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# Article Choosing a Price and Cost Combination—The Role of Correlation

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**Abstract**: Often, firms can choose from different combinations of price and cost processes. For example, they can choose between different production locations or technologies, between different products to produce, or between different locations for selling them. To study the choice of the optimal combination, we return to themodel that was developed by Dixit and Pindyck, where both output price and production cost are stochastic processes, and add a novel focus on how the correlation between these processes affects the firm's decision. We find that, ceteris paribus, the firm prefers the combination with the lowest correlation between the processes, as it seeks a greater profitability variance which maximizes its value.

Keywords: investment; real-options; correlation

JEL Classification: C61; D40

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**Copyright:** © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). *"Our goal is to produce cars where they're sold."* (Paul Nolasco, a spokesman for Toyota in Tokyo, New York Times, 3 September 2010)

# 1. Introduction

As in the statement above, firms can often choose among different combinations of price and cost processes. This variety of combinations springs, for example, from an ability to choose between different production locations or technologies, between different products or between different market locations. In this study, we model such a case, assume that both output price and production cost are stochastic processes and focus on the role of the correlation between them in choosing the optimal price–cost combination in which the firm is going to invest.

The typical models of the literature about investment under uncertainty study the optimal policy of a firm that can choose the investment timing optimally. They also assume that the investment enables the production of a certain good and that the demand for this good evolves stochastically across time. The main results of this study are:

- The optimal policy of the firm is to delay its investment and remain idle as long as the market price of the good is below a certain threshold and invest and become active when the price reaches this threshold.
- The entry threshold is above the long-run average cost, as by becoming active, the firm loses not merely the investment and production costs but also loses the option to delay its investment furthermore.
- Uncertainty delays investment—the higher the uncertainty, the higher the investment threshold.

In the current study, we take a step further from these typical models and assume that, alongside the demand, the production cost is a stochastic process too. Such cases have already been analyzed in a variety of studies, some purely theoretical and others which have applied this theory to a variety of fields such as renewable energy, mining, forestry, information technology and electricity systems reliability, to name a few.<sup>1</sup> However, those studies have focused on finding the profitability investment threshold and have not looked at how it is affected by the price–cost correlation. In the current study, we fill this gap by analyzing how the degree of this correlation affects the optimal investment policy and the value of the firm. We then use these results to look at a case where the firm can choose its price and cost processes out of several alternatives and to study how the price–cost correlation affects the choice of price and cost combination.

Our main findings are that:

- The higher (i.e., the closer to 1) the price-cost correlation, the lower the value of the firm
- In choosing between different combinations of price and cost processes, the firm prefers, ceteris paribus, the one with the lowest (i.e., the closest to -1) correlation.

The explanation for these results is as follows: With demand and cost being stochastic so is profit. The larger the correlation between the price and cost processes, the more likely it is that in each time period the price and the cost change at similar magnitudes and balance one another, which lowers the (absolute) size of the change in profitability. Likewise, the smaller (i.e., the closer to -1) the correlation between these processes, the more likely it is that changes in price and cost would go at different directions, making the profit process more volatile.

Thus, the higher the demand–cost correlation the lower the uncertainty in the profitability process and therefore the lower the profitability threshold that triggers investment and the faster the investment is expected to happen.

However, the actual interest of the firm is maximizing its value and not speeding up its investments. Therefore, it prefers the price–cost combination with the lowest correlation because it maximizes the uncertainty of the resulting profit process and thus maximizes the value of the firm, even though it delays its investment. The logic behind the positive effect that profit uncertainty has on value is that a greater uncertainty is comparable to a symmetric mean-preserving spread of the profit distribution at each future point in time. The behavior of the firm is not symmetric though: it invests and receives a stream of profits only when the profit reaches a sufficiently high threshold and remains idle if the profit levels dominates the rise in the likelihood of the relatively large profit levels and makes the spread raise the value of the firm.<sup>2</sup>

One of the main applications of this result regards the case of a firm that can choose where to sell and where to produce. Consider, for example, a Chinese producer that sells in the USA market and contemplates whether to produce in the USA or in China. Many factors influence this decision: production costs are (on average) lower in China; producing in China causes shipping cost; producing in the USA may have a positive influence on USA consumers' sentiments and raise local demand; and producing in the USA may also suffer from the difficulties of controlling production conducted abroad. There are other variables in this dilemma, and our focus is on the correlation between the swings in the prices at the USA market and the swings in production costs at the production location. Taking the reasonable assumption that the USA prices are more positively correlated with USA costs than with production costs in China yields that the intention to enjoy a smaller correlation is a factor that pushes toward producing in China.

This logic applies not merely in a spatial context but could apply also to other cases where a firm can choose between different combinations of price and cost processes. For example, when a firm can choose between different production technologies and should prefer, ceteris paribus, the one that generates the cost process with the smallest correlation with the price process.

Likewise, our results also apply to cases where the firm has a given production process that fits several different goods and contemplates which one to produce or to cases where the firm can choose at which market to sell its product. As in the examples before, ceteris paribus, it chooses the combination with the lowest price–cost correlation. The cases in which the firm can choose among different investments, each with its own price and cost processes, are not new to the literature about investment under uncertainty. Such cases have been analyzed in the theoretical studies of Dixit (1993); Décamps et al. (2006); Hagspiel et al. (2016) and in applied studies dealing with real-estate Capozza and Li (1994); Geltner et al. (1996), biodiesel energy Tareen et al. (2000), energy for agricultural uses Lima et al. (2013), and transmission cables Bakke et al. (2016), to name a few. However, none of these studies have uncertainty both in the price and in the cost processes and therefore cannot study the role of correlation in the choice of the firm. An important exception is Boomsma et al. (2012), which deals with renewable energy and different alternatives for the firm to choose from. The theoretical part of this study does have both price and cost uncertainty but it assumes that these processes are not correlated (and offers some empirical support for that).

The study that is perhaps the closet to ours in its results is Armada et al. (2013), which presents an investment model with two stochastic processes and, among other things, studies how the correlation between them affects the entry threshold and the value of the firm. However, the issue they study is different from the one studied here as they look at a case where both processes are on the revenue side. Specifically, in their model, the multiplication of the two processes forms the stream of proceeds from the investment, as in the case of price and quantity, or price in foreign currency and the exchange rate.

Section 2 presents the model and the optimal investment policy of the firm, for a given pair of price and cost processes. Section 3 studies how the correlation coefficient affects the value of the firm in that case. Section 4 adds the possibility of optimally choosing these processes and searches for the characteristics of the optimal choice. Section 5 concludes the study. Technical details are relegated to two short appendices.

### 2. The Model and Its Solution

Consider a risk neutral firm that can enter the market for a certain good at any time point. Time is continuous, and once the firm enters, it produces indefinitely at a rate of one unit of output per unit of time at the variable cost *W* per unit produced and sells this unit at the price *P*. Both *P* and *W* change stochastically over time in geometric Brownian motions, under the following rules of motion:

$$dP_t = \alpha_P \cdot P_t \cdot dt + \sigma_P \cdot P_t \cdot dZ_t \tag{1}$$

$$dW_t = \alpha_W \cdot W_t \cdot dt + \sigma_W \cdot W_t \cdot dH_t \tag{2}$$

where  $\alpha_P$  and  $\alpha_W$  are drift parameters,  $\sigma_P$  and  $\sigma_W$  are the standard deviation of the incremental changes in *P* and in *W*, respectively, and  $dZ_t$  and  $dH_t$  are the increments of standard Wiener processes, satisfying at each point in time:

$$E(dZ_t) = E(dH_t) = 0, (3)$$

$$E(dZ_t^2) = E(dH_t^2) = dt,$$
 (4)

$$E(dZ_t \cdot dH_t) = \rho \cdot dt \tag{5}$$

where the parameter  $\rho$ , satisfying  $-1 \le \rho \le 1$ , denotes the coefficient of contemporaneous correlation between  $dZ_t$  and  $dH_t$ .

The interest rate is constant and denoted by *r*. The convergence of the firm's revenues and expenses requires  $r > \alpha_P$  and  $r > \alpha_W$ , as shall be seen later. The firm is risk neutral and therefore maximizes its expected value.

To enter the project at time *t*, the firm must incur a sunk cost,  $k \cdot W_t$ , where *k* is a positive constant. This assumption is a simplifying one, and we take it primarily to ensure an analytical solution for the model.<sup>3</sup> However, it is also less restrictive and more plausible than the typical assumptions which are taken in the relevant literature with regard to this cost. Specifically, under this assumption the entry cost has the following two noteworthy

features: first, it is not constant but changes over time; and second, it is positively connected to the flow of the production cost. The second feature may reflect a rather plausible scenario where both construction of the project and the subsequent production conducted in it use similar inputs such as labor or electricity. By contrast, in almost all the models of the relevant literature, this cost is assumed constant.<sup>4</sup> The deviation from the typical form of the entry cost to this less restrictive form does not affect the main results of the analysis as it preserves the irreversibility of the investment. This irreversibility generates the main force underlying the results, namely the convex effect of the profit flow on the value of the firm. In addition, the assumption about the entry cost is not connected to the other main force underlying the results, namely the negative effect of the correlation between price and cost on the variance of the profitability flow.

Thus, the expected value of the total costs following entry at time *t* are:

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$$I_t \equiv k \cdot W_t + E\left[\int_t^{\infty} W_{\tau} \cdot e^{-r \cdot \tau} \cdot d\tau\right] = k \cdot W_t + \frac{W_t}{r - \alpha_W} = \left(k + \frac{1}{r - \alpha_W}\right) \cdot W_t, \quad (6)$$

where the calculation of the expectancy follows from the standard properties of the geometric Brownian motion. Since  $I_t$  is proportional to  $W_t$ , it follows from Itô's lemma that  $I_t$  is also a geometric Brownian motion with drift and standard deviation parameters identical to those of  $W_t$ , i.e.,  $\alpha_W$  and  $\sigma_W$ .

Thus constructed, the model is mathematically identical to the one studied in pp. 207–211 of Dixit and Pindyck (1994).<sup>5</sup> The analysis from here to the end of this section briefly presents their analysis.

Let V(P, W) denote the value function of an active firm, given the current levels of P and W.<sup>6</sup> Based on the assumptions above, V(P, W) satisfies:

$$V(P, W) = E\left[\int_{0}^{\infty} (P - W) \cdot e^{-rt} \cdot dt\right] = \frac{P}{r - \alpha_P} - \frac{W}{r - \alpha_W}$$
(7)

Let F(P, W) denote the value function of an idle firm, given the current levels of P and W. The inactive firm has no operating profit and its value springs from the opportunity to invest later. As Dixit and Pindyck (1994) point out, both V(P, W) and F(P, W) are homogenous of degree 1 since multiplying P and W by the same amount also multiplies both the expected value of discounted revenues and the expected value of total costs by the same amount. Based on this homogeneity, Dixit and Pindyck (1994) define the markup M and the functions v(M) and f(M) as follows:

$$M \equiv \frac{P}{W} \tag{8}$$

$$V(P, W) = W \cdot V(M, 1) = W \cdot v(M)$$
(9)

$$F(P, W) = W \cdot F(M, 1) = W \cdot f(M)$$
<sup>(10)</sup>

From (9) and (7), it follows that:

$$v(M) = \frac{M}{r - \alpha_P} - \frac{1}{r - \alpha_W},\tag{11}$$

In Appendix A, we show that as the ratio of two such processes, *M* is a geometric Brownian motion too, and with a variance parameter, denoted  $\sigma_M^2$ , which satisfies:

$$\sigma_M^2 = \sigma_P^2 + \sigma_W^2 - 2 \cdot \sigma_P \cdot \sigma_W \cdot \rho \tag{12}$$

In Appendix B, we show that f(M) has the following general form:

$$f(M) = B \cdot M^{\beta} \tag{13}$$

where *B* is a constant that is yet to be determined via boundary conditions and  $\beta$  is the positive root of the following quadratic:

$$\frac{\sigma_M^2}{2} \cdot X^2 + \left(\alpha_P - \alpha_W - \frac{\sigma_M^2}{2}\right) \cdot X - (r - \alpha_W) = 0.$$
(14)

We show in Appendix B that the other root is negative and that  $\beta > 1$ .

As Dixit and Pindyck (1994) show, the optimal policy in this case is based on a trigger value denoted  $M_H$  such that:

- if  $M < M_{H_{t}}$  then the firm remains inactive keeping the option to enter later
- if  $M > M_H$ , the firm exercises this option at a cost  $k \cdot W$  and becomes active.

The boundary conditions for finding *B* and  $M_H$  are the following *value matching condition* which refers to the investment applied when *M* hits the entry threshold  $M_H$ :

$$f(M_H) = v(M_H) - k, \tag{15}$$

and the high-order contact condition known as the smooth pasting condition:

$$f'(M_H) = v'(M_H).$$
 (16)

Using (11) and (13) in (15) and (16) gives the following solution for  $M_H$ :

$$M_H = \frac{\beta}{\beta - 1} \cdot \frac{r - \alpha_P}{r - \alpha_W} \cdot [1 + (r - \alpha_W) \cdot k].$$
(17)

# 3. Firm Value and Correlation

In this section, we go beyond the analysis of Dixit and Pindyck (1994) and find the value of the firm and how the correlation between price and cost affects this value.

We start by noticing that applying (11) and (13) in (15) and (16) yields, alongside the threshold  $M_H$ , the following expression for the parameter *B*:

$$B = \frac{1 + (r - \mu_W) \cdot k}{(\beta - 1) \cdot (r - \mu_W) \cdot M_H^\beta}$$
(18)

Applying it in (13) yields:

$$f(M) = \frac{1 + (r - \mu_W) \cdot k}{r - \mu_W} \cdot \frac{1}{\beta - 1} \cdot \left(\frac{M}{M_H}\right)^{\beta}$$
(19)

Proposition 1 states that the larger the correlation coefficient, the lower the entry threshold,  $M_H$ , and also the lower the value of the firm.

**Proposition 1.** (*i*) 
$$\frac{dM_H}{d\rho} < 0$$
. (*ii*)  $\frac{dF(P, W)}{d\rho} < 0$ .

**Proof.** From (17), it follows that:

$$\frac{dM_H}{d\beta} = -\frac{M_H}{\beta \cdot (\beta - 1)} < 0.$$
<sup>(20)</sup>

In addition, an implicit differentiation of (14), evaluated at  $X = \beta$ , yields:

$$\frac{d\beta}{d\sigma_M^2} = -\frac{\frac{1}{2} \cdot (\beta^2 - \beta)}{\sigma_M^2 \cdot \beta + \alpha_P - \alpha_W - \frac{\sigma_M^2}{2}} = -\frac{\frac{1}{2} \cdot (\beta^2 - \beta)}{\frac{r - \alpha_W}{\beta} + \frac{\sigma_M^2}{2} \cdot \beta} < 0.$$
(21)

The first equality follows from implicit differentiation of (14). The second equality follows from rearranging terms in (14). The inequality springs from  $\sigma_M^2 > 0$ ,  $\beta > 1$ , and  $r > \alpha_W$ .

From (12), it immediately follows that  $\frac{d\sigma_M^2}{d\rho} < 0$ . Thus:

$$\frac{dM_H}{d\rho} = \frac{dM_H}{d\beta} \cdot \frac{d\beta}{d\sigma_M^2} \cdot \frac{d\sigma_M^2}{d\rho} < 0,$$
(22)

which proves (i). To prove (ii), note that:

$$\frac{d\ln\left[\frac{1}{\beta-1}\cdot\left(\frac{M}{M_{H}}\right)^{\beta}\right]}{d\beta} = \frac{d\ln\left(\frac{1}{\beta-1}\right)}{d\beta} + \frac{d\left[\beta\cdot\ln\left(\frac{M}{M_{H}}\right)\right]}{d\beta} = -\frac{1}{\beta-1} + \ln\left(\frac{M}{M_{H}}\right) + \beta\cdot\frac{M_{H}}{M}\cdot\frac{-M}{M_{H}^{2}}\cdot\frac{dM_{H}}{d\beta} = \ln\left(\frac{M}{M_{H}}\right) < 0,$$
(23)

where the third equality springs from (20) and the inequality springs from  $M < M_H$  which holds throughout the definition range of F(P, W).

From (23), together with (19), and  $r > \alpha_W$ , it follows that  $\frac{df(M)}{d\beta} < 0$ .  $\frac{df(M)}{d\beta} < 0, \frac{d\sigma_M^2}{d\rho} < 0$ , (10) and (21) complete the proof of (ii) via:

$$\frac{dF(P, W)}{d\rho} = W \cdot \frac{df(M)}{d\beta} \cdot \frac{d\beta}{d\sigma_M^2} \cdot \frac{d\sigma_M^2}{d\rho} < 0.$$
(24)

#### 4. Choosing a Price-Cost Combination

In this section, we incorporate within the model an initial stage that takes place at time t = 0. In this stage, the firm is choosing the price process and the cost process relevant to its subsequent actions. The choice of the price process is from a finite set of price process  $\{P^A, P^B, P^C, \ldots\}$  from which the firm can choose at t = 0 where each process  $P^i$  follows (1), (3) and (4) and has its own parameters  $\alpha_p^i$  and  $\sigma_p^i$ , a specific Wiener process  $dZ_i$  attached to it, and a t = 0 value. Likewise, the choice of the cost process is from a finite set of cost processes  $\{W^a, W^b, W^c, \ldots\}$  from which the firm can choose at t = 0 where each process  $W^j$  has its own parameters  $\alpha_W^j$  and  $\sigma_W^j$ , a specific Wiener process  $dH_j$  attached to it, and a t = 0 value. In addition, each couple  $dZ_i$  and  $dH_i$  has their own coefficient of correlation  $\rho^{ij}$ .

From then on, the firm continues as in the model analyzed in the previous sections. In particular, the firm is idle if the t = 0 values of the price process and the cost process the firm has chosen make the relevant markup below the investment threshold  $M_H$  captured by (17). In that case, even though the firm is idle, it cannot replace the price or the cost processes it has chosen. Instead, it continues to monitor them and when the markup hits its threshold, it makes the investment and starts producing and selling.

The variety of price processes to choose from could represent a choice of a certain product to produce out of several different products that the firm can produce. It could also represent a choice of a certain market where the product will be sold, out of several locations. Likewise, the possibility of choosing a cost process could represent a choice of a certain technology or a choice of a production location, with the relevant factor prices of that place.

The assumption that the choice is made only at t = 0 simplifies the analysis greatly, as follows from the work by Décamps et al. (2006), who analyze a case where the firm can choose between two alternative price processes and maintain the option to choose from both as long as it is idle. As they show, the analysis in that case is rather complicated and cannot be reduced to a policy based on a single threshold. However, there is no reason to assume that the role played by the correlation coefficient is influenced, at least qualitatively, from this assumption, as the logic behind the main results, as explained in the introduction, is not based on this assumption.

Besides simplifying the analysis, this assumption can also be viewed as representing a situation where monitoring how the price and cost processes evolve over time is costly, such that the firm prefers to monitor only one price–cost combination of processes.

It is almost trivial that with all other things equal, the firm prefers:

- The price process with the highest  $P^i$  at t = 0
- The price process with the highest  $\alpha_p^i$
- The cost process the with lowest  $W^{j}$  at t = 0
- The cost process with the lowest  $\alpha_W^j$ .

Far less obvious, and even unintuitive, are the results regarding the three uncertainty parameters. Specifically, we find that with all other things equal, the firm prefers:

- The price process with the highest  $\sigma_p^i$
- The cost process with the highest  $\sigma_W^j$
- The pair of price and cost processes with the lowest (closest to -1)  $\rho^{ij}$ .

The logic behind each of these three results is that the firm wishes to achieve the pair with the highest variance in the profit process.

These results contrast with the intuition that uncertainty is bad for investment. This intuition is strengthened by the result that a higher profitability uncertainty ( $\sigma_M^2$ ) makes the firm delay its investments in the sense that it leads to a higher profitability threshold. In the current model, this is captured by the positive effect of  $\sigma_M^2$  on  $M_H$ , as follows from (17), (21) and (20). However, as explained in the introduction, the firm is interested in maximizing its value, and a higher profitability uncertainty promotes this objective, even though it is expected to delay the investment act.

Previous studies have already found, in similar models, that price or cost uncertainty (captured by  $\sigma_P^i$  and  $\sigma_W^j$ ) positively affects the firm's value.<sup>7</sup> In that sense, the novelty in the results regarding these parameters is in the application to the issue of choosing a price–cost combination.

By contrast, the result about how  $\rho$  affects the firm's choice, which is the main result of this article, is fully novel as its effect on the value of the firm has never been studied before. Specifically, in the current model, it follows from Proposition 1 of the previous section which shows that having the correlation as negative as possible maximizes the value of the firm by maximizing the variance of the profit process.

To illustrate this point, we use the following numerical example. A firm plans to sell at a certain market and has two possible production locations, denoted as A and B. Production in location *i*, where  $i \in \{A, B\}$ , entails the cost process  $W^i$  which follows the model's assumptions regarding *W*. While idle and waiting for the optimal time to become active, the firm cannot monitor two locations at the same time and therefore cannot efficiently preserve its two options of becoming active—whether by producing at A or by producing at B. Thus, it needs to decide already at time 0 about the location in which it will produce. Interested in maximizing its value, it chooses to commit at time 0 to producing at A if, and only if,  $F(P_0, W_0^A) \ge F(P_0, W_0^B)$ . Otherwise, it chooses to commit to production at B. After the time 0 choice of production location *i*, the firm waits until  $W^i$  and *P* are such that the entry threshold (17) is reached.

Indifference in time 0 between choosing production in A or in B happens if:

$$F\left(P_0, W_0^{\rm A}\right) = F\left(P_0, W_0^{\rm B}\right) \tag{25}$$

Note that there is no location index on *P*, implying that the selling location needs not be A or B but could be a different one.

To enhance the focus on the role of the difference between  $\rho^{A}$  and  $\rho^{B}$  in choosing between A and B, we assume that r,  $\alpha_{W}$ , k and  $\sigma_{W}$  are the same in both locations.

Applying (10), (13), (17) and (18) in (25) and rearranging the terms, the indifference condition becomes:

$$W_{0}^{A} = \left[\frac{\beta^{A} - 1}{\beta^{B} - 1} \cdot \frac{\left(M_{H}^{A}\right)^{\beta^{A}}}{\left(M_{H}^{B}\right)^{\beta^{B}}} \cdot P_{0}^{\beta^{B} - \beta^{A}} \cdot \left(W_{0}^{B}\right)^{1 - \beta^{B}}\right]^{\overline{1 - \beta^{A}}}.$$
(26)

Note that  $\rho^{A}$  and  $\rho^{B}$  appear in (26) within  $\beta^{A}$  and  $\beta^{B}$ , as follows from (12) and (14).

Equation (26) shows the "cost of correlation" by showing how higher levels of  $\rho^A$  require lower production cost in A at time 0 in order to prevent the firm from preferring B and to preserve indifference between choosing A or B.

We illustrate this "cost of correlation", as captured by (26), via the following parameter values:  $r^{A} = r^{B} = 0.025$ ,  $\alpha_{P} = \alpha_{W}^{A} = \alpha_{W}^{A} = 0$ ,  $\sigma_{P}^{2} = (\sigma_{W}^{A})^{2} = (\sigma_{W}^{B})^{2} = 0.01$ , and  $k^{A} = k^{B} = 4$ . In using these values, we follow Dixit (1989) and Dixit and Pindyck (1994), who use similar values in the variety of numerical examples they present for their general results and with no intention for these values to be representative of any particular industry.<sup>8</sup> We also assume for the purpose of the example that  $P_{0} = 1$ ,  $W_{0}^{B} = 1$  and  $\rho^{B} = 0$ .

assume for the purpose of the example that  $P_0 = 1$ ,  $W_0^B = 1$  and  $\rho^B = 0$ . From (26), it follows that if  $\rho^A = 0$  (just like  $\rho^B$ ), then the value of  $W_0^A$  required to preserve indifference is  $W_0^A = 1$ , i.e., the same as  $W_0^B$ .

However, with a medium-size positive correlation of  $\rho^A = 0.4$ , location A is as good as B only if its time 0 cost is  $W_0^A = 0.84$ , which represents a 16% "correlation cost".

As  $\rho^A$  approaches 1, the value of  $W_0^A$ , which is required to preserve indifference between A and B converges down to  $W_0^A = 0.73$ , implying that A can be as good as B, despite the large correlation that its cost has with the price process, only if its cost is 27% smaller than the cost in B.

The following Figure 1 shows this trade-off between correlation and cost, based on (26). Note that if  $\rho^A < 0$ , then indifference between A and B requires that production cost at A exceeds the production cost at B.



**Figure 1.** The correlation cost. The larger the correlation between *P* and  $W^A$ , the lower  $W_0^A$  should be for the firm not to prefer B and to remain indifferent between A and B.

#### 5. Conclusions

In this article, we have shown that when a firm can choose the price and cost processes it uses—it tends to choose the combination with the lowest price–cost correlation.

The reasons for that were explained already at the introduction for this article—the more positively correlated the price and cost processes are, the lower the volatility of profitability and, therefore, the lower the value of the firm. This negative effect of profit volatility on firm value reflects the asymmetric effect that a mean preserving spread of the distribution of profitability values has on the firm's value. The reason for this asymmetry is that the firm can optimally choose whether to invest or not and also to choose when to invest. Thus, it can enjoy the increase in the probability of high profits while the parallel decrease in the probability of low or negative profits has a smaller effect because the firm would not be in the market in this case.

We have also found that in choosing its price and cost processes, the firm also prefers, with everything else equal, the processes with the highest uncertainty. Once again, this follows from the positive effect that profit uncertainty has on the value of the firm. This effect has already been found in previous studies, but the application to the issue of choosing the price and cost processes is a novel application of this result. These previous studies have also found that uncertainty is bad for investment in the sense that it increases the profitability level for which the firm waits while it delays its investment. The current result shows that the uncertainty problem is even worse, as the firm prefers the processes with the highest uncertainty, when able to choose.

In the analysis, we have simplified the analysis by assuming that the firm cannot monitor two different markets and therefore has to choose the combination of price and cost already at time zero. From a methodological point of view, analyzing a more complicated case where the firm preserves both its options until making one of the possible investments may be an interesting topic for future research. However, such a change to the model is not expected to alter the main results derived here as it does not change the main forces in action: namely, the convexity of the value function with regard to the profit stream and the negative effect of correlation on future expected profits. Another simplification taken in the model is the assumption that once the firm invests, it is active indefinitely and with no option to temporarily suspend its operations when experiencing negative profitability. This assumption is crucial for having a closed form analytical solution for the model. Otherwise, the solution equations can only be solved numerically, even if the complete exit, or the temporary suspension, are at no cost at all. Adding temporary suspension option, or a complete exit option, to the model does not affect the qualitative results of the analysis, because it does not eliminate the two main forces underlying the results which were specified above. Moreover, the optimal conduct of the firm still makes the value of the firm a convex function of the potential profitability flow, and this feature would become even stronger with such suspension or exit options.

We have modeled the price and cost processes as geometric Brownian motions. While this assumption is very common in the relevant literature, some studies have also looked at cases involving other processes such as jump processes or mean reverting ones.<sup>9</sup> While such different choices for these processes are bound to affect the model quantitatively, they are not supposed to alter its qualitative results as all reasonable choices for such processes share the negative effect that the correlation between the price and cost processes has on the variance of the profitability.

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#### Appendix A. Properties of M

In order to find the stochastic process for the markup M, we apply Itô's lemma to M, as defined by (8), and simplify. This yields:

$$dM(P, W) = \alpha_M \cdot M \cdot dt + \sigma_M \cdot M \cdot dW, \tag{A1}$$

where:

$$_{M} \equiv \alpha_{P} - \alpha_{W} + \sigma_{W}^{2} - \sigma_{P} \cdot \sigma_{W} \cdot \rho \tag{A2}$$

$$\sigma_M^2 \equiv \sigma_W^2 + \sigma_P^2 - 2 \cdot \sigma_P \cdot \sigma_W \cdot \rho \tag{A3}$$

$$dW \equiv \frac{\sigma_P \cdot dZ - \sigma_W \cdot dH}{\sigma_M}.$$
 (A4)

Since dZ and dH are normally distributed, so is dW. From (3) and (A4), it follows that E(dW) = 0, and from (4), (5) and (A4), it follows that  $E[(dW)^2] = dt$ . Thus, dW is a standard Wiener process and, by (A1), M is therefore a geometric Brownian motion with the drift parameter  $\alpha_M$  and the variance parameter  $\sigma_M$ .

In addition, from (A3), it follows that:

$$\sigma_M^2 > \sigma_W^2 + \sigma_P^2 - 2 \cdot \sigma_P \cdot \sigma_W = (\sigma_W - \sigma_P)^2 > 0, \tag{A5}$$

where the first inequality follows from  $\rho < 1$ .

#### Appendix B. The Function *f*(*M*)

At each time instance dt, owning the opportunity to invest yields an expected capital gain dF(P, W) in response to the fluctuations of P and W during that instance. The standard no-arbitrage condition requires that the expectancy of this capital gain should equal the instantaneous normal return, i.e.,

$$E[dF(P, W)] = r \cdot dt \cdot F(P, W) \tag{A6}$$

Expanding dF(P, W) via Itô's lemma, taking the expectancy, applying (3), (4) and (5) and simplifying, turns (A6) into the following differential equation:<sup>10</sup>

$$F_P(P,W) \cdot \alpha_P \cdot P + F_W(P,W) \cdot \alpha_W \cdot W + \frac{1}{2} \cdot F_{PP}(P,W) \cdot \sigma_P^2 \cdot P^2 + \frac{1}{2} \cdot F_{WW}(P,W) \cdot \sigma_W^2 \cdot W^2 + F_{PW}(P,W) \cdot \sigma_P \cdot \sigma_W \cdot P \cdot W \cdot \rho - r \cdot F(P,W) = 0$$
(A7)

In general, this type of a multivariate differential equation cannot be solved. However, due to the homogeneity of F(P, W) it can be transformed to a differential equation which is based on M alone. To do so, note that from (8) and (10) it follows that:

$$F_P(P, W) = f'(M),$$
 (A8)

$$F_{PP}(P, W) = \frac{1}{W} \cdot f''(M),$$
 (A9)

$$F_W(P, W) = f(M) - M \cdot f'(M),$$
 (A10)

$$F_{WW}(P, W) = \frac{M^2}{W} \cdot f''(M), \qquad (A11)$$

$$F_{PW}(P, W) = -\frac{M}{W} \cdot f''(M).$$
(A12)

Applying (10) and (A8)–(A12) in (A7), dividing both sides by *W*, applying (8) and (10) and simplifying yields:

$$\frac{\sigma_M^2}{2} \cdot f''(M) \cdot M^2 + f'(M) \cdot M \cdot (\mu_P - \mu_W) - (r - \mu_W) \cdot f(M) = 0$$
(A13)

To solve this single-variable second-order homogenous differential equation, we first try the general solution  $f(M) = M^X$  which turns (A13) into:

$$\frac{\sigma_M^2}{2} \cdot X^2 + \left(\alpha_P - \alpha_W - \frac{\sigma_M^2}{2}\right) \cdot X - (r - \alpha_W) = 0$$
(A14)

The LHS of this equation is a quadratic function of *X*. This is a quadratic with a minimum point, because  $\sigma_M^2 > 0$ , as shown in Appendix A. In addition, from  $r > \alpha_P$  and  $r > \alpha_W$ , it follows that the LHS is negative both at X = 0 and at X = 1. This leads to the conclusion that (A14) has one negative root and one root which is larger than 1. We denote the negative root by  $\gamma$  and the positive one by  $\beta$ . Thus, the general solution of the differential Equation (A10) is

$$f(M) = A \cdot M^{\gamma} + B \cdot M^{\beta} \tag{A15}$$

where *A* and *B* are parameters to be determined via boundary conditions. The first one of these boundary conditions is:

$$\lim_{(P/W)\to 0} F(P, W) = 0$$
 (A16)

This condition implies that if P approaches 0 then, by the properties of the geometric Brownian motion, the probability of it ever rising above a much larger W so that the stream of profits becomes positive is zero, and therefore, the value of the option to become active is zero. A similar interpretation of this condition rises from the case where W goes to infinity (and in particular that it is infinitesimally larger than P).

From (10) and (A16) and from  $\gamma < 0$  and  $\beta > 1$ , it follows that A = 0, which leads to:

$$f(M) = B \cdot M^{\beta},\tag{A17}$$

where *B* is to be determined via additional boundary conditions. These conditions refer to the investment conducted when *P* and *W* are such that their ratio, the markup *M*, hits

its threshold level,  $M_H$ , which implies that P hits its entry threshold, which we define by  $P_H(W) \equiv W \cdot M_H$ . The first one is the following *value matching condition*:

$$F[P_H(W), W] = V[P_H(W), W] - k \cdot W,$$
(A18)

and dividing both its sides by *W* leads to Equation (15). The second condition is the high-order contact condition known as the *smooth pasting condition*:

$$F_P[P_H(W), W] = V_P[P_H(W), W],$$
 (A19)

and dividing both its sides by *W* leads to Equation (16).

#### Notes

- McDonald and Siegel (1985) or Dixit and Pindyck (1994, pp. 207–11) are the most prominent examples for purely theoretical works comprising uncertainty in both the price and cost processes. Others studies with both types of uncertainty have applied this theory to a variety of fields such as renewable energy (Boomsma et al. 2012), mining (Slade 2001), forestry (Wiemers and Behan 2004), information technology (Schwartz and Zozaya-Gorostiza 2003) and electricity systems reliability (Andreis et al. 2020), to name a few.
- <sup>2</sup> McDonald and Siegel (1985) do find that the price-cost correlation has a negative effect on the value of the firm. However, in that study there is no investment, as the production project is already active and therefore there is also no search for optimal investment policy, let alone a choice among different possible investment with different combinations of price and cost processes.
- <sup>3</sup> This assumption is crucial for using the homogeneity of the functions representing the value of the firm (which are presented later on in the analysis). Specifically, this homogeneity enables turning the two-variable PDE that captures the equilibrium pricing of the value of the idle firm into a single-variable PDE and thus paves the way toward a unique analytical solution for the model, as shown in Appendix B.
- <sup>4</sup> A few models did assume that this cost is stochastic but have not connected it to the production cost, with the model in pp. 207–211 by Dixit and Pindyck (1994) being a prominent example.
- <sup>5</sup> More specifically, our model is a particular case where  $\rho_{pm} = 0$  of the case that Dixit and Pindyck (1994) study. In their model, this parameter captures the correlation between *P* and the return on the whole market portfolio. For simplicity, we assume no such correlation.
- <sup>6</sup> Time indexes are omitted from now on.
- <sup>7</sup> See, for example, Dixit (1989) and chapter 6 in Dixit and Pindyck (1994).
- <sup>8</sup> See a discussion in p. 153 in Dixit and Pindyck (1994).
- <sup>9</sup> See, for example, Slade (2001); Andreis et al. (2020) and Nunes and Pimentel (2017).
- <sup>10</sup> For a clear presentation of Itô's lemma, in general and also for the case of two correlated processes, see pp. 79–81 in Dixit and Pindyck (1994).

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