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The optimal size for a minority ⁺

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ABSTRACT

We address the issue of the optimal size of an ethnic or religious minority from the minority's perspective. Externalities are introduced, whereby cooperation by members of one group and the *relative* size of that group, affect the incentives to cooperate by members of the other group. The model assumes the existence of altruism, which is independent of the affiliation of the trading partner with whom one is transacting. We find the following results. First, cooperation requires social segmentation. As a result, there will never be intergroup cooperation. Second, bigger is not necessarily better, and the minority will often be interested in limiting its size. It will, however, never be optimal for the minority to totally assimilate itself into the majority. Third, there are instances in which the minority will be interested in promoting cooperation within the majority despite the ensuing negative externality on its own members' welfare. The model provides insights on social conflicts both between groups and within groups.

JEL Classification: D64, J15, J61

Key Words: Altruism, Minorities, Ethnic groups, Religious Groups, Relative group size, Social Conflicts.

1. Introduction

The economic role of ethnic and religious minorities in various contexts and times is well documented. The performance of Protestants in Catholic countries, Catholics in Protestant countries, Muslims in India, Balts in Northern Italy, Armenians in the Ottoman Empire, and Jews and Chinese almost everywhere, has undergone extensive scrutiny.¹ Work by economic historians in this field has recently been reinterpreted through the prism of institutional economics, showing that minorities have a comparative advantage in a range of economic activities characterized by pervasive informational imperfections and,

⁺ Corresponding author: Hillel Rapoport. E-mail: hillel@mail.biu.ac.il; Fax: +972 - 3 - 535 31 80. We thank Gary Fields, Patricia Vornetti, David Weil, Yoram Weiss, Joseph Zeira, seminar audiences at Bar-Ilan University, Hebrew University, the University of Paris-Sorbonne, and participants at the Conference on "Social Interactions and Economic Behavior", Paris, December 1999, for useful remarks on a previous draft. The usual disclaimer applies.

¹ See the proceedings of a conference on "The Economic Role of Minorities in Europe, XIII-XVIII", *Il Datini Lectures*, Firenze, April 1999. For an interdisciplinary perspective, see BREZIS & TEMIN (1999), and, for a reassessment of the role of Chinese networks in international trade, RAUSCH (1999).

consequently, high transaction costs. The advantages of minorities stem from their ability to enforce trust and to credibly sanction opportunistic behavior, resulting in more cooperation within minorities than within majorities.

The economic explanations for this phenomenon can be roughly divided into four categories. The first is differences in preferences – a higher degree of altruism or of “morality” could be assumed to exist within minorities, presumably because minority members have not yet been “contaminated” by the market mentality (BOWLES, 1998). Another reason for this could be that minorities invest more in the shaping of their members’ preferences and beliefs (GUTTMAN *et al.*, 1992). Second, the essence of the interactions may vary across groups, with economic transactions within the minority being embedded in overlapping social networks (FAFCHAMPS, 1992). As a result, interactions within a minority are analyzed within the framework of a repeated game, while the majority is seen as carrying out a series of one-shot interactions.² A third category (related to the previous one) deals with informational issues; for example, minority groups may incur lower informational costs when checking for the reputation of an agent (GREIF, 1989). Finally, the mere difference in group size may be of consequence. Because a minority is, by definition, relatively small, and because organizational costs have often been incurred by these smaller groups for other (such as cultural or social) purposes, the occurrence of free riding can be expected to be less prevalent since a potential defector would incur high costs from being excluded from the group’s social and economic networks (OLSON, 1965).³

Along the lines of this last category, this paper investigates the performance and incentives of members of a population that is bifurcated into a minority and a majority. The only difference between members of the minority and members of the majority is group affiliation - all members of society have the same preferences, costs are identical, and transaction frequencies are the same for all. Individuals are all equally altruistic towards all members of society, and are randomly matched in transactions with other individuals.

Our results are driven by two non-standard assumptions. First, we assume that the relative size of a group is what determines the incentives to cooperate, and not the absolute size, as assumed in prior research. The result of this assumption is that an increase in the relative size of the minority (which leads to an automatic decrease in the relative size of the majority) can spark cooperation *within the majority*. The second assumption is that the payoff

² Indeed, the achievement of cooperation may well require repetition (the “Folk Theorem”).

from non-cooperative transactions is positively dependent on the number of non-cooperative transactions that occur, because larger markets are more competitive.

The first of these assumptions, in particular, requires justification. The justification we offer is not particularly economic in nature – it is taken mostly from the fields of social psychology and of the sociology of intergroup relations (especially the economic sociology of immigration).⁴ A central finding of social psychology is that the activation of group identities requires confrontation between groups. The “minimal group” experience, as presented in BREWER (1979), illustrates this nicely. In that experiment, people were artificially divided into two groups according to a “heads or tails” procedure, and were then asked to distribute gifts to other participants. It was found that the amounts distributed to members of the group were robustly and significantly higher when there was another group present than when there was no other group. Even when groups were identified, there were no behavioral implications until its members were confronted with the presence of another group. More often than not, behavior within a group is affected not only by the mere existence of another group, but also by the composition, size, etc. of that other group. For example, numerous sociological studies provide extensive evidence that the relative size of a minority is critical in determining racial attitudes of the majority (GILES, 1977; FOSSETT & KIECOLT, 1989) and between minorities (CUMMINGS & LAMBERT, 1997).⁵ These studies demonstrate the importance of the relative size of an ethnic minority as a key factor for understanding a majority’s attitudes and behaviors. Obviously, such experimental and empirical evidence cannot be accounted for by the absolute size hypothesis.

The main idea behind this hypothesis is that contact with other groups can have important behavioral implications. For example, the presence of a potential conflict with another group can alter the behavior of individuals *even when not interacting directly with members of the group in conflict*. The only economic analysis of this type of issue we are aware of is the insightful paper by CARLTON (1995), which considers the issue of conflict and hostility between groups, and discusses ways of possibly altering preferences in order to minimize such conflicts (see also the comments on that paper by Masten, 1995, and Franke,

³ Another category concerns evolutionary economics, which deals with the transmission and reproduction of types of individuals, characterized by their genetic makeup. In such a framework cooperation can also be sustained through the use of informational devices (STARK, 1995, Chapter 5).

⁴ For collections of articles in these fields, see AUSTIN & WORCHEL (1986) on the psychology of intergroup relations and PORTES (1995) on the economic sociology of immigration.

⁵ See DUSTMANN & PRESTON (1998) for both a survey of the sociology of racial attitudes and a convincing methodological appraisal of that literature. Their results for the United Kingdom show that high concentrations of ethnic minorities are indeed associated with more hostile attitudes.

1995). In our paper we do not consider hostility or conflict directly, but our model could be interpreted as being pertinent for these issues because of the relative size hypothesis.

A notable innovation of this research is to endogenize the size of the minority within the strategic framework of intergroup relations. Is the size of a group a legitimate choice variable? While most of the literature on minorities takes group size as exogenous (with the notable exception of IANNACCONE, 1992), there is no a priori reason for this to be so. Members of one group may decide that they would prefer switching affiliation to a different group, because of pecuniary or non-pecuniary incentives. Members of a minority may decide that the existing restrictions are too onerous, and leave the minority, or there may be a stigma attached with membership in the minority, which will discourage others from joining. Furthermore, groups may be able to take steps to enhance the desirability of joining the group, or, even, to make joining less desirable. For example, a group of immigrants can set-up a network that will make future migration easier, thus increasing the base of potential immigrants. Similarly, a religious order can make joining desirable because of positive intragroup interactions, or make it less desirable by legislating religious restrictions that make joining highly costly, and, hence, unattractive. Thus, the size of the minority could change as a result, say, of natural reproduction or defection, or through the actions of some central body (an organization or religious board) that takes decisions that affect incentives.

Note that the justifications just given are equally appropriate for endogenizing the absolute or the relative size of a group. Indeed, IANNACCONE (1992) studies the determinants of the absolute size of a religious club or sect. The use of relative size instead of absolute size has a number of benefits. First, it allows us to analyze the effects of externalities the size of one group inflicts on another group, and, hence, to account for the strategic dimension of intergroup relations in a simple manner. Second, it shows how a decision by an individual to switch affiliation affects both groups simultaneously. Therefore, the use of relative size is both analytically convenient and methodologically appropriate.

Our model yields the following results. First, cooperation requires social segmentation. As a result, there will never be intergroup cooperation. Second, bigger is not necessarily better, and the minority will often be interested in limiting its size. It will, however, never be optimal for the minority to totally assimilate itself into the majority. Third, there are instances in which the minority will be interested in promoting cooperation within the majority despite the ensuing negative externality on its own members' welfare. The model also provides insights on social conflicts both between groups and within groups.

The paper is organized as follows. Section 2 presents the model and details the main assumptions made. In Section 3 we show how the optimal size of the minority is determined. Section 4 considers how the size of the minority is likely to evolve as a result of spontaneous departures or arrivals and as a result of actions by minority leaders. Section 5 suggests some extensions and concludes.

2. A simple model

We consider a population containing two groups: N members of a majority and M members of a minority. For illustrative purposes, we assume that the majority is a group of Natives, and the minority is comprised of a group of Migrants, although this interpretation is not essential. The proportion of migrants is thus:

$$p_M = \frac{M}{N + M},$$

with $N > M > 0$. Hence, by definition, $p_M < 1/2$.

Every period, each individual engages in a number of economic transactions, with the number of transactions being the same for all individuals in both groups. Transactions take place both within the group to which the individual belongs and outside the group, with the individual being randomly matched with other individuals.⁶ Thus, a proportion p_i of the transactions consummated by an individual belonging to group i , $i=M,N$ are carried out within the group to which he belongs. Exchanges take place without recognition costs, i.e., when a transaction is entered into, the individual immediately knows the group affiliation of his trading partner.

Each transaction can be carried out through a market mechanism, henceforth denoted a non-cooperative interaction, or via a cooperative agreement between the sides. Transacting through the market involves significant transaction costs that can be saved through a cooperative agreement.⁷ The feasible outcomes of each interaction can therefore be described by a non-cooperative one-shot Prisoner's Dilemma game. For a given member of group i , $i=N,M$, the payoff matrix for each transaction is as presented in Table 1, with $A > B > C > D$ and

⁶ While there would seem to be some justification for assuming that the individual would choose to transact relatively more with people in his own group, we assume randomness for two reasons. First, there is the opposing effect that people in the same group may have similar comparative advantages and thus be employed in similar occupations. In this case, intragroup trade would be relatively marginal. Secondly, including a bias towards relatively more (or less) intragroup trading would not change the essence of the results.

⁷ For example, because of informational imperfections, it might be that market transactions require the writing of a contract by a lawyer, a cost that can be avoided if the parties agree via handshake, and each party keeps his side of the agreement.

$2B > A + D$. A result of (B, B) is the cooperative, non-market based result, and a result of (C, C) is the payoff from interacting via the market.

Table 1: The payoff matrix for each transaction

	Cooperation	No Cooperation
Cooperation	(B,B)	(D,A)
No Cooperation	(A,D)	(C,C)

We now proceed to set values for each of these potential payoffs.

Without loss of generality, we assume that $D=0$.

As stated above, C denotes the return from transacting via the market. We assume that this payoff is dependent on the density of market transactions – the more market transactions that occur, the greater the payoff from dealing through the market (for an early exposition of this argument, see SMITH, 1776). The justification for such an assumption is immediate, as the greater the number of players, the closer the economy is to the ideal of perfect competition, and, hence, the more efficient is the market. One consequence of this is that the reward for a market (non-cooperative) transaction is negatively affected by the incidence of cooperative transactions.⁸ Holding total population constant, cooperative transactions reduce the size of the market and, consequently, increase the transaction cost that are incurred, thereby decreasing the gains from market exchanges.

Assume each individual carries out X transactions, with random matching. The total number of transactions in this instance will be $X(M+N)/2$.⁹ If all of these transactions are carried out via the market, the market will reach its maximal level of efficiency, and C attains its maximum value. We normalize the maximal market payoff to be equal to 1. We further assume, for simplicity, that the payoff, C , is linearly dependent on the number of transactions carried out in the market. The total number of transactions can be divided as follows. Each of the M members of the minority will have $XM/(M+N)$ dealings with other minority members, so that a total of $XM^2/(2(M+N))$ intragroup transactions will be carried out within the

⁸ Continuing the example in the last footnote, the bigger the market, the greater the number of lawyers that can provide the specific contract needed, so the more competitive the market for lawyers, and the lower the transaction costs. When transaction costs are minimal but still positive, the gain from a market transaction is maximal, but still lower than the cooperative payoff.

⁹ The division by 2 appears so that when two individuals interact, it is counted only once.

minority.¹⁰ Similarly for majority members, $XN^2/(2(M+N))$ intragroup transactions will occur. Dividing this by the total number of transactions, a portion p_i^2 of all transactions are carried out within group i , $i=N,M$. The payoff from a market transaction will be $C = 1 - d_N p_N^2 - d_M p_M^2 - 2d_{NM} p_N p_M$, where d_i , $i=N,M$, is a dummy variable that equals 1 if members of the group in question cooperate when dealing with members of their own group, and 0 if they do not, and d_{NM} is a dummy variable equal to 1 if there is intergroup cooperation, and 0 if not. Thus, for any size groups, $0 \leq C \leq 1$, with the reward for a market transaction equaling unity only when *all* transactions take place through the market (i.e., there are no cooperative agreements), and equaling zero when there are no market transactions.

Finally, we turn to the gain an individual receives by deviating from a cooperative agreement (the payoff A).¹¹ This is an increasing function of the relative size of the *relevant* group (the size of the group as viewed by the individual for *that specific transaction*, as detailed below). This is due to the alleviation of social sanctions when agents get more anonymous, or, in other words, to the increasing incentives to free ride in larger groups. While this is a widely used assumption, it is usually based on absolute sizes.¹² As discussed at length in the introduction, we stray from that standard assumption, and posit, instead, that the size of one group affects behavior within the other group, so it is relative size, and not absolute size, that is of interest. The consequence of our assumption is that a change in the size of one group automatically changes the size of the other group (which would not occur if absolute sizes were considered). Such a change not only influences the payoff structure for transactions within that group, it also influences the payoff structure for transactions *within* the other group, as demonstrated below.

We now make the following assumption:

Assumption 1: *When an individual in group i , $i=M,N$, transacts with an individual in the same group, he views the size of the relevant group as p_i , but when he deals with someone from the other group, he views the size of the relevant group as the entire population.*

¹⁰ This is only approximately true, because one can never be drawn to play oneself, so the number of migrants in the draw for a migrant is only $M-1$.

¹¹ Continuing the lawyer example, this is the case where the players decided to transact without a contract, and one player reneged on the agreement.

¹² For experimental evidence, see for example ISAAC & WALKER (1988).

This is a reasonable assumption, that highlights the fact that only when dealing with someone from the same group is free riding relatively costly (i.e., the payoff from defecting is relatively low), while the cost of free riding is brought to a minimum when dealing with someone from another group.

With these understandings, we conclude that the payoff A is expressed as a function of the relative size of the relevant group. Assuming a linear form, $A_{ij} = B(1 + \rho_{ij})$, $i, j = M, N$, where $\rho_{ij} \equiv \rho_i$ for intragroup transactions, and $\rho_{ij} = 1$ for intergroup transactions. Table 1 can now be rewritten as follows:

Table 2: The payoff matrix

	Cooperation	No Cooperation
Cooperation	(B,B)	$(0, B(1 + \rho_{ij}))$
No Cooperation	$(B(1 + \rho_{ij}), 0)$	$(1 - d_N \rho_N^2 - d_M \rho_M^2 - 2d_{NM} \rho_N \rho_M, 1 - d_N \rho_N^2 - d_M \rho_M^2 - 2d_{NM} \rho_N \rho_M)$

Our final assumption deals with altruism. Agents are homogeneous in their preferences independent of the group to which they belong; and they are assumed to have identical altruistic utility functions, so that the individual is concerned with the utility attained by his trading partner. More precisely, the individual's utility is a weighted average of the monetary payoffs of both trading partners, with a weight of $(1 - a)$ placed on his/her own payment, and a weight of a placed on the payment to the other party. As a result of this specification, the payoff table needs to be modified only in those cells in which players play different actions, i.e., in the off-diagonal cells. Note in particular that the same degree of altruism applies for intragroup and intergroup transactions, i.e. it is independent of the identity of the trading partner. This means that the eventual emergence of cooperation within one group alone will not be attributed to differences in preferences; it will be attributed to differences in the incentive structure faced by the individual.

We now make the following reasonable assumption:

Assumption 2: $a \leq 1/2$.

Assumption 2 indicates that the value attached by an individual to the welfare of his partner in the exchange can never exceed the value he attaches to his own felicity. An extreme case is when $a = 1/2$, i.e. when the individual values his partner's gains as much as he values his own.

We rewrite the payoff matrix as follows:

Table 3: The payoff matrix with altruism

	Cooperation	No Cooperation
Cooperation	(B,B)	$(aB(1+p_{ij}), (1-a)B(1+p_{ij}))$
No Cooperation	$((1-a)B(1+p_{ij}), aB(1+p_{ij}))$	$(1 - d_N p_N^2 - d_M p_M^2 - 2d_{NM} p_N p_M, 1 - d_N p_N^2 - d_M p_M^2 - 2d_{NM} p_N p_M)$

Note that although preferences are homogeneous throughout the population, payoffs differ in different transactions solely as a result of the different relative sizes of the relevant groups. These differences could be strengthened by introducing an asymmetry in the quality of social ties (increased altruism) in favor of the minority. In this case, the bias between groups in their relative performance in the private provision of public goods would be further enhanced (VAN DIJK & VAN WINDEN, 1996).

We assume that when the Pareto superior cooperative outcome is also a Nash equilibrium, cooperation is chosen.¹³ Thus, cooperation will be observed if the payoff when cooperating is higher than the payoff when defecting, i.e. if $B > (1-a)B(1+p_{ij})$. A sufficient condition for cooperation to be a possible (but not unique) equilibrium in intragroup transactions is therefore:

$$a > \frac{p_i}{1+p_i} \equiv a_i^{\min}. \quad (1)$$

Thus, we arrive at the following propositions:

¹³ The possibility of multiple equilibria is ignored; we simply assume the existence of coordination procedures if necessary.

Proposition 1: *The altruistic threshold required for cooperation to prevail in intragroup transactions is an (less than proportionally) increasing function of the relative size of that group.*

Proof: This is immediate because

$$\frac{\partial a_i^{\min}}{\partial p_i} = \frac{1}{(1+p_i)^2} > 0 \text{ and } \frac{\partial^2 a_i^{\min}}{\partial p_i^2} = -\frac{2}{(1+p_i)^3} < 0.$$

Corollary 1: *It can never be the case that the majority cooperates, but the minority does not. The converse, however, can occur.*

Proposition 2: *Cooperation is never achieved in intergroup transactions; cooperation requires social segmentation.*

Proof: As discussed above, in intergroup transactions, the entire population is viewed as a single group ($p_{ij} = 1$). To get intergroup cooperation, we thus need $B > 2(1-a)B$, i.e. $a > 1/2$ which is ruled out by Assumption 2.

As stated in Corollary 1, there are instances in which transactions among members of the minority will be cooperative while those among members of the majority will be non-cooperative, and this despite the fact that both groups share the same degree of altruism. Indeed, since by definition $p_M < p_N$, it must be that $a_M^{\min} < a_N^{\min}$ for any size minority. Hence, there are three possible cases: 1) if $a < a_M^{\min}$ there are no cooperative transactions (Area 1 in Figure 1 below), 2) if $a_M^{\min} < a < a_N^{\min}$ intragroup transactions are cooperative within the minority but non-cooperative within the majority (Area 2 in Figure 1), and 3) if $a > a_N^{\min}$ all intragroup transactions will be cooperative (Area 3 in Figure 1).¹⁴

Figure 1 about here

3. The Optimal Size

Until this point, the relative size of the minority (and, consequently, of the majority) has been exogenous. As discussed in the introduction, there are many instances in which steps can be taken to purposefully alter the size of a group. With this understanding, we proceed as

follows. In this Section we show what the *optimal* relative size of the minority is when viewed from the minority's perspective. This done, we then turn in the next Section to look at the question of whether it is likely that this optimum will be attained.

The main result of this section is that bigger is not necessarily better – an increase in the relative size of the minority may have ambiguous effects on the welfare of minority members. To show why, assume intragroup cooperation prevails for the minority alone. In this case, a higher relative size means a higher share of intragroup transactions, which increases the number of transactions that attain the high payoff B . There are, however, two opposing effects. First, the number of transactions carried out through the market decreases, lowering the payoff from each of the intergroup transactions (which are always carried out through the market). Hence, the gain from an additional cooperative transaction may well be offset by numerous small losses on each non-cooperative transaction. Second, an increase in the group's size decreases the relative size of the majority. At some point this will act as a trigger that will spark cooperation within the majority. This would be detrimental for the minority, again because of the ensuing (substantial) decrease in the number of market transactions, and, concurrently, in the payoff from these transactions.

To find the optimal size, we first calculate the earnings per minority member in each situation. Assume X , the number of transactions per person, is of measure 1. Thus, p_i measures the percentage of transactions concluded with members of group i . From Table 3, the income if there is no cooperation (I_{nc}) in either group equals one.¹⁵ If the minority alone cooperates, the payoff for a member of the minority is

$$I_{c,m} = p_M B + p_N (1 - p_M^2), \quad (2)$$

and if both groups cooperate, the payoff is:

$$I_{c,mn} = p_M B + p_N (1 - p_M^2 - p_N^2). \quad (3)$$

In the calculations that follow we substitute $p_N = 1 - p_M$, and, for notational simplicity, we write p to symbolize the size of the minority. This yields:

$$I_{c,m} = p^3 - p^2 + (B - 1)p + 1, \quad (2')$$

and

$$I_{c,mn} = 2p^3 - 4p^2 + (B + 2)p. \quad (3')$$

¹⁴ To give a numerical illustration, if the minority is one third and the majority is two thirds, the minority will experience intragroup cooperation while the incumbent majority will not when $0.25 < a < 0.40$.

¹⁵ From Proposition 2 we know that there is *never* intergroup cooperation.

Equation (2) is depicted in Figure 2. Analysis of the function in Equation (2) shows that if $B < 4/3$ there is a local maximum at $p_1^* = \frac{2 - \sqrt{16 - 12B}}{6}$, and a local minimum at $p_2^* = \frac{2 + \sqrt{16 - 12B}}{6}$. After this latter point the income will again increase, and there is a value, denoted p_3 , at which $I_{c,m}(p_1^*) = I_{c,m}(p_3)$. Note that p_1^* is increasing in B , attaining a maximum at $p_1^* = 1/3$, while p_2^* is decreasing in B . Since, by definition, the minority can never comprise more than half the population, it is of interest to note that $p_2^* = 1/2$ when $B = 1.25$, and $p_3 = 1/2$ when $B \approx 1.32$. Finally, if $B > 4/3$ income is monotonically increasing in the size of the minority, so that p_1^* , p_2^* and p_3 do not exist.

Figure 2 about here

We use this information to find the optimal size for the minority *under the initial assumption that the minority alone is cooperating* (Area 2 in Figure 1). If $a < 1/3$, the majority (which must comprise more than half the population) will never cooperate. Hence, the largest minority that will yield cooperation within the minority can be found from Equation (1): $p_{c,m}^{\max} \equiv a/(1-a)$. If, however, $a > 1/3$, this size minority could well spark cooperation within the majority also. As a result, the largest minority that will yield cooperation for the minority *only* is $p_{c,m}^{\max} \equiv 1 - [a/(1-a)] = (1-2a)/(1-a)$.

The optimal size *assuming cooperation within the ranks of the minority only* is now found as follows. If $B > 4/3$, since $\frac{\partial I_{c,m}}{\partial p_M} > 0 \quad \forall p_M$, the optimal size, $p_{c,m}^*$, is $p_{c,m}^* = p_{c,m}^{\max}$.

This simply means that for sufficiently high returns to cooperation, it is always welfare improving for the minority to increase its size as long as this does not induce cooperation within the majority.¹⁶ If $B < 4/3$, however, then if $p_{c,m}^{\max} < p_3$, $p_{c,m}^* = \min(p_1^*, p_{c,m}^{\max})$, and if $p_{c,m}^{\max} > p_3$, then $p_{c,m}^* = p_{c,m}^{\max}$.¹⁷

Having set the optimal size for the minority when the minority alone cooperates, we must now address two questions. First, does the minority always prefer a situation where the

¹⁶ A comparison of this outcome with the outcome when the majority also cooperates is carried out below.

¹⁷ See Figures 3.1 to 3.3 for the different cases.

minority alone cooperates to a situation where no one cooperates? Second, does the minority always prefer a situation where the minority alone cooperates to a situation where both groups cooperate? As we now show, the first question is answered in the affirmative, while the second question is not.

To answer the first question, recall that the payoff from non-cooperation in either group, I_{nc} , equals 1. As long as the payoff at the optimum with minority cooperation only is greater than one, such cooperation will be preferred by the minority. As we show in the proof to the following Proposition, this always holds at the optimal size.

Proposition 3: *The optimal size for the minority is always strictly positive. That is, $I_{c,m} > 1$ if $p = p_{c,m}^*$.*

Proof: Note from Equation (2) that when $p = 0$, $I_{c,m} = 1$. Furthermore, when $p = 0$, $\partial I_{c,m} / \partial p = B - 1 > 0$. (As discussed above, $I_{c,m}$ attains a local maximum at p_1^* if $B < 4/3$, and continually increases if $B > 4/3$.) Therefore, the optimal minority size must yield at least as high a payoff as attained at $p = e$, so $I_{c,m} > 1$. Note that this is independent of the value of a , as long as it is bounded away from 0 and $1/2$ (see Figures 3.1-3.5)

To answer the second question posed above, we now compare the outcome without majority cooperation with the outcome if there is also cooperation within the majority. When $a < 1/3$, the majority will never cooperate so that optimality is achieved from the minority's perspective when its size is set as detailed above. However, when $a > 1/3$ there are instances (when p is sufficiently large) in which there is intragroup cooperation in both groups (Area

3). As seen from Equation (3), since $B > 1$, $\frac{\partial I_{c,mm}}{\partial p} > 0 \quad \forall p$, so if the economy is already in

Area 3, it is optimal for the minority to have the minority as large as possible ($p \rightarrow 1/2$). At that point, $I_{c,mm} = 0.25 + 0.5B$. Thus, we must compare the highest payoff when there is cooperation within the minority alone, to this value. To this end, we have the following proposition:

Proposition 4: a) If $B \leq 1.5$ the minority always prefers that the majority not cooperate. b) If $B > 1.5$, there exists a value of a , denoted a^* , such that if $a \leq a^*$ the minority prefers no

cooperation within the majority, and if $a > a^*$ the minority prefers that the majority also cooperate. c) In addition, this altruistic threshold is decreasing in B , i.e., $\partial a^*/\partial B < 0$.

Proof: a) If $B \leq 1.5$, $I_{c,mn}(p \rightarrow 1/2) \leq 1$. As proven in Proposition 3, the payoff at the optimal minority size with minority cooperation only is strictly greater than 1 ($I_{c,m}(p_{c,m}^*) > 1$). Consequently, the first part of the proposition holds.

b) If $B > 1.5$, since we are dealing with cases in which $a > 1/3$, the optimal size of the minority with minority cooperation only is $p_{c,m}^{\max}(a) = (1 - 2a)/(1 - a)$. Note that for values of a between $1/3$ and $1/2$ there is a one-to-one and onto relationship between $p_{c,m}^{\max}$ and a . In particular, when $a = 1/3$, $p_{c,m}^{\max}(a) = 1/2$, when $a = 1/2$, $p_{c,m}^{\max}(a) = 0$, and $p_{c,m}^{\max}(a)$ is strictly decreasing in a . Hence (from Equation (2)), the greatest payoff when the minority alone cooperates is $I_{c,m} = (p_{c,m}^{\max}(a))^3 - (p_{c,m}^{\max}(a))^2 + (B - 1)p_{c,m}^{\max}(a) + 1$. Recall that the maximal payoff when both groups cooperate is $I_{c,mn} = 0.25 + 0.5B$. a^* , as defined in the proposition, is found by finding the value of $p_{c,m}^{\max}(a)$ for which $I_{c,m} = I_{c,mn}$.

As shown earlier, for values of $B > 1.5$, $\partial I_{c,m}/\partial p > 0$, or, in the cases under consideration, $\partial I_{c,m}/\partial a < 0$. In addition $I_{c,mn}$ is not dependent on π . Thus, as a increases, $I_{c,m} - I_{c,mn}$ decreases. Now, when $a = 1/3$, $I_{c,m} - I_{c,mn} = 1/8 > 0$, and when $a = 1/2$, $I_{c,m} - I_{c,mn} = 0.75 - 0.5B < 0$ (since $B > 1.5$). Hence, there exists a value of a , a^* , for which $I_{c,m} = I_{c,mn}$.

c) With respect to the last part of the proposition, replace a^* in the desired equality to get the identity $L \equiv (p_{c,m}^{\max}(a^*))^3 - (p_{c,m}^{\max}(a^*))^2 + (B - 1)p_{c,m}^{\max}(a^*) + 1 - 0.25 - 0.5B \equiv 0$.

Totally differentiating L with respect to $p_{c,m}^{\max}$ and B and rearranging, we find that

$$\frac{dp_{c,m}^{\max}}{dB} = \frac{(0.5 - p_{c,m}^{\max})}{\partial L / \partial p_{c,m}^{\max}} > 0, \text{ since } \partial L / \partial p_{c,m}^{\max} = \partial I_{c,m} / \partial p_{c,m}^{\max} > 0, \text{ and } p_{c,m}^{\max} < 1/2. \text{ Hence,}$$

$$\partial a^* / \partial B < 0.$$

Figures 3.1-3.5 show all the relevant cases, with the darkened lines representing the optimum. In 3.1 and 3.2 $p_3 > 1/2$, so p_1^* is optimal when it can be attained. Because of the relatively low value of B , the minority always prefers to keep the majority from cooperating,

even for high values of a . In 3.3 $p_3 < 1/2$, so when $p_{c,m}^{\max} > p_3$, then $p_{c,m}^{\max}$ is optimal, but if $p_{c,m}^{\max} < p_3$, the minority is better off at p_1^* (if attainable without majority cooperation). In 3.4 p_1^* does not exist, but B is still low enough that the minority is better off by keeping the majority from cooperating through a reduction of its own size if necessary. In 3.5 B is large enough that, above some values of a , the cost of keeping the majority from cooperating is too great, and the minority is better off with a larger group (more cooperative transactions) despite the induced cooperation within the majority.

Figure 3 about here

4. Moving Towards the Optimal Size

With the task of finding the optimal size from the minority's perspective complete, we now ask whether this optimal size can be expected to be attained. We look at this question from three perspectives – one that can be described as an "invisible hand" perspective, and two that fit into the category of a "visible hand". The first is the personal choices of individuals considering changing affiliation. While the choice to change affiliation is most probably affected by many determinants, we assume that pecuniary issues alone are considered.¹⁸ Hence, *ceteris paribus*, individuals will desire to join (leave) a minority if and only if the expected income in the minority is greater (lower) than in the majority. The other two forces deal with the leadership of the minority, interested in maximizing income per member. We look at two kinds of leaderships – a "myopic" (or "local") leader, and an omniscient (or "global") leader. The difference between the two is that the myopic leader sees how a small (local) change in the relative size affects income, but does not see the overall picture, i.e., he knows only the sign of the derivative. An omniscient leader, however, knows the entire function, and can thus strive for the global optimum, although this may require a discrete jump in size. Consider, for instance, the case in Figure 2 in which $B < 4/3$. For values of p such that $p_1^* < p < p_2^*$, the local leader will desire a decrease in the size of the minority in order to return to the locally optimal size of p_1^* , while the global leader will push towards increasing the relative size past p_3 , assuming that $p_3 < 1/2$. We show the

¹⁸ In particular, we ignore issues related to other differences between the members of the two groups. For instance, differences in the average amount of human capital, and differences in average wealth from other income sources, are not dealt with. Human and physical capital are assumed to be equally valuable in market transactions as a member of either group.

directions of these three forces (individual incentives, local leader and global leader) with three sets of arrows in Figures 3.1-3.5.

Individual incentives can be best discerned from Figure 1. If the economy begins inside Area 1 of Figure 1 no one cooperates, so that everyone earns the same payoff. Thus, there is no incentive to switch affiliation (and, as a result, no arrow in the figures in this area). In Area 2 the minority alone cooperates, so the income per member is greater for the minority than for the majority. Hence if only pecuniary considerations are of concern, there will be a desire by majority members to join the minority. In Area 3, however, both groups cooperate, and because there are more cooperative intragroup interactions within the majority than within the minority, majority members earn more than minority members, so, *ceteris paribus*, there will be defection from the minority to the majority (assimilation).

A myopic leader in Area 1 will similarly see that local changes in the relative size have no effect on income. Hence, in Area 1 there is no pressure to increase or decrease the size of the minority (and, again, no arrow in the figures). With respect to Area 2, if p_1^* , p_2^* and p_3 exist (i.e., if $B < 4/3$, as in the lower curve in Figure 2), then for values of p_M between 0 and p_2^* the move will be towards p_1^* , and if $p_2^* < p_M < p_{c,m}^{\max}$ the move will be towards $p_{c,m}^{\max}$. If p_1^* , p_2^* and p_3 do not exist, the move will always be towards $p_{c,m}^{\max}$. In Area 3, however, the local leader will always desire a larger minority because, as discussed above, the payoff is locally increasing in minority size.

The omniscient leader, by definition, will always want to move towards the optimal size. Hence, the arrow will always point toward the darkened lines that denote the optimal size.

As can be seen in the Figures, there are numerous instances of opposing forces. When all three arrows point in the same direction, it is likely that the fate of the minority members will be improved over time. However, when there are opposing forces, there is no a priori way of determining what forces will prevail. While it is possible to make ad-hoc assumptions about the relative strengths of these forces, we do not do so. Rather, we simply show the directions of the forces in order to demonstrate the kinds of conflicts that can arise.

For instance, in Figure 3.2 in Area 2 when $p_2^* < p_M < 1/2$, members of the minority earn more than do members of the majority, so there will be a desire by members of the majority to join the minority. From a local perspective an increase in the size of the minority will increase the minority members' incomes, and so will be also be popular politically. But a

global leader will desire a decrease in the size of the minority, and will want to take measures to keep these outsiders out (and, most likely, even harsher measures to drive some of the insiders out). This global leadership, however, will have to fight both market forces from both inside and outside the group, and internal pressures from members who see their incomes initially falling as the group size is decreased.

A glance at Figures 3.1-3.5 shows that almost all possible combinations of forces exist. Determining which forces will prevail when the forces point in different directions, is, however, beyond the scope of this research.

5. Conclusion

In the model presented in this paper, members of a minority and a majority play repetitive rounds of a one-shot prisoner's dilemma game, and despite all individuals being identical from all respects except group affiliation, they may behave differently with different trading partners. Given this setting, the optimal size for the minority was discerned, and expected endogenous changes in the minority size were examined from the perspectives of individual members of the society, and of a planner for the minority. We have shown that when considering a change in size, the minority's planner must take two effects into account – the direct effect on welfare, and the indirect effect on the behavior of the majority. Assuming that cooperation initially prevails within the minority alone, an increase in the minority size negatively affects the welfare of the majority through a decrease in the breadth of market exchanges. This, in turn, affects the majority's incentives to adopt a cooperative mode in its internal transactions. The minority may wish to limit its size in order to prevent cooperation within the majority, since such cooperation will, in turn, decrease the breadth of market transactions yet more, and, consequently, harm the minority.

In our model we have abandoned the usual assumption that the absolute size of a group affects behavior within the group, and replaced it with the postulation that it is the relative size of a group that determines behavior. We do this for four reasons. First, when determining the optimal size for the minority, one needs to consider the environment in which the minority finds itself. One important characteristic of this environment is the existence of the majority, and the use of relative sizes in our model takes this characteristic into consideration in a clear and direct manner. Second, an immediate consequence of the use of relative size is that what is done within one group has effects on the other. Since members of

both groups recognize these effects, the use of relative size allows us to account for the strategic dimension at work in intergroup relations.

Third, we believe that our theory is of interest for the analysis of intergroup tensions. Such tensions, prevalent in both developed and developing countries, are generally ignored by economists (with the notable exception of CARLTON, 1995), and left to the sagacity of other social scientists. In our model, the relative size hypothesis provides economic foundations for the emergence of hostile attitudes and behaviors towards minorities on the part of members of a majority. Hostile attitudes emerge initially due to a decrease in the market payoff to majority members resulting from an increase in the minority size. Note that the negative impact on payoffs is initiated by the mere existence of a minority. Still, hostility will most probably not arise while the minority remains small, since the effect would be minute. However, when the minority increases, the negative effect increases exponentially, which can trigger such hostility.¹⁹ This conclusion is consistent with the findings of the sociology of immigration, as discussed in the introduction. In addition, hostile behavior could deepen if the majority switches from a non-cooperative to a cooperative mode its in internal transactions, since this would introduce discrimination into the economy. It is worth emphasizing that, in our model, hostile attitudes and behavior arise despite the presence of altruism in individual relations. That is, the source of social conflicts is not at the individual level but rather at the group level.

Fourth, we have also shown that the issue of minority size may give rise to internal conflicts within the minority. At the individual level (the invisible hand), people compare the personal costs and benefits of changing group affiliation, and of integrating new members into their group. Their willingness to integrate new members (that is, to make the group more attractive to potential newcomers) or to leave the group might be backed or opposed by the group authorities. Moreover, the group authorities themselves (the visible hand) may disagree on the relevant strategy for the group. Indeed, there are many instances in which a “local” planner (who knows only the effect of including or excluding one additional member) and a “global” planner for the minority (who knows the precise optimal size for the minority) will pull in opposing directions. Taken together, these potentially opposing forces may lead to strong internal conflicts and eventually to schisms, excommunications, etc.

¹⁹ Note that such a hostile attitude may be economically irrational in the case of ethnic minorities that grow through immigration flows. These flows tend to increase the size of the market and to consequently benefit the native population through an increase in the immigration surplus (BORJAS, 1995). However, at a cognitive level, people might retain the negative effect of the increase of the minority size as explicitly modeled in the paper.

The lessons from our model would seem to be widely applicable. For instance, we have not differentiated between ethnic and religious minorities, but could do so. A central difference between ethnic minorities and religious minorities, is that the relative size of the latter is primarily determined through competition among existing groups, while the relative size of the former depends mainly on immigration and assimilation flows. Applying our framework to these two issues could yield interesting insights on religious proselytism or exclusion, as well as on the degree of hospitality or lack of hospitality of existing ethnic communities towards newcomers. Another straightforward prediction of our theory is that the choice of location may be critical for the success of a minority – indeed, location choice might be a cost-efficient means for the minority to fix its relative size. For instance, the minority could prefer to locate itself in big cities rather than in small cities. Finally, a natural extension of the model would be to incorporate other channels through which the wellbeing of one group affects that of another group (e.g., social status, frustration, relative deprivation, etc.), and to consider how these alternative channels affect the degree of cooperation within the group.²⁰

²⁰ See BEAUDRY *et al.* (2000) for a general game theory framework in which cooperation within one group affects the occurrence of cooperation within other groups.

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Figure 1.

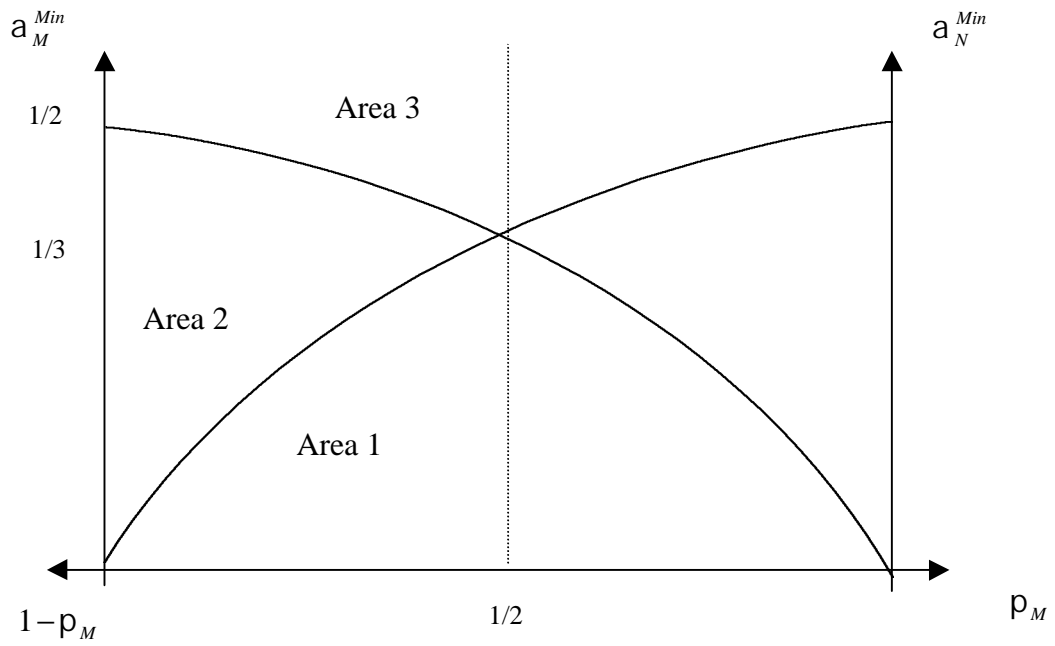


Figure 2

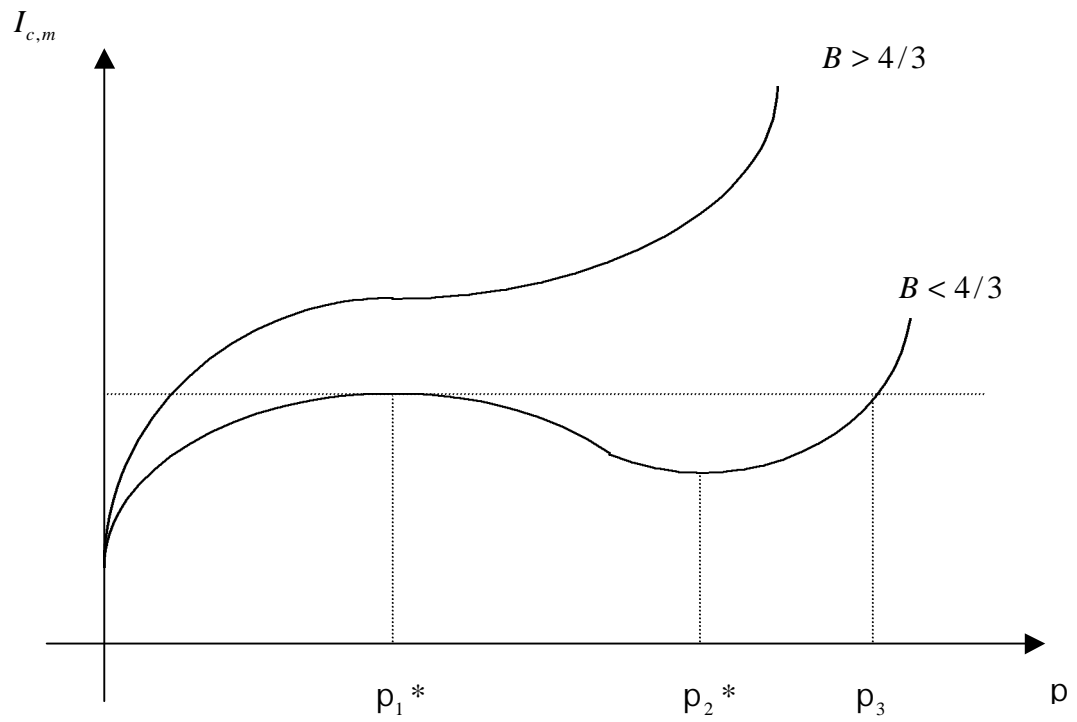
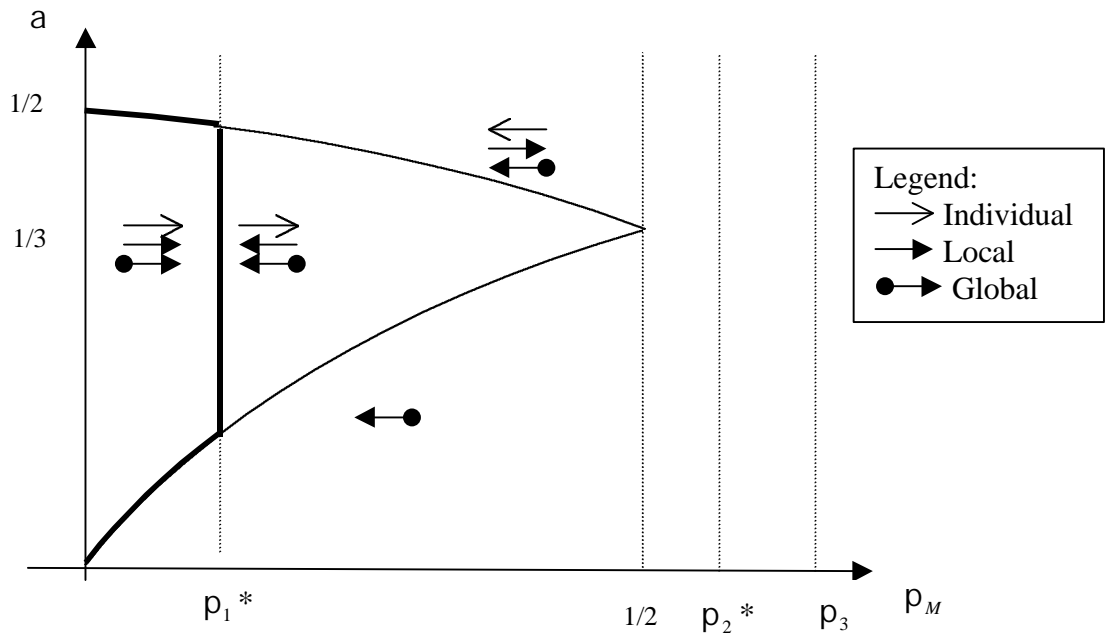
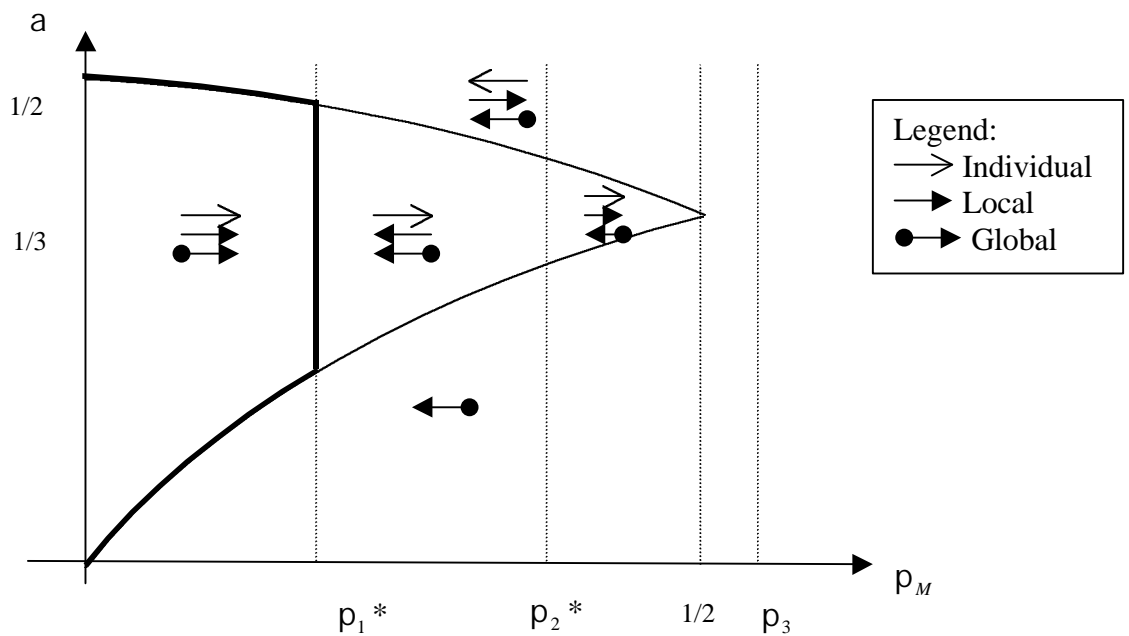


Figure 3.

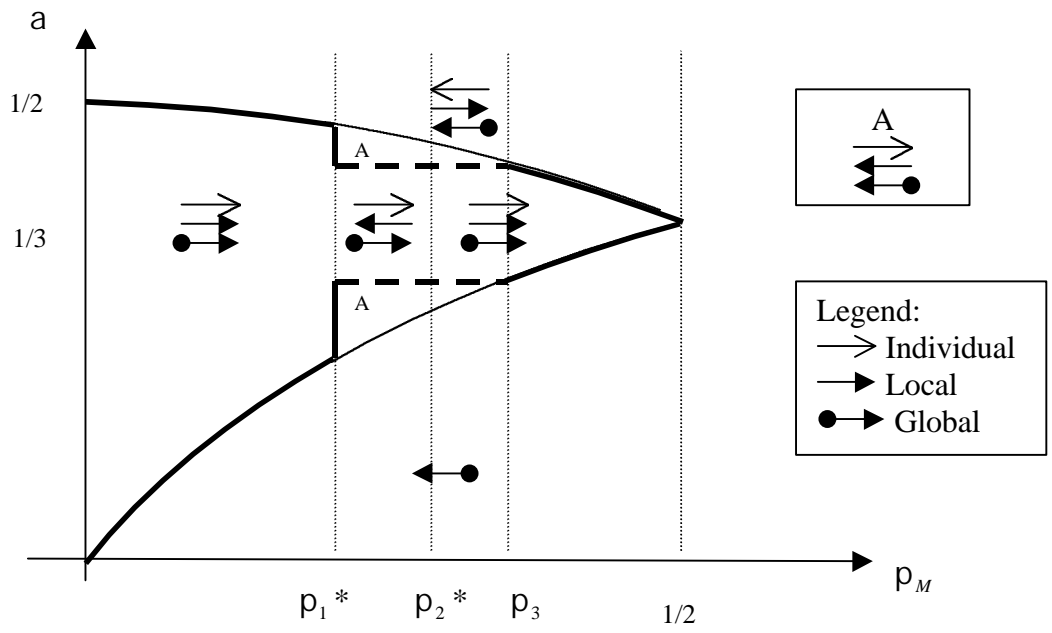
Case 1: $B < 1.25$, $p_1^* < 1/2 < p_2^* < p_3$.



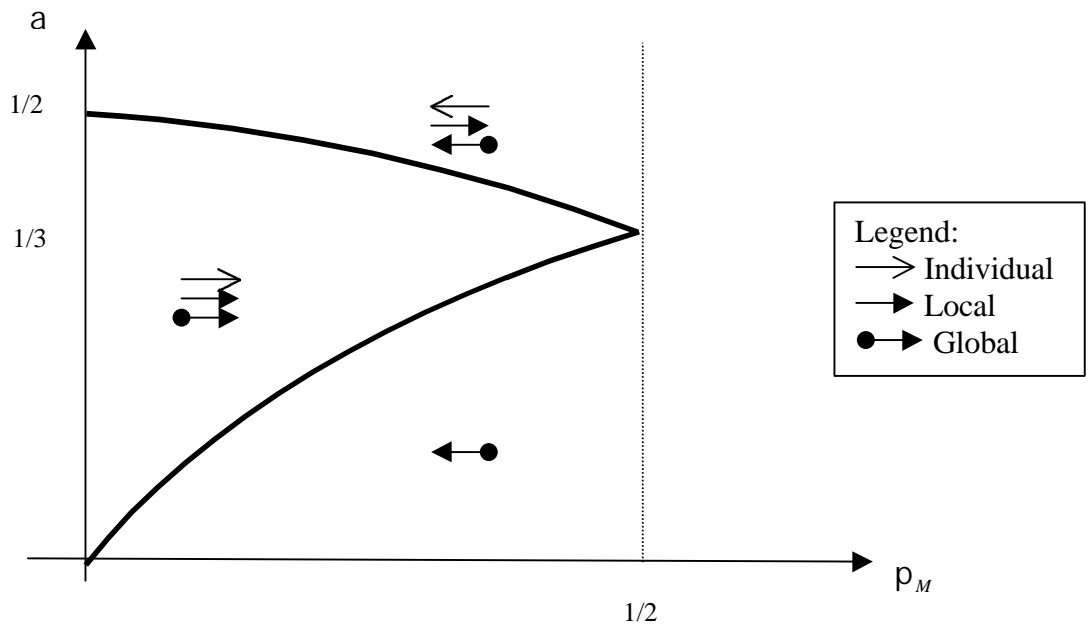
Case 2: $1.25 < B < B_c$, $p_1^* < p_2^* < 1/2 < p_3$.



Case 3: $Bc < B < 4/3$, $p_1^* < p_2^* < p_3 < 1/2$



Case 4: $4/3 < B < 1.5$



Case 5: $B > 1.5$

