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Progressive Pension Formula and Life Expectancy Heterogeneity

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Abstract: An increasing number of empirical studies have shown a positive relationship between lifetime income and life expectancy at retirement. One's income during the active part of one's career translates into the amount of retirement benefits one might receive, leading to actuarial unfairness inside cohorts of retirees. In order to discuss unfairness and sustainability issues, the Belgium pension reform committee issued a proposal for a point system designed to be both sustainable and adequate. In this paper, we use a similar defined benefit framework in order to set out a compensation mechanism linked to life expectancy heterogeneity during the active part of the career, aiming to reduce unfairness once reaching retirement. This method is based on the progressivity of pension benefit formulae. We implement these ideas in a simple demographic context in order to capture the constraints related to the model.

Keywords: heterogeneity in life expectancy; pension scheme design; progressivity



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1. Introduction

Many studies have observed different tendencies in the relationship between mortality and income. In a cross-country study covering 28 major western countries, (Neumayer and Plumper 2016) showed that income inequality before taxes and transfers was positively associated with inequality in the number of years lived. Unlike earlier times, the current increase in life expectancy of European nations seems to have been influenced by the development of the economy, as (Mackenbach and Looman 2013) showed.

This trend was also observed in Denmark in a 2008 study by (Brønnum-Hansen and Baadsgaard 2008), where their results showed that the gap in health expectancy between persons with low and high educational levels was increasing with startling constancy.

It has been shown that there are many factors influencing the differences in life expectancy, with income gained during the period of professional activity being one of them (Kreiner et al. (2018); Kinge et al. (2019)). On the same track, (Walczak et al. 2021) demonstrated that richer people live longer and that income has a considerably greater influence on life expectancy among men than among women. (Blanpain 2008) found that in France, people with better quality of life have higher life expectancy. For the period 2012–2016, among the wealthiest people, men had a life expectancy at birth of 84.4 years. In contrast, among the lowest 5% of people, men had a life expectancy of 71.7 years. The wealthiest men therefore live on average 13 years longer than the poorest (Blanpain 2016). Among women, this gap is smaller: the life expectancy at birth of women among the wealthiest 5% of people reaches 88.3 years, compared to 80.0 years among the lowest 5%, i.e., 8 years apart.

Across societies, life expectancy is significantly linked to income (Currie and Schwandt (2016a); Mackenbach et al. (2018)). The relationship between a society's socioeconomic class and life expectancy is critical for measuring equality and assessing the risks and benefits of healthcare and social security policies, (Snyder and Evans (2006); Waldron 2007). The relationship between income and life expectancy is well documented, (Currie and Schwandt (2016b); Waldron 2007). Using tax data, (Chetty et al. 2016) calculated non parametric estimates of the relationship between income class and life expectancy.

When ranked by socioeconomic level (SES), (Bosworth et al. 2016) discovered considerable death rate inequalities among elderly Americans, and these differences have expanded dramatically in recent years. It makes little difference whether we assess SES using education or midcareer earnings, though earnings are more directly linked to the income concept used to compute Social Security payments.

These studies demonstrate the influence of wealth disparity on life expectancy. When we apply the relationship between income and remaining life expectancy to public pension systems, we want to know what implications this relationship has on the contribution–benefit connection.

Public pension schemes transfer wealth over generations and, in some cases, among individuals. Individual pension benefits are not perfectly locked down by contributions, resulting in the latter sort of redistribution. Because individual pensions are given out as annuities regardless of life expectancy, the social security system transfers wealth from those with short lives to those with better life expectancies, according to (Borck 2007) and also (Belloni and Maccheroni 2013). When considering the life expectancy difference, Borck’s paper may be viewed as an investigation of the impact of pension schemes’ wealth transfer characteristics.

Pension systems may be progressive because lower-income persons often contribute less than higher-income people in order to gain a similar pension. Individuals with higher incomes, on the other hand, live longer. As a result, if the longer life expectancy outweighs the greater payments, the system may be regressive. There is evidence that actual pension plans can be regressive if individual variations in life expectancy are considered. E.g., (Coronado et al. (2000); (Lefebvre 2007); Reil-Held (2000). (Mitchell and Zeldes 1996) explain that: “Despite its intent, the pension system is less progressive than it might seem, because there is a positive correlation between lifetime earnings and length of life”. It is worth noting that this conclusion may be seen in both more Bismarckian systems, like the German system, and more Beveridgean systems, including the United States. More recently, (Economic Policy Committee 2020) stated that unless alternative sources of funding are found, a shrinking working-age population and an increasing number of retirees will place a double burden on future workers: greater contribution rates while working and smaller pensions when they retire. Furthermore, pension systems must ensure pension fairness and redistribution across income levels, as a shift in funding from social security contributions to taxation, as well as changes in progressivity rules, limits pension systems’ redistributive capacity and raises questions about their fairness across income classes.

Heterogeneity in longevity and the underlying unfairness issue in pension schemes motivated (Holzmann et al. 2017) to propose redesigning the Notional Defined Contribution (NDC) scheme. Here, two promising design alternatives were briefly presented: individualized annuities and the multiple contribution model. In comparison to the current system, both conceptual models succeed in minimizing tax disparities.

Longevity heterogeneity and pension fairness have been studied quantitatively since (Ayuso et al. 2017); (Ayuso et al. 2021b); (Bravo et al. 2021). In order to measure the overall intensity of the transfer mechanism, profiles for tax/subsidy rates are computed from variances in life expectancy. (Culotta 2021) quantified this intensity in Italy between 1995 and 2019 by gender and region. This research not only provided an up-to-date picture of differences in lifespan among Italian regions, but it also calculated the implications of such disparities in terms of an implicit transfer of pension resources. His conclusion was that the conventional architecture of Italian public pension systems has to be changed to differentiate structural characteristics, such as the longevity factor used to calculate pension annuities. In principle, this would diminish the intensity of an implicit but persistent tax/subsidy mechanism. The choice of the socioeconomic factor to tag will be critical in this case. Nonetheless, in order to make completely transparent redistributive performances of public pension systems, a closer and updated monitoring of lifespan heterogeneity along important socioeconomic characteristics is required.

Another approach would be taking into account the life expectancy differences directly in the pension formula, as (Breyer and Hupfeld 2008) did. In their work, they established the notion “distributive neutrality”, which considers disparities in life expectancy based on income group, and then they empirically analyzed the relationship between annual wages and life expectancy among retirees and demonstrated how the mechanism that ties benefits to contributions would need to be adjusted to ensure distributive neutrality.

In this paper, we consider a defined benefit pension system as well as a stationary demographic framework in which agents are differentiated in terms of salary and life expectancy. To study the redistributive features of such a pension system, we consider the actuarial fairness ratio. This ratio for an agent is the discounted value of pension benefit over the agent’s contributions. We introduce a progressive transformation of the pension formula that takes into account the life expectancy differential in order to satisfy two conditions: the sustainability condition of a PAYG scheme, and the actuarial fairness ratio equality condition for agents of different salary classes. The main contribution of this paper is to obtain the explicit form of this progressive transformation. We show in particular that under very special conditions (canonical case), the correction to apply in the pension formula is based on a simple ratio of life expectancies. However, in more general cases, especially when the discount rate used to estimate the level of fairness does not correspond to the actuarial return of the PAYG system, we use a more general transformation formula, based on two multiplicative factors: the ratio of annuities (longevity effect) and the ratio of aggregated salaries (salary effect). The study is structured as follows: In Section 2, we show the pension issue in our defined benefit framework as well as introducing the progressive concept, and we study the fairness properties in the general case. In Section 3, we apply our model to some specific cases before illustrating it with a simple numerical example. Section 4 allows us to conclude and discuss our results and highlight the questions that our study gives rise to.

2. The Pension Unfairness Issue

2.1. Defined Benefits System Pension Calculation

Let us imagine a career average re-evaluated earnings (CARE)-defined benefits scheme. Such system typically provides a retirement benefit based on a percentage of your average wages after taking inflation into account throughout the course of your participation in the scheme.

The pension benefit formula for a complete career at time t is:

$$P^{(t,x_r)} = \frac{\delta}{N} \cdot \sum_{x=x_0}^{x_r-1} S^x \prod_{i=1}^x (1 + \gamma_i)^{x_r-i}$$

where:

x_0 is the age of entry in the scheme and x_r is the retirement age

N is the normal length of contribution.

$P^{(t,x_r)}$ is the pension at retirement for a contribution period of $N = x_r - x_0$ years

γ_t is the indexation rate for year t .

δ is the target replacement rate of the scheme.

S^x is the individual salary taken into account at age x .

- At retirement

We assume that pensions in payment are adjusted each year according to the same coefficient as the indexation rate applied to the salary indexation of active workers:

$$P^{(t,x)} = P^{(t-1,x)} \cdot (1 + \gamma_t)$$

- Actuarial Inequality tied to life expectancy disparity

Let us imagine a scheme with only two salary classes, low incomes and high incomes ($S_1^x < S_2^x$), associated with low and high life expectancy at retirement ($e_1 < e_2$).

In this framework, salaries are indexed with inflation rate g .

$$S^{x+1} = S_i^x \cdot (1 + g)$$

The discount rate is equal to inflation.

In the DB framework, the pension formula at retirement is computed as a proportion of the salary:

$$P_j = \delta \cdot S_j^{x_r}$$

For each class j , the probability of surviving t years at age x is given by

$$p_j(x; x + t)$$

The discount rate used in the PAYG system is equal to the inflation rate. This implies that the present value of benefits at retirement B_i until death, denoted as ω , is given by:

$$B_j = \sum_{x=x_r}^{\omega} P_j \cdot p_j(x_r; x) \cdot \left(\frac{1+g}{1+g}\right)^{\omega-x}$$

$$B_j = P_j \cdot e_j$$

where $e_j = \sum_{x=x_r}^{\omega} p_j(x_r; x)$ is the remaining life expectancy.

The present value of contributions, with π being the contribution rate is:

$$C_j = \sum_{x=x_0}^{x_r-1} \pi \cdot S_j^x \cdot (1+g)^{x_r-x}$$

$$C_j = N \cdot \pi \cdot S_j^{x_r}$$

This leads to the the following results in term of the fairness ratio for low incomes:

$$\frac{B_1}{C_1} = \frac{P_1 \cdot e_1}{N \cdot \pi \cdot S_1} = \frac{\delta}{N \cdot \pi} \cdot e_1$$

and for high incomes:

$$\frac{B_2}{C_2} = \frac{P_2 \cdot e_2}{N \cdot \pi \cdot S_2} = \frac{\delta}{N \cdot \pi} \cdot e_2$$

The actuarial fairness ratio for an agent in this scheme is a proportion of the ratio between the remaining life expectancy at retirement of this agent and their contribution period.

When the two agents have contributed for the normal period, this ratio grows with life expectancy. This means that agents with higher life expectancy are expected to receive more benefits due to the fact that they receive their pension benefits for a longer period. The positive relationship of income with life expectancy leads us to think that people from low incomes are penalized for not living long enough.

This mechanism is a result of the proportionality of the pension benefit with the salary in the considered DB scheme. This conclusion is in line with the results of (Hachon 2009) regarding life expectancy inequalities and pension benefit unfairness.

The following section introduces the progressive transformation of the pension formula that takes into account life expectancy inequality in order to reach actuarial fairness for all salary classes.

Progressivity

(Biggs et al. 2009) defined progressivity as “The degree to which benefits are higher relative to lifetime payroll contributions for lower contributors than for higher contributors”. This is a translation of (Musgrave and Tun Thin 1948) approach regarding income tax.

The characterization of progressivity in pension benefits formulae is given in an OCED pension report (OCED 2011a). In this study, we show two pension systems operating according to a progressive formula: Switzerland and the USA.

- USA

The pension formula is progressive. Earning USD 895 a month gives a 90% replacement rate.

In 2018, the first 895 USD a month of relevant earnings attracts a 90% replacement rate. The income range is replaced at 32% from 895 USD and 5397 USD. The 2018 average national pay index's upper and lower bounds are 20% and 118%, respectively. Between the earlier cutoff and the earnings limitation, a replacement rate of 15% is applied.

$$P = \begin{cases} 0.9 \cdot S & \text{if } S < 10,740 \\ 9666 + 0.32 \cdot (S - 10,740) & \text{if } 10,740 < S < 64,764 \\ 20,725 + 0.15 \cdot (S - 5397) & \text{if } S > 64,764 \end{cases} \quad (1)$$

- Switzerland

The public earnings-related pension benefit is based on average lifetime earnings. Benefit payments include both superior and inferior limitations. The “two-branch” pension calculation benefits average earnings between these two levels. The pension computation tends to transfer income from rich to poor agents. Pension payouts ranged from CHF 14,100 to CHF 28,200 in 2016 with complete contributions. These figures correspond to 16% and 33% of mean worker wages, respectively. When the average income is CHF 84,600, the maximum profit is 99% of the national economy average income. The formula for the complete contribution period is as follows:

$$P = \begin{cases} 0.84 \cdot 16,800 & \text{if } S < 16,800 \\ 14,113 + 0.56 \cdot (S - 16,800) & \text{if } 16,800 < S < 84,600 \\ 0.33 \cdot 84,600 & \text{if } S > 84,600 \end{cases} \quad (2)$$

The existing system calibrates the progressive factors such that the pension benefit reaches certain thresholds such as quantiles of average salary; this approach only serves the accounting purpose of progressivity.

Our aim in the following part is to link the value of the progressive coefficients to the differences in life expectancy between salary classes in order to satisfy actuarial and accounting equilibrium.

2.2. Progressive Pension Model and Fairness

The defined benefit system is one of the most popular Pay-As-You-Go pension schemes, and it has been shown, under the premise of life expectancy inequalities related to income, that this system is unfair to low-income individuals.

Our aim is to use progressivity to calibrate the progressive coefficients with two conditions: actuarial fairness and Pay-As-You-Go equilibrium

2.2.1. Model Hypothesis

- The contribution period is denoted as N .
- There are m working classes determined upon entry into the scheme characterized by their salary $(S_1; \dots; S_m)$ and their proportion (w_1, \dots, w_m)
- The age of entry in the scheme is denoted as x_0 and the age of retirement is denoted as x_r

- $L(x, t)$ is the population function giving at date t the size of the population of age x . We can write $L(x, t) = \sum_{i=1}^m L_i(x, t)$, where $L_i(x, t) = w_i \cdot L(x, t)$ is the size of the population aged x at time t in the class i .
- Let us initially consider a population in an absolutely stationary state. The population function is therefore independent of time.
- We ignore the effects of mortality before retirement. The population function at any age before retirement $x < x_r$ is given by:

$$L(x, t) = L(x) = L \tag{3}$$

- After retirement, we consider that each class has a different mortality evolution. If $p_j(x; x + t)$ represents the probability of an agent aged x and belonging to class j surviving t years, the population of class j is given by:

$$L_j(x; t) = L_j(x_r, t) \cdot p_j(x_r; x) \tag{4}$$

- At any time t , let us imagine that the salary of an agent depends on their age (x) and time (t); this gives

$$S_j^{(t;x)} = S_j^{(0;x_0)} \cdot (1 + s_j)^{x-x_0} \cdot (1 + g)^t \tag{5}$$

where s_j is the career growth effect for the group j salary and g is the inflation effect depending on time t .

- The progressive salary transformation is given by:

$$\begin{aligned} X(S_2) &= \lambda_1 \cdot S_1 + \lambda_2 \cdot (S_2 - S_1) \\ X(S_j) &= \lambda_1 \cdot S_1 + \sum_{i=2}^j \lambda_i \cdot (S_i - S_{i-1}) \end{aligned}$$

The general formula is:

$$X(S) = \sum_{i=1}^m \lambda_i \cdot \max(\min(S, S_i) - S_{i-1}; 0), \tag{6}$$

where $\lambda_1, \dots, \lambda_m$ are the progressive coefficients.

- After retirement, pension benefits are indexed with the same rate γ , and we have the following relationship at any age $x > x_r$:

$$P^{(t,x)} = P^{(t,x_r)} \cdot (1 + \gamma)^{(x-x_r)} \tag{7}$$

2.2.2. Pension Formula

We implemented the progressive mechanism in the pension formula, aiming to correct the underlying unfairness issue. In order to transform a DB system into a progressive one, we chose to transform the salary in the pension formula using salary bandwidths, and based on those thresholds, we apply the progressive coefficients.

We can write the salary transformation for any class j as such with $S_0^{(t;x)} = 0$:

$$X(S_j^{(t;x)}) = X_j^{(t;x)} = \sum_{i=1}^j \lambda_i \cdot (S_i^{(t;x)} - S_{i-1}^{(t;x)})$$

For any $T > t$, $X_j^{(T;x)} = X_j^{(t;x)} \cdot (1 + g)^{T-t}$ we obtain the pension at the time of retirement $T = N$ with:

$$\begin{aligned}
 P_j^{(N,x_r)} &= \sum_{t=0}^{N-1} \frac{\delta}{N} \cdot X_j^{(t;x_0+t)} \cdot (1 + \gamma)^{N-t} \\
 &= \frac{\delta}{N} \cdot \sum_{i=1}^j \lambda_i \sum_{t=0}^{N-1} (S_i^{(t;x_0+t)} - S_{i-1}^{(t;x_0+t)}) \cdot (1 + \gamma)^{N-t}
 \end{aligned}$$

We also have

$$\begin{aligned}
 \widehat{S}_j^N(\gamma, g) &= \sum_{t=0}^{N-1} (S_i^{(t;x_0+t)}) \cdot (1 + \gamma)^{N-t} \\
 &= S_j^{(0;x_0)} \cdot (1 + \gamma) \cdot \frac{[(1 + g) \cdot (1 + s_j)]^N - (1 + \gamma)^N}{(1 + g) \cdot (1 + s_j) - (1 + \gamma)}
 \end{aligned}$$

The last term is the future value of a geometric annuity with growth $(1 + g) \cdot (1 + s_j)$ indexed at rate $1 + \gamma$ over a period of time N . It is noted that

$$Gs_j^{(\gamma)} = (1 + \gamma) \cdot \frac{[(1 + g) \cdot (1 + s_j)]^N - (1 + \gamma)^N}{(1 + g) \cdot (1 + s_j) - (1 + \gamma)}, \tag{8}$$

This leads to:

$$P_j^{(N,x_r)} = \frac{\delta}{N} \cdot \sum_{i=1}^j \lambda_i \cdot (\widehat{S}_i^N(\gamma, g) - \widehat{S}_{i-1}^N(\gamma, g)) = \frac{\delta}{N} \cdot \widehat{X}_j^N(\gamma, g) \tag{9}$$

with:

$$\widehat{X}_j^N(\gamma, g) = \sum_{i=1}^j \lambda_i \cdot (\widehat{S}_i^N(\gamma, g) - \widehat{S}_{i-1}^N(\gamma, g)) \tag{10}$$

Here, we give an explanation of the notations we have introduced so far:

- $\widehat{S}_j^N(\gamma, g)$ is the sum of all salaries throughout the career of an agent of class j indexed at rate γ , with g being the inflation effect.
- $\widehat{X}_j^N(\gamma, g)$ is the progressive transformation applied to the indexed sum of the salary.

2.2.3. Pay-As-You-Go Equilibrium

The number of active workers in the scheme is:

$$\sum_{x=x_0}^{x_r-1} L(x, t) = \sum_{x=x_0}^{x_r-1} L = N \cdot L \tag{11}$$

The total amount of contributions in the scheme for the agents of class i is:

$$\begin{aligned}
 \sum_{x=x_0}^{x_r-1} L_i(x, t) \cdot \pi \cdot S_i^{(t,x)} &= \sum_{x=x_0}^{x_r-1} w_i \cdot L \cdot \pi \cdot S_i^{(t,x)} \\
 &= w_i \cdot L \cdot \pi \cdot \sum_{x=x_0}^{x_r-1} S_i^{(t;x_0)} \cdot (1 + s_i)^{x-x_0} \\
 &= w_i \cdot L \cdot \pi \cdot S_i^{(t;x_0)} \cdot \sum_{x=x_0}^{x_r-1} (1 + s_i)^{x-x_0}
 \end{aligned}$$

where $S_i^{(t;x_0)} \cdot \sum_{x=x_0}^{x_r-1} (1 + s_i)^{x-x_0}$ is the sum of all salaries of class i from age x_0 to x_{r-1} .

For all retired agents of class i , the total of the pension benefits paid is:

$$\begin{aligned} \sum_{x=x_r}^{\omega} L_i(x, t) P_i^{(t,x)} &= L \cdot w_i \cdot P_i^{(x_r)} \cdot \sum_{x=x_r}^{\omega} p_i(x_r; x) \left(\frac{1+\gamma}{1+g}\right)^{(x-x_r)} \\ &= L \cdot w_i \cdot P_i^{(t,x_r)} \cdot a_j^{(\gamma,g)} \end{aligned}$$

where $a_j^{(\gamma,g)} = \sum_{x=x_r}^{\omega} p_i(x_r; x) \left(\frac{1+\gamma}{1+g}\right)^{(x-x_r)}$ is a life annuity discounted with rate $\frac{1+g}{1+\gamma}$.

The pay-as-you-go equilibrium is characterized by the following relationship:

$$\text{Contributions}^t = \text{Benefits}^t$$

We are looking for the π value that satisfies this equilibrium for any given replacement rate.

$$\begin{aligned} \sum_{x=x_0}^{x_r-1} \sum_{i=1}^m L_i(x, t) \cdot \pi \cdot S_i^{(t,x)} &= \sum_{x=x_r}^{\omega} \sum_{j=1}^m L_j(x, t) P_j^{(t,x)} \\ \pi \cdot \sum_{x=x_0}^{x_r-1} \sum_{i=1}^m w_i \cdot L \cdot S_i^{(t,x)} &= \sum_{j=1}^m w_j \cdot L \cdot a_j^{(\gamma,g)} \frac{\delta}{N} \cdot \widehat{X}_j(\gamma, g). \end{aligned}$$

We divide this expression by L .

$$\begin{aligned} \pi \cdot \sum_{x=x_0}^{x_r-1} \sum_{i=1}^m w_i \cdot S_i^{(t,x)} &= \frac{\delta}{N} \cdot \sum_{j=1}^m w_j \cdot \widehat{X}_j(\gamma, g) \cdot a_j^{(\gamma,g)} \\ \pi \cdot \sum_{i=1}^m w_i \cdot \sum_{x=x_0}^{x_r-1} S_i^{(t,x)} &= \frac{\delta}{N} \cdot \sum_{j=1}^m w_j \cdot \widehat{X}_j(\gamma, g) \cdot a_j^{(\gamma,g)} \end{aligned}$$

Here, we introduce $\bar{S} = \sum_{i=1}^m w_i \cdot \sum_{x=x_0}^{x_r-1} S_i^{(t,x)}$, being the average salary of all agents in the scheme.

This gives us the replacement rate and progressive factors for the sustainability conditions between the contribution rate.

$$\delta = \frac{N \cdot \pi \cdot \bar{S}}{\sum_{i=1}^m \lambda_1 w_i \cdot a_i^{(\gamma,g)} \cdot \widehat{S}_1^N(\gamma, g) + \sum_{i=2}^m \lambda_i \sum_{j=i}^m w_j \cdot a_j^{(\gamma,g)} \cdot (\widehat{S}_i^N(\gamma, g) - \widehat{S}_{i-1}^N(\gamma, g))} \tag{12}$$

The equilibrium is characterized by this equation for $(\lambda_1; \dots; \lambda_m; \delta)$ where $\delta > 0$

$$\pi = \frac{\delta \cdot \sum_{i=1}^m \lambda_i \cdot b_i}{N \cdot \bar{S}} \tag{13}$$

with $b_j = \sum_{j=i}^m w_j \cdot a_j^{(\gamma,g)} \cdot (\widehat{S}_i^N(\gamma, g) - \widehat{S}_{i-1}^N(\gamma, g))$.

This relationship expresses the fact that for a given replacement rate δ and a vector of progressive factors $\lambda' = (\lambda_1; \dots; \lambda_m)$, the contribution rate that allows for the sustainability of the scheme is given by the previous relationship (13).

Using this, we implement our second criterion, namely actuarial fairness for every agent in order to find the value of the progressive factors $(\lambda_1, \dots, \lambda_m)$.

2.2.4. Inter-Class Fairness Conditions

We reach inter-class fairness when the actuarial fairness ratio is the same for every agent, regardless of their salary class.

The PAYG condition allowed us to find a value for the system target contribution rate. In order to calibrate the progressive coefficients, we need to solve the inter-class fairness conditions:

- Present value of contributions

The present value of contributions is computed with the agents discount factor r . It is not necessarily an actuarial rate of return, but rather a rate which expresses the perception that agents have about the evolution of their contributions over time.

$$\begin{aligned}
 C_j^N &= \sum_{t=0}^{N-1} \pi \cdot S_j^{(t;x_0+t)} \cdot (1+r)^{N-t} \\
 &= \sum_{t=0}^{N-1} \pi \cdot S_j^{(0;x_0)} \cdot (1+g)^t \cdot (1+s_j)^t \cdot (1+r)^{N-t} \\
 C_j^N &= \pi \cdot S_j^{(0;x_0)} \cdot (1+r) \cdot \frac{[(1+g) \cdot (1+s_j)]^N - (1+r)^N}{(1+g) \cdot (1+s_j) - (1+r)}
 \end{aligned}$$

Using the geometric annuity form, we can write the present value of contributions as:

$$C_j^N = \pi \cdot S_j^{(0;x_0)} \cdot Gs_j^{(r)} = \pi \cdot \widehat{S}_j^N(r, g) \tag{14}$$

- Present value of benefits

At the time of retirement N , the pension benefit for the class j is $P_j^{(N,x_r)}$. The present value of benefits for the class j is indexed with rate $1 + \gamma$, which is computed with the survival probability p_j for the class j :

$$\begin{aligned}
 B_j^N &= \sum_{x=x_r}^{\omega} P_j^{(N,x_r)} \cdot p_j(x_r; x) \cdot \left(\frac{1+\gamma}{1+r}\right)^{x-x_r} \\
 B_j^N &= P_j^{(N,x_r)} \cdot a_j^{(\gamma,r)}
 \end{aligned}$$

where $a_j^{(\gamma,r)} = \sum_{x=x_r}^{\omega} p_j(x_r; x) \cdot \left(\frac{1+\gamma}{1+r}\right)^{x-x_r}$ is the life annuity calculated for the population with survival probability $p_j(x; x+t)$ ¹

- Fairness condition

We need to obtain the values of λ for each career by solving the following inter-class fairness conditions:

$$\frac{B_1^N}{C_1^N} = \dots = \frac{B_j^N}{C_j^N} = \dots = \frac{B_m^N}{C_m^N}$$

This leads to the following system

$$\left\{ \begin{aligned} \frac{B_1^N}{C_1^N} &= \frac{\delta \cdot \lambda_1}{N \cdot \pi} \cdot \frac{\widehat{S}_1^N(\gamma, g)}{\widehat{S}_1^N(r, g)} \cdot a_1^{(\gamma, r)} \\ \dots \\ \frac{B_j^N}{C_j^N} &= \frac{\delta}{N \cdot \pi \cdot \widehat{S}_j^N(r, g)} \cdot (\lambda_1 \cdot \widehat{S}_1^N(\gamma, g) + \sum_{i=2}^j \lambda_i \cdot (\widehat{S}_i^N(\gamma, g) - \widehat{S}_{i-1}^N(\gamma, g))) \cdot a_j^{(\gamma, r)} \\ \dots \\ \frac{B_m^N}{C_m^N} &= \frac{\delta}{N \cdot \pi \cdot \widehat{S}_m^N(r, g)} \cdot (\lambda_1 \cdot \widehat{S}_1^N(\gamma, g) + \sum_{i=2}^m \lambda_i \cdot (\widehat{S}_i^N(\gamma, g) - \widehat{S}_{i-1}^N(\gamma, g))) \cdot a_m^{(\gamma, r)} \end{aligned} \right. \quad (15)$$

The solution is given by:

$$\lambda' = (\lambda_1, \dots, \lambda_m)$$

λ is thus the vector of solution for our $(m - 1)$ conditions. The coordinates are all a scalar times λ_1 .

$$\lambda = \lambda_1 \cdot \left(\begin{array}{c} 1 \\ \frac{\widehat{S}_2^N(r, g) \cdot \frac{a_1^{(\gamma, r)}}{a_2^{(\gamma, r)}} - \widehat{S}_1^N(r, g) \cdot \frac{a_2^{(\gamma, r)}}{a_2^{(\gamma, r)}}}{\widehat{S}_2^N(\gamma, g) - \widehat{S}_1^N(\gamma, g)} \cdot \frac{\widehat{S}_1^N(\gamma, g)}{\widehat{S}_1^N(r, g)} \\ \vdots \\ \frac{\widehat{S}_i^N(r, g) \cdot \frac{a_1^{(\gamma, r)}}{a_i^{(\gamma, r)}} - \widehat{S}_{i-1}^N(r, g) \cdot \frac{a_1^{(\gamma, r)}}{a_{i-1}^{(\gamma, r)}}}{\widehat{S}_i^N(\gamma, g) - \widehat{S}_{i-1}^N(\gamma, g)} \cdot \frac{\widehat{S}_1^N(\gamma, g)}{\widehat{S}_1^N(r, g)} \\ \vdots \\ \frac{\widehat{S}_m^N(r, g) \cdot \frac{a_1^{(\gamma, r)}}{a_m^{(\gamma, r)}} - \widehat{S}_{m-1}^N(r, g) \cdot \frac{a_1^{(\gamma, r)}}{a_{m-1}^{(\gamma, r)}}}{\widehat{S}_m^N(\gamma, g) - \widehat{S}_{m-1}^N(\gamma, g)} \cdot \frac{\widehat{S}_1^N(\gamma, g)}{\widehat{S}_1^N(r, g)} \end{array} \right) \quad (16)$$

The progressive fair pension is then expressed as:

$$P_j^{(N, x_r)} = \frac{\delta}{N} \cdot \widehat{X}_j^N = \frac{\delta}{N} \cdot \sum_{i=1}^j \lambda_i \cdot (\widehat{S}_i^N(\gamma, g) - \widehat{S}_{i-1}^N(\gamma, g))$$

We then use the expression of the progressive factors in the solution of the inter-class fairness condition in the pension formula; this allows us to reduce the telescopic sum in this way for any class $j > 1$:

$$P_j^{(N, x_r)} = \delta \cdot \lambda_1 \cdot \frac{\widehat{S}_1^N(\gamma, g)}{\widehat{S}_1^N(r, g)} \cdot \frac{\widehat{S}_j^N(r, g)}{N} \cdot \frac{a_1^{(\gamma, r)}}{a_j^{(\gamma, r)}} \quad (17)$$

Here, we introduce

$$\beta_j = \frac{\widehat{S}_j^N(r, g)}{\widehat{S}_j^N(\gamma, g)} \cdot \frac{\widehat{S}_1^N(\gamma, g)}{\widehat{S}_1^N(r, g)}, \quad (18)$$

being the correction term for the indexation of the classes. The fair pension then becomes:

$$P_j^{(N,x_r)} = \delta \cdot \lambda_1 \cdot \frac{\widehat{S}_j^N(\gamma, g)}{N} \cdot \beta_j \cdot \frac{a_1^{(\gamma,r)}}{a_j^{(\gamma,r)}} \tag{19}$$

This quantity can be interpreted as a replacement rate multiplied by the average career salary with two corrections: the first one is an economic correction linked to the disparity between the evolution of the agent’s salary during their career and the scheme indexation rate, and the second one is a correction tied to the mortality differential between the classes.

Proposition 1. For any couple (δ, λ_1) and any discount rate r , there is a vector $\lambda = (\lambda_1; \dots; \lambda_m)$ satisfying at the same time the PAYG and inter-class fairness conditions given by (13) and (16).

The theoretical expression of the actuarial ratio is the same for every agent².

$$\frac{B_j}{C_j} = \frac{\delta \cdot \lambda_1}{N \cdot \pi} \cdot \frac{\widehat{S}_1^N(\gamma, g)}{\widehat{S}_1^N(r, g)} \cdot a_1^{(\gamma,r)} \tag{20}$$

This result demonstrates the fact the progressive fair pension makes it possible to obtain the same ratio of benefits to contributions for all classes. However, this ratio is not necessarily equal to one. In the following part, we explore the conditions regarding the discount rate in order for the system to be fair and reach equilibrium.

3. Canonical Actuarial and Indexation Rates

Aaron (1966) explained the relationship between the actuarial rate (equilibrium return) on PAYG social security and the income growth rate, as the sum of two rates: earnings growth per head and population growth rate.

In our framework, the canonical case will correspond to the two following conditions:

- The revaluation of pensions is consistent with the increase in salaries ($g = \gamma$);
- The discount rate r used to estimate the level of fairness is equal to the rate of return of the PAYG system. In the demographic stationary situation, this rate of return is equal to the rate of increase in the salaries (g).

Therefore, the canonical case leads to: $r = g = \gamma$.

3.1. Canonical Model

We chose the canonical values of the discount factor and the indexation rate of the scheme. In a stationary demographic framework, they are equal. We compute here the progressivity coefficients in the canonical case.

- Pension formula and present values.

The sum of the salaries becomes:

$$\widehat{S}_j^N = S_j^{(0;x_0)} \cdot \frac{1 - s_j^N}{1 - s_j}$$

Thus, the pension Formula (9) transforms into:

$$P_j^N = \frac{\delta}{N} \cdot \sum_{i=1}^j \lambda_i \cdot (\widehat{S}_i^N - \widehat{S}_{i-1}^N) = \frac{\delta}{N} \cdot \widehat{X}_j^N \tag{21}$$

with:

$$\widehat{X}_j^N = \sum_{i=1}^j \lambda_i \cdot (\widehat{S}_i^N - \widehat{S}_{i-1}^N) \tag{22}$$

The present value of benefits at the time of retirement N is:

$$\begin{aligned} B_j^N &= \sum_{x=x_r}^{\omega} P_j^N \cdot p_j(x_r; x) \cdot \left(\frac{1+\gamma}{1+r}\right)^{x-x_r} \\ &= P_j^N \sum_{x=x_r}^{\omega} p_j(x_r; x) \\ B_j^N &= P_j^N \cdot e_j \end{aligned}$$

The present value of contributions at the time of retirement N is given by:

$$\begin{aligned} C_j^N &= \sum_{t=0}^{N-1} \pi \cdot S_j^{(t;x_0+t)} \cdot (1+r)^{N-t} \\ C_j^N &= \pi \cdot \widehat{S}_j^N \end{aligned} \tag{23}$$

- PAYG equilibrium.

The PAYG equilibrium expression on the target replacement rate and contribution rate becomes independent of (γ, g) and becomes

$$\pi = \frac{\delta \cdot \sum_{i=1}^m w_i \cdot \widehat{X}_i^N \cdot e_i}{N \cdot \bar{S}} \tag{24}$$

- Inter-class fairness.

We express the actuarial fairness equality condition between the classes. This leads to the condition on the progressive factors:

$$\lambda = \lambda_1 \cdot \left| \begin{array}{c} 1 \\ \frac{\widehat{S}_2^N \cdot \frac{e_1}{e_2} - \widehat{S}_1^N \cdot \frac{e_2}{e_2}}{\widehat{S}_2^N - \widehat{S}_1^N} \\ \vdots \\ \frac{\widehat{S}_i^N \cdot \frac{e_1}{e_i} - \widehat{S}_{i-1}^N \cdot \frac{e_1}{e_{i-1}}}{\widehat{S}_i^N - \widehat{S}_{i-1}^N} \\ \vdots \\ \frac{\widehat{S}_m^N \cdot \frac{e_1}{e_m} - \widehat{S}_{m-1}^N \cdot \frac{e_1}{e_{m-1}}}{\widehat{S}_m^N - \widehat{S}_{m-1}^N} \end{array} \right| \tag{25}$$

The progressive fair pension calculated with the progressive coefficients is thus:

$$P_j^N = \frac{\delta}{N} \cdot \widehat{X}_j^N = \frac{\delta}{N} \cdot \sum_{i=1}^j \lambda_i \cdot (\widehat{S}_i^N - \widehat{S}_{i-1}^N)$$

We compute the pension with the progressive factors obtained via the inter-class fairness condition:

$$P_j^N = \delta \cdot \lambda_1 \cdot \frac{S_j^N}{N} \cdot \frac{e_1}{e_j} \tag{26}$$

In this case, since the discount rate and the indexation rate are the same, we notice that the fair pension has only one correction term, which is the mortality correction expressed as the ratio between two remaining life expectancies; the salary term has no effect with canonical rates:

$$\beta_j = 1 \tag{27}$$

Then, the actuarial ratio becomes, using (27) and (28):

$$\begin{aligned} \frac{B_j^N}{C_j} &= \frac{\delta \cdot \lambda_1 \cdot e_1}{N \cdot \pi} \\ &= \frac{\lambda_1 \cdot e_1 \cdot \bar{S}}{\sum_{i=1}^m w_i \cdot \hat{X}_i \cdot e_i} \end{aligned}$$

As this sum is telescopic, we thus obtain:

$$\sum_{i=1}^m w_i \cdot \hat{X}_i \cdot e_i = \sum_{i=1}^m w_i \cdot \lambda_1 \cdot S_i \cdot e_1 = \lambda_1 \cdot e_1 \cdot \bar{S}$$

Thus, with canonical rates, the system reaches perfect actuarial fairness for all of the classes:

$$\frac{B_j^N}{C_j^N} = 1 \tag{28}$$

3.2. No Salary Indexation Model and Illustration

We will consider here a special case of a canonical environment where $g = 0$ and $s_j = 0$,

- Salaries are constant during the career.
- There is no indexation.
- The annuity used to compute the present value of benefits is simply the life expectancy at retirement:

$$\begin{aligned} B_j &= \sum_{x=x_r}^{\omega} P_j \cdot p_j(x_r; x) \\ &= P_j \sum_{x=x_r}^{\omega} \cdot p_j(x_r; x) \\ B_j &= P_j \cdot e_j \end{aligned}$$

At any time t , the salary in a class j is $S_j^t = S_j$.

For an individual in a salary class j , their pension at retirement for a complete career is computed as such:

$$\begin{aligned} P_j &= \sum_{t=1}^N \frac{\delta}{N} \cdot X_j \\ P_j &= \delta \cdot X_j \end{aligned}$$

The PAYG equilibrium becomes:

$$\sum_{i=1}^m w_i \cdot P_i^N \cdot e_i = \sum_{i=1}^m w_i \cdot C_i^N \tag{29}$$

This leads to the condition on the contribution rate:

$$\pi = \frac{\delta \cdot \sum_{i=1}^m w_i \cdot X_i \cdot e_i}{N \cdot \bar{S}} \tag{30}$$

With the inter-class fairness, we have the following condition for the progressive factors for $j > 1$:

$$\lambda_j = \lambda_1 \cdot \frac{S_j \cdot \frac{e_1}{e_j} - S_{j-1} \cdot \frac{e_1}{e_{j-1}}}{S_j - S_{j-1}}$$

The fair pension is given by:

$$P_i = \delta \cdot \lambda_1 \cdot S_i \cdot \frac{e_1}{e_i} \tag{31}$$

This expression is very intuitive and underlines the fact that, in this particular situation, we can correct unfairness using the life expectancy ratio in the pension formula.

Numerical Illustration

Here, we used French data on the distribution of net monthly salaries in 2016 from “Tableaux de l’économie française Édition 2018”³. We chose eight quantiles of salary to create our classes, and each salary is associated with a life expectancy at 65. We assume that there is no salary indexation.

In our defined-benefit scheme, the target replacement rate is fixed: $\delta = 0.8$. We use the conditions of (4.1) linking the progressive coefficients with salary level and life expectancy from Table 1 in order to generate our progressive coefficients:

Table 1. Life expectancy with salary level.

Social Class	Salary	Life Expectancy (65)
Class 1	1171	19.47
Class 2	1288	20.36
Class 3	1396	21.26
Class 4	1512	22.18
Class 5	1830	23.1
Class 6	2073	24.03
Class 7	2432	24.98
Class 8	3149	25.93

We use (30) and the values of the progressive factors in Table 2 to find the equilibrium contribution rate: $\pi = 0.346$.

Table 2. Progressive coefficients generated with the Salary/life expectancy table.

λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8
1.00	0.51	0.43	0.42	0.47	0.82	0.60	0.65

If we apply the correction of those progressive factors on the salary thresholds in the pension formula, the inter-class fairness condition is respected, while the actuarial fairness ratio reaches 1 because there is no indexation.

4. Discussion

Income during one's career and the remaining life expectancy at retirement are inextricably linked. In pension schemes, this correlation induces certain inequalities. The average life expectancy tends to increase with increasing income, meaning that high-status agents obtain more benefits relative to their contributions in most Bismarckian pension designs (Hachon 2009).

In many nations, income distributions are getting increasingly uneven. As mentioned before, mortality and lifespan inequalities have been widening in many, but not all, nations. While broadening income distributions appear to be an apparent explanation for growing Socioeconomic Status (SES) mortality disparities, most empirical research is unable to throw light on this issue because they measure income or education by quantiles, which do not represent broadening or narrowing of distributions. Our methodology for the agents classification also relies on quantiles income. Although secular trends in differential mortality are considerable, their impact on the length of time people get benefits is muted by the fact that low-SES individuals tend to claim Social Security at earlier ages, and high-SES employees are more inclined to postpone retirement and benefit claiming. Differences in mortality throughout the earnings distribution negate part of the Social Security benefit formula's progressivity, but the pattern of lifetime benefits remains progressive (Bosworth et al. 2016).

This paper sought to introduce a progressive component in the pension formula of a defined-benefit scheme, offering a correction to the fairness linked to life expectancy inequalities in a stationary demographic framework. In comparison with (Breyer and Hupfeld 2008) and their "Distributive Neutrality" approach in relation to pension benefits, our method introduces an additional correction term, the mortality correction, which is quite similar. The progressive pension formula that we proposed used factors that are the solution of actuarial equilibria conditions on salary level thresholds that can be chosen. The main result is that under the absolute stationary hypothesis, fairness between the classes can be obtained for any discount rate.

Our contribution is based on a theoretical discussion of the redistributive features of pension systems when life expectancy is linked to salary levels. We expand on the sentence from (Mitchell and Zeldes 1996) that was mentioned in the beginning. PAYG DB pension systems are demonstrated to be less progressive than they appear to be. Furthermore, we demonstrate that, when the progressive pension formula is used, an equivalent redistribution of resources by wage distribution may be achieved.

Our progressive pension model links the pension formula to demographic and economical factors, with the immediate result being that the pension is reduced as life expectancy increases. This is an especially important issue for low-wage workers. As life expectancy rises, cutting their already inadequate payments could lead to a revival of old-age poverty. This link was studied in (OCED 2011b), where they demonstrated that there must be limits to tying benefit amounts to life expectancy. If benefit cuts force low-income workers to rely on social assistance and other safety-net programs in retirement, the savings from the life-expectancy link in public earnings-related benefits will be negated in part or entirely (notional accounts, defined-benefit or points). With private defined-contribution plans, it will mean more public spending. Parts of the pension system may have enhanced financial stability, but retirement income provision as a whole will not.

Our proposal introduces a modification of the benefit formula, it is vital to remember that modifying the retirement pension formula can have significant incentive consequences, which can affect resource allocation efficiency. In a dynamically efficient economy, every PAYG obligatory pension scheme imposes an implicit tax on labor supply since the present value of future retirement benefits is lesser than the matching payments. (Breyer and Hupfeld 2008) previously stated that increasing the retirement pension for poor workers and cutting it for rich workers will boost (uncompensated) labor supply while reducing the tax burden. Then as result, the proposed revision of the retirement benefit formula's allocation impacts would, if anything, be beneficial.

Pension age or career trajectory are usually a better option to tie life expectancy to than pension benefits, since the latter generates lower income in retirement, which raises pension adequacy and old age income inequality concerns. It is crucial to recall, that the usual work life is growing longer; expanding it is one way of reconciling sustainability and adequacy of the pension system in the context of aging populations by re-balancing working and retirement times. Knowing ahead of time that longer life expectancy involves working longer to make appropriate retirement payouts creates significant motivation for people to postpone retirement. Furthermore, the level of retirement benefits may have an indirect effect on mortality, particularly among the lower income categories. Since lower pension is a process that influences mortality itself, the proposed increase in pension savings in favor of low-income agents may improve life span within those categories, weakening the empirical association.

The demographic framework we used, although very simple, was necessary to understand how the correction was possible. The absolute stationary hypothesis forces the use of a periodic mortality table. Period life expectancy measurements have two drawbacks: first, they do not account for the likelihood that people' socioeconomic characteristics develop over time, namely, because membership in a particular demographic subgroup may change over time, so may the corresponding mortality rates. Except for education, this is usually the true for any metric of socioeconomic level. Second, life expectancy does not take into account the fact that mortality rates fall with time and at varying rates for people in different socioeconomic groups. Recent empirical evidence (Ayuso et al. 2021b); (Bravo et al. 2021); (Ayuso et al. 2021a) show that, at retirement ages, most countries have a significant and systematic divergence in cohort and period life expectancy measures, resulting in significant ex-ante tax/subsidies from generations to come to present generation and, as a result, an unjust actuarial relationship between contributions and pension benefits. In addition, this affects labor supply decisions, resulting in macroeconomic inefficiencies, and wrongly communicates solvency expectations, delaying pension changes. We are considering creating an extension of the model with a stochastic model of the effect of mortality and dynamic economic conditions on salaries and discount rates as well as including incomplete contribution periods.

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Abbreviations

The following abbreviations are used in this manuscript:

PAYG	Pay-As-You-Go
DB	Defined Benefits
NDC	Notional Defined Contribution

Appendix A

Here is displayed the details for the theoretical expression of the Actuarial fairness ratio (20):

$$\begin{aligned}\frac{B_j^N}{C_j^N} &= \frac{P_j^{(N;x_r)} \cdot a_j^{(\gamma;r)}}{\pi \cdot \widehat{S}_j^N(r, g)} \\ &= \frac{\delta \cdot \lambda_1 \cdot \frac{\widehat{S}_1^N(\gamma, g)}{\widehat{S}_1^N(r, g)} \cdot \frac{\widehat{S}_j^N(r, g)}{N} \cdot \frac{a_1^{(\gamma,r)}}{a_j^{(\gamma,r)}}}{\pi \cdot \widehat{S}_j^N(r, g)} \cdot a_j^{(\gamma;r)} \\ &= \frac{\delta}{N \cdot \pi} \cdot \frac{\widehat{S}_1^N(\gamma, g)}{\widehat{S}_1^N(r, g)} \cdot \lambda_1 \cdot a_1^{(\gamma,r)}\end{aligned}$$

This expression can be developed further to prove that the ratio is not at equilibrium. Using the Formula (13) of the contribution rate, it comes:

$$\frac{B_j^N}{C_j^N} = \frac{\bar{s}}{\sum_{i=1}^m \lambda_i \cdot b_i} \cdot \frac{\widehat{S}_1^N(\gamma, g)}{\widehat{S}_1^N(r, g)} \cdot \lambda_1 \cdot a_1^{(\gamma,r)}$$

Then, we need to use the solution for our progressive factors in the telescopic sum; this gives:

$$\begin{aligned}\frac{B_j^N}{C_j^N} &= \frac{\sum_{i=1}^m w_i \cdot \sum_{x=x_0}^{x_r-1} S_i^{(t,x)}}{\lambda_1 \cdot \frac{\widehat{S}_1^N(\gamma, g)}{\widehat{S}_1^N(r, g)} \cdot \sum_{i=1}^m w_i \cdot \widehat{S}_i^N(r, g) \cdot \frac{a_i^{(\gamma,g)}}{a_i^{(\gamma,r)}}} \cdot \frac{\widehat{S}_1^N(\gamma, g)}{\widehat{S}_1^N(r, g)} \cdot \lambda_1 \cdot a_1^{(\gamma,r)} \\ \frac{B_j^N}{C_j^N} &= \frac{\sum_{i=1}^m w_i \cdot \sum_{x=x_0}^{x_r-1} S_i^{(t,x)}}{\sum_{i=1}^m w_i \cdot \widehat{S}_i^N(r, g) \cdot \frac{a_i^{(\gamma,g)}}{a_i^{(\gamma,r)}}}\end{aligned}$$

In order for this ratio to reach equilibrium we need:

$$\sum_{i=1}^m w_i \cdot \sum_{x=x_0}^{x_r-1} S_i^{(t,x)} = \sum_{i=1}^m w_i \cdot \widehat{S}_i^N(r, g) \cdot \frac{a_i^{(\gamma,g)}}{a_i^{(\gamma,r)}}$$

If $r \neq g$ and the actuarial fairness ratio does not reach equilibrium.

Notes

- ¹ In particular when $\gamma = r$, $a_j^{(\gamma,r)} = e_j$.
- ² See Appendix A for proof.
- ³ <https://www.insee.fr/fr/statistiques/3303417?sommaire=3353488> (accessed on 30 June 2021).

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