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Article

An Equilibrium-Based Measure of Systemic Risk

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Abstract: This paper develops and implements an equilibrium model of systemic risk. The model derives a systemic risk measure, loss beta, in characterizing all too-big-to-fail banks using a capital insurance equilibrium. By constructing each bank's loss portfolio with a recent accounting approach, we perform a comprehensive empirical study of this loss beta measure and document all TBTF banks from 2002 to 2019. Our empirical findings suggest a significant number of too-big-to-fail banks in 2018–2019.

Keywords: systemic risk; too big to fail; capital insurance; loss beta

JEL Classification: G11; G12; G13



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1. Introduction

The financial crisis of 2007–2009 generates a significant amount of interest in measuring systemic risk and ensuring the health of the financial system. The most vivid response is the passing of the Dodd-Frank Act. Measuring systemic risk has been at the center of academic research since 2008. For example, Acharya (2009), Acharya et al. (2012), and Brownless and Engle (2016) show that time-varying correlation structure plays a crucial role in their systemic risk measurements and develop an expected shortfall measure approach. Adrian and Brunnermeier (2016) develop a CoVaR approach conditional on financial institutions being in a state of financial distress. Acemoglu et al. (2015), and Elliott et al. (2014) develop a network approach for systemic risk.¹ While these quantitative and statistical metrics are straightforward to apply, the precise economic channel to which the systemic risk enters the metrics is somewhat lacking. Moreover, Benoit et al. (2019) identify several shortcomings in the systemic-risk scoring methodology.

From an insurance equilibrium perspective, Panttser and Tian (2013) and Ivanov (2017) present a capital insurance approach to address systemic risk and identify *too-big-to-fail* (TBTF) banks. Developing a rational expectation equilibrium model of the capital insurance market to address systemic risk was initially proposed in Kashyap et al. (2008). In the equilibrium model of capital insurance, a central player (regulator or insurer) sells an insurance product, *capital insurance*, on the systemic risk of all banks. The insurer injects the guaranteed capital contingent upon a stressed period, while each bank pays insurance premium upfront in exchange for an *implicit guarantee subsidy*. In equilibrium, each bank predicts the optimal insured amount and the insurer determines the optimal pricing structure. As a result, those banks purchasing capital insurance are characterized as TBTF banks under this approach, so TBTF banks are identified endogenously. In characterizing the equilibrium, this approach introduces a new equilibrium-based systemic risk measure, *loss beta*.

Compared with numerous systemic risk measures in previous literature, the equilibrium-based systemic risk measure, such as loss beta, is unique since it captures the perspective from all banks together as well as an issuer about each bank's systemic risk. Moreover,

both the implicit guarantee subsidy and TBTF banks are determined endogenously and simultaneously.² Since the framework and its pricing structure are motivated by a classical insurance setting, the capital insurance equilibrium is similar to classical insurance equilibrium models such as [Borch \(1962\)](#), [Arrow \(1971\)](#), and [Raviv \(1979\)](#) for a new type of insurance product on the systemic risk. Nevertheless, the strong law of large numbers is no longer satisfied for capital insurance because of the particular correlated feature of systemic risk among banks ([Acharya et al. \(2012\)](#) and [Adrian and Brunnermeier \(2016\)](#)).

This paper extends the equilibrium analysis in [Panttser and Tian \(2013\)](#) and [Ivanov \(2017\)](#) in several theoretical and empirical aspects. First, this paper characterizes the equilibrium for a general class of capital insurance. We focus on identifying TBTF banks, whereas [Panttser and Tian \(2013\)](#) study the welfare analysis of capital insurance as a financial innovation. Second, and more importantly, we introduce a new methodology to construct an individual bank's loss portfolio with systemic risk components consistent with Basel documentations.³ Building on the deposit insurance model developed in [Diamond and Dybvig \(1983\)](#) and [Peck and Shell \(2003\)](#), we construct an appropriate loss portfolio with systemic risk exposure by modifying a recent accounting model in [Atkeson et al. \(2019\)](#) for commercial banks. Using deposit, loan, equity, and debt together, we construct a loss portfolio in every period and the time series of loss beta for each bank. In contrast, an asset-equity leverage ratio is used in [Ivanov \(2017\)](#) to construct the asset loss portfolio, but other specific information for commercial banks is not incorporated. Finally, we implement the proposed approach for U.S. banks and provide a comparison to the previously developed systemic risk measures.

Specifically, there are three significant contributions in this paper. *First*, we characterize the equilibrium of general capital insurance for the entire banking sector. This equilibrium model helps us identify TBTF and how much premium TBTF banks should pay upfront to hedge the systemic risk. Since our model is building on the insurance principle, it complements the equilibrium asset-pricing model of systemic risk in [Allen and Gale \(2000\)](#). At the same time, since our model focuses on the systemic risk only, we can construct an equilibrium-based systemic risk measure and identify TBTF simultaneously, which is not addressed in [Allen and Gale \(2000\)](#).

Second, we propose a new methodology to construct the loss portfolio and use this loss portfolio to develop an equilibrium systemic risk measure—bank loss beta. Given each bank's loss portfolio, a bank's loss beta is a ratio of the covariance between a bank's loss portfolio with the aggregate loss portfolio in the entire banking sector to the variance of the aggregate loss portfolio. The innovation of this methodology is to use all deposit, loan, and equity and debt information that is important to analyze a commercial bank's systemic risk exposure.

Third, we implement the equilibrium approach for U.S. banks.⁴ Using the "Call Report" of all U.S. banks with assets over \$50 billion in each quarter from 2002 to 2019, we identify TBTF banks in each quarter. Our empirical findings suggest highly concentrated TBTF banks in the financial crises period and a significant number of TBTF banks starting from 2019Q1. We also calculate CoVaR simultaneously and find a positive and significant relationship between CoVaR and loss beta for a comparative purpose. As a comparison, we further implement the asset loss beta approach in [Ivanov \(2017\)](#) for all institutions with a Standard Industrial Classification (SIC) code between 6000 and 6499 and assets over \$50 billion from quarter 1 of 2002 to quarter 2 of 2019. In our empirical findings, both approaches to constructing the loss portfolio generate a reasonably consistent group of TBTF banks during the same period, but the accounting approach covers a much larger group of commercial banks. Overall, the empirical implementation and comparison between these two approaches support the idea of using capital insurance to identify TBTF banks.

The paper contributes to the literature on TBTF banks and identifies such banks through a new equilibrium approach. For instance, in the network approach to systemic risk of [Acemoglu et al. \(2015\)](#) and [Elliott et al. \(2014\)](#), the connectedness amongst the banks

plays a key role. In other words, only the most relevant banks affect a bank's systemic risk. By contrast, whether one bank is TBTF in our approach relies on *all* loss betas of financial institutions in the market. Therefore, the capital insurance approach highlights the *relative weight* of loss beta in the whole financial system. In Acharya et al. (2012) and Adrian and Brunnermeier (2016), specific distribution-based measures are calculated and sorted, so all banks' systemic risks are compared with each other. In our approach, we use the loss beta as a criterion to measure and sort the systemic risk, so this approach provides an intuitive and robust way to measure the systemic risk in the spirit of CAPM. Eisenberg and Noe (2001) consider each bank's cash flow and outflow and study the clearing payment to ensure each system member clears the obligation (not default). We also use the deposit, loan, debt, and equity information in our approach, but this information is used to construct a loss portfolio with systemic risk exposure. Instead of examining the clearing payment vector (with a topology method) in Eisenberg and Noe (2001), we characterize the equilibrium of systemic risk based on economic and finance principles.

Since we study the appropriate amount of guarantee subsidy in exchange for a government bailout (injection), this paper also complements a recent paper by Berndt et al. (2021) on TBTF banks. Statistically speaking, it is impossible to estimate the government bailout probability since Lehman Brothers might be the only large bank that has not been bailed out. Therefore, Berndt et al. (2021) develop a structural model to compute a government bailout's market-implied (*risk-neutral*) probabilities, and these authors find a significant post-Lehman reduction in market-implied probabilities of government bailout for both globally systemically important banks (GSIBs) in the U.S. and other large banks that are not large enough to be classified as GSIBs.⁵ Kelly et al. (2016) also demonstrate the collective government guarantee for the financial sector using options data on the financial sector index and individual banks. Because Kelly et al. (2016) use the options data, the government guarantee is also interpreted under the risk-neutral probability measure. By contrast, our approach builds on the insurance principle to compute the insurance premium under the *real-world expectation* of the loss portfolio. In our approach, we do not estimate the real-world bailout probability directly. However, since the firm should pay a premium to buy insurance from the government in exchange for the government bailout, the capital insurance premium is essentially the *expected bailout amount*, a product of the bailout exposure and the bailout probability under the real-world probability measure. Like Berndt et al. (2021), we find that the number of TBTF banks gradually decreases post-Lehman until 2019.⁶

The paper proceeds as follows. Section 2 introduces a simple accounting model to construct each bank's loss portfolio with a systemic risk component. In Section 3, we characterize the capital insurance equilibrium and derive a general property about the loss beta measure. Finally, we report our empirical results in Section 4, and Section 5 concludes. Appendix A presents several properties of general capital insurance.

2. Bank's Loss Portfolio

In this subsection, we explain bank's loss portfolio in a Diamond-Dybvig framework (Diamond and Dybvig (1983) and Peck and Shell (2003)). We start with a motivated example. Then we present its formal description.

2.1. A Motivated Example

Consider a bank with equity investment \$100 at time zero. There are two future times $t = 1$ and $t = 2$. There are identical households who are ex-ante identical and endowed with \$1 at time $t = 0$. We assume that there are in total 800 such households to deposit in the bank, thus the bank has \$800 of short-term debt (deposit) at time $t = 0$. In addition, the bank issues a subordinated debt of face value \$100 at maturing time $t = 2$ and the coupon rate is 5%.⁷ On the other hand, the bank loans \$1000 to long-term borrowers with random gross return \tilde{R}_1 in the first time period (if liquid at $t = 1$) and random gross return \tilde{R}_2 at time $t = 2$. Depositors have no risk because of deposit insurance issued by the Federal

Deposit Insurance Corporation (FDIC). For simplicity, we assume that the deposit cost (including the deposit insurance premium and operational cost) for the bank is zero, and the deposit interest rate is also zero.

There are two types of depositors in the presence of idiosyncratic uncertainty. The first type of depositor needs to consume at time $t = 1$ while the second type of depositor only withdraws and consumes at $t = 2$. The depositor's type is revealed in $t = 1$ in equilibrium. Since we focus on systemic risk in the banking system, we assume the withdraw probability is $\lambda = 0.4$ at time $t = 1$ in a good (Diamond-Dybvig) equilibrium, and there is no bank run equilibrium in the presence of deposit insurance.

At time $t = 1$, \$320 ($=\lambda \times 800$) are withdrawn and a \$5 coupon payment for the subordinated debt is paid. We assume that the prepaid probability is μ and no loan defaults; thus, the cash inflow is $1000\mu\tilde{R}_1$ from the loan (project). Let

$$Y_1 = 1000\mu\tilde{R}_1 + E_1 - 325,$$

where E_1 is the market price of the equity at time $t = 1$. Y_1 presents a “profit and loss (P & L) portfolio” including the liquid common equity asset. If $Y_1 > 0$, there exists sufficient cash inflow to cover the cash outflow, thus no systemic risk to the economy.

If, however, $1000\mu\tilde{R}_1 + E_1 \leq 325$ at time $t = 1$, the bank might sell the long-term illiquid asset (and could be subject to asset fire-sales) to avoid the default. In this situation, the total sum of $1000\mu\tilde{R}_1$ and the fair value (at time $t = 1$) of the future payment $1000(1 - \mu)\tilde{R}_2$ is the “fair value” of the long-term loan. Similarly, the subordinated debt obligation is represented by the market value of the debt. Therefore, we replace the P & L portfolio Y_1 by

$$Z_1 = FL_1 + E_1 - FD_1 - B_1$$

where

- FL_1 is the fair value of the long-term loan at time $t = 1$;
- FD_1 is the fair value of the deposit at time $t = 1$;
- B_1 is the market price of the subordinated debt.

If Z_1 is positive, the bank does not generate systemic risk to the economy since the total asset value is greater than the total obligation to both short-term and long-term debt holders. On the other hand, if Z_1 is negative, since the asset value is not sufficient to meet its obligation to both depositor (short-term debt holder) and long-term debt holder, the bank not only defaults but also generates systemic risk to the economy. Therefore, the bank's loss portfolio is characterized as

$$L_1 = \max(-Z_1, 0),$$

the maximum between negative Z_1 and zero.

Similarly, let

$$Y_2 = 1000(1 - \mu)\tilde{R}_2 + E_2 - 580 - (100 + 5),$$

where E_2 is the market price of the equity at time $t = 2$. Conditional on no-default in the previous time period, we notice that

$$Y_2 = Z_2 = FL_2 + E_2 - FD_2 - B_2$$

where $FL_2 = 1000(1 - \mu)\tilde{R}_2$ is the fair value of the loan, $FD_2 = \$580$ is the fair value of the deposit and $B_2 = \$105$ is the market price of the subordinated debt at time $t = 2$. Hence, the banks' loss portfolio at time 2 is

$$L_2 = \max(-Z_2, 0).$$

2.2. Construction of Bank's Loss Portfolio

In this subsection we define the bank's loss portfolio by modifying a recent accounting model in Atkeson et al. (2019). The construction extends the idea in the last motivated example. For simplicity, we assume the Treasury interest rate is a constant r .

Specifically, D denotes the total face value (or the book value) of the deposit on the bank's balance sheet. In each time period, every dollar of deposits costs the bank c_D , including the interest paid on deposit, the serving cost and the deposit insurance premium paid to FDIC. The deposit is withdrawn with probability, μ_D , which depends on the depositor's type in Diamond-Dybvig's framework. Since a depositor's withdrawal decision relies on an idiosyncratic shock, his type is revealed upon new information, so the probability μ_D is a random variable at each time period. The fair value of the debt is denoted by FD_t at time t . Following Atkeson et al. (2019), the fair value FD_t is given by⁸

$$FD_t = \frac{c_D + \mathbb{E}_t^Q[\mu_{D,t+1}]}{r + \mathbb{E}_t^Q[\mu_{D,t+1}]} D \quad (1)$$

where $\mathbb{E}_t^Q[\cdot]$ denotes the conditional expectation operator under a risk-neutral probability measure.

On the other hand, L denotes the total face value of the loan. In every time period, every dollar of loan pays a coupon c_L , net of serving cost. The fair value of the loan depends on the prepaid probability μ_L , and the loan default probability δ_L on the face value. By a similar derivation, the fair value of the loan at time t is⁹

$$FL_t = \frac{c_L + \mathbb{E}_t^Q[\mu_L]}{r + \mathbb{E}_t^Q[\mu_L] + \mathbb{E}_t^Q[\delta_L]} L. \quad (2)$$

The bank issues a subordinated debt as well as the equity. At each time t , the common equity's market value is denoted by E_t , and the market value of the subordinated debt is written as B_t . By the discussion in the motivated example, the bank's P & L portfolio at time t is

$$Z_t = FL_t + E_t - FD_t - B_t, \quad (3)$$

and the bank's loss portfolio at time t is

$$L_t = \max\{FD_t + B_t - FL_t - E_t, 0\}. \quad (4)$$

According to its definition, L_t represents the bank's loss exposure to systemic risk. To see it, the loss exposure is positive if any only if

$$FD_t + B_t > FL_t + E_t. \quad (5)$$

There are two terms on the left side of the last formula, denoting the obligation to short-term debt holders (depositor) and long-term debt holders, while the right side is a sum of the (liquid asset) common equity and (illiquid asset) loan. Regardless depositors or debt holders, as long as the bank's obligation is greater than the total asset value, the bank endures a loss to the economy; thus a positive loss exposure L_t .

We notice that Atkeson et al. (2019) consider a representative bank with an additional loan-making arm and the deposit-taking arm. Then the government guarantee is one component in the bank market-to-book ratio. In contrast, we consider a group of heterogeneous banks and we argue that the bank's systemic risk and the government (implicit) guarantee are driven by the "correlated structure" of all banks' loss portfolios, in a theory of capital insurance below.

3. Loss Beta Measure

In this section, we introduce an equilibrium-based loss beta measure of a bank's systemic risk. We present an equilibrium model of a capital insurance market and characterize TBTF banks in this equilibrium approach. Our classification of TBTF relies on equilibrium-based loss beta measures of all banks simultaneously in the banking system. We further discuss several examples of the loss beta measures.

3.1. An Equilibrium Model

Following Panttser and Tian (2013) and Ivanov (2017), we present a model of capital insurance market.

There are N financial institutions, namely banks, indexed by $i = 1, \dots, N$, in a financial sector. Each bank is endowed with a loss portfolio (or exposure, and we do not distinguish between these two concepts in this paper), L_1, \dots, L_N , respectively. The aggregate loss portfolio is

$$L = L_1 + \dots + L_N. \quad (6)$$

While our empirical results are based on the bank loss portfolio introduced in the last section, we highlight that the loss portfolios are merely inputs in the presented equilibrium model.

To hedge the potential loss, each bank decides whether or not to purchase a capital insurance contract from an insurance company, which determines the insurance premium. The insurance contract is written on the aggregate loss portfolio L instead of individual loss to each bank. Specifically, the payoff of one unit of the insurance payoff is written as $I(L)$, where $I(\cdot)$ is a continuous monotonic increasing function.¹⁰ Thus, the insurance company and all banks are the market participants in the capital insurance market, in which the equilibrium is obtained if all participants achieve their highest expected utilities, respectively.

Following the standard insurance equilibrium literature (Arrow (1971) and Raviv (1979)), the insurance premium for each unit is $P = (1 + \rho)\mathbb{E}[I(L)]$, where ρ is a load factor that is determined by the issuer. Given the premium structure, each bank i decides how much insurance to purchase, namely, $a_i I(L)$, where $a_i \geq 0$, for a premium $a_i P$ accordingly. A bank decides the percentage coefficient a_i to optimize its expected utility. At the same time, the insurance company determines the load factor, i.e., the premium structure, by the market demand $a_i I(L)$ from each bank i . Therefore, $\{\rho, a_i, i = 1, \dots, N\}$ are determined in this equilibrium.

Specifically, we assume that each bank is risk-averse, and its risk preference is represented entirely by the mean and the variance of the wealth with the reciprocal of risk aversion parameter $A > 0$.¹¹ Given a premium structure ρ , bank i solves an optimal portfolio problem by choosing the best coinsurance coefficient:

$$\max_{\{a_i \geq 0\}} \left\{ \mathbb{E}[\tilde{W}^i] - \frac{1}{2A} \text{Var}(\tilde{W}^i) \right\}, \quad (7)$$

where $\tilde{W}^i = W_0^i - L_i + a_i I(L) - (1 + \rho)\mathbb{E}[a_i I(L)]$ is the *ex post* terminal wealth for the bank i after purchasing the capital insurance and W_0^i is the initial wealth of bank i . Similarly, $W^i = W_0^i - L_i$ represents the *ex ante* wealth of bank i before buying capital insurance.

By the first-order condition in (7), the optimal coinsurance coefficient for bank i is given by

$$a_i(\rho) = \max \left(\frac{\text{Cov}(L_i, I(L)) - \rho \mathbb{E}[I(L)]A}{\text{Var}(I(L))}, 0 \right). \quad (8)$$

The issuer is assumed to be risk-neutral.¹² Therefore, the expected terminal wealth of the issuer is

$$W^r = \sum_{i=1}^N (1 + \rho)\mathbb{E}[a_i I(L)] - \sum_{i=1}^N a_i I(L) - \sum_{i=1}^N c(a_i I(L)), \quad (9)$$

where $c(\cdot)$ denotes the issuance cost, which can be a fixed cost, a constant percentage of the indemnity or a general function of the indemnity. To focus on the equilibrium analysis of TBTF, we assume that the regulatory cost is a constant for each bank.

Given the optimal demand $a_i(\rho)$ for each bank with a load factor ρ in (8), the issuer maximizes the expected welfare $\mathbb{E}[W^r]$. By plugging Equation (8) into Equation (9), the issuer's optimal load factor is derived from the following optimization problem:

$$\max_{\{\rho>0\}} \rho \sum_{i=1}^N \max \left(\frac{\text{Cov}(L_i, I(L)) - \rho A \mathbb{E}[I(L)]}{\text{Var}(I(L))}, 0 \right) \quad (10)$$

and the optimal coinsurance coefficient for each bank $i = 1, \dots, N$ is given by $a_i(\rho^*)$, where ρ^* is the optimal load factor in (10). As a result, the capital insurance's payoff for each bank i , $a_i(\rho^*)I(L)$, relies on both demand (from all banks) and supply (from the regulator) in a rational expectation equilibrium. Both the optimal $a_i(\rho^*)$ and ρ^* are determined endogenously. Bank i is TBTF under the capital insurance $I(L)$ if and only if $a_i(\rho^*) > 0$.

Equation (10) is first discussed and solved for particular capital insurance contract in Pantser and Tian (2013) and Ivanov (2017). We next discuss in detail how to determine TBTF banks in this equilibrium approach for general capital insurance.

3.2. Characterizing TBTF Banks

In this subsection, we characterize the equilibrium for general capital insurance. Our new result is to demonstrate that this approach captures each bank's systemic risk to some extent (Proposition 1) for a general capital insurance contract.

We first assume that all banks are TBTF under this approach. Then, Equation (10) becomes

$$\max_{\{\rho>0\}} \rho \sum_{i=1}^N \frac{\text{Cov}(L_i, I(L)) - \rho A \mathbb{E}[I(L)]}{\text{Var}(I(L))},$$

yielding the optimal load factor¹³

$$\hat{\rho} \equiv \frac{\sum_{i=1}^N \text{Cov}(L_i, I(L))}{2AN \mathbb{E}[I(L)]} = \frac{\text{Cov}(L, I(L))}{2AN \mathbb{E}[I(L)]} > 0.$$

Since $a_i(\hat{\rho})$ must be positive, for each i , the covariance between the loss L_i with $I(L)$ satisfies

$$\text{Cov}(L_i, I(L)) > \frac{\text{Cov}(L, I(L))}{2N}.$$

Clearly, if all covariances $\text{Cov}(L_i, I(L))$ are sufficiently close so that each of them is greater than the half of their average, the last condition is satisfied, and the optimal load factor is $\hat{\rho}$. In particular, if $\text{Cov}(L_i, I(L)) = \text{Cov}(L_j, I(L))$, $\forall i \neq j$, all banks are TBTF.

From the above analysis, we see that the covariance $\text{Cov}(L_i, I(L))$ provides some insights about the bank's systemic risk. More importantly, the TBTF concept depends on all covariances $\text{Cov}(L_i, I(L))$, instead of on individual bank's loss portfolio. Indeed, by virtue of Equation (8), bank i is too-big-to-fail as long as

$$\text{Cov}(L_i, I(L)) > \rho^* A \mathbb{E}[I(L)]. \quad (11)$$

But the optimal load factor ρ^* in (11) is determined endogenously. The optimal load factor is solved by (10), and it depends on all loss portfolio information. Even the load factor $\hat{\rho}$ depends on all covariance terms $\text{Cov}(L_i, I(L))$. Therefore, an individual bank's loss beta is **not** sufficient yet to recognize whether it is too big to fail or not; rather, we have to study the entire financial sector as a whole to identify all TBTF banks simultaneously. Briefly speaking, a bank is TBTF only when its covariance with the aggregate loss portfolio

is relatively large compared with other banks' corresponding covariances in the same financial sector.

Therefore, we introduce the loss beta of bank i with a capital insurance $I(L)$,

$$\beta_i = \frac{\text{Cov}(L_i, I(L))}{\text{Var}(I(L))}, \quad (12)$$

in analyzing systemic risk. We reorder $\{\beta_1, \dots, \beta_N\}$ such that $\beta_1 \geq \dots \geq \beta_N > 0$. We omit those banks with negative or zero loss betas. Clearly,

$$\sum_{i=1}^N \beta_i = \frac{\text{Cov}(L, I(L))}{\text{Var}(I(L))}. \quad (13)$$

The total beta of all banks is the beta (regression) coefficient of the aggregate loss portfolio to the indemnity of the capital insurance. Appendix A presents an algorithm to characterize all TBTF banks. In particular, a bank with high loss beta is TBTF. Conversely, the next result shows that all TBTF banks' loss betas must be bounded below by $\frac{1}{2N} \frac{\text{Cov}(L, I(L))}{\text{Var}(I(L))}$, regardless of the distribution of loss exposure of each bank. Therefore, a TBTF must have a large loss beta, and vice versa to a certain extent. Proposition 1 thus justifies our systemic risk measurement in terms of loss betas.

Proposition 1. *For any capital insurance $I(L)$, the loss beta of a TBTF bank must be greater than or equal to $\frac{1}{2N} \frac{\text{Cov}(L, I(L))}{\text{Var}(I(L))}$.*

Proof. See Appendix A. \square

We present two examples below to illustrate the equilibrium approach to identify TBTF banks.

Example 1. *We assume that $N = 15$. The loss portfolios follow a one-factor model, $L_i = \alpha_i Y + \epsilon_i$; $\mathbb{E}[Y] = 1$ billion and $\text{Var}(Y) = 5\%$, ϵ_i has standard deviation moving from 40% to 12% equally from $i = 1$ to $i = 15$, while α_i moves from 4% to 6.8% equally. We also assume that $A = 1$ and $I(L) = L$.*

Table 1 represents loss betas, ρ^* and the optimal coinsurance $a_i(\rho^*)$. We note that when $i = 13, 14, 15$, $\text{Cov}(L_i, L) > \frac{\sum_{j=1}^i \text{Cov}(L_j, L)}{2i}$ for $i = 13, 14, 15$. Moreover, the first 12 banks are TBTF. The optimal load factor is $\rho^* = 8.1\%$. The expected total aggregate loss is $\sum_{i=1}^{12} a_i \mathbb{E}[Y] = 0.81$ billion. The coinsurance coefficient $a_i(\rho^*)$ increases with respect to the loss beta.

Example 2. *Assume that $N = 5$, and any two different banks' loss portfolios have the same correlation coefficient $\rho = 0.10$. $\text{Var}(L_i) = k^{2i} \sigma^2$ for $0 < k < 1$.*

Table 2 demonstrates that TBTF banks must have large betas, but the inverse statement is not completely valid. In fact, only the last bank has a small beta, but the first three banks are TBTF banks. Both examples provide a crucial insight into the equilibrium approach. Even though some banks contribute positively to systemic risk and banks are heavily correlated, those banks might still not be TBTF banks, as shown by the last example. The intuition is as follows. By insuring the bank with the most significant systemic risk exposure, other banks' systemic risks can be insured to some extent. As a consequence, other banks are not TBTF anymore.

Table 1. This table displays a bank sector with 15 banks and identifies “too-big-to-fail” banks following the presented capital insurance approach. The loss portfolios follow a one-factor model, $L_i = \alpha_i Y + \epsilon_i$; $\mathbb{E}[Y] = 1$ billion and $\text{Var}(Y) = 5\%$, ϵ_i has standard deviation moving from 40% to 12% equally from $i = 1$ to $i = 15$, while α_i moves from 4% to 6.8% equally. We also assume that each $A = 1$. We note that when $i = 13, 14, 15$, $\text{Cov}(L_i, L)$ is strictly greater than $\frac{\sum_{j=1}^i \text{Cov}(L_j, L)}{2i}$. Then, the last three banks, banks 13, 14, and 15, are not “too-big-to-fail”. The optimal load factor is $\rho^* = 8.1\%$. The expected total aggregate loss is $\sum_{i=1}^{12} a_i \mathbb{E}[Y] = 0.81$ billion.

Bank	$\text{Cov}(L_i, L)$	$\frac{\sum_{j=1}^i \text{Cov}(L_j, L)}{2i}$	$\frac{\text{Cov}(L_i, L)}{\text{Var}(L)}$	a_i	$(1 + \rho^*) a_i$
1	0.2106	0.1053	0.1226	8.46%	9.14%
2	0.1934	0.1010	0.1126	7.46%	8.06%
3	0.1770	0.0968	0.1031	6.50%	7.03%
4	0.1614	0.0928	0.0940	5.59%	6.04%
5	0.1466	0.0889	0.0854	4.73%	5.11%
6	0.1326	0.0851	0.0772	3.92%	4.23%
7	0.1194	0.0815	0.0695	3.15%	3.40%
8	0.1070	0.0780	0.0623	2.43%	2.62%
9	0.0954	0.0746	0.0556	1.75%	1.89%
10	0.0846	0.0714	0.0493	1.12%	1.21%
11	0.0746	0.0683	0.0435	0.54%	0.59%
12	0.0655	0.0653	0.0381	0.01%	0.01%
13	0.0571	0.0625	0.0332	0	0
14	0.0495	0.0598	0.0288	0	0
15	0.0427	0.0573	0.0248	0	0

Table 2. Example 2. This table displays a bank sector with 5 banks and identifies “too-big-to-fail” banks following the presented equilibrium approach. In this example, any two different banks have the same parameter $\rho = 10\%$, and $\text{Var}(L_i) = k^2 \sigma^2$ where $k = 0.8$. In this example, we see that “too-big-to-fail” banks can be found using beta vector entirely. In this example bank 5 is not “too-big-to-fail”

Bank	β_i	$\frac{\beta_i}{\sum_{k=1}^i \beta_k}$	Coinsurance Coefficient a_i
1	0.4690	1	35.24%
2	0.3206	0.4060	20.40%
3	0.2215	0.2191	10.49%
4	0.1548	0.1328	3.82%
5	0.1095	0.0859	0

3.3. Examples of Loss Beta

In the above equilibrium model of capital insurance, the payoff function (i.e., indemnity of the insurance contract) of the aggregative loss, $I(L)$, is an input element. In this subsection, we provide several examples of loss beta for various payoff functions of the aggregative loss portfolio.

3.3.1. Aggregate Loss Beta

We first specialize the capital insurance-aggregate capital insurance-by assuming that the indemnity, $I(L)$, is the aggregate loss L . It is motivated by the standard coinsurance contract in the insurance market.

Since the loss beta concept depends on the capital insurance contract, we call the loss beta for the aggregate capital insurance an *aggregate loss beta*. Clearly, the aggregate loss beta becomes the classical beta in classical portfolio choice literature (See Hogan and Warren (1974) and Bawa and Lindenberg (1977)),

$$\beta_i = \frac{\text{Cov}(L_i, L)}{\text{Var}(L)},$$

and the total loss beta, $\sum_{i=1}^N \beta_i = 1$. If each $\beta_i = \frac{1}{N}$, all banks are TBTF. In this case, Proposition 1 is proved in Ivanov (2017) that a TBTF bank's aggregate loss beta is greater than $\frac{1}{2N}$.

3.3.2. Deductible Loss Beta

In addition to the coinsurance, aL as we discuss above, it is natural to consider a deductible capital insurance with payoff $I(L) = \max\{L - M, 0\}$ where M is a deductible level. In other words, the bank does not receive a percentage of the aggregate loss, but receives a percentage of the deductible $L - M$. In the insurance literature, this deductible part (loss) $a_i M$ is borne by the bank itself. Arrow (1971) shows that the deductible is optimal for the insured under the linear premium principle. The corresponding beta is the deductible loss beta

$$\beta_{i,M} = \frac{\text{Cov}(L_i, \max(L - M, 0))}{\text{Var}(\max(L - M, 0))}.$$

3.3.3. Cap Loss Beta

If the deductible represents an optimal one for the insured, Raviv (1979) finds the optimal one from the insurer's perspective in classical insurance literature. In a principal-agent setting, Raviv (1979) shows that the cap insurance is optimal for the insurer. Therefore, we can also consider a cap capital insurance with the payoff $I(L) = \min(L, c)$, and the corresponding loss beta is

$$\beta_i^C = \frac{\text{Cov}(L_i, \min(L, c))}{\text{Var}(\min(L, c))}.$$

Despite the difference among aggregate, deductible, and cap loss beta, the empirical implications are reasonably similar, as shown in Ivanov (2017). Therefore, the payoff structure of capital insurance is only second-order, while the construction of the loss portfolio has first-order importance, as will be explained in detail in empirical Section 4 below.

3.3.4. Other Capital Insurance Contracts

Our equilibrium approach can be applied to a larger class of capital insurance concept. Instead of the aggregate loss portfolio L , we can consider a general specification in all individual loss such as

$$I(L_1, \dots, L_N)$$

where $I(x_1, \dots, x_N)$ is a multi-variable function which is increasing with respect to each component. By a similar analysis about multi-claim in Raviv (1979) we can show that the payoff structure of the optimal contract must be a function of the aggregate variable $L = L_1 + \dots + L_N$. Therefore, the capital insurance written on the aggregate loss portfolio is an optimal design from the insurance perspective.

However, other specifications of the capital insurance are also plausible. For instance, in their earlier proposal, Kashyap et al. (2008) suggest the following specification of the capital insurance for bank i ,

$$I(L - L_i)$$

in which this bank's loss portfolio is excluded. Its intuition is to reduce the asymmetric information effect so the bank has no incentive to misreport the loss information. In a welfare analysis of certain correlated structures of the loss portfolios, Peck and Shell (2003) document that this kind of capital insurance is dominated by the capital insurance on the aggregate insurance. Furthermore, Ivanov (2017) demonstrates that the moral hazard issue can be resolved effectively by implementing capital insurance. For these reasons,

we suggest the specification form $I(L)$ as a capital insurance in studying the systemic risk through this presented insurance approach.

3.4. Analytical Expression of Loss Betas

We examine the loss beta calculation when the loss portfolios are normally distributed. We assume that all P & L portfolios Y_i are represented in a factor model as follows.

$$Y_i = b_i + \sum_{j=1}^M b_{ij}f_j + \epsilon_i, i = 1, \dots, N$$

in which the factors $(f_1, \dots, f_K) \sim N(0_N, I_N)$, a standard multivariate normal distribution. Each noise, $\epsilon_i \sim N(0, \sigma_i^2)$, is independent from each other, and these noises are independent from all factors. Therefore, the distribution of the individual loss is given by $L_i = \max(-Y_i, 0)$, and

$$-Y_i \sim N\left(-b_i, \sum_{j=1}^M b_{ij}^2 + \sigma_i^2\right).$$

L_i is truncated and normally distributed. The aggregate loss $L = L_1 + \dots + L_N \sim \max(-Y_1, 0) + \dots + \max(-Y_N, 0)$. Therefore, the loss beta β_i can be obtained analytically or well approximated.

3.5. Conditional Loss Betas

At first sight, it is challenging to extend the loss beta concept in a dynamic setting due to the time-inconsistent feature of the mean-variance preference. However, we can consider the capital insurance contract repeatedly, so both the regulator and the financial market monitor systemic risk in a prompt manner. At each period, the existing capital insurance is expired, and the bank updates new loss portfolios. The insurance company issues the capital insurance at each period. Then each bank's systemic risk is updated, and all TBTF are also updated through the new capital insurance market.

Specifically, we use \mathcal{F}_t to represent the information set at time t , and the updated loss portfolios at time t are $L_1 = \max(-Y_1, 0), \dots, L_N = \max(-Y_N, 0)$. Therefore, the loss beta at time t is

$$\beta_i = \frac{\text{Cov}(L_i, I(L)|\mathcal{F}_t)}{\text{Var}(I(L)|\mathcal{F}_t)},$$

and the same discussion on (unconditional) loss beta in the previous section can be applied in this situation.

For instance, we consider the same factor model as in the last subsection, and a shock \tilde{s} arrives at time t . Assume that $(f_1, \dots, f_M, \epsilon)$ has a multivariate normal distribution. Therefore, the conditional P&L variables are

$$Y_i|\{\tilde{s} = s\} = b_i + \sum_{j=1}^M b_{ij}f_j|\{\tilde{s} = s\} + \epsilon_i|\{\tilde{s} = s\}, i = 1, \dots, N.$$

We assume that the covariance-variance matrix of $(f_1, \dots, f_M, \tilde{s})$ is

$$\begin{pmatrix} I_N & \alpha^\top \\ \alpha & \sigma_s^2 \end{pmatrix}.$$

Then, these factors f_1, \dots, f_M , are multivariate normally distributed conditional on $\tilde{s} = s$, with mean $\alpha' \frac{s}{\sigma_s^2}$, and the covariance-variance matrix $I_N - \frac{\alpha\alpha'}{\sigma_s^2}$. Analytical expression of the conditional loss betas follows from the discussion in the last subsection.

3.6. Loss Betas with Tail Risks

Since our equilibrium approach is model-free, any profit and loss portfolio distribution and the loss portfolio can be used.¹⁴ Therefore, we can incorporate tail risk or the skewness in the profit and loss portfolio, particularly for the portfolio with significant derivative positions. Moreover, the correlation structure between these loss portfolios can calibrate the correlated structure in the market.

Specifically, we let $F_i(x)$ denote the distribution of the loss variable L_i , and the joint distribution of $\{L_1, \dots, L_N\}$ is written as

$$F(x_1, \dots, x_N) = C(F_1(x_1), \dots, F_N(x_N)), x_1, \dots, x_N \geq 0$$

where $C(u_1, \dots, u_N)$ is a copula function. Sklar's theorem states that any joint distribution can be written in this way, so the copula function $C(\cdot)$ measures the correlation structure of the loss variables. In practice, the copula functions can be Gaussian copula and Archimedean copula. Those marginal distributions $F_i(x_i)$ can be chosen to capture the tail risk.

The loss beta in this setting can be calculated easily via a simulation method. For instance,

$$\mathbb{E}[L_i I(L)] = \int x_i I(x_1 + \dots + x_N) dC(F_1(x_1), \dots, F_N(x_N)).$$

4. Empirical Results

This section takes our model into the data to study the empirical implication of our equilibrium-based measure of systemic risk. Our objective is to identify TBTF banks in the U.S. financial market over the period 2002 to 2019. We first construct the loss portfolio of each bank based on an empirically accessible proxy. Then we calculate the loss beta and identify TBTF banks using the algorithm in Section 3.

We conduct two approaches in constructing the loss portfolio. In the first approach, we follow the accounting-based methodology introduced in Section 2. In the second approach, we follow [Ivanov \(2017\)](#) to construct the loss portfolio. Moreover, we compare with the CoVaR approach in [Adrian and Brunnermeier \(2016\)](#).

4.1. Data and Sample

In our first approach to construct the loss portfolio, we use data from FFIEC 031 Forms, also called "Call Reports". FFIEC 031 forms are the forms that every commercial bank or thrift institution must file quarterly, regardless of regulation by the Federal Deposit Insurance Corporation (FDIC), Office of the Comptroller of the Currency (OCC), or Federal Reserve. FFIEC 031 includes bank deposits, loans, and other bank characteristics of interest.

In our sample, we focus on all commercial U.S. banks with assets over \$50 billion, which is the threshold to be classified as Large Banking Organization by the Federal Reserve. Large Banking Organizations are subject to stricter examination for their systemic risk, as evident by an additional layer of examination in annual stress testing. We choose Large Banking Organizations to be in our sample of interest since they are the candidates for higher systemic risk. The final sample contains 2821 bank-quarter observations over 70 quarters, from the first quarter of 2002 to the second quarter of 2019. Since the data panel is unbalanced as banks exit and enter the sample throughout the sample period, there are 67 unique banks in total.

In this section, we compare our loss beta with an established measure of systemic risk, CoVaR.¹⁵ To calculate CoVaR, we extend the measure from [Adrian and Brunnermeier \(2016\)](#) to the second quarter of 2019 based on stock return data from CRSP and macroeconomic variables from the FRED.¹⁶ For tests on the correlation between loss beta and CoVaR, the sample is restricted to public banks since CoVaR requires stock return data that are only available to public firms. We use the CRSP-FRB link provided by the Federal Reserve Bank of New York to merge our loss data measures with the CoVaR measure.

Finally, in our second approach to construct the loss portfolio, we construct the bank's loss portfolio based on information available in Compustat and CRSP, following [Adrian and Brunnermeier \(2016\)](#) and [Ivanov \(2017\)](#). Specifically, we obtain the bank total asset from Compustat and bank total equity from CRSP. Our sample is limited to publicly traded financial institutions only in this approach.

4.2. Bank Loss Beta

Following the model provided in Section 2, we estimate the bank loss portfolio. From the Call Reports, we gather the book value of the bank's total loans, the book value of the bank's total equity, the book value of the bank's total deposits, and the book value of the bank's subordinated debt. We use these book values as proxies for the fair value of the loans and deposits, and the market value of the equity and the subordinated debt. Then, we use these book values to calculate each bank's loss portfolio for each quarter as explained in Section 2. The aggregate loss portfolio is also calculated for each quarter in the sample period.

Figure A1 shows the histogram of all banks' loss portfolios by quarter. As shown, the distribution of loss portfolio across all banks is highly skewed. Moreover, the probability of a large loss, such as an over \$800 billion loss, is not marginal, showing a clear tail risk in the banks' portfolios.

Given all loss portfolios and assuming that $A = 1$, we compute all aggregate loss betas, $\frac{Cov(L_i, L)}{Var(L)}$, for each quarter. Figure A2 reports both mean and the standard deviation of all loss betas for each quarter. Since the total number of banks in our sample is $N = 67$, the mean of loss beta is close to $1/67$. However, we observe a relatively high standard deviation, meaning that some banks have substantial systemic risk components. Therefore, some banks are TBTF and some are not.

TBTF banks are identified in each quarter using the algorithm described in Appendix A. Figure A3 reports the number of TBTF in each quarter from 2002 to 2019. The number of TBTF is time-varying. For example, there are 9 TBTF in 2004, then the number gradually decreases. During the 2007–2008 financial crisis period, there are 3–5 TBTF, but each TBTF's systemic risk is substantial. As shown in [Acharya et al. \(2012\)](#), 5 firms provide 50% of the entire systemic risk in the U.S. financial markets, and 15 firms account for 92% of the systemic risk. This highly concentrated feature of TBTF banks is thus consistent with [Acharya et al. \(2012\)](#). This finding is also confirmed by the second empirical approach below.

Table 3 reports all TBTF in a particular quarter. No bank is TBTF for the whole sample period, but four banks are TBTF the vast majority of the time. Not surprisingly, those banks are the "Big Four"—Bank of America, JP Morgan Chase, Wells Fargo, and Citibank. From the start of the data in 2001, the number of TBTF banks declined steadily even before the crisis, dropping to just three TBTF banks in the fourth quarter of 2008 (JP Morgan Chase, Bank of America, and Citibank). Interestingly, Wachovia is identified as TBTF over the period 2007 Q4 to 2008 Q3. The number of TBTF banks stayed low until 2014, when BNY Mellon, State Street, and Capital One briefly joined the Big Four before abruptly falling out after 2015. The number of TBTF banks reached a sample low of two (Bank of America and Wells Fargo) TBTF banks from 2017Q2 to 2018Q1, before rapidly expanding to 2019Q2, with a sample high of 11 banks.

Table 3. List of TBTF Bank based on Bank Loss Portfolio Approach. This table reports TBTF banks from the first quarter of 2002 to the second quarter of 2019 based on the bank loss portfolio approach. TBTF banks are identified base on the loss betas constructed following the models in Section 2 using Call Reports. The sample contains 2821 quarterly observations from the first quarter of 2002 to the second quarter of 2019 with 67 unique commercial U.S.banks with assets over \$50 billions.

Name	TBTF Quarters
Bank of America	2002Q1–2013Q2, 2014Q1–2019Q2
Citibank	2002Q1–2010Q4, 2012Q2–2015Q4, 2019Q1–2019Q2
JP Morgan Chase	2002Q1–2017Q1, 2018Q2–2019Q2
Fleet National	2002Q1–2004Q1
US Bank	2002Q1–2004Q1
Merrill Lynch	2002Q1–2004Q1
Bank One	2002Q1–2004Q3
Suntrust	2002Q1–2003Q4
Wachovia	2003Q2–2007Q2, 2007Q4–2008Q3
Wells Fargo	2004Q1–2008Q3, 2010Q1–2019Q2
Bank of New York Mellon	2014Q1–2015Q4
State Street	2014Q3–2015Q4
Capital One	2014Q4–2015Q4, 2018Q4–2019Q2
Charles Schwab	2018Q2–2019Q2
Goldman Sachs	2018Q4–2019Q2
Ally	2019Q1–2019Q2
TD Bank	2019Q1–2019Q2
American Express	2019Q1–2019Q2

4.3. Comparison between Loss Beta and CoVaR

As a comparison, we implement the CoVaR methodology in [Adrian and Brunnermeier \(2016\)](#). We calculate CoVaR for each quarter in the same time period. Figure A4 reports mean and standard deviation (SD) of CoVaR in each quarter. Table 4 reports summary statistics of bank assets, loss portfolios, loss beta, TBTF, and CoVaR. Bank assets are from Call Reports.

We also take a step further by running panel regressions of loss beta with respect to CoVaR. Table 5 reports the regression coefficients and statistical significance. The number of sample is slightly different, reducing from 2824 to 2018, because the stocks of some of the banks in the Call Report are not publicly traded. Table 5 show a positive and significant relationship between Loss Beta and CoVaR. The magnitude decreases once we control for quarter fixed effect and a bank-specific characteristic, total assets. As shown in Table 5, loss beta is positively correlated with an established systemic risk measure, CoVaR, and captures some additional aspects of systemic risk.

Table 4. Summary Statistics. This table reports summary statistics of bank assets, loss portfolios, loss beta, TBTF, and CoVaR. Bank assets are from Call Reports. Loss portfolio and loss beta following the models in Section 2 using Call Reports. TBTF is a dummy variable the equals one if the bank is identified as a TBTF bank based on the loss betas constructed. CoVaR is constructed following Adrian and Brunnermeier (2016). The sample is from the first quarter of 2002 to the second quarter of 2019 with U.S.banks with assets over \$50 billion.

	Observations	Mean	SD	Min	Max
Assets	2824	211.62	383.42	0.58	2354.81
Loss Portfolio	2824	129.74	237.00	0.00	1388.28
Loss Beta	2824	0.03	0.05	0.00	0.35
TBTF	2824	0.16	0.33	0.00	1.00
CoVaR	2018	0.01	0.01	−0.02	0.05

Table 5. Loss Beta and CoVaR. This table reports coefficients and standard errors of panel regressions of loss beta on CoVaR. Loss beta following the models in Section 2 using Call Reports. CoVaR is constructed following Adrian and Brunnermeier (2016). Controls include bank asset obtained from Call Report. The sample is from the first quarter of 2002 to the second quarter of 2019 with U.S.banks with assets over \$50 billions. Standard errors are reported in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively, based on a *t*-test.

	(1) CoVaR	(2) CoVaR	(3) CoVaR	(4) CoVaR
Loss Beta	0.494 *** (0.127)	0.438 *** (0.126)	0.330 * (0.187)	0.396 ** (0.185)
Controls			Yes	Yes
Quarter FE		Yes		Yes
Observations	2018	2018	2018	2018
R-squared	0.007	0.075	0.008	0.075

4.4. Alternative Approach: Asset Loss Beta

In this subsection, we present the result of loss beta using an alternative approach. Specifically, we calculate the loss beta using the total asset from Compustat and total equity from CRSP, in the spirit of Adrian and Brunnermeier (2016). This approach was implemented in Ivanov (2017) for 14 big financial institutions between 2004 to 2012.¹⁷ For a comparison purpose, we extend Ivanov (2017) to include all institutions with a Standard Industrial Classification (SIC) code between 6000 and 6499 and assets over \$50 billion from quarter 1 of 2002 to quarter 2 of 2019. This sample has significant overlap from our banking loss portfolio, in particular, the publicly traded Large Banking Organization. It includes financial institutions that are not banks, such as insurance companies and securities brokers and dealers.¹⁸ To distinguish, we name the loss beta in this approach as “asset loss beta”, whereas the first one is “bank loss beta”. Our purpose is to identify TBTF banks in this period with the asset loss beta and present the effect of loss portfolio input to the identification of TBTF banks. In particular, we explain that the bank loss beta might be more appropriate for commercial banks.

In this alternative approach, we estimate the key input parameters in this approach as follows. For each financial institution *i*. Let

- l_t^i : the leverage ratio of institution *i* at time *t*, the ratio of total asset value over the total equity value;
- M_t^i : the market capitalization of institution *i* at time *t*;
- Y_t^i : the profit and loss of institution *i* at time *t*, that is, $Y_t^i \equiv l_t^i \cdot M_t^i - l_{t-1}^i \cdot M_{t-1}^i$;
- L_t^i : the loss portfolio of institution *i* at time *t*, that is, $L_t^i \equiv \max\{-Y_t^i, 0\}$.

Table 6 shows the TBTF financial institutions identified using this alternative approach. Among the TBTF are traditional large commercial banks, such as JP Morgan Chase, Bank of America, Citigroup, and Wells Fargo, large investment banks such as Morgan Stanley and Goldman Sachs, and large insurance firms like American International (AIG) and Metlife. Interestingly, AIG, Wachovia, and Countrywide Financial were identified as TBTF financial institutions before they were bankrupt or acquired. Specifically, Table 6 identifies Countrywide Financial as TBTF in the third quarter of 2007, but this firm was not considered in [Ivanov \(2017\)](#). We also notice that American Express, which is missing in [Ivanov \(2017\)](#), is identified as a TBTF in 2001Q1–2003Q2 and 2004Q1–2005Q4. Table 6 provides additional empirical evidence on the validity of our measure.

Table 6. List of TBTF Financial Institutions based on Asset Loss Beta. This table reports TBTF financial institutions from 2002Q1 to 2019Q2 from the publicly listed firms with assets over \$50 billion by using asset loss portfolio. The asset loss portfolio is estimated based on the total asset from Compustat and total equity from CRSP in the spirit of [Adrian and Brunnermeier \(2016\)](#). This table extends [Ivanov \(2017\)](#) from 14 financial institutions between 2004 to 2012 to a more extensive set of institutions with a Standard Industrial Classification (SIC) code between 6000 and 6499 and assets over \$50 billion from quarter 1 of 2002 to quarter 2 of 2019.

Name	TBTF, Quarters
American Express	2002Q1–2003Q2, 2004Q1–2005Q4
Bank of America	2002Q1–2004Q2, 2008Q2–2019Q2
Bank One	2002Q1–2002Q2
Citigroup	2002Q1–2019Q2
JP Morgan Chase	2002Q1–2007Q3, 2008Q4–2019Q2
Morgan Stanley	2002Q1–2005Q3, 2006Q1–2006Q2, 2007Q3–2013Q3, 2014Q1–2019Q2
American International Group	2002Q3–2007Q3, 2008Q2–2008Q3
Goldman Sachs	2004Q2–2005Q3, 2007Q3, 2008Q1–2013Q3, 2014Q1–2019Q2
Countrywide Financial	2007Q3
Wachovia	2008Q3
Wells Fargo	2009Q1–2019Q2
Metlife	2014Q1–2018Q1

Compared with [Ivanov \(2017\)](#), the identified TBTF banks are relatively similar even though our dataset contains significantly more financial institutions than [Ivanov \(2017\)](#). We also find that the Big Four commercial banks—Bank of America, Citigroup, JP Morgan Chase, and Wells Fargo—are identified as TBTF since 2008. In addition, Morgan Stanley and Goldman Sachs are also consistently identified as TBTF after 2014.

A comparison of bank loss beta with asset loss beta yields interesting results. Asset loss beta can identify non-bank financial firms that are TBTF, such as AIG and Metlife, or the large investment banks before they reorganized in 2008. However, asset loss beta fails to take into account any firms that are not publicly traded. Since bank loss beta relies on only banks' data, it cannot identify non-bank financial institutions as TBTF. However, bank loss beta has the benefit of identifying TBTF regardless of the public status of the firm. It also identifies more banks as TBTF than the asset loss beta measure. Moreover, it is convenient from a policy perspective as banks fall under the same regulatory structure, unlike other non-bank financial institutions.

Table 3 identifies TBTF institutions using bank loss beta, and Table 6 uses asset loss beta. The findings are slightly different even when only looking at banks. Both approaches are fairly consistent in identifying the largest commercial banks, such as Bank of America,

JP Morgan Chase, Citigroup, Wachovia, and Wells Fargo, as TBTF. However, the bank loss beta, by not taking into consideration non-bank financial institutions, is more informative within the banking sector as it much more consistently and frequently identifies relatively smaller banks as TBTF, such as Bank One, Fleet National, US Bank, and Suntrust as TBTF before the 2008 Financial Crisis, and BNY Mellon and State Street after the crisis, and even Capital One, Charles Schwab, Ally, and TD Bank in the most recent quarters of the data sample. Notably, in the most recent quarter 2019Q2, using asset loss beta, only 6 institutions, Bank of America, Citigroup, JP Morgan Chase, Morgan Stanley, Goldman Sachs, and Wells Fargo, were TBTF. However, using bank loss beta, 11 institutions are TBTF, the six under asset loss beta, and Capital One, Charles Schwab, Ally, TD Bank, and American Express. Overall, the bank loss beta provides a more consistent and frequent signal that a commercial bank should be considered TBTF.

5. Conclusions

In this paper, we provide an equilibrium approach based on economic theories to measure the systemic risk of financial institutions and identify TBTF financial intermediaries such as banks, insurance companies, or other types of intermediaries. This paper sheds light on why and how financial crises such as the 2007–2009 form and grow in magnitude. The recent episode of Covid-19 also sparks considerable interests on the systemic risk due to unexpected shocks. By constructing each financial intermediary's expected loss portfolio in a recent accounting model, we suggest a capital-based insurance approach to address the systemic risk that is crucial to the health of the entire financial market. We derive a systemic risk measure, termed loss beta, in a characterization of the capital-based insurance equilibrium. We perform a comprehensive empirical implication and document U.S. TBTF financial intermediaries from 2002 to 2019. We demonstrate a time-series pattern of U.S. TBTF financial intermediaries, and a significant number of TBTF banks starting from 2019.

Compared with previous literature on systemic risk, our approach relies on an insurance equilibrium in which all banks with systemic risk exposures and a regulator with implicit guarantee obligation maximize expected utility separably. Previous important studies such as [Acharya et al. \(2012\)](#), [Billio et al. \(2012\)](#) and [Adrian and Brunnermeier \(2016\)](#) introduce some statistical measures that are easy to apply, but these measures are mostly constructed exogenously. On the other hand, other measures that provide specific channels such as liquidity mismatching or consumer/real property risk require future data, which is impossible to implement ex-ante. Our proposed approach advances the literature by providing a forward-looking systemic risk measure with an insurance equilibrium approach to determine the TBTF financial intermediaries endogenously. Moreover, since we develop this approach in an insurance framework, this approach can also be helpful to estimate the expected bailout and bailout probability under the real-world probability measure.

Our results have policy implications and practical appeals. First, the implementation of this approach is straightforward. By using a loss portfolio for each bank (provided by each bank or constructed by the regulator), the regulator can calculate each bank's loss beta and identify TBTF banks with a simple algorithm in this conceptual framework. Second, the regulator is allowed to choose capital insurance contracts in the framework. In this way, various loss betas can be used to check the robustness of a bank's systemic risk. Third, the regulator can use the capital insurance premium to assess the implied guarantee subsidy or another type of capital for TBTF banks. Finally, through the analysis of the loss portfolio, the bank can reduce the systemic risk exposure with this approach.

According to our theoretical and empirical study, a loss portfolio is a crucial input in our approach and affects the identification of TBTF banks. However, the construction of a loss portfolio is far from unique. While we suggest some accounting-based or asset-based methods, constructing the most appropriate loss portfolio with systemic risk exposure is not resolved yet. In other words, we need to understand which bank factors/characteristics contribute to the systemic risk before constructing the loss portfolio, and there are many studies in the literature to identify those factors. This paper only compares the loss beta

approach with CoVaR, but it is also essential to compare it with numerous other systemic risk measures to understand the loss beta measure better. More importantly, we do not consider the availability of this approach to other regions, particularly European banks. We leave these topics to a future study.

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Appendix A. Properties of Capital Insurance

In this appendix, we present several properties of a general capital insurance, extending [Pantser and Tian \(2013\)](#) and [Ivanov \(2017\)](#). Let $Z = I(L)$, $b = \frac{A\mathbb{E}[Z]}{\text{Var}(Z)} > 0$, and

$$f(\rho) = \sum_{i=1}^N \max(\beta_i \rho - \rho^2 b, 0). \quad (\text{A1})$$

Without loss of generality, we assume that $\beta_1 > \beta_2 > \dots > \beta_N > 0$. For each $m = 1, \dots, N$, we define

$$g_m(\rho) = \sum_{i=1}^m \rho(\beta_i - \rho b), \quad (\text{A2})$$

and

$$\begin{cases} \mathbb{I}_m = \left[\frac{\beta_{m+1}}{b}, \frac{\beta_m}{b} \right], m = 1, \dots, N-1, \\ \mathbb{I}_N = \left[0, \frac{\beta_N}{b} \right]. \end{cases} \quad (\text{A3})$$

It can be shown that (see [Ivanov \(2017\)](#), Appendix A) the insurer's optimization problem is reduced to

$$\max_{\rho > 0} f(\rho) = \max_{m=1, \dots, N} \max_{\rho \in \mathbb{I}_m} g_m(\rho). \quad (\text{A4})$$

Let $B_m \equiv \max_{\rho \in \mathbb{I}_m} g_m(\rho)$, $m = 1, \dots, N$, and $\tau_m = \frac{\beta_1 + \dots + \beta_m}{2mb}$, $m = 1, \dots, N$.

Case 1. $m = 1$.

In this case, $A_1 = g_1(\max(\tau_1, \frac{\beta_2}{b}))$. The function $g_1(\cdot)$ is decreasing over \mathbb{I}_1 if and only if $\frac{\beta_2}{b} \geq \tau_1$, that is, $\beta_2 \geq \frac{\beta_1}{2}$.

Case 2. $m = N$.

In this case, $A_N = g_N(\min(\tau_N, \frac{\beta_N}{b}))$. The function $g_N(\cdot)$ is increasing over \mathbb{I}_N if and only if $\frac{\beta_N}{b} \leq \tau_N$; that is, $\beta_N \leq \frac{\beta_1 + \dots + \beta_N}{2N}$. Otherwise, if $\beta_N > \frac{\beta_1 + \dots + \beta_N}{2N}$, then $A_N = g_N(\tau_N) = \frac{(\beta_1 + \dots + \beta_N)^2}{4Nb}$.

Case 3. $m = 2, \dots, N - 1$.

3.a. If $\frac{\beta_{m+1}}{b} < \tau_m < \frac{\beta_m}{b}$, then $A_m = g_m(\tau_m) = \frac{(\beta_1 + \dots + \beta_m)^2}{4mb}$.

3.b. If $\tau_m \leq \frac{\beta_{m+1}}{b}$, then $g_m(\cdot)$ is decreasing over \mathbb{I}_m .

3.c. If $\tau_m \geq \frac{\beta_m}{b}$, then $g_m(\cdot)$ is increasing over \mathbb{I}_m .

Characterization of TBTF banks and ρ^* :

Choose m^* as $\operatorname{argmax}_{1 \leq m \leq N} B_m$, and the least one if there are multiple solutions (see Panttser and Tian (2013)). Then,

- Bank $1, \dots, m^* - 1$ are TBTF;
- Bank $m^* + 1, \dots, N$ are NOT TBTF;
- Bank m^* is TBTF if and only if $\tau_{m^*} < \beta_{m^*}$, that is, $\beta_{m^*} > \frac{\beta_1 + \dots + \beta_{m^*}}{2m^*}$.

Moreover, the optimal load factor is

$$\rho^* = \frac{\beta_1 + \dots + \beta_{m^*}}{2m^*}. \quad (\text{A5})$$

The next three results follow from the above property of each function $g_m(\rho)$ over $\mathbb{I}_m, 1 \leq m \leq N$.

Proposition A1. Bank 1 is always a TBTF bank. If β_1 is sufficiently large such that following conditions hold,

$$\frac{\beta_1 + \dots + \beta_m}{2m} \geq \beta_m, m = 2, \dots, N$$

then Bank 1 is the only one TBTF bank.

Proof. The first part follows from the above characterization of TBTF banks; see also Panttser and Tian (2013). By the above discussion, under the proposed condition, the function $g_m(\rho)$ is increasing over $\mathbb{I}_m, m = 2, \dots, N$. Therefore $\max_{\rho} f(\rho) = g_1(\max(\tau_1, \frac{\beta_2}{b}))$, $m^* = 1$. Since $\max(\tau_1, \frac{\beta_2}{b}) < \frac{\beta_1}{b}$, Bank 1 is the only TBTF bank. \square

Proposition A2. Assume that for each $m = 1, \dots, N - 1$

$$\beta_{m+1} \geq \frac{\beta_1 + \dots + \beta_m}{2m},$$

and

$$\beta_N > \frac{\beta_1 + \dots + \beta_N}{2N},$$

then all banks are TBTF.

Proof. Under the first condition, the function $g_m(\rho)$ is decreasing over $\mathbb{I}_m, m = 1, \dots, N - 1$. Then $m^* = 1$. If $\beta_N > \frac{\beta_1 + \dots + \beta_N}{2N}$, then all banks are TBTF. If β_N is not large enough such that $\beta_N \leq \frac{\beta_1 + \dots + \beta_N}{2N}$, then bank N is not TBTF, but all other banks are TBTF. \square

Proposition A3. Assume $2 \leq m \leq N - 1$ and the following conditions holds,

$$\beta_i \leq \frac{\beta_1 + \dots + \beta_i}{2i}, i = m + 1, \dots, N,$$

and

$$\beta_j \leq \frac{\beta_1 + \dots + \beta_j}{2j}, j = 1, \dots, m - 1.$$

1. If $\frac{\beta_1 + \dots + \beta_m}{2m} < \beta_m$, then all TBTF banks are banks $1, \dots, m$.
2. If $\frac{\beta_1 + \dots + \beta_m}{2m} \geq \beta_m$, then all TBTF banks are banks $1, \dots, m-1$.

Proof. Given the condition we see that the function $g_i(\cdot)$ is increasing over \mathbb{I}_i for $i = m+1, \dots, N$, and the function $g_j(\cdot)$ is decreasing over \mathbb{I}_j for $j = 1, \dots, m-1$. Therefore $m^* = m$, $\max_{\rho} f(\rho) = \max_{\rho \in \mathbb{I}_m} g_m(\rho)$. If $\tau_m < \frac{\beta_m}{b}$, then $\max_{\rho} f(\rho) = g_m(\max(\tau_m, \frac{\beta_{m+1}}{b}))$. Then all TBTF banks are $1, \dots, m$. Otherwise, $\tau_m \geq \frac{\beta_m}{b}$ implies that $\max_{\rho} f(\rho) = g_m(\frac{\beta_m}{b})$. Then all TBTF banks are $1, \dots, m-1$. \square

To prove Proposition 1, we need the following lemma that is shown in Ivanov (2017), Appendix A.

Lemma A1. Given N positive numbers such that $b_1 \geq b_2 \geq \dots \geq b_N$ and $\sum_{i=1}^N b_i = 1$. If there exists an integer i such that

$$\frac{b_i}{\sum_{k=1}^i b_k} > \frac{1}{2i}, \quad (\text{A6})$$

then $b_i > \frac{1}{2N}$. Moreover, if “>” is replaced by \geq in (A6), then $b_i \geq \frac{1}{2N}$.

Proof of Proposition 1. The proof is given for aggregate capital insurance in Ivanov (2017). We extend its proof as follows.

Case 1. Assume $m^* = m$ and $\tau_m < \frac{\beta_m}{b}$.

In this case, $\beta_m > \frac{\beta_1 + \dots + \beta_m}{2m}$. Then by Lemma A1 for $b_m = \beta_m \frac{\text{Var}(Z)}{\text{Cov}(L, Z)}$ we obtain $b_m > \frac{1}{2N}$. That is, $\beta_m > \frac{1}{2N} \frac{\text{Cov}(L, Z)}{\text{Var}(Z)}$. Here, all TBTF banks are $1, \dots, m$. Hence the loss betas of all TBTF banks are greater than $\frac{1}{2N} \frac{\text{Cov}(L, Z)}{\text{Var}(Z)}$.

Case 2. Assume $m^* = m$ and $\tau_m \geq \frac{\beta_m}{b}$.

In this case, $\max_{\rho} f(\rho) = \max_{\rho \in \mathbb{I}_m} g(\rho) = g_m(\frac{\beta_m}{b})$. Therefore, the function $g_{m-1}(\cdot)$ must be decreasing over \mathbb{I}_{m-1} . Hence, $\frac{\beta_{m-1}}{b} > \frac{\beta_m}{b} \geq \tau_{m-1}$. It can be written as

$$\beta_{m-1} > \frac{\beta_1 + \dots + \beta_{m-1}}{2(m-1)}.$$

Then by Lemma A1 again and similar to Case 1, we obtain $\beta_{m-1} > \frac{1}{2N} \frac{\text{Cov}(L, Z)}{\text{Var}(Z)}$. In this case, all TBTF banks are $1, \dots, m-1$. Hence, the loss betas of all TBTF banks are greater than $\frac{1}{2N} \frac{\text{Cov}(L, Z)}{\text{Var}(Z)}$. \square

Finally, Ivanov (2017) Proposition 3 shows that the capital insurance market improves the market participants' utilities. See also Panttser and Tian (2013) for more welfare analysis of capital insurance and its application to classical insurance market.

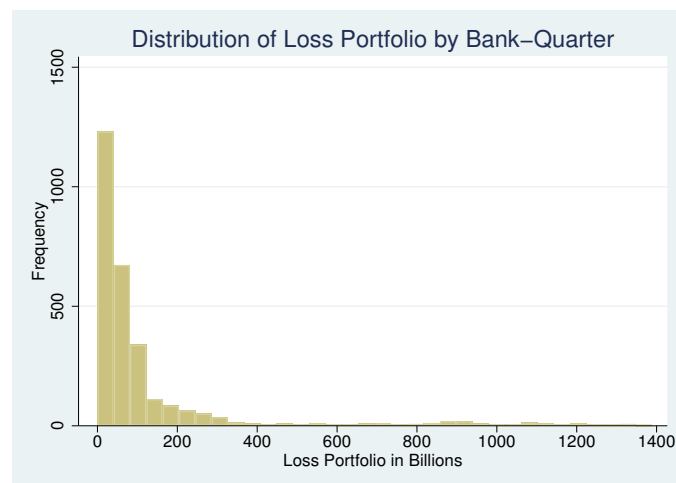


Figure A1. Histogram of Loss Portfolio. This graph displays the histogram bank's loss portfolios by quarter. Loss portfolios are constructed following the model in Section 2 using Call Reports. The sample contains 2821 quarterly observations from the first quarter of 2002 to the second quarter of 2019 with 67 unique commercial U.S. banks with assets over \$50 billions.

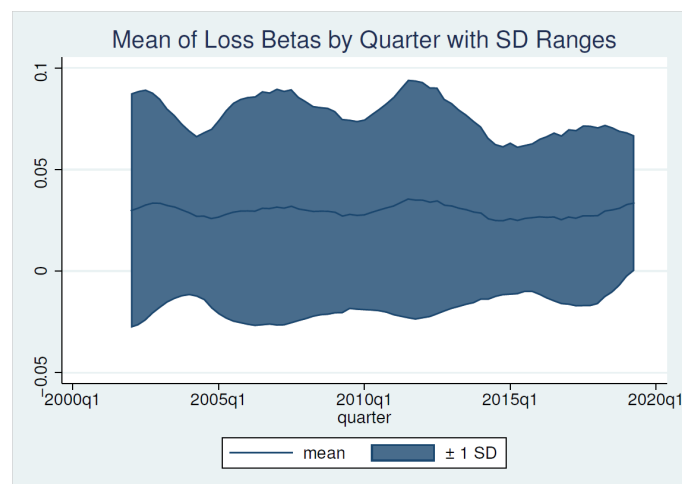


Figure A2. Loss Beta. This graph displays the mean and the standard deviation (SD) of all loss betas in each quarter. Loss betas are constructed following the models in Section 2 using Call Reports. The sample contains 2821 quarterly observations from the first quarter of 2002 to the second quarter of 2019 with 67 unique commercial U.S. banks with assets over \$50 billions.

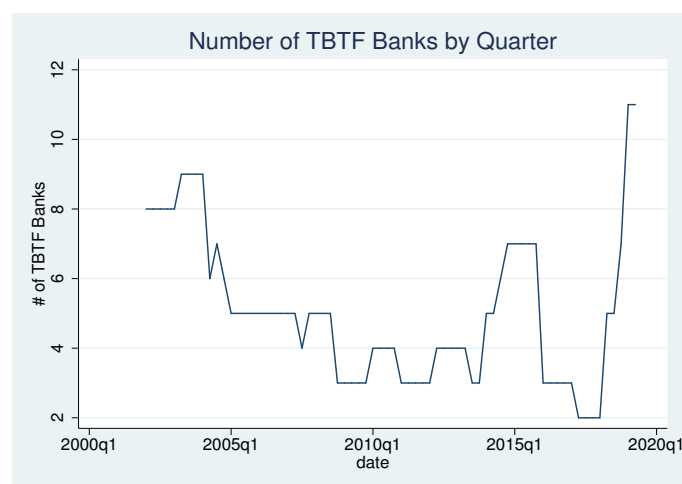


Figure A3. TBTF banks. This graph displays the number of TBTF time series by each quarter.

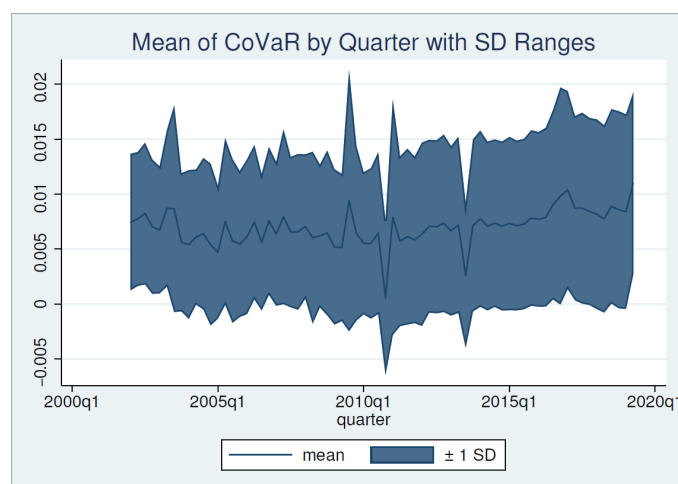


Figure A4. CoVaR. This graph displays the CoVaR time series in each quarter from the first quarter of 2002 to the second quarter of 2019. CoVaR is estimated following [Adrian and Brunnermeier \(2016\)](#) and extended to the second quarter of 2019. The sample contains 2018 bank-quarter observations.

Notes

- 1 Other important approaches include the default probability approach of the whole financial system developed by [Shin \(2008\)](#), and the CDS premium approach in [Zhou et al. \(2009\)](#). See also [Battiston et al. \(2012\)](#), [Billio et al. \(2012\)](#), [Eisenberg and Noe \(2001\)](#), [Hanson et al. \(2011\)](#), [Hellwig \(2009\)](#), [Lehar \(2005\)](#) and [Van Oordt and Zhou \(2016\)](#). [Hansen \(2012\)](#) explains the challenge in measuring systemic risk. A comprehensive survey of systemic risk measures is presented by [Bisias et al. \(2012\)](#).
- 2 In previous studies, systemic risk is primarily measured exogenously, and the implicit guarantee subsidy or capital surcharge is not addressed directly. See [IMF Report \(2014\)](#), [Greens/EFA Report \(2014\)](#), and [BCBS \(2013\)](#) for assessment methodology.
- 3 For a comprehensive discussion about the new Basel capital requirement, we refer to [Tian \(2017\)](#). See also [Allen and Carletti \(2013\)](#) about the classification of systemic risk.
- 4 The construction of a loss portfolio for European banks could be different from U.S. banks. Moreover, we also need to modify the capital insurance approach for European banks since there are at least two layers of regulators for European banks—the European central bank and the regulator in each country. Based on our available dataset, we consider U.S. banks over the period 2002 to 2019 in this paper. Therefore, our approach here only applies to U.S. commercial banks.
- 5 These GSIBs U.S. banks are Bank of America, Citigroup, Goldman Sachs, JP Morgan Chase, Morgan Stanley, and Wells Fargo.
- 6 We notice that our result has no implication to European TBTF banks and implicit government intervention (guarantee). See [Steinruecke \(2018\)](#) about European TBTF banks from the regulatory reform of expected bailout perspective.
- 7 Under these particular numbers, the bank meets Tier 1 capital requirement (at least 6 percent) of risk-weighted asset and total capital (Tier 1 and Tier 2) requirement (10.5%) of risk-weighted asset in Basel III, assuming risk-weight 50% of the subordinated debt.
- 8 The derivation is as follows. Let $v_{D,t}$ denote the ratio of the fair to book value of deposits at time t . Then in that time period $[t, t + 1]$,

$$v_{D,t} = \frac{1}{1+r} \mathbb{E}_t^Q [c_D + \mu_{D,t+1} + (1 - \mu_{D,t+1})v_{D,t+1}].$$

Assume that $\mu_{D,t+1}$ is independent from $v_{D,t+1}$ and $v_{D,t} = \mathbb{E}_t^Q[v_{D,t+1}]$, we obtain

$$v_{D,t} = \frac{1}{1+r} (c_D + \bar{\mu}_{D,t} + (1 - \bar{\mu}_{D,t})v_{D,t})$$

where $\bar{\mu}_{D,t} = \mathbb{E}_t^Q[\mu_{D,t+1}]$ is the conditional deposit withdrawn under risk-neutral probability at time t . Solving $v_{D,t}$ we obtain the expression of FD_t .

- 9 It follows from the same derivation as in the last footnote, or [Atkeson et al. \(2019\)](#), Section 3.1.
- 10 The monotonicity condition is consistent with the classical revelation principle and reduces the moral hazard issue. Similarly, the continuity condition helps to resolve some implementation issues. It is possible to obtain optimal discontinuous and non-monotonic insurance contracts. See [Bernard and Tian \(2009\)](#) and [Huberman et al. \(1983\)](#).

- 11 In other words, each bank is distinguished from the other by the loss portfolio instead of risk attitude. It is straightforward to extend to different risk-aversion parameters in this setting.
- 12 The issuer of capital insurance contracts can be a private-sector, reinsurance company, a central bank or a government entity such as Financial Stability Oversight Council (FSOC) in Dodd-Frank Act, which is universally named as an issuer.
- 13 It is well known that $Cov(L, I(L)) > 0$ for any increasing function $I(\cdot)$ and a general random variable L .
- 14 We only need the finite covariance $Cov(L_i, I(L))$ and finite variance $Var(I(L))$ in the framework.
- 15 Here, we use CoVaR as an example of other numerous systemic risk measures to compare with our approach for the following reasons. (1) The CoVaR approach is intuitive and also based on aggregate loss, (2) Similar to our approach, the CoVaR approach is robust and also simple to be implemented, and (3) the CoVaR approach also generates consistent empirical insights with other measures. See discussions in [Adrian and Brunnermeier \(2016\)](#).
- 16 We would like to thank Markus Brunnermeier for providing the codes to estimate CoVaR.
- 17 They are: Freddie Mac, Fannie Mae, American International Group, Merrill Lynch, Bank of America, Bear Sterns, Citigroup, Goldman Sachs, JP Morgan Chase, Lehman Brother, Metlife, Morgan Stanley, Wachovia, and Wells Fargo. We notice that Merrill Lynch, Bear Sterns, Lehman brother, and Wachovia have not existed since 2008.
- 18 For these reasons, Metlife, American International Group, and Countrywide Financial are included in this approach, while the first approach concentrates on commercial banks.

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