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Impact of tourism on regional economic growth: A global value chain perspective

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ONLINE APPENDIX

ADB Economics Working Paper No. 646

**Impact of Tourism on Regional Economic Growth:
A Global Value Chain Perspective**

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January 2022

Technical Notes

I. Multipliers

Multipliers measure the sensitivity of sectors given a demand change in their final products. The set of indicators recognized by this paper looks at a nominal change in final demand of a tourism sector that could induce changes to the economy's total output and value-added contributions of a sector. The analysis can be interregional, intraregional, or national.

Output Multipliers

Output multipliers give the total additional output necessary for production due to a US dollar's worth of increase in demand for a tourism sector's final products. This measure considers both direct and indirect effects. This is similar to the national multiplier such that it also considers the nationwide impact of the additional final demand to the economy. Suppose that sector j 's final demand increases, the total additional output induced in the economy $m(o)_j$ is given by sector j 's column sum in the Leontief inverse matrix \mathbf{B} , i.e.:

$$m(o)_j = \sum_{i=1}^n B_{ij}$$

In matrix form, it is computed as follows, where $\mathbf{i}' = [1 \ 1 \ \dots \ 1]$ and \mathbf{B} = sector demand-to-sector output multipliers:

$$\mathbf{m}(o) = \mathbf{i}'\mathbf{B} = [m(o)_1 \ \dots \ m(o)_n]$$

If tourism sector j 's final demand increases by \$1, then the additional output in the economy is $m(o)_j$. This multiplier is useful to see which sector contributes the greatest impact throughout the economy if the government is to spend to this sector.

Sectoral Multipliers

Sectoral multipliers give the total additional output of sector i necessary for production of tourism sector j to meet its additional final demand. This measure considers both direct and indirect effects. Denoted as $m(o)_{ij}$, this is the sector-demand-to-sector-output multipliers in the Leontief inverse matrix \mathbf{B} . In general:

$$\mathbf{m}(o)^{\text{sectoral}} = (\mathbf{i}' \otimes \mathbf{I}_n)\mathbf{B} = [\mathbf{I}_n \ \dots \ \mathbf{I}_n]\mathbf{B} = [m(o)^{\cdot 1} \ \dots \ m(o)^{\cdot n}]$$

If tourism sector j 's final demand increases by \$1, then the additional output of sector i is $m(o)_{ij}$.

Value-added Multipliers

Value-added multipliers give the economy-wide value added that is generated to satisfy demand from the tourism sector. This measure considers both direct and indirect effects. Starting with the value-added coefficient matrix \mathbf{V}' which is the sector's value added divided by the total output, i.e.:

$$\mathbf{V}' = \mathbf{v}'\hat{\mathbf{x}}^{-1} = \begin{bmatrix} \frac{v_1}{x_1} & \dots & \frac{v_n}{x_n} \end{bmatrix} = [a_{n+1,1} \quad \dots \quad a_{n+1,n}]$$

Then, the value-added multiplier for sector j is computed as the sum of the product of the value-added coefficients and the sectoral multiplier, i.e.:

$$m(v)_j = \sum_{i=1}^n a_{n+1,i} B_{ij}$$

Or in matrix notation:

$$\mathbf{m}(v) = \mathbf{i}'\mathbf{V}'\mathbf{B} = \mathbf{i}'\mathbf{v}'\hat{\mathbf{x}}^{-1}\mathbf{B} = [\mathbf{m}(v)_1 \quad \dots \quad \mathbf{m}(v)_n]$$

If sector j 's final demand increased by a dollar, then it would generate an additional value added throughout the economy of $m(v)_j$.

Type I Value-Added Multipliers

Type I value-added multipliers demonstrate the relative magnitude of total value added from the initial value-added impacts from each unit of spending. It is simply calculated as the ratio of the value-added multiplier the value-added coefficient, i.e.:

$$m(v)_j^I = \frac{\sum_{i=1}^n a_{n+1,i} B_{ij}}{a_{n+1,j}} = \frac{m(v)_j}{a_{n+1,j}}$$

In matrix notation:

$$\mathbf{m}(v)^I = \mathbf{m}(h)(\widehat{\mathbf{V}}')^{-1} = \mathbf{v}'\mathbf{B}(\widehat{\mathbf{V}}')^{-1}$$

This multiplier shows by how much the initial income effects are blown up when total effects are considered. For example, if $m(v)_j^I$ is 1.32, then the total value added generated by a change in sector j 's final demand amounts to 32% more than the initial value added generated in the economy.

Regional Multipliers

Multiregional Input–Output (MRIO) models summarize the interaction between goods produced and consumed in two or more regions in an economy. Assume a two-region two-sector economy:

Selling sectors		Purchasing sectors			
		Region r		Region s	
		1	2	1	2
Region r	1	z_{11}^{rr}	z_{12}^{rr}	z_{11}^{rs}	z_{12}^{rs}
	2	z_{21}^{rr}	z_{22}^{rr}	z_{21}^{rs}	z_{22}^{rs}
Region s	1	z_{11}^{sr}	z_{12}^{sr}	z_{11}^{ss}	z_{12}^{ss}
	2	z_{21}^{sr}	z_{22}^{sr}	z_{21}^{ss}	z_{22}^{ss}

From this MRIO, the technical coefficients matrix \mathbf{A} and Leontief inverse matrix \mathbf{B} can be derived, defined as follows:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^{rr} & \mathbf{A}^{rs} \\ \mathbf{A}^{sr} & \mathbf{A}^{ss} \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix}$$

Wherein subscript numbers for \mathbf{B} are basically the same as the superscript letters for \mathbf{A} .

With these, several multiplier effects can be calculated: (i) for a single region, (ii) for each of the other regions, (iii) for the “rest of the economy”, and (iv) for the total economy.

Intraregional Effects

This considers the intraregional transactions (i.e., rr and ss) in the MRIO. Impacts of exogenous changes in final demand for goods produced in region r is represented by the column sums of \mathbf{B}_{11} and in region s is by \mathbf{B}_{22} . If the final demand in sector j in region r increases by \$1, then the increase in the total output for all other sectors in region r is given by $\mathbf{m}(o)^{rr}$. Similar with the value-added measure, if the final demand in sector j in region r increases by \$1, then the additional value added generated for all other sectors in region r is given by $\mathbf{m}(v)^{rr}$.

The intraregional multipliers for region r is given by:

$$\mathbf{m}(o)^{rr} = \mathbf{i}'\mathbf{B}_{11} \text{ and } \mathbf{m}(v)^{rr} = \mathbf{i}'\mathbf{V}'\mathbf{B}_{11}$$

The intraregional multipliers for region s is given by:

$$\mathbf{m}(o)^{ss} = \mathbf{i}'\mathbf{B}_{22} \text{ and } \mathbf{m}(v)^{ss} = \mathbf{i}'\mathbf{V}'\mathbf{B}_{22}$$

To simultaneously estimate the intraregional multipliers, construct a ‘domestic’ Leontief inverse matrix which is the block diagonals of the matrix $\mathbf{B}^D = \begin{bmatrix} \mathbf{B}_{11} & 0 \\ 0 & \mathbf{B}_{22} \end{bmatrix}$. Then, the **intraregional multiplier for all regions** is given by:

$$\mathbf{m}(\mathbf{o})^{\text{intra}} = \mathbf{i}'\mathbf{B}^D = [\mathbf{m}(\mathbf{o})^{\text{rr}} \quad \mathbf{m}(\mathbf{o})^{\text{ss}}] \text{ and } \mathbf{m}(\mathbf{v})^{\text{intra}} = \mathbf{i}'\mathbf{V}'\mathbf{B}^D = [\mathbf{m}(\mathbf{v})^{\text{rr}} \quad \mathbf{m}(\mathbf{v})^{\text{ss}}]$$

Interregional Effects

This considers the interregional transactions (i.e., rs and sr) in the MRIO. Impacts in one region that are caused by changes in another region are reflected by the column sums of \mathbf{B}_{12} and \mathbf{B}_{21} . If there is a final demand change of sector j in region r by \$1, then the total output change of all sectors in region s is given by $\mathbf{m}(\mathbf{o})_j^{\text{sr}}$. Similar with the value-added measure, if the final demand in sector j in region r increases by \$1, then the additional value added generated for all other sectors in region s is given by $\mathbf{m}(\mathbf{v})_j^{\text{sr}}$.

The interregional multipliers from r to s is given by:

$$\mathbf{m}(\mathbf{o})_j^{\text{sr}} = \mathbf{i}'\mathbf{B}_{21} \text{ and } \mathbf{m}(\mathbf{v})_j^{\text{sr}} = \mathbf{i}'\mathbf{V}'\mathbf{B}_{21}$$

The interregional multipliers from s to r is given by:

$$\mathbf{m}(\mathbf{o})_j^{\text{rs}} = \mathbf{i}'\mathbf{B}_{12} \text{ and } \mathbf{m}(\mathbf{v})_j^{\text{rs}} = \mathbf{i}'\mathbf{V}'\mathbf{B}_{12}$$

If you want to simultaneously estimate the intraregional output multipliers, construct a ‘foreign’ Leontief inverse matrix \mathbf{B}^F .

To simultaneously estimate the interregional multipliers, construct a ‘foreign’ Leontief inverse matrix which is the off-block diagonals of the matrix $\mathbf{B}^F = \begin{bmatrix} 0 & \mathbf{B}_{12} \\ \mathbf{B}_{21} & 0 \end{bmatrix}$. Then, the **interregional multiplier for all regions** is given by:

$$\mathbf{m}(\mathbf{o})^{\text{inter}} = \mathbf{i}'\mathbf{B}^F = [\mathbf{m}(\mathbf{o})^{\text{rr}} \quad \mathbf{m}(\mathbf{o})^{\text{ss}}] \text{ and } \mathbf{m}(\mathbf{v})^{\text{inter}} = \mathbf{i}'\mathbf{V}'\mathbf{B}^F = [\mathbf{m}(\mathbf{v})^{\text{rr}} \quad \mathbf{m}(\mathbf{v})^{\text{ss}}]$$

Note: The sum of the intraregional and interregional effects gives the national effect.

II. Linkages

Production has two economic effects: (i) if sector j increases its output, this means there will be increased demand on sectors of which sector j purchase their inputs from, and (ii) if sector j increases its output, it also means that additional amounts of product j are available to be used as inputs to other sectors for their own production. The former effect is known as the backward linkage and the latter is known as the forward linkage.

Backward Linkage

This indicates an interconnection of a particular sector with those sectors from which it purchases inputs. It consists of two types: direct and total backward linkages.

Direct Backward Linkage

The direct backward linkages only measure interconnection with sectors that are directly tied with sector j . A \$1 increase in sector j 's total output will increase their demand for inputs to production by the estimated direct backward linkage $b(d)_j$. To get the direct backward linkage of sector j , get the column sum of the technical coefficients matrix \mathbf{A} :

$$b(d)_j = \sum_{i=1}^n a_{ij}$$

In matrix notation, $\mathbf{i}' = [1 \ 1 \ \dots \ 1]$ and \mathbf{A} = intermediate consumption-to-total output ratio:

$$\mathbf{b}(d) = [\mathbf{b}(d)_1 \ \dots \ \mathbf{b}(d)_n] = \mathbf{i}'\mathbf{A}$$

For a two-region economy, it can be computed as the sum of interregional and intraregional direct backward linkages for that region (=r):

$$b(d)_j^r = b(d)_j^{rr} + b(d)_j^{sr} = \sum_{i=1}^n a_{ij}^{rr} + \sum_{i=1}^n a_{ij}^{sr}$$

Or in matrix notation:

$$\mathbf{b}(d)^r = \mathbf{i}'\mathbf{A}^{rr} + \mathbf{i}'\mathbf{A}^{sr}$$

Total Backward Linkage

This measures both the direct and indirect effects since it utilizes the Leontief inverse matrix instead of the technical coefficients matrix. Therefore, the total backward linkage is equal to the output multiplier. To get the total backward linkage of sector j , get the column sum of the Leontief inverse matrix \mathbf{B} :

$$b(t)_j = \sum_{i=1}^n B_{ij}$$

In matrix notation:

$$\mathbf{b}(t) = [\mathbf{b}(t)_1 \ \dots \ \mathbf{b}(t)_n] = \mathbf{i}'\mathbf{B}$$

For a two-region economy, it is computed as the sum of interregional and intraregional total backward linkages for that region (=r):

$$b(t)_j^r = b(t)_j^{rr} + b(t)_j^{sr} = \sum_{i=1}^n l_{ij}^{rr} + \sum_{i=1}^n l_{ij}^{sr}$$

Or in matrix notation:

$$b(t)^r = \mathbf{i}' \mathbf{B}^{rr} + \mathbf{i}' \mathbf{B}^{sr}$$

Forward Linkage

This indicates an interconnection of a particular sector with those sectors to which it sells its output. If sector i 's total output increases by \$1, the additional output available to be used as input by the other sectors they supply to is the estimated forward linkage. It can measure either only the direct effects, or both the direct and indirect effects. Moreover, instead of the typical technical coefficients matrix \mathbf{A} and Leontief inverse matrix \mathbf{B} , measuring forward linkage involves using allocation coefficients matrix \mathbf{O} and Ghosh inverse matrix \mathbf{G} instead.

Direct Forward Linkage

The direct forward linkages only measure interconnection with sectors that are directly tied with sector i . To get the direct forward linkage of sector i , get the row sum of the allocation coefficients matrix \mathbf{O} , where it is the value of total intermediate sales by sector (\mathbf{Z}) as a proportion of the value of sector i 's total output (\mathbf{x}), i.e., $\mathbf{O} = \hat{\mathbf{x}}^{-1} \mathbf{Z}$.

$$f(d)_i = \sum_{j=1}^n O_{ij}$$

In matrix notation:

$$\mathbf{f}(d) = \begin{bmatrix} \mathbf{f}(d)_1 \\ \vdots \\ \mathbf{f}(d)_n \end{bmatrix} = \mathbf{O} \mathbf{i}$$

For a two-region economy, it can also be computed as the sum of interregional and intraregional direct forward linkages for that region (=r):

$$f(d)_i^r = f(d)_i^{rr} + f(d)_i^{sr} = \sum_{j=1}^n O_{ij}^{rr} + \sum_{j=1}^n O_{ij}^{sr}$$

Or in matrix notation:

$$\mathbf{f}(d)^r = \mathbf{O}^{rr} \mathbf{i} + \mathbf{O}^{sr} \mathbf{i}$$

Total Forward Linkage

This measures both direct and indirect effects since it utilizes the Ghosh inverse matrix \mathbf{G} instead of the allocation coefficients matrix. To get the total forward linkage of sector i , get the row sum of the Ghosh inverse matrix $\mathbf{G} = (\mathbf{I} - \mathbf{O})^{-1}$.

$$f(t)_i = \sum_{j=1}^n g_{ij}$$

In matrix notation:

$$\mathbf{f}(t) = \begin{bmatrix} \mathbf{f}(t)_1 \\ \vdots \\ \mathbf{f}(t)_n \end{bmatrix} = \mathbf{G}\mathbf{i}$$

For a two-region economy, it can also be computed as the sum of interregional and intraregional total forward linkages for that region (=r):

$$f(t)_i^r = f(t)_i^{rr} + f(t)_i^{sr} = \sum_{j=1}^n g_{ij}^{rr} + \sum_{j=1}^n g_{ij}^{sr}$$

Or in matrix notation:

$$\mathbf{f}(t)^r = \mathbf{G}^{rr}\mathbf{i} + \mathbf{G}^{sr}\mathbf{i}$$

Intraregional Effects

Assume a two-region, two-sector economy, where we are looking at region r .

The intraregional **backward linkage** effects for region r is given by $b(d)_j^r$ for the direct and $b(t)_j^r$ for the total backward linkage, i.e.:

$$b(d)_j^r = \sum_{i=1}^n a_{ij}^{rr} \text{ and } b(t)_j^r = \sum_{i=1}^n l_{ij}^{rr}$$

or in matrix notation, the column sums of \mathbf{A} and \mathbf{B} :

$$\mathbf{b}(d)^{rr} = \mathbf{i}'\mathbf{A}_{11} \text{ and } \mathbf{b}(t)^{rr} = \mathbf{i}'\mathbf{B}_{11}$$

A \$1 increase in region r 's sector j 's total output will increase their inputs demand from industries within region r by the estimated intraregional backward linkage.

To simultaneously estimate the intraregional direct backward linkages, construct "domestic" matrices \mathbf{A}^D and \mathbf{B}^D .

Then, the intraregional backward linkages for all regions is given by:

$$\mathbf{b}(d)^{\text{intra}} = \mathbf{i}'\mathbf{A}^D = [\mathbf{b}(d)^{\text{rr}} \quad \mathbf{b}(d)^{\text{ss}}] \text{ and } \mathbf{b}(t)^{\text{intra}} = \mathbf{i}'\mathbf{B}^D = [\mathbf{b}(t)^{\text{rr}} \quad \mathbf{b}(t)^{\text{ss}}]$$

The intraregional **forward linkage** effects for region r is given by $f(d)_i^r$ for the direct and $f(t)_i^r$ for the total forward linkage, i.e.:

$$f(d)_i^r = \sum_{j=1}^n O_{ij}^{\text{rr}} \text{ and } f(t)_i^r = \sum_{j=1}^n g_{ij}^{\text{rr}}$$

or in matrix notation, the row sums of \mathbf{O} and \mathbf{G} :

$$\mathbf{f}(d)^{\text{rr}} = \mathbf{O}_{11}\mathbf{i} \text{ and } \mathbf{f}(t)^{\text{rr}} = \mathbf{G}_{11}\mathbf{i}$$

Of the \$1 increase in region r 's sector i 's total output, the additional output available to be used as input by industries they supply to within region r by the estimated intraregional forward linkage.

To simultaneously estimate the intraregional direct forward linkages, construct “domestic” matrices \mathbf{O}^D and \mathbf{G}^D .

Then, the intraregional forward linkages for all regions is given by:

$$\mathbf{f}(d)^{\text{intra}} = \begin{bmatrix} \mathbf{f}(d)^{\text{rr}} \\ \mathbf{f}(d)^{\text{ss}} \end{bmatrix} = \mathbf{O}^D\mathbf{i} \text{ and } \mathbf{f}(t)^{\text{intra}} = \begin{bmatrix} \mathbf{f}(t)^{\text{rr}} \\ \mathbf{f}(t)^{\text{ss}} \end{bmatrix} = \mathbf{G}^D\mathbf{i}$$

Interregional Effects

Assume a two-region, two-sector economy, where we are looking at region r .

The interregional **backward linkage** effects from region s to r is given by $b(d)_j^{\text{rs}}$ for the direct and $b(t)_j^{\text{rs}}$ for the total backward linkage, i.e.:

$$b(d)_j^{\text{rs}} = \sum_{i=1}^n a_{ij}^{\text{rs}} \text{ and } b(t)_j^{\text{rs}} = \sum_{i=1}^n b_{ij}^{\text{rs}}$$

or in matrix notation, the column sums of \mathbf{A} and \mathbf{B} :

$$\mathbf{b}(d)^{\text{rs}} = \mathbf{i}'\mathbf{A}_{12} \text{ and } \mathbf{b}(t)^{\text{rs}} = \mathbf{i}'\mathbf{B}_{12}$$

A \$1 increase in the region r 's sector j 's total output will increase their demand for inputs supplied to them by industries from region s by the estimated interregional backward linkage.

To simultaneously estimate the interregional direct backward linkages, construct “foreign” matrices \mathbf{A}^F and \mathbf{B}^F .

Then, the interregional backward linkages for all regions is given by:

$$\mathbf{b}(d)^{\text{inter}} = [\mathbf{b}(d)^{\text{sr}} \quad \mathbf{b}(d)^{\text{rs}}] = \mathbf{i}'\mathbf{A}^F \text{ and } \mathbf{b}(t)^{\text{inter}} = [\mathbf{b}(t)^{\text{sr}} \quad \mathbf{b}(t)^{\text{rs}}] = \mathbf{i}'\mathbf{B}^F$$

The intraregional **forward linkage** effects from region s to r is given by $f(d)_i^{rs}$ for the direct and $f(t)_i^{rs}$ for the total forward linkage, i.e.:

$$f(d)_i^{rs} = \sum_{j=1}^n O_{ij}^{rs} \text{ and } f(t)_i^{rs} = \sum_{j=1}^n g_{ij}^{rs}$$

or in matrix notation, the row sums of \mathbf{O} and \mathbf{G} :

$$\mathbf{f}(d)^{rs} = \mathbf{O}_{12}\mathbf{i} \text{ and } \mathbf{f}(t)^{rs} = \mathbf{G}_{12}\mathbf{i}$$

Of the \$1 increase in the total output of sector i in region r , the additional output available to be used as input by industries they supply to in region s by the estimated interregional forward linkage.

To simultaneously estimate the intraregional direct forward linkages, construct 'foreign' matrices \mathbf{O}^F and \mathbf{G}^F .

Then, the interregional forward linkages for all regions is given by:

$$\mathbf{f}(d)^{\text{inter}} = \begin{bmatrix} \mathbf{f}(d)^{rs} \\ \mathbf{f}(d)^{sr} \end{bmatrix} = \mathbf{O}^F\mathbf{i} \text{ and } \mathbf{f}(t)^{\text{inter}} = \begin{bmatrix} \mathbf{f}(t)^{rs} \\ \mathbf{f}(t)^{sr} \end{bmatrix} = \mathbf{G}^F\mathbf{i}$$

Relative Strengths

Direct and Total Backward Linkages

This gives the dependency of a sector to other sectors since they rely on the other sectors for inputs.

To get the relative strength of interregional direct and total backward linkages of sector j in region r , it is measured by $\frac{b(d)_j^{sr}}{b(d)_j^r} \times 100$ and $\frac{b(t)_j^{sr}}{b(t)_j^r} \times 100$, respectively. If the relative strength is 6%, then the interregional dependency for inputs of region r 's sector j is 6%, such that 6% of its inputs were derived from industries within region s .

To get the relative strength of intraregional direct and total backward linkages of sector j in region r , it is measured by $\frac{b(d)_j^{rr}}{b(d)_j^r} \times 100$ and $\frac{b(t)_j^{rr}}{b(t)_j^r} \times 100$, respectively. If the relative strength is 6%, then the intraregional dependency for inputs of region r 's sector j is 6%, such that 6% of its inputs were derived from industries within region r .

Direct and Total Forward Linkages

This gives the dependency of sector i to other sectors since they serve as a seller of inputs to other sectors.

To get the relative strength of interregional direct and total forward linkages of sector i in region r , it is measured by $\frac{f(d)_i^{sr}}{f(d)_i^r} \times 100$ and $\frac{f(t)_i^{sr}}{f(t)_i^r} \times 100$, respectively. If the relative strength is 12%, then region r 's sector i supply 12% of their industry outputs as inputs to industries in region s .

To get the relative strength of intraregional direct and total forward linkages of sector i in region r , it is measured by $\frac{f(d)_i^{rr}}{f(d)_i^r} \times 100$ and $\frac{f(t)_i^{rr}}{f(t)_i^r} \times 100$, respectively. If the relative strength is 12%, then region r 's sector i supply 12% of their industry outputs as inputs to industries in region r .

III. Leakage Effects (Backward and Forward Import Multipliers)

As with linkage effects, there are two types of leakage effects: forward and backward. As linkage effects pertain to the demand and supply relationships of a sector i to another sector j in the domestic economy, leakage effects refer to the same relationships to sectors outside the economy. They are so-called "leakage" since, when imports or foreign inputs are included in production, the gains from increased production, in the form of demand and income, do not entirely accrue to the domestic economy; these "leak" onto foreign economies as well.

Backward Import Multiplier (Backward "Leakage" Effects)

On the demand-side perspective, this refers to the total amount of production that accrues outside the economy as a result of an exogenous change in final demand of sector i . This is a measure of the total imported inputs of product j required per unit of final demand in sector i .

$a_{ij}^m = \frac{z_{ij}^m}{x_j}$ is the imports direct input coefficient. An increase in output in sector i should reflect an increase of its demand for product j 's imported inputs. These imports are absorbed by sector j per unit of output of sector j . The matrix of a_{ij}^m elements is A_m .

The vector of backward (or demand-side) import multipliers is represented as the column sums of the matrix.

$$A_m(I - A_d)^{-1}$$

Where:

A^m is the input coefficients matrix of imported intermediates; and

A^d is the input coefficients matrix of domestically produced intermediates.

Each element in this matrix provides the additional imports of product i if final demand for sector j 's output increases by one dollar.

The sum of column vector j in this matrix represents the total "leakage" resulting from a dollar increase in the final demand for sector j 's output.

(A \$1 increase in sector j 's final demand, brought about by an increase in its production, will increase the total imported inputs of product i by the column sum of the matrix $A_m(I - A_d)^{-1}$).

Forward Import Multiplier (Forward "Leakage" Effects)

Looking at the supply-side perspective this time, this refers to the total amount of production that accrues outside the economy because of exogenous changes in the availability of primary inputs in sector j . This is a measure of the total imported inputs that are available to the economy due to changes in primary inputs.

$a_{ij}^{*m} = \frac{z_{ij}^m}{x_i}$ is the forward import multiplier. A one dollar increase in the primary inputs of sector j means there are more available inputs for production to be used by other sectors. Thus, this should reflect an increase in another sector's output. The matrix of a_{ij}^{*m} elements is A^{*m} .

The leakage matrix is:

$$(I - A^{*d})^{-1}A^{*m}$$

Where:

A^{*d} is the output coefficients matrix of domestically produced intermediates.

$(I - A^{*d})^{-1}$ is the output inverse matrix. May also be denoted as B^*

A^{*m} is the input coefficients matrix of imported intermediates; and

Each element in this matrix provides a forward import multiplier.

The sum of the elements in row vector i in this matrix represents the total "leakage" resulting from a dollar increase in the primary inputs for sector i . Each element in this matrix provides the additional imports of product i if final demand for sector j 's output increases by one dollar. The sum of column vector j in this matrix represents the total "leakage" resulting from a dollar increase in the final demand for sector j 's output.

(A \$1 increase in sector i 's primary inputs, will increase the total output and the inputs available for other sectors by the sum of the i th row of the matrix $(I - A^{*d})^{-1}A^{*m}$).

IV. Global Value Chain

In the framework of Wang et al. (2017), value added is embedded into the following:

1. Final Products and Intermediates absorbed domestically
2. Final Products for Exports, which we refer to as "traditional trade"
3. Intermediates for Export, which we refer to as "global value chain-related activities" or "GVCs".

GVC participation measures an economy's participation in global value chain-related activities. There are two perspectives or ways to measure GVC participation: by looking at value added consumed (backward GVCs) and value added produced (forward GVCs). Moreover, GVC activities may be broken down into simple and complex GVCs.

Simple GVCs are GVCs with only one border crossing (export once or import once), while complex GVCs are GVCs with two or more border crossings.

In matrix notation, the formula for total value-added decomposition is:

$$\hat{V}B\hat{Y} = VLY^D + VLY^F + VLA^FBY$$

Where:

$\hat{V}B\hat{Y}$ is the total value added (forward) or final goods and services (backward) of an economy-sector. It is the matrix product of the following:

- \hat{V} = value-added coefficients (diagonalized for forward GVCs: \hat{V})
- B = global Leontief inverse matrix $(I - A)^{-1}$
- \hat{Y} = final demand, sum of the domestic and foreign final demand (diagonalized for backward GVCs: \hat{Y})

The diagonalization of \hat{V} and \hat{Y} will vary depending on the perspective.

Backward Global Value Chain Participation Rate (GVC_B)

Of the total value added consumed (or total final consumption) in the economy, a portion of this comes from the imports of intermediates, or global value chain-related activities. The share of this component in the total final demand of an economy is the backward GVC participation rate. It also pertains to the share of value consumed that is sourced from GVCs.

Backward GVC Participation can be broken down into simple and complex backward GVCs.

In matrix notation, the formula for the decomposition of total value added consumed is:

$$VB\hat{Y} = VL\hat{Y}^D + VL\hat{Y}^F + VLA^FB\hat{Y}$$

Where:

$VB\hat{Y}$ is the production of final goods and services by an economy-sector. It is the matrix product of the following:

- V = vector of value-added coefficients
- B = global Leontief inverse matrix $(I - A)^{-1}$
- \hat{Y} = diagonalized final demand, sum of the domestic and foreign final demand

For the case of 3 economies (r) and 2 sectors (s):

$$\begin{bmatrix} v_{r_1s_1} \\ v_{r_1s_2} \\ v_{r_2s_1} \\ v_{r_2s_2} \\ v_{r_3s_1} \\ v_{r_3s_2} \end{bmatrix} \times \begin{bmatrix} b_{r_{11}s_{11}} & b_{r_{11}s_{12}} & b_{r_{12}s_{11}} & b_{r_{12}s_{12}} & b_{r_{13}s_{11}} & b_{r_{13}s_{12}} \\ b_{r_{11}s_{21}} & b_{r_{11}s_{22}} & b_{r_{12}s_{21}} & b_{r_{12}s_{22}} & b_{r_{13}s_{21}} & b_{r_{13}s_{22}} \\ b_{r_{21}s_{11}} & b_{r_{21}s_{12}} & b_{r_{22}s_{11}} & b_{r_{22}s_{12}} & b_{r_{23}s_{11}} & b_{r_{23}s_{12}} \\ b_{r_{21}s_{21}} & b_{r_{21}s_{22}} & b_{r_{22}s_{21}} & b_{r_{22}s_{22}} & b_{r_{23}s_{21}} & b_{r_{23}s_{22}} \\ b_{r_{31}s_{11}} & b_{r_{31}s_{12}} & b_{r_{32}s_{11}} & b_{r_{32}s_{12}} & b_{r_{33}s_{11}} & b_{r_{33}s_{12}} \\ b_{r_{31}s_{21}} & b_{r_{31}s_{22}} & b_{r_{32}s_{21}} & b_{r_{32}s_{22}} & b_{r_{33}s_{21}} & b_{r_{33}s_{22}} \end{bmatrix} \times \begin{bmatrix} y_{r_1s_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & y_{r_1s_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & y_{r_2s_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & y_{r_2s_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & y_{r_3s_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & y_{r_3s_2} \end{bmatrix}$$

$VL\hat{Y}^D$ is the domestic value added embedded in domestically consumed final goods and services. It is the matrix product of the following:

V = vector of value-added coefficients

L = local Leontief inverse matrix $(I - A^D)^{-1}$

Y^D = diagonalized final demand comprising only the domestic final demand component

For the case of 3 economies (r) and 2 sectors (s):

$$\begin{bmatrix} v_{r_1s_1} \\ v_{r_1s_2} \\ v_{r_2s_1} \\ v_{r_2s_2} \\ v_{r_3s_1} \\ v_{r_3s_2} \end{bmatrix} \times \begin{bmatrix} l_{r_{11}s_{11}} & l_{r_{11}s_{12}} & l_{r_{12}s_{11}} & l_{r_{12}s_{12}} & l_{r_{13}s_{11}} & l_{r_{13}s_{12}} \\ l_{r_{11}s_{21}} & l_{r_{11}s_{22}} & l_{r_{12}s_{21}} & l_{r_{12}s_{22}} & l_{r_{13}s_{21}} & l_{r_{13}s_{22}} \\ l_{r_{21}s_{11}} & l_{r_{21}s_{12}} & l_{r_{22}s_{11}} & l_{r_{22}s_{12}} & l_{r_{23}s_{11}} & l_{r_{23}s_{12}} \\ l_{r_{21}s_{21}} & l_{r_{21}s_{22}} & l_{r_{22}s_{21}} & l_{r_{22}s_{22}} & l_{r_{23}s_{21}} & l_{r_{23}s_{22}} \\ l_{r_{31}s_{11}} & l_{r_{31}s_{12}} & l_{r_{32}s_{11}} & l_{r_{32}s_{12}} & l_{r_{33}s_{11}} & l_{r_{33}s_{12}} \\ l_{r_{31}s_{21}} & l_{r_{31}s_{22}} & l_{r_{32}s_{21}} & l_{r_{32}s_{22}} & l_{r_{33}s_{21}} & l_{r_{33}s_{22}} \end{bmatrix} \times \begin{bmatrix} y_{r_1s_1}^D & 0 & 0 & 0 & 0 & 0 \\ 0 & y_{r_1s_2}^D & 0 & 0 & 0 & 0 \\ 0 & 0 & y_{r_2s_1}^D & 0 & 0 & 0 \\ 0 & 0 & 0 & y_{r_2s_2}^D & 0 & 0 \\ 0 & 0 & 0 & 0 & y_{r_3s_1}^D & 0 \\ 0 & 0 & 0 & 0 & 0 & y_{r_3s_2}^D \end{bmatrix}$$

$VL\hat{Y}^F$ is the domestic value added in final exports ("traditional trade"). It is the matrix product of the following:

V = vector of value-added coefficients

L = local Leontief inverse matrix $(I - A^D)^{-1}$

\hat{Y}^F = diagonalized final demand comprising only the foreign final demand component

For the case of 3 economies (r) and 2 sectors (s):

$$\begin{bmatrix} v_{r_1s_1} \\ v_{r_1s_2} \\ v_{r_2s_1} \\ v_{r_2s_2} \\ v_{r_3s_1} \\ v_{r_3s_2} \end{bmatrix} \times \begin{bmatrix} l_{r_{11}s_{11}} & l_{r_{11}s_{12}} & l_{r_{12}s_{11}} & l_{r_{12}s_{12}} & l_{r_{13}s_{11}} & l_{r_{13}s_{12}} \\ l_{r_{11}s_{21}} & l_{r_{11}s_{22}} & l_{r_{12}s_{21}} & l_{r_{12}s_{22}} & l_{r_{13}s_{21}} & l_{r_{13}s_{22}} \\ l_{r_{21}s_{11}} & l_{r_{21}s_{12}} & l_{r_{22}s_{11}} & l_{r_{22}s_{12}} & l_{r_{23}s_{11}} & l_{r_{23}s_{12}} \\ l_{r_{21}s_{21}} & l_{r_{21}s_{22}} & l_{r_{22}s_{21}} & l_{r_{22}s_{22}} & l_{r_{23}s_{21}} & l_{r_{23}s_{22}} \\ l_{r_{31}s_{11}} & l_{r_{31}s_{12}} & l_{r_{32}s_{11}} & l_{r_{32}s_{12}} & l_{r_{33}s_{11}} & l_{r_{33}s_{12}} \\ l_{r_{31}s_{21}} & l_{r_{31}s_{22}} & l_{r_{32}s_{21}} & l_{r_{32}s_{22}} & l_{r_{33}s_{21}} & l_{r_{33}s_{22}} \end{bmatrix} \times \begin{bmatrix} y_{r_1s_1}^F & 0 & 0 & 0 & 0 & 0 \\ 0 & y_{r_1s_2}^F & 0 & 0 & 0 & 0 \\ 0 & 0 & y_{r_2s_1}^F & 0 & 0 & 0 \\ 0 & 0 & 0 & y_{r_2s_2}^F & 0 & 0 \\ 0 & 0 & 0 & 0 & y_{r_3s_1}^F & 0 \\ 0 & 0 & 0 & 0 & 0 & y_{r_3s_2}^F \end{bmatrix}$$

$VLA^F B \hat{Y}$ is the domestic and foreign value added in intermediate imports ("GVCs"). It is the sum of simple GVCs ($VLA^F L \hat{Y}^D$), or the value added in the production of domestically used goods and services, and complex GVCs ($\hat{V}LA^F (B Y - L Y^D)$), or the value added in the production of exported goods and services. It is the matrix product of the following:

V = value-added coefficients

L = local Leontief inverse matrix $(I - A^D)^{-1}$

A^F = technical coefficients matrix comprising only the foreign components (foreign values in the off-diagonals)

B = global Leontief inverse matrix $(I - A)^{-1}$

\hat{Y} = diagonalized final demand, sum of the domestic and foreign final demand

For the case of 3 economies (r) and 2 sectors (s):

$$\begin{bmatrix} v_{r_1 s_1} \\ v_{r_1 s_2} \\ v_{r_2 s_1} \\ v_{r_2 s_2} \\ v_{r_3 s_1} \\ v_{r_3 s_2} \end{bmatrix} \times \begin{bmatrix} l_{r_1 s_11} & l_{r_1 s_12} & l_{r_1 s_21} & l_{r_1 s_22} & l_{r_1 s_31} & l_{r_1 s_32} \\ l_{r_1 s_21} & l_{r_1 s_22} & l_{r_2 s_11} & l_{r_2 s_12} & l_{r_2 s_21} & l_{r_2 s_22} \\ l_{r_2 s_11} & l_{r_2 s_12} & l_{r_2 s_21} & l_{r_2 s_22} & l_{r_2 s_31} & l_{r_2 s_32} \\ l_{r_2 s_21} & l_{r_2 s_22} & l_{r_3 s_11} & l_{r_3 s_12} & l_{r_3 s_21} & l_{r_3 s_22} \\ l_{r_3 s_11} & l_{r_3 s_12} & l_{r_3 s_21} & l_{r_3 s_22} & l_{r_3 s_31} & l_{r_3 s_32} \\ l_{r_3 s_21} & l_{r_3 s_22} & l_{r_3 s_31} & l_{r_3 s_32} & l_{r_3 s_41} & l_{r_3 s_42} \end{bmatrix} \times \begin{bmatrix} 0 & 0 & a_{r_{12} s_{12}} & a_{r_{12} s_{22}} & a_{r_{13} s_{11}} & a_{r_{13} s_{12}} \\ 0 & 0 & a_{r_{12} s_{21}} & a_{r_{12} s_{22}} & a_{r_{13} s_{21}} & a_{r_{13} s_{22}} \\ a_{r_{21} s_{11}} & a_{r_{21} s_{12}} & 0 & 0 & a_{r_{23} s_{11}} & a_{r_{23} s_{12}} \\ a_{r_{21} s_{21}} & a_{r_{21} s_{22}} & 0 & 0 & a_{r_{23} s_{21}} & a_{r_{23} s_{22}} \\ a_{r_{31} s_{11}} & a_{r_{31} s_{12}} & a_{r_{32} s_{11}} & a_{r_{32} s_{12}} & 0 & 0 \\ a_{r_{31} s_{21}} & a_{r_{31} s_{22}} & a_{r_{32} s_{21}} & a_{r_{32} s_{22}} & 0 & 0 \end{bmatrix} \\
 \times \begin{bmatrix} b_{r_{11} s_{11}} & b_{r_{11} s_{12}} & b_{r_{12} s_{11}} & b_{r_{12} s_{12}} & b_{r_{13} s_{11}} & b_{r_{13} s_{12}} \\ b_{r_{11} s_{21}} & b_{r_{11} s_{22}} & b_{r_{12} s_{21}} & b_{r_{12} s_{22}} & b_{r_{13} s_{21}} & b_{r_{13} s_{22}} \\ b_{r_{21} s_{11}} & b_{r_{21} s_{12}} & b_{r_{22} s_{11}} & b_{r_{22} s_{12}} & b_{r_{23} s_{11}} & b_{r_{23} s_{12}} \\ b_{r_{21} s_{21}} & b_{r_{21} s_{22}} & b_{r_{22} s_{21}} & b_{r_{22} s_{22}} & b_{r_{23} s_{21}} & b_{r_{23} s_{22}} \\ b_{r_{31} s_{11}} & b_{r_{31} s_{12}} & b_{r_{32} s_{11}} & b_{r_{32} s_{12}} & b_{r_{33} s_{11}} & b_{r_{33} s_{12}} \\ b_{r_{31} s_{21}} & b_{r_{31} s_{22}} & b_{r_{32} s_{21}} & b_{r_{32} s_{22}} & b_{r_{33} s_{21}} & b_{r_{33} s_{22}} \end{bmatrix} \times \begin{bmatrix} y_{r_1 s_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & y_{r_1 s_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & y_{r_2 s_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & y_{r_2 s_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & y_{r_3 s_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & y_{r_3 s_2} \end{bmatrix}$$

The backward GVC Participation is given by the share of GVCs in total value added consumed:

$$GVC_B = \frac{VLA^F B \hat{Y}}{V B \hat{Y}}$$

Forward Global Value Chain Participation (GVC_F)

Of the total value added produced in the economy, a portion of this is embedded in the exports of intermediates, or global value chain-related activities. The share of this component in the total value-added (GDP) of an economy is the forward GVC participation rate. It also pertains to the share of value produced absorbed into GVCs.

Forward GVC participation can also be broken down into simplex and complex backward GVCs.

In matrix notation, the formula for the decomposition of total value-added produced is:

$$\widehat{V}BY = \widehat{V}LY^D + \widehat{V}LY^F + \widehat{V}LA^F BY$$

Where:

$\widehat{V}BY$ is an economy-sector's total value added. It is the matrix product of the following:

\widehat{V} = diagonalized value-added coefficients

B = global Leontief inverse matrix $(I - A)^{-1}$

Y = final demand vector, sum of the domestic and foreign final demand

For the case of 3 economies (r) and 2 sectors (s):

$$\begin{bmatrix} v_{r_1s_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & v_{r_1s_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & v_{r_2s_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & v_{r_2s_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & v_{r_3s_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & v_{r_3s_2} \end{bmatrix} \times \begin{bmatrix} b_{r_{11}s_{11}} & b_{r_{11}s_{12}} & b_{r_{12}s_{11}} & b_{r_{12}s_{12}} & b_{r_{13}s_{11}} & b_{r_{13}s_{12}} \\ b_{r_{11}s_{21}} & b_{r_{11}s_{22}} & b_{r_{12}s_{21}} & b_{r_{12}s_{22}} & b_{r_{13}s_{21}} & b_{r_{13}s_{22}} \\ b_{r_{21}s_{11}} & b_{r_{21}s_{12}} & b_{r_{22}s_{11}} & b_{r_{22}s_{12}} & b_{r_{23}s_{11}} & b_{r_{23}s_{12}} \\ b_{r_{21}s_{21}} & b_{r_{21}s_{22}} & b_{r_{22}s_{21}} & b_{r_{22}s_{22}} & b_{r_{23}s_{21}} & b_{r_{23}s_{22}} \\ b_{r_{31}s_{11}} & b_{r_{31}s_{12}} & b_{r_{32}s_{11}} & b_{r_{32}s_{12}} & b_{r_{33}s_{11}} & b_{r_{33}s_{12}} \\ b_{r_{31}s_{21}} & b_{r_{31}s_{22}} & b_{r_{32}s_{21}} & b_{r_{32}s_{22}} & b_{r_{33}s_{21}} & b_{r_{33}s_{22}} \end{bmatrix} \times \begin{bmatrix} y_{r_1s_1} \\ y_{r_1s_2} \\ y_{r_2s_1} \\ y_{r_2s_2} \\ y_{r_3s_1} \\ y_{r_3s_2} \end{bmatrix}$$

$\widehat{V}LY^D$ is the value-added embedded in the production of final goods and services absorbed domestically. It is the matrix product of the following:

\widehat{V} = diagonalized value-added coefficients

L = local Leontief inverse matrix $(I - A^D)^{-1}$

Y^D = final demand vector comprising only the domestic final demand component

For the case of 3 economies (r) and 2 sectors (s):

$$\begin{bmatrix} v_{r_1s_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & v_{r_1s_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & v_{r_2s_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & v_{r_2s_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & v_{r_3s_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & v_{r_3s_2} \end{bmatrix} \times \begin{bmatrix} l_{r_{11}s_{11}} & l_{r_{11}s_{12}} & l_{r_{12}s_{11}} & l_{r_{12}s_{12}} & l_{r_{13}s_{11}} & l_{r_{13}s_{12}} \\ l_{r_{11}s_{21}} & l_{r_{11}s_{22}} & l_{r_{12}s_{21}} & l_{r_{12}s_{22}} & l_{r_{13}s_{21}} & l_{r_{13}s_{22}} \\ l_{r_{21}s_{11}} & l_{r_{21}s_{12}} & l_{r_{22}s_{11}} & l_{r_{22}s_{12}} & l_{r_{23}s_{11}} & l_{r_{23}s_{12}} \\ l_{r_{21}s_{21}} & l_{r_{21}s_{22}} & l_{r_{22}s_{21}} & l_{r_{22}s_{22}} & l_{r_{23}s_{21}} & l_{r_{23}s_{22}} \\ l_{r_{31}s_{11}} & l_{r_{31}s_{12}} & l_{r_{32}s_{11}} & l_{r_{32}s_{12}} & l_{r_{33}s_{11}} & l_{r_{33}s_{12}} \\ l_{r_{31}s_{21}} & l_{r_{31}s_{22}} & l_{r_{32}s_{21}} & l_{r_{32}s_{22}} & l_{r_{33}s_{21}} & l_{r_{33}s_{22}} \end{bmatrix} \times \begin{bmatrix} y_{r_1s_1}^D \\ y_{r_1s_2}^D \\ y_{r_2s_1}^D \\ y_{r_2s_2}^D \\ y_{r_3s_1}^D \\ y_{r_3s_2}^D \end{bmatrix}$$

$\widehat{V}LY^F$ is the value-added embedded in the production of goods and services of final exports ("traditional trade"). It is the matrix product of the following:

\widehat{V} = diagonalized value-added coefficients

L = local Leontief inverse matrix $(I - A^D)^{-1}$

Y^F = final demand vector comprising only the foreign final demand component

For the case of 3 economies (r) and 2 sectors (s):

$$\begin{bmatrix} v_{r_1s_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & v_{r_1s_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & v_{r_2s_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & v_{r_2s_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & v_{r_3s_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & v_{r_3s_2} \end{bmatrix} \times \begin{bmatrix} l_{r_{11}s_{11}} & l_{r_{11}s_{12}} & l_{r_{12}s_{11}} & l_{r_{12}s_{12}} & l_{r_{13}s_{11}} & l_{r_{13}s_{12}} \\ l_{r_{11}s_{21}} & l_{r_{11}s_{22}} & l_{r_{12}s_{21}} & l_{r_{12}s_{22}} & l_{r_{13}s_{21}} & l_{r_{13}s_{22}} \\ l_{r_{21}s_{11}} & l_{r_{21}s_{12}} & l_{r_{22}s_{11}} & l_{r_{22}s_{12}} & l_{r_{23}s_{11}} & l_{r_{23}s_{12}} \\ l_{r_{21}s_{21}} & l_{r_{21}s_{22}} & l_{r_{22}s_{21}} & l_{r_{22}s_{22}} & l_{r_{23}s_{21}} & l_{r_{23}s_{22}} \\ l_{r_{31}s_{11}} & l_{r_{31}s_{12}} & l_{r_{32}s_{11}} & l_{r_{32}s_{12}} & l_{r_{33}s_{11}} & l_{r_{33}s_{12}} \\ l_{r_{31}s_{21}} & l_{r_{31}s_{22}} & l_{r_{32}s_{21}} & l_{r_{32}s_{22}} & l_{r_{33}s_{21}} & l_{r_{33}s_{22}} \end{bmatrix} \times \begin{bmatrix} y_{r_1s_1}^F \\ y_{r_1s_2}^F \\ y_{r_2s_1}^F \\ y_{r_2s_2}^F \\ y_{r_3s_1}^F \\ y_{r_3s_2}^F \end{bmatrix}$$

$\hat{V}LA^F BY$ is the value added in the production of intermediates exported abroad ("GVCs"). It is the sum of simple GVCs ($\hat{V}LA^F LY^D$), or value added absorbed by the direct importer, and complex GVCs ($\hat{V}LA^F (BY - LY^D)$), or value added in re-exports and re-imports. It is the matrix product of the following:

\hat{V} = diagonalized value-added coefficients

L = local Leontief inverse matrix $(I - A^D)^{-1}$

A^F = technical coefficients matrix comprising only the foreign components
(foreign values in the off-diagonals)

B = global Leontief inverse matrix $(I - A)^{-1}$

Y = final demand vector, sum of the domestic and foreign final demand

For the case of 3 economies (r) and 2 sectors (s):

$$\begin{bmatrix} v_{r_1s_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & v_{r_1s_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & v_{r_2s_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & v_{r_2s_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & v_{r_3s_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & v_{r_3s_2} \end{bmatrix} \times \begin{bmatrix} l_{r_{11}s_{11}} & l_{r_{11}s_{12}} & l_{r_{12}s_{11}} & l_{r_{12}s_{12}} & l_{r_{13}s_{11}} & l_{r_{13}s_{12}} \\ l_{r_{11}s_{21}} & l_{r_{11}s_{22}} & l_{r_{12}s_{21}} & l_{r_{12}s_{22}} & l_{r_{13}s_{21}} & l_{r_{13}s_{22}} \\ l_{r_{21}s_{11}} & l_{r_{21}s_{12}} & l_{r_{22}s_{11}} & l_{r_{22}s_{12}} & l_{r_{23}s_{11}} & l_{r_{23}s_{12}} \\ l_{r_{21}s_{21}} & l_{r_{21}s_{22}} & l_{r_{22}s_{21}} & l_{r_{22}s_{22}} & l_{r_{23}s_{21}} & l_{r_{23}s_{22}} \\ l_{r_{31}s_{11}} & l_{r_{31}s_{12}} & l_{r_{32}s_{11}} & l_{r_{32}s_{12}} & l_{r_{33}s_{11}} & l_{r_{33}s_{12}} \\ l_{r_{31}s_{21}} & l_{r_{31}s_{22}} & l_{r_{32}s_{21}} & l_{r_{32}s_{22}} & l_{r_{33}s_{21}} & l_{r_{33}s_{22}} \end{bmatrix} \times \begin{bmatrix} 0 & 0 & a_{r_{12}s_{12}} & a_{r_{12}s_{12}} & a_{r_{13}s_{11}} & a_{r_{13}s_{12}} \\ 0 & 0 & a_{r_{12}s_{21}} & a_{r_{12}s_{22}} & a_{r_{13}s_{21}} & a_{r_{13}s_{22}} \\ a_{r_{21}s_{11}} & a_{r_{21}s_{12}} & 0 & 0 & a_{r_{23}s_{11}} & a_{r_{23}s_{12}} \\ a_{r_{21}s_{21}} & a_{r_{21}s_{22}} & 0 & 0 & a_{r_{23}s_{21}} & a_{r_{23}s_{22}} \\ a_{r_{31}s_{11}} & a_{r_{31}s_{12}} & a_{r_{32}s_{11}} & a_{r_{32}s_{12}} & 0 & 0 \\ a_{r_{31}s_{21}} & a_{r_{31}s_{22}} & a_{r_{32}s_{21}} & a_{r_{32}s_{22}} & 0 & 0 \end{bmatrix} \times \begin{bmatrix} b_{r_{11}s_{11}} & b_{r_{11}s_{12}} & b_{r_{12}s_{11}} & b_{r_{12}s_{12}} & b_{r_{13}s_{11}} & b_{r_{13}s_{12}} \\ b_{r_{11}s_{21}} & b_{r_{11}s_{22}} & b_{r_{12}s_{21}} & b_{r_{12}s_{22}} & b_{r_{13}s_{21}} & b_{r_{13}s_{22}} \\ b_{r_{21}s_{11}} & b_{r_{21}s_{12}} & b_{r_{22}s_{11}} & b_{r_{22}s_{12}} & b_{r_{23}s_{11}} & b_{r_{23}s_{12}} \\ b_{r_{21}s_{21}} & b_{r_{21}s_{22}} & b_{r_{22}s_{21}} & b_{r_{22}s_{22}} & b_{r_{23}s_{21}} & b_{r_{23}s_{22}} \\ b_{r_{31}s_{11}} & b_{r_{31}s_{12}} & b_{r_{32}s_{11}} & b_{r_{32}s_{12}} & b_{r_{33}s_{11}} & b_{r_{33}s_{12}} \\ b_{r_{31}s_{21}} & b_{r_{31}s_{22}} & b_{r_{32}s_{21}} & b_{r_{32}s_{22}} & b_{r_{33}s_{21}} & b_{r_{33}s_{22}} \end{bmatrix} \times \begin{bmatrix} y_{r_1s_1} \\ y_{r_1s_2} \\ y_{r_2s_1} \\ y_{r_2s_2} \\ y_{r_3s_1} \\ y_{r_3s_2} \end{bmatrix}$$

The forward GVC participation is given by the share of GVCs in total value added produced:

$$GVC_F = \frac{\hat{V}LA^F BY}{\hat{V}BY}$$

V. Revealed Comparative Advantage

Traditional Comparative Advantage

This measures the relative advantage or disadvantage of an economy-sector as "revealed" by trade patterns. It is the share of an economy-sector's gross exports in the economy's total gross exports divided by sector's gross exports from all economies as a share of world total gross exports.

In mathematical notation:

$$TRCA_i^r = \frac{\frac{e_i^r}{\sum_{i=1}^N e_i^r}}{\frac{\sum_{k=1}^G e_i^k}{\sum_i \sum_{k=1}^G e_i^k}}$$

Where:

e_i^r is the gross exports of economy r sector i

$\sum_{i=1}^N e_i^r$ is the total gross exports of economy r

$\frac{e_i^r}{\sum_{i=1}^N e_i^r}$ is the share of economy r sector i 's gross exports in economy r 's total gross exports

$\sum_{k=1}^G e_i^k$ is the gross exports of sector i from all economies

$\sum_i \sum_{k=1}^G e_i^k$ is the total world gross exports

$\frac{\sum_{k=1}^G e_i^k}{\sum_i \sum_{k=1}^G e_i^k}$ is the share of sector i 's gross exports from all economies in the world total gross exports

If $TRCA_i^r > 1$, economy r is said to have a comparative advantage in sector i , based on this framework

If $TRCA_i^r < 1$, economy r does not have a comparative advantage in sector i

New Revealed Comparative Advantage

Using gross exports, an economy's value added may be exported indirectly through exports to other sectors, and ignore foreign value-added content. These two components are not captured in gross exports. Compared to the TRCA, the new revealed comparative advantage includes indirect value-added exports and removes foreign value-added content. So instead of using gross exports, the NRCA utilizes all domestic value-added (DVA_F) exported based on forward linkages, regardless of where it is absorbed.

The NRCA is the share of an economy-sector's forward-linkage based measure of DVA in exports in the economy's total DVA, divided by that sector's total forward-linkage based DVA in exports as a share of global value added in exports.

DVA can be obtained through:

$$DVA_F = VAX_F + RDV_F$$

Where:

DVA_F is the total domestic value added exported based on forward linkages

VAX_F is the domestic value added exported or absorbed outside of the economy

RDV_F is the domestic value-added from a specific sector embodied in source economy s 's intermediate gross exports to economy r , but eventually return to and is absorbed in economy s , via all possible routines through third economies and other sectoral linkages.

The mathematical notation for the NRCA is:

$$NRCA_i^r = \frac{\frac{DVA_{F_i}^r}{\sum_{i=1}^N DVA_{F_i}^r}}{\frac{\sum_{k=1}^G DVA_{F_i}^k}{\sum_i^N \sum_{k=1}^G DVA_{F_i}^k}}$$

Where:

$DVA_{F_i}^r$ is the forward-linkage based measure of DVA in exports of economy r sector i

$\sum_{i=1}^N DVA_{F_i}^r$ is the total forward-linkage based measure of DVA in exports of economy r

$\frac{DVA_{F_i}^r}{\sum_{i=1}^N DVA_{F_i}^r}$ is the share of economy r sector i 's DVA in economy r 's total forward-linkage based measure of DVA in exports

$\sum_{k=1}^G DVA_{F_i}^k$ is the forward-linkage based measure of DVA in exports of sector i from all economies

$\sum_i^N \sum_{k=1}^G DVA_{F_i}^k$ is the total global value added in exports

$\frac{\sum_{k=1}^G DVA_{F_i}^k}{\sum_i^N \sum_{k=1}^G DVA_{F_i}^k}$ is the share of sector i 's DVA in all economies in total forward-linkage based measure of global value-added in exports

If $NRCA_i^r > 1$, economy r has a revealed comparative advantage in sector i , based on this framework

If $NRCA_i^r < 1$, economy r does not have a revealed comparative advantage in sector i