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## Article

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Games

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## Article

# Self-Enforcing Collective Counterterror Retaliation

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**Abstract:** Motivated by recent examples of collective effort on the war on terror, we examine the incentives that retaliation may produce for the endogenous formation of an international counterterror coalition. We show that there are quite reasonable circumstances under which any nation that is a target of a terrorist attack finds it desirable to be a member of the international counterterror coalition, holding the choices of all other nations as given. The incentives to join the coalition are the group-specific benefits from retaliation enjoyed by each coalition member, the relatively lower spillover benefit from retaliation enjoyed by each stand-alone nation, and the inability of pre-emptive measures to avert terrorist attacks. The disincentive to join is the anticipated backlash from retaliation, which targets coalition members only.

**Keywords:** retaliation; counterterror; coalition; backlash

**JEL Classification:** C72; D74; F53; H87



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## 1. Introduction

Governments often retaliate after some citizens they represent become victims of terrorist attacks (see, e.g., Crenshaw [1], Merari [2], Lee [3], Lee and Sandler [4], Kydd and Walter [5], Benmelech et al. [6], Carter [7], and Gaibullov and Sandler [8]). The U.S. bombed targets in Tripoli and Benghazi, Libya, on 15 April 1986 as retaliation for the Libyan sponsored terrorist attack in Berlin that killed two and injured sixty-two U.S. citizens on 4 April 1986 (Lee and Sandler [4]). Since 1967, Israel has demolished houses in areas occupied by Palestinians as retaliation for Palestinian terrorist attacks (Benmelech et al. [6]). In response to the 9/11 attacks, the United States and Britain conducted airstrikes on October 2001 and, later, together with many other allies in Operation Enduring Freedom, engaged in other military operations, as retaliation against the Taliban and al-Qaeda in Afghanistan. In coordination with the United States, France bombed ISIS targets in Raqqa, Syria, on 15 November 2015, following a number of ISIS terrorist attacks in Paris on 13 November 2015.

Terrorists, however, may respond to retaliatory actions with further attacks (see e.g., Lee and Sandler [4], Jacobson and Kaplan [9], Argomaniz and Vidal-Diez [10], Benmelech et al. [6], Gaibullov and Sandler [8], Matthews et al. [11], and Kattelman [12]). The U.S. retaliatory strikes in Libya in 1986, which received partial support from Britain, produced several terrorist attacks against U.S. and British interests soon after the airstrikes (Lee and Sandler, [4]). The Israeli policy of demolishing houses as retaliation against Palestinian terrorists generated an increase in terrorist attacks after precautionary house demolitions in 2004 and 2005, because properties of some non-terrorists (i.e., neutrals) were demolished (Benmelech et al. [6]). Both examples reveal that terrorist attacks following retaliation are likely if retaliation generates large or nondiscriminatory collateral damages. Airplane and drone strikes, for example, are prone to cause collateral damages owing to inaccuracy of information about precise location or signatures of terrorist targets (see, e.g., Gaibullov and

Sandler [8] and Allen et al. [13]). Owing to negative publicity, moral outrage, and the desire for vengeance, counterterror proactive policies, of which retaliation is an example, may facilitate a terrorist group's acquisition of resources as well as induce neutrals to become leaderless jihadists (see, e.g., Enders, Sandler and Cauley [14], Enders and Sandler [15], Pape [16], Kaplan et al. [17], Faria and Arce [18,19], and Sageman [20]). With a larger resource endowment, the terrorist organization may supply a greater amount of terrorist attacks in response to retaliatory actions.

In this paper, we consider the pros and cons of collective retaliation effort against a terrorist organization. As the paragraphs above reveal, retaliation for terror attacks is common even though there is evidence that it causes backlash—the phenomenon that counterterror policies expand the resources available to terrorists (Faria and Arce [19]). Motivated by Lee [3], Lee and Sandler [4], Cárcelos-Poveda and Tauman [21], de Oliveira et al. [22], Kattelman [12], and the recent examples of collective effort on the war on terror, we examine the incentives that retaliation may produce for the endogenous formation of an international counterterror coalition. Lee [3] notes that retaliation against transnational terrorists yields country-specific and international benefits. An example of a country-specific benefit is the increased security level enjoyed by citizens of a retaliating nation whenever retaliation reduces the incidence of terrorist attacks. In addition, a nation's retaliation effort generates international benefits whenever it leads to a subsequent overall reduction in terror attacks produced by the targeted terrorist organization. Lee and Sandler [4] characterize retaliation against transnational terrorists as an action that produces country-specific and global, purely public, benefits. Unlike Lee [3], they argue that retaliation yields global consumption benefits that are both nonrival and nonexcludable. This purely public characteristic motivates free-riding behavior, which makes voluntary cooperation in the provision of retaliation effort difficult, if not impossible. More recently, Cárcelos-Poveda and Tauman [21] point out that proactive counter-terror measures generate group benefits from cooperation to members of an international counter-terror coalition, which are not enjoyed by non-coalition members. Examples of group benefits from cooperation are international recognition and trade benefits enjoyed by trading agreements among members of the coalition only. Another important contribution to the study of the effectiveness of collective counterterror effort is provided by de Oliveira et al. [22]. They show that a coalition containing three nations is stable if the nations are symmetric and utilize defensive measures to prevent terrorist attacks promoted by a common terrorist organization.

In our analysis, the coalition engages in defensive and proactive measures. The latter include pre-emptive actions, which occur prior to terrorist attacks and are observed by the terrorist organization, and retaliatory actions, which occur after the counterterror coalition observes terrorist attacks. Coordinated retaliatory actions are desirable because pre-emptive actions are unable to completely deter attacks from a terrorist organization. Retaliation affects the terrorist organization as a pecuniary externality, yielding a monetary increase in its resources. As retaliation is known to cause substantial backlash, which subsequently may lead to an increase in the terrorist organization's available resources, we include this effect in the model. For the perpetrators of retaliatory actions, we postulate that retaliation, per se, may yield group-specific benefits originating from at least three sources: (1) an internationally coordinated tough position on terrorist attacks in order to deter future terror from the attacker or other terrorist organizations (i.e., reputation for counterterror leadership); (2) the sense of increased safety, being avenged (i.e., retribution), well represented by their elected officials (i.e., politics), or globally empowered (i.e., global prestige) felt by citizens of coalition members (as in Lee [3]); and (3) as in Cárcelos-Poveda and Tauman [21], the possibility of exclusive trade deals among coalition members. As retaliation effort carried out by the coalition should produce future global benefits in terms of reduced terrorist activity, it yields a positive spillover to non-coalition nations. This is in line with the view advanced by Lee and Sandler [4] that retaliation generates purely public global benefits.

All nations and the terrorist organization play a sequential game of complete, but imperfect information as follows. In stage 0, each nation makes a choice to join or not

to join an international counterterror coalition, taking the choices of all other nations as given. The choices are observed by all nations and the terrorist organization prior to the subsequent stage of the game. After being formed, the coalition represents its members and makes choices to maximize the sum of its members' payoffs. In stage 1, the coalition and the stand-alone nations choose their pre-emptive activities, taking each other's actions as given. In stage 2, the coalition and the stand-alone nations choose their defensive measures, taking each other's choices as given. In stage 3, the terrorist organization makes its choices concerning terrorist attacks. In stage 4, the coalition decides on the level of retaliation. The equilibrium concept is subgame perfect equilibrium.

We show that there are quite reasonable circumstances under which each nation in the globe, holding the choices of all other nations as given, finds it desirable to be a member of the international counterterror coalition. The incentives to join the coalition are the group-specific benefits from retaliation enjoyed by each coalition member, the relatively lower spillover benefit from retaliation enjoyed by each stand-alone nation, and the inability of pre-emptive measures to avert terrorist attacks. The disincentive to join is the anticipated backlash from retaliation, which targets coalition members only.

To the best of our knowledge, this is the first paper in the game-theoretic terrorism literature that explicitly separates retaliatory actions from other proactive actions and examines the incentives associated with retaliation to the endogenous formation of a counterterror coalition.

From this point on, the paper is organized as follows. Section 2 presents the simple model. Section 3 examines the solution to the game played from stages 1 to 4. Section 4 considers the choice made by each nation of whether to join the coalition. Section 5 offers concluding remarks.

## 2. Model

We consider a complete information coalition formation game with five stages, one terrorist organization, and  $I$  nations. In the coalitional stage (stage 0), nations decide whether or not to join a coalition. A generic coalition is denoted by  $S$  and has cardinality (# of members)  $|S| \equiv s$ . In stage 1, the coalition chooses the preemptive measures of its members and non-coalition members decide their pre-emptive measures independently. Stage 2, where each nation decides the level of its defensive measures, is followed by the terrorist organization's selection of a set of countries to attack as well as the magnitudes of the attacks, in stage 3. Finally, in stage 4, the coalition chooses whether to carry out retaliatory measures. We call this game a retaliation game.

Pre-emptive measures are actions that increase the costs of or reduce the resources available to a terrorist organization, reducing the terrorist threat for all potential targets. In our model, decisions about pre-emptive actions precede those about defensive actions. The latter can be thought of as measures that improve the nation's homeland security.

The terrorist organization derives benefits from its attacks according to the function  $b(\mathbf{t}) = \sum_{i=1}^I b_i t_i$ , where  $\mathbf{t} = (t_1, t_2, \dots, t_I)$  is the vector of attacks (damage inflicted) on countries  $i = 1, \dots, I$  and  $b_i$  is the marginal effect of an attack on nation  $i$ . It incurs a specific cost  $c_i(t_i, d_i) = \frac{1}{2}(d_i + t_i)^2$  when it carries out an attack of magnitude  $t_i$  on country  $i$ , whose defensive effort is  $d_i$ . It also sustains a (common) cost  $c_c(\mathbf{p}) = \left( \sum_{i=1}^I t_i \right) \zeta(u)$ , where  $\zeta(u) = \alpha u$ ,  $\alpha \in (0, 1]$  and  $u = \sum_{i=1}^I p_i$  (In addition to reducing the burden of notation, the advantage of specifying the common cost this way is that it makes clear how our findings would change with the specification of  $\zeta(u)$ ), which is imposed on the organization by the pre-emptive measures  $\mathbf{p} = (p_1, p_2, \dots, p_I)$  chosen in stage 1. The parameter  $\alpha$  can be interpreted as the sensitivity rate of the terrorist organization's common cost to pre-emptive actions.

The objective of the terrorist organization is to maximize

$$\pi^T(t, d, p) = \sum_{i=1}^I b_i t_i - \sum_{i=1}^I \frac{1}{2} (d_i + t_i)^2 - \left( \sum_{i=1}^I t_i \right) \xi(u) \quad (1)$$

where  $d = (d_1, d_2, \dots, d_I)$ . The impact of retaliatory actions by the coalition on the terrorist organization happens through the marginal benefit parameters. We define the marginal benefit of attacking nation  $i$  as  $b_i = \begin{cases} b + \omega R, & \text{if } i \in S \\ b, & \text{if } i \notin S \end{cases}$ , where  $\omega R$  is the pecuniary externality caused by retaliation. The parameters  $\omega \in [0, 1]$  and  $R$  are the marginal external gain and the magnitude of the retaliation carried out by the coalition, respectively. Retaliation generates a gain (positive  $\omega$ ) for the terrorist organization when it attacks a coalition member because backlash leads to an increase in the terrorist organization's available resources<sup>1</sup>.

The terrorist organization knows the size and composition of the counterterror coalition when it makes its choices, as well as the identities of stand-alone nations and the pre-emptive and defensive actions  $p$  and  $d$  undertaken by all nations. In addition, it knows how the pecuniary externality associated with retaliation affects its resources and fully anticipates the amount of retaliation that it will face if it attacks the counterterror coalition.

The payoff of nation  $i$  is given by

$$\begin{aligned} \pi(t_i, r, d_i, p_i) &= \beta R - \frac{1}{2} r_i^2 - \theta t_i - \frac{1}{2} (d_i^2 + p_i^2), \text{ if } i \in S \\ \pi(t_i, r, d_i, p_i) &= \frac{\gamma}{2} \beta R - \theta t_i - \frac{1}{2} (d_i^2 + p_i^2), \text{ if } i \notin S \end{aligned} \quad (2)$$

where  $R = \sum_{i=1}^I r_i$ ,  $r_i$  is the retaliatory effort of nation  $i$ ,  $\beta > 0$  is marginal benefit of retaliation,  $\gamma \in [0, 2]$  is a scale parameter that controls how the benefit of retaliation to stand-alone nations compares to that of coalition members, and  $\theta > 0$  is the marginal damage from a terrorist attack suffered by each nation. The first two terms in a member nation's payoff comprise the benefit it gets from retaliatory actions and a variable cost that includes monetary expenditures. For simplicity, we assume that stand-alone nations do not find it desirable to carry out retaliatory actions—thus, only the coalition retaliates, implying  $r_i = 0$  for  $i \notin S$  and  $R = \sum_{i \in S} r_i$ .

The coalition faces different scenarios in stages 1 and 4 of the game. Its objective function in the first stage is to maximize the sum of its members' payoffs, that is,

$$\Pi_C(r, d, p, t) = \sum_{k \in S} \pi(t_k, r, d_k, p_k) = \sum_{k \in S} \left[ \beta R - \frac{1}{2} r_k^2 - \theta t_k - \frac{1}{2} (d_k^2 + p_k^2) \right] \quad (3)$$

given the optimal values of  $r = (r_{i_1}, r_{i_2}, \dots, r_{i_s})$ ,  $p$  and  $t$ , where  $i_1, i_2, \dots, i_s$  are the members of coalition  $S$ . In the last stage, the coalition chooses  $r_i$ ,  $i \in S$ , which maximizes  $\sum_{k \in S} \left[ \beta R - \frac{1}{2} r_k^2 \right] = s\beta R - \frac{1}{2} \sum_{k \in S} r_k^2$  if  $\sum_{k \in S} t_k > 0$ , and  $R = 0$  otherwise.

To finalize the description of our model, we need to take a closer look at the coalition formation stage 0. Nations simultaneously choose whether they want to join a coalition  $S$ ,  $|S| \equiv s \leq I$ , or play the game independently. To test which coalitions are stable, we follow D'Aspremont et al. [23] and apply the internal and external stability criteria:

$$\begin{aligned} \text{Internal stability : } \pi_m^*(S) &\geq \pi_m^*(S \setminus \{m\}), \forall m \in S \\ \text{External stability : } \pi_n^*(S) &\geq \pi_n^*(S \cup \{n\}), \forall n \notin S \end{aligned} \quad (4)$$

### 3. Equilibrium Analysis

Our equilibrium concept is subgame perfection. Utilizing backward induction, we start the analysis with an examination of the last stage of the game. In the last stage,

the coalition chooses  $r_i$ ,  $i \in S$ , in order to maximize  $s\beta R - \frac{1}{2} \sum_{k \in S} r_k^2$  if  $\sum_{k \in S} t_k > 0$ , and  $R = 0$  otherwise. If  $\sum_{k \in S} t_k > 0$ , which is the case under our assumptions, the first-order condition yields

$$s\beta - r_i = 0 \Rightarrow r_i = s\beta, \quad (5)$$

which implies  $R = \sum_{i \in S} r_i = s^2\beta$ . The optimal retaliation level for each coalition member is equal to the sum of the marginal benefits of retaliation enjoyed by the entire coalition. As the objective function of the coalition is strictly concave, the unique solution is a maximum.

In stage 3, the terrorist organization chooses the attack levels that maximize its payoff under the constraint that the retaliation of the coalition follows the formula above. It will thus solve the maximization problem below:

$$\max \pi(t, d, p) = \sum_{i \in S} (b + s^2\omega\beta) t_i + \sum_{i \notin S} b t_i - \sum_{i=1}^I \frac{1}{2} (d_i + t_i)^2 - \left( \sum_{i=1}^I t_i \right) \xi(u) \text{ s.t. } t_i \geq 0, \forall i \quad (6)$$

The Lagrangean function is

$$L(t, \mu) = \sum_{i \in S} (b + s^2\omega\beta) t_i + \sum_{i \notin S} b t_i - \sum_{i=1}^I \frac{1}{2} (d_i + t_i)^2 - \left( \sum_{i=1}^I t_i \right) \xi(u) + \sum_{i=1}^I \mu_i t_i, \quad (7)$$

where  $\mu = (\mu_1, \mu_2, \dots, \mu_I)$  is the vector of Lagrange multipliers. Before we look at the first-order conditions, we show that this Lagrangean is concave. All the terms of  $L(t, \mu)$  that depend on the  $t_i$ 's are linear, with the exception of  $-\sum_{i=1}^I \frac{1}{2} (d_i + t_i)^2$ , so it suffices to show that the latter is a concave function of  $t$ . It is easy to see that this is the case, as its Hessian matrix is negative definite:

$$\begin{bmatrix} -1 & 0 & \cdots & 0 \\ 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 \end{bmatrix} \quad (8)$$

The necessary and sufficient first-order conditions are as follows:

$$\begin{aligned} (i) \quad \frac{\partial L}{\partial t_i} &= b + s^2\omega\beta - d_i - t_i - \xi(u) + \mu_i = 0, \text{ if } i \in S \\ (ii) \quad \frac{\partial L}{\partial t_i} &= b - d_i - t_i - \xi(u) + \mu_i = 0, \text{ if } i \notin S \\ (iii) \quad t_i &\geq 0, \mu_i \geq 0 \text{ and } \mu_i t_i = 0 \forall i \end{aligned} \quad (9)$$

We assume that the terrorist organization chooses a positive level of attack for every nation (and later check if this assumption is satisfied in equilibrium). Then,  $\mu_i = 0 \forall i$  and

$$\begin{aligned} b + s^2\omega\beta - d_i - t_i - \xi(u) &= 0 \Rightarrow t_i = b + s^2\omega\beta - d_i - \xi(u), \quad i \in S \\ b - d_i - t_i - \xi(u) &= 0 \Rightarrow t_i = b - d_i - \xi(u), \quad i \notin S \end{aligned} \quad (10)$$

Note that the sole difference in attack levels is a function of the pecuniary externality that retaliation produces. As the externality is non-negative, the amount of terrorist activity in a nation that is a member of the counterterror coalition is at least as large as the activity in a stand-alone nation.

In stage 2, nation  $i$  maximizes its payoff with respect to  $d_i$ . It knows that the terrorist organization will behave according to the reaction functions derived above. The other arguments in its payoff functions are given. Therefore, we can rewrite their payoff functions as follows:

$$\begin{aligned} \pi(d_i) &= \frac{s^2\beta^2}{2} - \theta [b + s^2\omega\beta - d_i - \xi(u)] - \frac{1}{2} (d_i^2 + p_i^2), \text{ if } i \in S \\ \pi(d_i) &= \frac{\gamma s^2\beta^2}{2} - \theta [b - d_i - \xi(u)] - \frac{1}{2} (d_i^2 + p_i^2), \text{ if } i \notin S \end{aligned} \quad (11)$$

As the payoff functions are strictly concave, the necessary and sufficient conditions for a unique maximum are as follows:

$$\frac{\partial \pi}{\partial d_i} = \theta - d_i = 0 \Rightarrow d_i = \theta, \quad \forall i \quad (12)$$

Each nation finds it optimal to set its level of defensive effort equal to its marginal damage from terrorism.

In stage 1, the coalition and stand-alone nations choose their pre-emptive measures. Let us start with a non-member nation. It chooses its pre-emptive measure  $p_i$  to maximize

$$\pi(p_i) = \frac{\gamma}{2}s^2\beta^2 - \theta[b - \theta - \xi(u)] - \frac{1}{2}(\theta^2 + p_i^2) = \frac{\gamma}{2}s^2\beta^2 + \frac{\theta^2}{2} - \theta[b - \xi(u)] - \frac{p_i^2}{2} \quad (13)$$

Clearly, this is a concave function of  $p_i$  for any  $\xi(u)$ , such that  $\partial^2 \xi(u) / \partial p_i^2 = 0$  (which is the case under the functional form of  $\xi(u)$  in our model). The first-order conditions give us, for all  $i \notin S$ ,

$$\frac{\partial \pi}{\partial p_i} = \theta \frac{d\xi(u)}{du} - p_i = 0 \Rightarrow p_i = \alpha\theta, \quad (14)$$

where we used the facts that  $\partial \xi(u) / \partial p_i = d\xi(u) / du$ , because  $\partial u / \partial p_i = 1$  and  $d\xi(u) / du = \alpha$ . Each stand-alone nation sets its amount of pre-emptive effort equal to its marginal effective damage from terrorism avoided with pre-emptive action. The latter is proportional to the terrorist organization's sensitivity rate to pre-emptive actions.

The coalition wishes to maximize

$$\Pi_C(r, t_{-i \notin S}, d_{-i \notin S}, p_{-i \notin S}) = \sum_{k \in S} \left[ \beta R - \frac{1}{2}r_i^2 - \theta t_k - \frac{1}{2}(d_k^2 + p_k^2) \right] \quad (15)$$

subject to the expressions for the optimal levels of  $t_{-i \notin S}$ ,  $d_{-i \notin S}$ , and  $r$ . After some algebra, the objective function becomes

$$\Pi_C(p) = \sum_{k \in S} \left[ \frac{s^2\beta^2}{2} - \theta \left[ b + s^2\omega\beta - \theta - \xi(u) \right] - \frac{1}{2}(\theta^2 + p_k^2) \right] \quad (16)$$

The coalition wants to maximize the expression above with respect to  $p_k$ ,  $k \in S$ . Notice that the objective function is concave. The first-order conditions are

$$\frac{\partial \Pi_C(p)}{\partial p_k} = s\theta \frac{d\xi(u)}{du} - p_k = 0 \Rightarrow p_k = s\alpha\theta, \quad k \in S \quad (17)$$

As externalities within the coalition are internalized, each coalition member provides pre-emptive effort equal to the sum of effective marginal damages. As pre-emptive efforts are a global public good, stand-alone nations "easy ride" on the higher provision levels of coalition members. This is a disincentive to join the counterterrorism coalition.

The proposition below summarizes the equilibrium of the retaliation game for  $\xi(u) = \alpha u$ .

**Proposition 1:** The unique pure strategy subgame-perfect Nash equilibrium of the retaliation game is given by the following:

- i. Retaliation:  $r_i^* = s\beta$ .
- ii. Defensive measures:  $d_i^* = \theta$ ,  $\forall i$ .
- iii. Preemptive measures:  $p_i^* = \alpha\theta$ ,  $i \notin S$ ;  $p_i^* = s\alpha\theta$ ,  $i \in S$
- iv. Terrorism activities:

$$t_i^* = b + s^2\omega\beta - \theta - \alpha^2\theta(I - s + s^2) = b + s^2\omega\beta - \theta[1 + \alpha^2(I - s + s^2)], \quad i \in S$$

$$t_i^* = b - \theta - \alpha^2\theta(I - s + s^2) = b - \theta[1 + \alpha^2(I - s + s^2)], \quad i \notin S$$

Before we proceed, we need to check under what conditions our assumption that the terrorist organization chooses a positive level of attack for every nation is valid. It is easy to see from part (iv) of Proposition 1 that  $t_i^* > 0 \forall i$  if  $b > \theta + \alpha^2\theta(I - s + s^2)$ . As  $I - s + s^2$  reaches a maximum at  $s = I$ , this condition is satisfied if  $b > \theta + \alpha^2\theta I^2$ , which depends only on model parameters. In words, the benefit the terrorist organization enjoys when it attacks a non-member nation needs to be sufficiently high. This condition, which we assume holds true, does not affect the coalition stability results of the next section, where the parameter  $b$ , as it turns out, plays no role.

We will now highlight some important features of the equilibrium allocation. First, note that an increase in  $\omega$ , the marginal external transfer from retaliation, increases terrorist attacks on nations that are members of the counterterrorism coalition, but does not affect attacks on stand-alone nations. The impact of  $\omega$  is augmented by the size of the coalition because the larger the number of coalition



members attacked, the larger the effects of retaliation on the terrorist organization. An increase in  $b$ , the marginal benefit of a terrorist attack, increases the terrorist organization's attacks on both coalition members and stand-alone nations at the same rate. As one expects, terror attacks decrease with the effectiveness of pre-emptive measures (i.e.,  $\alpha$ ) and with the marginal damage caused by terror (higher  $\theta$ ) owing to defensive and pre-emptive measures. As pre-emptive and retaliatory measures are members of a family of proactive measures, they are naturally competing measures to achieve the same goal—namely, to reduce the terrorist organization's available resources. The key difference is the timing at which they occur. Pre-emptive actions occur before attacks and retaliatory actions occur afterwards. A necessary condition for retaliation is the failure of pre-emptive actions to completely deter terrorist attacks, because retaliation occurs only if coalition members are attacked. The incentive to retaliate and thus to join the counterterror coalition is higher the lower the effectiveness of pre-emptive actions. We will clearly demonstrate this connection below in our analysis of coalitional stability and size.

An increase in  $\beta$ , the marginal benefit to a member nation of retaliatory actions, increases the amount of retaliation and increases terrorist attacks on a member nation. Terrorist attacks on stand-alone nations are not affected by changes in  $\beta$ .

#### 4. Stable Coalitions

We start this section with a proposition providing the conditions for internal and external stability.

**Proposition 2:** The internal and external stability conditions are as follows, where  $|S| \leq I$ :

Internal stability:

$$(\beta[\beta(1 - \gamma) - 2\theta\omega] - \alpha^2\theta^2)s^2 + 2(\beta^2\gamma + 2\alpha^2\theta^2)s - (\beta^2\gamma + 3\alpha^2\theta^2) \geq 0 \text{ for all } i \in S.$$

External stability:

$$(\beta[\beta(1 - \gamma) - 2\theta\omega] - \alpha^2\theta^2)s^2 + 2[\beta(\beta - 2\theta\omega) + \alpha^2\theta^2]s + \beta(\beta - 2\theta\omega) \leq 0 \text{ for all } i \notin S.$$

**Proof.** See Appendix A.  $\square$

This proposition has important implications for the size of stable coalitions. For instance, it implies that full cooperation in the form of a grand coalition is possible under certain conditions. We will establish these conditions momentarily, but first, we introduce new notation.

Define the internal and external stability functions, respectively, as

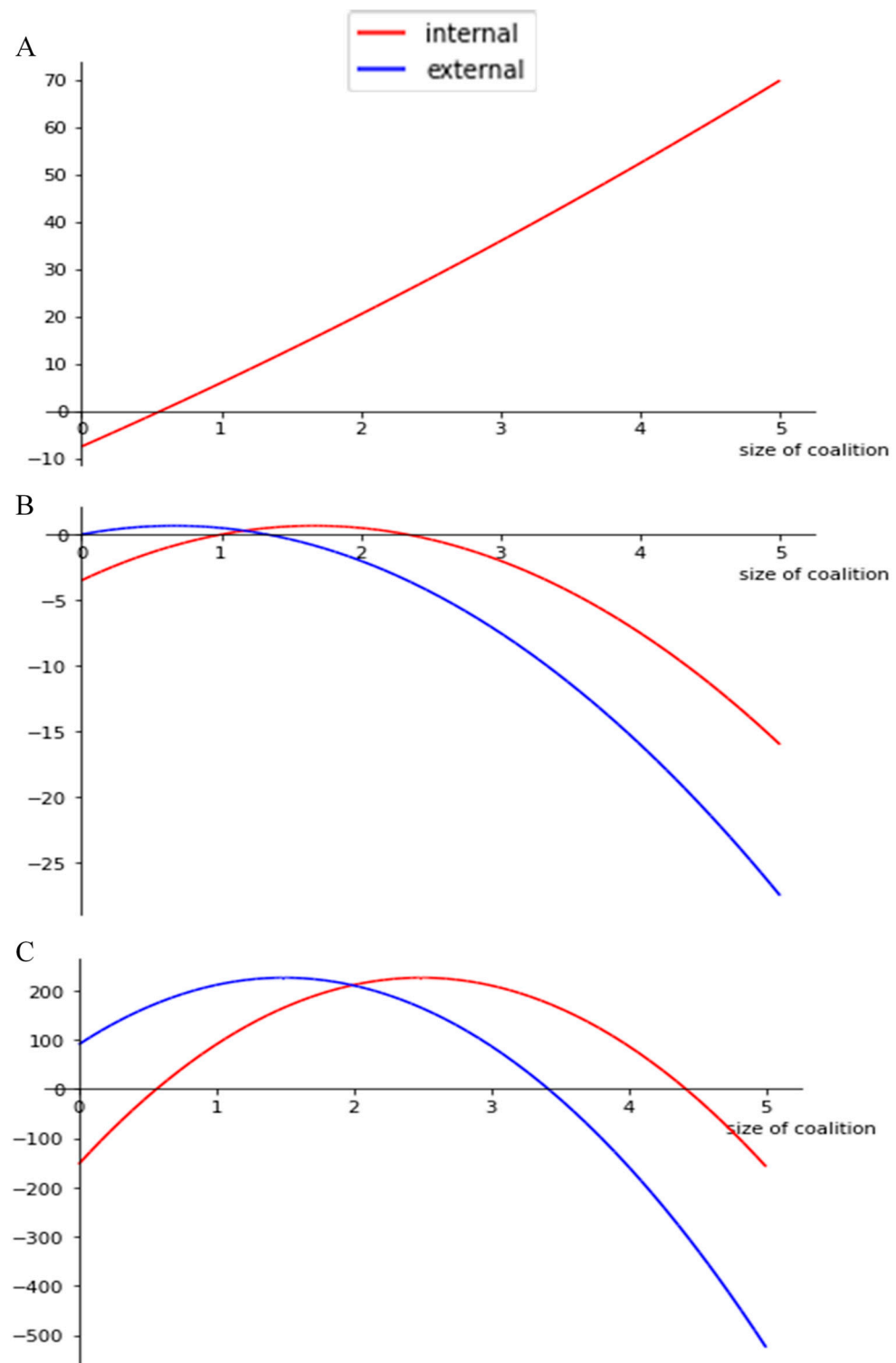
$$\begin{aligned} \psi(s|\beta, \theta, \omega, \alpha) &= (\beta[\beta(1 - \gamma) - 2\theta\omega] - \alpha^2\theta^2)s^2 + 2(\beta^2\gamma + 2\alpha^2\theta^2)s - (\beta^2\gamma + 3\alpha^2\theta^2) \\ \text{and} \\ \varphi(s|\beta, \theta, \omega, \alpha) &= (\beta[\beta(1 - \gamma) - 2\theta\omega] - \alpha^2\theta^2)s^2 + 2[\beta(\beta - 2\theta\omega) + \alpha^2\theta^2]s + \beta(\beta - 2\theta\omega) \end{aligned} \quad (18)$$

Both functions are quadratic in  $s$ , and thus can be written in the generic form  $ax^2 + bx + c$ . We define

$$\begin{aligned} a &= a_{\text{int}} = a_{\text{ext}} = \beta[\beta(1 - \gamma) - 2\theta\omega] - \alpha^2\theta^2 \\ b_{\text{int}} &= 2(\beta^2\gamma + 2\alpha^2\theta^2) \\ c_{\text{int}} &= -(\beta^2\gamma + 3\alpha^2\theta^2) \\ b_{\text{ext}} &= 2[\beta(\beta - 2\theta\omega) + \alpha^2\theta^2] \\ c_{\text{ext}} &= \beta(\beta - 2\theta\omega) \\ \Delta_{\text{int}} &= b_{\text{int}}^2 - 4ac_{\text{int}} \\ \Delta_{\text{ext}} &= b_{\text{ext}}^2 - 4ac_{\text{ext}} \\ s_{\text{int}}^- &= \text{smallest root of } \psi(s|\beta, \theta, \omega, \alpha) \\ s_{\text{int}}^+ &= \text{largest root of } \psi(s|\beta, \theta, \omega, \alpha) \\ s_{\text{ext}}^- &= \text{smallest root of } \varphi(s|\beta, \theta, \omega, \alpha) \\ s_{\text{ext}}^+ &= \text{largest root of } \varphi(s|\beta, \theta, \omega, \alpha) \end{aligned} \quad (19)$$

To better understand how  $\psi(s|\beta, \theta, \omega, \alpha)$  and  $\varphi(s|\beta, \theta, \omega, \alpha)$  behave, we generate a few pictures for different values of the parameters, shown in Figure 1. In the first two,  $\alpha = \theta = 1$  and  $\gamma = \omega = 0.5$ , with  $\beta = 3$  in Figure 1A and  $\beta = 1$  in Figure 1B. The values of the parameters in Figure 1C are  $\alpha = \theta = 1$ ,  $\omega = 0.5$ ,  $\gamma = 1.5$ , and  $\beta = 10$ . The graphs of the internal and external stability functions are shown in red and blue, respectively. For a coalition to be stable, the red curve needs to be on or above the  $x$  axis, and the blue curve needs to be on or below the  $x$  axis.





**Figure 1.** Graphs of stability conditions. (A) Grand coalition is stable, (B) Coalition of size 2 is stable, (C) Coalition of size 4 is stable.

In Figure 1A, the grand coalition is stable (The blue, external stability curve is not shown because the grand coalition satisfies external stability by default). In Figure 1B,C, coalitions of sizes 2 and 4, respectively, are the only stable coalitions. This shows that there is a variety of possible scenarios as far as coalition stability is concerned. The size of a stable coalition can be quite small or as large as the total number of nations, depending on specific combinations of values of the parameters. It is also possible for parameter values to be such that no coalition of any positive size is stable (these cases will be identified and discussed at the end of this section).

The corollaries below systematize our findings in this regard, starting with the grand coalition.

**Corollary 1:** The grand coalition is stable under the following conditions:

- (i)  $a \geq 0$ .
- (ii)  $a < 0$ ,  $\Delta_{\text{int}} \geq 0$ , and  $s_{\text{int}}^- \leq I \leq s_{\text{int}}^+$ .

**Proof.** See Appendix A.  $\square$

Before we explore scenarios where conditions (i) and (ii) hold, it is important to stress that the findings in the corollary depart from the results frequently obtained in the literature on internal and external stability of coalitions, which Barrett [24] refers to as the “paradox of cooperation”: Stable coalitions are either small or, if they are large, the full cooperation aggregate payoff is not much larger than the no cooperation aggregate payoff. The reason for this phenomenon is the positive externality generated by players’ actions. As coalitions become larger, the payoffs of outsiders increase, making it more difficult to sustain stability.

Barrett [24] studies a pollution abatement game with identical countries with independent cost functions and shows, through simulations, that large coalitions are only stable when the cost of abatement is relatively small compared with its benefit. However, when this is true, coalitions with many countries do not increase net benefits by very much compared with the non-cooperative outcome. Cooperation would increase net benefits considerably when the cost and benefit of abatement are both large, but in this case, large coalitions are not stable. Yi [25] also analyses a game with identical countries, but considers a more general framework where several coalitions of different sizes can be formed. He considers a variety of endogenous coalition formation rules and shows that the grand coalition is usually not an equilibrium outcome in the presence of positive externalities. More recently, Finus and McGinty [26] show analytically that the largest stable coalition in a pure public good game with no transfers and where coalition members have identical individual benefit and cost functions is comprised of three nations.

Our findings show that the grand coalition is stable under a variety of conditions. Condition (i) in the corollary requires  $\gamma < 1$ , which means that the marginal benefits generated by the coalition’s retaliatory actions are substantially larger for coalition members. That is not sufficient for  $a \geq 0$  though, which can be written as  $\beta[\beta(1 - \gamma) - 2\theta\omega] \geq \alpha^2\theta^2$ . According to this inequality, the marginal benefit of retaliation  $\beta$  has to be high enough with respect to factors that measure sensitivity to terrorist activities ( $\theta$ ), backlash ( $\omega$ ), and the impact of pre-emptive measures on terrorist costs ( $\alpha$ ).

The fact that the grand coalition is stable for high enough  $\beta$  (modulated by  $\gamma$ ) is surprising. A high  $\beta$  is associated with strong positive externalities, in which case nations have a strong incentive to free ride, typically leading to a violation of internal stability. What is happening here is that retaliation also has a private good component, measured by  $\gamma$ . When the private benefit to coalition members is high enough ( $\gamma$  is low enough), it pays to stay in the coalition.

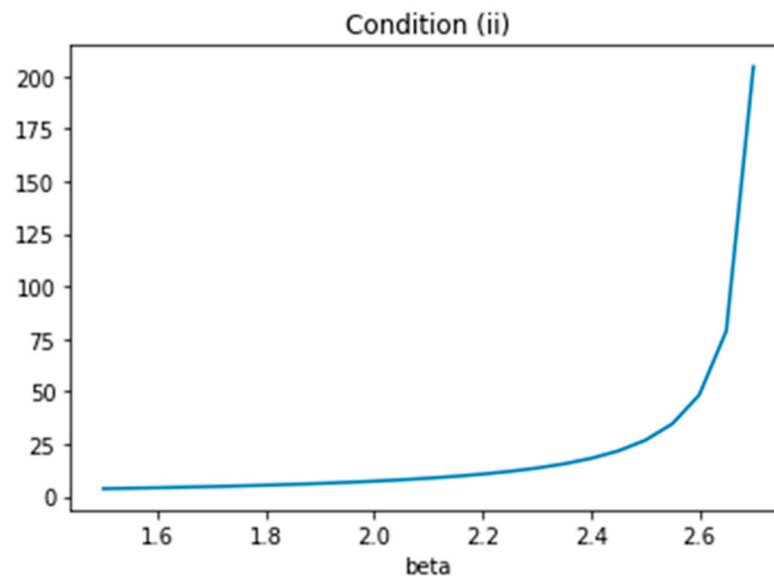
Put differently, part (i) of Corollary 1 states that, when the benefit stand-alone nations enjoy from retaliation is relatively small compared with that of coalition members ( $\gamma < 1$ ), the internal stability condition will be satisfied for any coalition size if  $\beta$  is large enough. The rationale is that, if a coalition member stays in the coalition, it stands to benefit substantially from the retaliatory actions of the coalition, whereas as a stand-alone nation, it still benefits from retaliation, but to a considerably smaller extent<sup>2</sup>.

The condition  $a \geq 0$  can also be satisfied when  $\theta$ ,  $\omega$ , and  $\alpha$  are sufficiently low. The parameter  $\theta$  measures the marginal damage suffered by a nation when it is attacked. When  $\theta$  is low enough, coalition members do not care much about the fact that retaliation increases the likelihood they will suffer a terrorist attack, making internal stability easier to satisfy. A similar reasoning applies to the parameter  $\omega$ , which captures the marginal external gain (due to backlash) from retaliation enjoyed by the terrorist organization<sup>3</sup>. A smaller  $\omega$  translates into fewer attacks (or attacks of smaller magnitude) on member nations, which makes them less sensitive to the negative effects of retaliation. Finally, the impact of the parameter  $\alpha$  on the possibility of full cooperation (and thus maximal coordinated retaliation effort) is also reasonable. As this parameter measures the sensitivity rate of the terrorist organization’s common cost to preemptive actions, a decrease in  $\alpha$  means that pre-emptive actions become less effective as deterrence instruments, generating weaker positive externalities enjoyed by free riders.

Full cooperation is also feasible when condition (ii) in Corollary 1 is satisfied. Let us assume that  $\Delta_{\text{int}} \geq 0$ , which can be shown to be true, with a little bit of algebra, when  $\beta > 2\theta\omega$ <sup>4</sup>. One scenario where condition (ii) may hold is  $\gamma < 1$  and yet  $a < 0$ . In this case, it can be shown that there is a  $\beta > 0$

such that  $s_{\text{int}}^- \leq I \leq s_{\text{int}}^+$  (see proof in the Appendix A). The interpretation is similar to that of part (i) of Corollary 1; that is, if the positive spillover of retaliation on stand-alone nations is limited ( $\gamma < 1$ ), full cooperation is possible when the marginal benefit of retaliation ( $\beta$ ) is high enough in relation to the following: (a) factors that channel potential negative effects of retaliation on coalition members, namely, the marginal damage caused by terrorist attacks ( $\theta$ ) and backlash ( $\omega$ ), which increase terrorist attacks in member nations; and (b) the effectiveness rate of pre-emptive measures in making the terrorist activities costly ( $\alpha$ ).

Figure 2 below illustrates this scenario. We set  $\alpha = \theta = 1$  and  $\gamma = \omega = 0.5$ , and let  $\beta$  vary from 0.5 to 2.732<sup>5</sup> in increments of 0.05. The graph shows how the maximum number of stable coalitions depends on  $\beta$ .



**Figure 2.** Maximum size of stable coalition as a function of  $\beta$ —condition (ii).

The maximum number of stable coalitions is an increasing function of  $\beta$ , and it is always possible to find a  $\beta$  such that the grand coalition is stable.

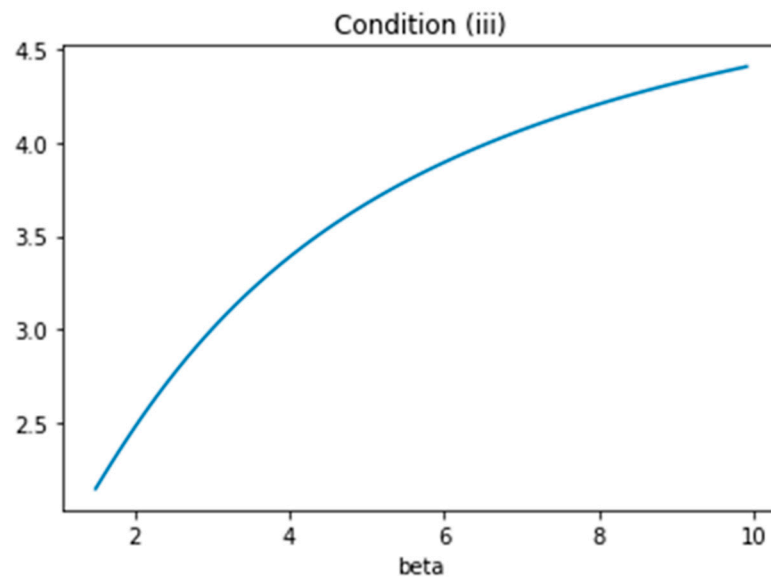
Another scenario under which condition (ii) of Corollary 1 may be satisfied is  $\gamma \geq 1$ , for it implies  $a < 0$ . However, in this case, the number of nations  $I$  cannot be too high. In fact, as  $s_{\text{int}}^+$  has a limit as  $\beta$  increases without bound (see Appendix A), it is possible for  $I > s_{\text{int}}^+$ , and then the grand coalition is not stable. In sum, even if the marginal benefit of retaliation becomes larger and larger compared with the other parameters, there is a limit to the size of stable coalitions. If the total number of nations is higher than that limit, full cooperation will not be possible. It is important to point out that this result is driven by the fact that the spillover effect of retaliation on stand-alone nations is relatively high in this case ( $\gamma \geq 1$ ), making it harder to satisfy internal stability.

A possible combination of parameters in this scenario is shown in Figure 3 below. We again set  $\alpha = \theta = 1$  and  $\omega = 0.5$ , but now  $\gamma = 1.5$ .  $\beta$  varies from 1.5 to 10 in increments of 0.1. The graph shows how the maximum number of stable coalitions depends on  $\beta$ .

Now that we are done discussing the stability of the grand coalition, we shift gears and focus on situations where only smaller coalitions or even no coalitions are stable.

**Corollary 2:** When  $a < 0$  and  $\Delta_{\text{int}} \geq 0$ , a coalition of size  $|S| = s < I$  is stable if  $s_{\text{ext}}^+ \leq s \leq s_{\text{int}}^+ < I$ .

**Proof.** See Appendix A.  $\square$



**Figure 3.** Maximum size of stable coalition as a function of  $\beta$ —condition (iii).

The importance of Corollary 2 is that it shows it is possible to obtain different degrees of cooperation between nations depending on how large the marginal benefit of retaliation  $\beta$  is in comparison with the potential negative effects of retaliation on coalition members, measured by  $\theta$  and  $\omega$ , and the effect of pre-emptive measures on the terrorist's common cost, measured by  $a$ <sup>6</sup>. As mentioned in our discussion about stability under  $\gamma < 1$  and  $a < 0$ , larger values of  $\beta$ , all else the same, increase the maximum size of a stable coalition. We can always ensure full cooperation for large enough  $\beta$ , but, given  $\beta$ , only coalitions smaller than the grand coalition will be stable if the number of nations is such that  $s_{\text{int}}^+ < I$ .

A similar reasoning applies to the case  $\gamma \geq 1$ , but now there is a limit to the maximum size of a stable coalition. It is still possible for the grand coalition to be stable, but only if the total number of nations is relatively small.

Our last corollary establishes conditions under which there are no stable coalitions with more than one nation. In this case, only the non-cooperative solution is viable.

**Corollary 3:** There is no stable coalition with size  $|S| = s > 1$  if one of the following conditions is satisfied: (i)  $a < 0$  and  $\Delta_{\text{int}} < 0$ ; (ii)  $a < 0$ ,  $\Delta_{\text{int}} \geq 0$  and  $s_{\text{int}}^+ < 2$ .

**Proof.** See Appendix A.  $\square$

Once again, stability hinges on the relationship between the benefit of retaliation parameter  $\beta$ , the “cost” of retaliation (from the perspective of coalition members) parameters  $\theta$  and  $\omega$ , and the effectiveness of pre-emption parameter  $\alpha$ . When  $\beta$  is not large enough with respect to the other parameters, no coalition is stable. Figure 4 illustrates this scenario.

In Figure 4, the parameters were set at  $\alpha = \theta = 1$ ,  $\gamma = 1.5$ ,  $\omega = 0.5$ , and  $\beta = 0.5$ . Notice how internal stability is not satisfied for any  $s$  greater than approximately 1.7.

Table 1 collects all the results of this section.

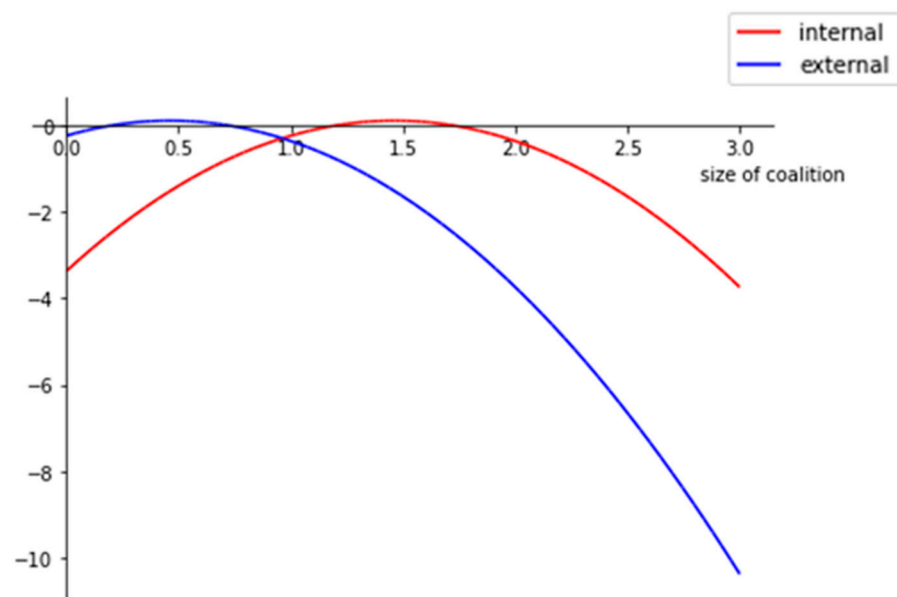


Figure 4. Only the grand coalition is stable—Corollary 3.

Table 1. Summary of stability conditions.

Condition	Internally Stable	Externally Stable	Stable
$a \geq 0$	All coalitions	Only the grand coalition (by default). No other coalition is externally stable because $s_{ext}^+ < s_{int}^+ < 1$ .	Only the grand coalition
$a < 0, \Delta_{int} < 0$	No coalition	This case was not investigated.	No coalition
$a < 0, \Delta_{int} \geq 0, s_{int}^+ < 2$	No coalition	Grand coalition (by default) and coalition of size $s < I$ if $s \geq s_{ext}^+$ .	No coalition
$a < 0, \Delta_{int} \geq 0, s_{int}^+ \geq 2$	Coalition of size $s_{int}^- \leq s \leq s_{int}^+$ .	Grand coalition (by default) and coalition of size $s < I$ if $s \geq s_{ext}^+$ .	Grand coalition, if $s_{int}^- \leq I \leq s_{int}^+$ , or a coalition of size $s < I$ if $s_{ext}^+ \leq s \leq s_{int}^+$ .

If we take a closer look at the scenarios where stable coalitions are possible, we realize there are two possibilities: (1) only the grand coalition is stable (when  $a \geq 0$ ); (2) either the grand coalition or a coalition of size  $s < I$  is stable, but not both (when  $a < 0, \Delta_{int} \geq 0$ , and  $s_{int}^+ \geq 2$ ). If the stable coalition in this scenario is of size  $s < I$ , there are no stable coalitions of other sizes  $s' < I$  (this follows from the fact that  $s_{int}^+ - s_{ext}^+ = 1$ ).

To summarize, we have shown how coalition formation in the retaliation game depends on the intricate relationship between the overall and private marginal benefits of retaliation, the marginal damage caused by a terrorist attack, the backlash after retaliation, and the impact of pre-emptive measures on terrorist's costs. The key aspect of our findings is that, despite being members of the same category of proactive measures, retaliatory and preventive actions have differentiated effects on the incentives to join a coalition.

## 5. Conclusions

We build a simple model to capture key factors that influence potential strategic coalitional counterterror retaliatory effort by multiple nations that fight a common, strategic, terrorist organization. The key factors that we consider are as follows: (i) the sequential nature of strategic moves, with retaliation occurring at the last stage; (ii) the group-specific and internalized public benefits from retaliation enjoyed by coalition members; (iii) the external public benefit from retaliation enjoyed by stand-alone nations; (iv) the external backlash benefit produced by retaliation and enjoyed by the terrorist organization; and (v) the effective rate of pre-emptive counterterror measures in producing a cost to terrorist activities. Motivated by various observations of joint international retaliation triggered by terrorist attacks, we focus on retaliation by a potential counterterror coalition only. Stand-alone nations do not engage in retaliation.

Because retaliation and pre-emptive measures are members of the same family of proactive measures, retaliation becomes a viable and necessary additional weapon in the combat of terrorism when it proves to be a sufficiently different product if compared with pre-emptive measures. Retaliation is a desirable differentiated product in any of the various circumstances under which counterterror coalitions, including the grand coalition, emerge in equilibrium. We demonstrate that the grand coalition is stable depending on the factors that yield a positive net gain to any nation of being a member of the counterterror coalition relative to being a single free rider. In such circumstances, the group-specific marginal benefit from retaliation enjoyed by coalition members and the lower private marginal benefit from retaliation enjoyed by each stand-alone nation as a spillover are fundamentally important. For example, the subgame perfect equilibrium involves full cooperation whenever the group-specific marginal benefit from retaliation enjoyed by each coalition member is sufficiently large, while the external marginal benefit from retaliation enjoyed by a stand-alone nation is sufficiently small. The factors that may hinder the emergence of the grand coalition in equilibrium are backlash and the effectiveness rate of pre-emptive measures in making terrorist activities costly to the terrorist organization. The lower the effectiveness of pre-emptive measures, the more desirable retaliation becomes as a collective instrument to fight terror.

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## Appendix A

**Proof of Proposition 2.** In order to check the internal stability of a coalition, we compare the payoff of a member  $m$  of the coalition  $S$  when it stays in the coalition:

$$\frac{s^2\beta^2}{2} + \frac{\theta^2}{2} - \theta(b + s^2\omega\beta - \alpha u^S) - \frac{1}{2}(s\alpha\theta)^2 \quad (A1)$$

to its payoff when it leaves:

$$\frac{\gamma}{2}(s-1)^2\beta^2 + \frac{\theta^2}{2} - \theta(b - \alpha u^{S \setminus \{m\}}) - \frac{1}{2}(\alpha\theta)^2 \quad (A2)$$

Therefore, a nation stays in the coalition if

$$\begin{aligned} \frac{s^2\beta^2}{2} + \frac{\theta^2}{2} - \theta(b + s^2\omega\beta - \alpha u^S) - \frac{1}{2}(s\alpha\theta)^2 &\geq \frac{\gamma(s-1)^2\beta^2}{2} + \frac{\theta^2}{2} - \theta(b - \alpha u^{S-1}) - \frac{1}{2}(\alpha\theta)^2 \\ \Rightarrow \beta^2[s^2 - \gamma(s-1)^2] - 2\theta(b + s^2\omega\beta - \alpha u^S) - s^2\alpha^2\theta^2 + 2\theta(b - \alpha u^{S-1}) + \alpha^2\theta^2 &\geq 0 \\ \Rightarrow \beta^2[s^2 - \gamma(s-1)^2] - 2\beta\theta\omega s^2 + 2\alpha\theta(u^S - u^{S-1}) - \alpha^2\theta^2(s^2 - 1) &\geq 0 \end{aligned} \quad (A3)$$

Now, we turn to external stability. The payoff of a stand-alone nation is

$$\frac{\gamma}{2}s^2\beta^2 + \frac{\theta^2}{2} - \theta(b - \alpha u^S) - \frac{\alpha^2\theta^2}{2} \quad (A4)$$

and its payoff if it joins the coalition is

$$\frac{(s+1)^2\beta^2}{2} + \frac{\theta^2}{2} - \theta(b + (s+1)^2\omega\beta - \alpha u^{S+1}) - \frac{(s+1)^2\alpha^2\theta^2}{2} \quad (A5)$$

External stability requires

$$\begin{aligned} \frac{\gamma}{2}s^2\beta^2 + \frac{\theta^2}{2} - \theta(b - \alpha u^S) - \frac{\alpha^2\theta^2}{2} &\geq \frac{(s+1)^2\beta^2}{2} + \frac{\theta^2}{2} - \theta(b + (s+1)^2\omega\beta - \alpha u^{S+1}) - \frac{(s+1)^2\alpha^2\theta^2}{2} \\ \Rightarrow (s+1)^2\beta^2 - \gamma s^2\beta^2 - 2\theta(b + (s+1)^2\omega\beta - \alpha u^{S+1}) - (s+1)^2\alpha^2\theta^2 + 2\theta(b - \alpha u^S) + \alpha^2\theta^2 &\leq 0 \\ \Rightarrow \beta^2[(s+1)^2 - \gamma s^2] - 2\beta\theta\omega(s+1)^2 + 2\alpha\theta(u^{S+1} - u^S) - \alpha^2\theta^2[(s+1)^2 - 1] &\leq 0 \end{aligned} \quad (A6)$$

In order to evaluate the stability conditions, we need to find  $u^S = \sum_{i=1}^I p_i^*$ . We plug in the optimal values of the  $p_i$ 's to obtain the following:

$$u^S = \sum_{i \notin S} p_i + \sum_{i \in S} p_i = (I-s)\alpha\theta + s(s\alpha\theta) = \alpha\theta(I-s+s^2) \quad (A7)$$

Given these expressions, we can write

$$\begin{aligned} u^S - u^{S-1} &= \alpha\theta(I-s+s^2) - \alpha\theta(I-(s-1)+(s-1)^2) \\ &= \alpha\theta(I-s+s^2-I+s-1-s^2+2s-1) = 2(s-1)\alpha\theta \\ \text{and, similarly,} \\ u^{S+1} - u^S &= \alpha\theta(I-(s+1)+(s+1)^2) - \alpha\theta(I-s+s^2) \\ &= \alpha\theta(I-s-1+s^2+2s+1-I+s-s^2) = 2s\alpha\theta \end{aligned} \quad (A8)$$

Plugging the expressions above into the internal and external stability conditions, we obtain

$$\begin{aligned} \beta^2[s^2 - \gamma(s-1)^2] - 2\beta\theta\omega s^2 + 2\alpha\theta[2(s-1)\alpha\theta] - \alpha^2\theta^2(s^2-1) &\geq 0 \\ \Rightarrow \beta^2(s^2 - \gamma s^2 + 2\gamma s - \gamma) - 2\beta\theta\omega s^2 + \alpha^2\theta^2(-s^2 + 4s - 3) &\geq 0 \\ \Rightarrow \beta^2 s^2 - \beta^2 \gamma s^2 - 2\beta\theta\omega s^2 + 2\beta^2 \gamma s - \beta^2 \gamma - \alpha^2 \theta^2 s^2 + 4\alpha^2 \theta^2 s - 3\alpha^2 \theta^2 &\geq 0 \\ \Rightarrow (\beta[\beta(1-\gamma) - 2\theta\omega] - \alpha^2\theta^2)s^2 + 2(\beta^2\gamma + 2\alpha^2\theta^2)s - (\beta^2\gamma + 3\alpha^2\theta^2) &\geq 0 \end{aligned} \quad (A9)$$

and

$$\begin{aligned} \beta^2[(s+1)^2 - \gamma s^2] - 2\beta\theta\omega(s+1)^2 + 2\alpha\theta(2s\alpha\theta) - \alpha^2\theta^2[(s+1)^2 - 1] &\leq 0 \\ \Rightarrow \beta^2(s^2 + 2s + 1 - \gamma s^2) - 2\beta\theta\omega(s^2 + 2s + 1) + \alpha^2\theta^2(-s^2 + 2s) &\leq 0 \\ \Rightarrow \beta^2 s^2 + 2\beta^2 s + \beta^2 - \beta^2 \gamma s^2 - 2\beta\theta\omega s^2 - 4\beta\theta\omega s - 2\beta\theta\omega - \alpha^2 \theta^2 s^2 + 2\alpha^2 \theta^2 s &\leq 0, \\ \Rightarrow (\beta^2 - \beta^2 \gamma - 2\beta\theta\omega - \alpha^2 \theta^2)s^2 + (2\beta^2 - 4\beta\theta\omega + 2\alpha^2 \theta^2)s + (\beta^2 - 2\beta\theta\omega) &\leq 0 \\ \Rightarrow (\beta[\beta(1-\gamma) - 2\theta\omega] - \alpha^2\theta^2)s^2 + 2[\beta(\beta - 2\theta\omega) + \alpha^2\theta^2]s + \beta(\beta - 2\theta\omega) &\leq 0 \end{aligned} \quad (A10)$$

respectively, which are expressions on the parameters only.  $\square$

**Proof of Corollary 1.** First, it is helpful to recall that  $\alpha \in (0, 1]$ ,  $\beta > 0$ ,  $\gamma \in [0, 2]$ ,  $\theta > 0$ , and  $\omega \in [0, 1]$ .

(i) Notice that, for  $s \geq 1$ ,

$$\begin{aligned} as^2 &\geq 0 \\ b_{\text{int}}s + c_{\text{int}} &= 2s\beta^2\gamma + 4s\alpha^2\theta^2 - \beta^2\gamma - 3\alpha^2\theta^2 \\ &= (2s-1)\beta^2\gamma + (4s-3)\alpha^2\theta^2 > 0 \end{aligned} \quad (A11)$$

which implies  $\psi(s|\beta, \theta, \omega, \alpha) > 0$ . This means that internal stability is satisfied for all  $s \geq 1$ . As the grand coalition is externally stable by default, we conclude that it is stable.

(ii) If  $a < 0$ , the quadratic function  $\psi(s|\beta, \theta, \omega, \alpha)$  is concave. If its discriminant  $\Delta_{\text{int}}$  is negative, then it has no real roots, and thus is everywhere below the  $x$  axis. In this case, no coalition is internally stable, including the grand coalition.

If  $\Delta_{\text{int}} \geq 0$ , we have, for  $s \geq 1$ :

$$\begin{aligned} as^2 &< 0 \\ b_{\text{int}}s + c_{\text{int}} &= 2s\beta^2\gamma + 4s\alpha^2\theta^2 - \beta^2\gamma - 3\alpha^2\theta^2 \\ &= (2s-1)\beta^2\gamma + (4s-3)\alpha^2\theta^2 > 0 \end{aligned} \quad (A12)$$

This means that the first term in  $\psi(s|\beta, \theta, \omega, \alpha)$  is negative and quadratic in  $s$ , while the sum of the second and third terms is positive and linear in  $s$ . Thus,  $\psi(s|\beta, \theta, \omega, \alpha) < 0$  for large enough  $s$ . Given that  $\psi(s|\beta, \theta, \omega, \alpha)$  is concave, internal stability will be satisfied for  $s_{\text{int}}^- \leq s \leq s_{\text{int}}^+$ . Therefore, the grand coalition will be stable if  $s_{\text{int}}^- \leq I \leq s_{\text{int}}^+$ .  $\square$

**Proof of Corollary 2.** We have already seen in the proof of Corollary 1 that (a) when the discriminant  $\Delta_{\text{int}}$  is negative no coalition is internally stable, so  $\Delta_{\text{int}} \geq 0$  is required; (b) under the conditions  $a < 0$  and  $\Delta_{\text{int}} \geq 0$ , internal stability holds for  $s_{\text{int}}^- \leq s \leq s_{\text{int}}^+$ .



External stability, on the other hand, is satisfied for large enough  $s$ , because

$$\begin{aligned} & (\beta[\beta(1-\gamma) - 2\theta\omega] - \alpha^2\theta^2)s^2 < 0 \\ & 2[\beta(\beta - 2\theta\omega) + \alpha^2\theta^2]s + \beta(\beta - 2\theta\omega) \\ & = (2s+1)\beta(\beta - 2\theta\omega) + \alpha^2\theta^2s \geq 0 \end{aligned} \quad (A13)$$

implies that, even if the sum of the two last terms in  $\varphi(s|\beta, \theta, \omega, \alpha)$  is positive, that is,  $2[\beta(\beta - 2\theta\omega) + \alpha^2\theta^2]s + \beta(\beta - 2\theta\omega) > 0$ ,  $\varphi(s|\beta, \theta, \omega, \alpha)$  will be negative for large enough  $s$ . Moreover, as  $\varphi(s|\beta, \theta, \omega, \alpha)$  is concave when  $a < 0$ , external stability is satisfied for  $s \leq s_{ext}^-$  and  $s \geq s_{ext}^+$ .

Next, we need to show that  $s_{ext}^+ < s_{int}^+$ , for then there will exist an  $s$  between  $s_{ext}^+$  and  $s_{int}^+$ . The algebra to obtain this result looks very complicated, but the result is incredibly simple. Using Sympy, a Python library for symbolic mathematics, we calculated the difference between  $s_{ext}^+$  and  $s_{int}^+$ , obtaining  $s_{int}^+ - s_{ext}^+ = 1$ . Therefore, there exists a stable coalition  $s$ , and if  $s_{int}^+ < I$ , this coalition is smaller than the grand coalition.  $\square$

**Proof of Corollary 3.** We already know from Corollary 1 that there are no internally stable coalitions when  $a < 0$  and  $\Delta_{int} < 0$ . When  $a < 0$  and  $\Delta_{int} \geq 0$ , we know from the proof of Corollary 2 that a coalition of size  $s$  will be stable if  $s_{ext}^+ \leq s \leq s_{int}^+$ . Thus, if  $s_{int}^+ < 2$ , there is no stable coalition larger than a singleton.  $\square$

**Proof of implications of condition (ii) of Corollary 1.**

Consider  $a$  as a function of  $\beta$ , i.e.,

$$\begin{aligned} a(\beta) &= \beta[\beta(1-\gamma) - 2\theta\omega] - \alpha^2\theta^2 \\ &= (1-\gamma)\beta^2 - 2\theta\omega\beta - \alpha^2\theta^2 \end{aligned} \quad (A14)$$

This quadratic function in  $\beta$  has the following roots:

$$\begin{aligned} \beta^+ &= \frac{2\theta\omega + \sqrt{4\theta^2\omega^2 + 4(1-\gamma)\alpha^2\theta^2}}{2(1-\gamma)} = \frac{2\theta\omega + \sqrt{4\theta^2[\omega^2 + (1-\gamma)\alpha^2]}}{2(1-\gamma)} \\ &= \frac{\theta(\omega + \sqrt{\omega^2 + (1-\gamma)\alpha^2})}{(1-\gamma)} \\ \beta^- &= \frac{\theta(\omega - \sqrt{\omega^2 + (1-\gamma)\alpha^2})}{(1-\gamma)} \end{aligned} \quad (A15)$$

Under the assumption that  $\gamma < 1$ , the discriminant is positive, so there are two real roots. Moreover,  $a(\beta)$  is convex and  $a(\beta) < 0$  when  $\beta^- < \beta < \beta^+$ .

When  $\Delta_{int} \geq 0$  (see proof of Corollary 1), it is clear that  $s_{int}^+ = (-b_{int} - \sqrt{\Delta_{int}})/2a \geq -b_{int}/2a$ . Because  $\lim_{\beta \rightarrow \beta^+} a = 0$  (remember the constraint that  $a < 0$ ) and  $b_{int}$  increases with  $\beta$ , we can always find a  $\beta$  such that  $s_{int}^- \leq I \leq s_{int}^+$ .

If  $\gamma \geq 1$ ,  $a(\beta)$  is always negative. Notice that  $s_{int}^+ = (-b_{int} - \sqrt{\Delta_{int}})/2a$  can be written as  $s_{int}^+ = p(\beta)/q(\beta)$ , where

$$\begin{aligned} p(\beta) &= -2(\beta^2\gamma + 2\alpha^2\theta^2) - [4(\beta^2\gamma + 2\alpha^2\theta^2)^2 + 4(\beta[\beta(1-\gamma) - 2\theta\omega] - \alpha^2\theta^2)(\beta^2\gamma + 3\alpha^2\theta^2)]^{1/2} \\ &\text{and} \\ q(\beta) &= 2\beta[\beta(1-\gamma) - 2\theta\omega] - \alpha^2\theta^2 \end{aligned} \quad (A16)$$

are polynomials of degree 2 in  $\beta$ . Therefore,  $s_{int}^+$  is a rational function in  $\beta$  whose limit is equal to the ratio of the leading coefficients of  $p$  and  $q$ .  $\square$

## Notes

- <sup>1</sup> The parameter  $\omega$  captures the differential benefit of attacking a nation that belongs to the coalition. If  $\omega$  is negative, non-coalition members will have less incentive to free ride, making cooperation easier. By assuming a non-negative  $\omega$ , we are making it harder for cooperation to take place, which increases the robustness of our coalition stability results.
- <sup>2</sup> We can show that the impact of a larger  $\beta$  on external stability is similar. Because the (positive) impact of retaliation on the payoff of coalition members is substantially higher than that on the payoff of stand-alone nations, it is impossible to prevent entry into a coalition smaller than the grand coalition.
- <sup>3</sup> In this paper, we do not consider the possibility that retaliation will lead to a reduction of terrorist attacks in nations that are members of the coalition.
- <sup>4</sup> The condition  $\beta > 2\theta\omega$  is not necessary for  $\Delta_{int} \geq 0$ . In fact,  $\Delta_{int} > 0$  in all the numerical simulations we carried out.
- <sup>5</sup>  $\beta^+ = 2.73205 \dots$  in this case (see Appendix A for definition of  $\beta^+$ ).
- <sup>6</sup> Notice first that, if  $\gamma < 1$  and  $a \geq 0$ , only the grand coalition is stable.
- <sup>7</sup> If  $s_{int}^- = s_{int}^+ = s_{int}^*$ ,  $\psi(s|\beta, \theta, \omega, \alpha)$  has a unique real root ( $\Delta_{int} = 0$ ) and the grand coalition is stable only when  $I = s_{int}^*$ .

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