The result is thus yet another manifestation of the *trickling down* of incentives.
- In the environment considered so far, the constraint on cost-saving incentives is the only distortion. The shift of effort is therefore unambiguously a bad thing from a welfare point of view (unless, of course, there are social preferences diverging from the principal’s preferences). The important thing to note here, however, is the positive aspect: by tilting incentives away from cost savings at the management level, the activities “on the ground” are tilted in the same direction due to the trickling-down effect. Thus, the result does, in our view, corroborate commonsensical notions of, for examples, the consequences of privatization.

### 4.3 Manipulability of performance measures

In this section we have, up to this point, assumed that the principal and the management observe the same performance measures,  $(x_1, x_2)$ . Now, we will assume that the principal has an informational disadvantage concerning the observation of the quality-related performance measure,  $x_2$ . More precisely we will assume that while the management observes  $x_2 = a_2 + \varepsilon_2$ , the principal observes

$$z_2 = a_2 + qd_2 + \varepsilon_2^z \tag{10}$$

where  $q$  is a non-negative constant and  $d_2$  is a distortion of the signal controlled by the management;  $\varepsilon_2^z$  is an error term that is independent of other random variables. The management can thus distort the performance measure, but doing so carries a cost  $d_2^2/2$  for  $M$ .

**Comparative statics with respect to manipulability.** The separability properties implied by this formulation renders us a problem which is structurally quite similar to the original one. In the Appendix the problem is stated, followed by an analysis of the comparative statics of the principal’s optimal contract,  $(\beta_1, \beta_2)$ , with respect to  $q$ . Again unsurprisingly, we find that  $q$  affects  $\beta_2$  negatively, while the effect on  $\beta_1$  has the opposite sign of  $\kappa$ , i.e.,  $q$  leads

to an attenuation of cost-saving incentives too when there is “competition” between the two components of effort.

When manipulability of quality is introduced while, at the same time,  $\beta_1$  is constrained, the comparative statics are similar – i.e.  $\beta_2$  depends negatively on  $q$  – while the exact expression is somewhat different.

**Comparative statics with respect to the valuation of quality.** In the presence of manipulability, equilibrium incentives to the management depend non-trivially on other variables, such as the valuation of quality,  $p$ , and the incentive problem faced by the management (as measured by  $v_1$  and  $v_2$ ). The dependence on  $p$  is clear cut: For  $q > 0$  – i.e. in the presence of manipulability – the incentive intensity for quality,  $\beta_2$ , depends positively on  $p$ ; i.e., a higher valuation of quality by the principal increases the optimal reward for quality to the management. More interestingly,  $\partial\beta_1/\partial p$  has the opposite sign of  $\kappa$ ; i.e., when efforts are substitutes a higher valuation of quality makes the principal blunt cost-saving incentives. While this is not entirely unsurprising, it is quite important since it says that *there is a trickling-down effect in the effort-allocation dimension as well*: In the presence of an incentive problem between the  $P$  and  $M$ , blunting of cost-saving incentives *for the management* helps shifting *the agent’s* effort towards the quality dimension; moreover, this blunting of cost-saving incentives is an optimal response to a higher valuation of the quality of output.

#### 4.4 The tradeoffs

The comparative statics of the optimal contract offered by the principal to the management can straightforwardly be translated into statements about how the optimal mode of governance – i.e. make or buy – are affected by manipulability,  $q$ , and the importance of quality,  $p$ .

The choice between “make” (employing the management) and “buy” (contracting with an independent management) is a choice between muted cost-saving incentives,  $\beta_1 \leq b < 1$ , and full cost-saving incentives,  $\beta_1 = 1$ .<sup>16</sup> Since the objective function is well-behaved in being concave and, in fact, quadratic in  $(\beta_1, \beta_2)$  – as is shown in the Appendix – an exogenous shift reducing  $\beta_1$  can be identified with a shift that makes choosing an employed management more attractive. The point at which employing the management becomes optimal depends, obviously, on parameters such as  $b$ ; at the current stage, however, we confine ourselves to a

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<sup>16</sup>One could of course allow some room for letting  $\beta_1 \neq 1$  in the case of an independent management; it would not add any insight, however.

purely qualitative analysis. The following proposition follows directly from the analysis in the previous subsection.

**PROPOSITION 2** *Suppose that cost-saving effort and quality effort are substitutes in the agent's utility function ( $\kappa > 0$ ). Then choosing in-house production (an employed management) is more attractive relative to outsourcing (contracting with an independent management): (i) the more severe is the quality-control problem faced by the principal in dealing with the management (i.e. the higher is  $q$ ), and, for  $q > 0$ , (ii) the more valuable is quality for the principal (the higher is  $p$ ).*

An additional question to ask would be how the optimal contract between the principal and the management depends on the measurement problem, i.e. how  $(\beta_1, \beta_2)$  depends on  $v_2$ . This question turns out not to have a clear answer; analyzing the expressions it is easy to verify that the contract can move in either direction.

## 5 Applications and elaborations

In this section we will elaborate on the results and discuss robustness and extensions.

**Robustness.** The assumption perhaps most likely to catch the reader's eye as strong is the assumption that the management's possibilities to manipulate the quality measure is equal across the two regimes. While this may or may not be reasonable, the results only depend on the much weaker property that manipulability across activities varies similarly in the two regimes: As long as this is the case the notion of an activity with a high degree of manipulability is well defined, and the prediction that the higher this degree, the more likely is the activity to be performed in-house, stands.

The reader may also question the generality of the information structure. In a way, the assumed information structure is simple; most of the potential informational asymmetries that may arise among three parties are assumed away. In particular, the principal and the management observe performance measures of the agent's activity that are closely related to one another: that observed by the principal is the same as that observed by the management plus manipulation plus additional noise. Apart from being in the interest or tractability, however, the resulting vertical structure is, in our view, conceptually appealing. Complicating matters by, for example, giving the management substantive private information that would open up for collusion between the agent and the management (or making it risk averse) might be interesting,

but it does not seem to raise issues that are critical to our goal of illuminating measurement-related factors affecting the make-or-buy decision.<sup>17</sup> The tenet of our specification is that the principal by necessity delegates the undertaking of the project to the management; this tenet is captured, albeit perhaps a bit starkly, by letting the management be risk neutral.

**Empirical implications.** There are two sets of empirical implications tied to Propositions 1 and 2 respectively. The main implication of the first proposition – that in-house production is a way of securing quality when quality measurement is imprecise – is, in our view, that there is some truth in the often-heard argument that privatization or outsourcing may be a threat to quality. This is interesting and important but does not lead much further since it does not tell anything about optimal contracts.

The implications of Proposition 2 – that in-house production is more likely the more severe the quality-control problem between the principal and the management, and the more important is quality – on the other hand, produce empirically testable hypotheses about governance of activities. To give a simple example, it would seem to predict that management-type activities and research-and-development activities be organized in-house rather than independently. It also provides a potential rationale for the pervasiveness of publicly provided elementary education, although this view is sometimes challenged as we will note below.<sup>18</sup>

**Privatization and outsourcing.** Although we believe that the framework developed can prove useful in systematic empirical investigations of determinants of privatization and outsourcing, this is not the place for explorations in such directions. Instead, we will confine ourselves to discussing a salient example. It is generally held that garbage collection is a prime example of an activity, often performed in-house by local governments, that can be contracted out in a way that leads to substantial cost savings without jeopardizing quality.<sup>19</sup> Snow removal is an activity that may superficially look similar to garbage collection. There are, however, scattered evidence from Sweden – in particular from a major overhaul of snow removal in the city of Stockholm – indicating that contracting out is likely to work much less well in this case.<sup>20</sup>

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<sup>17</sup>See Tirole (1986) for an analysis of a three-layer hierarchy where the intermediate layer – the *supervisor* – has private information about the state of the world and is risk averse. In such a framework, collusion – in particular between the agent and the supervisor – becomes the key issue.

<sup>18</sup>This example is pushed by Acemoglu, Kremer and Mian (2003) too.

<sup>19</sup>See e.g. Savas (1977).

<sup>20</sup>This point is made more elaborately in Swedish in Andersson, F. (2002), “Konkurrens på kommunala villkor – Om konkurrensutsättning och gränsen mellan marknad och byråkrati”, Kommunförbundets, Stockholm.

There are, we believe, two distinguishing features that may explain the difference: uncertainty and measurement problems. While uncertainty clearly plays a role, the measurement issue is fundamental: In order to establish a contractually viable relationship between the effort exerted in snow removal and “snowfreeness,” an elaborate measurement apparatus is necessary. However ambitious – and costly – such an apparatus is construed, it is still bound to rest heavily on vague criteria. The upshot is that in-house provision – with the accompanying weak incentives – is likely to be a substitute for some of the measurement effort and, in the end, preferable.<sup>21</sup> This example illustrates, in our view, two important points: first, measurement is key; secondly, whether or not an activity is suitable to outsourcing/privatization/contracting out has little to do with its production technology and all to do with contracting possibilities.<sup>22</sup>

**Firms, governments and non-profits.** In this paper, our focus is on the make-or-buy decision concerning a particular activity, and how the decision is affected by the nature of the activity. The driving mechanism is the muted cost-saving incentives induced by in-house production. A related question is whether the constraint that cost-saving incentives in-house not be too strong may be different for different types of organizations. There is, for example, reasons to believe that incentives in non-profit organizations are weaker than in for-profit organizations.<sup>23</sup> The difference between firms and governments in this regard is, moreover, corroborated by Acemoglu, Kremer and Mian (2003). In terms of our model,  $b$  might differ across types of organizations, and this would have relatively straightforward implications for organizational choice (provided that, plausibly,  $b$  was constrained both upwards and downwards for intermediate modes of organization). This may, for example, throw light on the prevalence of non-profits in some sectors, like hospitals and schools, and it may also rationalize calls for prohibiting for-profit actors in certain types of activities. In pursuing this line of thought, the trickle-down property of incentives seems particularly pertinent.

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<sup>21</sup>Note that the endogeneity of the imprecision of measurement alluded to takes us a little bit beyond the model, but not in a consequential way.

<sup>22</sup>The last point is re-inforced by noting that steps towards privatization in schooling in the sense of student/parent choice combined with some extent of free entry seems to work well in many circumstances; see e.g. Hoxby (2002). The reason seems to be that quality control can be decentralized to students/parents under voucher-type competition. The model in this paper is not directly applicable to such an environment, but the example illustrates the point that the production technology and the “softness”/“hardness” of the activity are *not* the key determinants of its suitability to choice and private provision.

<sup>23</sup>See Roomkin and Weisbrod (1999) for an empirical explorations of hospitals, and Glaeser and Shleifer (2001) for a simple theoretical model.

## 6 Concluding remarks

In this paper we have tried to approach the make-or-buy decision in a comprehensive-contracting framework emphasizing the measurement aspects of cost savings and quality. We have shown that incentives trickle down from the principal-management contract to the management-agent contract, and that this produces the result that outsourcing, roughly, is less attractive, the harder is the quality-control problem. Finally, we have discussed implications for privatization and outsourcing.

There are a number of substantive questions raised but unanswered by this paper. First, while we believe that the foundations for the distinction between weak incentives in-house and strong incentives in contracting should be sought in contracting possibilities – the results of Acemoglu, Kremer and Mian (2003) notwithstanding – the current justification is very tentative; this deserves further investigation. Second, the current theory seems – as we have noted – readily extended to analyzing the choice among, for example, for-profit, non-profit and government operation; this extension seems worthwhile, and it may offer tractable empirical implications as a side benefit. Finally, one would like to see some integration between measurement-based and asset-ownership-based theories of make-or-buy.<sup>24</sup> This model, or variants of it, seem potentially useful in such an undertaking.

## Appendix

*Optimal contracts in the multitask principal-agent model.*

The problem can, evaluating expectations, be written

$$\begin{aligned} & \max [(\beta_1 - m_1)a_1 + (\beta_2 - m_2)a_2 - F] \\ \text{s.t. } & -\exp(-r(F + m_1a_1 + m_2a_2 - \frac{r}{2}m_1^2v_1 - \frac{r}{2}m_2^2v_2 - [a_1^2 + 2\kappa a_1a_2 + a_2^2])) \geq u_0 \end{aligned}$$

and optimality for the agent, the first-order conditons for which are

$$m_1 - 2(a_1 + \kappa a_2) = 0; \quad m_2 - 2(\kappa a_1 + a_2) = 0.$$

Maximization yields

$$a_1^* = \frac{m_1 - \kappa m_2}{2(1 - \kappa^2)}; \quad a_2^* = \frac{m_2 - \kappa m_1}{2(1 - \kappa^2)}$$

and the objective function is (with  $a_1^*$  and  $a_2^*$  inserted,  $\tilde{u} = -\ln(-u_0)/r$  and after substituting

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<sup>24</sup>Holmström (1999) takes some preliminary steps.

the constraint)

$$\begin{aligned}\phi(\beta_1\beta_2) &= (\beta_1 - m_1)\frac{m_1 - \kappa m_2}{2(1 - \kappa^2)} + (\beta_2 - m_2)\frac{m_2 - \kappa m_1}{2(1 - \kappa^2)} + \\ &\quad m_1\frac{m_1 - \kappa m_2}{2(1 - \kappa^2)} + m_2\frac{m_2 - \kappa m_1}{2(1 - \kappa^2)} - \frac{r}{2}m_1^2v_1 - \frac{r}{2}m_2^2v_2 \\ &\quad - \frac{1}{4(1 - \kappa^2)^2}[(m_1 - \kappa m_2)^2 + 2\kappa(m_1 - \kappa m_2)(m_2 - \kappa m_1) + (m_2 - \kappa m_1)^2] - \tilde{u}.\end{aligned}$$

Noting that  $\lambda = 1$  and simplifying by multiplying by  $2(1 - \kappa^2)$ , we have

$$\begin{aligned}\phi(\beta_1\beta_2) &= \beta_1(m_1 - \kappa m_2) + \beta_2(m_2 - \kappa m_1) - r(1 - \kappa^2)m_1^2v_1 - r(1 - \kappa^2)m_2^2v_2 \\ &\quad - \frac{1}{1 - \kappa^2}\left[\frac{1}{2}(m_1 - \kappa m_2)^2 + \kappa(m_1 - \kappa m_2)(m_2 - \kappa m_1) + \frac{1}{2}(m_2 - \kappa m_1)^2\right] - \widehat{u}.\end{aligned}$$

The first-order conditions w.r.t.  $(m_1, m_2)$  are:

$$\begin{aligned}\beta_1 - \beta_2\kappa - 2r(1 - \kappa^2)v_1m_1 - \frac{1}{1 - \kappa^2}[(m_1 - \kappa m_2) + \kappa[(m_2 - \kappa m_1) - \kappa(m_1 - \kappa m_2)] - \kappa(m_2 - \kappa m_1)] &= 0, \\ \beta_2 - \beta_1\kappa - 2r(1 - \kappa^2)v_2m_2 - \frac{1}{1 - \kappa^2}[-\kappa(m_1 - \kappa m_2) + \kappa[(m_1 - \kappa m_2) - \kappa(m_2 - \kappa m_1)] + (m_2 - \kappa m_1)] &= 0.\end{aligned}$$

Or, simplifying,

$$\begin{aligned}\beta_1 - \beta_2\kappa &= \left(2r(1 - \kappa^2)v_1 + \frac{1 - \kappa^2}{1 - \kappa^2}\right)m_1 + \frac{\kappa^3 - \kappa}{1 - \kappa^2}m_2, \\ \beta_2 - \beta_1\kappa &= \frac{\kappa^3 - \kappa}{1 - \kappa^2}m_1 + \left(2r(1 - \kappa^2)v_2 + \frac{1 - \kappa^2}{1 - \kappa^2}\right)m_2;\end{aligned}$$

simplifying further

$$\begin{aligned}\beta_1 - \beta_2\kappa &= (2r(1 - \kappa^2)v_1 + 1)m_1 - \kappa m_2, \\ \beta_2 - \beta_1\kappa &= -\kappa m_1 + (2r(1 - \kappa^2)v_2 + 1)m_2.\end{aligned}$$

This can be written

$$\begin{pmatrix} 2r(1 - \kappa^2)v_1 + 1 & -\kappa \\ -\kappa & 2r(1 - \kappa^2)v_2 + 1 \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = \begin{pmatrix} \beta_1 - \beta_2\kappa \\ \beta_2 - \beta_1\kappa \end{pmatrix},$$

and the solution is

$$\begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = \frac{1}{D} \begin{pmatrix} 2r(1 - \kappa^2)v_2 + 1 & \kappa \\ \kappa & 2r(1 - \kappa^2)v_1 + 1 \end{pmatrix} \begin{pmatrix} \beta_1 - \beta_2\kappa \\ \beta_2 - \beta_1\kappa \end{pmatrix}.$$

The determinant is

$$\begin{aligned}D &= 4r^2(1 - \kappa^2)^2v_1v_2 + 2r(1 - \kappa^2)v_1 + 2r(1 - \kappa^2)v_2 + 1 - \kappa^2 = \\ &\quad (1 - \kappa^2)[4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1];\end{aligned}$$

note that it is positive as it should be. Solving, we obtain

$$m_1 = \frac{2rcv_2(\beta_1 - \beta_2\kappa) + \beta_1}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1},$$



and

$$m_2 = \frac{2rcv_1(\beta_2 - \beta_1\kappa) + \beta_2}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1}.$$

The principal's problem in the three-layer model.

Principal's problem is

$$\max_{\beta} E[x_1 + px_2 - (\beta_0 + \beta_1x_1 + \beta_2x_2)] = E[(1 - \beta_1)x_1 + (p - \beta_2)x_2 - \beta_0]$$

$$\text{s.t. } E(\beta_0 + (\beta_1 - m_1)x_1 + (\beta_2 - m_2)x_2 - F) \geq \bar{u}_M$$

and  $m$  maximizes  $E(\beta_0 + (\beta_1 - m_1)x_1 + (\beta_2 - m_2)x_2 - F)$  s.t.  $M$ 's constraints

where  $\bar{u}_M$  is  $M$ 's reservation utility. This can be written

$$\max_{\beta} E[x_1 + px_2 - (\beta_0 + \beta_1x_1 + \beta_2x_2)] = E[(1 - \beta_1)x_1 + (p - \beta_2)x_2 - \beta_0]$$

$$\text{s.t. } E(\beta_0 + (\beta_1 - m_1(\beta))x_1 + (\beta_2 - m_2(\beta))x_2 - F) \geq \bar{u}_M,$$

with  $m_1$  and  $m_2$  chosen optimally, and  $F$  determined by the participation constraint; taking expectations we have

$$\max_{\beta} \phi = (1 - \beta_1)a_1 + (p - \beta_2)a_2 - \beta_0$$

$$\text{s.t. } \beta_0 + (\beta_1 - m_1(\beta))a_1 + (\beta_2 - m_2(\beta))a_2 - F(m_1(\beta), m_2(\beta)) \geq \bar{u}_M.$$

Substituting the constraint, the objective function is

$$\phi(\beta_1, \beta_2) = (1 - \beta_1)a_1 + (p - \beta_2)a_2 + (\beta_1 - m_1(\beta))a_1 + (\beta_2 - m_2(\beta))a_2 - F(m) - \bar{u}_M$$

where (with  $\tilde{u}$  the agent's (re-normalized) reservation utility)

$$F(m) = \tilde{u} + \left(\frac{m_1 - \kappa m_2}{2(1 - \kappa^2)}\right)^2 + 2\kappa \frac{m_1 - \kappa m_2}{2(1 - \kappa^2)} \frac{m_2 - \kappa m_1}{2(1 - \kappa^2)} + \left(\frac{m_2 - \kappa m_1}{2(1 - \kappa^2)}\right)^2 \quad (\text{A.1})$$

$$+ \frac{r}{2}m_1^2v_1 + \frac{r}{2}m_2^2v_2 - m_1 \frac{m_1 - \kappa m_2}{2(1 - \kappa^2)} - m_2 \frac{m_2 - \kappa m_1}{2(1 - \kappa^2)}. \quad (\text{A.2})$$

Substituting for the  $a$ 's and simplifying a bit:

$$\phi(\beta_1, \beta_2) = (1 - m_1(\beta)) \frac{m_1 - \kappa m_2}{2(1 - \kappa^2)} + (p - m_2(\beta)) \frac{m_2 - \kappa m_1}{2(1 - \kappa^2)} - \bar{u}_M - F(m).$$

Importantly, the objective is jointly concave in  $\beta$ . This follows from the fact that  $\phi$  is concave in  $m$ , while  $m$  is linear – and thus weakly concave with a zero second derivative – in  $\beta$  from (5) and (6).

Differentiating w.r.t.  $(m_1, m_2)$  gives (following the optimality conditions for the agent's incentives):

$$\frac{\partial \phi}{\partial m_1} = 1 - p\kappa - (2rc(1 - \kappa^2)v_1 + 1) m_1 + \kappa m_2,$$

$$\frac{\partial \phi}{\partial m_2} = p - \kappa + \kappa m_1 - (2rc(1 - \kappa^2)v_2 + 1) m_2.$$

This should be inserted into

$$\frac{\partial \phi}{\partial \beta_1} = \frac{\partial \phi}{\partial m_1} \frac{\partial m_1}{\partial \beta_1} + \frac{\partial \phi}{\partial m_2} \frac{\partial m_2}{\partial \beta_1} = 0, \quad (\text{A.3})$$

$$\frac{\partial \phi}{\partial \beta_2} = \frac{\partial \phi}{\partial m_1} \frac{\partial m_1}{\partial \beta_2} + \frac{\partial \phi}{\partial m_2} \frac{\partial m_2}{\partial \beta_2} = 0, \quad (\text{A.4})$$

and at this stage we may note that any solution to

$$\frac{\partial \phi}{\partial m_1} = \frac{\partial \phi}{\partial m_2} = 0$$

is clearly a solution to the principal's problem too, and with  $\beta_1 = 1$  and  $\beta_2 = p$  the  $m$ 's solving the system will coincide with equilibrium  $m$ 's. This confirms the already-noted fact that setting  $\beta_1 = 1$  and  $\beta_2 = p$  is optimal for the principal in the absence of distortions.

To say something about cases where there are distortions or constraints making a first-best contract between  $P$  and  $M$  infeasible, we need to develop (A.3) and (A.4) explicitly, however. To do this, we note

$$m_1 = \frac{2rv_2(\beta_1 - \beta_2\kappa) + \beta_1}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1}; \quad m_2 = \frac{2rv_1(\beta_2 - \beta_1\kappa) + \beta_2}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1},$$

and

$$\begin{aligned} \frac{\partial m_1}{\partial \beta_1} &= \frac{2rv_2 + 1}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1}; \\ \frac{\partial m_1}{\partial \beta_2} &= \frac{-\kappa 2rv_2}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1}; \\ \frac{\partial m_2}{\partial \beta_1} &= \frac{-\kappa 2rv_1}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1}; \\ \frac{\partial m_2}{\partial \beta_2} &= \frac{2rv_1 + 1}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1}. \end{aligned}$$

For notational convenience, denote the denominator of these expressions:

$$N = 4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1.$$

The first-order conditions are can be written:

$$\begin{aligned} \frac{\partial \phi}{\partial \beta_1} &= \frac{1}{2(1 - \kappa^2)N} \left\{ \begin{aligned} &[1 - p\kappa - (2r(1 - \kappa^2)v_1 + 1) m_1 + \kappa m_2] (2rv_2 + 1) \\ &+ [p - \kappa + \kappa m_1 - (2r(1 - \kappa^2)v_2 + 1) m_2] (-\kappa 2rv_1) \end{aligned} \right\} = 0, \\ \frac{\partial \phi}{\partial \beta_2} &= \frac{1}{2(1 - \kappa^2)N} \left\{ \begin{aligned} &[1 - p\kappa - (2r(1 - \kappa^2)v_1 + 1) m_1 + \kappa m_2] (-\kappa 2rv_2) \\ &+ [p - \kappa + \kappa m_1 - (2r(1 - \kappa^2)v_2 + 1) m_2] (2rv_1 + 1) \end{aligned} \right\} = 0. \end{aligned}$$

The second derivatives of the objective function are:

$$\frac{\partial^2 \phi}{\partial \beta_1^2} = \frac{1}{2(1 - \kappa^2)N^2} \left\{ \begin{array}{l} - [(2r(1 - \kappa^2)v_1 + 1)(2rv_2 + 1) + \kappa^2 2rv_1](2rv_2 + 1) \\ - [\kappa(2rv_2 + 1) + (2r(1 - \kappa^2)v_2 + 1)\kappa 2rv_1] \kappa 2rv_1 \end{array} \right\} < 0,$$

or, simplifying,

$$\frac{\partial^2 \phi}{\partial \beta_1^2} = -\frac{\kappa^2 2rv_1 + 2rv_2 + 1}{2(1 - \kappa^2)N} < 0,$$

$$\frac{\partial^2 \phi}{\partial \beta_2^2} = \frac{1}{2(1 - \kappa^2)N^2} \left\{ \begin{array}{l} - [(2r(1 - \kappa^2)v_1 + 1)\kappa 2rv_2 + \kappa(2rv_1 + 1)] \kappa 2rv_2 \\ - [\kappa^2 2rv_2 + (2r(1 - \kappa^2)v_2 + 1)(2rv_1 + 1)](2rv_1 + 1) \end{array} \right\} < 0,$$

or, simplifying,

$$\frac{\partial^2 \phi}{\partial \beta_2^2} = -\frac{2rv_1 + \kappa^2 2rv_2 + 1}{2(1 - \kappa^2)N} < 0,$$

and

$$\frac{\partial^2 \phi}{\partial \beta_1 \partial \beta_2} = \frac{1}{2(1 - \kappa^2)N^2} \left\{ \begin{array}{l} [(2r(1 - \kappa^2)v_1 + 1)\kappa 2rv_2 + \kappa(2rv_1 + 1)](2rv_2 + 1) \\ + [\kappa^2 2rv_2 + (2r(1 - \kappa^2)v_2 + 1)(2rv_1 + 1)](\kappa 2rv_1) \end{array} \right\},$$

or, simplifying,

$$\frac{\partial^2 \phi}{\partial \beta_1 \partial \beta_2} = \frac{\kappa(2rv_1 + 2rv_2 + 1)}{2(1 - \kappa^2)N} > 0.$$

Using this, we can write down the Hessian,

$$H = \frac{1}{2(1 - \kappa^2)N} \begin{pmatrix} -(\kappa^2 2rv_1 + 2rv_2 + 1) & \kappa(2rv_1 + 2rv_2 + 1) \\ \kappa(2rv_1 + 2rv_2 + 1) & -(2rv_1 + \kappa^2 2rv_2 + 1) \end{pmatrix}$$

and its inverse,

$$H^{-1} = \frac{2(1 - \kappa^2)N}{\tilde{D}} \begin{pmatrix} -(2rv_1 + \kappa^2 2rv_2 + 1) & -\kappa(2rv_1 + 2rv_2 + 1) \\ -\kappa(2rv_1 + 2rv_2 + 1) & -(\kappa^2 2rv_1 + 2rv_2 + 1) \end{pmatrix},$$

where  $\tilde{D}$  is the determinant of the Hessian when  $1/(2(1 - \kappa^2)N)$  is factored out. Thanks to the concavity of  $\phi$ , we know that it is positive, and it is relatively easily found to be  $\tilde{D} = (1 - \kappa^2)N$ .

*Comparative statics with respect to  $b$ .*

- $\beta_2$  w.r.t.  $b$ . Since the constraint that  $\beta_1 \leq b$  binds, comparative statics can be performed on the one-equation problem  $\partial \mathcal{L} / \partial \beta_2 = 0$ . Straightforwardly,

$$\frac{\partial \beta_2}{\partial b} = \frac{\kappa(2rcv_1 + 2rcv_2 + 1)}{2rcv_1 + \kappa^2 2rcv_2 + 1}, \quad (\text{A.5})$$

and it is readily seen that

$$\text{sign} \frac{\partial \beta_2}{\partial b} = \text{sign} \kappa.$$

- $a$ 's dependence on  $\beta$ : We have

$$a_1^* = \frac{m_1 - \kappa m_2}{2(1 - \kappa^2)}; \quad a_2^* = \frac{m_2 - \kappa m_1}{2(1 - \kappa^2)},$$

and

$$m_1 = \frac{(2rv_2 + 1)\beta_1 - \kappa\beta_2}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1}; \quad m_2 = \frac{(2rv_1 + 1)\beta_2 - \kappa\beta_1}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1}.$$

Thus:

$$\begin{aligned} \frac{\partial a_1^*}{\partial \beta_1} &= \frac{1}{2(1 - \kappa^2)} \frac{1}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1} [(2rv_2 + 1) + \kappa^2] > 0, \\ \frac{\partial a_1^*}{\partial \beta_2} &= \frac{1}{2(1 - \kappa^2)} \frac{1}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1} [-\kappa [1 + (2rv_1 + 1)]] < 0, \\ \frac{\partial a_2^*}{\partial \beta_1} &= \frac{1}{2(1 - \kappa^2)} \frac{1}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1} [-\kappa [(2rv_2 + 1) + 1]] < 0, \\ \frac{\partial a_2^*}{\partial \beta_2} &= \frac{1}{2(1 - \kappa^2)} \frac{1}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1} [\kappa^2 + (2rv_1 + 1)] > 0. \end{aligned}$$

Looking at the effect on the constraint on cost-saving incentives,

$$\frac{da_2^*}{db} = \frac{\partial a_2^*}{\partial \beta_1} + \frac{\partial a_2^*}{\partial \beta_2} \frac{\partial \beta_2}{\partial \beta_1}$$

where, obviously,  $\partial \beta_2 / \partial \beta_1 = \partial \beta_2 / \partial b$  from (A.5) above, and therefore, factoring out the positive constant, we have

$$\frac{da_2^*}{db} = \text{const.} \cdot \left( -\kappa [(2rv_2 + 1) + 1] + [\kappa^2 + (2rv_1 + 1)] \frac{\partial \beta_2}{\partial \beta_1} \right). \quad (\text{A.6})$$

The main conclusion coming from this is that the first term is leading for large enough  $v_2$ , i.e. when the quality dimension is indeed hard to measure. We also have analogously,

$$\frac{da_1^*}{db} = \text{const.} \cdot \left( [(2rv_2 + 1) + \kappa^2] + [-\kappa [1 + (2rv_1 + 1)]] \frac{\partial \beta_2}{\partial \beta_1} \right), \quad (\text{A.7})$$

and we find that here, too, the first term is leading for large enough  $v_2$ , implying – expectedly – that cost-saving incentives are attenuated.

*Comparative statics with respect to  $q$ .*

The principal receives a different quality signal,  $z_2 = a_2 + \varepsilon_2^z + \alpha d_2$ , and the principal's problem is then

$$\begin{aligned} \max_{\beta} E[x_1 + px_2 - (\beta_0 + \beta_1 x_1 + \beta_2 z_2)] &= E[(1 - \beta_1)x_1 + px_2 - \beta_2 z_2 - \beta_0] \\ \text{s.t. } E(\beta_0 + (\beta_1 - m_1)x_1 + \beta_2 z_2 - m_2 x_2 - F) - \frac{d_2^2}{2} &\geq \bar{u}_M \end{aligned}$$

and  $m, e$  maximizes  $E(\beta_0 + (\beta_1 - m_1)x_1 + \beta_2 z_2 - m_2 x_2 - F - \frac{d_2^2}{2})$  s.t. constraints

which can be reduced to

$$\begin{aligned} & \max_{\beta} (1 - \beta_1)a_1 + (p - \beta_2)a_2 - \beta_2qd_2 - \beta_0 \\ \text{s.t. } & \beta_0 + (\beta_1 - m_1)a_1 + (\beta_2 - m_2)a_2 + \beta_2qd_2 - \frac{d_2^2}{2} - F(m(\beta)) \geq \bar{u}_M. \end{aligned}$$

The management's problem is now

$$\begin{aligned} & \max_{m_1, m_2, d_2} \beta_0 + (\beta_1 - m_1)a_1 + (\beta_2 - m_2)a_2 + \beta_2qd_2 - \frac{d_2^2}{2} - F \\ \text{s.t. } & -\exp\left\{-r\left[F + m_1(a_1 + \varepsilon_1) + m_2(a_2 + \varepsilon_2) - (a_1^2 + 2\kappa a_1 a_2 + a_2^2)\right]\right\} \geq u_0 \\ & \text{and } a \text{ maximizes } -\exp\left\{-r\left[F + m_1(a_1 + \varepsilon_1) + m_2(a_2 + \varepsilon_2) - (a_1^2 + 2\kappa a_1 a_2 + a_2^2)\right]\right\}. \end{aligned}$$

It is readily seen that  $M$ 's optimal choice of  $d_2$  is  $d_2 = \beta_2q$ , and that this is independent of other objects; thus, the total impact on  $M$ 's objective function can be summarized by adding the net effect,  $\beta_2qd_2 - d_2^2/2 = (\beta_2q)^2/2$ . Thus, the principal's problem is (noting that  $(m, F)$  depends on  $\beta$ )

$$\begin{aligned} & \max_{\beta} (1 - \beta_1)a_1 + (p - \beta_2)a_2 - q^2\beta_2^2 - \beta_0 \\ \text{s.t. } & \beta_0 + (\beta_1 - m_1(\beta))a_1 + (\beta_2 - m_2(\beta))a_2 + \frac{q^2\beta_2^2}{2} - F(m(\beta)) \geq \bar{u}_M. \end{aligned}$$

The objective function is, after substituting the constraint

$$\phi = (1 - \beta_1)a_1 + (p - \beta_2)a_2 - q^2\beta_2^2 + (\beta_1 - m_1(\beta))a_1 + (\beta_2 - m_2(\beta))a_2 + \frac{q^2\beta_2^2}{2} - F - \bar{u}_M$$

where  $F(\beta)$  is the same as in (A.1). This gives,

$$\phi = (1 - m_1(\beta))a_1 + (p - m_2(\beta))a_2 - \frac{q^2\beta_2^2}{2} - \bar{u}_M - F(\beta),$$

substituting for the  $a$ 's,

$$\phi = (1 - m_1)\frac{m_1 - \kappa m_2}{2(1 - \kappa^2)} + (p - m_2)\frac{m_2 - \kappa m_1}{2(1 - \kappa^2)} - \frac{\alpha^2\beta_2^2}{2} - \bar{u}_M - F(\beta).$$

The first-order conditions w.r.t.  $(m_1, m_2)$  are the same as in the case with  $q = 0$ ,

$$\frac{\partial\phi}{\partial m_1} = 1 - p\kappa - (2r(1 - \kappa^2)v_1 + 1)m_1 + \kappa m_2; \quad \frac{\partial\phi}{\partial m_2} = p - \kappa + \kappa m_1 - (2r(1 - \kappa^2)v_2 + 1)m_2,$$

and this should be inserted into

$$\begin{aligned} \frac{\partial\phi}{\partial\beta_1} &= \frac{\partial\phi}{\partial m_1} \frac{\partial m_1}{\partial\beta_1} + \frac{\partial\phi}{\partial m_2} \frac{\partial m_2}{\partial\beta_1} = 0, \\ \frac{\partial\phi}{\partial\beta_2} &= \frac{\partial\phi}{\partial m_1} \frac{\partial m_1}{\partial\beta_2} + \frac{\partial\phi}{\partial m_2} \frac{\partial m_2}{\partial\beta_2} - q^2\beta_2 = 0, \end{aligned}$$

and we are thus interested in how  $\beta_1$  and  $\beta_2$  depend on  $q$ . The modified Hessian is straightforwardly,

$$H = \frac{1}{2c(1 - \kappa^2)N} \begin{pmatrix} -(\kappa^2 2rcv_1 + 2rcv_2 + 1) & \kappa(2rcv_1 + 2rcv_2 + 1) \\ \kappa(2rcv_1 + 2rcv_2 + 1) & -(2rcv_1 + \kappa^2 2rcv_2 + 1) - q^2 \end{pmatrix},$$

and the inverse is

$$H^{-1} = \frac{2(1 - \kappa^2)N}{\tilde{D} + q^2(\kappa^2 2rcv_1 + 2rcv_2 + 1)} \begin{pmatrix} -(2rv_1 + \kappa^2 2rv_2 + 1) - q^2 & -\kappa(2rv_1 + 2rv_2 + 1) \\ -\kappa(2rv_1 + 2rv_2 + 1) & -(\kappa^2 2rv_1 + 2rv_2 + 1) \end{pmatrix}.$$

The comparative-statics equation is

$$H \begin{pmatrix} \frac{\partial \beta_1}{\partial q^2} \\ \frac{\partial \beta_2}{\partial q^2} \end{pmatrix} = \begin{pmatrix} 0 \\ \beta_2 \end{pmatrix}.$$

Inverting and inserting, we have

$$\begin{pmatrix} \frac{\partial \beta_1}{\partial q^2} \\ \frac{\partial \beta_2}{\partial q^2} \end{pmatrix} = \frac{2(1 - \kappa^2)N}{\tilde{D} + q^2(\kappa^2 2rcv_1 + 2rcv_2 + 1)} \begin{pmatrix} -\beta_2 \{\kappa(2rv_1 + 2rv_2 + 1)\} \\ -\beta_2 \{\kappa^2 2rv_1 + 2rv_2 + 1\} \end{pmatrix}$$

where  $\tilde{D} > 0$ . The key observations to make are that – as long as  $\beta_2 > 0$  which is always true in cases of interest – the sign of  $\partial \beta_1 / \partial q^2$  (and hence, obviously,  $\partial \beta_1 / \partial q$ ) is opposite to the sign of  $\kappa$ , and that  $\partial \beta_2 / \partial q^2$  is unambiguously negative.

*Comparative statics with respect to  $p$ .*

The analysis follows similar lines as those underlying the comparative statics with respect to  $q$ , and therefore the derivation is omitted. The final expressions are

$$\begin{pmatrix} \frac{\partial \beta_1}{\partial p} \\ \frac{\partial \beta_2}{\partial p} \end{pmatrix} = \frac{1}{\tilde{D} + q^2(\kappa^2 2rcv_1 + 2rcv_2 + 1)} \begin{pmatrix} -q^2 \kappa(2rcv_1 + 2rcv_2 + 1) \\ \tilde{D} \end{pmatrix}.$$

It is worth noting that for  $q = 0$  there is no effect on  $\beta_1$ , while  $\partial \beta_2 / \partial p = 1$ .

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