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Empirical Probability Distributions of Real Return from Swedish Stock and Bond Portfolios

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Abstract

This paper introduces a new non-parametric approach to integrate empirical probability functions of the real return for different investment horizons for five portfolios of Swedish stocks and bonds. In our setting the problem reduces to generating new generalizations from a known empirical Markov chain. We find that the stocks yield a real return of about 7.5% and bonds about 3.0%. Our results suggest that an investor ought to avoid bonds in the long run. Finally if the investors goal is to minimize the risk of capital destruction the preferable long-run passive portfolio is a mix of bonds and stocks.

1 Introduction

The most important issue when investing is the concern of what real value growth one is likely to receive. Or as important, what are the odds the investment falls in real wealth when investing? Today almost eight out of ten Swedes invest in the stock market.[1] The number of shareholders and investments in mutual funds have increased dramatically during the last decade. In 2000 the Swedish government floated a new public pension system, *the premium pension*, in which the individual decides how 10 percent of her retirement funds should be invested in a selection of about 450 mutual funds.[2] Altogether this underlines the importance of what real return an investor can expect to receive when investing in Swedish stocks and bonds.

The purpose of this paper is to study the empirical distributions of annualized real return from Swedish stock and bond portfolios for different investment horizons. Jones and Wilson (1999) studied the annualized real returns from different portfolios of US stocks and bonds for different horizons by fitting theoretical lognormal distributions. In their approach they make the assumption that returns actually are lognormal distributed and discard the fact that empirical research have found that the real world is more complex as financial assets often exhibit fat tail distributions and skewness (see Sweden: Frennberg and Hansson (1993) and US: Ibbotson and Singuefield (1976)).

The issue in this paper is to find the distribution of the annualized real return, $F(x)$, for different investment horizons. However the distribution $F(x)$ is generally unknown. In this paper we introduce a new Markovian moving block bootstrap methodology that enables us to replace the unknown distribution $F(x)$ by its empirical distribution $F_n(x)$. This approach have several advantages as it enables us to capture possible fat tails and skewness of the returns. Our contribution is that we introduce a kernel that governs the transition probabilities in the Markov chain. The idea being similar to the recent research by Carlstein, Do, Hall, Hesterberg and Kunsch (1998) and Paparoditis and Politis (2001a, 2001b). In the empirical part of the paper states, or blocks of return, with similar variance have a higher resampling probability. This is analogous to the well-known time varying volatility often found in financial time series (for Sweden see Hansson and Hördahl (1997)).

The economic contribution is that we analyze the empirical probability distributions

[1] According to Temo-Aktiespararnas förening January 21th 2002. see Dagens Industri.

[2] The Swedish public pension funds have today an inflow of about 18.5 percent of the individuals annual taxable income. 16 percent is managed by the authorities in income retirement funds, *allmäna pensionsfonderna*. The remaining 2.5 percent are invested according to the individuals choice of a selection of about 450 mutual funds. See also `www.PPM.nu`.

of the annualized real return for different investment horizons for five mixed portfolios of Swedish stocks and bonds. Hence we are able to find the probability of actually receiving a specific return or more. Further, under the assumption that an investor prefers more wealth to less wealth we can rank our portfolios according to first-degree stochastic dominance.

We find that the most likely real return from stocks is around 7.4 percent to 8.2 percent and the real bond returns varies between 2.5 percent and 3.0 percent. Further, the best solution to avoid capital erosion for a passive long-term investor is to diversify into mixed stock-bond portfolios. Notable is that for a long-run investment horizon the bond portfolio has about 10 percent probability of falling in real value. This is significantly higher compared to the other portfolios in the study.

The outline of the paper is as follows. Section 2 specifies the data. The computation of returns are presented in section 3. The methodology are thoroughly described in section 4 along with the Markovian bootstrap framework. Risk and stochastic dominance are discussed in section 5. The results and the empirical evidence are presented in section 6 and section 7 concludes the paper. The resampling methodology is presented in the appendix.

2 Description of Data

The data consists of monthly consumer price index, CPI, and nominal price series of Swedish bonds and the Swedish stock market portfolio, including dividends, P_t. All data are from the Frennberg and Hansson database (see Frennberg and Hansson (1992)). Our sample covers the period January 1919 – December 1999. A total of 80 years of monthly observations. Some descriptive statistics of the monthly real return is presented in Table 1. The null hypothesis of normal distribution is rejected and the explanation is the very high kurtosis in the data. This also verifies that fitting a normal distribution to the data is not the correct approach.

Table 1
Descriptive statistics of monthly real return from Swedish stocks and bonds 1920-1999.

Asset	Mean	Median	Standard-deviation	Skewness	Kurtosis	Doornik-Hansen test	P-value of DH-test
Bonds	0.28	0.12	2.20	0.45	13.40	875.82	0.00
Stocks	0.72	0.73	4.71	0.02	6.89	283.77	0.00

3 Returns

The monthly real return of the original data, $\mathbf{x}$, is computed as:

$$x_t = \frac{P_t}{P_{t-1}} - \frac{CPI_t}{CPI_{t-1}} \tag{1}$$

From the monthly bond and stock returns we construct the following five portfolios; all equity, sixty percent equity forty percent bonds, fifty-fifty equities and bonds, forty percent equity sixty percent bonds and all bond. We will compute non-overlapping annualized real returns, as overlapping returns will exhibit a strong autocorrelation with increasing investment horizon. This will produce $n = \lfloor \frac{T}{q} \rfloor$ numbers of non-overlapping returns for investment horizon q. The drawback is obvious as q, the investment horizon in months increases, the number of observed returns, n, decreases, and for long investment horizons we will have too few returns in order to make statistical inference. A solution is to resample the portfolios on the original data, $\mathbf{x}$, in order to generate new vectors of resampled return $\mathbf{r}^*$, and from these construct new asset price paths, P^*, of the portfolios as:

$$P_t^* = P_{t-1}^* \left(1 + r_t\right) = \prod_{i=1}^{t} \left(1 + r_t^*\right) \tag{2}$$

Now we can compute a new set of non-overlapping annualized q-month returns for each new asset price path, P^*, as:

$$r_{mq}^* = \left(\left(\frac{P_{mq}^* - P_{mq-q}^*}{P_{mq-q}^*} + 1 \right)^{\frac{12}{q}} - 1 \right) \cdot 100 \quad \text{for} \quad m = 1, ..., n \tag{3}$$

This results in $N \times n$ number of returns for investment horizon q and the returns will converge to the empirical distribution when repeated N number of times and as $N \to \infty$. The real returns of the portfolios are computed for eight different investment horizons, q, of one, two, three, five, ten, fifteen, twenty, and twenty-five years.

4 Markov Chain Monte Carlo Methodology

Resampling time series with serial dependence raises the question of how to keep these properties, as ordinary wild bootstrap would destroy this dependence.[3] One solution is

[3] An excellent description of resampling techniques can be found in Hjort (1994) "Computer Intensive Statistical Methods"; Shao Jun and Dongsheng Tu (1995) "The Jackknife and Bootstrap ," and Davidson A.C. and D.V. Hinkley (1998) "Bootstrap Methods and Their Application".

to bootstrap blocks of the original data as first suggested by Carlstein (1986) or utilize a moving block bootstrap, see Kunsch (1989), and resample overlapping blocks of data.[4] However, both approaches have later been criticized by Carlstein et al (1996) as the dependence between the generated blocks is ignored. Here we suggest an improvement in line with the ideas presented by Carlstein et al (1996) and Paparoditis and Politis (2001a, 2001b), and construct an empirical Markov chain in which transition probabilities depend on the data.

4.1 States

We define a state S_i as a set, or a block of b number of observations $S_i = \{r_i, ..., r_{i+b}\}$. The total number of states is $k = T - b$, where b is the block length and T is the sample size. This is a first order Markov Chain by construction as a random state, S_i, conditional upon all of the past events only depend on the previous state S_{i-1}.

4.2 Block length

We do not address the question of an optimal block length. However we conclude that one should be cautious of using block lengths exceeding the shortest investment horizon as this introduces a discretization of the non-overlapping returns. Notable is that the problem becomes more severe as the block length increases. A discussion of this statement is given in appendix 2.[5] Therefore we utilize a block length of six months $b = 6$ as this is long enough to preserve some of the serial dependence and yet shorter than the shortest investment horizon.

4.3 Transition kernel

The idea behind the transition kernel is to match states with similar information and rule out the possibilities of moving between the extreme states. Our transition kernel have two advantages. First, the information set $\mathbf{y}$, that governs the transition probability, can easily be modified to account for dependence between the states, such as return or variance. In this kernel, the probability that the next random generated state S_{i+1} is S_j depends on how close the information y_j is to the information in the current generated

[4] See also Davidson and Hinkley (1998) Ch. 8.2.3.

[5] We have also done estimations with block lengths ranging up to 120 months. The results are available upon request.

state y_i. The closer y_j to y_i the higher the resampling probability (see also Carlstein et al (1996) and Paparoditis and Politis (2001a, 2001b)). Second, the strength of our beliefs in the information, $\mathbf{y}$, is determined by a measure raised to the power, z. If we question the non-equal dependence between the states, then z is set to zero and all the transition probabilities of the Markov chain is equal. This special case is the moving block bootstrap introduced by Kunch (1989). Further, our transition kernel is that of an ordinary, wild, bootstrap if we set z to zero and the number of observation in each state to one, $b = 1$.

The transition kernel describes the probability to move from a given state S_i to a given state S_j conditional upon an information criteria $\mathbf{y}$ in the states. v_{ij}, denotes the resampling probability of moving to a given state S_j conditional upon the current state S_i.

$$v_{ij} = \left(1 - \frac{\left|y_{i,b} - y_{j,1}\right|}{\left|y_{\max} - y_{\min}\right|}\right)^z \tag{4}$$

The suffix i, b denotes the last observation of the information set, $\mathbf{y}$, in state i, and $j, 1$ denotes the first observation of the information set in state j. The factor z determines the probability of moving to states with a different value of $\mathbf{y}$. The measure, v_{ij}, can easily be modified to account for time variations in volatility and variance, where $\sum_{l=1}^{b} \frac{r_{il}^2}{b}$ can be employed to proxy variance, σ_i^2, at state, S_i, of block length b. In this case each state is associated with a single value of variance and the subscript denotes the state.

$$v_{ij} = \left(1 - \frac{\left|\sigma_i^2 - \sigma_j^2\right|}{\left|\sigma_{\max}^2 - \sigma_{\min}^2\right|}\right)^z \tag{5}$$

4.4 Transition probabilities

The transition probabilities p_{ij}, i.e. the probability of moving from one state S_i, to another state S_j, is gathered in a $k \times k$ transition matrix $\mathbf{P}$. The transition probabilities in a row vector of the transition matrix $\mathbf{P}$ always sum to unity. Each state is represented by row vector of transition probabilities for the Markov chain to move to another state. Given the fact that each realization of a bootstrap is a state in a Markov chain and v_{ij}, denotes the strength of moving to a given state j conditional upon the previous state i, we can compute the transition probabilities, p_{ij}, in this k-state Markov chain as:

$$p_{ij} = \frac{v_{ij}}{\sum_{j=1}^{k} v_{ij}}, \qquad i, j = 1, ...k \tag{6}$$

$$\sum_{j=1}^{k} p_{ij} = 1 \tag{7}$$

It can easily be seen that this is a Markov chain with no absorbing states as: $p_{ii} < 1$.

4.5 Generation via accept-rejection method

According to Robert et al (1999) an ideal accept/reject density $g(x)$, is a density such that $h(x) \leq Mg(x)$ for all x and for which the ratio $h(x)/Mg(x)$ is relatively constant over the range of x where $h(x)$ has most of its mass, and where M is a scaling factor. We let $g(x)$ be a uniform density, $U(0,1)$, $M = 1$ and $h(x)$ is a density describing the strength, v_{ij}, of moving from a given state, S_i to a given state, S_j. Note that each and every, v_{ij}, is bounded by the closed unit interval.

Let r denote the vector of the originally 1-month return series. Determine the block length b and compute the number of states, i.e. blocks, $n = \lfloor \frac{T}{b} \rfloor$, that will fit in the original return series.

Step 1: Set $N = 1$
Step 2: Set $a = 1$ and the first random state as i.
Step 3: Draw a random number from $U(1,k)$
and pick the corresponding candidate block j of returns.
Compute v_{ij} and draw a random number $c \sim U(0,1)$.
$v_{ij} = \left(1 - \frac{|\sigma_i^2 - \sigma_j^2|}{|\sigma_{\max}^2 - \sigma_{\min}^2|}\right)$, $c \sim U(0,1)$.
accept if; $c < v_{ij}$,
and set $a = a + 1$,
else generate a candidate, j.
Repeat until $a = n$. and create a vector r_N^* of returns.
Step 4: Compute a price series P_N^* from the resampled r_N^*.
Step 5: Compute the non-overlapping returns r_q^* $\forall$ q and set $N = N + 1$;
Step 6: Go to step 2. Repeat until say, N=20.000.

The non-overlapping q-horizon returns will converge to their empirical distributions if this is repeated a large number of times. In our case N is set to 20.000 and we compute $N \times n$ number of returns for each investment horizon q.

5 Risk and First Order Stochastic Dominance

It is reasonable to assume that investors always prefer more wealth to less wealth. Thus investors have non-decreasing utility functions with respect to wealth. A portfolio decision

rule corresponding to the information of non-decreasing utility in wealth is the first degree stochastic dominance, FSD, see Levy (1998). As we are able to compute the numerical integrals of the empirical cumulative density functions we can rank the portfolios according to the FSD.

Let F and G be two cumulative probability densities such that $F(r) \leq G(r)$ for all returns r. It the follows:

$$F(r) \leq G(r) \Longleftrightarrow E_F U(r) \geq E_G U(r) \tag{8}$$

The condition of FSD of F over G is that the difference between $G(r) - F(r) = I(r); \geq 0$ for all r and $G(r_*) - F(r_*) = I(r_*); > 0$ for some r_*. In other words portfolios with more probability mass to the right of origo, i.e. positive returns, dominate portfolios with more probability mass on the negative returns. If an investor expects the investment to yield a certain real return, target return, then risk can be stated as the probability of receiving a return below target, τ. This is referred to as downside risk in the literature (see Fishburn (1977)). We also refer to capital erosion as the probability that the investment yields a negative real return. The probability of receiving at least specified return level τ, $P(r \geq \tau)$ is computed as:

$$P(r \geq \tau) = \frac{\#\{r_i \geq \tau\}}{n} \tag{9}$$

A portfolio F first order stochastically dominates G if and only if:

$$P_F(r \geq \tau) \geq P_G(r \geq \tau) \qquad \forall r \qquad \text{and} \qquad P_F(r_* \geq \tau) > P_G(r_* \geq \tau) \tag{10}$$

This should be read as given that portfolio F and G have the same probability of achieving all the specified target returns. Then F dominates G and is the preferred choice of the investor if there exist at least one target return for which portfolio F have a higher probability than G.

6 Results

6.1 Rejections from the Accept-Reject method

We generate a total of 20.000 monthly return series from which we construct 20.000 asset price paths. From these asset price paths we calculate annualized non-overlapping returns

for each of the eight investment horizons. This procedure is then repeated for each of the five portfolios.

The accept-rejection methodology has been criticized as being computational inefficient as samples are being rejected and does not contribute with information.

The bond portfolio has the highest number of accepted candidates and the stock portfolio the highest number of rejected candidates. The rejections are presented in Table 2. The accept-rejection algorithm rejects between 3.48 percent and 5.40 percent of the proposed blocks. In our case this is not a problem as the numbers of rejections are quite low and tolerable. This suggests that the algorithm is efficient.

Table 2
Rejected blocks in number and percent.

Asset mix	Stocks	60-40	50-50	40-60	Bonds
# of rejections	175078	127100	118281	111353	109254
Rejections in %	5.40	3.92	3.65	3.44	3.42

Note: In our case the number of accepted blocks is always 3.200.000.

6.2 The empirical distributions of the real return

For each of the investment horizons we compute the empirical probability density functions and the empirical cumulative density functions. In fact this is a method of numerical integration that enables us to compute not only the integral of the distribution but also the probability density surface, in other words the probability density as a function of both investment horizon and return.

Figure 2 presents the probability density surface of the real return for the Swedish stock market for investment horizons up to 25 years. It clearly shows how the probability mass is more and more centered, or peaked with increasing investment horizon, see also Table 4 to Table 8.

The empirical PDF's for stocks and bonds are presented in Figure 3 and Figure 4. Stocks have dispersed distributions and bonds more peaked distributions which are slightly skewed to the left. Again we can see how the distributions are more peaked with increasing investment horizon. Notice that stocks, see Figure 3, have more probability mass for the positive returns for the 20-year and 25-year horizons compared to bonds, Figure 4.

Table 3 presents the mean annualized real return along with the 2.5 upper and lower percentiles for the five analyzed portfolios and the eight investment horizons. This presents us with some interesting results. Stocks yield the highest average real return, between

7.40 percent and 8.17 percent, and bonds the lowest, between 2.48 percent and 3.04 percent. Jones and Wilson (1999) find that US stocks yielded an annualized real return of approximately 7.49 % and bonds 2.68 %. We find that the mean annualized real return from Swedish stocks and bonds are roughly the same. Notable in Table 3 is the 2.5 lower percentile for the mixed stock bond portfolios, when compared to the stocks and bonds. For investment horizons exceeding two years the return is higher, less negative, for the mixed portfolios and at 25 year horizon the lower percentile is a positive return whereas both stocks and bonds have a negative return. This is the effect of diversification between stocks and bonds.

Table 3
Mean annualized real return for the investigated portfolios and investment horizons

	Stocks	60/40 Stocks/Bonds	50/50 Stocks/Bonds	40/60 Stocks/Bonds	Bonds
1-Year	8.17 [−27.07, 51.75]	6.25 [−19.35, 36.79]	5.68 [−17.80, 33.95]	5.04 [−16.66, 31.73]	2.48 [−17.15, 31.17]
2-Year	7.79 [−17.61, 37.44]	6.14 [−12.21, 26.68]	5.66 [−11.36, 24.53]	5.16 [−10.90, 22.84]	2.86 [−12.53, 20.92]
3-Year	7.62 [−13.29, 31.29]	6.09 [−9.00, 22.52]	5.66 [−8.36, 20.67]	5.17 [−8.03, 19.24]	2.96 [−9.73, 17.19]
5-Year	7.51 [−8.89, 25.59]	6.04 [−5.70, 18.57]	5.61 [−5.30, 17.11]	5.16 [−5.12, 15.90]	3.03 [−6.79, 13.65]
10-Year	7.44 [−4.28, 20.03]	6.00 [−2.34, 14.76]	5.59 [−2.17, 13.59]	5.15 [−2.12, 12.62]	3.04 [−3.94, 10.41]
15-Year	7.42 [−2.20, 17.70]	5.99 [−0.83, 13.06]	5.59 [−0.77,12.03]	5.14 [−0.78, 11.21]	3.04 [−2.64, 8.99]
20-Year	7.39 [−0.93, 16.23]	5.98 [0.02, 12.13]	5.58 [0.07, 11.19]	5.12 [−0.06, 10.38]	3.04 [−1.88, 8.14]
25-Year	7.40 [−0.12, 15.27]	5.98 [0.68, 11.51]	5.58 [0.68, 10.57]	5.12 [0.50, 9.82]	3.04 [−1.35, 7.55]

Note: 2.5 and 97.5 percentiles within brackets

6.3 Target returns and first order stochastic dominance

Table 4 to Table 8 presents specified target returns along with the probabilities of achieving the specified target returns or more for each of the investment horizons. The probability that the investment yields at least the money back, $P(r \geq 0)$, is of special economic importance as probability mass below this return is associated with capital erosion.

Stocks have the highest probability of rendering a high return for all investment horizons in comparison to the other investigated portfolios. Stocks also stand the highest probability of capital erosion for investment horizons up to 10 years. However for the long investment horizons, more than 15 years, bonds is the riskiest asset in this aspect. The later is a somewhat surprising result. Moreover from Table 4 to Table 8 the diversification effect between stocks and bonds is evident as the mixed 40-60 stock/bond portfolio

dominates the bond portfolio even at short horizons. As the investment horizon increases all stock/bond portfolios dominate the bond portfolio and at long investment horizons also the stock portfolio dominates the bond portfolio. It is interesting to note that at a 25 year investment horizon the bond portfolio just stand a 91.4 percent chance of yielding any annual value growth whereas stocks have a 97.5 percent probability of value growth.

Table 4: Probability of achieving at least Specified Stock Market Return.

Return in %	1-Year	2-Year	3-Year	5-Year	10-Year	15-Year	20-Year	25-Year
≥ 100	0.001	0	0	0	0	0	0	0
≥ 95	0.002	0	0	0	0	0	0	0
≥ 90	0.002	0	0	0	0	0	0	0
≥ 85	0.003	0	0	0	0	0	0	0
≥ 80	0.004	0	0	0	0	0	0	0
≥ 75	0.006	0	0	0	0	0	0	0
≥ 70	0.008	0	0	0	0	0	0	0
≥ 65	0.010	0.001	0	0	0	0	0	0
≥ 60	0.014	0.002	0	0	0	0	0	0
≥ 55	0.020	0.003	0	0	0	0	0	0
≥ 50	0.028	0.005	0.001	0	0	0	0	0
≥ 45	0.041	0.010	0.002	0	0	0	0	0
≥ 40	0.059	0.018	0.006	0.001	0	0	0	0
≥ 35	0.086	0.033	0.014	0.003	0	0	0	0
≥ 30	0.124	0.060	0.032	0.009	0.001	0	0	0
≥ 25	0.180	0.108	0.068	0.029	0.004	0.001	0	0
≥ 20	0.254	0.183	0.137	0.082	0.025	0.008	0.003	0.001
≥ 19	0.271	0.202	0.156	0.099	0.035	0.014	0.006	0.002
≥ 18	0.290	0.223	0.177	0.119	0.048	0.021	0.010	0.004
≥ 17	0.309	0.244	0.200	0.142	0.066	0.032	0.016	0.009
≥ 16	0.328	0.268	0.225	0.167	0.087	0.048	0.028	0.017
≥ 15	0.349	0.293	0.252	0.196	0.114	0.070	0.045	0.029
≥ 14	0.370	0.319	0.281	0.229	0.147	0.099	0.068	0.049
≥ 13	0.391	0.346	0.312	0.264	0.186	0.137	0.103	0.079
≥ 12	0.413	0.374	0.345	0.302	0.231	0.183	0.149	0.120
≥ 11	0.436	0.403	0.379	0.343	0.282	0.240	0.208	0.179
≥ 10	0.458	0.432	0.414	0.386	0.339	0.306	0.279	0.253
≥ 9	0.481	0.463	0.450	0.431	0.400	0.378	0.358	0.341
≥ 8	0.504	0.493	0.486	0.478	0.464	0.455	0.444	0.439
≥ 7	0.527	0.524	0.523	0.524	0.529	0.533	0.536	0.542
≥ 6	0.550	0.555	0.559	0.570	0.594	0.611	0.626	0.641
≥ 5	0.573	0.585	0.596	0.616	0.656	0.685	0.709	0.732
≥ 4	0.596	0.615	0.631	0.660	0.715	0.752	0.784	0.808
≥ 3	0.618	0.644	0.666	0.703	0.768	0.812	0.845	0.870
≥ 2	0.640	0.672	0.699	0.742	0.816	0.861	0.894	0.918
≥ 1	0.661	0.699	0.730	0.779	0.857	0.901	0.931	0.950
≥ 0	**0.682**	**0.726**	**0.760**	**0.813**	**0.891**	**0.933**	**0.957**	**0.972**
≥ -1	0.702	0.751	0.788	0.843	0.919	0.955	0.975	0.986
≥ -2	0.722	0.775	0.814	0.870	0.941	0.971	0.986	0.993
≥ -3	0.741	0.797	0.838	0.894	0.958	0.983	0.993	0.997
≥ -4	0.759	0.818	0.860	0.914	0.971	0.990	0.996	0.999
≥ -5	0.776	0.837	0.880	0.931	0.980	0.994	0.998	0.999
≥ -6	0.793	0.855	0.898	0.946	0.987	0.997	0.999	1
≥ -7	0.809	0.872	0.914	0.958	0.992	0.998	1	1
≥ -8	0.824	0.887	0.928	0.968	0.995	0.999	1	1
≥ -9	0.838	0.901	0.940	0.975	0.997	1	1	1
≥ -10	0.851	0.914	0.950	0.981	0.998	1	1	1
≥ -11	0.863	0.925	0.959	0.986	0.999	1	1	1
≥ -12	0.875	0.936	0.967	0.990	0.999	1	1	1
≥ -13	0.886	0.945	0.973	0.993	1	1	1	1
≥ -14	0.896	0.953	0.978	0.995	1	1	1	1
≥ -15	0.905	0.960	0.983	0.996	1	1	1	1
≥ -16	0.914	0.967	0.987	0.998	1	1	1	1
≥ -17	0.922	0.972	0.990	0.998	1	1	1	1
≥ -18	0.929	0.977	0.992	0.999	1	1	1	1
≥ -19	0.936	0.981	0.994	0.999	1	1	1	1
≥ -20	0.942	0.984	0.995	1	1	1	1	1
≥ -25	0.967	0.995	0.999	1	1	1	1	1
≥ -30	0.983	0.999	1	1	1	1	1	1
≥ -35	0.993	1	1	1	1	1	1	1
≥ -40	0.997	1	1	1	1	1	1	1
≥ -45	0.999	1	1	1	1	1	1	1
≥ -50	1	1	1	1	1	1	1	1

Table 5: Probability of achieving at least specified five-year bond returns.

Return in %	1-Year	2-Year	3-Year	5-Year	10-Year	15-Year	20-Year	25-Year
≥ 100	0	0	0	0	0	0	0	0
≥ 95	0	0	0	0	0	0	0	0
≥ 90	0	0	0	0	0	0	0	0
≥ 85	0	0	0	0	0	0	0	0
≥ 80	0	0	0	0	0	0	0	0
≥ 75	0	0	0	0	0	0	0	0
≥ 70	0	0	0	0	0	0	0	0
≥ 65	0	0	0	0	0	0	0	0
≥ 60	0.001	0	0	0	0	0	0	0
≥ 55	0.001	0	0	0	0	0	0	0
≥ 50	0.002	0	0	0	0	0	0	0
≥ 45	0.005	0	0	0	0	0	0	0
≥ 40	0.009	0	0	0	0	0	0	0
≥ 35	0.016	0.001	0	0	0	0	0	0
≥ 30	0.028	0.004	0.001	0	0	0	0	0
≥ 25	0.044	0.011	0.003	0	0	0	0	0
≥ 20	0.073	0.030	0.012	0.002	0	0	0	0
≥ 19	0.082	0.036	0.015	0.003	0	0	0	0
≥ 18	0.091	0.044	0.020	0.005	0	0	0	0
≥ 17	0.101	0.053	0.026	0.007	0	0	0	0
≥ 16	0.112	0.063	0.034	0.010	0	0	0	0
≥ 15	0.126	0.075	0.043	0.015	0.001	0	0	0
≥ 14	0.142	0.089	0.055	0.021	0.002	0	0	0
≥ 13	0.159	0.106	0.070	0.031	0.005	0.001	0	0
≥ 12	0.179	0.126	0.088	0.045	0.009	0.002	0	0
≥ 11	0.201	0.149	0.111	0.063	0.017	0.005	0.002	0.001
≥ 10	0.226	0.176	0.139	0.088	0.031	0.011	0.004	0.002
≥ 9	0.254	0.207	0.172	0.119	0.053	0.024	0.011	0.006
≥ 8	0.284	0.242	0.211	0.160	0.085	0.048	0.027	0.016
≥ 7	0.317	0.283	0.256	0.209	0.134	0.089	0.060	0.042
≥ 6	0.352	0.329	0.308	0.270	0.202	0.155	0.122	0.096
≥ 5	0.391	0.379	0.366	0.340	0.288	0.248	0.220	0.192
≥ 4	0.432	0.434	0.430	0.419	0.392	0.369	0.351	0.334
≥ 3	0.475	0.492	0.497	0.502	0.505	0.505	0.506	0.506
≥ 2	0.523	0.552	0.566	0.588	0.619	0.641	0.663	0.680
≥ 1	0.571	0.612	0.636	0.670	0.724	0.765	0.795	0.819
≥ 0	**0.621**	**0.670**	**0.701**	**0.743**	**0.811**	**0.859**	**0.889**	**0.912**
≥ -1	0.669	0.725	0.759	0.807	0.879	0.922	0.947	0.964
≥ -2	0.716	0.775	0.811	0.858	0.927	0.960	0.977	0.988
≥ -3	0.758	0.818	0.853	0.898	0.957	0.981	0.991	0.996
≥ -4	0.796	0.854	0.887	0.928	0.977	0.992	0.997	0.999
≥ -5	0.829	0.884	0.914	0.950	0.988	0.997	0.999	1
≥ -6	0.857	0.907	0.934	0.966	0.994	0.999	1	1
≥ -7	0.881	0.926	0.949	0.977	0.997	1	1	1
≥ -8	0.901	0.940	0.960	0.984	0.999	1	1	1
≥ -9	0.917	0.951	0.969	0.990	0.999	1	1	1
≥ -10	0.930	0.960	0.976	0.994	1	1	1	1
≥ -11	0.941	0.967	0.982	0.996	1	1	1	1
≥ -12	0.950	0.972	0.987	0.998	1	1	1	1
≥ -13	0.957	0.977	0.990	0.999	1	1	1	1
≥ -14	0.962	0.980	0.993	0.999	1	1	1	1
≥ -15	0.967	0.984	0.995	0.999	1	1	1	1
≥ -16	0.971	0.987	0.997	1	1	1	1	1
≥ -17	0.975	0.990	0.998	1	1	1	1	1
≥ -18	0.977	0.992	0.998	1	1	1	1	1
≥ -19	0.979	0.994	0.999	1	1	1	1	1
≥ -20	0.981	0.996	0.999	1	1	1	1	1
≥ -25	0.988	0.999	1	1	1	1	1	1
≥ -30	0.993	1	1	1	1	1	1	1
≥ -35	0.998	1	1	1	1	1	1	1
≥ -40	1	1	1	1	1	1	1	1
≥ -45	1	1	1	1	1	1	1	1
≥ -50	1	1	1	1	1	1	1	1

Table 6: Probability of achieving at least specified portfolio return, 60-40 percent.

Return in %	1-Year	2-Year	3-Year	5-Year	10-Year	15-Year	20-Year	25-Year
≥ 100	0	0	0	0	0	0	0	0
≥ 95	0	0	0	0	0	0	0	0
≥ 90	0	0	0	0	0	0	0	0
≥ 85	0	0	0	0	0	0	0	0
≥ 80	0	0	0	0	0	0	0	0
≥ 75	0	0	0	0	0	0	0	0
≥ 70	0.001	0	0	0	0	0	0	0
≥ 65	0.001	0	0	0	0	0	0	0
≥ 60	0.002	0	0	0	0	0	0	0
≥ 55	0.003	0	0	0	0	0	0	0
≥ 50	0.006	0	0	0	0	0	0	0
≥ 45	0.011	0.001	0	0	0	0	0	0
≥ 40	0.018	0.002	0	0	0	0	0	0
≥ 35	0.030	0.005	0.001	0	0	0	0	0
≥ 30	0.051	0.013	0.004	0	0	0	0	0
≥ 25	0.085	0.034	0.013	0.002	0	0	0	0
≥ 20	0.146	0.079	0.045	0.015	0.001	0	0	0
≥ 19	0.162	0.093	0.056	0.021	0.002	0	0	0
≥ 18	0.179	0.110	0.070	0.030	0.004	0.001	0	0
≥ 17	0.198	0.129	0.087	0.041	0.007	0.001	0	0
≥ 16	0.219	0.150	0.106	0.056	0.013	0.003	0.001	0
≥ 15	0.242	0.174	0.130	0.076	0.022	0.007	0.002	0.001
≥ 14	0.266	0.201	0.157	0.100	0.036	0.014	0.005	0.002
≥ 13	0.292	0.231	0.187	0.130	0.057	0.027	0.012	0.006
≥ 12	0.319	0.263	0.222	0.165	0.086	0.047	0.027	0.015
≥ 11	0.348	0.298	0.262	0.208	0.126	0.081	0.053	0.035
≥ 10	0.378	0.336	0.304	0.257	0.179	0.131	0.098	0.074
≥ 9	0.409	0.376	0.351	0.312	0.245	0.198	0.166	0.137
≥ 8	0.442	0.419	0.401	0.372	0.323	0.284	0.255	0.232
≥ 7	0.475	0.462	0.452	0.436	0.409	0.387	0.369	0.356
≥ 6	0.509	0.506	0.504	0.503	0.500	0.499	0.498	0.497
≥ 5	0.542	0.551	0.557	0.569	0.592	0.613	0.627	0.641
≥ 4	0.576	0.595	0.609	0.634	0.680	0.715	0.743	0.765
≥ 3	0.609	0.637	0.659	0.696	0.759	0.803	0.835	0.862
≥ 2	0.641	0.678	0.707	0.752	0.826	0.873	0.905	0.927
≥ 1	0.673	0.717	0.751	0.802	0.879	0.924	0.950	0.966
≥ 0	**0.702**	**0.753**	**0.791**	**0.845**	**0.920**	**0.956**	**0.975**	**0.986**
≥ -1	0.731	0.787	0.826	0.882	0.950	0.977	0.989	0.996
≥ -2	0.758	0.817	0.858	0.911	0.970	0.989	0.996	0.998
≥ -3	0.782	0.844	0.886	0.935	0.982	0.995	0.999	1
≥ -4	0.806	0.868	0.908	0.954	0.990	0.998	1	1
≥ -5	0.828	0.890	0.928	0.967	0.995	0.999	1	1
≥ -6	0.847	0.908	0.944	0.977	0.998	1	1	1
≥ -7	0.864	0.924	0.957	0.985	0.999	1	1	1
≥ -8	0.880	0.938	0.967	0.990	0.999	1	1	1
≥ -9	0.894	0.949	0.975	0.994	1	1	1	1
≥ -10	0.907	0.959	0.982	0.996	1	1	1	1
≥ -11	0.919	0.967	0.987	0.998	1	1	1	1
≥ -12	0.929	0.974	0.990	0.999	1	1	1	1
≥ -13	0.938	0.979	0.993	0.999	1	1	1	1
≥ -14	0.945	0.984	0.995	0.999	1	1	1	1
≥ -15	0.952	0.987	0.997	1	1	1	1	1
≥ -16	0.959	0.990	0.998	1	1	1	1	1
≥ -17	0.964	0.993	0.998	1	1	1	1	1
≥ -18	0.969	0.994	0.999	1	1	1	1	1
≥ -19	0.974	0.996	0.999	1	1	1	1	1
≥ -20	0.977	0.997	1	1	1	1	1	1
≥ -25	0.990	0.999	1	1	1	1	1	1
≥ -30	0.996	1	1	1	1	1	1	1
≥ -35	0.999	1	1	1	1	1	1	1
≥ -40	1	1	1	1	1	1	1	1
≥ -45	1	1	1	1	1	1	1	1
≥ -50	1	1	1	1	1	1	1	1

Table 7: Probability of achieving at least specified portfolio return, 50-50 percent.

Return in %	1-Year	2-Year	3-Year	5-Year	10-Year	15-Year	20-Year	25-Year
≥ 100	0	0	0	0	0	0	0	0
≥ 95	0	0	0	0	0	0	0	0
≥ 90	0	0	0	0	0	0	0	0
≥ 85	0	0	0	0	0	0	0	0
≥ 80	0	0	0	0	0	0	0	0
≥ 75	0	0	0	0	0	0	0	0
≥ 70	0	0	0	0	0	0	0	0
≥ 65	0	0	0	0	0	0	0	0
≥ 60	0.001	0	0	0	0	0	0	0
≥ 55	0.002	0	0	0	0	0	0	0
≥ 50	0.003	0	0	0	0	0	0	0
≥ 45	0.006	0	0	0	0	0	0	0
≥ 40	0.012	0.001	0	0	0	0	0	0
≥ 35	0.022	0.003	0	0	0	0	0	0
≥ 30	0.039	0.008	0.002	0	0	0	0	0
≥ 25	0.069	0.023	0.008	0.001	0	0	0	0
≥ 20	0.123	0.060	0.031	0.008	0	0	0	0
≥ 19	0.138	0.072	0.040	0.012	0.001	0	0	0
≥ 18	0.154	0.086	0.050	0.018	0.002	0	0	0
≥ 17	0.172	0.103	0.064	0.026	0.003	0	0	0
≥ 16	0.192	0.123	0.081	0.037	0.006	0.001	0	0
≥ 15	0.213	0.145	0.101	0.053	0.012	0.003	0.001	0
≥ 14	0.237	0.170	0.126	0.073	0.021	0.006	0.002	0.001
≥ 13	0.262	0.198	0.155	0.098	0.035	0.014	0.005	0.002
≥ 12	0.289	0.230	0.188	0.130	0.057	0.027	0.013	0.006
≥ 11	0.319	0.265	0.226	0.169	0.090	0.051	0.030	0.017
≥ 10	0.350	0.304	0.270	0.217	0.136	0.090	0.061	0.042
≥ 9	0.382	0.346	0.317	0.271	0.197	0.149	0.114	0.089
≥ 8	0.416	0.390	0.369	0.334	0.272	0.229	0.195	0.168
≥ 7	0.452	0.436	0.424	0.401	0.360	0.331	0.306	0.288
≥ 6	0.488	0.484	0.480	0.471	0.459	0.448	0.441	0.435
≥ 5	0.525	0.532	0.538	0.545	0.560	0.573	0.581	0.591
≥ 4	0.562	0.581	0.595	0.616	0.658	0.688	0.712	0.735
≥ 3	0.598	0.628	0.650	0.685	0.745	0.789	0.820	0.847
≥ 2	0.634	0.674	0.702	0.747	0.820	0.866	0.898	0.923
≥ 1	0.669	0.717	0.750	0.802	0.879	0.922	0.948	0.966
≥ 0	**0.703**	**0.757**	**0.794**	**0.848**	**0.922**	**0.957**	**0.977**	**0.987**
≥ -1	0.735	0.793	0.832	0.887	0.952	0.979	0.990	0.995
≥ -2	0.764	0.826	0.865	0.917	0.972	0.990	0.997	0.999
≥ -3	0.792	0.854	0.893	0.941	0.985	0.996	0.999	1
≥ -4	0.817	0.879	0.917	0.959	0.992	0.998	1	1
≥ -5	0.840	0.900	0.936	0.972	0.996	0.999	1	1
≥ -6	0.860	0.919	0.951	0.981	0.998	1	1	1
≥ -7	0.878	0.934	0.963	0.988	0.999	1	1	1
≥ -8	0.894	0.947	0.972	0.992	1	1	1	1
≥ -9	0.908	0.957	0.979	0.995	1	1	1	1
≥ -10	0.920	0.966	0.985	0.997	1	1	1	1
≥ -11	0.931	0.973	0.989	0.998	1	1	1	1
≥ -12	0.940	0.979	0.992	0.999	1	1	1	1
≥ -13	0.948	0.983	0.995	0.999	1	1	1	1
≥ -14	0.955	0.987	0.996	1	1	1	1	1
≥ -15	0.961	0.990	0.997	1	1	1	1	1
≥ -16	0.967	0.992	0.998	1	1	1	1	1
≥ -17	0.972	0.994	0.999	1	1	1	1	1
≥ -18	0.976	0.996	0.999	1	1	1	1	1
≥ -19	0.979	0.997	1	1	1	1	1	1
≥ -20	0.982	0.998	1	1	1	1	1	1
≥ -25	0.992	1	1	1	1	1	1	1
≥ -30	0.996	1	1	1	1	1	1	1
≥ -35	0.999	1	1	1	1	1	1	1
≥ -40	1	1	1	1	1	1	1	1
≥ -45	1	1	1	1	1	1	1	1
≥ -50	1	1	1	1	1	1	1	1

Table 8: Probability of achieving at least specified portfolio return, 40-60 percent.

Return in %	1-Year	2-Year	3-Year	5-Year	10-Year	15-Year	20-Year	25-Year
≥ 100	0	0	0	0	0	0	0	0
≥ 95	0	0	0	0	0	0	0	0
≥ 90	0	0	0	0	0	0	0	0
≥ 85	0	0	0	0	0	0	0	0
≥ 80	0	0	0	0	0	0	0	0
≥ 75	0	0	0	0	0	0	0	0
≥ 70	0	0	0	0	0	0	0	0
≥ 65	0	0	0	0	0	0	0	0
≥ 60	0	0	0	0	0	0	0	0
≥ 55	0.001	0	0	0	0	0	0	0
≥ 50	0.002	0	0	0	0	0	0	0
≥ 45	0.004	0	0	0	0	0	0	0
≥ 40	0.008	0	0	0	0	0	0	0
≥ 35	0.016	0.001	0	0	0	0	0	0
≥ 30	0.031	0.005	0.001	0	0	0	0	0
≥ 25	0.058	0.015	0.004	0	0	0	0	0
≥ 20	0.105	0.045	0.020	0.004	0	0	0	0
≥ 19	0.118	0.055	0.027	0.007	0	0	0	0
≥ 18	0.132	0.068	0.035	0.011	0.001	0	0	0
≥ 17	0.148	0.082	0.046	0.016	0.001	0	0	0
≥ 16	0.166	0.099	0.060	0.024	0.003	0	0	0
≥ 15	0.186	0.119	0.078	0.035	0.006	0.001	0	0
≥ 14	0.208	0.142	0.099	0.051	0.011	0.002	0.001	0
≥ 13	0.232	0.168	0.125	0.071	0.020	0.006	0.002	0.001
≥ 12	0.258	0.198	0.155	0.098	0.036	0.014	0.005	0.002
≥ 11	0.287	0.233	0.191	0.134	0.061	0.029	0.015	0.007
≥ 10	0.318	0.271	0.233	0.177	0.098	0.056	0.034	0.020
≥ 9	0.351	0.312	0.280	0.230	0.151	0.102	0.072	0.050
≥ 8	0.386	0.357	0.332	0.291	0.221	0.174	0.139	0.111
≥ 7	0.423	0.406	0.389	0.360	0.308	0.270	0.239	0.211
≥ 6	0.462	0.456	0.448	0.434	0.408	0.387	0.369	0.355
≥ 5	0.502	0.509	0.511	0.513	0.516	0.519	0.518	0.521
≥ 4	0.542	0.561	0.573	0.590	0.622	0.647	0.666	0.683
≥ 3	0.583	0.613	0.635	0.666	0.720	0.761	0.791	0.817
≥ 2	0.623	0.664	0.692	0.734	0.803	0.850	0.883	0.907
≥ 1	0.663	0.711	0.746	0.795	0.869	0.913	0.942	0.959
≥ 0	**0.700**	**0.755**	**0.793**	**0.846**	**0.917**	**0.954**	**0.973**	**0.984**
≥ -1	0.735	0.795	0.834	0.886	0.951	0.978	0.989	0.995
≥ -2	0.768	0.830	0.870	0.919	0.972	0.990	0.996	0.998
≥ -3	0.799	0.861	0.898	0.943	0.985	0.996	0.999	1
≥ -4	0.826	0.886	0.922	0.961	0.992	0.998	1	1
≥ -5	0.850	0.908	0.940	0.974	0.996	0.999	1	1
≥ -6	0.872	0.926	0.955	0.983	0.998	1	1	1
≥ -7	0.890	0.940	0.966	0.989	0.999	1	1	1
≥ -8	0.906	0.952	0.975	0.993	1	1	1	1
≥ -9	0.919	0.962	0.982	0.996	1	1	1	1
≥ -10	0.931	0.970	0.987	0.997	1	1	1	1
≥ -11	0.941	0.976	0.990	0.999	1	1	1	1
≥ -12	0.950	0.981	0.993	0.999	1	1	1	1
≥ -13	0.957	0.985	0.995	1	1	1	1	1
≥ -14	0.963	0.988	0.997	1	1	1	1	1
≥ -15	0.968	0.991	0.998	1	1	1	1	1
≥ -16	0.972	0.993	0.999	1	1	1	1	1
≥ -17	0.976	0.995	0.999	1	1	1	1	1
≥ -18	0.979	0.996	0.999	1	1	1	1	1
≥ -19	0.982	0.997	1	1	1	1	1	1
≥ -20	0.985	0.998	1	1	1	1	1	1
≥ -25	0.992	1	1	1	1	1	1	1
≥ -30	0.996	1	1	1	1	1	1	1
≥ -35	0.999	1	1	1	1	1	1	1
≥ -40	1	1	1	1	1	1	1	1
≥ -45	1	1	1	1	1	1	1	1
≥ -50	1	1	1	1	1	1	1	1

We can summarize the result of the empirical analysis as follows:

- For investment horizons exceeding two years we find that a mixed bond/stock portfolio have higher probability of yielding a positive value growth than the bond portfolio

- Our results clearly suggests that a passive investor should avoid bonds in the long run and ought to avoid them even at moderate investments horizons.
- However, the bond portfolio is the less risky portfolio for horizons less than two years.
- The stock- and diversified portfolios dominate the bond portfolio for long investment horizons by the first order stochastic dominance criteria.
- The potential gains in the long run of the stock portfolio far exceeds the mixed portfolios at the cost of slightly higher probability of capital erosion.

7 Conclusion

This paper examines the empirical distributions of the real return from a number of portfolios of Swedish stocks and bonds. The contribution of this paper is twofold. First, we are able to answer the question of what real return an investor can expect to receive if investing in Swedish stocks and bonds. Further we are able to find the probability of actually receiving a specific return, which makes our approach applicable in risk management. Second we propose an improvement to the moving block bootstrap methodology by introducing a new Markov chain Monte Carlo method based on the empirical properties of the data for generating samples.

We find the most likely real return from stocks to be around 7.4 percent to 8.2 percent depending on the investment horizon and the real bond returns vary between 2.5 percent and 3.0 percent.

It is interesting to note that at a 25-year investment horizon, the bond portfolio stand a 91.2 percent chance of yielding positive annual value growth whereas stocks have a 97.2 percent probability. The effect of diversification is evident as our 60-40, 50-50 and 40-60 stock/bond portfolios have a higher probability of yielding a positive value growth than the bond portfolio. The overall conclusion is that when investing in a long-run passive portfolio a mix of bonds and stocks is important if ones goal is to minimize the risk of capital destruction.

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Appendix

A1 Generation via random numbers

Another approach to generate samples from a Markov Chain is to first construct the transition matrix $\mathbf{P}$. Given the state S_i the move to state S_j is determined by comparing a random draw against the cumulative probability distribution of the ith row in the transition matrix such that:

$$u \sim U(0,1)$$

$$S_j := \min \left\{ j : \sum_{l=1}^{j} p_{il} \geq u \right\} \tag{A1}$$

This approach have the advantage that all generations are accepted. Hence this is a less computer intensive approach than the accept-reject method.

A2 Block length

The idea behind block bootstrap is to keep the (time) serial properties of the original sample in the generated series. A long block length assures that a large fraction of the serial properties is kept within the generated series. However, long block lengths keep the serial properties at a cost of less possible outcomes of the generated series. One can think of the extreme case application of a block length equal to the original sample that would render the same sample. In the approach of this paper the bootstrapped monthly return series is utilized to construct price paths. Thus, the total number of possible outcomes or price series, P_b, from a moving block bootstrap is a function of the total number of observations, T, and the block length, b.

$$P_b = (T-b)^{\frac{T}{b}} \tag{A2}$$

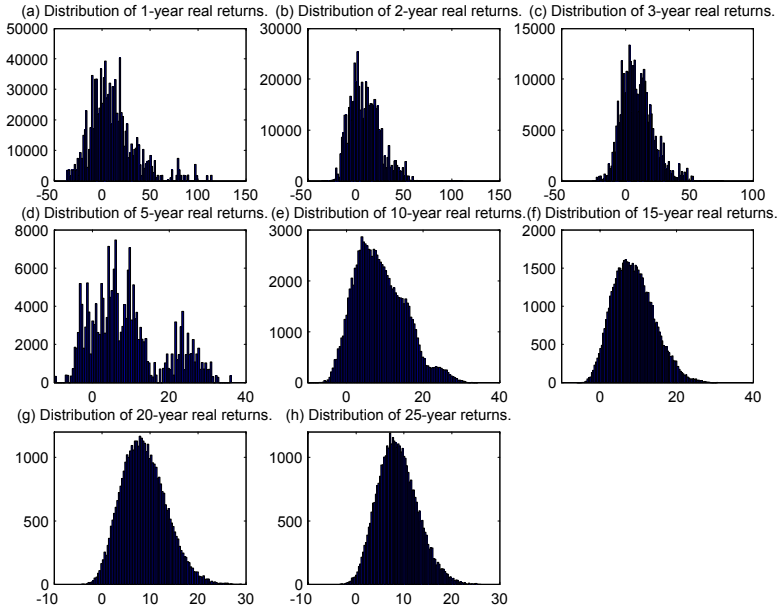

Figure 6: A visualization of the discretization of the empirical distributions of real stock returns for investment horizons shorter than the block length, 60 months.

This discretization occurs for investment horizons shorter that the block length, see Figure 6. The number of possible non-overlapping q month returns, n_q, for block lengths, b, longer than investment horizon, q, is given by:

$$n_q = \frac{b}{q}(T-b) \quad \text{for} \quad b \geq q \tag{A3}$$

To avoid the discretization of the generated series block lengths should be shorter than the investment horizon. This increase the number of possible outcomes, n_q^* as:

$$n_q^* = (T-b)^{\frac{q}{b}} \quad \text{for} \quad b \leq q \tag{A4}$$

Below we compare the possible outcomes for a long block, $b = 60$, versus short block, $b = 6$.

Example 1: If we have 600 observations and a block length of 60 months the number of possible trajectories are $P_{60} = 540^{10} = 2.10 \cdot 10^{27}$. The number of possible 60-month returns are 540 and the number of possible 12 month return are $5 \cdot 540 = 2700$.

Example 2: If we have 600 observation and a shorter block length of 6 months the number of possible trajectories are $P_{12} = 594^{100} = 2.39 \cdot 10^{277}$. The number of possible 60-

month returns increases to $n^*_{60} = 594^{10} = 5.46 \cdot 10^{27}$ and 12-months returns increases to $n^*_{12} = 594^2 = 352836$.

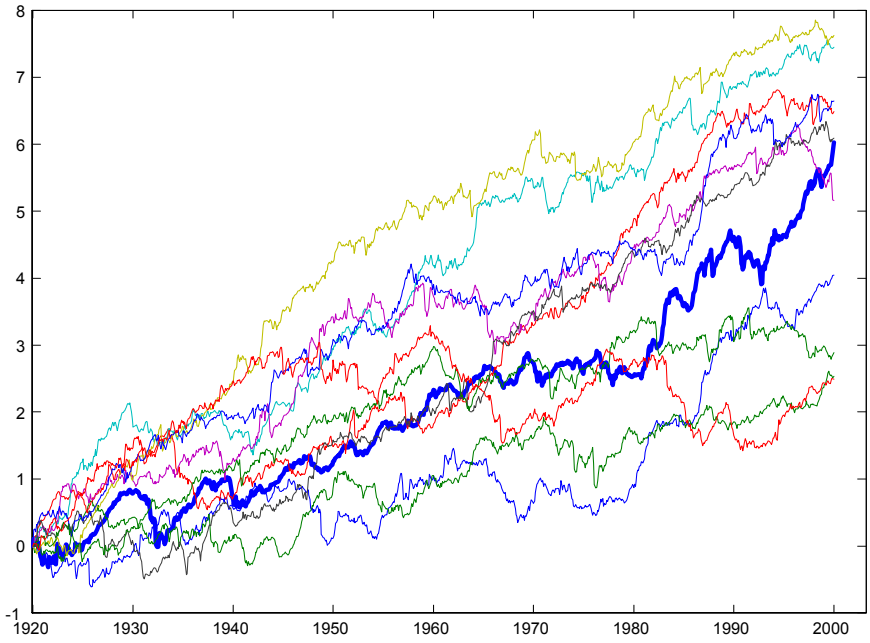

Figure 1: Logarithm of resampled real stock price paths vs. Swedish stock market index. Note: The time scale refers to the Swedish stock market index in real prices, thick line.

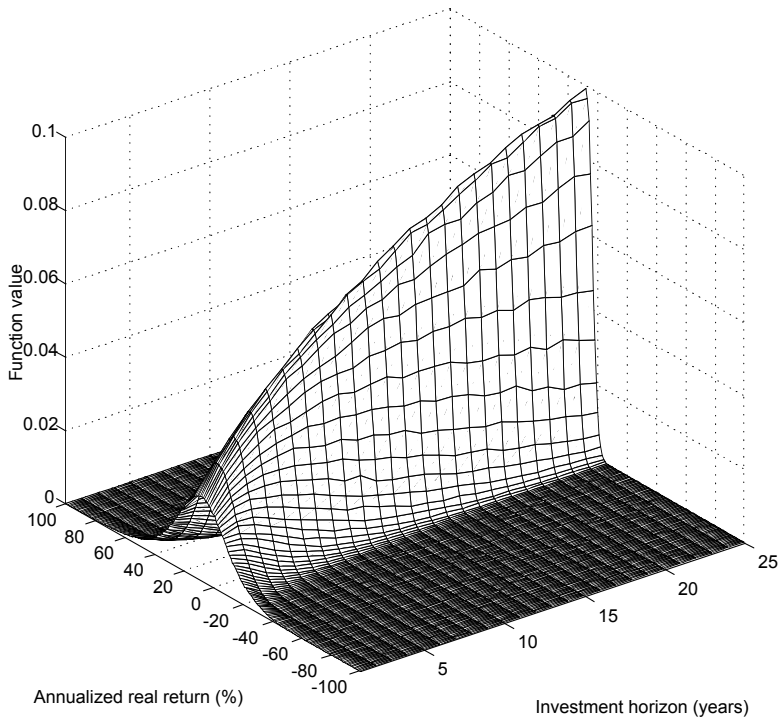

Figure 2: Probability density surface of annualized real return versus investment horizon for Swedish stocks

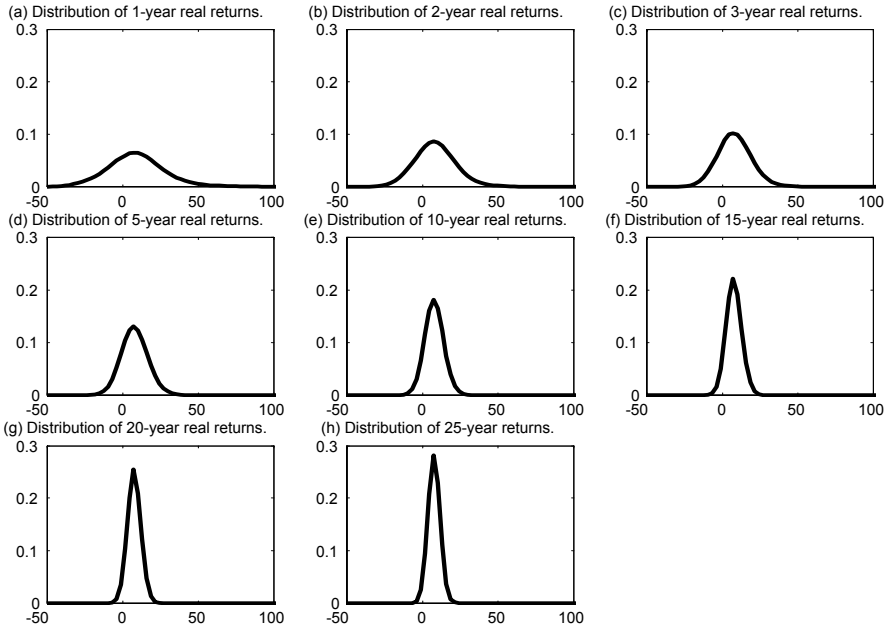

Figure 3: Empirical probability distributions of annualized real stock returns.

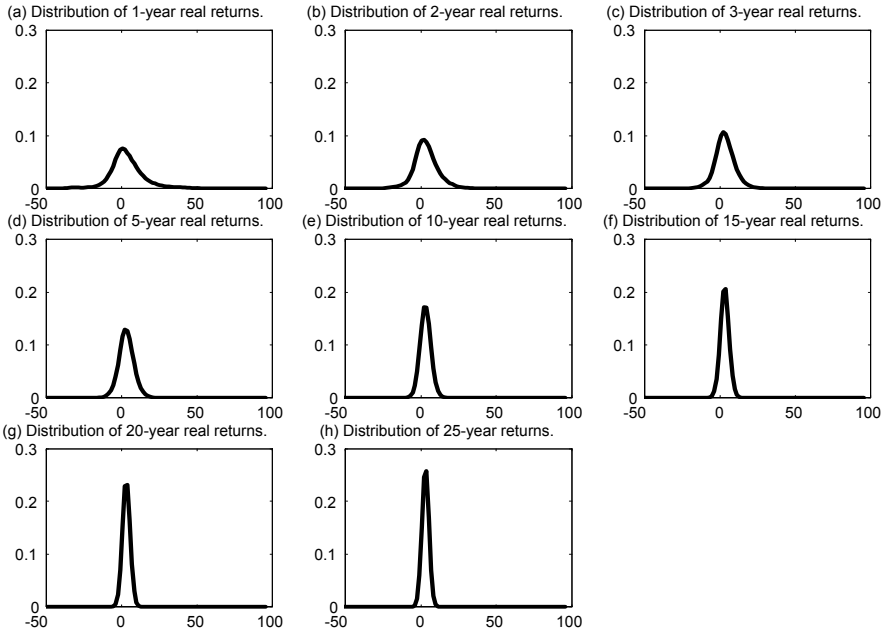

Figure 4: Empirical probability distributions of annualized real bond returns.

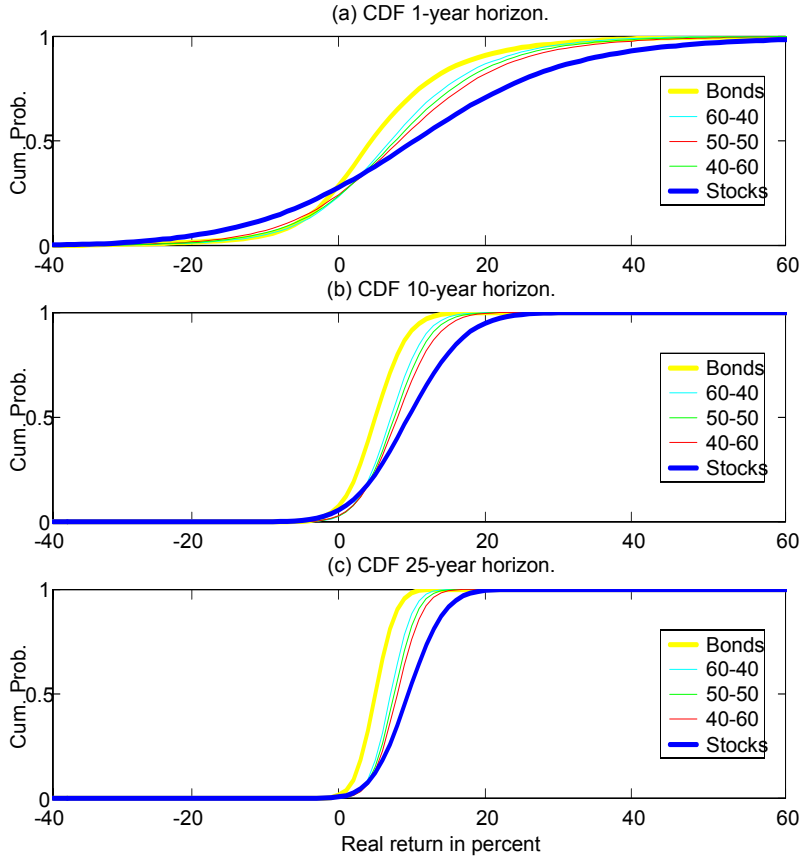

Figure 5: Cumulative distributions of the investigated portfolios. **(a)** 1-year horizon, **(b)** 10-year horizon, **(c)** 25-year horizon.